Scale laws

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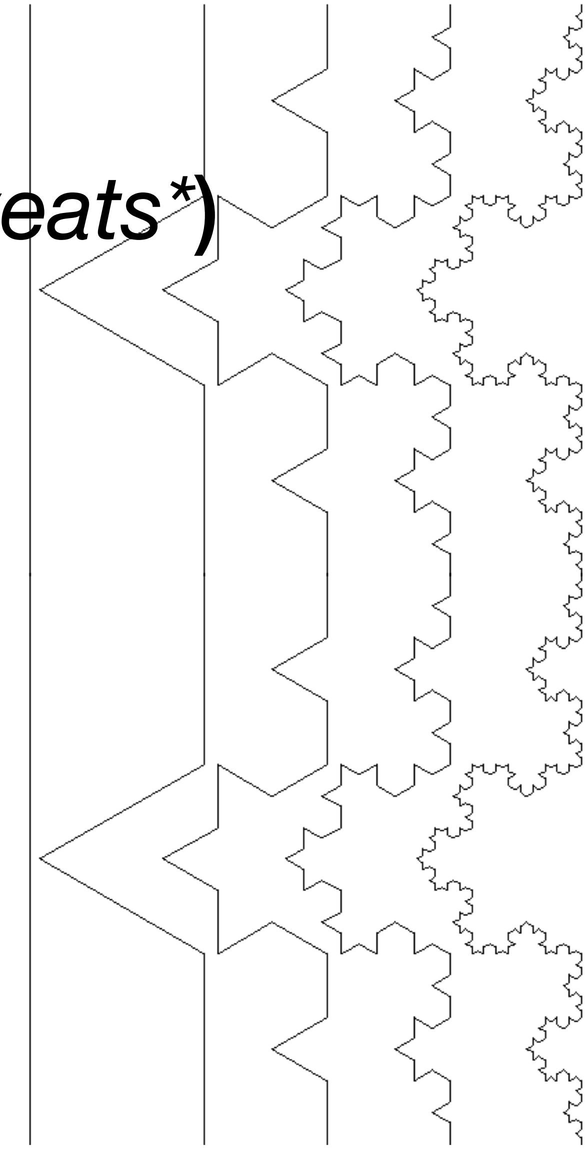
Outline

- Power-law distributions in nature
- Types of noise
- 1/f as a signature of complexity
- Debate between brain oscillations and noise
- New perspectives on 1/f: functional significance and temporal organization

Power law distributions in nature: pervasiveness of the phenomenon (with *caveats**)



- <u>In natural sciences/physics</u>: **natural disasters** (earthquakes magnitude vs occurrence, snow and sand avalanches), Stefan–Boltzmann law, **inverse-square law** (Newton's law, Columbus'law)…

- <u>In social sciences/economics</u>: **Pareto's law** (80-20 rule for income) and Yule's process= "richer get richer", stock market fluctuations, **Gibrat's law** (rule of proportionate growth of city sizes)…

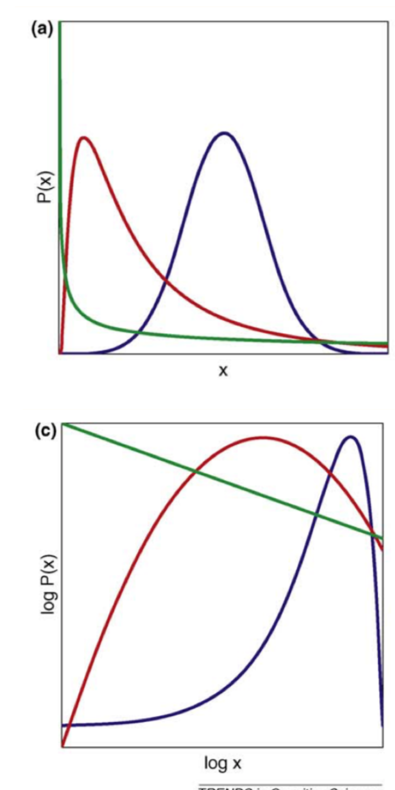
- <u>In linguistics</u>: **Zipf's law** (the frequency of occurrence of a certain words in corpus of natural language utterances: rank of 'importance' $\propto$ frequency).

- <u>In biological systems</u>: allometric scaling laws (body mass vs metabolic rate -**Kleiber's law**- or brain matter), scale-free network models, neuronal avalanches, geometry of axonal dendritic tree, fluctuations of neuronal membrane potentials, statistics of neurotransmitter release, neuronal firing…

- <u>In psychology/motor science</u>: **Stephen's/Weber's law** (between the magnitude of a physical stimulus and its perceived intensity or strength), retrospective and prospective memory, reaction times, daily self-esteem fluctuations, **time estimation**, speech production, **finger tapping**, forearm oscillation, **heart beat rate**, stride series during walking, synchronization to metronome…

*same name for different type of functions… (Biyu He 2014)

Kello et al. 2010, Torre-Wagenmakers 2009, Gilden 2001



figure(a),(c) from Kello et al. 2010

Power law distribution (in log-log plot): the hallmark of scale-free systems

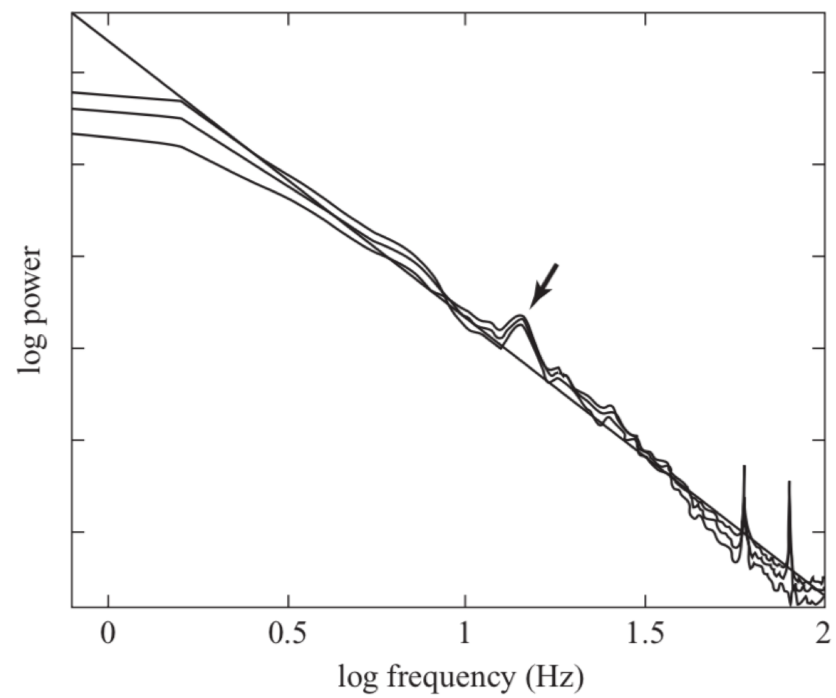


Figure 5.2. Power spectrum of EEG from the right temporal lobe region in a sleeping human subject (subdural recording). Note the near-linear decrease of log power with increasing log frequency from 0.5 to 100 hertz, the characteristic feature of "pink" or "complex" noise. The arrow indicates the peak at alpha (~11 hertz). Reprinted, with permission, from Freeman et al. (2000).

- Temporal power spectrum of arhythmic brain activity: the amplitude increases as the frequency decreases ($A \sim 1/f^{\alpha}$)
- The **speed at which the power decreases** from low to high frequency measures the *length of the correlations* or […] the **'temporal memory effects' in the signal**
- Small perturbations=microscopic (at low frequency for example) cause large=macroscopic dissipation of energy at all frequency scales

Buzsaki (2006)

“Crash course on noise”

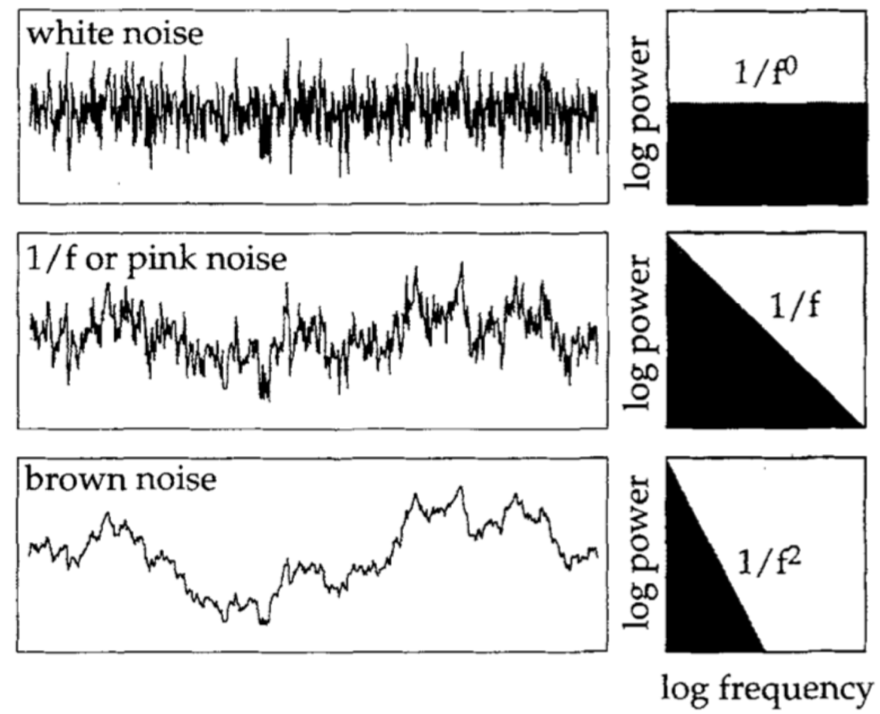


Figure 3. The three canonical types of fractional Brownian motion. Examples of each motion are shown along with its power spectrum. These motions are self-affine fractals; frequency scaling can be offset by amplitude scaling.

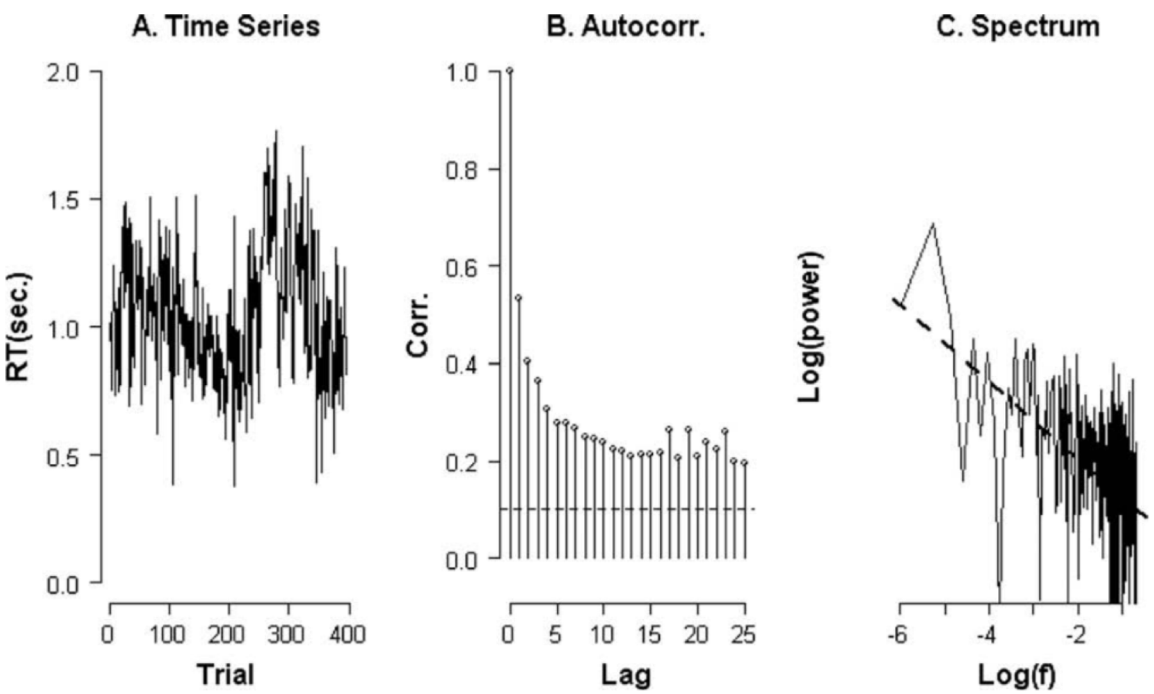
- The *correlation function* expresses the effect of distance on similarity (Kello et al. 2010)

- **White noise ($1/f^0$):** uncorrelated noise (=no interactions), total lack of predictability.
- **Pink noise (1/f):** balance between irregularity/unpredictability and over-regularity/overpredictabilty
- **Brown noise ($1/f^2$):** integration over uncorrelated noise =integral of a white noise Gaussian process, ‘random walk’, high temporal predictability.

Time series on the left **log-log plots** on the right **- figure from Gilden 2001**

“[…]”the 1/f behavior of EEG and magnetoencephalogram (MEG) is the golden means between the ***disorder with high information content*** (**white noise**) and the ***predictability with low information content*** (**brown noise**). The cerebral cortex with its most complex architecture generates the most complex noise known to physics.” (Buzsaki 2006)

Analyses of dependencies in time series



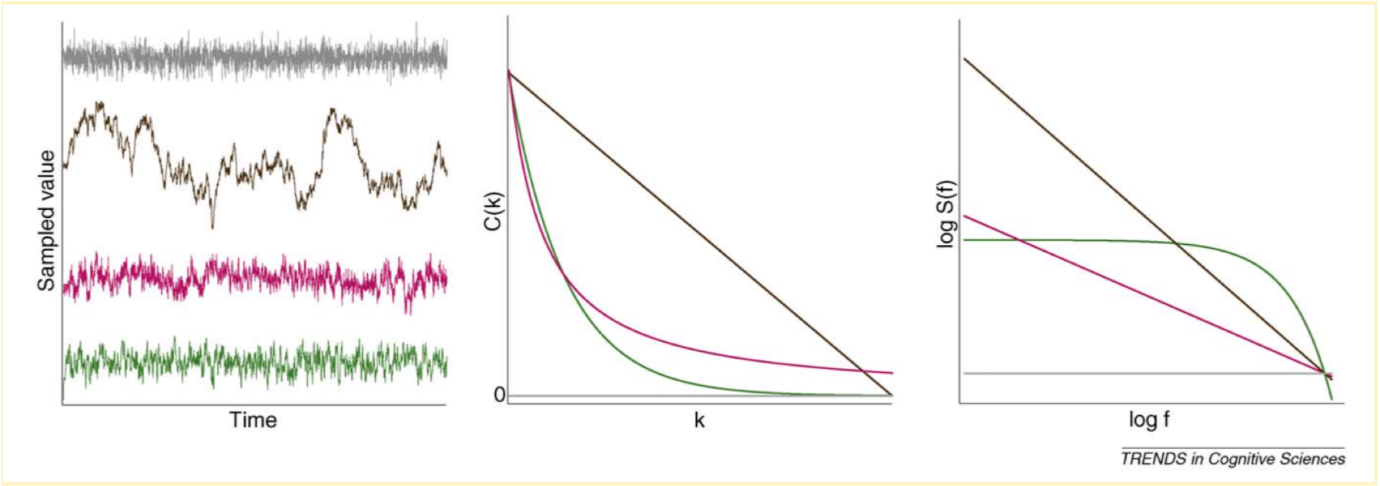


Figure from Kello et al. 2010

- **Serial correlations** describe better human behavior (vs. idea by standard statistics that consecutive behaviors are unrelated)
- **Short range temporal correlations** (**ARMA models** or AutoRegressive Moving Average): autocorrelation function that <u>decays very quickly</u> (= *weak interactions*) and spectral density function that levels off at low frequencies.
- **Long range temporal correlations** (**pink noise** or 1/f): autocorrelation function that <u>decays so slowly</u> that its sum doesn't converge to a finite number (=*strong interactions*) and log-log power spectrum described by a linear with slope -b.

Diniz et al. 2011

Formalization of 1/f noise

- Time domain analysis: **autocorrelation function** gives the autocorrelation between variables of the process at 2 different times. Formally, a stationary process* is said to have a long memory if its autocorrelation function satisfies the power law. The stochastic **memory of the process**= given by the speed of the decay of the autocorrelation function. The function in this case *decays to zero very slowly*. The process is said to have *persistent long memory*: the remote past has a strong influence into the present.

  *mean is constant across time and autocorrelation function depends only on the time lag between the variables.

- Frequency domain analysis: **spectral density function** gives the amount of variance accounted for by each frequency in the process - corresponds mathematically to the Fourier transform of the autocorrelation function. Formally, a long-memory process can be defined as a process whose spectral density function satisfies the power law.

- In continuous time, a long memory process is **self-similar***.

- 1/f noise or long range dependence: also **long-term memory** or **fractal process**

Diniz et al. 2011

Methods for estimation of 1/f exponents

- R/S (Rescaled range methodology): developed by Hurst (1951) to study the level of water of the river Nile: c and b two constants such that $c>0$ and $0.5<b<1$ (for short-memory, $b=0.5$) —> **Hurst exponent or self-similarity parameter H** [methods in Diniz et al. 2011]

- DFA (**Detrended fluctuation analysis**): developed by Peng (1993) -[methods in Diniz et al. 2011]

- GPH (Gewake and Porter-Hudak regression (1983) -[methods in Diniz et al. 2011]

- MLE (Maximum Likelihood estimation: a parametric technique (vs heuristics technique of the other 3) - [methods in Diniz et al. 2011]

- CGSA (**Coarse-graining spectral analysis**) by Yamamoto & Hughson (1991,1993): it separates the harmonic/oscillatory components from the fractal/scale-free ones. It takes advantage of the self-affinity property (=self-similarity) of scale-free time series -not true for harmonic time series.

Diniz et al. 2011, Biyu He et al. 2010

1/f as a signature of complexity

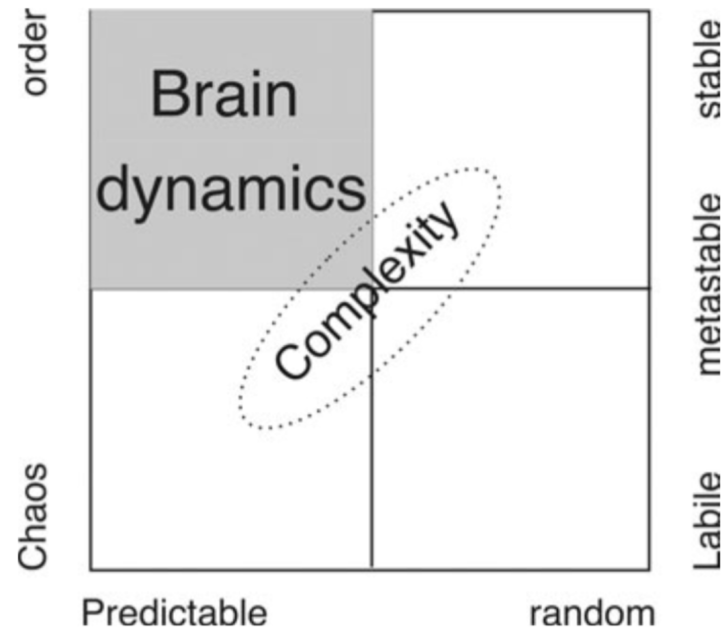


Figure 2.7. Complexity occupies the boundary between disorder (maximum entropy) and order. It is neither fully predictable nor fully random. Complex systems are governed by simple power laws. The dynamic range of the brain varies between complex and predictable.

- Simple systems vs Complex systems: component dominance vs interaction dominance, self-organization, emergence etc…
- **Nomothetic approach**: 1/f as a general principle (or even law of the universe!)-> Self similarity hypothesis, from Bak (1987,1996). But the ubiquity of the phenomenon doesn't make it psychologically meaningful!
- **Mechanistic approach:** 1/f is just a useful constraint for modeling underlying processes of a particular psycho-physiological system. Concrete models (hopping model, shifting strategy model) for specific tasks (self-paced tapping, synchronization tapping, bimanual tapping).
- Other characteristics of complexity: **scale invariance** or self-similarity (fractals'property) and **metastability** (= being close to the point of criticality) -> SOC hypothesis (**Self-organized criticality**) by Bak.

Diniz et al. 2011,Torre-Wagenmakers 2009

1/f as a signature of complexity

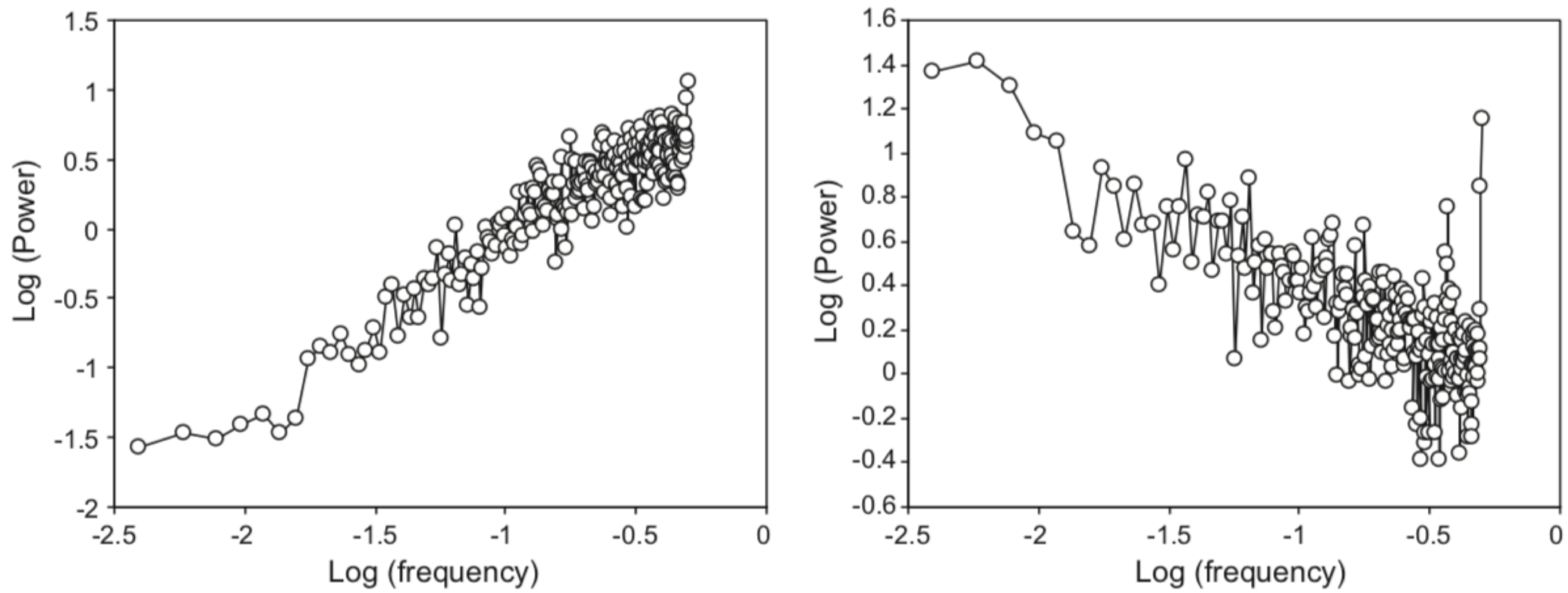


Fig. 2. Log–log power spectra of series of inter-tap intervals (left) and asynchronies (right), in synchronization finger tapping (Torre & Delignières, 2008a). The positive slope, for inter-tap interval series, reveals the presence of negative serial correlations between successive values. The negative slope for asynchronies series is indicative of the presence of 1/*f* noise.

- 1/f noise is **ubiquitous in human nature**: any behavioral phenomenon will reveal long range dependence if measured over a sufficient duration in time (cfr. EEG measurement dilemma discussed in Buzsaki 2006).
- 1/f is **obscured when sources of external variation are increased** (cfr. Series of tap-interval vs series of asynchrony to a metronome in synchronization tapping; online feedback during experiment and other experimental constraints that reduce voluntary control).
- **more stable and coordinated behavior** reveal a clearer 1/f signature (**healthy baseline** of 1/f and disorders or deficiencies when deviation towards randomness or over-regularity; also task performances improved by learning correlates with 1/f).
- 1/f should be accompanied by **additional evidence for emergency and self-organization** (reduced entropy, decrease in system dimensionality…).
- indefinite numbers of 1/f exists in any behavior (if under condition of intrinsic fluctuations).

Diniz et al. 2011

Neglecting variance, neglecting 1/f noise

- **Variability** considered 'as the expression of methodological and experimental errors or of the presence of unmeaning noise in the system': **discarded by averaging** over participants and trials **or by filtering** in the case of time series.
- Implicit **assumption that fluctuations are white noise** (= uncorrelated over time): noise is usually removed from measured brain signal by normalizing with **pre-task baseline** or '**whitening**' (= removal of pink noise) to emphasize oscillations.
- "the ***variability of the responses across trials*** is generally downplayed ***as unexplained variance or 'noise'*** that needs to be averaged out to reveal the brain's true representation of invariant input" (Buzsaki 2006). Instead, importance of trial-to-trial dynamics!
- '**Systematic disregard for 1/f noise**', partly due to its ubiquitous presence in nature and general idea that 1/f noise originates from instrument noise (Biyu He 2010).

The rhythms or the 1/f: which came first?

Are brain dynamics characterized better by various oscillators or pink noise?

- **Oscillations**: periodic rhythmic brain activity before filtering = with a particular temporal frequency.
- **Scale-free brain activity**: band-pass-filtered aperiodic/arhythmic brain activity = without a predominant temporal frequency.
- Periodic brain oscillations appear as **local peaks** above the power-law distribution.
- **DILEMMA**: If we have long EEG recordings, spectra without clear peaks*: **are recorded brain rhythms the extreme states of neuronal noise?** Or, instead, **could scale-free brain activity be produced by the sum of many oscillations?**

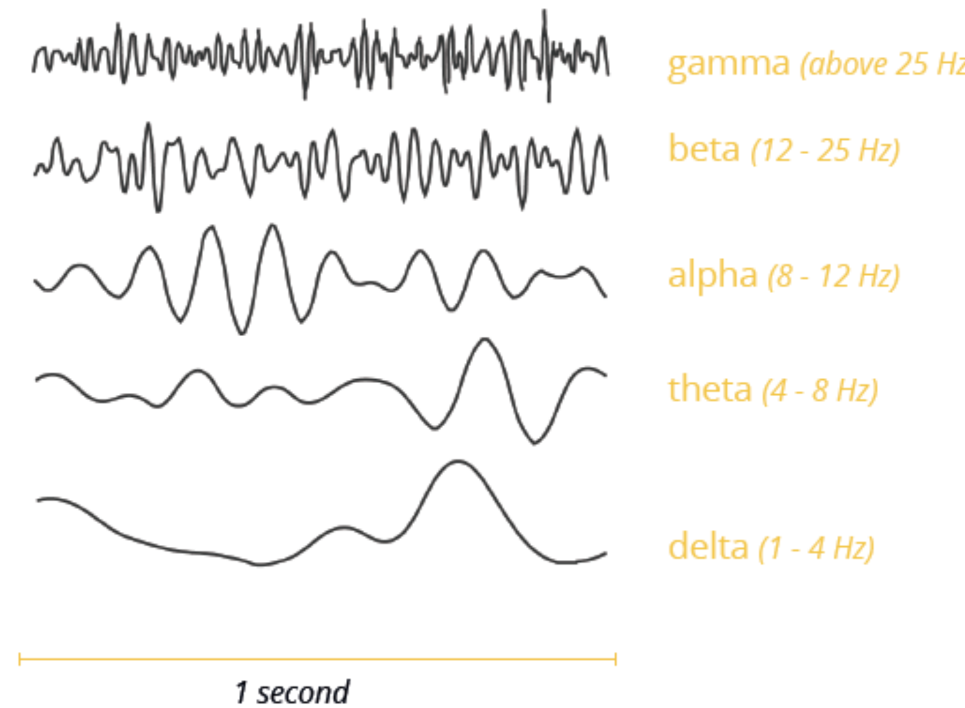
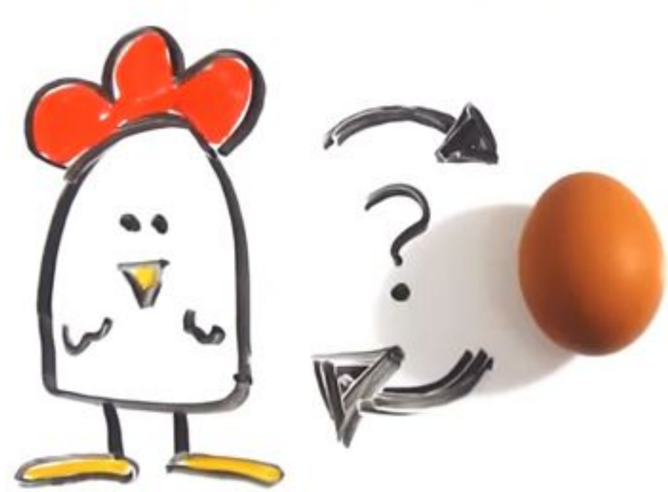


image from https://imotions.com/blog/eeg/

"THE Oscillations-OR-THE **Neuronal noise**



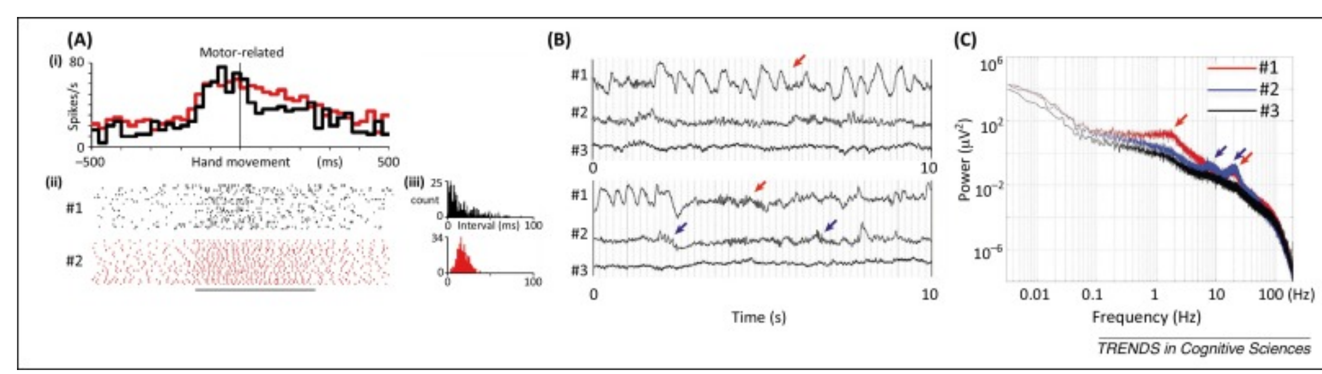
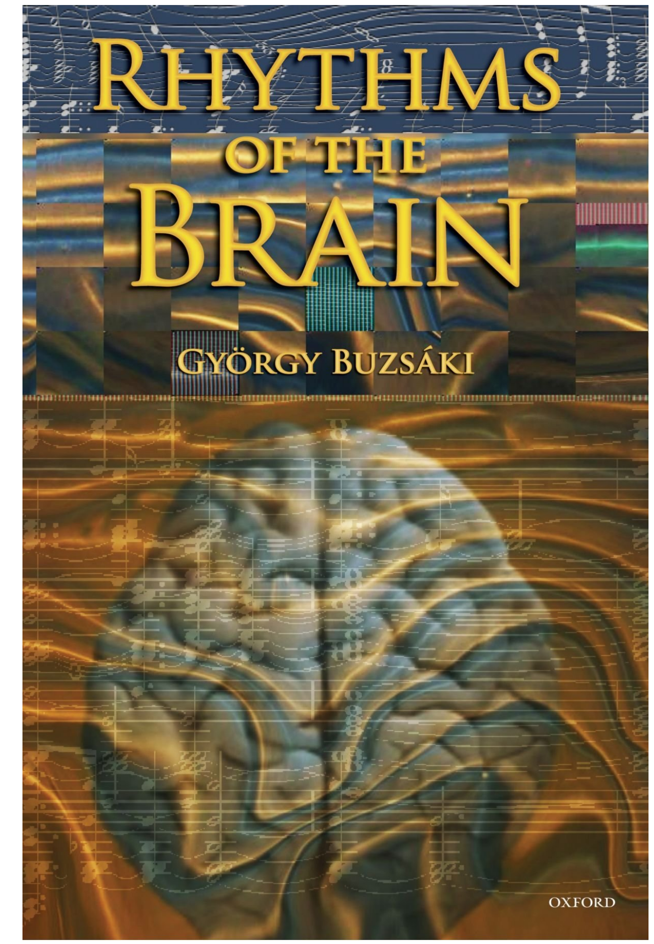


Figure 1. Example rhythmic and arrhythmic activity in neuronal firing and field potentials (Biyu He 2014)

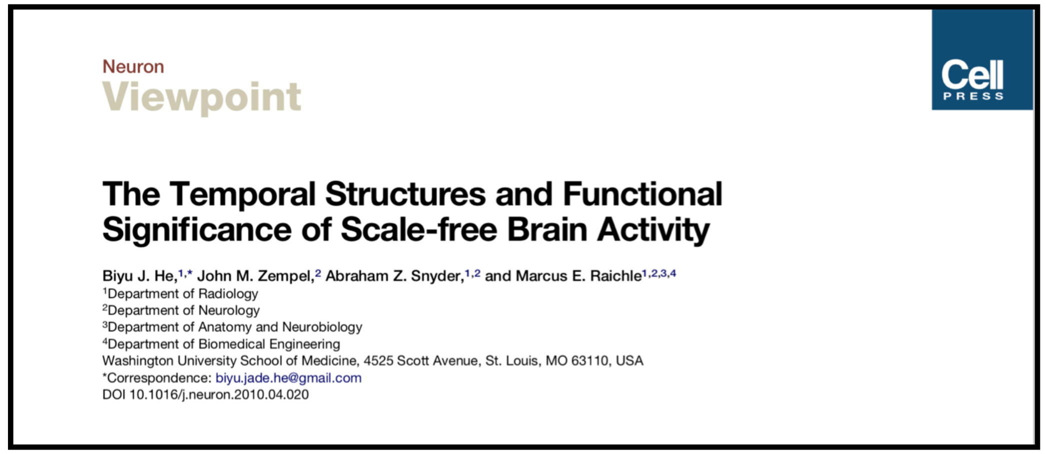
"Rhythms": a Buzsaki perspective

"Put bluntly, **the brain does not generate complex noise directly**. Instead, it generates **a large family of oscillations** whose spatial temporal integration **gives rise to the 1/f** statistics.This is in fact, the simplest way of producing complex noise."(Buzsaki 2006)



- Brain dynamics are in a state of self-organized criticality, displaying perpetual state transitions (***metastability***): the system is in a **critical state** that can be reduced by **perturbations** (sensory stimuli, motor output) and enter a **transient stability** via oscillations
- The cortical brain dynamics constantly shifts from the **high complexity regime of metastability (1/f)** -where it's highly responsive to minimal environmental perturbations- to an **oscillatory regime** characterized by temporal scales
- The brain needs this **transition from complexity to prediction**

“Both coexist”: a parsimonious explanation?



Neuron
Viewpoint

The Temporal Structures and Functional Significance of Scale-free Brain Activity

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- Two types of brain activity coexist: the **broadband**, arrhythmic activity, and the **narrow-band**, rhythmic oscillations
- Randomly picked three electrodes, two with and one without rhythmic oscillations. Electrode #33 contains oscillations at ~1.5 Hz and ~20 Hz, electrode #43 contains oscillations at 7–8 Hz and ~20 Hz, and electrode #64 contains no periodic oscillations but only arrhythmic, scale-free activity.
- Two 20 s segments of raw data separated in time by >1 hr are randomly selected. The power spectra of these **two short data segments recapitulate the power spectra averaged over the entire 83 min record** (with 1/f spectrum and oscillatory bumps)
- 1/f type of spectrum **doesn’t seem to be the result of a summation over many narrow-band oscillations.**

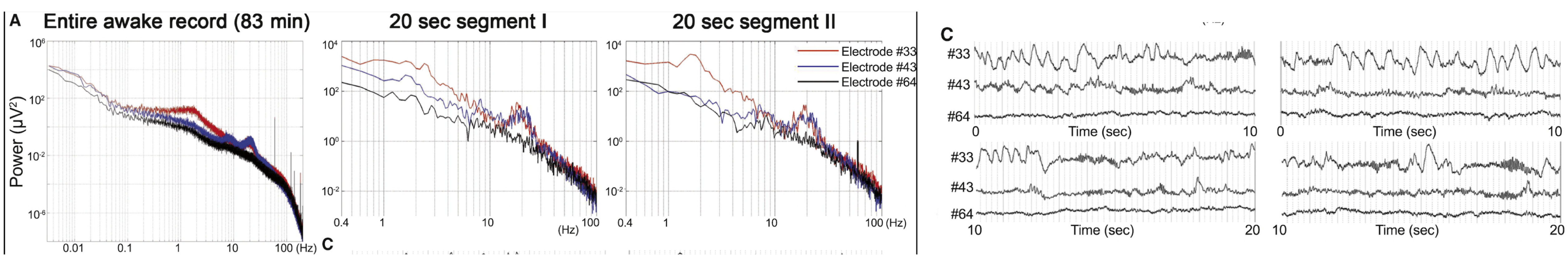


Figure 2. Stability of the 1/fβ Power Spectrum. (A) Power spectra from three example electrodes in Patient #3. (C) The raw data records for the two 20 s segments.

Biyu He et al. 2010

Fine temporal structure

- Contrary to common assumptions, arrhythmic brain activity contains a **rich temporal organization**
- **Nested frequency analysis** shows that *the phase of lower frequencies modulates the amplitude of higher frequencies* in an upward progression across the frequency spectrum (***nested frequencies*** usually found in brain oscillations, for example between the phase of theta and the amplitude of gamma).
- **Cross-frequency phase-amplitude coupling**: extensive nested frequencies **across the entire frequency spectrum.**
- **Stability** of nested frequencies **over time** also for electrodes with only scale-free activity.

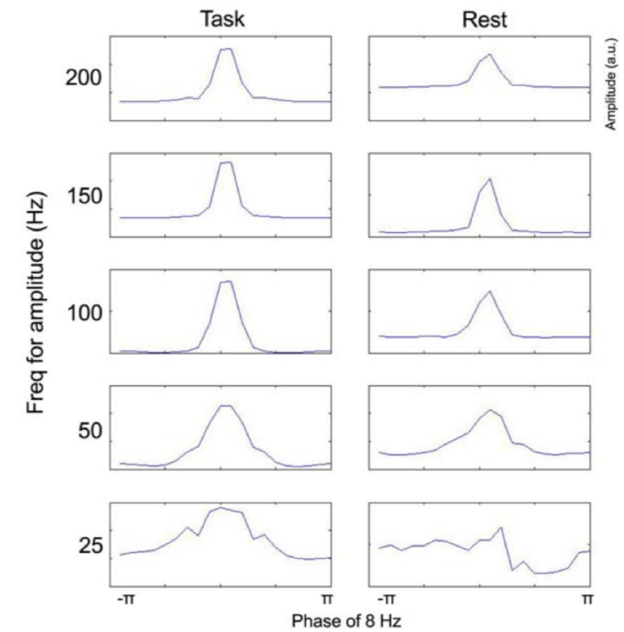


Figure S5. Complement to Fig. 4D, results from electrode #64 in Patient #3. Amplitude of 5-Hz-width bands centered at 25, 50, 100, 150 and 200 Hz were each averaged at different phases of the 8-Hz activity (extracted from a 1-Hz-width band). The left column was from ECoG data during task performance, averaged across all task blocks (~35 min in total). The right column was from spontaneous ECoG data in the waking state (~83 min).

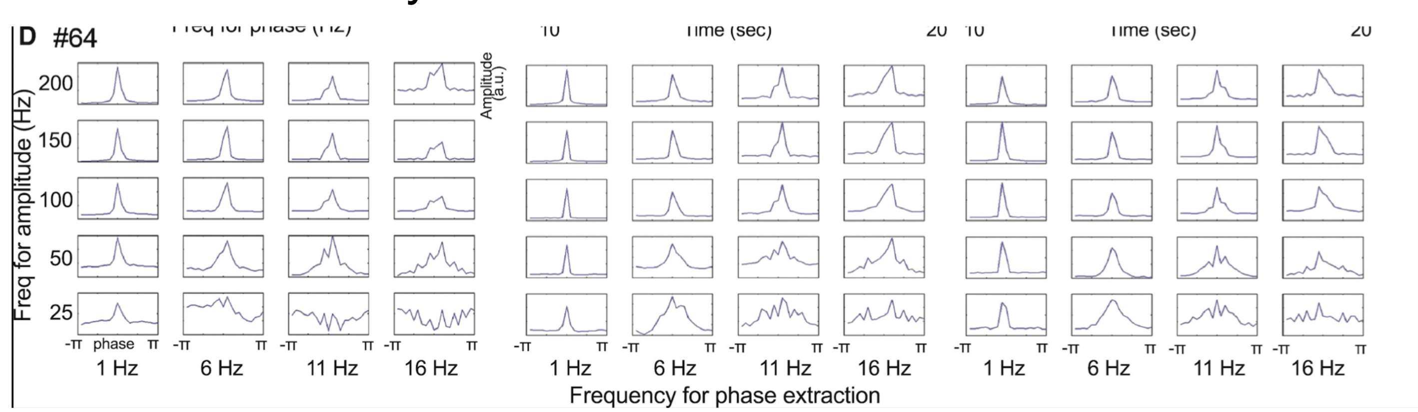
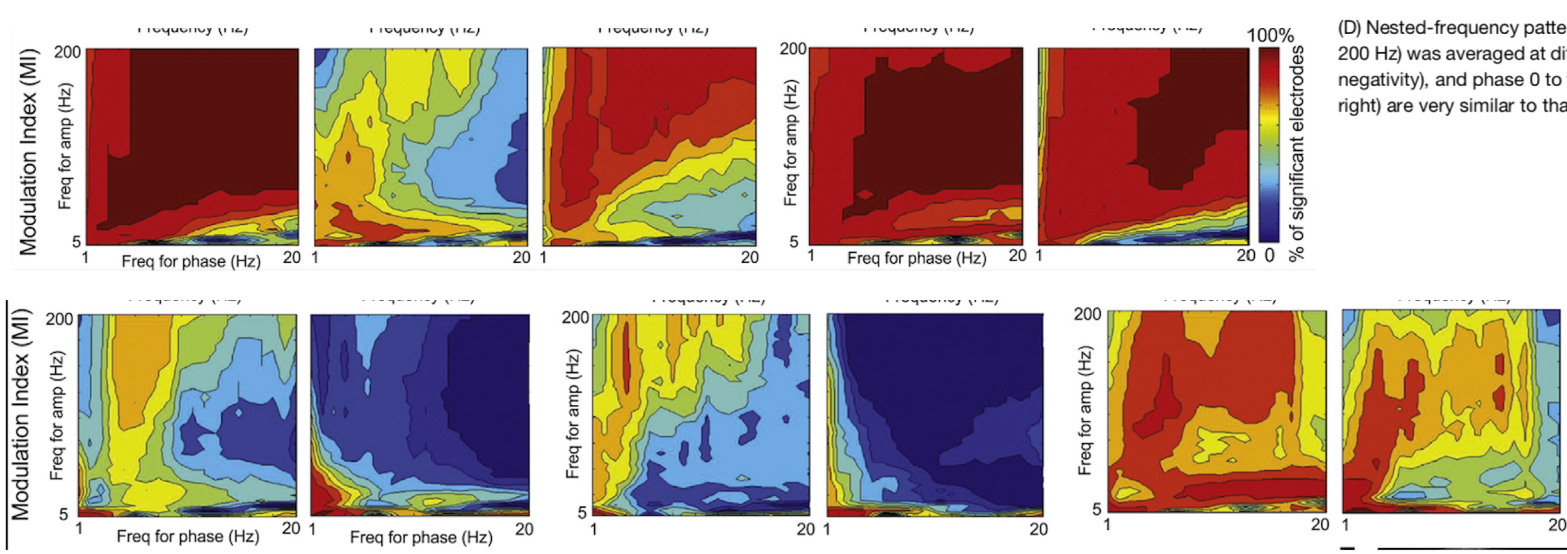
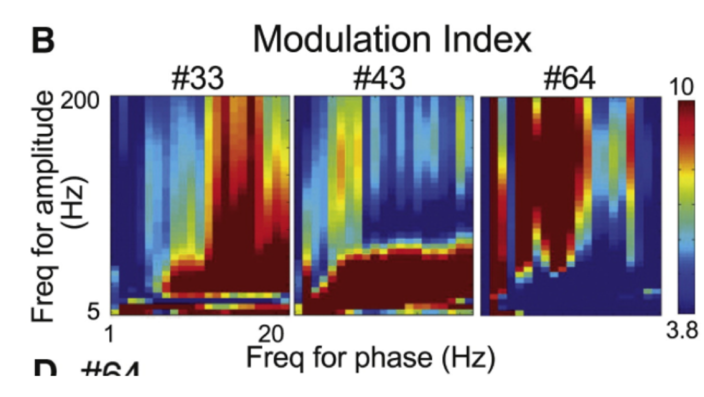


Figure 1. Power Spectra and **Cross-frequency Coupling** in ECoG Data
(A–E) **Bottom:** The percentage of electrodes with significant phase-amplitude cross-frequency coupling. Phase was extracted from 1 Hz-width bins with center frequencies from 1 to 20 Hz in 1 Hz steps. Amplitude was extracted from 5 Hz-width bins with center frequencies from 5 to 200 Hz in 5 Hz steps. The percentage of electrodes with significant MI Z score (p < 0.05 after Bonferroni correction) is plotted as color for each frequency pair.



(D) Nested-frequency patterns for selected frequency pairs in electrode #64. Amplitude of the higher frequencies (5 Hz width bands centered at 25, 50, 100, 150, 200 Hz) was averaged at different phases of the lower frequencies (1 Hz width bands centered at 1, 6, 11, 16 Hz). Phase ± π corresponds to the trough (surface negativity), and phase 0 to the peak (surface positivity) of the lower-frequency fluctuation. Nested-frequency patterns from the two 20 s segments (middle and right) are very similar to that from the entire awake record (left).

Figure 2. Stability of the 1/fβ Power Spectrum and Nested-Frequency Patterns.
(B) Phase-amplitude cross-frequency coupling for each of the three electrodes computed from the entire awake record. MI Z score is plotted as color for each frequency pair. Only significant values (p < 0.05 after Bonferroni correction) are shown.



Fine temporal structure

Biyu He et al. 2010

- **Control analyses (1):** 1/f noise is **not caused by instrumental noise** -> Dummy ECoG recording conducted in a standard in-patient monitoring room vs real
- **Control analyses (2):** broadband activity is **not caused by microsaccades** -> Plot of the strength of cross-frequency coupling across the cortical surface confirms that significant nested frequencies were present across wide cortical regions and did not exhibit an anterior-posterior gradient.

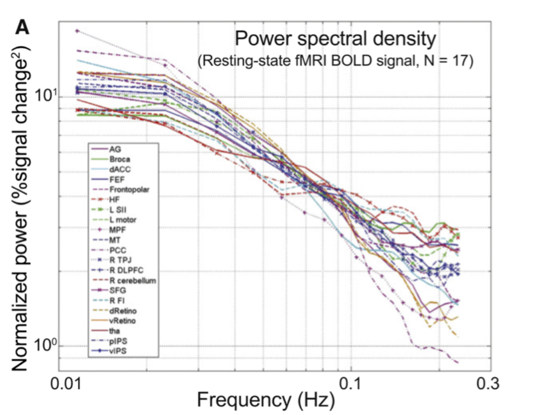


Figure 5. Scale-free Dynamics in Spontaneous fMRI Signals
(A) Normalized power spectrum (total power/variance = 1) for each brain region is plotted in a log-log plot.

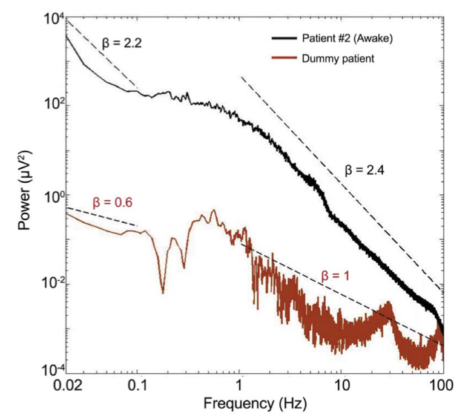


Figure S6. Power spectrum of a dummy recording conducted in a standard ECoG-video in-patient monitoring room (duration: 46 min) compared to the power spectrum of brain ECoG activity. Red: a resistor (5 kΩ) was connected into the amplifier (input impedance 10 GΩ) to record the internal noise in the recording system. Black: Average ECoG power spectrum across all electrodes in Patient #2 during the awake state. The low (< 0.1 Hz) and high (1-100 Hz) frequency ranges of both spectra were fitted with a power-law function $P(f) \propto 1/f^{\beta}$, the obtained exponents β are indicated in the graph.

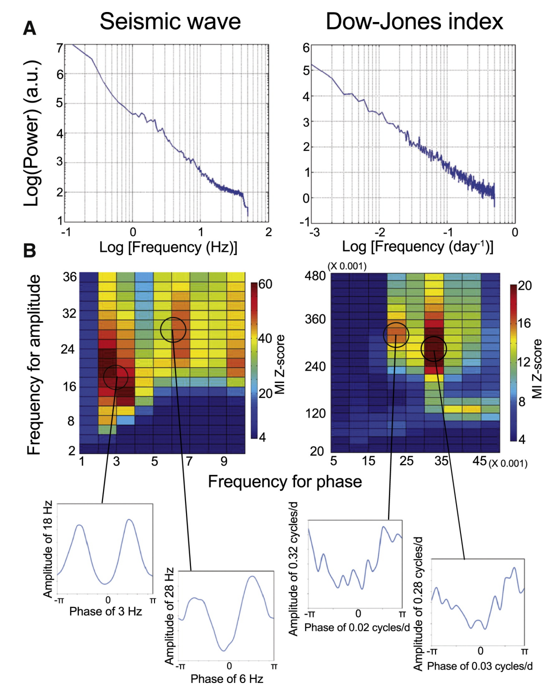


Figure 6. Scale-free Dynamics in Earth Seismic Waves (Left Column) and Stock Market Fluctuations (Right Column)
(A) Power spectra plotted in log-log plots.
(B) **Top:** Phase-amplitude cross-frequency coupling assessed by MI Z score, plotted as color in the 2D frequency space. Only significant values ($p < 0.05$ after Bonferroni correction) are shown. **Bottom:** Example nested-frequency patterns for selected frequency pairs. Amplitude of the higher frequency was averaged at different phases of the lower frequency and plotted.

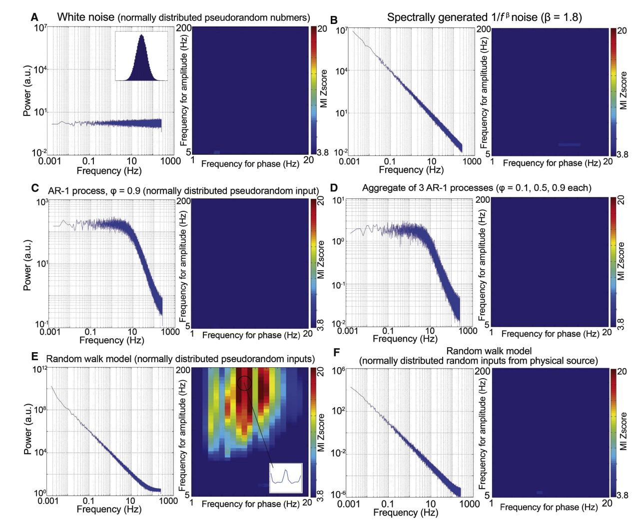


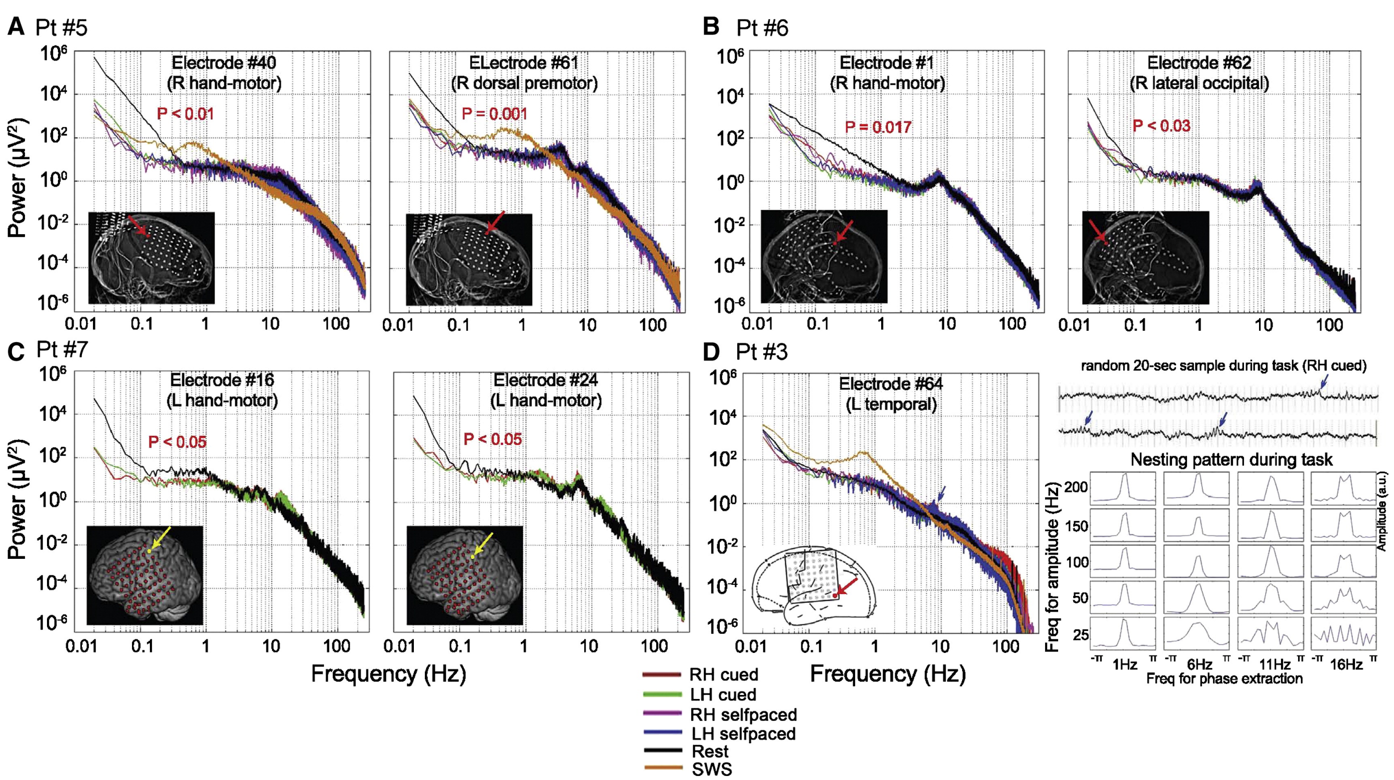
Figure 7. Power Spectra and Nested-Frequency Patterns of Simulated Scale-free Dynamics
(A) White-noise time series following Gaussian distribution from a pseudorandom number generator (mean = 0, variance = 10).
(B) Spectrally generated scale-free time series. This time series does not have nested frequencies.
(C) A first-order autoregressive (AR-1) process
(D) Aggregate of three AR-1 processes. Neither (C) nor (D) has significant nested frequencies.
(E) A random-walk model. **This random-walk time series does have significant nested frequencies across many frequency pairs.** The inset shows, for one example frequency pair, the higher-frequency amplitude averaged at different phases of the lower frequency.
(F) A random-walk model. **This random-walk model does not have nested frequencies.**

- **Comparative analyses (3):** nested-frequency analysis on spontaneous earth conducted on seismic waves and stock market fluctuations -> the exact patterns of nested frequencies in these signals differed from those in brain activity (*extensive nested frequencies observed in ECoG signals were contributed primarily by scale-free brain activity and not by periodic brain oscillations*).

- **Comparative analyses (4):** simulations of scale-free dynamics.

Functional significance

- Tested if scale-free dynamics activity has any functional significance by recording ECoG signals during both **quiet wakefulness** and **task performance**
- **Task (1):** cued **= visual-cued button press condition**
- **Task (2):** self-paced = **self-paced button press condition** (cfr. *readiness potential task* by Kornhuber & Deecke)
- Significant level of difference between power-law exponents of rest and task condition
- During task condition the β value decreased during all 4 trials types (left-hand cued, right-hand cued, left-hand self-paced, right-hand self-paced).
- No systematic difference between task co



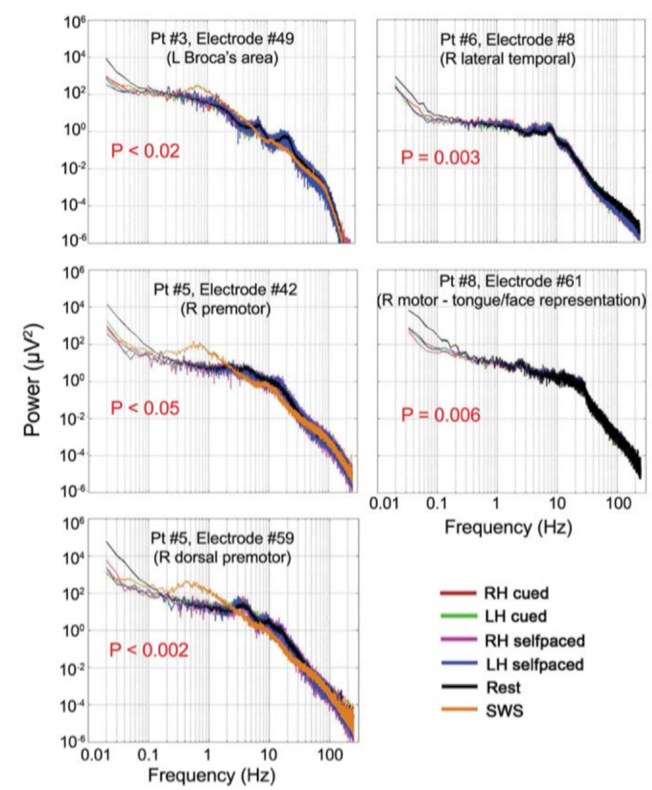


Figure S4. Complement to Fig. 4A-C, power-law exponent changes during task performance. The task consisted of a 2 × 2 design: visual-cued button press ("cued") or self-paced button press ("selfpaced") condition; and the button press was performed by either the left (LH) or right (RH) index finger. Significance levels of the difference of power-law exponent between rest and task conditions are indicated in the graphs (t-tests). In Patient #3 & #5, SWS power spectrum was presented for comparison, but not used for statistical analysis. The anatomical locations of each electrode are specified in the graphs.

Summary

- Scale free dynamics and scale invariance are universal characteristics of complex systems in nature
- 1/f or long-range temporal correlations or pink noise is also a self-generated/correlated brain noise that coexists with periodic brain oscillations in a non predominant temporal fashion
- For a long time this kind of noise has been ignored in experimental manipulations or analyses because thought to be the expression of instrumental noise
- Recent evidence speaks instead in favor of the functional significance of the 1/f and its spatio-temporal organization at different levels
- Further investigation is needed in order to unveil the relationship between each specific 1/f noise (across individuals, behavioral tasks, across different natural phenomena etc…)

Wanna know more?

Review

CellPress

Scale-free brain activity: past, present, and future

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“Thank you for your attention.”