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Abstract

We study mechanisms for selecting up to m out of n projects. Project managers' private information on quality is elicited through transfers. Under limited liability, the optimal mechanism selects projects that maximize some function of the project's observable and reported characteristics. When all reported qualities exceed their own project-specific thresholds, the selected set only depends on observable characteristics, not reported qualities. Each threshold is related to (i) the outside option level at which the cost and benefit of eliciting information on the project cancel out and (ii) the optimal value of selecting one among infinitely many ex ante identical projects.

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Many individuals, firms and government agencies face situations in which they need to choose between a number of projects. Often, when making this choice the decision maker (from now on "the firm") is not fully informed and needs to rely on better informed agents whose interests are not aligned with the firm's. Our paper studies the firm's project selection problem under asymmetric information. For example, a firm, or a government agency, deciding which R&D projects to pursue, from projects proposed by managers working for the firm; or a corporate board deciding which capital investment project to finance.

Projects can be risky, the likelihood of success (or the net expected return) might not be known to the decision maker who is not sufficiently familiar with the technical details. Project managers hold private information about the quality of their projects, and typically prefer that their own project be selected. Hence, their interests are not perfectly aligned with those of the decision maker, and an agency problem arises. As Paul Sharpe (vice president at SmithKline Beecham) and Tom Keelin (1998) described (page 45):

> Major resource-allocation decisions are never easy. For a pharmaceuticals company like SB [SmithKline Beecham], the problem is this: How do you make good decisions in a high-risk, technically complex business when the information you need to make those decisions comes largely from the project champions who are competing against one another for resources?

In our model, a firm can only choose to pursue a limited number of projects (up to m projects) from a given selection of ($n \geq m$) projects. Each project, if selected, would yield some return to the firm. Projects of higher quality yield higher returns. There is a fixed set of projects to choose from and the firm knows the distribution of qualities for each project. Managers have private information about the quality of their project. For example, a manager might know the probability of success of his project. A project manager enjoys a private benefit if his own project is selected, and competes with other managers to be financed. Hence, unless given the right incentives, a project manager might overstate the quality of his project. It is costly for the firm to elicit information on project quality.

We take a mechanism design approach and look for an optimal mechanism. A direct mechanism consists of a project selection rule and a transfer scheme (which depend on reported project qualities). Transfers are restricted to be nonnegative – a limited liability constraint. This constraint distinguishes our problem from the classic design of an optimal auction, where transfers are bounded by an individual rationality constraint. It is appropriate in contexts where the firm cannot take away money from project managers, or when managers are entitled to some sort of base salary, as may be the case within a firm.

Regarding limited liability, the following remark is in order. The limited liability constraint does not lead to interesting and new solutions in all problems. For example, if the firm only cares about (minimizing) transfers, the solution is trivial. It should select projects randomly and never compensate any manager. Or if the firm only cares about selecting a good project, it should use a Vickrey-Clarke-Groves mechanism. The constraint is theoretically interesting in the case where the firm cares both about selecting a good project and about not spending too much on transfers, as is the case in our model.

We focus on mechanisms that only depend on reported project qualities and not on the realized outcome of the selected projects. Such schemes may be appealing for R&D projects that take a long time to be completed. For example, based on an interview of Merck's CFO Judy Lewent, Nicholas (1994) states that it takes about 10 years to bring a drug to market, and once there, 70 percent of the products fail to return the cost of capital. It may also be that the outcome of the project is only observable by the firm (and not by the project managers or by a third party). This would be the case, for example, if the project is part of a bigger scheme. Moreover, in some cases, for example in basic research projects, verifying success might prove difficult. We also characterize the optimal mechanism when transfers can be contingent on the realization of project selection and returns. We show that in general mechanisms that allow such state dependent transfers can yield higher profits than the simple mechanisms we discussed earlier. But, when private benefits of project managers do not depend on project quality, the firm cannot improve its profits by making transfers state dependent.

If the firm knew the projects' qualities, the optimum would be to choose the m highest quality projects. However, in the presence of information asymmetry, the optimal mechanism is such that sometimes the firm selects inferior projects. When project managers are ex-ante identical, the optimal project selection rule involves a cutoff quality such that if at most

m project managers report a quality above the cutoff, the highest quality projects would be selected. But if more project managers report high quality, the firm's allocation rule randomizes between those projects whose quality exceeds the cutoff. Hence, suboptimal projects may be selected. When projects are ex-ante asymmetric, the inefficiency of the optimal mechanism takes a more subtle form. For any given realized qualities of the other projects, the probability that any particular project is selected is constant with respect to its own quality when it is higher than some predetermined project-specific quality cutoff. Each project's quality cutoff only depends on the project's own observable characteristics.

Intuitively, the firm faces a trade-off between choosing the most promising project, and saving on costs of eliciting information. When project managers' private benefits from being selected are very high, the cost of eliciting information may be too high, and the optimal mechanism would be to choose a project independent of reported quality. A mechanism that always chooses a fully efficient allocation is only optimal if project managers have no private benefits (or if information is complete). Between these two extremes, there is a subset of the project quality space (one with multiple promising projects) over which the firm's selection is independent of the quality of projects in that range. Within this region, the marginal cost of eliciting additional quality information exceeds the marginal value of this information.

The optimal mechanism involves transfers that help the firm to elicit information. Managers of projects that were not selected may get transfers that compensate them for truthfully reporting lower quality. While rewarding a manager of a project that is not selected may seem surprising at first, such mechanism can be important for providing incentives and can be implemented in practice. Rewards for nonselected projects may be monetary or of other nature. For example, within an organization, an R&D project manager that was not selected may be transferred to a (possibly better paid, more prestigious or more secure) position involving administrative duties.

While our paper's main motivation is project selection, our model may also be applicable to other situations. For example, the model can also capture the problem of a constrained employer who needs to decide who should retire. The employees ("project managers") benefit from continuing to work. Their benefits are positively correlated with their productivity, and some aspects of it is private information (such as the person's health). Those who retire receive a monetary incentive to do so. Similarly, our model might capture the decision of an employer who needs to decide whom among n employees to promote to $m \leq n$ desirable positions. In order to select the best candidates for these positions, the others may be offered some compensation. Or consider parents who need to decide who of their kids to send to college, those not sent to college may receive other monetary rewards. In all these examples, it is crucial that the set of candidate projects and their quality distributions is exogenous and fixed. Otherwise, the commitment to pay those not selected could attract bad projects or reduce incentives of a project manager to either improve or acquire information about the quality of his project.

The rest of the paper is organized as follows: in section I we present the model; in section II we consider the simple choice between a single project and an outside option; the optimal mechanism for the general problem is derived in section III. In section IV we present comparative statics results; in section V we discuss extensions of the model, the formal derivations of these are available in an online appendix. In section VI we review related literature; Section VII provides concluding remarks. All proofs are in the appendix.

1 Model

A firm faces a choice between n projects $i \in \mathcal{N} = \{1, ..., n\}$. Being resource constrained, the firm can choose to select up to $m \leq n$ projects. Each project is represented by an agent (a researcher or project manager). Project i's true quality q_i, which is its net expected return for the firm, is drawn from a distribution $F_i(q_i)$ on $[\underline{q_i}, \overline{q_i}] \subset \mathbb{R}$ with a positive density function $f_i(q_i) > 0$. We assume that q_i are independent random variables. The quality of each project is private information of its manager. In addition to the n projects, the firm has an outside option that must be chosen if no other project is selected. The outside option is the best alternative to selecting a project. It can be that the best alternative is not to select any project which yields $q^\circ = 0$, or a project whose quality $q^\circ \in \mathbb{R}$ is known to the firm. If not selecting a project is possible then the outside option satisfies $q^o \geq 0$.

Project managers enjoy a private benefit $b_i(q_i) \geq 0$ if their own project is selected, which represents an expected value of payoffs.[1] Because we assume it is the firm who finances the project, and not the managers, we find it reasonable to assume that benefits are nonnegative. We also assume the private benefit is nondecreasing in the quality of the project $b_i'(q_i) \geq 0$. The benefit is assumed weakly increasing to captures the fact that managers may get a higher benefit from being financed when their projects are more profitable. This can result from a grater feeling of satisfaction, pride or reputation from working on a project that is more likely to succeed.[2] Higher quality projects can also be easier to implement. The benefit functions are common knowledge. In our model, a project manager cannot benefit from the project unless it is selected and financed. For example, a researcher working for a pharmaceutical company is not likely to be able to develop a drug on her own, and may be limited in her ability to secure other funding for a project (possibly because of "covenant not to compete" agreements with the company she works for, or because the value of the project is firm specific).

The firm decides which projects to select, and whether to offer additional payments to project managers. An allocation $\mathbf{p} = (p_1, ..., p_n)$ is a vector of probabilities, with p_i representing project i's probability to be selected. A *transfers vector* $\mathbf{t} = (t_1, ..., t_n)$ indicates how much money each manager receives from the firm. The firm cannot take money from the managers, i.e. managers have limited liability, $\mathbf{t} \geq 0$. In a direct mechanism, each project manager reports her quality and the firm selects projects and transfers money to managers depending on the reports. Thus, a direct mechanism is a function $\mathbf{q} \mapsto (\mathbf{p}(\mathbf{q}), \mathbf{t}(\mathbf{q}))$.[3]

In the game played, the firm first chooses and commits to a direct mechanism. The managers of the n projects simultaneously report to the firm their projects' qualities and the allocation $\mathbf{p}(\mathbf{q})$ and transfers $\mathbf{t}(\mathbf{q})$ are realized. We look for a dominant-strategy incentive

[1]When project quality represents the probability of success of a binary random variable, we can write $b(q_i) = q_i b_i^s + (1 - q_i) b_i^f$ where b_i^s is the private benefit in case the project succeeds and b_i^f is the private benefit in case it fails. It is natural to assume in this example that $b_i^f < b_i^s$.

[2]Stern (2003) found, using survey data on job offers made to PhD biologists, that offers which contained science-oriented provisions, were associated with lower monetary compensation and starting wages. Such result lends support to the intuitive assumption that project managers enjoy private benefits from having their own projects financed.

[3]In section V, we also consider more general mechanisms with transfers that depend on observed realized returns of selected projects, when the quality of a project is its expected return.

compatible mechanism. The firm's objective is to maximize its expected profits:

$$\max_{\{p_i(\mathbf{q}),t_i(\mathbf{q})\}_i} \left\{ \mathbb{E}_{\mathbf{q}} \left[\sum_{i=1}^{n} [(q_i - q^\circ)\, p_i(\mathbf{q}) - t_i(\mathbf{q})] \right] \right\} \tag{1}$$

subject to

$$\text{feasibility} : \sum_{i=1}^{n} p_i(\mathbf{q}) \leq m \;\text{ and } 0 \leq p_j(\mathbf{q}) \leq 1 \text{ for all } \mathbf{q} \text{ and } j; \tag{2}$$

$$\text{limited liability } (LL_i) : t_i(\mathbf{q}) \geq 0 \text{ for all } i \text{ and for all } \mathbf{q}; \tag{3}$$

and the following incentive compatibility constraints: for all $\mathbf{q}$, for all i, and for all q_i',

$$(IC_i) : p_i(\mathbf{q}) b_i(q_i) + t_i(\mathbf{q}) \geq p_i(q_i', \mathbf{q}_{-i}) b_i(q_i) + t_i(q_i', \mathbf{q}_{-i}), \tag{4}$$

where $\mathbf{q} = (q_1, ..., q_n)$, with its j-th component being project j's quality, and $(q_i', \mathbf{q}_{-i})$ equals $\mathbf{q}$ everywhere excepts in its i-th coordinate which is replaced with q_i'.[4]

Finally, we introduce notation that would be helpful in later analyses. For any quality of project i, q_i, let

$$G_i(q_i) = q_i + b_i(q_i) + b_i'(q_i) \frac{F_i(q_i)}{f_i(q_i)} \tag{5}$$

be the *virtual return* that is obtained from selecting project i. Throughout the paper, we assume that this function is strictly increasing. Note that the sum of the first two terms is necessarily strictly increasing. The function is monotone as long as the derivative of the third term is not too negative. This holds true for a wide range of parametric assumptions for example, if $b_i(q_i)$ is convex (or linear) in q_i and the reverse hazard $f_i(q_i)/F_i(q_i)$ is nonincreasing. The latter holds, for nearly all the commonly used distributions (see Moor 1985). Additionally, the function $G_i(q_i)$ is strictly increasing for any distribution when $b_i(q_i)$ is constant.

[4]The constraint (4) is a dominant-strategy incentive compatibility condition, as opposed to a Bayesian incentive compatibility constraint often used. In an online appendix we show that the optimal mechanism we find is also optimal among Bayesian incentive compatible mechanisms. This result is related to Manelli and Vincent (2010) although we could not directly apply their findings due to the presence of the limited liability constraint.

2 One Project

In this section we derive the optimal mechanism when the firm faces the choice between selecting a unique project i or the outside option. The analysis of this case, where $m = n = 1$, will facilitate the analysis of the general problem in section IV. The unique project is denoted by i rather than 1 to avoid repetitions in the rest of the paper.

2.1 Threshold mechanisms

A natural family of direct mechanisms to consider is to select the project whenever its reported quality exceeds a given threshold, and choose the outside option otherwise.

Definition 1 *A threshold mechanism a, for $a \in [\underline{q_i}, \overline{q_i}]$ is defined by:*

$$p_i(q_i) = \begin{cases} 1 & \text{if } q_i \geq a, \\ 0 & \text{if } q_i < a. \end{cases} \quad \text{and } t_i(q_i) = \begin{cases} 0 & \text{if } q_i \geq a, \\ b_i(a) & \text{if } q_i < a. \end{cases}$$

The outside option is selected with probability $1 - p_i(q_i)$.

The transfers make truthful reports incentive compatible. There is no need to give a transfer to the manager whose project is selected. However, whenever the project is not selected, the manager is compensated an amount equal to $b_i(a)$. By falsely reporting a quality high enough to be selected, the manager could get at most $b_i(q_i) \leq b_i(a)$, so there is no incentive to overstate the quality. In a threshold mechanism with $a = \underline{q_i}$, the project is always selected.

Always selecting the outside option, without transfers, is not a threshold mechanism.[5] We refer to this mechanism as the *outside option mechanism.* In the remainder of the section, we look for the optimal mechanism among all threshold mechanisms and the outside option mechanism. In section III, we establish that this restriction is without loss of generality.

In a threshold mechanism a, when the project quality is $q_i < a$, the firm selects the outside option and pays $b_i(a)$ to the project manager. In this range, the firm's payoff is constant,

[5] If we redefine transfers so that $p_i(q_i) = 1$ for $q_i > a$, there is a threshold mechanism that always selects the outside option. However, by definition 1, it would be accompanied by positive transfers. The outside mechanism defined here involves no transfers.

$q^\circ - b_i(a)$. When the project quality is $q_i \geq a$ the project is selected and no transfers are given. In this range, the firm's payoff is q_i. Thus, the expected profit for the firm under the threshold mechanism a and the outside option q° equals

$$V(a, q^\circ) = (q^\circ - b_i(a)) F(a) + \int_a^{\overline{q_i}} q_i f_i(q_i) \, dq_i.$$

Substituting $b_i(a)F(a) = b_i(\overline{q_i}) - \int_a^{\overline{q_i}} [b_i(q_i) f_i(q_i) + b_i'(q_i) F(q_i)] \, dq_i$ we obtain

$$V(a, q^\circ) = q^\circ F(a) + \int_a^{\overline{q_i}} [b_i(q_i) f_i(q_i) + b_i'(q_i) F(q_i)] \, dq_i - b_i(\overline{q_i}) + \int_a^{\overline{q_i}} q_i f_i(q_i) \, dq_i.$$

Rearranging and using the definition of the virtual return $G_i(\cdot)$ in (5) we obtain

$$V(a, q^\circ) = \int_a^{\overline{q_i}} G_i(q_i) f_i(q_i) \, dq_i - b_i(\overline{q_i}) + q^\circ F(a). \tag{6}$$

We look for the quality threshold $a_i^*(q^\circ)$ that maximizes this value for a given q°. Because $G_i(\cdot)$ is increasing, the firm's profit $V(\cdot, \cdot)$ is single-peaked in a. In an interior solution,

$$\frac{\partial V}{\partial a}(a, q^\circ) = -G_i(a) f_i(a) + q^\circ f_i(a) = 0,$$

which implies $G_i(a_i^*(q^\circ)) = q^\circ$. That is, the profit $V(a, q^\circ)$ is maximized at a threshold quality a that equalizes the virtual return $G_i(a)$ with the quality of the outside option q°. For low values of the outside option, $q^\circ < G_i\left(\underline{q_i}\right)$, $\frac{\partial V}{\partial a}(a, q^\circ) < 0$ for all a, which implies that the optimal threshold is $\underline{q_i}$. Similarly for high values of the outside option, $q^\circ > G_i(\overline{q_i})$ the optimal threshold is $\overline{q_i}$. Thus,

$$a_i^*(q^\circ) = \begin{cases} \overline{q_i} & \text{if} \quad q^\circ > G_i(\overline{q_i}), \\ G_i^{-1}(q^\circ) & \text{if} \quad q^\circ \in \left[G_i\left(\underline{q_i}\right), G_i(\overline{q_i})\right], \\ \underline{q_i} & \text{if} \quad q^\circ < G_i\left(\underline{q_i}\right). \end{cases} \tag{7}$$

From these expressions, we see that the optimal threshold $a_i^*(q^\circ)$ is a nondecreasing function of the outside option q°, intuitively, the principal will be more selective when her outside option is better. We also note that if the value of the outside option is at least as large as the worse project $q^\circ \geq \underline{q_i}$ then the threshold is smaller than or equal to the outside option $a_i^*(q^\circ) \leq q^\circ$. This means that some projects may be selected even if their reported return is lower than that of the outside option. The reason for this is that a wider range of selection reduces the need to offer transfers.

Let the value of the optimal threshold mechanism to the firm, given q° be:

$$V^{*}\left(q^{\circ}\right)=\max_{a} V\left(a, q^{\circ}\right)=V\left(a_{i}^{*}\left(q^{\circ}\right), q^{\circ}\right). \tag{8}$$

2.2 Best threshold mechanism vs. the outside option mechanism

The outside option mechanism is not a threshold mechanism. Therefore, we still need to compare the value of the optimal threshold mechanism with the outside option q°. For any project i (characterized by a distribution of project qualities and a private benefit function) we find the value of the outside option g_i° that makes the firm indifferent between the best threshold mechanism and the outside option mechanism. We refer to this value of the outside option as the *cap*. If the outside option exceeds the cap g_i°, then the firm should stick to the outside option mechanism – always select the outside option and never provide transfers. If the outside option is lower than the cap, then the firm should use the optimal threshold mechanism which sometimes selects the project and other times selects the outside option. The cap g_i° only depends on project i's observable characteristics. It will play an important role in the analysis of the general case in section IV.

Lemma 1 *There exists a quality* $g_i^{\circ} \in \left[\mathbb{E}\left[q_i\right], G_i\left(\overline{q_i}\right)\right]$, *unique in* $\mathbb{R}$, *such that*

$$V^{*}\left(g_i^{\circ}\right)-g_i^{\circ}=0. \tag{9}$$

The threshold mechanism can be thought of as if the firm offers the candidate a choice between being selected for the project and being paid a fixed monetary amount. The optimal monetary amount that the firm should offer and the expected transfer to the candidate are nondecreasing functions of the quality of the outside option. Essentially, the firm buys information on the quality of the project. The better the outside option, the higher the price the firm must pay to elicit this information. If the quality of the outside option is high enough (exceeds the cap g_i°), it is better to chose the outside option without eliciting any information.

2.3 Firm's optimal profit

We now derive a simple expression of the firm's optimal profit. Consider first the case $q^\circ \leq g_i^\circ$, so that the optimal threshold mechanism dominates the outside option mechanism. Using (8) at q° and g_i° and taking the difference between the two expressions, we obtain

$$V^*(q^\circ) - V^*(g_i^\circ) = \int_{a_i^*(q^\circ)}^{a_i^*(g_i^\circ)} G_i(q_i) f_i(q_i)\, dq_i + q^\circ F(a_i^*(q^\circ)) - g_i^\circ F(a_i^*(g_i^\circ)).$$

Using $V^*(g_i^\circ) = g_i^\circ$ and rearranging terms,

$$\begin{aligned} V^*(q^\circ) &= q^\circ F(a_i^*(q^\circ)) + \int_{a_i^*(q^\circ)}^{a_i^*(g_i^\circ)} G_i(q_i) f_i(q_i)\, dq_i + g_i^\circ (1 - F(a_i^*(g_i^\circ))) \\ &= \int_{\underline{q_i}}^{a_i^*(q^\circ)} q^\circ f_i(q_i)\, dq_i + \int_{a_i^*(q^\circ)}^{a_i^*(g_i^\circ)} G_i(q_i) f_i(q_i)\, dq_i + \int_{a_i^*(g_i^\circ)}^{\overline{q_i}} g_i^\circ f_i(q_i)\, dq_i \\ &= \mathbb{E}(\max\{\min\{G_i(q_i), g_i^\circ\}, q^\circ\}). \end{aligned}$$

The third equality is true because $q^\circ \leq G_i(q_i)$ if and only if $a_i^*(q^\circ) \leq q_i$ and because $g_i^\circ \leq G_i(q_i)$ if and only if $a_i^*(g_i^\circ) \leq q_i$.

Let the *virtual quality* of the project be denoted by

$$x_i = \min\{G_i(q_i), g_i^\circ\}. \tag{10}$$

The project is selected if its virtual quality exceeds the outside option and otherwise the outside option is chosen. Hence

$$V^*(q^\circ) = \mathbb{E}(\max\{x_i, q^\circ\}). \tag{11}$$

In the case where the outside option is better than the optimal threshold mechanism, i.e. $q^\circ \geq g_i^\circ$, the profit of the firm at the optimal mechanism is q°, which also equals the expression (11). Therefore, the expression (11) describes the optimal profit of the firm for any outside option q°.

An implication of these derivations is that there are projects for which the decision maker never elicits information from the manager regardless what the outside option is, and for this reason, the manager will also never receive any compensation. For such project the virtual quality is constant $x_i = g_i^\circ = \mathbb{E}(q_i)$ for all q_i and the project will be selected if an only if its expected value $\mathbb{E}(q_i)$ exceeds the outside option q°. We provide a necessary and sufficient condition for the manager's report to be ignored in the following proposition.

Proposition 1 *A project's virtual quality is constant and equals $g_i^\circ = \mathbb{E}(q_i)$ if and only if $f_i(\cdot)$ and $b_i(\cdot)$ satisfy the inequality $b_i\left(\underline{q_i}\right) \geq \mathbb{E}(q_i) - \underline{q_i}$.*

According to the condition in proposition 1, the decision maker is more likely to ignore the manager's reports when the manager has a lot to benefit even from the lowest quality project, or when the distribution of project qualities exhibits less uncertainty in a mean preserving spread order. In other words, transfers are used to elicit information when the cost (the need to compensate) is not too high, and the benefit (avoiding the need to finance low quality projects) is high.

In the next section we use the derivations we made for the single project case to solve the problem of choosing m out of n projects. When multiple projects are available, for each project we define the virtual quality as in (10). We generalize the expression in (11) for the optimal value and suggest a candidate optimal mechanism – the highest m virtual quality projects should be selected when these exceed the outside option. Our proof for the general case confirms that the optimal threshold mechanism we derived for the single project case is optimal among all feasible direct mechanisms.

3 Optimal Mechanisms in the General Case

We search for a profit maximizing incentive compatible mechanism with a feasible project selection rule and transfers that satisfy the limited liability constraint. For simplicity, and without loss of generality, we will assume that the outside option has quality $q^\circ = 0$.

3.1 Optimal mechanisms

If information was symmetric, the firm would have always selected the m highest quality projects (or as many that are better than the outside option). However, the manager's private information on project qualities is costly for the firm to elicit. Because each project manager benefits if his project is selected, absent compensation, selecting the highest quality projects creates an incentive for managers to overstate the quality of their projects. In any incentive compatible mechanism the firm needs to compensate project managers for truth telling. Thus, when choosing projects the firm takes into account not only the quality of projects but also the cost of eliciting information. This is done by choosing the m highest virtual quality projects instead of simply the highest m quality projects.

Given a vector of real numbers $\mathbf{x} = (x_1, ..., x_{n+m})$, let $\left(x_{(1)}, ..., x_{(n+m)}\right)$ denote the order statistics of $\mathbf{x}$, i.e. the vector obtained by sorting the coordinates of $\mathbf{x}$ in a nonincreasing order, so that $x_{(1)} \geq ... \geq x_{(n+m)}$. Let $S_m(x_1, ..., x_{n+m})$ be the sum of the first m coordinates of the order statistics vector, i.e. the sum of the m highest coordinates of $\mathbf{x}$.

The firm needs to choose at most m out of n projects, it obtains its outside option of $q^\circ = 0$ for a project not selected. Let us represent its problem as a choice of exactly m projects from the set of $n + m$ project that contains the original n projects and in addition m projects that have a known quality $q^\circ = 0$. For any (n-dimensional) vector of project qualities $\mathbf{q}$ we define an $(n + m)$-dimensional vector of *virtual project qualities*

$$\mathbf{x}(\mathbf{q}) = \left(\min\left\{G_1(q_1), g_1^\circ\right\}, ..., \min\left\{G_n(q_n), g_n^\circ\right\}, \underbrace{0, ..., 0}_{m \text{ times}} \right). \tag{12}$$

where G_i and g_i° are defined in (5) and (9). The first n coordinates represent the virtual qualities of the n projects as we derived for the single project case in (10) in section III. The last m coordinates represent the outside option which can replace each of the m selections.

For the vector of virtual qualities $\mathbf{x}(\mathbf{q})$ in (12) $x_{(m)}(\mathbf{q})$ is its m-th highest coordinate. There may be multiple projects with this virtual quality. The allocation we propose selects with probability 1 any project that has a virtual quality which is strictly higher than the m-th highest virtual quality (there are less than m such projects). For the remaining selections, the mechanism chooses with equal probabilities projects from the set of (one or more) projects that have the m-th highest virtual quality. Other projects are not selected.

$$p_i^*(\mathbf{q}) = \begin{cases} 1 & \text{if} \quad x_i(\mathbf{q}) > x_{(m)}(\mathbf{q}) \\ \frac{m - \left|i \in \left\{i \in \mathcal{N} : x_i(\mathbf{q}) > x_{(m)}(\mathbf{q})\right\}\right|}{\left|\left\{i \in \mathcal{N} : x_i(\mathbf{q}) = x_{(m)}(\mathbf{q})\right\}\right|} & \text{if} \quad x_i(\mathbf{q}) = x_{(m)}(\mathbf{q}) \\ 0 & \text{if} \quad x_i(\mathbf{q}) < x_{(m)}(\mathbf{q}) \end{cases}. \tag{13}$$

With this allocation, $x_{(m)}(\mathbf{q})$ is the lowest virtual quality that is selected with a positive probability. To find transfers that ensure this allocation is incentive compatible we first find the quality of project i such that $G(q_i^m)$ yields the virtual quality $x_{(m)}(\mathbf{q})$ (when it exists). For each $i \in \mathcal{N}$, let $q_i^m = a_i^* \left(x_{(m)}(\mathbf{q}) \right)$. For the allocation rule $p_i^* (\mathbf{q})$ to be incentive compatible transfers are as follows

$$t_i^* (\mathbf{q}) = \left(p_i \left(\overline{q_i}, \mathbf{q}_{-i} \right) - p_i \left(\mathbf{q} \right) \right) b_i(q_i^m). \tag{14}$$

With these transfers, a project that is selected with probability $p_i = 1$ receives no transfer $t_i^* = 0$. For a project selected with probability $p_i = 0$, the transfer equals the highest expected benefit from misreporting. A manager with quality lower but arbitrarily close to q_i^m is not selected and his transfer should be arbitrarily close to $p_i \left(\overline{q_i}, \mathbf{q}_{-i} \right) b_i(q_i^m)$, because he can misreport $\overline{q_i}$. Hence, $p_i \left(\overline{q_i}, \mathbf{q}_{-i} \right) b_i(q_i^m)$ must be the transfer of any manager with $p_i = 0$ who could report a quality arbitrarily close to q_i^m. A manager with $p_i \in (0, 1)$ receives a positive transfer only if $p_i \left(\overline{q_i}, \mathbf{q}_{-i} \right) = 1$.

The following theorem generalizes the results obtained for the case $m = n = 1$ in section III to the general case $n \geq m \geq 1$. Let V_m denote the optimal profit for the firm that can select up to m projects. The optimal profit for the firm is the expected value of the sum of the highest m virtual qualities out of the $n + m$ available virtual qualities.

Theorem 1 *The project selection mechanism defined by the allocation rule (13) and the transfers (14) solves the problem stated in (1)-(4). It gives the firm the optimal profit*

$$V_m = \mathbb{E}_{\mathbf{q}} \left[S_m \left(\mathbf{x}(\mathbf{q}) \right) \right]. \tag{15}$$

In the following Corollary we describe the allocation rule of the optimal mechanism when all projects are ex-ante symmetric.

Corollary 1 *In the symmetric problem, $g_1^\circ = ... = g_n^\circ := g^\circ$. (i) If all projects have negative virtual qualities $x_i = \min \left\{ G_i \left(q_i \right), g^\circ \right\} < 0$, then the outside option is selected; (ii) If there are at least m projects in the sample such that $G \left(q_i \right) \geq g^\circ \geq 0$, then in an optimal mechanism, m of these projects are selected at random; (iii) Otherwise, the m (or less) projects satisfying $G \left(q_i \right) \geq 0$ whose qualities rank among the m highest are selected.*

In the symmetric case, when sufficiently many projects have a quality that exceeds a threshold quality, selection among these top candidates is random, and therefore a lower quality project can be selected over a higher quality one. The mechanism we propose in Theorem 1 is essentially the only symmetric optimal mechanism, except in the boundary case where $g^{\circ} = 0$. However, even in the generic case ($g^{\circ} \neq 0$), nonsymmetric generalizations of the proposed mechanism, which assign fixed (not necessarily equal) selection probabilities to agents whose virtual qualities equal the m-th highest virtual quality, are also optimal. The results for the symmetric case are illustrated in the following example.

Example 1 *Suppose that the projects are ex-ante symmetric, and that the quality is uniformly distributed on* $[0, 1]$ *and that the private benefit is a constant* $b \in (0, 0.5)$. *In the optimal mechanism (see Figure 1), when at least one project manager reports a quality which is lower than the cutoff* $a^* = 1 - \sqrt{2b}$, *the highest quality project is selected. However, in the top region, where* $q_1, q_2 \geq a^*$, *the two projects are selected with equal probabilities* $p_i = 0.5$. *Hence, a less efficient project may be selected. Figure 1a illustrates the project selection probabilities. Transfers are shown in figure 1b. Compensation for the manager whose project was not selected is just enough to ensure incentive compatibility. Hence, it equals zero if this manager's reported quality was higher than* a^*, *otherwise, it equals* b *if the selected project had a quality below* a^* *and* $0.5b$ *if it exceeded* a^*.

[Figure 1]

3.2 Optimal vs. constant mechanisms

In section II, we showed that when there is only one project to evaluate against the outside option, it is sometimes optimal not to elicit any information from its manager. We ask here a similar question, when there are several candidate projects. We consider the family of simpler mechanisms that do not attempt to elicit information from managers. In such mechanisms the allocation does not depend on reported qualities and no transfers are made to the managers. We refer to these mechanisms as *constant mechanisms*. The optimal mechanism we derived before clearly generates at least as high a profit to the firm as any constant mechanism (constant mechanisms are a subset of all mechanisms we considered). Following Theorem 1, we ask, when is the optimal mechanism strictly better than the best constant mechanism. We first provide conditions for a mechanism to be optimal among all constant mechanisms.

Lemma 2 *For any $\mathcal{M} \subset \mathcal{N}$ with $|\mathcal{M}| \leq m$, the constant mechanism that selects the subset of projects $\mathcal{M}$ is optimal among all constant mechanisms if and only if*

$$\max\left\{\max_{j\in\mathcal{N}\setminus\mathcal{M}} \mathbb{E}\left[q_j\right], 0\right\} \leq \min_{i\in\mathcal{M}} \mathbb{E}\left[q_i\right]$$

and

$$|\mathcal{M}| < m \implies \max_{j\in\mathcal{N}\setminus\mathcal{M}} \mathbb{E}\left[q_j\right] \leq 0.$$

The first condition states that the least attractive project that is selected is better than the best project that is not selected and also better than the outside option (zero return). The second condition states that if less than the m projects are selected, the best project that is not selected is less preferred than the outside option. From Theorem 1 and Lemma 2, we now derive conditions under which a constant mechanism is optimal among all mechanisms.

For the best constant mechanism $\mathcal{M}$ to coincide with the optimal mechanism, the allocations of the two mechanisms must coincide for any realization of $\mathbf{q}$. This is equivalent to say that the virtual quality of any project i in $\mathcal{M}$ must be greater than the virtual quality of any project j outside of $\mathcal{M}$, and also greater than the outside option 0, for any realization of $\mathbf{q}$. For this condition to hold it is sufficient (and clearly also necessary) that it holds for the realization of $\mathbf{q}$ which are worst for projects in $\mathcal{M}$ and at the same time, are best for projects not in $\mathcal{M}$. In the worst case scenario for a project i in $\mathcal{M}$, its virtual quality either equals $\underline{q_i} + b_i\left(\underline{q_i}\right)$, if for project i, $b_i\left(\underline{q_i}\right) + \underline{q_i} \geq \mathbb{E}\left(q_i\right)$ or else it equals $\mathbb{E}\left(q_i\right)$. In the best case scenario for a project j outside of them $\mathcal{M}$, its virtual quality equals g_j°. In the case where $\mathcal{M}$ contains less than m projects, optimality among all mechanisms also requires that in the best realization for any project j outside of $\mathcal{M}$, its virtual quality g_j° is also smaller than the outside option 0. We summarize these conditions in the following result.

Corollary 2 *An optimal constant mechanism that selects a subset of projects $\mathcal{M}$ is optimal among all mechanisms if and only if*

$$\max\left\{\max_{j\in\mathcal{N}\setminus\mathcal{M}} \left\{g_j^{\circ}\right\}, 0\right\} \leq \min_{i\in\mathcal{M}} \min\left\{\mathbb{E}\left[q_i\right], \underline{q_i} + b_i\left(\underline{q_i}\right)\right\}$$

and

$$|\mathcal{M}| < m \implies \max_{j \in \mathcal{N} \setminus \mathcal{M}} g_j^{\circ} \leq 0.$$

When the conditions of the corollary are not satisfied for any subset $\mathcal{M}$, the optimal mechanism is not a constant one and involves eliciting private information through transfers. Interestingly, one can construct examples where the optimal mechanism improves upon the outside option mechanism, even though the outside option mechanism happens to be the optimal constant mechanism. In such situations, using the optimal mechanism enables the firm to select a project, although it wouldn't have selected any if only constant (independent of reported qualities) mechanisms were allowed. The following result provides the necessary and sufficient conditions, as a direct implication of Corollary 2.

Corollary 3 *The outside option is the best constant mechanism and is inferior to the optimal mechanism if and only if for all j in $\mathcal{N}$, $E(q_j) \leq 0$ and for some i in $\mathcal{N}$, $g_i^{\circ} > 0$.*

The first inequality ensures that the outside option mechanism is the best constant mechanism. The second inequality ensures that it is inferior to the optimal mechanism. These conditions are likely to be satisfied if all projects have a negative expected quality, but at least one of the candidate projects has a highly uncertain quality relative to its manager willingness to be selected.[6]

4 Comparative Statics

In this section, we provide comparative statics results on the number of projects, the private benefit function and the distribution of quality.

4.1 The number of projects

We have computed the optimal project selection mechanism for a fixed number of projects n. The caps g_i°, defined in (9), that we use to define the optimal mechanism only depends on the characteristics of project i, not on other projects' characteristics, nor on the number of

[6]See the comparative statics on mean preserving spreads at the end of section 5.

candidate projects. However, the profit of the firm does depend on the number of available projects to choose from. We first observe that when the set of projects the firm can select from expands, for a fixed m, profit (weakly) increases. To see this, consider a firm that originally faced selection between n projects now faces selection between these same projects and one additional $n+1$-th project. A mechanism that treats the first n projects just like when these where the only projects and ignores the last project, $t_{n+1}(\mathbf{q}) = p_{n+1}(\mathbf{q}) = 0$ for all $\mathbf{q}$, is feasible and achieves the same profit as before. Hence, with the optimal mechanism profit is at least as high.

In the symmetric case and for a fixed m, one can show that the optimal expected profit (15) is concave in n.[7] In the general case, while profit increases as the set of available projects expands, as long as project quality is bounded, so is the expected profit. In the following proposition we derive a limit result. Suppose that the set of projects is randomly chosen from some population. The distribution of the caps g_i° defined in (9) determines the profit per project in the limit as the number of projects to choose from goes to infinity.

Proposition 2 *(i) Suppose that the vectors (g_i°, q_i) of caps and project qualities are independently and identically drawn from a continuous distribution.*[8] *Let $\left[\underline{g_i^\circ}, \overline{g_i^\circ}\right]$ be the support of the marginal distribution of g_i°. Suppose that the conditional cdf $F(\cdot \mid g_i^\circ)$ are such that any distributions conditional on a higher value of g_i° first order stochastically dominates a distributions conditional on a lower g_i°. Then for a fixed m, in the limit where n goes to infinity, the profit of the firm at the optimal mechanism converges in probability to $m\overline{g_i^\circ}$.*

(ii) In the symmetric case, where all thresholds equal g°, the profit of the firm as the number of projects increases to infinity converges in probability to mg°.

The second part of this proposition provides an asymptotic interpretation of the threshold g_i° of a project i. It is the limit of the per selected project return when the number of available

[7]This is because the expected value of each of the k-th order-statistics for $k = 1, ..., m$ of an $n+m$ dimensional i.i.d. random vector is concave in n. The firm's profit is the sum of the first m order-statics of the vector of virtual qualities.

[8]Note that the assumption require independence across projects, but a project i's quality and the cap associated with that project may be dependent.

projects approaches infinity and all candidate projects are ex-ante identical to project i, i.e. they all have the same threshold g_i°.

The number of projects available to a firm to choose from is likely however to be finite. The number of skilled researchers in the relevant technology might be limited. It is likely costly to increase the pool of potential projects identifying candidates and the relevant distribution from which their quality is drawn. There can also be administrative costs to consider more projects. In the following example we derive the profit from choosing one project ($m = 1$) as a function of the number of projects n in the symmetric model when project qualities are uniformly distributed and benefits are constant. This profit is increasing and concave in n. Hence accounting for some convex cost $c(n)$ there would be a unique optimal finite number of projects.

Example 2 *Suppose* $b(q_i) = b < 0.5$ *for all* q_i *and that the distribution is uniform on* $[0, 1]$. *The firm faces a choice of* $m = 1$ *out of* n *projects. Then,*

$$g_i^\circ = 1 - \sqrt{2b} + b. \tag{16}$$

$$a^* \equiv a^*(g_i^\circ) = 1 - \sqrt{2b} \tag{17}$$

In the optimal mechanism, if at most one project has a quality that exceeds $1 - \sqrt{2b}$, *the highest quality project would be selected. If two or more project qualities exceed this cutoff, one project among the projects with a quality higher than* $1 - \sqrt{2b}$ *would be selected at random. The expected profit for the firm is*

$$V(n) = \frac{1}{2}(1 + a^{*2}) - \frac{a^{*n+1}}{n+1}. \tag{18}$$

The profit $V(n)$ *is increasing and concave in the number of projects* n. *As the number of projects approaches infinity,* $V(n) \to 0.5(1 + a^{*2}) = 1 - \sqrt{2b} + b = g_i^\circ$.

4.2 Changes in private benefits

We study here the effect on the optimal mechanism of changes in the benefit functions b_i.

4.2.1 Asymmetric benefits

In the general asymmetric case, let i be an arbitrary project. Consider a change in project i's benefit function from $b_i(\cdot)$ to another benefit function $\widetilde{b}_i(\cdot)$ for project i, such that $\widetilde{b}_i(q_i) < b_i(q_i)$ for all q_i, but no other change in the fundamentals of the model. How does this change affect the optimal mechanism and profit?

Let V and $\widetilde{V}$ respectively denote the profit of the firm under the benefit functions $b_i(\cdot)$ and $\widetilde{b}_i(\cdot)$. We will show that $V \leq \widetilde{V}$. To see this, observe that under the benefit function $b_i(\cdot)$, the value V is achieved by the optimal mechanism defined by (13) and (14). Under the benefit function $\widetilde{b}_i(\cdot)$, this same allocation $\mathbf{p}^*(\cdot)$ defined by (13) is implemented, provided that transfers (14) are replace by transfers such that

$$\widetilde{t}_i(\mathbf{q}) = (p_i(\widehat{q}, \mathbf{q}_{-i}) - p_i(\mathbf{q}))\,\widetilde{b}_i(q_i^m).$$

This mechanism is feasible under the benefit function $\widetilde{b}_i(\cdot)$, i.e. it satisfies the constraints (2), (3) and (4). Since $\widetilde{b}_i(q_i) < b_i(q_i)$ for all q_i, this mechanism gives the firm a higher profit than the initial one. This mechanism may not be optimal under the benefit function $\widetilde{b}_i(\cdot)$, but the optimal expected profit of the firm is at least the profit achieved by this mechanism. Thus, the firm's optimal profit is higher under $\widetilde{b}_i(\cdot)$ than under $b_i(\cdot)$, which shows that a decrease of the benefit function raises the firm's optimal profit.

We now study the effect of b_i on the cap g_i° for project i. Suppose that manager i's private benefit depends on a real parameter λ so that the benefit function is $b_i(q_i, \lambda)$ which is increasing in q_i and in λ. From the analysis in section III, we know that the cap g_i° is determined by the following equation.

$$\max_a \left\{ g_i^\circ F_i(a) - b_i(a,\lambda) F_i(a) + \int_a^{\overline{q}_i} q_i f_i(q_i)\, dq_i \right\} - g_i^\circ = 0.$$

Abusing notations, let $a_i^* \equiv a_i^*(g_i^\circ, \lambda)$. By implicit differentiation,

$$\frac{dg_i^\circ}{d\lambda} = -\frac{F_i(a_i^*)}{(1 - F_i(a_i^*))} \frac{\partial b_i(a_i^*, \lambda)}{\partial \lambda} < 0.$$

Thus, increasing λ decreases the cap g_i° for project i.

What is the effect of increasing λ on the quality threshold a_i^*? Since this function is increasing in g_i°, the indirect effect of increasing λ is to decrease a_i^*, via g_i°. However, the direct effect is unclear as it also depends on how the shift changes the slope of b_i. One can sign this direct effect as well, under the additional assumption that increasing λ (weakly) increases both the private benefit b_i and its slope $\partial b_i/\partial q_i$. Notice that this includes the case of a vertical shift of the form $b_i(q_i, \lambda) = b_i(q_i) + \lambda$. Under these assumptions, the threshold a_i^* is decreasing in the shift λ.

Fixing the other project's reported qualities $\mathbf{q}_{-i}$, the effect of an increase in λ (and thus in $b_i(\cdot)$) on the set of qualities q_i at which project i is selected (and thus on the selection probability of project i) is nonmonotonic. It first enlarges it, but at some point may discontinuously reduce it to the empty set. To see this, consider the case where project i is the unique candidate project evaluated only against an outside option q°, as in section III. An increase in λ decreases the optimal quality threshold $a_i^*(q^\circ)$ above which project i is selected, and therefore increases the selection probability of project i. But the value of the optimal threshold mechanism also decreases and at some point becomes inferior to the outside option mechanism. When this occurs, the selection probability of project i jumps down to zero.

The above results also enable us to compare caps and thresholds in an asymmetric problem. If two project have the same quality distributions, but one has a higher private benefit $b_i(\cdot)$ than the other, it has a lower cap g_i°. If in addition it also has a higher benefit slope $\partial b_i/\partial q_i$, it then also has a lower threshold a_i^*. These results are illustrated in the following example.

Example 3 *Let $n = 2$ and $m = 1$. Consider the optimal mechanism when qualities are drawn from the same distribution $F_1(.) = F_2(.)$; private benefits are constant $b_2 > b_1 \geq 0$; also assume $E(\mathbf{q}) > \underline{q}_i + b_i$ so that $a_i^* > \underline{q}_i$. Then $g_1^\circ \geq g_2^\circ$ and $a_1^* > a_2^*$. We observe some interesting properties of the optimal mechanism in this example. When both qualities exceed their corresponding cutoffs $q_i \geq a_i$, project 1, belonging to the manager whose private benefit is lower, is selected. This would give project manager 2 (who is most eager to be selected) less of an incentive to overstate his success probability. Therefore, there is a range of qualities for which project 1 is selected even though it has a lower quality. There is also a range of probabilities where project 2 has a lower quality than project 1 yet project 2 is selected. This will occur in the range where both probabilities are below their corresponding cutoffs,*

$q_i < a_i^*$, and (q_1, q_2) lies below the 45^o but above the $G_1(q_1) = G_2(q_2)$ curve (which is given by $q_2 = q_1 - (b_2 - b_1)$). In this case project 2 has a lower quality but a higher virtual quality than project 1 ($q_2 < q_1 < q_2 + b_2 - b_1$) and it is selected despite the fact that project 1 has a higher quality.

4.2.2 Symmetric benefits

In the symmetric case, consider an upward shift on the common private benefit $b(\cdot)$, while the distribution $F(\cdot)$ is held constant. Let the private benefit of all managers be $b(q_i, \lambda)$ which is increasing in q_i and in λ. From the analysis of the asymmetric case, increasing λ decreases both the common cap g_i° and the optimal expected profit of the firm. If in addition $\partial b/\partial q_i$ is also increasing in λ, then increasing λ also decreases the common quality threshold a^*. This results in a larger pooling region at the top. The set of quality profiles at which the highest quality project is not necessarily selected grows.

4.3 Changes in the distribution

One can study the effects of a first-order stochastic-dominant shift of the distribution $F_i(\cdot)$ for an arbitrary project i, while holding all other parameters constant. Using similar arguments as in the previous subsection, one can show that such a shift increases the expected profit of the firm and project i's cap g_i°. The effect of the shift on the quality threshold $a_i^* \equiv a_i^*(g_i^\circ)$ is ambiguous, but under the additional assumption that $b_i(\cdot)$ is a constant, one can show that the shift increases a_i^*. Notice however that the probability $F_i(a_i^*)$ that the quality is lower than a_i^* can move in either direction, since a_i^* moves up but $F_i(\cdot)$ moves down. An application of these results is given in the following example.

Example 4 *Let $n = 2$ and $m = 1$. Consider the optimal mechanism when $F_1(.)$ first-order stochastically dominates $F_2(.)$ but private benefits are constant and equal $b_2 = b_1 = b$. Then $g_1^\circ > g_2^\circ$ and $a_1^* > a_2^*$. When both qualities exceed their corresponding cutoffs, the virtual qualities are g_i° and so project 1 is selected.*

Similarly, one can study the effect of a mean preserving spread on the distribution $F_i(\cdot)$ for an arbitrary project i, while holding all other parameters constant.[9] Little can be said in this case, if the benefit b_i depends nontrivially on q_i. If b_i is constant, using again the same type of arguments as in the previous subsection, and the fact that the function $\max\{q^\circ - b_i, q_i\}$ is convex in q_i for any fixed q°, one can show that such a spread increases the expected profit of the firm, project i's cap g_i° and the quality threshold $a_i^* \equiv a_i^*(g_i^\circ)$. Here also, the probability $F_i(a_i^*)$ that the quality is lower than a_i^* can move in either direction, since $F_i(q_i)$ moves up or down depending on q_i. An application of these results is given in the following example.

Example 5 *Let $n = 2$ and $m = 1$. Consider the optimal mechanism when $F_1(.)$ is a mean preserving spread of $F_2(.)$ but private benefits are constant and equal $b_2 = b_1 = b$. Then $g_1^\circ > g_2^\circ$ and $a_1^* > a_2^*$. When both qualities exceed their corresponding cutoffs, the virtual qualities are g_i° and so project 1 is selected.*

5 Generalizations

In this section we informally describe directions in which our model can be generalized, and how the optimal mechanism would change. We provide corresponding formal derivations in an online appendix. We first allow transfers to be contingent on realization of project selection and outcomes. We next consider agents that have interests that are at least partially aligned with those of the firm. Finally we allow negative benefit functions so that some agents do not want to be selected.

5.1 State contingent transfers

In the analyses in the previous sections we have considered mechanisms that only depend on reported qualities – the only information available to the firm when it is making a selection. In this section we ask whether the firm can do better if it were able to make transfers contingent on the realized return of the projects that were selected. For example, if project

[9] Recall that $\widetilde{F}$ is a mean-preserving spread of F if the two distributions have the same expected value and for any concave function $u(\cdot)$ of q_i, we have $E_{\widetilde{F}}[u(q_i)] \le E_F[u(q_i)]$.

quality represents the probability of success of the project, and the firm can observe ex-post whether the selected project succeeded, then the transfer can depend on success. If state dependent mechanisms are available, the firm would generally achieve a higher profit than when transfers can only depend on reported probabilities.

To consider state contingent transfers we introduce a state space describing possible realized returns of the financed projects. The project quality which is known to the managers is the expected return of the project. We provide an upper bound on the value that the firm can obtain when it can pay transfers contingent on the realization of both the project stochastic allocation, and the selected projects' outcomes. A state $\mathbf{s} = (\mathcal{M}, \mathbf{R})$ indicates the set of projects (if any) that were selected $\mathcal{M} \subset \mathcal{N}$ and the realization of the return of the projects which were selected $\mathbf{R} \in \mathbb{R}^{|\mathcal{M}|}$. A mechanism is a function $(\widehat{\mathbf{q}}, \mathbf{s}) \mapsto (\mathbf{p}(\widehat{\mathbf{q}}), \mathbf{t}(\widehat{\mathbf{q}}, \mathbf{s}))$. We then write the constraint optimization problem, accounting for this larger space of mechanisms. We impose incentive constraints and the limited liability constraints which have to hold state by state.

The solution method for this problem follows similar steps as in sections II and III. We derive an upper bound for the optimal value, and then propose a mechanism that satisfies the constraints and (here only approximately) achieves the upper bound.

The optimal mechanism with state dependent transfers is characterized by a similar project selection rule as that with deterministic transfers. In particular, in the symmetric case, project selection is inefficient in the region where several managers report project qualities that exceed a cutoff. However, while with deterministic transfers the highest quality projects are selected for sure outside this region, with state dependent transfers, the firm could approach maximum profits if it allows low reported probability projects to be selected with some small probability, and highly rewards a favorable outcome in the unlikely event that the low quality project is selected. This can allow the firm to pay lower expected transfers to managers who report low quality and obtain a higher profit than with deterministic transfers. Interestingly, however, when private benefits are constant $(b_i'(q_i) = 0)$ the optimal mechanism with state dependent transfers is identical to that when transfers can only depend on reported probabilities. Hence, in this special case, nothing is gained from allowing the richer set of mechanisms.

5.2 Agents with partially aligned interests

In some environments, an agent might have interests that are at least partially aligned with those of the firm and so the agent might prefer some other agent's project to be implemented over his own, if the quality difference is high. In such situation, a manager enjoys some private benefit not only when his own project is selected, but also when another manager's project is selected. It is natural to assume that the benefit that manager i receives increases with the quality of the financed project. But for a given project quality, the manager's payoff is higher if his own project is selected. In the extreme case where manager i's benefit functions are the same function whether his own project is implement or another manager's project, the agent prefers the highest quality project to be selected, regardless of whether it is his own project. In such a case, the interests of the agents and the financing agency are perfectly aligned. A mechanism that chooses the highest reported quality project and never offers transfers would be optimal. However, when at least for some qualities q_i, an agent enjoys his own project being financed more than someone else's project with the same quality, an adverse selection problem remains.

In section C of the online appendix we formally derive the optimal mechanism for this version of the model. We solve this model under the assumption that when another project's quality is not much higher, the benefit from being selected is higher than the benefit from the other project being selected. The analysis and results are similar to those in the previous sections. Any allocation implementable in the main model is implementable in the model with partially aligned interests, with lower transfers. This is because when another agent is selected, that serves as partial compensation to the nonselected agents. Thus, profits in model with partially aligned interests are higher.

5.3 Agents that prefer not to be selected

In our model we have assumed that $b_i(q_i)$ is positive so that agents always want to be selected. We did not include an individual rationality constraint because it was implied by the limited liability constraint. If the benefit function could take negative values for low project qualities, then the individual rationality constraint would bind in a range of project qualities. Negative benefit functions could arise for example when the agent suffers a loss from a project that fails, and the probability of success of the project is low. If the benefit functions are always negative, then the individual rationality constraint implies limited liability. In this case, the problem becomes similar to that described in Vincent and Manelli (1995).

Maintaining our assumption that benefit functions increase with project quality, we considered the case where $b_i(q_i)$ is negative for low project qualities and positive for high qualities. In this problem, the individual rationality constraint binds when agents have low project qualities and the limited liability constraint binds when agents have high qualities. To guarantee that the incentive constraints hold, transfers need to compensate managers who want to be selected but are not selected, so that they would not overstate their probability of success; and transfers need to compensate managers who do not want to be selected but are selected so that they would not understate the quality of their project to avoid being selected. In section D of the online appendix we formally derive the optimal mechanism for this version of the model.

6 Related Literature

Our work is related to a large body of literature on auctions and mechanism design. In terms of our main application of interest, our work also relates to a literature on project selection. The main problem we study is similar to Myerson's (1981) optimal auction design, which maximizes a seller's utility. His problem is a linear program with incentive compatibility and individual rationality (IR) constraints. One important difference between our problem and Myerson's is that we consider a limited liability (LL) constraint, which, in our model, is stronger than individual rationality and therefore replaces it. We show how the optimal mechanism changes when IR is replaced by LL.

As in Myerson (1981) our solution selects the projects (bidders) with the highest "virtual" quality. The virtual quality for each player is a measure of the social value of selecting this player, net of the costs of providing truth-telling incentives. Importantly, it is a function of the type and commonly known characteristics of this particular player, but not of others.

The exact form of the function that transforms type (quality) into a virtual type in our model differs from Myerson's in two ways. The first is that his virtual valuation distorts the social value of selecting a player at the bottom, while our virtual quality distorts it at the top. The reason for this is that while IR is binding for low types (who contemplate on whether or not to participate), LL is binding for high types (who are never compensated). The second, more interesting, difference is that in our model, each project has its own specific quality threshold above which the virtual quality is constant. This interesting feature is not observed in Myerson's model and is the main consequence of replacing IR by LL. As a result, when a project's quality exceeds its threshold, the probability that it gets selected may be constant and less than one, which never occurs in Myerson's model.

Laffont and Robert (1996) and Maskin (2000) study optimal auctions under positive commonly-known budget constraints and IR. Laffont and Robert (1996) study revenue maximization. Maskin (2000) studies efficient auctions. In addition to these constraints, Maskin assumes that players cannot receive funds from the seller. One important difference with our work is that, unlike LL in our model, their budget constraint is not stronger than IR. Both constraints bind in different regions of the type space. The interaction between the two constraints makes these models quite complex to analyze in general. While the optimal mechanism can still be described as allocating the object to the player with the highest virtual valuation, the virtual valuations of the players are jointly determined. Each of them depends not only on the player's type and own observable characteristics but also on the observable characteristics of the other players. Laffont and Robert (1996) solve their model by restricting attention to the case where the players are ex-ante symmetric. Maskin (2000) only considers two asymmetric players. Because only LL binds in our model, we are able to analyze the general problem of selecting up to m among n ex-ante asymmetric projects. These authors show (like we do) that the virtual valuation functions are constant at the top. But because in our model, the virtual quality functions of the projects are determined separately, we can provide an interpretation for the quality thresholds and caps on virtual quality that has no counterpart in their models. In particular, we relate each project's quality threshold and virtual quality cap to the outside option level at which the cost and benefit of eliciting information on the project cancel out (section III) and to the optimal value of selecting one among infinitely many ex-ante identical projects (Proposition 2).[10]

Manelli and Vincent (1995) (MV) study the problem of a firm procuring a good, when the potential sellers have private information on the quality of their good. The buyer in their model, as the firm in our model, has a payoff that increases with quality and decreases with transfers. An important difference is that in their model, sellers incur an opportunity cost when selected, which increases with quality. In contrast, in our model, managers benefit from being selected, and more so when quality is higher. Their assumption that seller utility decreases with quality makes sense in applications where the seller finds it is more costly to part from a higher quality good that he owns. In the applications we have in mind, the project manager has no way to benefit from his good (the project) unless it is financed, and conditional on being financed, the manager benefits more from working on a higher quality project. Another important difference between our models is that MV use an individual rationality constraint, while our model uses the limited liability constraint. We believe that the limited liability constraint is appealing for some application. It might not be feasible for a company to extract funds from an employee researcher. Note that because the benefit function is nonnegative, the LL constraint in our model is a stronger constraint than the IR constraint would be.

Combined, these differences in our models result in different optimal mechanisms. In our model, the probability of project selection is nondecreasing in project quality, while in MV (the expected value of) the probability of project selection is nonincreasing in quality. MV find conditions under which either a sequential offer institution or an auction are optimal, these are not optimal mechanisms in our setting, except when a constant mechanism is optimal.

Finally, we note that our solution concept (using dominant strategy incentive compatibility) is also different than MV (who use Bayesian incentive compatibility). However, it can be shown (see proof in the online appendix) that the dominant strategy incentive compatible optimal mechanism also solve the problem with Bayesian incentive compatibility constraints. Thus, this different approach does not contribute to the differences in results.

[10] A large literature, starting with Che and Gale (1998 and 2000) analyze auctions with budgets constraints, where bidders hold private information on their budget, which is not the case here.

Recently, a few papers studied the problem of choosing one of multiple projects, in the presence of asymmetric information. In Armstrong and Vickers (2010), the principal and the agent have different preference ordering over projects, the principal delegates the project choice to the agent, the principal can influence the agent's behavior by specifying the set of projects from which the agent can choose.[11] In Che, Dessein and Kartik (2010) the preferences of the principal and agent are the same, except that the agent does not value the principal's outside option. Projects have observed and unobserved characteristics. The agent sends cheap talk messages, and the principal has no commitment power. They find that the agent biases his recommendation toward better-looking projects. These papers differ from ours in several important ways, in particular they assume that a single agent holds information about the set of all projects, while in our model there are multiple agents. Additionally, their analysis does not allow transfers. In Mylovanov and Zapechelnyuk (2013) a firm needs to select among agents who hold private information about their value to the decision maker and want to be selected. There are no transfers, but the firm can impose ex post a cost on the selected agent, once his type is revealed. An important question they address is the minimal number of agents that the firm should allow to enter the mechanism, in order to achieve the optimal value.

There is a large literature that studies other aspects of project selection, in particular the incidence of moral hazard issues on the selection process. Some consider moral hazard before the selection process, when effort must be invested to improve the projects (Sappington, 1982). Others study moral hazard at the selection stage, when information about projects must be acquired at a cost (Lambert, 1986; Shin, 2008). Finally, others consider moral hazard after the selection is made, when the selected project is undertaken. Moral hazard after the selection process distorts incentives to reveal private information on quality at the selection stage (Antle and Eppen, 1985; Harris and Raviv, 1996; Zhang, 1997). In this paper, we abstract from moral hazard considerations, and focus on adverse selection alone.

[11] Nocke and Whinston's (2013) model is related to Armstrong and Vickers (2010), but focuses on mergers.

7 Concluding Remarks

Private companies as well as government agencies often face the need to decide how to allocate limited resources to projects. The information that is needed to make decisions may be in the hands of project managers whose interests are typically not aligned with those of the firm. Project managers have more to gain if their own project is selected.

We derived an optimal mechanism to acquire information and decide which of a given set of projects to select. If the firm were fully informed it would always choose the highest quality projects, but in the presence of adverse selection, when information acquisition is costly, this is no longer true. In the optimal mechanism there is a region of reported probabilities in which more managers than can be selected report high qualities, such that in this region project selection is not sensitive to the project qualities. In the symmetric case, projects in this region are randomly selected. In the asymmetric model, a constant deterministic selection is made. Hence, the optimal mechanism involves inefficient project selection where a lower quality project can be selected over a high quality one. This practice allows the firm to save on costs of eliciting information (the incentive transfers) as it reduces the incentive to over state project qualities.

Information asymmetry in our analysis is of an adverse selection nature. Because managers compete for resources and want their own project to be selected, the firm must worry about managers overstating the quality of their project. In contrast in a situation with moral hazard and absent competition between managers, understating the quality of the project might be a concern as it can help hide low effort. In a moral hazard model, when transfers are used to induce effort, they typically reward success, while, in our setting, the firm might reward managers that are not selected to ensure truth-telling and achieve better allocation of resources. The optimal mechanism in our model may reward good outcomes of a manager who reported low quality, when state dependent mechanisms are possible.

A crucial feature of our model is that the firm is facing a selection from a prequalified pool of candidate projects. The firm is assumed to have some information about the distribution of qualities of each of the projects, and the number of candidates is fixed. If anyone could apply for the funds, a mechanism like we found would attract low quality candidates who would want to enter the competition for funds, not in hope of winning but rather with the intention of loosing and collecting a consolation prize – the transfers that reward truthful disclose. Facing such a pool of low expected quality candidates, not selecting any projects might be the optimal solution. A fixed pool of prequalified project managers can arise, for example, in a private company where project managers are prescreened employees of the company; when application for funding requires sufficient knowledge and there is a limited number of candidates with the capabilities to engage in a relevant research project (as is often the case with defense contracting); or as a final stage of a research funding process when the firm was able to employ a two (or more) stage procedure with early stages screening project managers whose quality comes from unfavorable distributions.

8 Appendix: Proofs

Proof of Lemma 1. By (7), we know that for $q^{\circ} < G_i\left(\underline{q_i}\right)$, $a_i^*\left(q^{\circ}\right) = \underline{q_i}$. Substitution into (6), we find that in this range, the function $V^*\left(q^{\circ}\right) = \mathbb{E}\left[q_i\right]$, and it does not depend on q°. Thus, $V^*\left(q^{\circ}\right) - q^{\circ}$ is strictly decreasing in q°. Similarly, by (6) and (7), for $q^{\circ} > G_i\left(\overline{q_i}\right)$, we have $V^*\left(q^{\circ}\right) = -b_i\left(\overline{q_i}\right) + q^{\circ}$ and so $V^*\left(q^{\circ}\right) - q^{\circ} = -b_i\left(\overline{q_i}\right)$ is a negative constant. For $q^{\circ} \in \left[G_i\left(\underline{q_i}\right), G_i\left(\overline{q_i}\right)\right]$, from the envelope theorem, we have $V^{*\prime}\left(q^{\circ}\right) = \partial V\left(a, q^{\circ}\right)/\partial q^{\circ} = F\left(a_i^*\left(q^{\circ}\right)\right) < 1$. The difference $V^*\left(q^{\circ}\right) - q^{\circ}$ is decreasing in q° in this interval as well. Hence, everywhere on $\mathbb{R}$ this difference is either strictly decreasing or it equals a negative constant which implies that $V^*\left(q^{\circ}\right) - q^{\circ}$ equals 0 at most once in $\mathbb{R}$.

For low values of the outside option, $q^{\circ} < \mathbb{E}\left[q_i\right]$, we have $V^*\left(q^{\circ}\right) \geq V\left(\underline{q_i}, q^{\circ}\right) = \mathbb{E}\left[q_i\right] > q^{\circ}$, i.e. the optimal threshold mechanism dominates the outside option. For high values of the outside option, $q^{\circ} \geq G_i\left(\overline{q_i}\right)$, we have $a_i^*\left(q^{\circ}\right) = \overline{q_i}$. In this case, $V^*\left(q^{\circ}\right) = -b_i\left(\overline{q_i}\right) + q^{\circ} < q^{\circ}$. Thus, for such values, the outside option mechanism dominates the optimal threshold mechanism. By the intermediate values theorem, there is a value of the outside option in the interval $\left[\mathbb{E}\left[q_i\right], G_i\left(\overline{q_i}\right)\right]$ for which the firm is indifferent between the outside option and the optimal threshold mechanism. By the previous paragraph, this value is unique in $\mathbb{R}$. ■

Proof of proposition 1. Suppose that $x_i = g_i^{\circ} = \mathbb{E}\left(q_i\right)$ for all q_i, then in particular for $\underline{q_i}$ we have $x_i = \min\left\{\underline{q_i} + b_i\left(\underline{q_i}\right), \mathbb{E}\left(q_i\right)\right\} = \mathbb{E}\left(q_i\right)$. This implies that $\underline{q_i} + b_i\left(\underline{q_i}\right) \geq \mathbb{E}\left(q_i\right)$, and thus $b_i\left(\underline{q_i}\right) \geq \mathbb{E}\left(q_i\right) - \underline{q_i}$. Conversely, if $b_i\left(\underline{q_i}\right) \geq \mathbb{E}\left(q_i\right) - \underline{q_i}$, then by (ai) $a_i^*\left(\mathbb{E}\left(q_i\right)\right) = \underline{q_i}$ and $V^*\left(\mathbb{E}\left(q_i\right)\right) = V\left(\underline{q_i}, \mathbb{E}\left(q_i\right)\right) = \mathbb{E}\left(q_i\right)$. Hence, $g_i^{\circ} = \mathbb{E}\left(q_i\right)$, and $b_i\left(\underline{q_i}\right) \geq g_i^{\circ} - \underline{q_i}$. Therefore, for all q_i, $G\left(q_i\right) \geq G\left(\underline{q_i}\right) = \underline{q_i} + b_i\left(\underline{q_i}\right) \geq g_i^{\circ}$, which implies by definition of x_i that $x_i = g_i^{\circ}$ for all q_i. Combining these results, $x_i = g_i^{\circ} = \mathbb{E}\left(q_i\right)$ for all q_i. ■

The following results are useful in the proof of Theorem 1.

Lemma 3 *Let the payoff of manager i in the optimal mechanism be*

$$M_i\left(\mathbf{q}\right) = t_i(\mathbf{q}) + b_i(\mathbf{q})p_i(\mathbf{q}) \tag{A1}$$

The target function in (1) can be written as

$$\mathbb{E}_{\mathbf{q}}\left[\sum_{i=1}^{n}\left[G_i(q_i)p_i(\mathbf{q}) - M_i(\overline{q_i}, \mathbf{q}_{-i})\right]\right].$$

Proof of Lemma 3. We apply the "Mirlees trick" (see Mirlees, 1974 and Fudenberg and Tirole, 1992) to prove this lemma. The series of transformations in this proof are familiar for this type of models, but we include them here for completeness.

Step 1. Consider first the incentive compatibility constraints (4) which we refer to as IC_i. The utility of project manager i under a mechanism in which he reports truthfully is $M_i(\mathbf{q})$, which was defined in (A1). His utility of misreporting a type q_i' equals

$$t_i(q_i', \mathbf{q}_{-i}) + b_i(q_i)p_i(q_i', \mathbf{q}_{-i}) = M_i(q_i', \mathbf{q}_{-i}) + (b_i(q_i) - b_i(q_i'))\, p_i(q_i', \mathbf{q}_{-i}).$$

From the incentive compatibility constraint, we know that

$$M_i(\mathbf{q}) \geq M_i(q_i', \mathbf{q}_{-i}) + (b_i(q_i) - b_i(q_i'))\, p_i(q_i', \mathbf{q}_{-i})$$

i.e.

$$M_i(q_i', \mathbf{q}_{-i}) - M_i(\mathbf{q}) \leq (b_i(q_i') - b_i(q_i))\, p_i(q_i', \mathbf{q}_{-i}).$$

Using the last inequality twice (once switching the roles of q_i and q_i'), we get

$$(b_i(q_i') - b_i(q_i))\, p_i(\mathbf{q}) \leq M_i(q_i', \mathbf{q}_{-i}) - M_i(\mathbf{q}) \leq (b_i(q_i') - b_i(q_i))\, p_i(q_i', \mathbf{q}_{-i}). \tag{A2}$$

Step 2. Consider now two qualities q_i, q_i' such that $b_i(q_i) < b_i(q_i')$ and fix $\mathbf{q}_{-i}$, then $p_i(q_i, \mathbf{q}_{-i}) \leq p_i(q_i', \mathbf{q}_{-i})$ holds.[12] Indeed, by (A2), we have

$$[b_i(q_i') - b_i(q_i)]\,[p_i(q_i', \mathbf{q}_{-i}) - p_i(\mathbf{q})] \geq 0.$$

Because $b_i(q_i') - b_i(q_i) > 0$, this implies $p_i(\mathbf{q}) \leq p_i(q_i', \mathbf{q}_{-i})$.

Step 3. Dividing (A2) by $q_i' - q_i$ and taking the limit as $q_i' \to q_i$ we find that

$$\frac{\partial M_i(\mathbf{q})}{\partial q_i} = b_i'(q_i)\, p_i(\mathbf{q}).$$

[12] Under the assumption that $b(\cdot)$ is strictly increasing, one can show that IC_i holds if and only if $p_i(\cdot,\cdot)$ is nondecreasing in q_i and transfers satisfy the formula of Lemma 1. This result is classic for this type of mechanism design models (e.g. see Myerson, 1981). Here, the analysis is complicated by the fact that $b(\cdot)$ can be constant on a subset of its domain.

The equality holds almost everywhere, and the right-hand side is continuous in q_i almost everywhere.[13] From the Fundamental Theorem of Calculus, we obtain:

$$M_i\left(q_i, \mathbf{q}_{-i}\right) = M_i(\overline{q_i}, \mathbf{q}_{-i}) - \int_{q_i}^{\overline{q_i}} b_i'(\widetilde{q}_i) p_i(\widetilde{q}_i, \mathbf{q}_{-i}) d\widetilde{q}_i,$$

substituting this into (A1) and rearranging yields the following expression.

$$t_i(\mathbf{q}) = -\left(b_i(q_i) p_i(\mathbf{q}) - M_i(\overline{q_i}, \mathbf{q}_{-i}) + \int_{q_i}^{\overline{q_i}} b_i'(\widetilde{q}_i) p_i(\widetilde{q}_i, \mathbf{q}_{-i}) d\widetilde{q}_i \right). \tag{A3}$$

Step 4. Fix $\mathbf{q}_{-i}$. We use integration by parts to show that:

$$\int_{\underline{q_i}}^{\overline{q_i}} \left(\int_{q_i}^{\overline{q_i}} b_i'(\widetilde{q}_i) p_i(\widetilde{q}_i, \mathbf{q}_{-i}) d\widetilde{q}_i \right) f(q_i) dq_i = \int_{\underline{q_i}}^{\overline{q_i}} b_i'(q_i) p_i(q_i, \mathbf{q}_{-i}) F_i\left(q_i\right) dq_i. \tag{A4}$$

Step 5. Now, we substitute the transfers (A3) into the expected value of the profit made from agent i, conditional on $\mathbf{q}_{-i}$:

$$\begin{aligned}
& \mathbb{E}_{q_i}\left[q_i p_i(\mathbf{q}) - t_i\left(\mathbf{q}\right)\right] \\
= \; & \mathbb{E}_{q_i}\left[q_i p_i(\mathbf{q}) + \left(b_i(q_i) p_i(\mathbf{q}) - M_i(\overline{q_i}, \mathbf{q}_{-i}) + \int_{q_i}^{\overline{q_i}} b_i'(\widetilde{q}_i) p_i(\widetilde{q}_i, \mathbf{q}_{-i}) d\widetilde{q}_i \right) \right] \\
= \; & \mathbb{E}_{q_i}\left[q_i p_i(\mathbf{q}) + b_i(q_i) p_i(\mathbf{q}) - M_i(\overline{q_i}, \mathbf{q}_{-i}) \right] + \int_{\underline{q_i}}^{\overline{q_i}} \left(\int_{q_i}^{\overline{q_i}} b_i'(\widetilde{q}_i) p_i(\widetilde{q}_i, \mathbf{q}_{-i}) d\widetilde{q}_i \right) f(q_i) dq_i \\
= \; & \mathbb{E}_{q_i}\left[q_i p_i(\mathbf{q}) + b_i(q_i) p_i(\mathbf{q}) - M_i(\overline{q_i}, \mathbf{q}_{-i}) \right] + \int_{\underline{q_i}}^{\overline{q_i}} b_i'(q_i) p_i(q_i, \mathbf{q}_{-i}) F_i\left(q_i\right) dq_i \\
= \; & \mathbb{E}_{q_i}\left[G_i\left(q_i\right) p_i(\mathbf{q}) - M_i(\overline{q_i}, \mathbf{q}_{-i}) \right].
\end{aligned}$$

Taking the expectation over $\mathbf{q}_{-i}$ and adding up over i yields the desired equality. ■

Lemma 4 *At any mechanism that satisfies the constraints, the following holds, for each agent $i \in \mathcal{N}$.*

$$\mathbb{E}_{q_i}\left[G_i(q_i) p_i(\mathbf{q}) - M_i(\overline{q_i}, \mathbf{q}_{-i})\right] \leq \mathbb{E}_{q_i}\left[\min\left\{G_i\left(q_i\right), g_i^o\right\} p_i\left(\mathbf{q}\right)\right].$$

[13] The limit of $\frac{\left(b(q_i') - b(q_i)\right)}{q_i' - q_i} p_i(q_i', \mathbf{q}_{-i})$ exists and equals $b'\left(q_i\right) p_i(q_i', \mathbf{q}_{-i})$ for almost all q_i. Indeed, at any point of the set $\{q_i : b'\left(q_i\right) = 0\}$, the limit exists and equals 0, since $p_i\left(\cdot\right)$ is bounded. As for the other points, a corollary of Step 2 is that $p_i\left(\cdot\right)$ is locally nondecreasing at any q_i that is such that $b'\left(q_i\right) > 0$ and thus is continuous almost everywhere in the set $\{q_i : b'\left(q_i\right) > 0\}$.

Proof of Lemma 4. Let $\mathbf{q}$ be a vector of qualities. First, by IC_i we have $t_i(\mathbf{q}) + b_i(\overline{q_i})\, p_i(\mathbf{q}) \leq M_i(\overline{q_i}, \mathbf{q}_{-i})$, otherwise agent i's type $\overline{q_i}$ would have an incentive to misreport as q_i. By LL_i, we have $t_i(\mathbf{q}) \geq 0$, thus for all $\mathbf{q}$,

$$b_i(\overline{q_i})\, p_i(\mathbf{q}) \leq M_i(\overline{q_i}, \mathbf{q}_{-i}) \tag{A5}$$

For all $q_i > a_i^*(g_i^\circ)$ we have $G_i(q_i) > g_i^\circ$, where $a_i^*(\cdot)$ is defined in (7). Therefore,

$$\begin{aligned}
& \left\{ \int_{a_i^*(g_i^\circ)}^{\overline{q_i}} G_i(q_i) p_i(\mathbf{q}) f_i(q_i) dq_i - M_i(\overline{q_i}, \mathbf{q}_{-i}) \right\} - \left\{ \int_{a_i^*(g_i^\circ)}^{\overline{q_i}} g_i^\circ p_i(\mathbf{q}) f_i(q_i) dq_i \right\} \\
= & \int_{a_i^*(g_i^\circ)}^{\overline{q_i}} \left(G_i(q_i) - g_i^\circ\right) p_i(\mathbf{q}) f_i(q_i) dq_i - M_i(\overline{q_i}, \mathbf{q}_{-i}) \\
\leq & \left[\int_{a_i^*(g_i^\circ)}^{\overline{q_i}} \left(G_i(q_i) - g_i^\circ\right) f_i(q_i) dq_i - b_i(\overline{q_i}) \right] \frac{M_i(\overline{q_i}, \mathbf{q}_{-i})}{b_i(\overline{q_i})} = 0.
\end{aligned}$$

The inequality between the second and third lines is an implication of the constraint (A5). The last equality holds by definition of the cap g_i° (in (9)). Therefore,

$$\int_{a_i^*(g_i^\circ)}^{\overline{q_i}} G_i(q_i) p_i(\mathbf{q}) f_i(q_i) dq_i - M_i(\overline{q_i}, \mathbf{q}_{-i}) \leq \int_{a_i^*(g_i^\circ)}^{\overline{q_i}} g_i^\circ p_i(\mathbf{q}) f_i(q_i) dq_i,$$

which further implies

$$\begin{aligned}
& \int_{\underline{q_i}}^{\overline{q_i}} G_i(q_i) p_i(\mathbf{q}) f_i(q_i) dq_i - M_i(\overline{q_i}, \mathbf{q}_{-i}) \\
\leq & \int_{\underline{q_i}}^{a_i^*(g_i^\circ)} G_i(q_i) p_i(\mathbf{q}) f_i(q_i) dq_i + \int_{a_i^*(g_i^\circ)}^{\overline{q_i}} g_i^\circ p_i(\mathbf{q}) f_i(q_i) dq_i \\
= & \mathbb{E}_{q_i}\left[\min\left\{G_i(q_i), g_i^\circ\right\} p_i(\mathbf{q})\right].
\end{aligned}$$

The last equality holds because, for all $q_i < a_i^*(g_i^\circ)$, we have $G_i(q_i) < g_i^\circ$, and for all $q_i > a_i^*(g_i^\circ)$, we have $g_i^\circ > G_i(q_i)$. ■

Proof of Theorem 1. The profit of the firm at some arbitrary mechanism satisfying

(2)-(4) equals

$$\begin{aligned}
& \mathbb{E}_{\mathbf{q}}\left[\sum_{i=1}^{n}\left[G_i(q_i)p_i(\mathbf{q}) - M_i(\overline{q_i}, \mathbf{q}_{-i})\right]\right] \\
\leq\ & \mathbb{E}_{\mathbf{q}}\left[\sum_{i=1}^{n}\min\left\{G_i\left(q_i\right), g_i^{\circ}\right\} p_i\left(\mathbf{q}\right)\right] \\
\leq\ & \mathbb{E}_{\mathbf{q}}\left[S_m\left(\min\left\{G_1\left(q_1\right), g_1^{\circ}\right\}, ..., \min\left\{G_n\left(q_n\right), g_n^{\circ}\right\}, \underbrace{0, ..., 0}_{m \text{ times}}\right)\right] \\
=\ & \mathbb{E}_{\mathbf{q}}\left[S_m\left(\mathbf{x}(\mathbf{q})\right)\right].
\end{aligned}$$

where the first expression for the profit was derived in Lemma A1, the first inequality holds by Lemma A2, and the second holds because of the feasibility constraints (2). Under the proposed mechanism, all the weak inequalities above hold as equalities, thus the mechanism achieves the optimal profit. The mechanism also satisfies the constraints (2)-(4), therefore it is optimal. ■

Proof of Proposition 2. Recall $x_i = \min\left\{G_i\left(q_i\right), g_i^{\circ}\right\}$. Note that $\Pr\left(x_i \leq \overline{g_i^{\circ}}\right) = 1$. Let $\varepsilon > 0$. We have

$$\begin{aligned}
& \Pr\left(x_i \geq \overline{g_i^{\circ}} - \frac{\varepsilon}{m}\right) \\
=\ & \Pr\left(G_i\left(q_i\right) \geq \overline{g_i^{\circ}} - \frac{\varepsilon}{m} \mid g_i^{\circ} \geq \overline{g_i^{\circ}} - \frac{\varepsilon}{m}\right) \Pr\left(g_i^{\circ} \geq \overline{g_i^{\circ}} - \frac{\varepsilon}{m}\right) \\
\geq\ & \Pr\left(G_i\left(q_i\right) \geq \overline{g_i^{\circ}} - \frac{\varepsilon}{m} \mid g_i^{\circ} = \overline{g_i^{\circ}} - \frac{\varepsilon}{m}\right) \Pr\left(g_i^{\circ} \geq \overline{g_i^{\circ}} - \frac{\varepsilon}{m}\right).
\end{aligned}$$

The last inequality is an implication of first order stochastic domination. Both terms of the product in the last line are positive. Therefore $\Pr\left(x_i \geq \overline{g_i^{\circ}} - \varepsilon/m\right) > 0$. Since the x_i are iid, by the law of large numbers $\lim_{n\to+\infty}\Pr\left(x_{(m)}\left(\mathbf{q}\right) \geq \overline{g_i^{\circ}} - \varepsilon/m\right) = 1$. Therefore $\lim_{n\to+\infty}\Pr\left(\left|S_m\left(\mathbf{x}\left(\mathbf{q}\right)\right) - m\overline{g_i^{\circ}}\right| \leq \varepsilon\right) = 1$, as needed. ■

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