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Gaming the system: An opportunity to analyse difficulties in arithmetical problem solving

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Gaming the system is a behaviour that must be avoided when interactive learning environments are designed. However, from the point of view of the research in mathematics education, the observation of this behaviour may bring to light the students' knowledge. In this paper, we provide results of a study in which primary education students (10–11 years old), grouped in pairs, solved problems in an arithmetical way using an intelligent tutoring system. We analyse cases in which the students were able to refine a fuzzy idea of how to calculate a quantity based on the belief that they necessarily had to use certain quantities, operations or conceptual schemes. We also provide examples of how sometimes such students' certainties can become obstacles.

Keywords: Word problems, intelligent tutoring systems, gaming, primary education, arithmetic.

INTRODUCTION AND AIMS

One-to-one tutoring (one teacher for one student) produces better results than instruction in ordinary groups (Bloom, 1984). However, the high cost of this kind of teaching prevents a widespread use. One way to circumvent these difficulties is to use intelligent tutoring systems (ITSs) that reproduce some of the actions that are carried out by a human tutor. The learning of arithmetical word problem solving has been a field where the design of new ITSs has abounded. Some examples would be AnimalWatch (Beal, Arroyo, Cohen, & Woolf, 2010); HERON (Reusser, 1993); or MathCal (Chang, Sung, & Lin, 2006).

Gaming the system is an unwanted behaviour when ITSs are used in teaching situations. We use the definition of gaming provided by Baker and colleagues (2009): “attempting to succeed in an interactive

learning environment by exploiting properties of the system rather than by learning the material (by systematically guessing or abusing hints)” (p. 475). To avoid this behaviour, some control elements should be introduced during the ITSs design. However, from the point of view of the research in mathematics education, the observation of systematic guessing in a computer environment can provide us with information about students' prior knowledge when they make a mistake and the degree of certainty associated with this knowledge (see examples about the importance of analysing these performances in Mavrikis, 2010; Shih, Koedinger, & Scheines, 2008).

In this report we briefly describe the operation of the ITS called Hypergraph Based Problem Solver (HBPS) when it is used to solve word problems in an arithmetical way. We put forward an analysis of the performances of primary education students (10–11 years old) when they solve problems arithmetically using HBPS. It is an exploratory study that is the first step of an ongoing research about the effect of gaming when teaching to solve word problems with an ITS. In particular, in this paper, our analysis focuses on the moments in which the students resort to gaming the system to advance into the resolution process when a verbal cue acts as a source of confusion. The aim of this report is to show that the analysis of this behaviour provides relevant information about students' knowledge and misconceptions.

THEORETICAL FRAMEWORK

The arithmetical solving of word problems implies the repeated execution of analytical-synthetic processes (Bogolyubov, 1972). During the analytical process, solvers try to link the unknown quantities with the known ones through conceptual schemes that typical-

ly arise from the described situations in the statement (Riley, Greeno, & Heller, 1983). For example, a problem in which the time spent by a person going from one point to another is mentioned will usually evoke typical conceptual schemes of distance-rate-time (Mayer, 1981) such as the distance covered by a person is obtained through multiplying the speed of this person by the time spent. In an arithmetical solution, conceptual schemes let solvers link one unknown quantity to other known ones, and therefore make possible directly calculating the unknown quantity. Obviously, when students solve word problems, they may find difficulties in matching up the conceptual scheme to the right operation. These difficulties are reflected in the construction of incorrect operations.

On the other hand, when solvers read a problem statement some key words, which act as verbal cues, can lead them to identify not only the conceptual scheme, but also the arithmetical operation necessary for finding the solution: e.g., altogether, lost, gained, etc. However, as Neshet and Teubal (1975) pointed out, these key words also might act as a source of confusion leading the solver to an incorrect mathematical operation. In this way, Lewis and Mayer (1987) called statements such as “Joe runs 6 miles a week. He runs 3 times as many miles a week as Ken does. How many miles does Ken run in 4 weeks?” (p. 366) inconsistent compare problems, because the solution requires division of two numbers but the structure *times as many as* is involved. Several studies have shown that students tend to generate more solution errors in word problems that contain key words that suggest the opposite operation to that which is required in the solution process (Hegarty, Mayer, & Green, 1992; Lewis & Mayer, 1987; Neshet & Teubal, 1975).

MATERIAL AND METHODS

Material

In this study we used the program HBPS (Arnau, Arevalillo-Herráez, Puig, & González-Calero, 2013; Arnau, Arevalillo-Herráez, & González-Calero, 2014; González-Calero, Arnau, Puig, & Arevalillo-Herráez, 2014), which is able to supervise both the arithmetical and algebraic ways of solving word problems [1]. In fact, HBPS was designed around the main actions that an expert human tutor would do when supervising a student at solving an arithmetical problem. In particular, HBPS can infer the line of reasoning that a student is following and provide him/her adapted

feedback messages using natural language. In this sense, results from Arnau, Arevalillo-Herráez, and González-Calero (2014) back the idea that HBPS is able to provide conceptual support as human experts do. These capabilities enable the system to offer a human-like personalized tutoring. This feature may be especially relevant taken into account that one of the potentials of ITSs is to make individualized tutoring widely and inexpensively available (Woolf, 2009). For this study we employed a version of HBPS, in which *help on demand* were deactivated and error messages were reduced to a simple notification. To illustrate the operation of HBPS we use the problem *The plans*, whose statement and a brief analysis is offered below.

The plans: Ann and Mike earned 36,000 € for designing a bridge. Since they did not work the same amount of time it should be distributed so that Ann will receive five parts of what they earned and Mike, seven parts. How much money did Mike earn?

The problem may be reduced to the known quantities (total of money, M (36000); number of parts that corresponds to Ann, Pa (5); and number of parts that corresponds to Mike, Pm (7)) and to the unknown ones (total of number of parts, P ; money that corresponds to one part, Mp ; and money that Mike receives, Mm). These quantities can be linked by the relations: $P = Pm + Pa$, $Mp = M/P$ and $Mm = Mp \cdot Pm$ [2]. If a solver follows this solution path, the last operation, which is used to calculate the part of money that corresponds to Mike, would be multiplication. When students are introduced to the solving of proportional sharing problems, this may cause difficulties due to the fact that in equal sharing problems the determination of these quantities usually implies dividing. In addition, in these problems it is common to find the key word *part*, which is usually associated with the mathematical operation division.

In the version of HBPS used in this study, the students can see the statement of the problem and find the name of the quantities that must be employed in a drop-down menu. When they select a name of a known quantity, they must assign the appropriate numerical value to it (see screenshot on the left in Figure 1). When they select a name of an unknown quantity they must assign an arithmetic operation that HBPS calculates automatically. To assign this operation, the expressions are introduced by using a calculator-like component. It is composed of a button

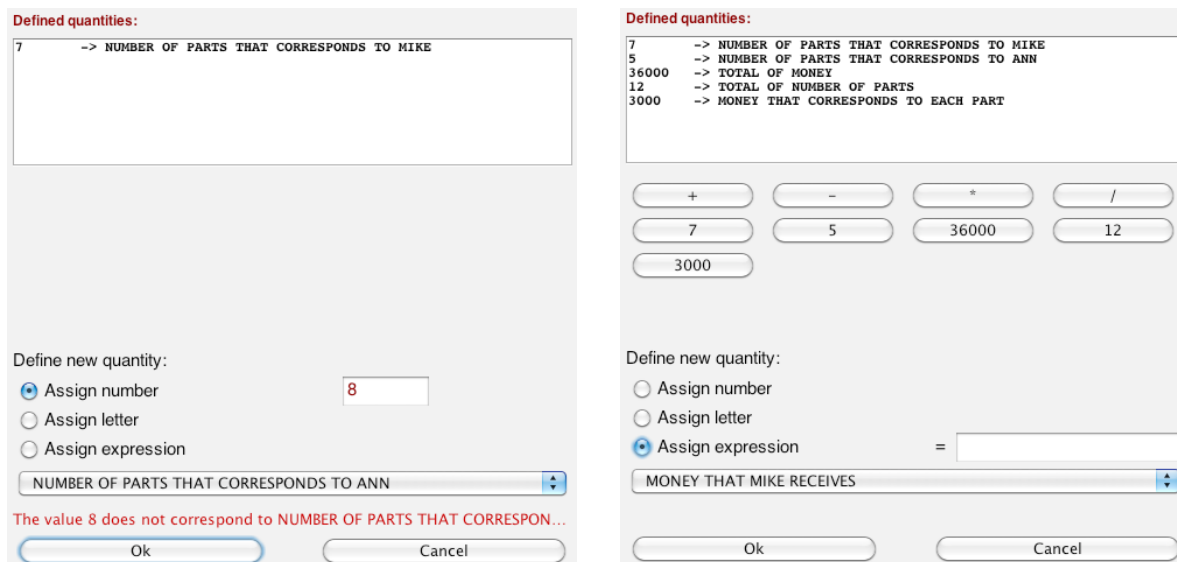


Figure 1: Error message (left) and construction of an arithmetical expression (right)

for each arithmetic operation, and another button for each different quantity that has been introduced into the system by the student (see screenshot on the right in Figure 1). In case of incorrect assignment of value or operation, this version of HBPS generates a simple error message by which the user is reported that last action is not correct but no additional information is offered (see screenshot on the left in Figure 1).

Empirical study design

The empirical study was carried out in a natural group of primary education composed by 20 students between 10 and 11 years old. As part of a wider study aimed at analysing how HBPS influences the students' proficiency in arithmetic word problem solving and that is not described in this report, the students had used HBPS to solve word problems in the classroom. Then, six students were chosen and they were grouped in pairs. The criterion adopted to select the students pursued to pair students who had shown difficulties in the same problems during the former phase of the study. Each pair was offered the problems that both members had previously failed to solve when they had been working individually in paper and pencil at the end of the instructional phase with HBPS. One of the problems the three pairs had to solve was *The plans*. They were asked to try to solve it using HBPS and they were encouraged to verbalise their actions. The sessions were video-recorded and afterwards the performances were transcribed to written language. As our investigation aimed to observe the process of problem solving, the researcher had a very low degree of intervention (Schoenfeld, 1985). The researcher just provided the sequence of problems and helped

with technical problems that could appear. The analysis of these transcriptions makes possible to identify at each moment what relationship among quantities was established by the pairs and which mathematical operation was employed to link quantities in order to evaluate how students translate information from the statement into the mathematical language over the resolution process.

RESULTS

Next, we describe episodes of systematic guessing produced when the three pairs were solving *The plans* problem. All cases start from a situation in which the unknown quantities "total of number of parts" (12) and "money that corresponds to each part" (3000) have been already determined and the only remaining quantity to be determined is "money that Mike receives". At this point the determination of this quantity requires the use of the conceptual scheme "total money = money per part \times number of parts". First we offer the sequence of operations that each pair introduced from that moment onwards in order to calculate the only unknown quantity.

Lola-Juan: (1) $36000 - 3000$; (2) $36000/3000$; (3) $36000/12$; (4) $3000/12$; (5) $3000-12$; (6) $3000/7$; (7) $3000 - 7$; (8) $3000 - 12$; (9) $3000 \cdot 7$.

Julia-Roberto: (1) $36000/7$; (2) $3000/12$; (3) $3000/7$; (4) $3000/12$; (5) $3000/7$; (6) $3000/7$; (7) $36000/12$; (8) $3000/12$.

Pablo-Helena: (1) $36000/3000$; (2) $36000/7$; (3) $3000/7$; (4) $36000 - 3000$; (5) $3000/12$; (6) $3000 \cdot 7$.

The pair Lola-Juan

The pair Lola-Juan starts the sequence performing three operations in which the quantity “total of money” is involved. In the dialog that follows, there are evidences that they are initially convinced that that quantity must be employed to calculate “money that Mike receives”. After rereading the statement, Juan says that the quantity that must be used is “money that corresponds to each part” (3000) (item 7, Lola-Juan). By contrast, Lola is still convinced of having to use “total of money” (36000) (item 6, Lola-Juan) and, in fact, she introduces 36000/12 (item 10, Lola-Juan).

- 1 Lola: Then...
- 2 Juan: 36000 divided by 3...? ... divided by 3000 (he introduces 36000 – 3000 and an error message is shown).
- 3 Juan: I think it is divided by.
- 4 Lola: So divide it. (Juan introduces 36000/3000 and an error message is shown.)
- 5 (It is heard how they reread the statement quietly.)
- 6 Lola: It certainly has to be 36000 (she introduces 36000).
- 7 Juan: No, it has to be 3000.
- 8 Lola: But then...
- 9 Juan: But then, divided by 12?
- 10 Lola: 36000 divided by 12 (Lola introduces 36000/12).
- 11 Juan: You have already done it and you got 3000! (He says so before the error message appears.)

From the fourth attempt on (item 14, Lola-Juan) the quantity “money that corresponds to each part” (3000) appears in all the operations they perform. However, as it is shown in the following dialogue, the erratic sequence of operations seems to point out that they are not so sure about which quantity will be the other one or which operation they will have to use.

- 12 Lola: Let’s see... 3000 divided by 12.
- 13 Juan: Or 3000 times 12 or minus 12.
- 14 (Lola introduces 3000/12 and an error message is shown.)

During the rest of the resolution Lola seems to have no doubts about the necessity of using the quantity “number of parts that corresponds to Mike” (7). In actual fact, it seems like she were trying to make use of the scheme $\text{money} = \text{money by part} \times \text{number of parts}$

(item 15, Lola-Juan). However, she divides by 7 instead of multiplying by it. This could be a consequence of associating sharing with division. After introducing 3000/7, Lola deduces that a multiplication must be done, whereas Juan carries on trying operations with the only criterion that “money that corresponds to each part” (3000) must appear.

- 15 Lola: Wait. If he receives 7 parts: 3000 divided by 7.
- 16 (Juan introduces 3000/7 and an error message is shown.)
- 17 Lola: Or multiplying, it will be multiplying.
- 18 (Juan introduces 3000 – 7 and an error message is shown.)
- 19 (Juan introduces 3000 – 12 and an error message is shown.)
- 20 Lola: 3000 times 7 (she says so before Juan introduces the last expression).
- 21 Juan: That’s true, minus 7 or minus 12 (Juan introduces 3000 – 12 and an error message is shown).
- 22 Lola: 3000 multiplied by 7.
- 23 Juan: Yes (he introduces 3000·7 and it is shown the message of problem solved correctly).

The pair Julia-Roberto

This pair opted to use division in all the attempts. This fact suggests that the pair erroneously considers a certainty, beyond any doubt, that the money an individual receives must be calculated using the mathematical operation division. Maybe the pair thinks that, given the fact that “the money Mike receives” is a part of “the total of money” and that there is a quantity “number of parts that corresponds to Mike”, the problem could be interpreted as a partition division one. As it shown in the following excerpt, the solving process is conditioned by this belief, which causes the pair not to manage to solve the problem. The dialogue starts when they introduce the first expression of their sequence.

- 1 (Julia introduces 36000/7 and an error message is shown.)
- 2 Roberto: So then, that number (he points out 3000 on the screen) must be there for some reason.
- 3 Julia: Yes. This one divided by 12 (while she is saying this, she writes 3000/12).

- 4 Roberto: 3000 divided by 12?
 5 Julia: I don't know (she introduces 3000/12 and an error message is shown).
 6 Roberto: Divided by 7, I think so.
 7 Julia: Yes.
 8 Roberto: And if it isn't like that, nothing.
 9 (Julia introduces 3000/7 and an error message is shown.)
 10 (They seem to read the name of the quantities inaudibly.)
 11 Julia: It is divided by 7.
 [...]
 12 Roberto: It would be money divided by the parts that Mike receives and it doesn't come to anything (while he is speaking, he introduces 3000/7 and an error message is shown).

The pair Pablo-Helena

After performing, at the request of Helena, the operation 3000/7 (third in their sequence) and obtaining an error message, they decide to read, in the window *Defined quantities* (see screenshot on the right in Figure 1), the name of the quantities and their values. Then, the following dialogue started in which it is evidenced how the Pablo's suggestion of performing a multiplication was the trigger for solving the problem (item 9, Pablo-Helena). Indeed, Helena immediately reacts to this idea by identifying the correct operation (item 10, Pablo-Helena). It should be underlined that when Pablo mentions the operation multiplication to Helena she straightaway turns her attention to the quantity "number of parts that corresponds to Mike".

- 1 Pablo: Ah, I think it is (he introduces 36000-3000 and an error message is shown)...!
 2 Helena: There is nothing else, only this one (she says this while Pablo is opening the drop-down menu where the name of the only non-defined quantity appears).
 3 Pablo: So we have to find it out, because we have already done a lot.
 4 Helena: Let's see.
 5 Pablo: And can't it be 3000 divided by 5?
 6 Helena: And 3000 divided by 12?
 7 Pablo: It may be.
 8 (Pablo introduces 3000/12 and an error message is shown.)
 9 Pablo: It may be multiplying.

- 10 Helena: By 7!!
 11 Pablo: I'll try it then (he introduces 3000-7 and a message of problem solved correctly is shown).

DISCUSSION

All the three cases can be classified as gaming the system performances. However, this does not mean that all the students' actions correspond to trial and error efforts without any trace of reflection. In fact, what is observed is a repeated analytical-synthetic process by which students try to establish the relation that the quantity "money that Mike receives" has with the rest of quantities already determined. The process makes visible the students' conceptions about what quantities should be used to solve the problem. Additionally the protocols reveal how the pairs' certainties evolve. In particular, the three pairs turn their attention to the quantity "money that corresponds to each part" (3000) after having been working with the quantity "total of money" (36000) (for example, at first Lola stated that this quantity had to be used).

The pairs that solved the problem correctly posed a multiplication just after having proposed a division. However, the trigger for writing the correct operation wasn't the same. In the case of the pair Lola-Juan, Lola proposes multiplying after the system had reported an error when introducing the operation 3000/7, which had been proposed by her. This may be attributed to Lola's capacity of synthesising the correct operation once she had identified the conceptual structure that linked the quantities. In the case of the pair Pablo-Helena, Pablo merely suggested using a multiplication and it was Helena who proposed to multiply by "number of parts that corresponds to Mike" (7). Perhaps, Pablo's suggestion acted as a hint for Helena, since, up to that moment, she had seemed to be sure about the need of performing a division.

The pair that did not solve the problem correctly insisted that it was necessary to divide by "number of parts that corresponds to Mike" (7). In fact, they posed the operation 3000/7 up to three times. This suggests that they had identified part of the conceptual scheme that linked the quantities, but they were not able to make a synthesis correctly. Possibly, the fact that a share of the total money had to be done could induce them to consider that a division had to be performed.

In this case, the pair's certainty that a division was mandatory could become an insurmountable obstacle.

This report illustrates how students' conceptions play an important role in gaming the system behaviours. The analysed cases are far from being irrational trial and error sequences in which the pairs abuse of the system's capabilities without engaging with the problem. Although, even in the case of the pairs that finally solve the problem, more evidences apart from the excerpts are needed to state that learning took place, these pairs seems to have gain insight into the conceptual structure of the problem either by refining their previous certainties or by completing them. It should be recognised that the fact that some pairs managed to solve the problem can be favoured by the setting of the study in which the students worked collaboratively in pairs. However, according to the purpose of our work, we opted to use this setting because two-person protocols are appropriate for analysing decision-making behaviour instead of using one-person protocols that could give rise to the purest students' cognitions (Schoenfeld, 1985). Besides these considerations, the reported cases demonstrate that gaming behaviours can be an important source of information about students' knowledge and the difficulties associated with some problems. Often, this kind of performances is classified as mere trial and error attempts and little attention is paid to it. However, intelligent tutoring systems are now capable of logging and processing all the user's actions when he/she is solving a problem. In this context, future studies may address how this massive information could be employed in intelligent learning environments in order to scaffold students' learning.

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ENDNOTES

1. We consider that a solving process will be arithmetical when the solver calculates all unknown quantities going from the known to the unknown quantities. On the other hand, a solution will be algebraic when the solver uses one or more letters to represent unknown quantities, which will result in one or more equations.

2. Although relations have been represented by using algebraic language, we are not asserting that the relations are represented in this way on the mental level. We have just offered a schematic representation of the net of quantities and relations that can be taken from the statement. To do so we have represented the relations by algebraic language although this does not mean anything about the algebraic or arithmetical character of the solving process.