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Abstract: Global historical series spanning the last two centuries recently became available for primary energy consumption (PEC) and gross domestic product (GDP). Based on a thorough analysis of the data, we propose a new, simple macroeconomic model whereby physical power is fueling economic power. From 1820 to 1920, the linearity between global PEC and world GDP justifies basic equations where, importantly, PEC incorporates unskilled human labor that consumes and converts energy from food. In a consistent model, both physical capital and human capital are fed by PEC and represent a form of stored energy. In the following century, from 1920 to 2016, GDP grows quicker than PEC. Periods of quasi-linearity of the two variables are separated by distinct jumps, which can be interpreted as radical technology shifts. The GDP to PEC ratio accumulates game-changing innovation, at an average growth rate proportional to PEC. These results seed alternative strategies for modeling and for political management of the climate crisis and the energy transition.

1. Introduction

Energy resources are essential to provide wealth and quality of life to human societies[1,2]. Any economic process consumes energy, i.e. turns energy from a valuable, low-entropy form into a high-entropy, waste form[1]. Nowadays, fossil carbon sources still provide about 85% of primary energy consumption (PEC). Thus, the energy sector remains the principal contributor to climate change through carbon dioxide (CO_2) emissions, as identified since the 1970s[3,4]. Replacing CO_2 emitting technologies by carbon neutral solutions is now an urgent goal. New renewable energy technologies (photovoltaic panels, wind turbines) that have been deployed for decades, have only recently become competitive with fossil fuels in several sectors: electricity[5], urban mobility, thermal management of well-insulated buildings, to name a few. Additional technologies in other services, notably chemical production from non-fossil resources like atmospheric CO_2,[6] justify strong incentive programs to spread low-emission energies[7]. Despite propositions for all-out deployment of fossil-free sources[8,9] that will be crucial to our future wealth and wellbeing, these technologies still only represent a modest piece of the energy pie[10]. To overcome this worrying gap, a thorough understanding of the reliance of the global economy on energy is a clear research objective. Currently, the macroeconomic field is lacking a universally accepted model that would give energy its proper share as a systemic input. Scenarios such as those reviewed by the Intergovernmental Panel on Climate Change[11] are based on "integrated assessment models" where energy is a mere sector of the economy, no larger than its nominal cost share (5 to 10%). Common economic textbooks[12,13,14] generally script capital and labor as the two key factors of production, with a correction factor (residual in the neoclassical Solow model[12-14]) accounting for the positive effect of knowledge development. Based on national accounting (USA, Japan, Germany etc…) indicating that energy consumption is as important as physical capital for production[15,16], ecological and biophysical economists have long criticized[17,18] the negligible role given to energy. Knowing this, macroeconomic models for the energy transition should cease to ignore the systemic role of energy embodied in capital and labor, as repeatedly demonstrated[19,20,21,22,23,24].

In this paper we present a simple, energy-based macroeconomic model to study the world economy, as an essential step to unravel the **economy-environment nexus**. Inspired by ecological economics[1,17-24] and neoclassical macroeconomics[12-14], this model aims to reconcile both schools. Since the subject of an energy-GDP relation has been treated by many in the last decades, we will start in section 2 with a review of the literature from which a few teachings and issues emerge. This section will remain short because no previous works used a physical approach like ours that encompasses the subject simultaneously on a worldwide basis and on a long time scale of two centuries. We will then briefly present the data in section 3 (with a longer discussion given in Appendix A). In section 4, the model will be described and applied to the data; its significance will be discussed and perspectives will be drawn as to its usefulness in section 5.

2. A short review of a large literature corpus on the energy-GDP relation

Our endeavor benefits from the recent availability of global estimates that can be used to establish a deep underlying relationship between GDP and PEC (primary energy consumption). With its wide geographical and historical spirit, our approach complements previous works[15-24] that established the importance of energy consumption in the economy. Note that several authors use the word exergy to signify "usable energy"; here we use the popular definition of

the word "energy" to have the same meaning as "exergy". These papers[15-24] generally focused on a local or regional level using available national accounting data. For example, Kümmel[15] studied Germany in 1960-1981 and the USA in 1960-1978, Stern[20] the USA, 1960-1999, Ayres and Warr[21] the USA and Japan, 1900–2005, excluding 1941–47, Ayres and Voudouris[23] the USA and Japan, adding the UK for the same time period, and Voudouris et al.[24] European countries, 1970-2012. These works showed that a production function had to include energy as one of its main factors, together with the more canonical capital and labor factors, in order to fit the national GDP data. These authors stressed that introducing energy as a factor notably diminished the magnitude of the Solow residual[12-14]. Although this demonstration was conceptually very important, one can criticize the approach from a physicist's point of view relative to the openness of the system: the production function does not include energy (or labor or capital) that entered a production process from abroad, although imported products may have subsequently enabled GDP generation inside the country. The terms of this exchange would deserve attention on political economy grounds, for example by examining the balance of power behind it. Mostly, by neglecting any consumption or production abroad, these national/regional analyses are not well-founded on ecological grounds, as is now commonly accepted with the emphasis on grey/embedded energy consumption and CO_2 emissions. On the one hand, focusing on environmental issues is most consistent with our (and Kümmel's[15]) consideration of primary energy. This contrasts with other approaches (Ayres et al employing useful energy[21,23,24]; Stern defining a "Divisia index" of energy use[20]). On the other hand, to make the point formally, we consider a geographical domain having no economic exchanges (so far) with the rest of the universe, and exchanging only energy with the environment. In doing so, we aim at legitimately neglecting the political economy question and correctly setting a global scale for environmental issues.

Much of the abundant literature on the PEC-GDP relationship looks for signs of causality from PEC to GDP or vice versa[20,25,26,27,28] which is a legitimate science issue and a very important question for governments confronted with an economic crisis and/or an energy shock. Consequently, most of these papers scrutinized individual nations benefiting from bona fide accounting and are therefore mainly restricted to half-century time spans or less. Causality tests obtained through statistical treatments (of the Granger-type in particular) often produced opposite conclusions in separate works: the recent compilation and careful analysis by Bruns, Gross and Stern[29] of more than 100 papers could not find a genuine causal effect as a whole. However, they found a robust genuine effect from output to energy use when energy prices are controlled for. Moreover, they also made interesting remarks about the possibility of evolution over time of the relationship between PEC and GDP which they then discarded. A variation over time of the relationship between energy and output was also noted in the case of Sweden (1850-2000) by Stern and Enflo[30], and for the USA by Ayres and Warr[21]. Through our global assessment in the long-run, we thus expect to deal with such variations in the constitutive relations between PEC and GDP. We will come back to causality as seen from our model rationale. As for energy prices, their effect will remain screened in a model strictly without money or social structures. Furthermore, considering that we will use reconstructed time series that are generated a posteriori by compiling very diverse data from sometimes-scarce sources with many years lacking any measurements (cf. Section 2 and Appendix A), a large work would be needed to make statistical correlations fully significant.

3. A new assessment of global production and energy data

The characteristic time scale of the aggregated world economy is of the order of decades: thus, its understanding can only rest on long-run observations, spanning centuries, as done for the climate system itself. Prompted by the current climate crisis and the emerging Anthropocene concept, developing and probing a new macroeconomic model makes sense especially in light of the recent availability of thorough, painstaking data collections and authoritative analyses of energy consumption[31,32,33]. The same holds for the estimates of global scale economic production in the long-run, pioneered by Angus Maddison[34] and his successors[35]. Appendix A discusses these various data sources. GDP data show an overall homogeneity while PEC sources suffer from some discrepancies: data from ref. 31 by Malanima are used in this paper, and are preferred over other published series[36,37] for reasons detailed in appendix A.
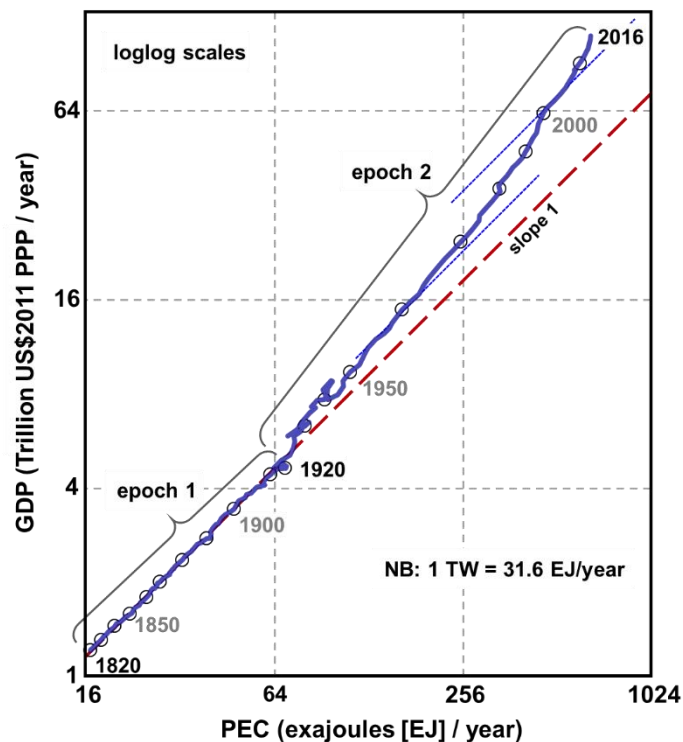

Fig. 1. **Log-log plot of global annual GDP** Y **vs. Annual Primary Energy Consumption (PEC)** E. GDP is in US\$-2011 at parity of purchasing power (see appendix A). The time scale is marked as empty disks at decades 1820, 1830 up to 2010. Energy aggregates all types of PEC, by humans as food, by draft animals as fodder, by diverse fireplaces or machines as fuels of any sort or wind or sun streams. In epoch 1, from 1820 to 1920, both Y and E vary by a factor ~4; a dashed red line extrapolates their linear relation (i.e. slope = 1) in this log-log plot, through two successive factors 4. During the following century (epoch 2, 1920-2016), Y grows quicker than E, while still following E quasi linearly for periods of a few years (unit slope segments, e.g. marked by blue dotted lines). See §3.1 and §3.2 for explanation of both features. (1 Exajoule [EJ] is 10^{18} J; a Trillion \$ equals 10^{12} \$; a Terawatt [TW] is 10^{12} W).

Figure 1 shows the sustained growth pattern of the world annual PEC E and GDP Y. During the last 200 years, GDP grew by a factor of about 90, while PEC was multiplied by 40. Both GDP and PEC have grown exponentially with time, at variable rates in different periods. Choosing period durations of a few decades (between 20 to 36 years, see table 1), the average annual growth rate never fell below ∼1 %.

Table 1. Relative growth rates of annual PEC E, global population L and world annual GDP Y .

periods	Average Annual Growth Rate			epochs (see Fig. 1)	Relative growth		
	$E =$ Global PEC	$L =$ Global Population	$Y =$ Global GDP				
1820-1850	1.0%	0.5%	0.9%	epoch 1	$\frac{E_{1920}}{E_{1820}} = 4.2$	$\frac{L_{1920}}{L_{1820}} = 1.8$	$\frac{Y_{1920}}{Y_{1820}} = 3.8$
1850-1870	1.1%	0.3%	1.2%				
1870-1900	1.8%	0.7%	1.8%				
1900-1920	1.9%	0.8%	1.5%				
1920-1950	1.6%	1.0%	2.4%	epoch 2	$\frac{E_{2016}}{E_{1920}} = 9.5$	$\frac{L_{2016}}{L_{1920}} = 4.0$	$\frac{Y_{2016}}{Y_{1920}} = 24$
1950-1980	3.7%	1.9%	4.5%				
1980-2016	1.9%	1.4%	3.1%				

The main point emerging from the careful treatment of GDP and PEC data (see details in appendix A) is the overall nearly linear trend on a log-log scale, over about 2 orders of magnitude, with a near-unity exponent[31] in epoch 1 (1820-1920, Fig. 1 and Table 1). Note that in this epoch there are already booms and busts, but they only scatter the underlying time along the line. The trend differs in epoch 2 (1920-2016), consisting of a piecewise patch of segments of similar unit slope, interspersed with jumps. We now lay out a modeling frame that accounts for the distinct features of both epochs.

4. Results and model significance

4.1 Epoch 1 (1820-1920): reassessment of production factors

The regular, linear relation observed in epoch 1 is interpreted using rather standard macroeconomic growth modeling[12-14]. Y is described by an aggregate production function $Y = f(K, H, L)$ which a priori depends on physical capital K, human capital[38] H and labor L, L being the number of workers, usually identified with the population number when specific data are missing, as in our case. As introduced in appendix B, and as generally adopted elsewhere[12-14], H capitalizes the knowledge and skills of workers, while K represents the tangible manufactured goods and infrastructure that support production. In refs. 12-14, the standard treatment considers an exponential growth of L which induces a similar proportional growth of Y. However, as evidenced in Fig. 1 and table 1, the proportionality lies here between Y and E, not L. Explicitly, if one tries to explain Y growth from L only, as neoclassical economists did at the beginning[12-14], one gets a huge discrepancy – which contains enhanced energy use – hardly described as a residual. Considering, as we do, PEC as a main factor restores a sensible residual, as already noticed by ecological economists[15-24] (see section 2).

A novel and very simple, although bold operation – embedding L into E – provides a strong link between standard modeling and global data. This new viewpoint that we propose here springs from the understanding that labor – unskilled human work fueled by food – assimilates any of the other forms of energy consumption[22] as the physics term "work" points out. Indeed, human labor has continuously been enhanced by new energy sources and converters throughout the Homo genus records. Enhancing L by energy consumption per capita $\varepsilon = \dfrac{E}{L}$, we consider a production function of the "Cobb-Douglas"[12-14,16,22] type:

$$\frac{Y}{Y_{1820}} = A \left(\frac{K}{K_{1820}} \right)^{\alpha} \left(\frac{H}{H_{1820}} \right)^{\beta} \left(\frac{\varepsilon L}{\varepsilon_{1820} L_{1820}} \right)^{\gamma} = A \left(\frac{K}{K_{1820}} \right)^{\alpha} \left(\frac{H}{H_{1820}} \right)^{\beta} \left(\frac{E}{E_{1820}} \right)^{\gamma} \quad (1)$$

where factors are normalized to their value in 1820. α, β and γ are the elasticities of production of K, H and E; inequalities $0 < \alpha < 1$, $0 < \beta < 1$ and $0 < \gamma < 1$ are standard hypotheses of diminishing returns; the condition $\alpha + \beta + \gamma = 1$ provides for constant return-to-scale with respect to the three factors. Interpreting Eq. (1) according to standard macroeconomic theory[12-14], γ is the global cost share of energy needs (including basic needs of workers and families, mainly food) while β (resp. α) is the part of production rewarding human (resp. physical) capital.

The *residual* A [12-14], also called *total factor productivity*, takes care of any discrepancy between the products of factors and Y. A equals unity in 1820 – by definition – and we keep it so for epoch 1. As we will discuss later, A evolves in epoch 2.

K and H are *stock* variables whereas E stands for a *flow*, the consumption of energy – actually a consumed *power* expressed in power units, Exajoule/year (EJ/year) or TW (1 TW=31.6 EJ/year). The dynamics in this system is set by the evolution equations of the

accumulated capital K and H. K (resp. H) grows with time t as a share $s_K Y$ (resp. $s_H Y$) of the production, saved and invested, while a part $\delta_K K$ (resp. $\delta_H H$) is lost by depreciation.

$$\frac{dK}{dt} = s_K Y - \delta_K K \qquad \& \qquad \frac{dH}{dt} = s_H Y - \delta_H H \tag{2}$$

with s_K and s_H (dimensionless, between 0 and 1) and δ_K and δ_H (inverse times) the savings and depreciation rates. Here, since H accounts for the part of skills in skilled labor, the second equation points to the investment necessary to maintain and develop the skills against classical depreciation dynamics. At a given time, K and H "store" the results of history through the integral of Y, through energy use (PEC E) as well as previous investments in machine or plant building and worker education resulting in new skills. In the view whereby Y and E are proportional, it follows that K and H actually store energy.

Equations 1 and 2, with $A = 1$, constitute a model[12-14,38] worth further studies (cf. appendix B). However, since data for K and H in epoch 1 is scarce, we opt for simplicity in a system where normalized production $\mathcal{Y}$ is a function of only two factors,

$$\frac{Y}{Y_{1820}} = \mathcal{Y} = \mathcal{J}^\alpha \, \mathcal{E}^{1-\alpha} \quad . \tag{3}$$

$\mathcal{E}$ is the normalized PEC, to be exogenously defined later, and $\mathcal{J}$ is an aggregated capital factor accumulating according to:

$$\frac{d\mathcal{J}}{dt} = s\mathcal{Y} - \delta\mathcal{J} \tag{4}$$

$\mathcal{J}$ (as well as its non-normalized companion J) is a combination of K and H, and can stand for both of them, or for any factor that requires PEC to accumulate and build up. Here, an extra but reasonable assumption is that the aggregation of K and H into J still lends itself to the same dynamics as above: such an assumption is easily proven when s and δ parameters for K and H are constant in time.

Equations (3) and (4) constitute a simple system thoroughly studied in macroeconomics[12-14].

Setting $\mathcal{E}$ to grow exponentially as $\exp[gt]$ (see exact solution in appendix B), the equations give rise to a steady-state *balanced growth path*[12-14] where both $\mathcal{Y}$ and the aggregated factor $\mathcal{J}$ grow exponentially at the same rate as $\mathcal{E}$ after a short transient period, maintaining a constant ratio $\dfrac{\mathcal{J}}{\mathcal{Y}} = \dfrac{s}{g + \delta}$, whatever the value of α in Eq. (3). In a nutshell, as $\mathcal{J}$ capitalizes production, it boils down to storage of energy. However, coming back to the question of causality evoked in section 2, Eq. (3) and (4) imply a bidirectional relation, a kind of "chicken and egg" situation, already noted by Ayres and Warr (cf. ref. 21, p. 204). In this balanced growth path with $\mathcal{E}$, $\mathcal{J}$ and $\mathcal{Y}$ growing proportionally and exponentially, any of the three drives the other two. PEC growth induces GDP growth from Eq. (3); simultaneously, a rise in

GDP causes capital to rise quicker (Eq. (4)); finally, a larger capital implies a bigger PEC to feed it, so as to maintain production proportionally according to Eq. (3). This last assertion may stop being true when new, radically more efficient technologies replace old ones, a situation that is relevant to epoch 2, discussed in §*3.2*.

Epoch 1 in Fig. 1 exhibits such a balanced growth phenomenon, with the iconic decades of the coal-iron-railway "development block" thoroughly described by Kander et al.[32]: coal transforms iron, iron makes rail and locomotives, railways transport coal, in a growing spiral fueled by coal burning. To grasp the specific role of K and H growth, it is interesting to take the log-differential of Eq. (1) which, since $Y \propto E$ here, leads to $\frac{\Delta K}{K} \simeq \frac{\Delta Y}{Y} + \frac{\beta}{\alpha}\left(\frac{\Delta Y}{Y} - \frac{\Delta H}{H}\right)$.

Since Piketty[39] reports that global capital K grows slightly quicker than Y in epoch 1 (see ref. 39, p. 196), we infer that H should have experienced a slower growth than Y in those periods, which is a testable assumption. It should translate into a variation of saving and/or depreciation parameters.

4.2 Epoch 2 (1920-2016): how energy feeds Schumpeterian innovation

In epoch 2, a different regime of economic growth settles. The 1920-1950 period, including the roaring twenties, the great depression and WWII appears in Fig. 1 as a transient towards a new energy/GDP relation. After exploring some alternatives, we deduce from the structure of Fig. 1 and the physical justification of the linearity of Y and E a closed-form relation applying to the whole timescale, which is a boldly simplified version of Eq. (1):

$$\frac{Y}{Y_{1820}} = A\frac{E}{E_{1820}} \qquad \text{or} \qquad \mathcal{Y} = A\mathcal{E} \quad , \tag{5}$$

The time-dependent residual A carries the whole flow of innovation, spurred on by the use of energy (e.g. tapping new resources, efficiency gains in converting PEC to *useful work*[21], inventing new conversion modes, machines, materials or distribution schemes) but also any revolutionary progress of universal knowledge (electricity, internal combustion engine, air travel). $A = \mathcal{Y}/\mathcal{E}$ can be considered as the *normalized productivity of energy*[31]. In Eq. 5, where the linearity between Y and E hides a relation like Eq. (1), H and K are implicitly present, merged into E along the exponential, balanced growth path. A more complex modeling is explored in Appendix B, keeping capital $\mathcal{J}$ with PEC $\mathcal{E}$ altogether, which turns out to produce similar results. However, the modeling in the previous subsection, with a normalized constant $A = 1$, and constant parameters (s, δ, α), showed that GDP, capital and energy are proportional to one another once energy grows exponentially. It is easy to show that if anyone of the three grows exponentially, all of Y, J and E do also and remain indefinitely proportional. The proportionality is extended also to a model with two types of capital, H and K, if parameters of investment equations (4) are kept fixed.

There is a vast macroeconomic literature on residuals in Cobb-Douglas-type production functions. We favor an endogenous definition of A,[40] and look for a simple factor dependency. The constraints we inherit from the data are that (i) the variation of A should be very modest in Epoch 1 where proportionality is observed and (ii) it should account, cumulated on Epoch 2, for a moderate factor of a few units. Thus, in terms of smooth functions, we can rely on very simple laws to obey such constraints. Imitating similar considerations on the long run human history[41], and guided by the linear Y to E relation at work during epoch 1, we merely conjecture that A depends on E only; the graphical representation in Fig. 2 leads to a linear relation between E and the growth rate of A. By contrast, other parametric dependencies for A were tested: when we used Y or Population instead of E, a linear fit of Log[A] variations was much less satisfying. Our key assumption thus reads:

$$\frac{dLog[A]}{dt} = \chi \mathcal{E}, \tag{6}$$

which could also be written $\frac{dA}{dt} = \chi \mathcal{E} A = \chi \mathcal{Y}$ with χ directly obtained from the fit. Since A contains all irreversible progress eventually shared by the whole of humanity (i.e. the so-called *nonrival goods*) it seems logical that any effort (measured in energy unit) anywhere on the planet has the same average, long-term effect on A growth. If one remembers that here $\mathcal{E}$ contains unskilled labor L, then Eq. (6) resembles Kremer's bold assumption[41], i.e. that $\frac{dA}{dt}$ was roughly proportional to LA.

From Eq. (6), we derive $A[t] = \exp\left[\int_{1820}^{t} \chi \mathcal{E} \, dt\right]$ since $A[1820]=1$. Figure 2 shows that, between the ratio $\mathcal{Y}/\mathcal{E} = A$ and the time integral of PEC ($\int_{1820}^{t} \mathcal{E} \, dt$), both derived from the data, an exponential relation holds from 1930 on, with $\chi \approx 0.00046 \, \text{year}^{-1}$ (this value hinges on the 1820 normalization of PEC in $\mathcal{E}$), hence the nonlinear skew only in epoch 2. We insist once again that the time integral yields a bona fide energy, since the PEC E is an energy flow, i.e. a physical power. During the 1920s and 1930s, $\mathcal{Y}/\mathcal{E}$ jumps from the epoch 1 conditions to the new epoch 2 regime, with the full-fledged range of modern techniques substituting older ones. Projecting the exponential relation between A and $\int_{1820}^{t} \mathcal{E} \, dt$ to the past, we see that the new regime integrates the effect of energy consumption on invention from the beginning of 19th century. Innovation in energy efficiency, and introduction of new machines etc. existed in epoch 1. Its first 50 years, described as the age of steam and railways, would rightly be characterized by a single trend in energy usage, with the deployment of steel and electrical power in the 1870s and 1880s qualifying as a new stage of industrial revolution[2,21,42].
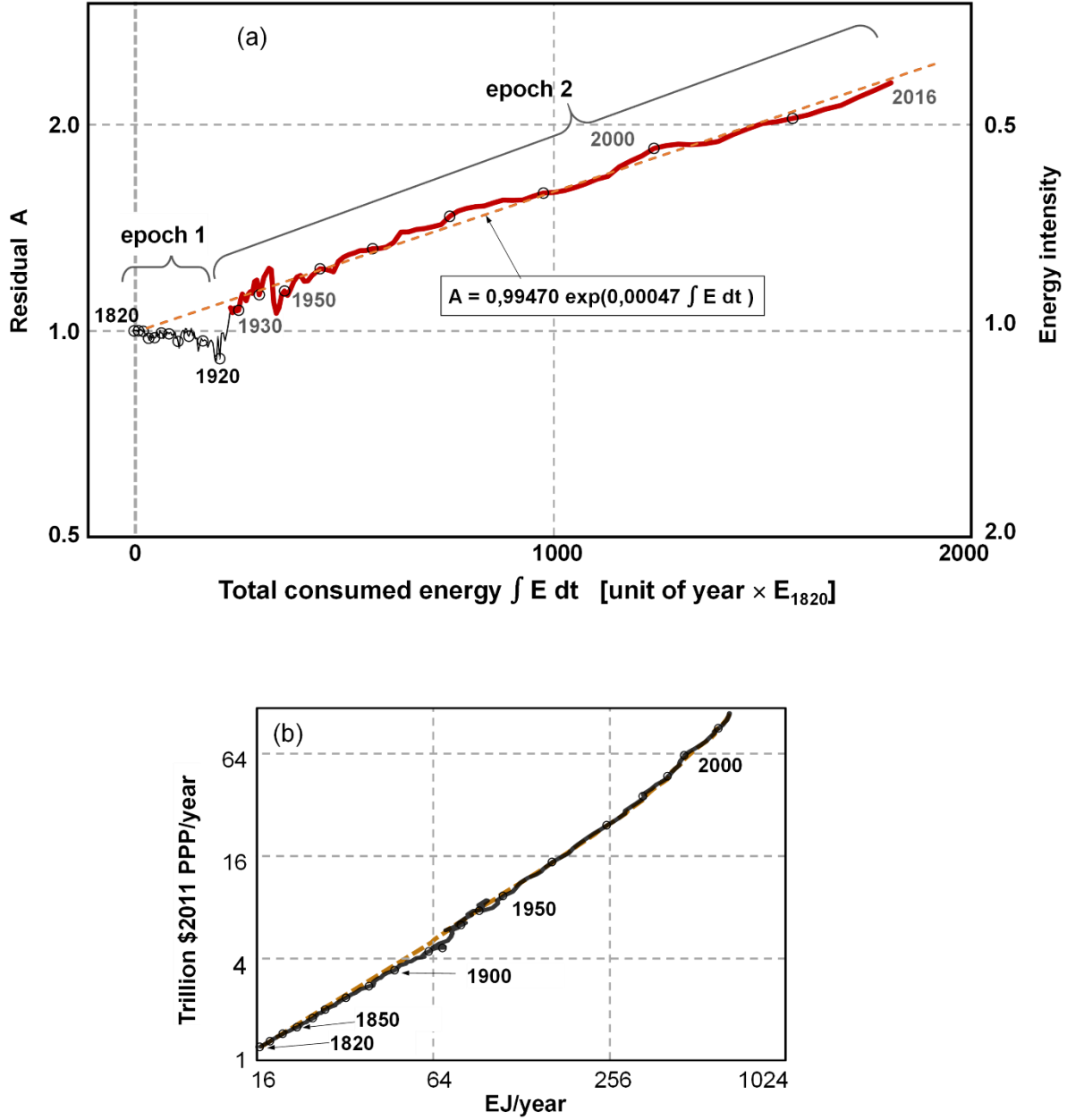
Fig. 2. (a) Semilog plot of $\dfrac{y}{\varepsilon}$ **, which is the residual** A **in Eq. (5) (left axis) and**

its inverse $\dfrac{\varepsilon}{y}$ **, the energy intensity of GDP (right axis, downward)** vs. the time-

integral of normalized PEC ε. Both are calculated from the data in Fig. 1. An exponential relation holds in the period from 1930 to the present between A and the time integral of PEC, consistent with Eq. (6). From 1910 to 1930, A jumps from its previous value of 1 in the 19th century to the new regime. The exponential fit is done between 1926 and 2016. Decennial data are marked with black circles.

(b) GDP Y **as a function of PEC** E**,** same data as in Fig. 1 (black line with decennial circles), fitted with Eq. (7) (brownish dashed line). The evolution of A in (a) accounts for the curvature in (b).

However, these new devices and industrial organization did not change the overall energy efficiency of the global economic system, maybe because a higher amount of energy had to be invested in the machines. Actually, table 1 shows that the growth rate of PEC grew steadily throughout epoch 1, evidencing a global rebound effect prompting the addition of energy thirsty processes. We would interpret this by saying that radical innovation was hindered in 1820-1920 and its path suddenly tipped in the 1920s, spurred on in all domains from e.g., air transport to agriculture[43] and to energy distribution. Several successive jumps occur during epoch 2. In the second half of the 20th century, A grows by steps of a few percent as shown by the peaks of annual $\frac{\Delta A}{A}$ in Fig. 3. We account for these as radical, "Schumpeterian" technology shifts[2,21], bringing a sudden increase in energy efficiency by the abrupt replacement of less efficient schemes or devices (even though singling out specific items, however iconic, is irrelevant, as triggering effects arise from their intertwined use). However, in the last 10 years, the growth of A tends to be more regular, maybe revealing a more decentralized innovation system. Nevertheless, during the last 90 years of epoch 2, the quantitative dependency of economic growth on energy does not experience any change, thus following Eq. (5) which can be re-written as

$$\frac{Y}{Y_{1820}} = \exp\left[\int_{1820}^{t} \chi \frac{E}{E_{1820}} \, dt \right] \frac{E}{E_{1820}} . \tag{7}$$

Visually, we refer to the plot of Fig. 2(b) where we fit with Eq. (7) the data from Fig. 1.
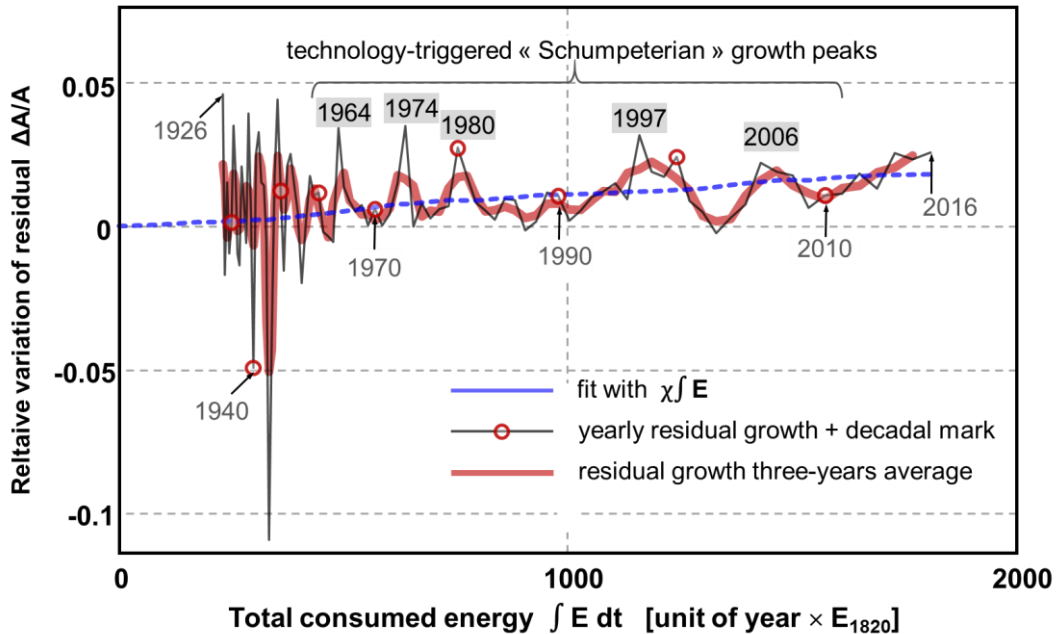

Fig. 3. Annual relative growth rate of A, the residual exhibiting the effect of innovation on GDP, plotted versus the time integral of consumed energy in epoch 2 (1926-2016). Jumps identified in Fig. 1 appear here as peaks of $\frac{\Delta A}{A}$. Five of these peaks are marked with the years of highest innovation growth, in 1964, 1974, 1980,

1997 and 2006. Thin line: year to year relative growth rate of A; thick line: 3-year average. Dashed line: linear relation in $\mathcal{E}$ according to Eq. (7).

Let us now comment on Fig. 3 in more detail, focusing on the peaks in the residual's growth $\frac{\Delta A}{A}$. Apart from the 1940 and 1945 expected dips (obvious for WWII, whereby all factors including knowledge align to physically destroy the capital of the belligerents, e.g., energy networks, and labor), the average growth is around 1% (see the dashed line $\chi \mathcal{E}$ in Fig. 3) with a baseline around 0.5%. Schumpeterian peaks appear as well, corresponding typically to booms of 1.5-2% extra growth in the moving average. We shall argue elsewhere about the collective "swarming" nature of these booms, as no single sector of economy can create them alone, mainly because economic success in epoch 2 spawns so many new objects[44].

5. Conclusion and Perspectives

To summarize our findings, we want to stress the two main contributions contained in this paper. First of all, based on the study of long term series of GDP and PEC, we propose to consider three major production factors at a global level, namely PEC E, Human capital H and Physical capital K. The traditional factor of Labor does not appear, being absorbed partly in PEC for its brute force component and in H for the skilled labor part. The observed proportionality of GDP and PEC during epoch 1, up until 1920, is described by a simplified system of equations with two factors, PEC and a single aggregated capital J, without the need of any residual. J is proportional to PEC and GDP in this period. Epoch 2, where a decoupling of GDP from PEC is observed, is then analyzed with a further simplified scheme capitalizing on this proportionality. The residual A behind this decoupling is found to grow exponentially with the accumulated PEC, i.e. the time integral of energy consumption. This relation between A, the productivity of primary energy, and integrated PEC is our second finding. In a broad socio-economic picture, the residual A captures the historic trend of accretion and development of universal, non-rival knowledge, a feature becoming salient in epoch 2. The apparent lack of progress in the productivity of energy in epoch 1 commands further research.

An interesting and legitimate future direction for this work is to extrapolate it in terms of possible trajectories for the energy transition. In this spirit, a deceivingly simple exercise consists of prolonging the long-time trend with Eq. (7) and ignoring all the jumps. We acknowledge that the linear Y to E relation masks a complex function involving all capitalizing factors (see appendix B). However, we prefer a simple algebra that better emphasizes our scope, through the derivative form of Eq. (7) $\frac{\Delta Y}{Y} = \frac{\Delta E}{E} + \chi \mathcal{E}$. Then, a *business as usual* scenario, shown as a black curve in Fig. 4, merely extrapolates the ~ 1.9% PEC growth observed during the last forty years.
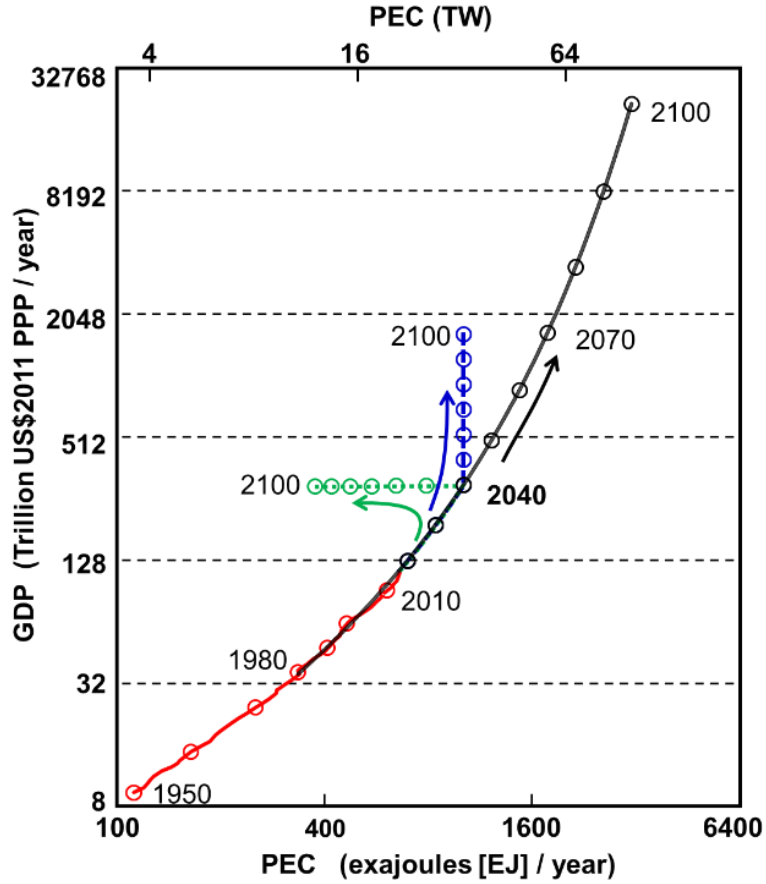
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Fig. 4. Prospective in the E, Y plane (loglog scales), according to Eq. (7). The black line is business-as-usual with PEC growing exponentially until 2100 at the same average rate as observed (red line) in 1980-2016. The two scenarios discussed in the text are illustrated, assuming a sharp turn, arbitrarily from 2040 on, with either constant PEC (dashed blue) or constant GDP (dotted green).

Fig. 4 also shows two other scenarios, better addressing the GDP-environment nexus. Firstly, allowing for a continued growth in PEC E during a short period (until 2040 in Fig. 4), we could envisage a time when the GDP "overgrowth" $\chi \mathcal{E}$ is large enough to sever any further PEC growth. This scenario would eventually usher in a new circular economy era, whereby GDP growth solely results from game-changing innovations (vertical blue segment up until 2100). Secondly, Eq. (7) could also introduce a *de-growth* trajectory, whereby E decreases while Y is maintained constant from a future time t_0 (2040 again in Fig. 4). Such a trend in

Eq. (7) would induce an algebraic decay of PEC $E = \left(\dfrac{1}{E_0} + \dfrac{\chi (t - t_0)}{E_{1820}} \right)^{-1}$, innovation gains

compensating for capital and PEC degrowth. These stylized scenarios are meant as draft tools

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to elaborate policies at a regional or global level, adding to recent work on a possible decoupling of GDP from resources consumption[45,46].

We finally situate our work in terms of its capability to initiate alternative views of the GDP-energy nexus. The difference between the two epochs can serve as a useful indication in this respect. The tale of epoch 1 is an energy-geared capital accumulation (K and H) while epoch 2 sees game-changing innovation enhancing the impact of PEC. This double role of energy consumption during a time of disquieting trends in global warming, biodiversity, etc., appears to be a key driver worth considering to address the economy-environment nexus. Thanks to its dual inspiration from ecological economics[1,17-24] and neoclassical macroeconomics[12-14,38,40,41], and substantiated by historical accounts[2,31-34,36,37,39,42], the model presented here offers an opportunity for reconciling both schools. It shall also foster operational tools to manage the planet's human future, which is part of the conversation on a desirable energy transition trajectory. In addition to taking into account the energy-based mechanisms suggested by this study, an improved model might add money and financial flows as effective levers for capital and knowledge in essential global shifts[42].

Data and materials availability: all data are publicly available as detailed in appendix A.

Appendices

A. *Choice and description of historical series*

In appendix A, we present the various sources of historical series for primary energy consumption (PEC) and GDP, and justify the choice that we made among them.

A.1. *Global primary energy consumption (PEC)*

We have been using Paolo Malanima's recently published (Supplementary materials and downloadable database in ref. 31) database, which compiles global primary energy consumption (PEC) of all energy types and sources from 1820 to 2016. Previously, the 19th century period was described only by scarce or partial data, apart from a compilation by Victor Court[36] that was also considered for our work (See appendix of ref. 36 p. 216-224 for the database), and will be compared to ref. 31 and its supplementary materials in the following. Malanima's main paper[31] rightly focusses onto PEC up to 1913, putting them in a geographical and historical perspective. The epoch 2 period (1920-2016) was much better known than epoch 1 (1820-1920), which justifies concentrating on the latter. However, Malanima's database provide historical series for the whole period (1820-2016) for the 8 comprehensive types of primary energy sources:

1. food for humans
2. fuelwood (which includes all solid biofuels, crop residues and derivates from wood, e.g. charcoal)
3. fodder for working animals
4. coal (all solid fossil fuels, also including peat)
5. oil
6. natural gas
7. electricity (from water, wind, geo, Sun and other renewables)
8. nuclear

Since these data relate to primary energy, electricity from coal (resp. nuclear or other fossil sources) will appear as spent coal (resp. nuclear …) and be accounted as coal's thermal energy content. Food, fuelwood and fodder (1 to 3) are traditional energy sources; we call items 4 to 8 "modern energy sources".

The energy crisis in the seventies and concerns about global warming prompted the survey of fossil fuel consumption in the long run, as well as new fossil free alternatives. Etemad and Luciani published[47] in 1991 an estimate of modern energy consumption since 1800 that has been used as a reference/basis by all authors afterwards. Fuelwood – solid biofuels – were discarded by most economic surveys up to the 2010s (being either invisible in trade or plainly ignored by surveys), although wood was a key energy resource. This is especially true during

the first century of the industrial revolution, when wood fueled e.g. a major part of the 19th century development of the USA[48] (cf. data on p.341), a would-be superpower. Smil[37] added fuelwood (biomass) consumption since 1800, based on the work of Fernandes et al[49] who assessed the use of biofuels from 1850 to 2000. Ref. 49 is the original source of most recent estimate of fuelwood/biofuel energy use. However, Fernandes et al provide a mass of consumed biofuels – wood, wood derivatives and crop residues - by macro-area (different geographical divisions than Malanima's). A conversion coefficient is necessary to convert those masses into their primary energy content. Malanima used 12.5 MJ/kg (3000 kcal/kg) for wood, which corresponds to wood with about 33% humidity. We come back to this a little later.

Up to the collective work of Kander, Malanima and Warde[24], most published estimates of energy consumption did not include food nor fodder. The inclusion of food – energy directly consumed by human workers – is crucial for a physical understanding of the role of energy in the economy, as previously claimed in §3. The same is true for fodder – energy necessary for draft animals to fuel their [horse] power – and fuelwood, which was the main energy source before the fossil fuels era. When draft animals replace humans, or when coal-fired locomotives replace horse traction, the economic impact must be assessed together with the increase of consumption that these changes induce.

Before the inclusion of traditional energy sources into the aggregate consumption, the productivity of energy ($A = \dfrac{y}{\varepsilon}$ in §3.2) showed a U-shape vs time on the long run – or an inverse U-shape for energy intensity of GDP ($= \dfrac{1}{A} = \dfrac{\varepsilon}{y}$) seen e.g. p. 350 of ref. 48. The inclusion of biofuels, fodder and food effectively levels the first part of the historical curve as seen at a global level in Fig. A1. A similar feature was also observed for energy intensity in the USA[50], where the hitherto much commented increase of energy intensity disappears and reverses into a steady decrease when traditional energies are included. Whereas some individual countries still exhibit inverse U-shape for time series of energy intensity, it seems that this is a geographical bias: energy intensity never notably increases when consumption is averaged out on large enough areas (energy productivity never decreases). In the European case[51], a similar but partial leveling of energy intensity was obtained by integrating international trade, i.e. subtracting from the UK and German account the energy spent for heavy industrial products sent abroad.

Let us expand on the methodology of traditional fuel estimates. Whereas usage of modern sources of energy (4 to 8) has been accounted for and can be known at the level of individual countries, the consumption of those traditional sources of energy (1 to 3) has been estimated/assessed by Malanima[31] on a macro-area basis (8 geographical macro-areas: Western Europe, Eastern Europe, etc.) as per capita averages. Apart from well tabulated modern energy consumption in the last 100 years, most data are assessed on a decennial basis, or every 5 years[47], and later interpolated on a yearly basis. For details, we refer the reader to Malanima's work, which encompasses/compiles many bibliographical, industrial (e.g. BP) and administrative (e.g. FAO) sources. Malanima also discusses differences with previous long-run estimates especially for fuelwood and compares his aggregate data with other available series. Victor Court also published a complete historical series covering 1800-2014 and the same categories of energy sources, but his work[36] is not cited in Malanima's[31]. Figure A2 compares both aggregate data sets.

Figure A2 shows that data from ref. 31 and 36 are very similar for the whole 20th and 21st centuries, and agree with other sources e.g. BP[10]. However, a noticeable relative gap shows up in the 19th century in Fig. A2. Separate comparisons of the eight energy types reveals that differences between the two sets come essentially from fuelwood. Figure A3 compares specifically those two energy resources.

As previously mentioned, Malanima[31], who does not cite Court[36], compared his fuelwood data with Smil's[37] and noticed that these were much higher than his in the 19th century. The trouble with Court's or Smil's data for biofuels energy consumption – very similar, as shown in Fig. A3a – lies in the per capita numbers, shown in Fig. A4. We remark that Smil's or Court's data imply a decrease of per capita biofuel consumption from 1820 to 1880, at odds with what was happening in the USA, and in numerous other colonial regimes. Moreover, in per capita figures, Court's average global biofuel energy consumption appears higher than the French total PEC per capita, which is very improbable, France being one of the dominant power in the beginning of the period and at the same time consuming mainly wood for heating and industry.

As a whole, like Malanima, we conclude that Smil or Court estimated excessive energy quantities for traditional sources. Concerning food, however, there is a very good agreement between Malanima and Court. For this reason, we retain the data of ref. 31. We can take into account the uncertainty of fuelwood energy figures by using intervals instead of isolated numbers: the curve of Fig. 1, reproduced with error margins in Fig. A5 associated to a 10-50% humidity range, can be seen to shift only gently from this assumption, so such adjustments do not jeopardize our main messages.

A.2 Global GDP

Measurement of Gross National Products, then Gross Domestic Products, began during the 1930s in the USA, and generalized progressively to other nations after WWII. Nowadays several international organization (the World bank, OECD, …) provide data for almost every country, which are the basis of a global world product[52].

Gross domestic product (GDP) for a geographical entity using only one currency (e.g. one country) during a short period (one year) is nearly merely a matter of accounting. "GDP is a monetary measure of the market value of all the final goods and services produced in a specific time period"[53]. However, when a complex ensemble of countries, using many different currencies, is surveyed on the long run, the accounting unit need to be corrected[54] for fluctuating exchange rates, inflation and differences in local purchasing power of specific moneys. After these diverse recalibrations of the measurement scale, GDP data are then said to be expressed in a constant currency, usually US$ of a given year, at parity of purchasing power.

For period predating the existence of national accounting, historians try to determine first the income of an average individual in the said country and time. Per capita GDP data are then combined with population data (taken from population census, or from estimates for the periods when no population census was available) to retrieve national GDP, and then global GDP. Within about 1%, there is a consensus over world population estimates from the beginning of the 19th century. We have been using Malanima's population series[31].

Angus Maddison[34] published in 2008 a first estimate of the world product, or global GDP, at selected years since year since the beginning of the Common Era, and for our need: 1820, 1870, 1900, 1913, 1940 and every year since 1950. After his passing away in 2010, his colleagues

and followers at University of Groningen continued and improved the work in the so-called "Maddison Project"[55]. New global estimates of GDP were recently released by the team[56,57]. The database provides GDP per capita for nearly every country in US$ 2011 at Parity of Purchasing Power (US$2011 PPP) at selected years. After 1820, data begin to be denser in the (year, country) table. Two different GDP per capita estimates are given: RGDPNA, preferred by the authors for growth measurements, and CGDP, preferred to compare the level of development in different countries. Another database provides aggregate figures for continental regions and the entire world economy[58], shown in Fig. A6.

To study the relations between energy consumption and GDP, yearly data are requested, or at least data at a regular decennial or half-decennial frequency. Thus, several scholars used the Maddison Project databases by year and country to establish yearly global GDP data, from 1870[59], or 1820[31]. Gapminder[60], a well-established statistician enterprise founded by the late Hans Rosling, tackled the task of completing Maddison project database using also other sources: they recently released a new set of data[61]. The new global GDP per capita estimate is labeled gdppc_cppp. Multiplied by the world population, it provides a series of world GDP, shown in Fig. A6. Figure A6 compiles the available world GDP data, showing a convergence of numbers for the part after 1950, and very similar figures for the rest of the studied period. Minute differences observed between series in Fig. A6 would not change the appearance of Fig. 1, and the conclusion of the paper. We retained Gapminder series since it is the only one available on a yearly basis.
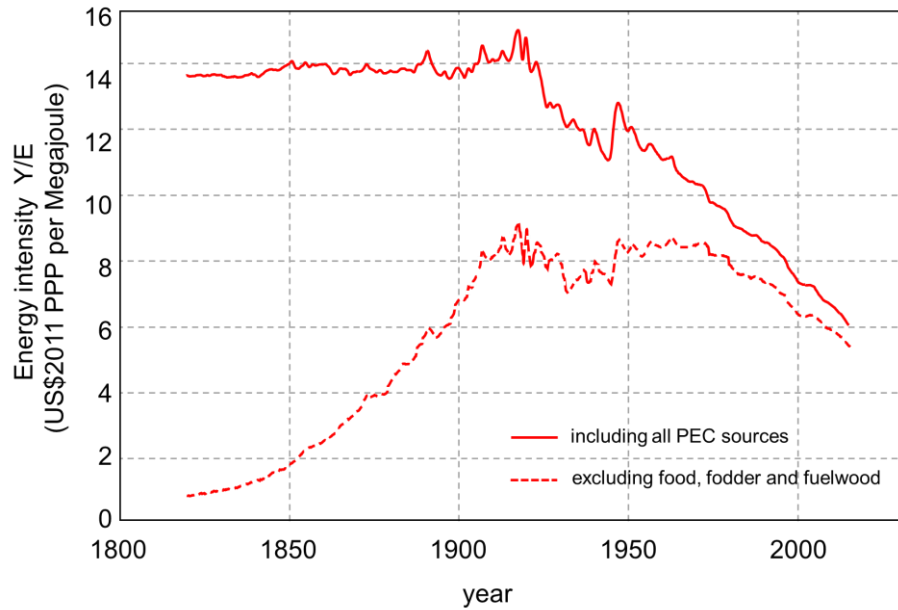
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Fig. A1.

Fig. A1: Effect of the inclusion of traditional energy sources (items 1 to 3, food, fuelwood and fodder) on Energy Intensity – i.e. the ratio of primary energy consumption (PEC) to GDP; same data as Fig. 1. Without traditional energies, the curve shows an inverse U-shape. This shape, found very often at the national level, especially when energy sources are only partially accounted for, was the subject of many comments and several theories. When traditional energies are included, plain line, the left part of the curve is levelled.
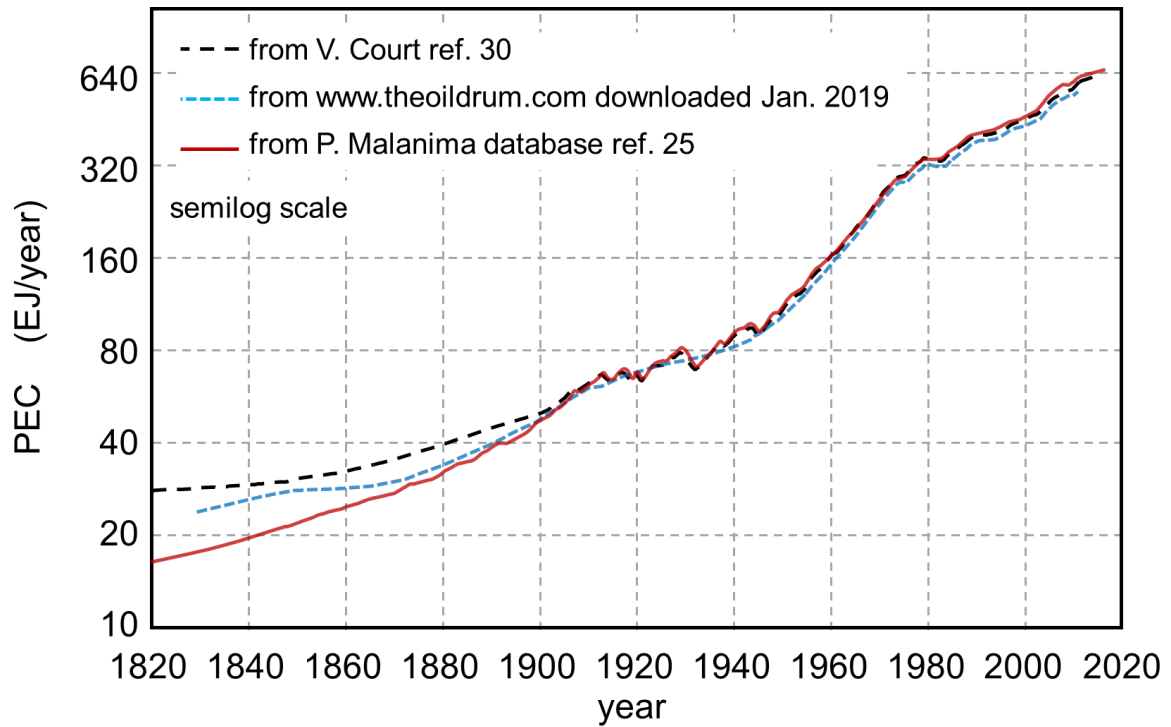
Fig. A2.

Fig. A2: Global Aggregate Energy Consumption from different database - EJ/year. Comparison of Paolo Malanima's database[31], red line, with Victor Court's data[36], dashed black line, and another dataset available on the web, dotted blue line. Energy in EJ/year, semi-log scale. Court's and Malanima's data are very similar in the 20th and 21st centuries. Differences in the 19th century come mainly from fuelwood. Cf. §A.1 and Fig. A3 and A4.
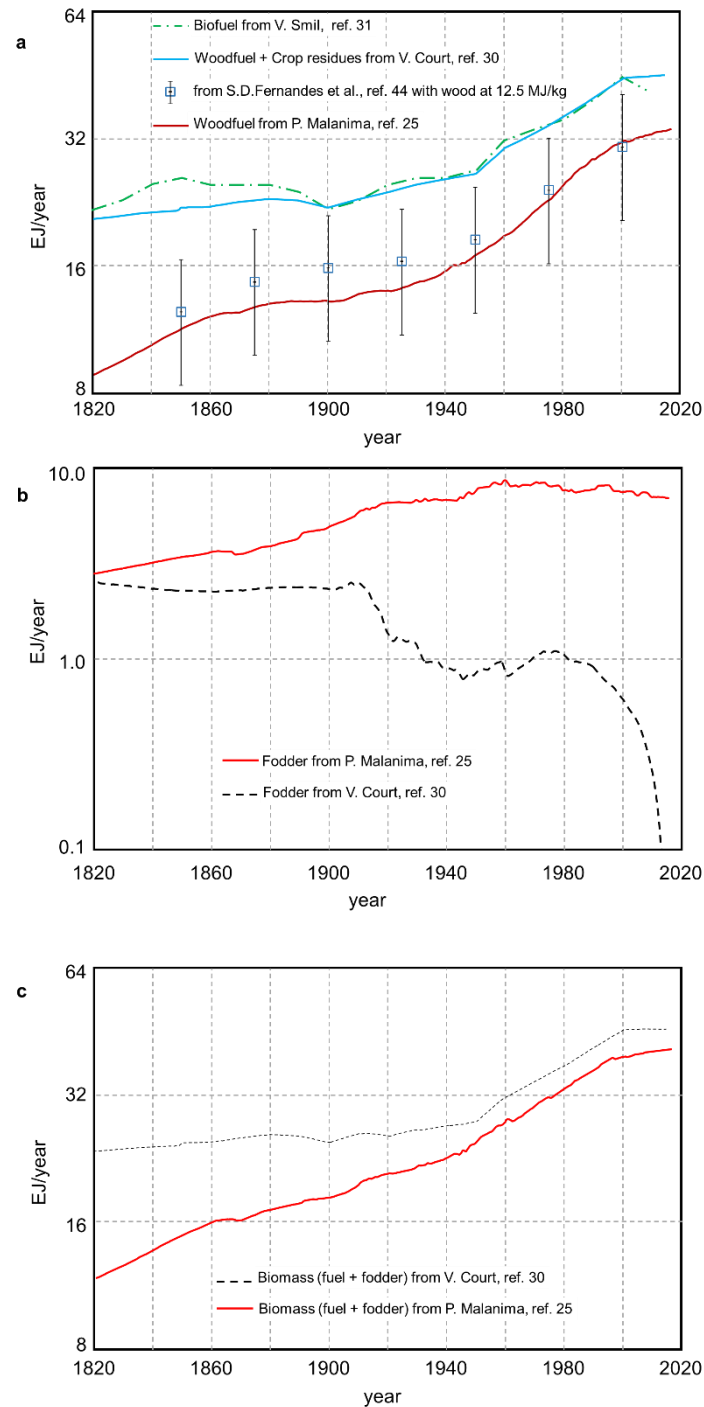
Fig. A3.

Fig. A3: comparison of traditional biomass energy data from Malanima[31] and Court[36]. **a**: solid biofuels series, also compared to Smil's estimate[37] (which only considered solid fuels, not fodder) and to Fernandes's[49] mass of biofuels converted with 12.5 MJ/kg (and a proportional error bar of +/- 20%). **b**: fodder series. **c**: sum of solid fuels and fodder. Court's data are very similar to Smil's for solid fuels (**a**), much higher than Malanima's, whereas the gap between Court's and Malanima's series is reversed for fodder (**b**).
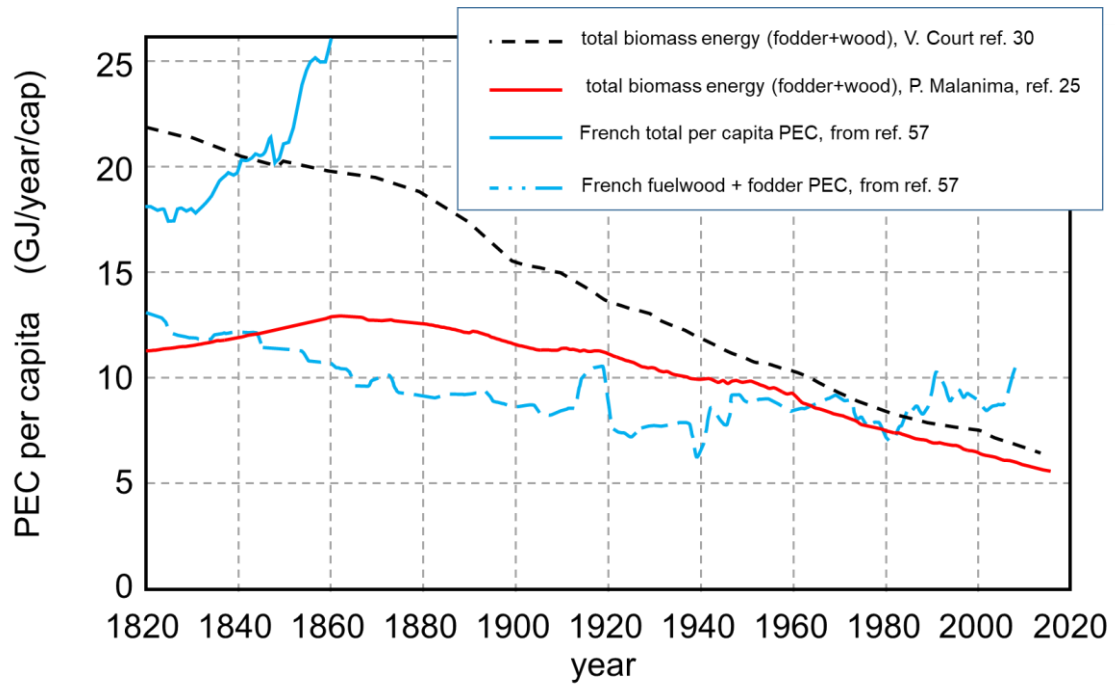
Fig. A4.

Fig. A4: traditional biomass energy per capita. Global data from ref. 31 and 36. French data from the Energy History Project[62]. Court's average global bioenergy consumption per capita is higher than French consumption in the early 19th century, which appears very improbable.
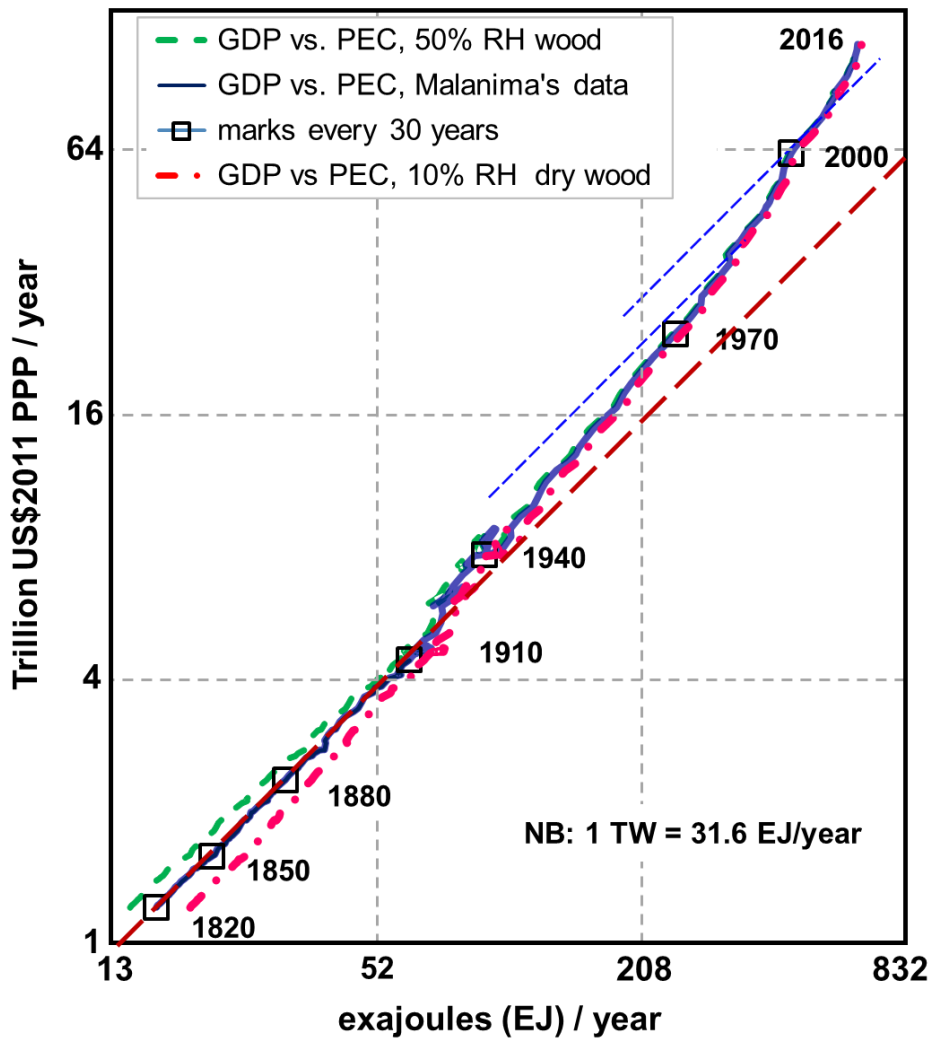
Fig. A5.

Fig. A5: reproduces Fig. 1, allowing for uncertainty in fuelwood energy. This uncertainty is expressed as variability in the average relative humidity [RH] retained to convert Fernandes *et al.*[49] biomass data into energy. Using about 33% RH in wood, which translates into 12.5 MJ/kg wood, Malanima established the aggregate energy consumption series that has been used in the main document. GDP (see §A.2 for a discussion of the data) is plotted vs PEC. The green (respectively red) dashed (resp. point-dashed) line is the same GDP vs aggregate energy using 50% RH, i.e. 9.5 MJ/kg (resp. 10% RH, i.e. 17 MJ/kg). The relative horizontal gap between green and red lines is about ±20% around the Malanima's series. The gap shrinks with time, and growth of modern energy consumption

Fig. A6.

Fig. A6: time series of world GDP, from different sources. A6a: semi logarithmic scale, from 1800 to 2016 ; A6b: linear scale, 1950-2016. All sources, except for Maddison 2008, use US$2001 PPP as unit. For Maddison 2008, we rescaled the original data expressed in US$ 1990 dollars PPP to US$2011, using a 1.57 factor correcting inflation, obtained from US inflation data[52]. Apart from a proportional gap of about 10% – which may be due to an incorrect rescaling from our side – between Maddison 2008 and the three other sets of numbers, a general agreement is found. Note that world bank data begin in 1990.

B. *modeling*

In appendix B, we go into more details about the resolution of differential equations employed in the main document. In the balanced growth case ($A = 1$) corresponding to epoch 1, we solve exactly the system of differential equations for a single capital factor $\mathcal{J}$ in addition to PEC in the Cobb-Douglas function; we briefly discuss the case of two capital factors H and K. When A is variable, as in the case of epoch 2, the simplified form used in §3.2 (only energy was kept) is justified here by solving numerically a system with a single capital $\mathcal{J}$ and PEC.

B.1 *balanced growth*

Here we give the basic elements to perform the macroeconomic modeling presented in the main document. We follow the spirit and basic mathematics of macroeconomic textbooks[12-14]. Our first attempt used Labor L and capital K as production factors – as usually introduced in textbooks – together with PEC E: the production function is then a function of these three factors:

$$Y \propto K^\alpha L^\beta E^\gamma \qquad \text{(B.1)}$$

However, the linearity between Y and E (such as in epoch 1), considered together with the investment equation $\frac{dK}{dt} = s_K Y - \delta_K K$ (already seen in Eq. (2) [in appendix B, equations are marked with a B, as above for Eq. (B.1)]) ends up giving a minute role to L, in accordance with many studies in the past[16-18]. The disappearance of workers from the equation is counterintuitive and suggests an awkward relation to reality and entailed policies. Two steps heal this problem, as explained in the article. First of all, every human contribution is divided on the one hand into its mere brute force, unskilled labor which only depends on the energy converted by muscles, and on the other hand into a cultural part, prone to improve due to education and knowledge growth. Mankiw et al[38] provided such a theory, where a human capital H appears, at par with the physical, usual capital K: it is thus not a disappearance of Labor but a skill-based redefinition. A second step consists in integrating unskilled labor in the aggregate PEC. The combination of both steps produces Eq. (1), repeated here:

$$\frac{Y}{Y_{1820}} = A \left(\frac{K}{K_{1820}} \right)^\alpha \left(\frac{H}{H_{1820}} \right)^\beta \left(\frac{E}{E_{1820}} \right)^\gamma \qquad \text{(1)}$$

with $\alpha + \beta + \gamma = 1$. Equation (1) is combined with Eq. (2)

$$\frac{dK}{dt} = s_K Y - \delta_K K \ \& \ \frac{dH}{dt} = s_H Y - \delta_H H \qquad \text{(2)}$$

where Y, K and H need to be normalized. Using $\mathcal{Y} = \frac{Y}{Y_{1820}}$, $\mathcal{K} = \frac{K}{K_{1820}}$, $\mathcal{H} = \frac{H}{H_{1820}}$, $\mathcal{E} = \frac{E}{E_{1820}}$

it is straightforward to obtain the system:

$$\mathcal{Y} = A\mathcal{K}^{\alpha}\,\mathcal{H}^{\beta}\,\mathcal{E}^{\gamma} \qquad\qquad \text{(B.2a)}$$

$$\frac{d\mathcal{K}}{dt} = \frac{s_K}{\beta_K}\mathcal{Y} - \delta_K\,\mathcal{K} \qquad\qquad \text{(B.2b)}$$

$$\frac{d\mathcal{H}}{dt} = \frac{s_H}{\beta_H}\mathcal{Y} - \delta_H\,\mathcal{H} \qquad\qquad \text{(B.2c)}$$

with $\beta_K = \dfrac{K_{1820}}{Y_{1820}}$ and $\beta_H = \dfrac{H_{1820}}{Y_{1820}}$ the physical and human capital to production ratios at the

beginning of the studied period. In the following, we will write $\sigma_K = \dfrac{s_K}{\beta_K}$ and $\sigma_H = \dfrac{s_H}{\beta_H}$. With

PEC growing exponentially as $\exp[gt]$, it follows logically from the investment equations (2)

that $\mathcal{K}$ and $\mathcal{H}$ will grow when energy is consumed. In macroeconomic models with labor L,

it is usual to set exogenously an exponential growth of L, approximated by the global
population. In our views, since energy is considered as the primary input, it suffices to set PEC
growth. If we define an exponential growth of energy *per capita*, as $\exp[g_\varepsilon t]$, it then implies

that population L grows as $\exp[(g - g_\varepsilon)t]$. As usual in macroeconomic textbooks (e.g. ref.

12-14), it is practical to study ratios of production and capital(s) to PEC that are noted $y_\varepsilon = \dfrac{\mathcal{Y}}{\mathcal{E}}$

, $k_\varepsilon = \dfrac{\mathcal{K}}{\mathcal{E}}$, $h_\varepsilon = \dfrac{\mathcal{H}}{\mathcal{E}}$. Eq. (B.2a) becomes, still with $\alpha + \beta + \gamma = 1$.

$$y_\varepsilon = Ak_\varepsilon^{\alpha}h_\varepsilon^{\beta} \qquad\qquad \text{(B.3a)}$$

Eq. (B.2b) can be rewritten

$$\frac{dk_\varepsilon}{dt} = \frac{d\mathcal{K}}{\mathcal{E}\,dt} - \frac{\mathcal{K}}{\mathcal{E}^2}\frac{d\mathcal{E}}{dt} = \frac{\sigma_K\mathcal{Y} - \delta_k\mathcal{K}}{\mathcal{E}} - \frac{\mathcal{K}}{\mathcal{E}}\frac{d\mathcal{E}}{\mathcal{E}\,dt}$$

which simplifies into

$$\frac{dk_\varepsilon}{dt} = \sigma_K y_\varepsilon - (\delta_K + g)k_\varepsilon \qquad\qquad \text{(B.3b)}$$

and the same holds for Eq. (B.2c)

$$\frac{dh_\varepsilon}{dt} = \sigma_H y_\varepsilon - (\delta_H + g)h_\varepsilon \qquad\qquad \text{(B.3c)}$$

For a constant $A = 1$, and for constant values of parameters (saving and depreciation rates), the
system of three equations (B.3) can be solved, at least numerically. The resolution of (B.3) with
the two types of capital is beyond the scope of this appendix. The simpler case with a unique

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capital factor is used in §3.1 for a simpler discussion of the dynamics of capital formation, and solved exactly in the following to explicit the steady-state *balanced growth* conditions.

We present here the basics of a simplified system with only one capital noted $\mathcal{J}$ in normalized form. $\mathcal{J}$ and $\mathcal{Y}$ are linked by Eqs. (3) and (4), reproduced here:

$$\mathcal{Y} = \mathcal{J}^\alpha \, \mathcal{E}^{1-\alpha} \tag{3}$$

$$\frac{d\,\mathcal{J}}{dt} = s\,\mathcal{Y} - \delta\,\mathcal{J} \tag{4}$$

For constant values of s and δ, this system is exactly solvable; most textbooks present a discussion of the fixed point only, but an exact solution of the dynamics can also be found elsewhere[13]. A solution can be reached rather easily using production to energy ratio and capital to energy ratio, $y_\mathcal{E} = \dfrac{\mathcal{Y}}{\mathcal{E}}$ and $j_\mathcal{E} = \dfrac{\mathcal{J}}{\mathcal{E}}$, with which the system is written:

$$y_\mathcal{E} = j_\mathcal{E}^{\,\alpha} \tag{B.4a}$$

$$\frac{dj_\mathcal{E}}{dt} = sy_\mathcal{E} - \left(\delta + g\right) j_\mathcal{E} \tag{B.4b}$$

Combining both equations, we derive a differential equation on $j_\mathcal{E}$ alone,

$$\frac{dj_\mathcal{E}}{dt} = sj_\mathcal{E}^{\,\alpha} - \left(\delta + g\right) j_\mathcal{E} \tag{B.5}$$

A few trivial lines bring:

$$\frac{1}{1-\alpha} \frac{d\left(j_\mathcal{E}^{\,1-\alpha}\right)}{dt} + \left(\delta + g\right) j_\mathcal{E}^{\,1-\alpha} = s \tag{B.6}$$

This first order equation on $j_\mathcal{E}^{\,1-\alpha}$ is simplified by definition of a new variable $\varphi[t] = j_\mathcal{E}^{\,1-\alpha} e^{(1-\alpha)(\delta+g)t}$, which obeys a trivial equation:

$$\frac{d\varphi}{dt} = \left(1 - \alpha\right) s.e^{(1-\alpha)(\delta+g)t} \tag{B.7}$$

easily integrated as

$$\varphi[t] = 1 + \frac{s}{\delta+g}\left(e^{(1-\alpha)(\delta+g)t} - 1\right) \tag{B.8}$$

since the initial condition (in 1820) is $\varphi[0] = 1$. It is now straightforward to express $j_\mathcal{E}$

$$j_{\varepsilon}[t] = \left(e^{-(1-\alpha)(\delta+g)t} + \frac{s}{\delta+g}\left(1 - e^{-(1-\alpha)(\delta+g)t}\right) \right)^{\frac{1}{1-\alpha}} \quad (B.9)$$

Capital to energy ratio tends exponentially to a constant $j_{\varepsilon}^{*} = \left(\dfrac{s}{\delta+g} \right)^{\frac{1}{1-\alpha}}$ with characteristic

time $\dfrac{1}{\delta+g}$. From j_{ε} and (B.4a), one easily obtains the production to energy ratio:

$$y_{\varepsilon}[t] = \left(e^{-(1-\alpha)(\delta+g)t} + \frac{s}{\delta+g}\left(1 - e^{-(1-\alpha)(\delta+g)t}\right) \right)^{\frac{\alpha}{1-\alpha}} \quad (B.10)$$

which converges with a characteristic time $\dfrac{1}{\alpha(\delta+g)}$ to

$$y_{\varepsilon}^{*} = \left(\frac{s}{\delta+g} \right)^{\frac{\alpha}{1-\alpha}} \quad (B.11)$$

In the asymptotic regime, called "balanced growth" in macroeconomic textbooks[12-14], capital and production are proportional to PEC and grow exponentially according to $\exp[gt]$.

Their asymptotic ratio is $\dfrac{J}{Y} = \dfrac{j_{\varepsilon}^{*}}{y_{\varepsilon}^{*}} = \dfrac{\left(\dfrac{s}{\delta+g} \right)^{\frac{1}{1-\alpha}}}{\left(\dfrac{s}{\delta+g} \right)^{\frac{\alpha}{1-\alpha}}} = \dfrac{s}{\delta+g}$ which is fixed when the parameters δ,

s and g are constant. Variation of these parameters, on the other hand, would produce an evolution of the capital to production ratio: its mathematical study calls for a new, more complex resolution of (B.4a) and (B.4b), which will not be tackled here. We will only give a brief discussion of epoch 1, extending the resolution of (B.4) to the case of the system (B.3) of three equations.

Epoch 1 is characterized by such a balanced growth dynamics. Similarly to system (B.4), (B.3b) and (B.3c) imply the convergence of k_{ε} to a fixed point $k_{\varepsilon}^{*} = \left(\dfrac{\sigma_K}{g+\delta_K} \right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\dfrac{\sigma_H}{g+\delta_H} \right)^{\frac{\beta}{1-\alpha-\beta}}$, while h_{ε} converges to $h_{\varepsilon}^{*} = \left(\dfrac{\sigma_H}{g+\delta_H} \right)^{\frac{1-\alpha}{1-\alpha-\beta}} \left(\dfrac{\sigma_K}{g+\delta_K} \right)^{\frac{\alpha}{1-\alpha-\beta}}$.

Compared to the simpler case with one capital, the dynamics will be made richer due to several different characteristic times of convergence. Since empirical data imply that y_{ε} is constant, $y_{\varepsilon} = \dfrac{Y}{Y_{1820}} \dfrac{E_{1820}}{E} = 1$, the actual asymptotic relations can be expressed, without much surprise,

by simpler relations, namely: $k_\varepsilon^* = \dfrac{\sigma_K}{g+\delta_K}$ and $h_\varepsilon^* = \dfrac{\sigma_H}{g+\delta_H}$. Reintroducing the non-normalized K and H, as well as Y and E, remembering $\beta_K = \dfrac{K_{1820}}{Y_{1820}}$ and $\sigma_K = \dfrac{s_K}{\beta_K}$, we retrieve $\dfrac{K}{Y} = \dfrac{s_K}{\delta_K + g}$ the fixed-point ratio of physical capital to production, which goes also here with the same relation for the human capital $\dfrac{H}{Y} = \dfrac{s_H}{\delta_H + g}$.

B.2 case of a variable A in epoch 2

In epoch 2, there is no more linearity of GDP with PEC. This can be modeled with a variable A in Eq. (1). In §3.2, this situation is studied in a very simplified version where the capital factors K and H are put aside, and where E only is kept: see Eq. (5) to (7). Here we consider a compromise solution, extending the single capital model (Eq. (3) and (4)) to the case of a variable residual A.

$$\mathcal{Y} = A \mathcal{J}^\alpha \mathcal{E}^{1-\alpha} \qquad (B.12a)$$

$$\frac{d\mathcal{J}}{dt} = s\mathcal{Y} - \delta\mathcal{J} \qquad (B.12b)$$

where we used normalized quantities $\mathcal{Y} = \dfrac{Y[t]}{Y[t_0]}$, $\mathcal{J} = \dfrac{J[t]}{J[t_0]}$, etc. The simplified case treated in §3.2 can be retrieved by taking $\alpha = 0$. As previously, we anticipate that A is related to PEC by

$$\frac{dLog[A]}{dt} = \chi_\alpha \mathcal{E} \qquad (B.12c)$$

with a constant χ_α. We have $\chi_{\alpha=0}$, the value of χ_α for $\alpha = 0$, that corresponds to χ obtained above in §3.2. We take $\mathcal{E}$ as exogenously defined and growing exponentially according to $\exp[g(t-t_0)]$ from a time $t_0 = 0$. (B.12c) is directly transformed into $A[t] = \exp[\int_{t_0}^t \chi_\alpha \mathcal{E} dt] = \exp[\int_0^t \chi_\alpha \exp[gt] dt]$. To solve numerically the system (B.12), we go through the same process as above: we study $y_\varepsilon = \dfrac{\mathcal{Y}}{\mathcal{E}}$ and $j_\varepsilon = \dfrac{\mathcal{J}}{\mathcal{E}}$ which obey the system of equations (B.13)

$$y_\varepsilon = \exp[\frac{\chi_\alpha}{g} \exp[gt]] j_\varepsilon^\alpha \qquad (B.13a)$$

$$\frac{dj_{\mathcal{E}}}{dt} = sy_{\mathcal{E}} - (\delta + g) j_{\mathcal{E}} \qquad\qquad\qquad \text{(B.13b)}$$

At the beginning of the period, at time t_0, we suppose that the conditions of balanced growth are met: this implies that $\dfrac{j_{\mathcal{E}}[t_0]}{y_{\mathcal{E}}[t_0]} = \dfrac{s}{\delta + g}$. But we also have $y_{\mathcal{E}}[t_0] = j_{\mathcal{E}}[t_0] = 1$. Thus, we have $s = \delta + g$. In a last simplifying step, we define the temporal scale such that s equals 1.

Figure B1 shows the evolution of $y_{\mathcal{E}} = \dfrac{\mathcal{Y}}{\mathcal{E}}$ $j_{\mathcal{E}} = \dfrac{\mathcal{J}}{\mathcal{E}}$ and A for $\alpha = 0.5$. On B, we see that all quantities scale like an exponential function of $\int_{t_0}^{t} \mathcal{E}\,dt$ within less than a percent. The residual A is such through our initial hypothesis ($A[t] = \exp[\int_{t_0}^{t} \chi_\alpha \mathcal{E}\,dt]$). Figure B1b proves for $\alpha = 0.5$ that this hypothesis results in the expected behavior for $y_{\mathcal{E}} = \dfrac{\mathcal{Y}}{\mathcal{E}}$ which was found to scale like $\exp[\int_{t_0}^{t} \mathcal{E}\,dt]$ (see §3.2, especially Fig. 2). The parameter $\chi_{0.5}$ in Fig. 7 was chosen to provide a factor 2.3 increase of $y_{\mathcal{E}} = \dfrac{\mathcal{Y}}{\mathcal{E}}$ as in the case of the historical period 1820-2016. The behavior of $y_{\mathcal{E}}$ when α varies is very stable. Its limited range can be seen for example in Fig. B2a where the same functions as Fig. B1b are plotted, here for $\alpha = 1$ with $\chi_1 = 0.0011$. In Fig. B2b, we plot in semilog scale the production to energy ratio $y_{\mathcal{E}} = \dfrac{\mathcal{Y}}{\mathcal{E}}$ for the two values of α : a minute gap, smaller than 3%, is found between the two curves.

We performed simulations for other values of α in the range 0 to 1, with very minor changes in the results. This shows that the hypothesis $A[t] = \exp[\int_{t_0}^{t} \chi_\alpha \mathcal{E}\,dt]$ with a constant χ_α correctly renders the historical period 1820-2016, whatever the value of α in Eq. (B.12a).

If one would reproduce the final discussion of §3.2 in the more general case of a model including a capital factor $\mathcal{J}$, similar conclusions would be reached.
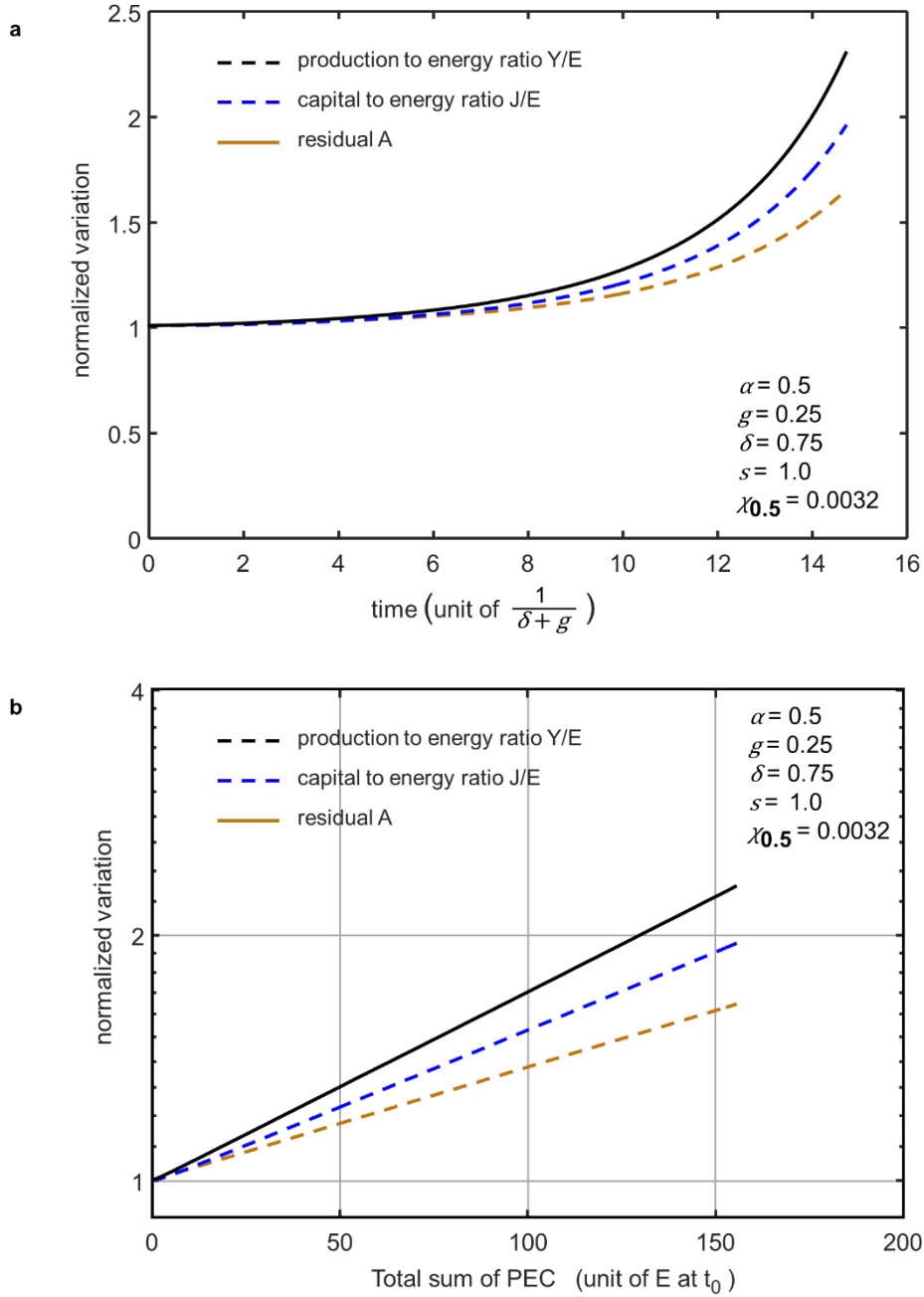
Fig. B1.

Fig. B1: Production to energy ratio $y_\varepsilon = \dfrac{\mathcal{Y}}{\mathcal{E}}$, the same ratio for its capital factor $j_\varepsilon = \dfrac{\mathcal{J}}{\mathcal{E}}$ and the residual A, calculated for $\alpha = 0.5$ according to Eq. (B.13). **a**: plotted on a linear scale as a function of time, normalized to $\dfrac{1}{\delta + g}$, the effective depreciation time. **b**: plotted on a semilog scale as a function of the total time integral of PEC, normalized to PEC at $t_0 = 0$, with the same time unit as B1a.
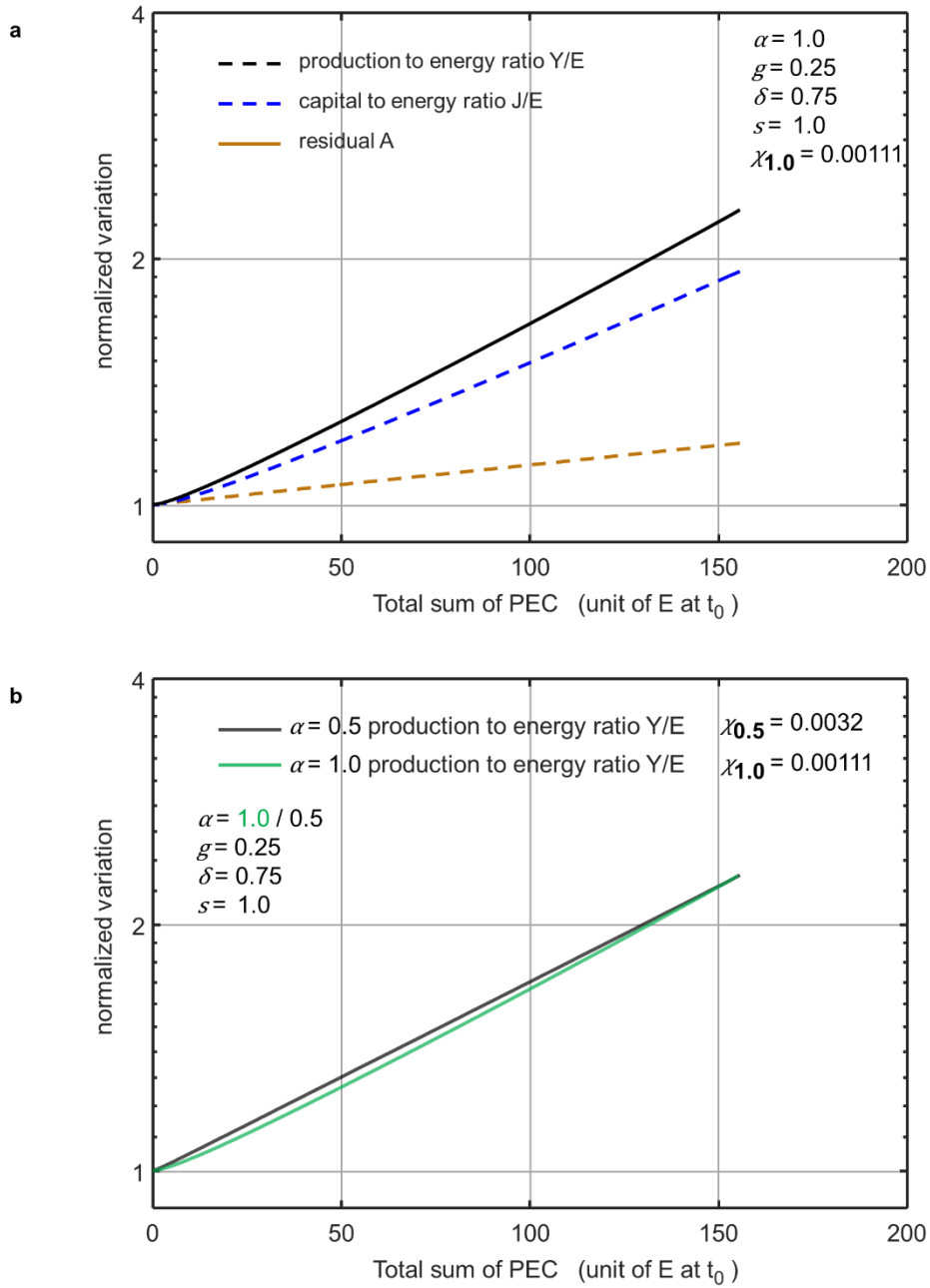
Fig. B2.

Fig. B2: Production to energy ratio $y_\varepsilon = \dfrac{y}{\varepsilon}$, the same ratio for its capital factor $j_\varepsilon = \dfrac{j}{\varepsilon}$ and

the residual A, calculated for $\alpha = 1$, according to Eq. (B.13). **a**: plot on a semilog scale as a function of the total sum of PEC, same abscissa unit (energy) as Fig. B1b. **b**: comparison of

$y_\varepsilon = \dfrac{y}{\varepsilon}$ on a semilog scale for the two values of α ; both curves are close to a straight line.

References:

[1] Cleveland, C. J., Costanza, R., Hall, C. A., & Kaufmann, R. Energy and the US economy: a biophysical perspective. *Science* **225**, 890-897 (1984).

[2] Smil, V. *Energy and civilization: a history* (MIT Press, Cambridge, MA, 2017).

[3] Keeling, C. D. Is carbon dioxide from fossil fuel changing man's environment? *Proc. Am. Phil. Soc.* **114**, 10-17 (1970).

[4] Sawyer, J. S. Man-made carbon dioxide and the "greenhouse" effect. *Nature* **239**, 23-26 (1972).

[5] Kos, C., Shammugam, S., Jülch, V., Nguyen, H.-T. & Schlegl, T. "Levelized Cost of Electricity Renewable Energy Technologies" (Fraunhofer Institute for Solar Energy Systems ISE, 2018; www.ise.fraunhofer.de/content/dam/ise/en/documents/publications/studies/EN2018_Fraunhofer-ISE_LCOE_Renewable_Energy_Technologies.pdf).

[6] Kätelhön, A., Meys, R., Deutz, S., Suh, S. & Bardow, A. Climate change mitigation potential of carbon capture and utilization in the chemical industry. *Proc. Natl. Acad. Sci. U.S.A.* **116**, 11187-11194 (2019).

[7] "The European Green Deal" (European Commission, COM/2019/640 final, 2019; available at https://eur-lex.europa.eu/).

[8] Jacobson, M. Z., Delucchi, M. A., Bauer, Z. A., Goodman, S. C., Chapman, W. E., Cameron, M. A., Bozonnat, C., Chobadi, L., Clonts, H. A., Enevoldsen, P., Erwin, J. R., Fobi, S. N., Goldstrom, O. K., Hennessy, E. M., Liu, J., Lo, J., Meyer, C. B., Morris, S. B., Moy, K. R., O'Neill, P. L., Petkov, I., Redfern, S., Schucker, R., Sontag, M. A., Wang, J., Weiner, E. & Yachanin, A. S. 100% clean and renewable wind, water, and sunlight all-sector energy roadmaps for 139 countries of the world, *Joule* **1**, 108-121 (2017).

[9] Hansen, K. Breyer, C. & Lund, H. Status and perspectives on 100% renewable energy systems. *Energy* **175**, 471-480 (2019).

[10] "BP Statistical Review of World Energy 2020", 69th edition (British Petroleum, 2020; https://www.bp.com/content/dam/bp/business-sites/en/global/corporate/pdfs/energy-economics/statistical-review/bp-stats-review-2020-full-report.pdf).

[11] Masson-Delmotte, V., Zhai, P., Pörtner, H.-O., Roberts, D., Skea, J., Shukla, P. R., Pirani, A., Moufouma-Okia, W., Péan, C., Pidcock, R., Connors, S., Matthews, J. B. R., Chen, Y., Zhou, X., Gomis, M. I., Lonnoy, E., Maycock, T., Tignor, M. & Waterfield, T. (eds.), *"Global Warming of 1.5°C. An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty"* (IPCC, 2018).

[12] Aghion, P. & Howitt, P. W. *The economics of growth* (MIT press, Cambridge MA, 2008).

[13] Jones, C. & Vollrath, D. *Introduction to Economic Growth* (WW Norton, New York, 2013).

[14] Romer, D. *Advanced macroeconomics,* fourth edition (McGraw-Hill, New York, 2012).

[15] Kümmel, R., Strassl, W., Gossner, A. & Eichhorn, W. Technical progress and energy dependent production functions. *Zeitschrift für Nationalökonomie/Journal of Economics* **45**, 285-311 (1985).

[16] Kahraman, Z. & Giraud, G. How Dependent is Growth from Primary Energy? An Empirical Answer on OECD Countries. In *Energy & the Economy, 37th IAEE International Conference, June 15-18, 2014*. International Association for Energy Economics. (2014).

[17] Ayres, R. U. & Miller, S. M. The role of technological change. *J. Environ. Econ. Manag*. **7**, 353-371. (1980).

[18] Kümmel, R. The impact of energy on industrial growth. *Energy* **7**, 189-203 (1982).

[19] Costanza, R. Embodied energy and economic valuation. *Science* **210**, 1219-1224 (1980).

[20] Stern, D. I. Energy and economic growth in the USA: a multivariate approach. *Energy economics* **15**, 137-150 (1993).

[21] Ayres, R. U. & Warr, B. *The economic growth engine: How energy and work drive material prosperity* (Edward Elgar Publishing, Northampton, MA, 2010).

[22] Keen, S., Ayres, R. U. & Standish, R. A Note on the Role of Energy in Production. *Ecol. Econ.* **157**, 40-46 (2019).

[23] Ayres, R., & Voudouris, V. The economic growth enigma: Capital, labour and useful energy? *Energy Policy*, **64**, 16-28 (2014).

[24] Voudouris, V., Ayres, R., Serrenho, A. C., & Kiose, D. The economic growth enigma revisited: The EU-15 since the 1970s. *Energy Policy*, **86**, 812-832 (2015).

[25] Kraft, J. & Kraft A., On the Relationship Between Energy and GNP, *The Journal of Energy and Development*, **3**, 401-403 (1978).

[26] Yu, E. S. H. & Choi, J.-Y. The Causal Relationship between Energy and GNP: an international comparison, *The Journal of Energy and Development*, **10**, 249-272 (1985).

[27] Soytas, U. & Sari, R. Energy consumption and GDP: causality relationship in G-7 countries and emerging markets. *Energy economics*, **25**(1), 33-37 (2003).

[28] Ozturk, I., A literature survey on energy–growth nexus, *Energy Policy*, **38**, 340-349 (2010).

[29] Bruns, S. B., Gross, C. & Stern, D. I. Is there really Granger causality between energy use and output?. *The Energy Journal*, **35**(4) (2014).

[30] Stern, D. I., & Enflo, K. (2013). Causality between energy and output in the long-run. *Energy economics*, **39**, 135-146.

[31] Malanima, P. The limiting factor: energy, growth, and divergence, 1820–1913. *The Economic History Review* **73**, 486-512 (2020).

[32] Kander, A., Malanima, P. & Warde, P. Power to the people: energy in Europe over the last five centuries (Princeton University Press, Princeton, NJ, 2015).

[33] Fouquet, R. Lessons from energy history for climate policy: Technological change, demand and economic development. *Energy Res. Soc. Sci.* **22**, 79-93 (2016).

[34] Maddison, A. *The World Economy: Volume 1: A Millennial Perspective and Volume 2: Historical Statistics* (OECD Development Center Studies, Paris, 2006).

[35] Internet resources (see also appendix A) : https://www.gapminder.org/data/ ; https://www.rug.nl/ggdc/historicaldevelopment/maddison ; https://data.worldbank.org/indicator/

[36] Court, V. "Énergie, EROI et croissance économique dans une perspective de long terme", thesis, Université Paris Nanterre (2016).

[37] Smil, V. *Energy transitions: history, requirements, prospects* (ABC-CLIO, 2010).

[38] Mankiw, N. G., Romer, D., & Weil, D. N. A contribution to the empirics of economic growth. *Q. J. Econ.* **107**, 407-437 (1992).

[39] Piketty, T. *Capital in the Twenty-First Century* (Harvard University Press, Cambridge, MA, 2014).

[40] Romer, P. M. Endogenous technological change. *J. Political Econ*. **98**, 71-102 (1990).

[41] Kremer, M. Population growth and technological change: One million BC to 1990. *Q. J. Econ.* **108**, 681-716 (1993).

[42] Perez, C. *Technological revolutions and financial capital*. (Edward Elgar Publishing, Northampton, MA, 2003).

[43] Harchaoui, S. & Chatzimpiros, P. Energy, nitrogen, and farm surplus transitions in agriculture from historical data modeling. France, 1882–2013. *J. Ind. Ecol.* **23**, 412-425 (2019).

[44] Benisty, H. & Bercegol H., unpublished.

[45] Haberl, H., Wiedenhofer, D., Virág, D., Kalt, G., Plank, B., Brockway, P., Tomer Fishman, T., Hausknost, D., Krausmann, F., Leon-Gruchalsk, B., Mayer, A., Pichler, M., Schaffartzik, A., Sousa, T., Streeck, J. & Creutzig, F. A systematic review of the evidence on decoupling of GDP, resource use and GHG emissions, part II: synthesizing the insights. *Environmental Research Letters*, *15*(**6**), 065003 (2020).

[46] Vadén, T., Lähde, V., Majava, A., Järvensivu, P., Toivanen, T., Hakala, E. & Eronen, J. T. Decoupling for ecological sustainability: A categorisation and review of research literature. *Environmental Science & Policy* **112**, 236-244 (2020).

[47] Etemad, B. & Luciani, J. *World Energy Production 1800-1985* (Droz, Genève, 1991).

[48] Energy Information Administration (US), *Annual energy review 2011* (Government Printing Office, 2012).

[49] Fernandes, S. D. *et al.*, Global Biofuel Use, 1850-2000. *Global Biogeochemical Cycles* **21**(2), pp.1–15. (2007).

[50] O'Connor, P. A. & Cleveland, C. J. US energy transitions 1780–2010. *Energies* **7**(12), 7955-7993. (2014).

[51] Kander, A. *et al.*, International trade and energy intensity during European industrialization, 1870–1935. *Ecol. econ.* **139,** 33-44 (2017).

[52] https://data.worldbank.org/indicator/

[53] https://en.wikipedia.org/wiki/Gross_domestic_product

[54] The processes of recalibrating the accounting units employ thousands of economists worldwide. Detailing their work is not the purpose of this appendix.

[55] https://www.rug.nl/ggdc/historicaldevelopment/maddison/data/mpd_2013-01.xlsx, cited by ref. 31.

[56] Bolt, J., Inklaar, R., de Jong, H. & Van Zanden, J. L. Rebasing 'Maddison': new income comparisons and the shape of long-run economic development. *GGDC Research Memorandum*, *174* (2018).

[57] https://www.rug.nl/ggdc/historicaldevelopment/maddison/releases/maddison-project-database-2018; https://www.rug.nl/ggdc/historicaldevelopment/maddison/data/mpd2018.xlsx accessed on 2020/06/10

[58] https://www.rug.nl/ggdc/historicaldevelopment/maddison/data/mpd2018_region_data.xlsx accessed on 2020/06/10

[59] Korotayev, A. V. & Tsirel, S. V. A spectral analysis of world GDP dynamics: Kondratieff waves, Kuznets swings, Juglar and Kitchin cycles in global economic development, and the 2008–2009 economic crisis. *Structure and Dynamics* **4**(1) (2010).

[60] https://www.gapminder.org/

[61] https://github.com/Gapminder-Indicators/gdppc_cppp/raw/master/gdppc_cppp-by-gapminder.xlsx provides regional and world data. GDP per capita for individual countries can be found at https://www.gapminder.org/data/ looking for Income per person (GDP/capita, PPP$ inflation-adjusted, in $2011 PPP). See also information about how the database was built at https://www.gapminder.org/data/documentation/gd001/ ; web sites accessed on 2020/06/11.

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[62] https://sites.fas.harvard.edu/~histecon/energyhistory/energydata.html accessed 2020/06/10