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Paradigm depletion, knowledge production and research effort

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Abstract: This paper deals with two elements of Thomas Kuhn (1962) ideas regarding paradigm: Depletion and resiliency. The possibility of paradigm depletion taking resilience into account, given the hierarchy among scientists, is modeled as a Stackelberg differential game between editors [leaders] and authors [followers]. A number of results emerge from the model: i) Paradigm depletion can be optimal; ii) The optimal editor’s shadow price of potential knowledge must be non-positive, if it is positive, the editor is just a keeper of the orthodoxy rather than a scientist; iii) Editor’s and/or researcher’s impatience is always bad for science; iv) In equilibrium editor’s behavior does not matter for optimal research effort, while only editor’s behavior matter for the paradigm.

Key words: Paradigm; Economics of science; Research effort.

JEL Classification Numbers: C79; J39; Q39.

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1. Introduction

In this paper we deal with two important elements of Thomas Kuhn (1962) theory of scientific revolutions related to the idea of paradigm: Paradigm depletion and resiliency. A paradigm is a set of beliefs, theories, empirical methodologies, communication practices shared by a scientific community that serves as a foundation for future scientific endeavors. The paradigm allows scientists to identify, frame and solve problems. When a problem is solved there is knowledge production. The first element of our interest in Kuhn’s theory is that when a paradigm fails to solve problems and explain anomalies, it results in crises that lead to the emergence of new theories and paradigm change. The second element is that paradigms are resilient; they are difficult to change because the scientific community has invested a lot in them.

The possibility of a paradigm failing to solve problems and, therefore, stopping producing scientific knowledge, leads to the idea that a paradigm can be thought as potential knowledge, as a stock of a resource of ideas that can be exploited, harvested, to produce science. If this stock of ideas is exhausted, depleted, then a paradigm becomes unable to solve problems.

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1 The philosophically minded reader may be quite critical of our interpretation and simplification of Kuhn’s ideas. We do not claim to have followed or modeled Kuhn’s theory. Sterman and Wittenberg (1999) provide a Kuhnian dynamic model in which completion and succession among paradigms are conditioned by positive feedback loops.

2 For the paradigm in modern economics, see e.g., Lazear (2000).

3 A scientific revolution generally results from paradigm exhaustion and an alternative set of new ideas. Morgenstern (1976), for instance, thought that game theory was a fundamental break with conventional economics.
Concerning the resiliency of paradigms; the scientific community resists changes in paradigms because of the costs associated with these changes. Researchers have invested a lot of time and effort doing normal science, working within a given paradigm, and any paradigmatic changes imply greater costs to learn, adapt, and develop new theories, methodologies and scientific practices. Paradigms are self-consistent communities of like-minded scientists, sharing a worldview [Sterman, 2000, p.849]. Paradigms transform groups of researchers into a profession, and lead to the formation of professional bodies, such as professional associations, schools of thought, departments, and specialized journals\(^4\). This social structure of science\(^5\) has its own hierarchy, developed, among other things, to organize and select what is in accordance with the paradigm, i.e., the hierarchy among scientists serves to preserve the paradigm, to create and keep orthodoxy.

One of the main guardians of the orthodoxy is, naturally, the editors of academic journals. They can be thought as the ones who decide the direction of research\(^6\). Editors set the scientific standards, choosing topics of interest, methods of analysis, and establishing the selection criteria of what constitutes new and valuable knowledge [Laband and Piette ,1994; Stigler et al., 1995; Dalen,1999; Chew et al. 2007; Macdonald and Kam, 2007; Baccini and

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\(^4\) Academic markets are prone to network effects [e.g. Faria, 2002] and network externalities can produce path dependence and lock-in [see Sutter, 2009].

\(^5\) By social structure of science we do not consider the background of the structure of the society in which they occur as Karl Mannhein (1936) did in his classic Ideology and Utopia, but a much smaller unit that consists in a scientific society. For a recent history of the economics profession, see Fourcade (2009).

\(^6\) According to Frey (2003), there are more than two groups of actors in the academic publishing system: notably publishers, editors, referees and authors. Each group owns different property rights in a scientific journal.
Barabesi, 2009\(^7\). In this paper editors can accelerate or slow down knowledge production\(^8\), and thus, ultimately, decide upon the life or death of a paradigm.

Editors of academic journals are generally selected among the leading researchers of a given field. If editors are leaders in their respective fields of knowledge, then the followers are the ordinary researchers. Given a set of academic incentives, researchers try to do their job putting more effort to earn, among other things, income and reputation [Samuelson 1995]. By producing knowledge through research, researchers aim at gaining professional promotion, peer recognition, respect and reputation by publishing their research in academic journals.

In this paper the possibility of paradigm depletion taking resilience into account is modeled as a hierarchical differential game between editors and authors, where editors are leaders and researchers are followers. This paper deals with two elements of Thomas Kuhn (1962) ideas regarding paradigm: Depletion and resiliency. A number of results emerge from a simplified version of the general model: i) Paradigm depletion can be optimal; ii) The optimal editor’s shadow price of potential knowledge must be non-positive, if it is positive, the editor is just a keeper of the orthodoxy rather than a scientist; iii) Editor’s and/or researcher’s impatience is always bad for science; iv) Editor’s behavior does not matter for optimal research effort, while only editor’s behavior matter for the paradigm.

The paper is organized as follows. In Section 2 we present the open-loop general model. In section 3 we study a particular solution of the model. The stability analysis of the equilibrium

\(^7\) We will not discuss problems with the referee process and the role of the editors, such as editorial favoritism [Medoff, 2003; Berg and Faria, 2008], plagiarism [Enders and Hoover, 2004; Arce et al., 2008], and academic dishonesty [Chichilnisky, 1995].

\(^8\) See Ellison (2002) for an exhaustive study of the slowdown of the publishing process in economics.
appears in Section 4. Section 5 discusses the possibility of paradigm depletion. Section 6 presents the comparative statics analysis. The concluding remarks are in Section 7.

2. The Model

The general model presented in this section is a Stackelberg differential game between editors [leaders] and researchers [followers]. As usual, we start presenting and solving the follower problem and then examine the leader’s.

The production of scientific knowledge, called actual knowledge, $k$, can be thought as an extraction of potential knowledge, $S$, which is the stock of knowledge available in a given paradigm. The dynamics of remaining reserves of potential knowledge, $\dot{S} = dS / dt$, describes the evolution of a paradigm. If the stock of ideas in a paradigm is nonrenewable, then it may be exhausted through extraction. The more knowledge a paradigm produces in the present, the less knowledge it may produce in the future until it gets exhausted. An exhausted paradigm no longer provides new knowledge by solving problems and dealing with anomalies.

An excellent illustration of potential and actual knowledge is given by the asymmetric information literature in economics. Arrow (1984, [1969]) set the research agenda following his general equilibrium model also known as Arrow-Debreu model. He listed a series of topics, problems and issues to be addressed, such as adverse selection, moral hazard, and agency problems; strategic behavior, increasing returns, and incomplete markets. At that point in time,

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9 For a related framework see Faria (2005), and Goel and Faria (2007). For an introduction to differential games see Dockner et al. (2000).
this research agenda was just potential knowledge, since no actual model or knowledge addressing these issues existed. However, in the next couple of decades there was an explosion of papers dealing with these issues. Some of these papers were published and became actual and widely accepted knowledge in the profession, such as Akerlof’s market for lemons, and Grossman and Hart’s principal-agent model [for a collection of essays on this literature see Eatwell et. Al.,1989].

The simple relationship between the time evolution of a paradigm, \( S \), and knowledge production, \( k \), can be written as: \( S = f(k), f_s < 0 \).

Notice, however, that there are two additional important factors that may retard or accelerate the paradigm evolution. The first factor is the research effort, \( e \), put forward by researchers to produce scientific knowledge. The expected impact of effort is to accelerate the depletion of a given paradigm.

The second factor is technology. Technology, \( T \), may have two different effects on the evolution of a paradigm. On the one hand, technology can make researchers more productive in producing knowledge, and, therefore, accelerate paradigm depletion. On the other hand, technology may preserve the paradigm by allowing researchers to further or recycle knowledge produced in the past.

By taking research effort, \( e \), and technology, \( T \), into account the time evolution of a paradigm is given by:

\[
\dot{S} = f(e, k, T), f_e < 0, f_{ee} = 0; f_k < 0, f_{kk} = 0; f_T > 0, f_{TT} > 0; f_{et} = f_{te} = f_{TT} = 0 \quad (1)
\]
Total benefits of a typical researcher, \( U \), depend on a composite consumption good \( X \), and actual knowledge, \( k \): \( U = U(X, k) \), \( U_X > 0, U_k > 0, U_{XX} < 0; U_{kk} \leq 0; U_{x} = 0 \). By normalizing the price of the composite consumption good to unity, and assuming the researcher consumes all his income, \( y \), at every point in time, we have: \( X = y \). Researcher’s income is assumed to be an increasing function of research effort: \( y = g(e) \), \( g_e > 0; g_{ee} = 0 \). Therefore, \( X = y = g(e) \), and \( U(X, k) = U(e, k) \).

Total costs of research, from the researcher viewpoint, \( C \), are positively associated with research effort, \( e \), and negatively associated with potential knowledge, \( S \), since for a researcher, when the level of potential knowledge \( S \) is high, the cost of finding a good idea should be lower. The impact of technology, \( T \), is ambiguous, since in the short run it is costly for a researcher to learn and manage a new technology, but it may make her more productive in the long run:

\[
C = C(e, S, T), \quad C_e > 0, C_{ee} \geq 0, C_S < 0, C_{SS} = 0; C_T > 0, C_{TT} \leq 0; C_{eT} = C_{ST} = C_{eS} = 0.
\]

At time \( t \), the welfare, \( W \), of a representative researcher is given by the difference between total benefits and total costs: \( W = U(e, k) - C(e, S, T) \).

The optimization problem of the representative researcher [author] is to:

\[
\text{Max } \int_{0}^{\infty} [U(e, k) - C(e, S, T)] \exp(-rt) dt
\]

s.t. \( S = f(e, k, T) \), \( S(0) \) given

where \( r \) is the researcher’s rate of time preference, i.e., researcher’s impatience.
The current value Hamiltonian of this problem is:

\[ H = U(e,k) - C(e,S,T) + \lambda f(e,k,T) \]  \hspace{2cm} (2)

where the variable \( \lambda \) is the researcher’s co-state variable [shadow price] of potential knowledge, \( S \). The first order conditions [F.O.C] are given by:

\[ U_e - C_e + \lambda f_e = 0 \Rightarrow \frac{C_e - U_e}{f_e} = \lambda \Rightarrow e = e(\lambda) \]  \hspace{2cm} (3)

\[ \dot{\lambda} - r \lambda = C_S \]  \hspace{2cm} (4)

And the transversality condition \( \lim_{t \to \infty} \lambda(t)S(t)e^{-rt} = 0 \).

Notice that from equation (3) the dynamics of \( S \) taken into account by the leader become:

\[ \dot{S} = f(e(\lambda),k,T) \]  \hspace{2cm} (5)

The job of editors, as leaders, is to select what is to be considered as knowledge in their fields of expertise. Total benefits of the representative editor depend on actual and potential knowledge, \( V = V(k,S), V_k > 0, V_{kk} < 0, V_s < 0, V_{ss} < 0, V_{ss} = 0 \). Note we assume that editors are honest and rather have creation of knowledge than preserve a given paradigm \([V_k > 0, V_s < 0]\). Editors face a cost associated with the selection of the appropriate actual knowledge given the potential knowledge: \( \phi(k,S), \phi_k > 0, \phi_s \leq 0, \phi_{ss} \leq 0, \phi_{ks} = \phi_{kk} = 0 \). The negative impact of \( S \) on editor’s costs has the same rationale as in the author’s costs, it becomes increasingly costly for an editor to identify valuable knowledge when the pool of ideas within a given paradigm decreases.
As leaders, editors take the reaction of authors given by eqs. (3) and (4) to:

$$\text{Max} \int_{0}^{\infty} [V(k,S) - \phi(k,S)] \exp(-\rho t) dt$$

s.t. $S = f(e(\lambda), k, T)$

$$\lambda = r\lambda + C_s$$

where $\rho$ is the editor’s rate of time preference, i.e., editor’s impatience.

The current value Hamiltonian of the editor’s problem is:

$$H = V(k, S) - \phi(k, S) + qf(e(\lambda), k, T) + \mu [r\lambda + C_s] \quad (6)$$

where the variables $q^k$ and $\mu$ are the editor’s co-state variables [shadow prices] of potential knowledge, $S$, and author’s shadow price of potential knowledge, $\lambda$. The first order conditions are given by:

$$V_k - \phi_k + qf_k = 0 \Rightarrow k = k(q) \quad (7)$$

$$q - \rho q = -[V_s - \phi_s] \quad (8)$$

$$\mu - \mu \rho = -[qf_s e_s + \mu r] \quad (9)$$

In addition, the transversality condition:

$$\lim_{t \to \infty} S(t)q(t)e^{-\rho t} = 0 ; \lim_{t \to \infty} \lambda(t)\mu(t)e^{-\rho t} = 0$$
must be satisfied.

There is an interesting result in equation (8). If the editor’s marginal benefit cancels out the marginal cost of potential knowledge, \( \phi_s = V_s \), then it follows that \( q = \rho q \), i.e., the editor’s opportunity cost [shadow price] of a paradigm [potential knowledge], grows at his own rate of time preference. This condition is analogous to the famous Hotelling rule [Hotelling, 1931]. This condition says that when the editor’s net marginal cost in selecting knowledge from a given paradigm is zero, the paradigm’s opportunity cost, from the viewpoint of the editor, must appreciate at the editor’s rate of time preference. For the editor this condition means that the sensitivity of the present value of his optimal total net benefit to the paradigm grows with his impatience.

The next section solves this general model using a set of very simple explicit equations so as to illustrate how the model works, and what type of insights and information we can obtain from it.

3. A Particular Solution

An example with explicit functions is given below. It allows us in this section to derive the steady-state equilibrium of the model, and in the next section to study its local stability, and then to do the comparative statics analysis.

The explicit functions to be considered are the following:

\[
U(e, k) = \log e + k^\xi, \quad 0 < \xi < 1
\]  

(10)
\[ V(k, S) = \log k - \frac{\sigma}{2} S^2, \quad \sigma > 0 \] (11)

\[ C(e, S, T) = ce - \gamma S + T^\beta, \quad c > 0, \quad \gamma > 0, \quad \beta > 0 \] (12)

\[ f(e, k, T) = T^\alpha - ak - be, \quad a > 0, b > 0, \quad \alpha > 0 \] (13)

\[ \phi(k, S) = \phi k - \frac{\theta}{2} S^2, \quad \phi > 0, \quad \theta > 0 \] (14)

By taking equations (10)-(14) into account and explicitly solving the author’s and editor’s problems we derive in Appendix 2 [corresponding to eqs. (A.3), (A.2), (A.5), (A.6)] the following canonical system:

\[ \dot{S} = T^\alpha - ak(q) - b e(\lambda) \] (15)

\[ \dot{\lambda} = r \lambda - \gamma \] (16)

\[ \dot{q} = \rho q - [\theta - \sigma]S \] (17)

\[ \dot{\mu} = \mu(\rho - r) + qbe(\lambda) \] (18)

from this canonical system we find the steady-state equilibrium of the model. The steady state equilibrium is denoted with an asterisk over the variable.

From equation (16) steady state value of the researcher’s shadow price of potential knowledge, \( \lambda^{\bullet \bullet \bullet \star} \), is:
\[ \dot{\lambda} = 0 \Rightarrow \lambda^* = \gamma / r \]  

(19)

Using (19) into (A. 1) we find the researcher’s steady state effort, \( e^* \):

\[ e^* = e(\lambda^*) = \left( \frac{1}{c + b(\gamma / r)} \right) \]  

(20)

From (20) and \( S = 0 \) in (15) we determine the optimal actual knowledge, \( k^* \) [expressed as a function of \( e^* \)]:

\[ k^* = \frac{T^\alpha - b \ e^*}{a} \]  

(21)

From (19), (A.4) and \( S = 0 \) in (15) we find the steady state value of the editor’s shadow price of potential knowledge, \( q^* \):

\[ \frac{1}{\phi + aq} = \frac{T^\alpha - b \ e(\lambda)}{a} \Rightarrow q^* = \left[ \frac{1}{T^\alpha - b \ e^*} - \frac{\phi}{a} \right] \]  

(22)

The determination of the steady state value of the editor’s shadow price of potential knowledge, \( q^* \), allows us to determine the steady state value of potential knowledge, \( S^* \). From \( q = 0 \) in equation (17) we derive the steady state value of potential knowledge, \( S^* \) [expressed as a function of \( q^* \)]:

\[ S^* = \frac{p q^*}{[\theta - \sigma]} \]  

(23)
From $\mu = 0$ in equation (18), given $q^*$ and $e^*$, the optimal editor’s shadow price of academic incentive, $\mu^*$, is given by:

$$\mu^* = \frac{-be_\lambda q^*}{[\rho - r]}$$

(24)

The next section analyses the stability of the model at this steady state equilibrium.

4. The Stability of Equilibrium

In order to do the comparative statics analysis of the model we need to investigate whether the steady state equilibrium found in the previous section [in expressions (19)-(24)] is saddle point stable.

Linearizing the canonical system [(15)-(18)] at the steady state, we obtain the following Jacobian matrix:

$$J = \begin{bmatrix}
0 & -be_\lambda & -ak_q & 0 \\
0 & r & 0 & 0 \\
0 & r & 0 & 0 \\
0 & qbe_\lambda''(\lambda) & be_\lambda & (\rho - r)
\end{bmatrix}$$

(25)

We define $M$ as the sum of the principal minors of $J$ of dimension 2 minus $\rho^2$:

$$M = \begin{bmatrix}
0 & -ak_q \\
(\sigma - \theta) & \rho
\end{bmatrix} + \begin{bmatrix}
r & 0 \\
qbe_\lambda''(\lambda) & (\rho - r)
\end{bmatrix} + 2\begin{bmatrix}
-be_\lambda & 0 \\
0 & 0
\end{bmatrix}$$

(26)
According to Dockner (1985) [see also Long, 1992] the steady state equilibrium is saddle point stable, i.e., there are two positive real roots and two negative real roots, if the following conditions are satisfied:

i) $\det J \in (0, M^2 / 4]$

ii) $M < 0$

Where $\det J$ stands for the determinant of the Jacobian matrix $J$. The following proposition 1 summarizes the stability conditions:

**Proposition 1.** The steady state equilibrium is stable: a) if the researcher is more impatient than the editor, $\rho < r$; and b) the editor’s marginal benefit of potential knowledge is greater than its marginal cost [in absolute terms]: $\sigma S > \theta S$.

**Proof.** The $\det J$ and $M$ are given by the following expressions:

$$
\det J = a r k_y (\rho - r)(\sigma - \theta) \quad (27)
$$

$$
M = r(\rho - r) + a k_y (\sigma - \theta) \quad (28)
$$

Since $k_y < 0$, it is easy to see that $\rho < r$; and $\sigma S > \theta S \Rightarrow \sigma > \theta$ are necessary and sufficient conditions for positive $\det J$ and negative $M$. It is straightforward to check that $\det J \leq M^2 / 4$ holds true.

One wonders whether the condition of the researcher being more impatient than the editor, $\rho < r$, is really necessary. The answer is no. The reason lies in the structure of the
canonical system [eqs. (15) to (18)]. Since only the initial value $S(0)$ is given, and $\lambda(0)$, $\mu(0)$, and $q(0)$ are free, a one-dimensional stable manifold is sufficient for reaching the steady state.

Notice that from the canonical system [eqs. (15) to (18)] the differential equation for $\lambda$ given by eq. (16) can be solved leading to:

$$\dot{\lambda}(t) = \left(\lambda(0) - \frac{\gamma}{r}\right)e^u + \frac{\gamma}{r}$$  \hspace{1cm} (16')

So $\lambda(t)$ diverges unless $\lambda(t)=\gamma/r$ [notice that the transversality condition $\lim_{t \to \infty} \lambda(t)S(t)e^{-rt} = 0$ only holds when choosing $\lambda(t)=\gamma/r$, except when $S(t) \to 0$]\(^{10}\).

Moreover, in the canonical system $\mu$ does neither appear in the dynamic equations of the other variables nor in the first order optimality condition [i.e. $k$ is independent of $\mu$].

Therefore it is sufficient to analyze the reduced dynamic system:

$$\dot{S} = T'' - \frac{a}{\phi + aq} - b e(\gamma/r)$$ \hspace{1cm} (15)

$$\dot{q} = \rho q - (\Theta - \sigma)S$$ \hspace{1cm} (17)

Which has a unique steady state at:

\(^{10}\) As the differential equation for $\lambda$ can explicitly be solved, we do not have to consider $\lambda$ as a second state variable to the leader’s problem. In the appendix 2, $e(t)$ in (A.3) becomes a function of $\lambda(t)$ as expressed in (16’), and we ignore (A.2) as a dynamic constraint. The corresponding maximized Hamiltonian is concave in the state variable, provided $\sigma \geq \theta$ and therefore Arrow’s sufficiency condition applies.
\[ q^* = \left[ \frac{1}{T^a - b \epsilon(y/r) - \phi} \right] \quad (22) \]

\[ S^* = \frac{\rho q^*}{\theta - \sigma} \quad (23) \]

The Jacobian is given as:

\[ J = \begin{bmatrix} 0 & \left( \frac{a}{\phi + aq} \right)^2 \\ \sigma - \theta & \rho \end{bmatrix} \]

If the editor’s marginal benefit of potential knowledge is greater than its marginal cost [in absolute terms], \( \sigma S > \theta S \Rightarrow \sigma > \theta \), then the determinant is negative, \( \det J = (\theta - \sigma) \left( \frac{a}{\phi + aq} \right)^2 < 0 \), and this model displays saddle-path stability. Therefore the assumption on the differences between editor’s and author’s impatience is not necessary for the stability of the model’s equilibrium.

**5. Is it Possible to Deplete a Paradigm?**

This is a very important question, since the production of actual knowledge, \( k \), is related to the reduction of the stock of ideas [potential knowledge], \( S \); it follows that the stock of ideas may be exhausted by the evolution of science. However, if the stock of ideas is exhausted, depleted, then the paradigm becomes unable to solve problems and then a revolution in science, understood by a sudden change in paradigm, may become necessary.
The issue regarding the possibility of paradigm depletion is associated with two possible different paths of science: Whether science evolves i) smoothly or ii) by sudden radical changes. It is a logical consequence of paradigm exhaustion that the only possible path of science is for a revolution in paradigm, i.e., a complete, and sudden, change in paradigm. Alternatively, if the paradigm does not get fully exhausted, then there is room for a smoother change of paradigm, a shift that may take time.

In order to analyze the possibility of paradigm exhaustion, let us investigate whether paradigm exhaustion, \( S^* = 0 \), can be a steady state solution of our model. The steady state value of \( S \) is positive or nil depending on the steady state value of the editor’s shadow price of potential knowledge, \( q^* \). From the stability conditions we have \( \sigma > \theta \), it follows from eq. (23) that the paradigm is exhausted, \( S^* = 0 \), only if \( q^* = 0 \), otherwise, the steady state value of potential knowledge is positive, \( S^* > 0 \), provided that \( q^* < 0 \). Therefore the paradigm \( S^* \) is positive or nil, \( S^* \geq 0 \), if and only if \( q^* \leq 0 \). It is important to stress that a non-positive \( q^* \) is consistent with the assumption that the representative editor is honest in the sense that he rather have more actual knowledge produced than to preserve a given paradigm.

In eq. (22) the editor’s shadow price of potential knowledge being non-positive, \( q^* \leq 0 \), implies that:

\[
q^* = \left[ \frac{1}{T^\alpha - b e^*} - \frac{\phi}{a} \right] \leq 0 \iff \frac{a}{\phi} + b e^* \leq T^\alpha
\]

(29)

Critical for inequality (29) is the role of one endogenous variable, author’s optimal effort, \( e^* \), one exogenous variable, technology, \( T \), and one parameter, \( \phi \), the editor’s marginal cost of
actual knowledge. For a given exogenous technology we may have by coincidence the equality in (29). However, given that author’s effort has to be constant, as $\lambda$ is constant, the inequality in (29) may hold when technology grows [assuming a positive $\alpha$].

As seen above, inequality (29) does not rule out the possibility of paradigm depletion. As a consequence, taking into account the possibility of paradigm depletion, one wonders whether we may have a paradoxical and inconsistent result in which the paradigm is exhausted, $S^* = 0$, and at the same time, by equation (15), the stock of ideas decreases, $\dot{S} < 0$. The issue is that $S^* = 0$, can only occur for a very special choice of parameters in a hairline case. To be more precise: If $S^* = 0$, it must follow from eq. (23) that $q^* = 0$. Plugging $q^* = 0$ into eq. (29) yields:

$$T^* - b e^* = a / \phi,$$

this relation can only hold in case that:

$$T^* - a / \phi = b e^* = br / [cr + b \gamma],$$

as $e^* = e(\lambda^*) = e(\gamma / r)$.

In sum, this model is consistent with different paths for science, which depend on whether the paradigm gets fully exhausted or not. If paradigm gets exhausted, science can evolve through jumps, resulting from revolutions in which paradigms have to change abruptly given the paradigm depletion. Alternatively, science can evolve smoothly, when there is time to shift from one paradigm to another. Science evolution, as given by the evolution [or revolution] from one paradigm to another, however, is not modeled and is beyond the scope of this paper, and remains an interesting issue for future research.

6. The Comparative Statics Analysis
A number of interesting results emerge from the comparative statics analysis of the steady-state equilibrium given by expressions (19)-(24).

According to (20) the optimal amount of research effort, $e^*$, increases with researcher’s impatience, $r$. It decreases with the extraction rate of research effort, $b$; the researcher’s marginal cost of actual knowledge, $\gamma$; and researcher’s marginal cost of effort, $c$. It is interesting to notice that optimal research effort does not depend on editor’s behavior.

According to (21), the optimal actual knowledge, $k^*$, decreases with extraction rate of knowledge, $a$; and with the researcher’s steady state effort, $e^*$. Consequently, it follows that parameters $c$, $r$, $b$, and $\gamma$, have the inverse impact on $k^*$, they have on $e^*$. Although researcher’s impatience $r$, increases research effort $e^*$, ultimately it is bad for science since it decreases actual knowledge $k^*$.

From (22) the steady state value of the editor’s shadow price of potential knowledge, $q^*$, increases with $a$, the extraction rate of knowledge; $b$, the extraction rate of research effort, and with optimal effort, $e^*$. It decreases with $\phi$, the editor’s marginal cost of actual knowledge; and with technology, $T$, if $\alpha > 0$.

Finally, the steady state value of the stock of ideas, $S^*$, given by (23), provided it is not nil, and recalling that $q^* < 0$, increases with $\rho$, editor’s impatience, and with $\theta$, editor’s marginal cost of potential knowledge. It decreases with $\sigma$, the editor’s marginal benefit of potential knowledge. It is important to stress that researcher’s behavior does not affect $S^*$, only editor’s behavior matter for the paradigm.
It is curious to note the negative effect of impatience in the model. Researcher’s impatience increases research effort, and by expression (21) for a given constant level of stock of ideas \( S \) is constant, there is a trade-off between research effort and the production of actual knowledge. According to expression (23) an increase in the editor’s impatience leads to preservation of the paradigm. As a consequence, researcher’s and editor’s impatience are bad for science.

7. Concluding Remarks

This paper elaborates upon two elements of Kuhn’s (1962) idea of paradigm. The first element is that a paradigm allows scientists to solve problems, which is knowledge production. When a paradigm fails to solve problems the production of scientific knowledge ends. So a paradigm can be thought as potential knowledge, as a stock of a resource of ideas that can be exploited or harvested to produce science. The second element is that paradigms are resilient; scientists are averse to changes in paradigms because they have invested a lot in them.

Paradigms have a social dimension, since they transform groups of researchers into a profession. The social structure of science has its own hierarchy, developed, among other things, to organize and preserve the paradigm, creating orthodoxy. Academic journals editors can be regarded as guardians of orthodoxy. They are generally selected among the leading researchers of a given field of expertise. In this hierarchical world researchers are followers of editors, since editors shape and direct research. Researchers are motivated by academic incentives to put forward more effort to produce knowledge.
This paper investigates the possibility of paradigm depletion given an hierarchy among scientists. It examines a Stackelberg differential game between editors [leaders] and authors [followers] that generates a number of results, among them we have: i) Paradigm depletion can be optimal; ii) The optimal editor’s shadow price of potential knowledge must be non-positive, if it is positive, the editor is just a keeper of the orthodoxy rather than a scientist; iii) Editor’s and/or researcher’s impatience is always bad for science; iv) In equilibrium editor’s behavior does not matter for optimal research effort, while only editor’s behavior matter for the paradigm.
Appendix 1: List of Variables and Parameters

Endogenous variables:

$S =$ potential knowledge [paradigm]; $e =$ research effort; $k =$ actual knowledge; $\lambda =$ researcher’s shadow price of potential knowledge; $q =$ editor’s shadow price of potential knowledge; $\mu =$ researcher’s shadow price of research effort.

Exogenous variables:

$T =$ technology

Parameters:

$\xi =$ researcher’s marginal utility of potential knowledge; $a =$ extraction rate of knowledge; $b =$ extraction rate of research effort; $c =$ researcher’s marginal cost of effort; $r =$ researcher’s impatience; $\rho =$ editor’s impatience; $\theta =$ editor’s marginal cost of potential knowledge; $\phi =$ editor’s marginal cost of actual knowledge; $\gamma =$ researcher’s marginal cost of actual knowledge; $\alpha =$ partial impact of technology on paradigm evolution; $\beta =$ partial impact of technology on researcher’s costs.

Appendix 2: Solving the Model

Author’s problem:

$$\begin{align*}
\text{Max} & \int_0^\infty [\log e + k^{\xi} - ce + \gamma S - T^\beta] \exp(-rt) dt \\
\text{s.t.} & \; S = T^\alpha - ak - be
\end{align*}$$
The current value Hamiltonian of this problem is:

\[ H_1 = \log e + k \tilde{e} - ce + \gamma S - T^\theta + \lambda [ T^a - ak - be ] \]

The first order conditions [F.O.C] are the following:

\[ \frac{\partial H_1}{\partial e} = 0 \Rightarrow e^{-1} - c - \lambda b = 0 \Rightarrow e = e(\lambda) = \left( \frac{1}{c + b\lambda} \right) \]  

(A.1)

\[ \dot{\lambda} - r \lambda = -\frac{\partial H_1}{\partial S} \Rightarrow \dot{\lambda} = r \lambda - \gamma \]  

(A.2)

And the transversality condition: \( \lim_{t \to \infty} \lambda(t) S(t) e^{-r t} = 0 \)

Editor’s problem:

\[ \text{Max} \int_0^\infty \left[ \log k - \frac{\sigma}{2} S^2 - \phi k + \frac{\theta}{2} S^2 \right] \exp(-\rho t) dt \]

s.t. \[ S = T^a - ak - be(\lambda) \]  

(A.3)

\[ \dot{\lambda} = r \lambda - \gamma \]  

(A.2)

The current value Hamiltonian of this problem is:

\[ H_2 = \log k - \phi k + (\theta - \sigma) \frac{S^2}{2} + q[T^a - ak - be(\lambda)] + \mu[r\lambda - \gamma] \]

The first order conditions [F.O.C] are the following:
\[ \frac{\partial H}{\partial k} = 0 \Rightarrow k^{-1} - \phi - aq = 0 \Rightarrow k = k(q) = \frac{1}{\phi + aq} \quad (A.4) \]

\[ q - \rho q = - \frac{\partial H}{\partial S} \Rightarrow q = \rho q - (\theta - \sigma)S \quad (A.5) \]

\[ \mu - \rho \mu = - \frac{\partial H}{\partial \lambda} \Rightarrow \mu = (\rho - r)\mu + qbe \quad (A.6) \]
References


Berg, Nathan and João R. Faria (2008) Negatively correlated author seniority and the number of acknowledged people: Name-recognition as a signal of scientific merit?, *Journal of Socio-Economics* 37, 1234-1247


