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Abstract

Project-financing and public-private partnership schemes are joint projects of investment that are generally submitted to investment valuation criteria based on compound discounting. However, the theoretical basis of these criteria is at issue nowadays. According to recent studies on relational contracting economics and behavioral finance, joint projects of investment can be considered as special relational environments where the project's returns improve on alternative replacement opportunities. This article aims to bridge the gap between new theories and widespread valuation techniques by providing a generalised approach to investment valuation. This article suggests new valuation criteria that fit those theoretical developments, including an endogenous optimal duration that the project's contractual agreement may integrate.

JEL classification: H43, H50, D61, D70, D90, G31, L20

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Introduction

Recent theoretical developments on relational contracting and behavioural finance renew the old debate on investment valuation criteria based on discounting (Biondi 2006). However, decision-makers are accustomed to this kind of valuation criteria, which are also central to the process of "value for money" (or "VfM") evaluation advocated by new public policies of investment such as "Private Finance Initiative" and "Public Private Partnerships" (Marty-Trosa-Voisin 2006).

Expanding upon Hirshleifer (1987) as well as the relational contracting literature (Williamson 1991), this paper aims to bridge the gap between recent theoretical insights and these influential techniques of valuation. New conventional rules are developed on the basis of the dynamic relational context involving all the potential undertakers of the joint project that must be evaluated and undertaken. These rules may be applied and enforced even in absence of perfect and complete competitive conditions. As the applied literature on the matter usually does, this paper outlines a framework for the generalisation but gives up huge formalisation, combining a commonly understandable analysis with viable investment valuation criteria. Therefore, some footnotes shall discuss further theoretical points, and some numerical samples shall help to understand the generalisation and its application to joint project-financing schemes, including the so-called "Public Sector Comparator" (PSC).

The rest of the paper is organised as follows. The first section questions the "value for money" valuation and sketches the new framework of reference and analysis. On this basis, a viable generalisation of discounting is suggested. The second section provides some reduced forms from this generalisation, which will be applied to the valuation of public sector investment projects under private-financing schemes such as PFI and PPP. Generalised investment valuation criteria include a new measure of *endogenous optimal duration (EOD)*. The third section briefly expands the application to the traditional case of private sector capital budgeting. All the applications are explained by means of numerical examples, some of them being based on actual PFI cases from UK.

1 – Cost of capital, discounting, and the “value for money” valuation

§ The discount rate issue in the valuation of public sector investment projects under private financing schemes

Recent public policies of investment (fostered by the UK Government under the “Private Finance Initiative” and later “Public Private Partnership” policies) have advocated and applied new project-financing schemes for public sector investment. Traditionally, the public procurement is used to make public sector investments by selecting private sector bids either for providing facilities that are thus financed on conventional government debt and exploited directly by the public agency (the procuring public authority) having launched the bid process, or for organising the whole activity of financing and exploiting under concession schemes. Under the new project-financing scheme, the public procurement asks private sector bids for financing and managing the facility whilst recovering the related investment on the subsequent cash flows from the exploitation of the overall project's activity. The project's arrangement is carried out by a “special purpose vehicle” (or “SPV”), which formally constitutes the project by bringing together a group of private sector companies, often including a construction company, a facility management company and a financier. The SPV is typically highly leveraged through 80%-90% debt and 20-10% equity, both provided by the member companies. Because of this huge debt's engagement, the SPV structure is then shaped in a way securing the debt repayment, whilst the constructor may gain directly from construction costs whilst multiplying its capacity to bid by reducing its equity involvement in each SPV. In UK, at least 56.88 billions £ have been invested on 622 projects arranged under this basis to 23 October 2007 (HM Treasury 2007c). This project-financing was “off-balance sheet” in the 80.7% of the projects, that is, it was not submitted to formal constraints on borrowing and spending imposed on the government's activity.

The choice of investing through these new project-financing schemes has been submitted to investment valuation based on conventional methods of economic analysis. In 1997, HM Treasury (1997) produced a first document on the matter, the so-called

"Green Book", which recommended comparative cost savings, in discounted terms, between the "public sector comparator" (PSC), i.e., the project developed according to the traditional public scheme of facility acquisition, and the "private finance initiative" offer (PFI) that implies the financing of the facility investment under a project-financing scheme. Since 2004, the HM Treasury suggests to utilise, especially in the first phase of comparative evaluation, a standardised procedure called "quantitative spreadsheet analysis", based on a valuation model dealing with a set of key hypotheses on the project cash flow profile (HM Treasury 2006). This models aims to calculate the internal rate of return on private *equity* investment (before tax) that may be agreed by a private sector bidder (estimated between 13% and 18%), and utilises the same nominal discount rate of reference for PSC and PFI (estimated to 6.09%), which "is based on the Green Book real discount rate of 3.5% and GDP Deflator assumption of 2.5%" (HM Treasury 2007a, 2007b).

All along, the underlying idea is to assess and compare the two alternatives by testing the "value for money" for the tax payer. The test is based on a compound discounting logic of valuation, called "cost-effectiveness", which selects the alternative having the lesser equivalent cost today, in discounted terms. In this context, the choice of the discount rate of reference is especially sensitive, since one of the key differences between PSC and PFI rests on the distinctive cash flow profile of each alternative through time. The PSC is expected to spend public money in few years and thus acquire and exploit the facility, whilst the PFI is expected to go on paying for the use of the facility (and provision of related services) for generally thirty years, whilst nothing will be paid during the construction phase of generally three years.

The "value for money" valuation follows the usual logic of economic analysis, which relates investment valuation to the fundamental relationship between investment decision-making and time. All investment decisions implicitly handle with the future. Discounting is then the practical way to take into account the timing of the investment projects that have to be assessed, looking for two valuation outcomes: (i) a *conventional* net economic value, and (ii) a *comparative* ranking of different available opportunities of investment. The usual scores of this approach are the net present value (NPV) and the internal rate of return (IRR). They incorporate the time element of the project by means of a *financial* assessment of its timing, generally based on the reference cost of capital, and compute this element with all the other costs and revenues in order to rank the different alternatives. The value of the time element, or better, its *values* contribute to

frame the current decision into a peculiar understanding of the future. Discounting is then called to enhance the decision-makers' understanding of time horizons.

Not surprisingly, the impact of this discounting logic of valuation on the public policies of investment has been discussed thoroughly and vividly (Mayston 1993; Spackman 2002; Broadbent and Laughlin 2003; Shaoul 2005; Grimsey and Lewis 2005). Concerns are expressed on the profit motive in the public service provision, especially when accountability is critical, cost-shifting involves problems, the time-frame is long, and qualitative societal needs are more important than costing. In particular, alternative frameworks for the appropriate discount rate of reference have been reviewed by Spackman (2002, 2004, 2006). Some have stressed the need of estimating the actual cost of capital invested, including opportunity cost and risk pricing, whilst others have argued for a discounting logic based on natural or social time preferences. Further issues were whether the same discount rate applies either from the public and the private viewpoints, or for near and distant future. Compound discounting was scarcely criticised, even though "indefinite compounding is not possible at a rate which exceeds the long term growth rate of the whole economy" (Spackman 2006: 10; see also Voinov and Farley 2007: 109). Notwithstanding all these theoretical efforts, the applied debate is often at risk of being influenced by ideological bias in favour or against the public intervention, since the choice of the appropriate discount rate factually has a huge impact on the evaluation outcomes.

Theoretically speaking, the preference for one unique compound rate for discounting is generally justified by advocating the virtue of the price system generated by complete and perfect markets. This efficient market hypothesis implies the existence of a unique discount rate for financing and investing (or replacing) cash flows, since a unique financial market is supposed to exist and perfectly operate. Following Williamson (1991), this framework describes an *unrelated* economic environment where every actor is independent of each other, and "money talks" among them. The price system implies then an idealized representation of the financial process of investing as an instantaneous affair beyond time and context.

Under real dynamics and complexity, however, this framework leaves practical fundamental problems of valuation unsolved when uncertainties, joint projects of investment and relational contracting are combined, as in the case of PFI and PPP. According to leading financial economist Fama (1996: 427),

> given the massive uncertainties inherent in all aspects of project valuation, does a discounting rule produce [reliable] value estimates […]? […] The [usual affirmative] conclusion is based more on faith than evidence.

When a *relational* economic framework is considered (Eisenberg 1998, Goldberg 1998), the theoretical tenure of usual discounting is at issue,[1] since, as another leading economist Hirshleifer (1987: 994a) states,

> only under complete and perfect markets is the concept of wealth or Present Value unambiguously defined, so that the choice of productive investments can be entirely disconnected from individuals' personal time-preferences, risk-preferences, beliefs, ecc.

In this dynamic relational context, the usual IRR score means not only a measure of the financial return expected to be earned from the project, but plays a role as "return sharing device" integrated in the institutional structure of the project that potential undertakers are concerned with. From the latter viewpoint, the IRR is the discounting technique that computes the biggest financial return with the smallest rate level. Therefore, together with the discount rate of reference, the IRR can be considered as the keystone of the "value for money" valuation, and one of the key measures of return in every project-financing scheme such as PFI and PPP (Vecchi et al. 2009). As the Private Finance Unit of the Office of Government Commerce (OGC-PFU, 2002: 26) recognises:[2]

> The Internal Rate of Return (IRR) is most commonly used in PFI Contracts as a measure of the rate of return expected to be earned by private sector capital in the project. It is therefore fundamental to the negotiations, and Authorities should take care to understand thoroughly the definitions, methodology for calculation and correct usage of IRRs in PFI Contracts.

Even tough its theoretical foundations are at issue, decision-makers are accustomed to consider rules based on compound discounting (such as present value and internal rate of return) as the benchmark for investment decisions. Accordingly, a new interpretation of discounting appears to be required to bridge the gap between recent theoretical developed and so widespread techniques. A renewed approach may provide valuation

[1] Further critical insights are provided by behavioural finance (Kahneman-Tversky 1979; Kahneman-Riepe, 1998; Frankfurter-McGoun, 1999; Marzo 2002), and by other influential theoretical works (Laibson 1997; Loewenstein and al. 1989 and 1992; Weitzman 2001). Ainslie (1992) stressed hyperbolic (simple interest) computation for inter-temporal valuation, referring to the seminal work of Herrenstein (1961) that Ainslie (1992) and Mazur (1986) further refine and formalise. Shane et al. (2002) provided a review of the economic literature on the matter.

[2] See also HM Treasury (2004, A145-A148: 38-39).

rules fitting the dynamic relational context that involves all the undertakers of the joint project that must be assessed and whose returns must be shared.

§ A tentative generalisation of the discounting approach

A tentative generalisation of the usual logic of compound discounting may start from the critique of the alleged hypothesis of one perfect financial market. Under this hypothesis, the return for financing, investing (or replacing) is the same, and a unique rate of interest/return exists for all financial opportunities (Fisher's separation principle). This is supposed to happen in a world of complete and perfect markets, where all practical problems involved by the dynamic complexity of a joint project of investment - having its own managerial needs, concerned with a peculiar economic context, and grappling with bounded knowledge - are abstracted away from time and context.

On the contrary, the special economics of institutions and organisations provides a different understanding of joint projects of investment such as project-financing schemes (including PFI and PPP). These projects are then understood as relational economic environments, different from markets, and featured by contract incompleteness, relational (transactional) specificities, forbearance rules and legal arbitration (Williamson 1991). In particular:

- The complex contractual arrangement underlying the joint project, and its sophisticated process of valuation and negotiation as well, provide strong security from usual competitive dynamic adjustments, especially *ex post*;
- The *ex ante* agreed return from the joint project is expected to be relatively higher than every outside placement at alternative rate, and all the constraining arrangements on future decisions, cash-flows claims and control rights make so that the so-called Fisher's separation principle[3] may hardly be true;
- The typical cash flows profile (provided by the quantitative spreadsheet analysis that is part of the agreement signed by the parties) starts with a phase of *development, construction and start-up* characterised by negative outflows and goes on with relatively steady positive operating inflows[4] (OGC 2002, point 3.1.2: 26; HM Treasury 2006).

[3] According to this principle, investment (and replacement) and financing rates are the same.
[4] They may be steady-increasing according to some revenues' inflating indexation.

Therefore, the joint project of investment constitutes a special economic environment shaped by relational contracting features. The buyer (the procuring public Authority) faces with a make (own) or buy (hire) decision in such a relational economic context. He can either make the investment through in-house financing, or buy the services provided by the promoter and pay the related (usually higher) financing costs. Reciprocally, the promoter -as financial investor- may either invest and operate the facility under the joint-project financing scheme, or finance the buyer by acquiring securities on his debt. In this context, investment valuation criteria play a role in the project's *accounting device* that establishes conventional benchmarks for the bilateral bargaining involved by the ongoing process of negotiating, renegotiating, arbitrating and forbearing, fraught with market failures, and limited competitive conditions (Williamson 1991).

In this context, every joint process of investing requires judgement, skills and knowledge of the specifics of complex situations. The joint project represents a special mode to co-ordinate the joint economic activity, because it cannot be controlled by outside market forces or incentives. In short, the relationship underlying the joint project must to be *managed* (for instance by inner decision-making rules), because the project's governance cannot replicate the market. In particular, the role plaid by the price system in a market environment is then plaid by an inner "accounting system", which represents the special mode of allocating and sharing resources controlled through the joint project[5]. This system is an integral part of the "institutional structure" of the project (in Coase's terms), and comprehends the quantitative valuation devices such as present value and internal rate of return. In this context, however, the usual logic of discounting is not shaped by an alleged unique rate of reference, since the managed coordination of the joint project allows improving *on market outcomes* through time[6]. Therefore, the joint project of investment is expected and generally has returns of reference that are superior to the outside returns from market alternative (re)placements.

§ Distinctive discount rates for investing and financing

In this context, the traditional logic of compound discounting appears to be unsound. Especially the hypothesis of a unique rate of reference results at odds with the intentions

[5] Biondi (2005) applies this reasoning to the business firm.
[6] Baker-Gibbons-Murphy (2001 and 2002) apply this reasoning to the business firm.

and the actual conditions experienced by decision-makers from the public and the private sides of the project-financing scheme[7]. As Baldwin (1959: 98b-99a, italics added) early claimed,

> it is to one critical assumption underlying the usual procedure [related to present value and IRR] that I take strong exception. The future receipts and payments are reduced to their present value by discounting them at the same rate as that which the proposed investment is estimated to provide.[8]
>
> *In other words, management assumes that, for the period between the base point and the time when the funds are spent or collected, the funds are, or could be, invested at the rate of return being calculated for the proposal*.
>
> This is simply not true. Indeed, it is only by coincidence that the two would be at all alike.

Every relational enduring economic activity (such as joint projects of investment or business firms) generates a special financial process of investing, which allows the inside returns to improve on the outside returns from alternative (re)placements. As valuation and profit-sharing device involved in this process, discounting cannot longer abstract away from time and context. A generalised approach may then take into account both different returns of reference (instead of one unique), and the replacement structure of future inflows. In this way, discounting may better accomplish two fundamental tasks: (i) to assist potential undertakers in estimating the *values of timing*, establishing conventional criteria to compare alternative investment opportunities that have different cash flow profiles; (ii) to better fit the return sharing device integrated in the institutional structure of the project.

Accordingly, discounting may better provides potential undertakers with scores that can express concisely and substitute each cash flows profile without muddling the overall understanding of the investment project, and without loosing relevant information about it. These scores finally rejoin the statistical notion of *mean of the cash flow profile* (Flemming-Wright 1971), whose weights must be established by agreement between undertakers and according to the ongoing sustainability of the joint process of investment.

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[7] For example, in a study commissioned by the Office of Government Commerce (OGC), PriceWaterhouseCoopers (2002) reports a significant superior return of PFI projects relative to the cost of capital based on the market reference (Weighted average Cost of Capital – WACC based on Capital Asset Pricing Model - CAPM), even after inclusion of the swap cost, and of an eventual provision for bidder's costs for failed bids on other projects.

[8] That is, the discount rate for NPV, the IRR for IRR. See note 9.

In some way, this approach rediscovers an early generalisation of discounting to a family[9] of financial measures based on at least two discount rates of reference, one for the inside investment (y) and another for outside replacement (i)[10]:

$$\sum_{t=0}^{n} f(1+i;1+y;t;a_t)\cdot a_t \text{ or, in a continuous time: } \int_0^n f(y;i;t;a_t)\cdot a_t dt \,.$$

This generalisation differs from the usual approach based on compound discounting alone[11]:

$$\sum_{t=0}^{n}(1+y)^{-t}\cdot a_t \quad \text{or} \quad \int_0^n e^{-y\cdot t}\cdot a_t dt \,.$$

Usual measures with one unique discount rate for investment and (re)placement become special cases of the generalised function *f(y;i;t;a).* This function declares the underlying discounting logic and describes the relation between the inside investment return and time horizons that decision-makers apply to compare alternative investment opportunities.[12] Further studies on the relevant properties of *f(y;i;t;a)* may be developed.[13]

Stating the conventional nature of forecasted flows (and related discount rates), the cash flow profile must be interpreted as a series of expected measures of each flow, i.e., $E_t(a_t)$ instead of a_t. Potential undertakers agree this expected cash flow profile that provides financial representation for the institutional structure of the joint project of investment. In this way, the generalised approach accounts for risk by estimates of future cash flows, and does not reduce risk appreciation merely to increasing the discount rate of reference on a market basis. Including risk and duration into the discounting logic is made ambitious by the underlying hypotheses that underpin this technique, that are, the permanence of the initial capital invested, and the existence and permanence of successive periodic flows that shall be reinvested. Under a project-financing scheme,

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[9] Where $a_0 \ldots a_n$ is the cash flow profile of the joint project of investment through time.

[10] That is, an inside rate of investment (y) for outflows (negative cash flows) and an outside rate of replacement (i) for inflows (positive cash flows).

[11] NPV and IRR differ each other in reason of the different replacement rate assumptions. The NPV supposes that the replacement rate is the discount rate, whilst the IRR fixes it to same rate as IRR. Thus, the generalised approach vindicates the return-based measures like IRR and theoretically unifies discounted *values* and discounted *rates of return*.

[12] Of course, a change in the replacement rate modifies the project's GIRR, but it does not modify its comparative ranking.

[13] For instance, Rubinstein (2000) suggests a function where the discount factor f_t is decreasing in t, and increasing in a_t (the larger the sum of money at stakes, the higher (closer to 1) the discount factor). She suggests indeed a procedural rationality approach, framed with non-expected utility theory. See also the references provided by the note 1. A framework for this kind of normative economics is suggested by Sudgen (2004).

therefore, risks are managed by the whole institutional structure of the joint project of investment, which is not reducible to a market structure under special conditions of complexity and hazard.

This approach treats risks through estimated cash flows so that a risk-free discount rate is applied to these forecasts that have been adjusted to their "certainty-equivalents", in the terminology of Brealey and Myers (2003). Therefore, the discount rates of reference will be considered for estimating the (opportunity) *costs* of capital and not the implied risks. Moreover, a series of discount rates, instead of one constant value for all the periods (that is: y_t instead of y, and i_t instead of i), may be utilised for bettering the estimated temporal profile of these implied costs of capital. The estimate(s) of *y* should represent the project's return expected by potential undertakers from the financial viewpoint, whilst the estimate(s) of *i* should represent the return that may be expected from an alternative investment scheme that is taken as outside option for the project.

In the case of a PFI/PPP scheme, the rate *y* may be the expected internal rate of return, whilst the rate *i* may be the government bond yield on the same duration as the project (the so-called risk-free discount rate). Under the generalised approach, the internal rate *y* is still dependent on (endogenous to) the cash flow profile related to either the "availability charge" or the "annual unitary charge", but it is constructed under more realistic assumptions about the replacement rate *i* and the related replacement structure. Both rates should be considered as implied *costs* of capital, not as a time preference rates, whatever social or natural. This is because of the the financial viewpoint adopted by the return measurement, based on the logic of investing and replacing for a profit motive. This justification for discounting is appropriate to business firms and to individuals as investors, and assumes they do invest and keep reinvesting at some rates of return (Voinov and Farley 2007: 109). In particular, it would not be appropriate to warrant a higher replacement rate *i* for some interest premia (whatever equity, risk, or uncertainty premium), since this higher rentability is already accounted for in the internal rate *y* generated by the estimated cash flow profile. As well, present and future values of the joint project will be based on explicit hypotheses on the rates of reference for investing and replacing.

2 - Application to project-financing schemes

§ Reduced forms for estimating the cost of capital in project-financing schemes

To resume, the usual assumption of the same rate of reference for the joint-project's and the replacement returns would be true only under the alleged hypothesis of one perfect financial market. However, if financial markets are segmented and potential undertakers are conditioned by capital rationing, the usual discounting approach based on this hypothesis will lead to under (over) evaluate the discounted value of low (high) return projects. *This is because the cash inflows from the projects are supposed to be reinvested at the same rate as the cash outflows, while the actual rates of reference differ*. In particular, this implies that cash flows from projects with high (low) return will be reinvested at a higher (lower) rate, hence leading to over (under) valuation of the project.[14] The usual approach fails indeed to consider more realistic replacement opportunities available and might mislead the whole process of investing, especially whenever applied to rank alternative investment projects.

On the contrary, the generalised approach provides decision-makers with viable valuation scores computed under realistic assumptions that shall be agreed by undertakers as a part of the institutional structure of the joint project. This approach is based on a recursive *flows method*. Each cash flow is discounted according to its sign at one of the discount rates of references, negative cash flows being assumed to be for investment, positive cash flows for earning. The *generalised internal rate of return* (GIRR)[15] will be:

$$\sum_{j=0}^{t} \bar{a}_j \cdot (1+GIRR)^{t-j} = \sum_{j=0}^{t} \dot{a}_j \cdot (1+i)^{t-j} \qquad \text{[GIRR]}$$

where each outflow $\bar{a}$ (negative cash flow for investment) is discounted at *GIRR* rate (the special value of the investment rate *GIRR* derived by the above equation), and each inflow $\dot{a}$ (positive cash flow from investment) is discounted at replacement rate *i*. Instead, the usual IRR would be:

$$\sum_{j=0}^{t} \bar{a}_j \cdot (1+IRR)^{t-j} = \sum_{j=0}^{t} \dot{a}_j \cdot (1+IRR)^{t-j} \qquad \text{[IRR]}$$

[14] We assume here that the rate of reference for discounting is the investment rate. Furthermore, the usual IRR over (under) evaluate investment projects with high (low) rate of return.

[15] Usual relation between GIRR, GNFV and GNPV applies: GIRR is the discount rate that makes both GNFV and GNPV equal to zero.

That generalised reduced form has been already developed by scholars like Solomon (1956), Hirshleifer (1958), Baldwin (1959), Lin (1976), and Athanapoulos (1978). They consider it as an appropriate score even for the private sector, notably since it is unique and easy to calculate in every case.

> The salient advantage of ERR method [here, GIRR], compared to all others, is that it combines the recognized reliability of the present worth criteria with the ease of interpretation and understanding inherent in a rate of return (percentage) analysis (…).
>
> In view of this clear advantage of the Effective Rate of Return [ERR] technique, serious consideration as the sole criterion for investment profitability is merited. (Athanapoulos 1978: 132b).

Nevertheless, the generalisation could not be completed if the replacement structure would not be considered. When time and context do matter, the different kinds of financial placements are no longer equivalent on successive periods. For instance, the compound rate usually expected from the project is no longer equivalent to the simple interest provided by the alternative government's gilt on the same duration. The alleged existence of systematic replacement at the rate of reference for the project's investment is conceivable only under the hypothesis of a unique financial market. Neither would the procuring public authority intent to, nor may potential financiers of the joint project expect to have a series of investments at the project's rate of return, which is supposed to improve on the usual rate of return available for government's borrowing. In particular, the relevant alternative scheme or the eventual replacements available to the project-financing scheme may be at simple rate of return. The hypothesis of a simple rate of replacement (rather than a compound one) is reasonable from the viewpoint of the procuring public authority, who is confronted with the issuance of an alternative gilt to finance the traditional acquisition scheme (so-called PSC), and does not necessarily intent to provide potential financiers with a series of replacements beyond that issuance. But that hypothesis may also be reasonable from the viewpoint of potential financiers, which are not expected to reinvest systematically all the inflows from the project through time, but to consume a part and only replace the rest of them.

Finally, under the generalised approach, the variety of possible returns (and replacements) implies a range of rates instead of a unique one. In particular, the GIRR is complemented by the SIRR, that is, the internal rate of return estimated under bullet replacements at simple interest rate:

$$\sum_{j=0}^{t} \bar{a}_j \cdot (1+SIRR)^{t-j} = \sum_{j=0}^{t} \overset{+}{a}_j \cdot (1+i \cdot (t-j)) \qquad \text{[SIRR]}$$

Furthermore, the generalised approach provides decision-makers with a score that excludes replacement, i.e., when the replacement rate *i* is zero, labelled Basic-IRR (B*IRR*). This measure excludes the "time value of money" and does not establish any financial preference between the present and the future. The BIRR may be considered as a basic score based only on returns *internally generated* by the joint project of investment.

$$\sum_{j=0}^{t} \bar{a}_j (1+BIRR)^{t-j} = \sum_{j=0}^{t} \overset{+}{a}_j \qquad \text{[BIRR]}$$

In sum, under the generalised approach, each score (GIRR, SIRR or BIRR) can be considered as "the equivalent constant interest rate at which a given series of cash outflows must be invested in order for the investor to earn a given series of cash inflows as income. It is in this sense a measure of the underlying [compound] return the private sector expects to achieve by investing in the project." (OGC-PFU: 26). In particular, the generalised approach provides decision-makers with a synthetic spectrum of scores, and contrasts with the short-cut unique score provided by the conventional compound discounting approach. Further rules shall be applied to rank and accept projects (or bids) according to that spectrum. For this, the issue of the overall duration of the joint project shall be further inquired in the following.

§ The special case of the public sector comparator (PSC)

The adoption of a project-financing scheme (such as PFI and PPP) by a procuring public authority is submitted to the comparative analysis with a "Public Sector Comparator" (PSC), which would happen when the project is developed under the traditional public scheme of facility acquisition. This analysis is based on the cost-effectiveness between the two alternatives: the proposal having the lesser equivalent cost today will be retained.

The estimation of the implied costs of capital is fundamental to the comparative analysis. In order to show the contribution of the generalised approach to this case, several conditions apply. First of all, both alternatives must provide analogous facility and flow of services through time, and the relevant cash flow profile will be that before

all investment-financing outflows, including debt-related flows (the so-called project's operational cash flow profile available for debt service and dividend distribution, typically included in the debt arrangement of the project, and computed after taxes). Then, if the procuring public Authority aims to compare the PFI alternative with the usual provision of the facility alone, only the "availability charge" shall be taken as the (financial) cost of reference for the PFI scheme. On the contrary, if she aims to compare the PFI alternative under its comprehensive form, and the PSC shall be constructed as if the overall flow of services is provided and the related costs shall then be included. In addition, if provisions for specific risks are added to the PSC, analogous provisions for specific risks shall be added to the PFI scheme. Moreover, if ancillary revenues are granted to the PFI project, they may be considered for inclusion in the overall cost of this alternative, whenever they might be appropriated by the public sector otherwise. Finally, the construction cost deserves further consideration, since it generally includes capitalisation of interests that may be significant. The PSC shall capitalise this interest charge at its own alternative rate, instead of the financing rate adopted under the PFI scheme.

Anyway, the generalised approach may provide reliable estimations of the internal rates of return implied by whatever cash flow profile retained as benchmark for the PFI scheme. The following numerical samples may illustrate as the generalised approach works[16].

The following analysis is based on the provision of the facility alone under the PFI and the PSC schemes. In particular, the construction under the PFI scheme is supposed to cost 64.6 mill £, and the exploitation will last for 30 years thereafter. The PFI charge of reference is of 8.7 mill £ during the first year, is limited to the "availability charge", and is further escalated by an annual factor of 1.25%, which is supposed to be one half of the inflation rate on the non-employment annual costs (HM Treasury 2007b). The construction costs are supposed to be the same in both alternatives, what is expected to partly undermine the PSC[17]. The discount rate of reference is fixed to 3.5%, which is supposed to be the cost of capital of reference for government's borrowing on the same duration as the project. Since the availability charge is already inflated, the discount rate

[16] Data come from actual PFI project-financing schemes treated by Gaffney-Pollock-Price-Shaoul (1999), and from HM Treasury (2007a, 2007b, 2007c).

[17] At the same time, this could be justified by the opportunity for the procuring public authority to enter the construction arrangement on the same basis (including insurance and security devices), but not to enter the financing arrangement, providing liabilities to service PFI contracts are as binding as the servicing of conventional government debt.

is nominal and does not include inflation, in order to avoid double counting. The first five years of this project are reported in table I.

[Insert Table I here]

The report clearly shows the different weight (value) that each inflow receives according to its position in time (timing). For instance, under the usual compound discounting, the discount rate is supposed to be the IRR, which is 14.29 % on the whole duration. During the first year, this discounting approach supposes that only 2.08% of the flows are generated by the project, whilst 97.92% come from the expected replacements. On the whole duration, only 8.71% (308.31) of the cash inflow are supposed to come from the project, and 91.29% (3 230.32) from replacements. On the contrary, the generalised approach will accord more weight to internally generated returns. In the case of generalised compound discounting (GIRR), the first year's and the overall weights of replacements are respectively 63.13% (14.89) and 40.29% (208). In the case of generalised simple discounting (SIRR), they are respectively 50.37% (8.83) and 32.42% (147.91). The rates of reference are then an IRR of 14.29%, a GIRR of 7.17% and a SIRR of 6.73%. Furthermore, the rate of return without replacement (BIRR) is 5.35%, all rates being computed on the whole duration.

At the same time, the PSC should expect to borrow for financing the same construction cost and then repay bullet gilt on the same duration (30 years). This implies a usual IRR of 3.5% (the same as the borrowing rate). The difference between the project's rates and the PSC's rate is then an estimate of the financial advantage/disadvantage of the PFI scheme relative to the PSC alternative, computed under the above assumptions on the level and structure of the reference rates. If the alternatives are equivalent, especially for qualities of facility and flow of services through time, this score may declare either the superior rank of the PSC option, from the financial costing viewpoint, in that case, or a measure of the financial profit (surplus) generated by PFI that should be shared among undertakers.

It should be noted that, since time and context are not longer abstracted away, the project’s IRRs and the long-term government bond yield are not directly comparable, since their replacement structure is different: compound interest in the first case, simple interest in the second one. This means that, from the government’s viewpoint, the required series of cash outflows is not equivalent, since it must pay for compound

returns in the case of a private sector investment, whilst it would pay just a simple return in the case of direct public borrowing through long-term government gilts. Following the generalised approach, then, the PSC should be based on the BIRR, i.e., the rate of return under simple replacement. In this case, the borrowing rate of reference would be 3.21% instead of the 3.50% under the PSC scheme. From the financier's viewpoint, the benchmark return for the outside option shall depend on the latter level and structure.

§ Endogenous optimal duration

The generalised approach provides decision-makers with a further outcome: the temporal sequence of generalised returns, computed period by period, can generate an optimal value through time.[18] This optimal value represents a sort of inner financial term depending on the replacement rate level and structure. This term may be considered as an *endogenous optimal duration* (EOD) for the joint project of investment, "an issue about which economists have had remarkably little to say" (Hart, 2003: 275). If multiple optimal values exist, decision-makers shall choice among the *earliest* value and the *dominant* optimum during the expected life of the joint project.

In a recent article, Tirole (1999)[19] suggests to provide BOT (Build, Operate and Transfer) projects[20] with an endogenous termination rule related to the successful achievement of the least present value of gross revenues. This rule may be agreed by the undertakers and enforced by the final agreement that includes the pro-forma financial statements of the joint project. According to Spackman (2002: 287), this rule was actually applied to tolled water crossings during the 1980s in UK. The monitoring of the crossing project was then based on maintenance and condition of the crossing on its eventual handover, and a maximum concession period based on a fixed real present value of total toll revenue (rather than tied to uncertain traffic flows). Generally speaking, even under more recent PFI schemes, payments are usually made only for

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[18] The EOD is related to the temporal evolution of the GIRRs period by period. Sufficient conditions for the existence of at least one optimal value may be easily found in the case of steady or steady-increasing positive inflows following strictly initial negative outflows. In the case of steady inflows, a unique optimal value might exist if each annual inflow is greater than the alternative benchmark return provided by $i \cdot \sum_{j=0}^{t} \bar{a}_j$.

[19] Reference is especially made to Engel-Fischer-Galetovic (1997). Guriev and Kvasov (2005) modelled relational contracting with termination or renegotiation clauses through time.

[20] A kind of project-financing arrangement between the buyer of final services and the promoter of the investment project.

availability and for the effect of heavy use on maintenance, whilst revenue uncertainties have very limited impact on the PFI payments, if any, because of the securitisation of the project's debt service.

Rules like that relate to the second role of investment valuation criteria, that is, their function as part of the return sharing devices provided by the institutional structure of the project. In this context, the EOD depends on the structure of such relevant parameters as cash flow profile, alternative replacement opportunities, and so on. The alternative replacement rate may be fixed according to the interest rate that the buyer (i.e., the procuring public Authority) could paid on financial markets for long-term borrowing, a rate generally inferior to the internal rate of return the promoter expects from the project-financing scheme. The EOD can then be applied as a financial benchmark to negotiate the actual term established by the final agreement.

For assisting the negotiation, the "quantitative spreadsheet analysis" (HM Treasury 2007a, 2007b) may compute the EOD by a simple recursive method applied to the cash flows profile, period by period.[21] Following the GIRR, SIR and BIR definitions, each annual cash flow (a_t) will be discounted either at internal rate if negative (it is then assumed to be a cash outflow for investment), or at the alternative outside rate *i* otherwise. Generally speaking, all the investment cash flows are disbursed during the construction phase (including interests capitalised). The construction cost may be expressed by just one negative flow (a_0) disbursed on the first period ($t = 0$). Then, the working rule for calculating every annual rate will be simplified as follows:

$$GIRR_t = \sqrt[t]{\frac{\sum_{j=1}^{t} a_j^{+} \cdot (1+i)^{t-j}}{-a_0}} - 1$$

$$SIRR_t = \sqrt[t]{\frac{\sum_{j=1}^{t} a_j^{+} \cdot (1+i \cdot (t-j))}{-a_0}} - 1$$

$$BIRR_t = \sqrt[t]{\frac{\sum_{j=1}^{t} a_j^{+}}{-a_0}} - 1$$

[21] Simulators like that developed by HM Treasury (2007b) or the INFRISK software developed by The World Bank Group (Dailami-Lipkovich-Van Dyck 1999) may easily integrate an EOD measure worked out on statistical basis.

In the case discussed above, starting from the beginning of operations, the sequence of $GIRR_t$ will have an optimal value of 13.28% at the seventh year. The $SIRR_t$ will have an optimal value of 10.18% at the tenth year, and the $BIRR_t$ will have an optimal value of 5.67% at the twentieth year.

Table II and III report the data summary and the results obtained by replicating this analysis on a sample of analogous PFI projects under the same assumptions for discounting.

[Insert Table II and III here]

In all cases, the project's endogenous optimal duration (EOD) is less than the legal duration of 25 or 30 years[22]. From the financial viewpoint, if the project stops at the EOD, buyer will pay lesser for acquiring project's flow of services and the facility, and promoters will gain the maximum value allowed by investing in the project. Thereafter, promoters simply may utilise their proceeds in refinancing the buyer (or in investing elsewhere) at the alternative replacement rate of return (supposed to be 3.5% under GIRR and SIRR). In this way, they will acquire their biggest financial result according to the cash flow profile and to the replacement structure included in the final agreement. On the contrary, if the project lasts until the legal duration, the burden paid by the buyer will ever increase, even tough the return generated for the promoter has been decreasing. Even though the promoters may wish to gain from the project beyond its *financial* rentability, it seems hard to justify legal durations longer than the optimal duration under BIRR, since the latter corresponds to the rate of return entirely generated by the project alone (without replacements). Only the project C appears to be in line with this latter consideration, whilst the other legal durations materially exceed the respective optima under BIRR.

This calculation is surely trivial, whilst the actual development, financing and management of the joint project of investment are so complex that they first require judgement, skills and knowledge of the specifics of complex situations. Financial scores and quantitative measures may surely assist, but can not replace the decision-making process required for undertaking such a project. Focus is nevertheless required, and the generalised approach offers a focused perspective on the project's optimal term from the

[22] Generally speaking, a replacement rate of 4,5% will reduce the optimal duration of one-two years under GIRR and SIRR, whilst it obviously does not affect the optimal duration under the BIRR.

financial viewpoint. In this context, the EOD means that the potential undertakers cannot simply rely on the mechanics of compound discounting to share the project's benefits. The replacement structure concerned with timing and uses of financial withdraws from the project has to be explicitly taken into account by procuring public authorities and their advisers "as part of their bidder appraisal processes and subsequent negotiations with the private sector" (OGC-PFU 2002: 1).[23] Starting from the EOD date, for instance, the project's agreement may arrange a simultaneous switch in the rate of reference and in the financial structure of the joint project whose benefits have to be shared. The score is obviously not a compulsory term of the project. The agreement may establish a different (longer) duration, based on remuneration of such relevant factors as the quality of facility and services, the promoter's special competences, risk-bearing compensation, and so on. All the undertakers shall agree with this profit-sharing arrangement, and the EOD may constitute a benchmark that estimates the optimal duration from a financial viewpoint based on realistic assumptions.

3 - The private sector capital budgeting

§ The critique of the usual compound discounting for the private sector

The previous sections of the article have been concerned with the public sector, but even for the private sector the usual logic of discounting overarching capital budgeting techniques has been questioned (Porter 1998). In this latter context, neoclassical economics based on the market approach stresses a unique positive compound discount rate (Samuelson 1976: 473-474), considered either as the "*market* standard of performance", or the *competitive* opportunity rate for investment projects. On the contrary, welfare economics questions the level of the discount rate in case of welfare choices involving the "claims of posterity" (Samuelson 1976: 487-488; Arrow-Lind 1970). Some industrial economists -like Hayes and Garvin (1982, summary)- further criticise the logic underlying compound discounting and argue that:

[23] This document refers to the IRRs as part of those processes and negotiations.

> an over-reliance on analytic techniques like discounting future cash flows leads managers to defer critical investments in the capital stock on which their companies depend. Such techniques and the assumptions on which they rest, claim these authors, inevitably bias managers against investment and thus short-change the future of American industry.

Samuelson (1976) calls the critique of discounting the "bogey of compound interest". According to the bogey, decision-makers are unable to cope with the future of the ongoing activity when a preference for today is computed by a negative compound weight on future states. The usual discounting approach abstracts the financial process of investing away from time and context. Investment opportunities, indeed, could be mis-evaluated and the whole process of investment might be muddled. As a matter of fact, empirical researches deny that even widespread measures like present value and internal rate of return play a clear-cut role. According to Sangster (1993), business decision-makers still refer to criteria like paybacks and accounting rates, even though votaries of the discounting approach harshly criticised them. Although the criteria based on compound discounting (together with its underlying logic) are widespread and influential, decision-makers factually appear to utilise several scores based on different methods (even one theoretically incompatible with another) for gathering relevant information about the investment project coupled with an effective picture of its context and implications from different viewpoints.

§ Numerical sample of the generalised approach

The following numerical sample illustrates how the generalised approach operates in the capital budgeting process for the private sector. A joint project of investment has an expected economic life of five years, an initial investment of 1000 units and a cash inflow profile of 110, 120, 130 et 1000 units, received at the end of each period. Table IV discounts the profile at a discount rate $r = 9\%$, either at a compound rate (case A) or at a simple one (case B).

[Insert Table IV here]

This report clearly explains the different weight (value) that each inflow receives according to its position in time (timing). In this case, the usual NPV provides a little

but positive score (IRR being 9.5% > *r*), whilst the generalised approach would be more cautious. If a replacement opportunities rate of 5% is considered, the usual approach does over-evaluate the overall economic return, leading to accept a project that should be rejected under more realistic hypotheses.

The generalised approach provides potential undertakers with a spectrum of rates of return generated by different replacement opportunities structures. Table V shows three different cases: (a) the usual compound discount rate (IRR), (b) the GIRR with a compound replacement at 5%, and (c) the BIRR completely excluding replacement.

[Insert Table V here]

In every case (see Table VI), thus, 460 units are *earned* within the project (accrued income), whilst each case provides different estimate for the replacement of proceeds. The case (a) assumes 114 units, the case (b) 58 units, and the case (c) zero. Under the case (a), around 80% of total income[24] to make a 9.5% return comes directly from the project, and 20% from replacements. When inflows are reinvested at 5% (case b), the total inflows accumulation is 1518 and the over-all rate of return drops to about 8.7%; only 11% of total income is generated by replacements. Under the case (c), all the income is generated by the project alone, and its industrial rate of return will be then 7.86%.

[Insert Table VI here]

In this sample, the relative weight of each inflow does not change very much between the cases (a) and (b),[25] but the conventional value of cumulated replacement from each inflow varies relevantly between the different ways of discounting. Following the generalised approach, management may evaluate a range of scores according to the different assumptions of replacement at compound rate (see Table VII).[26]

[Insert Table VII here]

[24] That is accrued income and cumulated inflows from replacement.

[25] According to the sole replacement structure, the weights attributed to preceding inflows is more important in IRR than in GIRR, than in SIRR, than finally in BIRR.

[26] Each score makes the compound cumulated returns of the initial outflow (investment) equal to the cumulated cash flows including replacements.

In sum, this range constitutes the "return spectrum" of the financial return of the joint project. The generalised approach provides decision-makers with a synthetic range of values, and contrasts with the short-cut unique score provided by the usual approach. Further rules shall be applied to rank and accept projects according to the spectrum. For instance, decision-makers may accept either the project having the biggest BIRR or the biggest GIRR, or all the projects having at least some minimal score.

Furthermore, future and present values may be generalised by specifying distinctive rates of references for investing and replacing. The *generalised net future value* (GNFV) at (*y, i*) rates is:

$$GNFV_{y,i} = \sum_{j=0}^{t} \overset{+}{a}_j (1+i)^{t-j} - \sum_{j=0}^{t} \overset{-}{a}_j \cdot (1+y)^{t-j} .$$

Whilst the *generalised net present value* at (*y, i*) rates is:

$$GNPV_{y,i} = \sum_{j=0}^{t} \overset{+}{a}_j \cdot (1+i)^{-j} - \frac{\sum_{j=0}^{t} \overset{-}{a}_j (1+y)^{t-j}}{(1+i)^t}$$

where the first term is the future value of cash inflows reinvested at the replacement rate *i* and discounted back to time zero at the same rate, and the second term is the present value of all cash outflows discounted back at the replacement rate *i*.[27] The generalised net present value clearly shows the relevance of investment and replacement rates in the discounting method. When the rate *y* and the rate *i* are the same, $GNPV_y$ will be equivalent to the usual NPV_y.

Conclusion

Discounting-based valuation, including the "value for money" approach, is a widespread and influential socio-economic technology having a significant impact on policies of investment, both in the public and private sectors. Its importance calls for a

[27] We assume here that the generalised future value *from the project* (cash earnings) and the cash outflows *for the project* (investment) are discounted at the same compound rate *i*. This assumption may be released by taking three different rates: one for financing (related to cash outflows), one for investing (the risk-adjusted discount rate of the investment), one for replacement (related to cash inflows). In fact, concerning the discounting of sources of financing, we should consider a generalised weighted average cost of capital (G-WACC) based on the target capital and target financial temporal structures, since equity finances (compound interest as reference) usually have not the same temporal structure as debt finances (simple interest as reference).

comprehensive perspective that cannot be bound to mere financial measurement outcomes.

Investment valuation scores are nevertheless significant in investment decision-making, and can influence the whole process of investing, especially when applied to assessing and ranking alternative opportunities. In turn, recent theoretical insights cast doubts on the usual criteria, either present value or internal rate of return, because of their reliance on compound discounting and a unique return of reference.

Starting from more realistic assumptions and distinctive rates of return for investing and replacing, this article has developed a viable generalisation of the usual discounting techniques, and provided more appropriate valuation criteria to better approach the delicate matter of investment valuation even from the financial viewpoint.

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Table I – Summary and Analysis of one PFI scheme (called Project A) -Mill £

Year (t):	Construction	1	2	2	4	5
Cash Flow (a_t)	**-64,60**	**8,70**	**8,81**	**8,92**	**9,03**	**9,14**
Generalised Compound Discounted Future Cash Flow		*23,59*	*23,08*	*22,58*	*22,08*	*21,59*
Generalised Simple Discounted Future Cash Flow		*17,53*	*17,44*	*17,34*	*17,24*	*17,13*
Usual Compound Discounted Cash Flow		*418,27*	*370,56*	*328,24*	*290,71*	*257,43*

Table II – Summary of some PFI cases (including Project A from Table I)

Project	Legal Duration (years)	Construction Cost (Mill £)	Availability Charge (Mill £)	IRR (on the whole duration)
A	30	64.6	8.7	14.29%
B	30	66.7	8.0	12.68%
C	25	94	10.5	11.31%
D	30	84	11	13.89%
E	30	61	7.1	12.29%

Table III – Optimal rates and durations for some PFI cases

Project	Optimal GIRR	Optimal Duration under GIRR (year)	Optimal SIRR	Optimal duration under SIRR (year)	Optimal BIRR	Optimal Duration under BIRR (year)
A	13,28%	7	10,18%	10	5,67%	20
B	11,62%	8	8,97%	11	5,10%	23
C	8,74%	11	7,42%	13	4,78%	24
D	12,86%	8	9,88%	10	5,53%	21
E	11,21%	9	8,68%	11	4,96%	23

Table IV – Capital Budgeting in the private sector: the "Generalised Project"

Period	*1*	*2*	*3*	*4*	*5*
Cash inflow profile	**100**	**110**	**120**	**130**	**1000**
Case A (r compound)	91,74	92,58	92,66	92,10	649,93
Case B (r simple)	91,74	93,22	94,49	95,59	689,66

Table V - The financial analysis of the "Generalised Project"

Period	***Project inflows***	**Inflows from replacement**			**Total cash inflows**		
		(a) at IRR	*(b) GIRR at 5%*	*(c) BIRR - without replacement*	*(a)*	*(b)*	*(c)*
1	*100*	0	0	0	100,0	100,0	100
2	*110*	9,5	5,0	0	119,5	115,0	110
3	*120*	20,9	10,8	0	140,9	130,8	120
4	*130*	34,2	17,3	0	164,2	147,3	130
5	*1000*	49,9	24,7	0	1.049,9	1.024,7	1.000
Cumulated	**1.460**	**114,5**	**57,7**	**0**	**1.574,5**	**1.517,7**	**1.460**
				IRRs	9,50%	8,70%	7,86%

Table VI - The different replacement hypotheses compared in the "Generalised Project"

Period	**Project inflows**	**Cumulated replacement from each inflow**			**Weights for each inflow**		
		(a)	*(b)*	*(c)*	*(a)*	*(b)*	*(c)*
1	100	43,8	21,6	0	1,38	1,37	1
2	110	34,4	17,3	0	1,30	1,30	1
3	120	23,9	12,3	0	1,21	1,21	1
4	130	12,4	6,5	0	1,11	1,11	1
5	1000	0	0	0	1,00	1,00	1
	Totals	114,5	57,7	0	6,00	6,00	5,00

Table VII - The return spectrum of the "Generalised Project"

(a) usual IRR at *r* (related to $NPV_r = 0$)	9,50%
(b) GIRR (compound replacement at 5%)	8,70%
(c) BIRR (Basic-IRR, without replacement)	7,86%