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TOLLS VERSUS MOBILITY PERMITS: A COMPARATIVE ANALYSIS

André de Palma ¹
CES, ENS Cachan, Université Paris-Saclay
94235 Cachan, France
andre.depalma@ens-cachan.fr

Stef Proost
KU Leuven
FEB (Faculty of Economics and Business)
stef.proost@kuleuven.be

Ravi Seshadri (corresponding author)
Singapore-MIT Alliance for Research and Technology (SMART)
CREATE Tower
1 Create Way
Singapore - 588177
Email: ravi@smart.mit.edu

Moshe Ben-Akiva
Massachusetts Institute of Technology (MIT)
Room 1-181
77 Massachusetts Avenue
Cambridge, MA, 02139

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ABSTRACT

To address traffic congestion, two categories of instruments are used: price regulation (for instance, road pricing or congestion tolling) and quantity regulation (credit-based mobility schemes). Although the comparison of price and quantity regulation has received significant attention in the economics community, the literature is relatively sparse in the context of transportation systems. This paper develops a methodology to compare the toll and mobility permit instruments using a simple transportation network consisting of parallel highway routes and a public transport alternative. The permits can be traded across roads. The demand for each route is determined by a mixed logit route choice model and the supply consists of static congestion. The comparison is based on the optimum social welfare which is computed for each instrument by solving a non-convex optimization problem involving the mixed logit equilibrium constraints. Equity considerations are also examined.

Numerical experiments conducted across a wide range of demand/supply inputs indicate that the toll and mobility permit instruments perform very closely in efficiency terms. The permit system is on average more efficient, but only by a small margin.

Keywords: Tolls, Mobility Permits, Mixed Logit, Social Welfare, Equity, Stochastic Demand.

INTRODUCTION

As traffic tolls turn out to be difficult to implement, the traffic engineering profession turns its attention also to mobility permit systems. Mobility permit systems can come in different forms, but can, with the right technology, cope with complex route systems and variation within days. The literature on the use of mobility permits has been reviewed by Fan and Jiang [1]. They point to the many variants of tradable mobility permits: they can be per mile, per day, specific for one route or period within the day, bankable over days or limited to one day, traded via a central trading organism or not etc.

In the absence of uncertainty, transaction costs and revenue constraints, both systems perform equally well in terms of economic efficiency. The road traffic system is however characterized by many uncertainties and in that case toll and permit systems perform differently. We focus on the short term uncertainty where the road agency knows *ex ante* the distribution and needs to set its controls before the realization. Demand is uncertain because of weather conditions, strikes or other foreseen events in the CBD. Supply can be uncertain because of weather, accidents and incidents which change road capacity. This is particularly important for road use which depends on demand and supply (capacity) at a specific time of day, such as the morning peak, when congestion is high. Intuitively, toll regulation fixes the access price but usage can vary strongly. In contrast, a credit based system fixes the maximal usage but the trading price can vary strongly from day to day.

This paper is among the first to compare the efficiency properties of tolls and permit systems for a road system. This paper develops an algorithm to compute optimal *ex ante* tolls and permits for a system of parallel highways and a public transport alternative. Users are characterized by a mixed logit system and the road system has static congestion using BPR functions. The efficiency of tolls and permits is compared considering consecutively demand uncertainty, supply uncertainty and the combination of both types of uncertainty. Equity aspects are also dealt with. The paper's main result is that permit systems tend, on average, to outperform toll systems in efficiency terms but only by a very small margin. Permit system's comparative advantage is larger when congestion is more severe due to either high expected demand or steep BPR functions.

The paper is divided into eight sections. Section 2 briefly reviews relevant literature. Section 3 describes the network model for the case of fixed demand and supply and introduces the toll and mobility permit systems and establishes their equivalence at the social optimum. Section 4 describes the network model and toll/mobility permit instruments for the case of variable demand and discusses the methodology for comparison. Section 5 describes the variable capacity case while 6 adds the variable demand case to the variable capacity case. Section 7 discusses results from numerical experiments that compare the performance of the two mechanisms in the presence of variability in demand and/or supply. Finally, Section 8 presents concluding remarks.

LITERATURE REVIEW

Congested roads are in the end a scarcity problem and this can be dealt with by either a price instrument, a quantity instrument or a combination of both. With a quantity control the number of permits to use a congested road is limited (and exogenously determined). In this case, the number of distributed permits (and volume of car use) is given, but these permits can be traded among the users. The equilibrium price of permits is endogenous and clears the supply and the demand. With a price system, the price (or toll) to be paid is given and exogenous, while the number of cars willing to use a road is endogenous.

Prices versus quantity regulation is a question that has occupied economists for a long time.

In the seminal paper of Weitzman [2], the author was mainly comparing "planning" solutions and "market and pricing" solutions to production. Consumers decide on the quantity they want to buy and profit maximising firms produce in function of prices. Under perfect information, both systems are equivalent in efficiency terms (see also [3] on this issue). The road setting we study is different. We have consumers decide to use the road or not and use of road space is free if there is no toll or permit system in place. As long as there is no uncertainty, tolls and quantity regulation are equivalent as in the regular market case studied by Weitzman. A toll equal to the marginal external congestion costs limits the use of the road to those trips that are valued higher than the external congestion cost. This optimal number of trips can be translated into permits. When these permits are distributed to all potential drivers and these drivers can trade these permits, one ends up in the same equilibrium because those who value the trip most will buy the available permits.

There are two main differences between a price system and a quantity system. The first is the transaction cost of permits between users. This transaction cost is higher when there are many agents involved and when different types of permits are needed for each road and period in the day. Nie [4] studied the efficiency costs of a permit system and derived conditions under which the transaction costs are prohibitive. As economists thought for a long time that transaction costs of mobility permits would hamper the efficient trading of permits [5], they focused much more on the toll instrument (see [6] for a review). The second difference between toll and permit regulation is that toll systems generate revenues while permits are usually handed out for free and generate no revenues for the road authority or the state. The revenue generating function of toll systems has three implications: The first implication is that tolls tend to be less politically acceptable as they are perceived as a tax, even if the revenues are redistributed somewhere (see [7]). For this reason, environmental problems (like climate change) are more often controlled via permits rather than via taxes. If transaction costs are low, one may see more road permit systems developing than toll systems in the future. One of the main problems of road pricing is that users are often worse-off when part of the whole toll revenue is not distributed to the drivers. This result is true even if the toll revenues can be transferred at no cost to the drivers. Moreover, and more importantly, tolls may have distortive effects in the sense that, with tolls, high income drivers can be better off, while low income users can be worse off. Of course, this is not acceptable politically. Transfers can remedy that, but such transfers, announced ex-ante, have to be credible.

The second implication is that the absence of revenue generation in a free permit system will require that other tax revenues need to be generated in order to maintain and extend the road and PT system. However, to compare different ways to raise revenue to finance infrastructure, a general equilibrium approach should be considered. This topic remains to be studied in the context of comparison of tolls and permits in a stochastic environment.

The third implication is that both tolling and permit systems increase the price of a road trip as peak road trips are often commuting trips, this may decrease the incentives to work and decrease income tax revenues. This drawback can be compensated by returning the toll revenues as lower income taxes but this does not work for permit systems. This second order efficiency effect can be important as it can wipe out the efficiency gain on the transport market [8]. We will not include transaction costs, acceptability issues or second order effects associated with the use of toll revenues.

This paper will focus on the effect of uncertainty on the comparative advantage of toll and permit systems. Weitzman showed that, under ex ante uncertainty, for a regular market, price and quantity regulation are not equivalent in efficiency terms. Weitzman used a linear approximation

of demand and supply functions to analyze the effects of demand uncertainty and supply uncertainty. Both uncertainties are symmetric and correspond to vertical displacements of demand and supply functions. These simplifications allowed Weitzman to derive analytical results. When the demand function is uncertain, there is no comparative advantage of prices over quantities and the variance of demand does not affect the efficiency loss. When the supply function is uncertain, he shows that price regulation is better if the demand function has a flatter slope than the supply function. Weitzman's analytical results cannot be transferred to our road control problem. First, in our model, demand is discrete and the aggregate demand function, produced with a lognormal distribution of values of time, is not linear. Second, we deal with congestion so the availability of road space depends on demand level itself. Third, the marginal external cost that can be seen as a supply function, is not linear when BPR functions with higher exponentials are used. Therefore, the comparison of the efficiency properties of toll and permit systems needs to be performed numerically.

Although uncertainty is important, there are only a very limited number of papers comparing price and quantity control under uncertainty for roads. Both demand uncertainty (weather, transit supply interruptions) as well as road supply uncertainty (weather, accidents, road works) need to be dealt with. Shirmohammadi et al. [9] deal explicitly with the tradable permit systems under uncertainty. They simulate a six link network under demand uncertainty and capacity uncertainty. They compare the performance of a link differentiated toll system and a mobility credit system that is differentiated by link. However, the performance is not measured in efficiency terms but by the difference with a given target volume of cars. They find strong variations in the permit prices as the prices of permits have to make sure the demand matches the set volume targets. Inspired by [10], the authors advocate the use of a hybrid system with a maximum permit price close to the optimal toll level. This keeps permit prices low and makes the system more acceptable. The authors focus mainly on the permit price fluctuations and do not compare the efficiency properties of both systems. Our paper is the first to focus specifically on the efficiency differences, assuming that each type of instrument is optimized.

STATIC NETWORK WITH FIXED DEMAND

The static transportation network of interest consists of a single origin destination pair connected by $i = 1 \dots n$ routes for private vehicles and a single public transportation route denoted by $i = 0$ (Figure 1). It is noted that the methodology applies also to general static networks which will be considered in future studies. The demand is assumed to be fixed and inelastic and is denoted by N and the travel time on route i ($i \in \{1 \dots n\}$), t_i is an increasing function of the route flow X_i . A BPR type function is used to model the dependence of route travel time on flow,

$$t_i(X_i) = t_i^{ff} (1 + \alpha_i (X_i/C_i)^{\beta_i}) \quad i = 1 \dots n, \quad (1)$$

where t_i^{ff} , C_i are the free flow travel time and capacity for route i , and α_i, β_i are BPR function parameters. For convenience, the public transport alternative is assumed to have no congestion and a fixed travel time t_0 . This is clearly a simplification but it helps to understand the results.

Equilibrium

Under the price instrument, the government imposes a toll τ_i on route i ($\tau_0 = 0$, without loss of generality). A mixed-logit discrete choice model is used to characterise the route choice of

Notation

Symbol	Description
n	Number of private vehicle routes
X_i	Flow on private route i ($i = 1 \dots n$)
X_0	Flow on public transport alternative
N	Number of travellers in fixed demand/supply setting
t_i	Congested travel time on private route i ($i = 1 \dots n$)
t_0	Travel time on public transport alternative
t_i^{ff}	Free flow travel time on private route i ($i = 1 \dots n$)
C_i	Capacity of private route i ($i = 1 \dots n$)
α_i	BPR function parameter for private route i ($i = 1 \dots n$)
β_i	BPR function parameter for private route i ($i = 1 \dots n$)
U_{ij}	Utility of private route i for individual j
γ	Value of time (VOT) variable
$f(\cdot)$	Probability density function of VOT variable
$\bar{\gamma}$	Mean of VOT variable γ
σ_γ	Standard deviation of VOT variable
γ_j	Value of time of individual j
ϵ_i	Error term in utility for route i
μ	Scale parameter of error term ϵ_i
V_0	Systematic utility of public transport alternative
B_i	Alternative specific benefit on route i ($i = 0 \dots n$)
τ_i	Toll on route i
δ_i	Number of permits required for route i
\bar{y}	Initial permit endowment per person
p_y^*	Equilibrium price of permits in fixed demand/supply
Ω_F^o	Optimum welfare in fixed demand/supply case
X_i^o	Optimum flow (with tolls or permits) on route i in fixed demand/supply setting
τ_i^o	Optimum toll on route i in fixed demand/supply case
δ_i^o	Optimum number of permits required for route i in fixed demand/supply setting
N^+	Number of travellers on good day in variable demand case
N^-	Number of travellers on bad day in variable demand case
X_i^{po+}	Optimum flow on the good day on route i for the price instrument under variable demand
X_i^{po-}	Optimum flow on the bad day on route i for the price instrument under variable demand
X_i^{qo+}	Optimum flow on the good day on route i for the quantity instrument under variable demand
X_i^{qo-}	Optimum flow on the bad day on route i for the quantity instrument under variable demand
Ω_{VP}^o	Optimum welfare for price instrument in variable demand case
Ω_{VQ}^o	Optimum welfare for quantity instrument in variable demand case

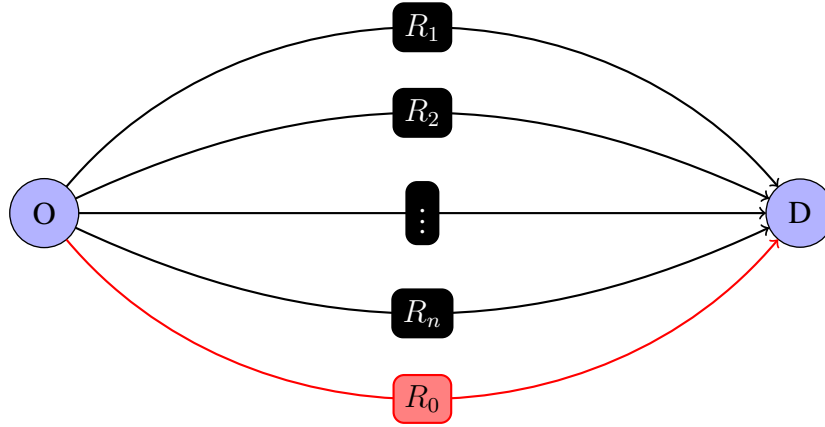


FIGURE 1 Transportation Network

travellers wherein the utility for a commuter j on route i is given by,

$$U_{ij} = B_i - \gamma_j t_i(X_i) - \tau_i + \epsilon_i, \quad (2)$$

where B_i is an alternative specific benefit, γ_j is the value of time of individual j and t_i is given by Equation 1. ϵ_i is a random error term that is assumed to be *i.i.d* gumbel distributed with scale parameter μ . The value of time γ is assumed to be randomly distributed across the population of travellers and follows a lognormal distribution with mean $\bar{\gamma}$ and standard deviation σ_γ .

The systematic utility of the public transport alternative for an individual with value of time γ is given by $V_0(\gamma) = B_0 - \gamma t_0$, where B_0 and t_0 are the alternative specific benefit and travel time for the public transport alternative.

The equilibrium route flows $\mathbf{X}^* = (X_1^*, X_1^* \dots X_n^*)$ are given by the solution of the following fixed point problem (note that $X_0^* = N - \sum_{i=1 \dots n} X_i^*$),

$$X_i = N \int_{\gamma} \frac{\exp[(1/\mu)(B_i - \gamma t_i(X_i) - \tau_i)]}{\sum_{j=1 \dots n} \exp[(1/\mu)(B_j - \gamma t_j(X_j) - \tau_j)] + \exp[V_0(\gamma)/\mu]} f(\gamma) d\gamma, \quad i = 1 \dots n. \quad (3)$$

In Equation 3 above, $f(\cdot)$ is the probability density function of the value of time variable γ . Note that the fixed point problem in Equation 3 can be viewed as a stochastic user equilibrium with heterogeneous travellers.

Lemma 1 *For the network topology considered here and an arbitrary toll vector, under the assumption that the travel time functions are increasing and the jacobian of the link travel time vector is positive semi-definite, it can be shown that a unique solution exists to the fixed point problem in Equation 3 (refer [11, 12, 13]).*

Social Optimum

The social optimum is obtained by maximizing the social welfare (which in the absence of tolls consists of the consumer surplus or the *logsum* measure) with respect to the route flows subject to the mixed logit equilibrium constraints as,

$$\underset{X_1, \dots, X_n}{Max} \Omega(\mathbf{X}) = \mu \sum_{j=1..N} \log \left[\sum_{i=1..n} \exp \left[\frac{1}{\mu} (B_i - \gamma_j t_i(X_i)) \right] + \exp \left[\frac{V_0(\gamma_j)}{\mu} \right] \right] \quad (4)$$

s.t

$$X_i = N \int_{\gamma} \frac{\exp \left[\frac{1}{\mu} (B_i - \gamma t_i(X_i)) \right]}{\sum_{j=1..n} \exp \left[\frac{1}{\mu} (B_j - \gamma t_j(X_j)) \right] + \exp \left[\frac{V_0(\gamma)}{\mu} \right]} f(\gamma) d\gamma, \quad i = 1 \dots n.$$

Can tolls and permits realize the social optimum?

Price Instrument (Tolls)

To decentralize the social optimum by using prices, it suffices to set the tolls on the different roads as follows: $\tau_1 = \tau_1^o, \dots, \tau_n = \tau_n^o$, or $\boldsymbol{\tau} = \boldsymbol{\tau}^o$, where the optimal toll $\boldsymbol{\tau}^o$ is obtained by maximizing the social welfare which consists of two components: the consumer surplus (logsum measure) and toll revenue, subject to the mixed logit equilibrium constraints given by Equation 3. Thus, we have,

$$\underset{\tau_1, \dots, \tau_n}{Max} \Omega(\boldsymbol{\tau}) = \mu \sum_{j=1..N} \log \left[\sum_{i=1..n} \exp \left[\frac{1}{\mu} (B_i - \gamma_j t_i(X_i(\boldsymbol{\tau})) - \tau_i) \right] + \exp \left[\frac{V_0(\gamma_j)}{\mu} \right] \right] \quad (5)$$

$$+ \sum_{i=1..n} \tau_i X_i(\boldsymbol{\tau})$$

s.t

$$X_i = N \int_{\gamma} \frac{\exp \left[\frac{1}{\mu} (B_i - \gamma t_i(X_i) - \tau_i) \right]}{\sum_{j=1..n} \exp \left[\frac{1}{\mu} (B_j - \gamma t_j(X_j) - \tau_j) \right] + \exp \left[\frac{V_0(\gamma)}{\mu} \right]} f(\gamma) d\gamma, \quad i = 1 \dots n.$$

The optimal vector of tolls and associated optimum welfare are denoted by $\boldsymbol{\tau}^o = (\tau_1^o, \dots, \tau_n^o)$ and $\Omega_F^o \equiv \Omega(\boldsymbol{\tau}^o)$ respectively. In addition, the route flows at the optimum are denoted by $\mathbf{X}^o = (X_1^o, \dots, X_n^o)$.

Lemma 2 *The optimum tolls are non-negative, i.e. $\tau_i^o \geq 0, i = 1 \dots n$ if there is congestion on the private vehicle routes and no congestion on the public transport route.*

Quantity Instrument (Mobility Credits)

Under the mobility credit instrument, the government or central planner distributes a certain number (\bar{y}) of credits (hereafter permits) to each potential traveller. In order to use a given private route i , the traveller requires a specified quantity of permits δ_i which is set by the government. Permits can be exchanged between travellers and the price of each permit (denoted by p_y) is endogenously determined by the supply and demand equilibrium in the permit market. The utility for a commuter j on route i in this case is given by,

$$U_{ij} = B_i - \gamma_j t_i(X_i) - p_y \delta_i + \epsilon_i. \quad (6)$$

Subsequently, in the case of deterministic demand/supply, we show that the two instruments are equivalent at the social optimum.

Assume that the number of permits required to use route i , δ_i^o is proportional to the optimal toll on route i . We are now required to find the equilibrium price of the permits, p_y^* . If the value of p_y is assumed to be given exogenously, then the number of permits required on route i is $\delta_i^o(p_y) = \tau_i^o/p_y$. The total number of permits to be given away is given by:

$$\sum_{i=1\dots n} \delta_i^o(p_y) X_i^o = N\bar{y}(p_y), \quad (7)$$

where N is the number of potential travellers. Substituting $\delta_i^o(p_y) = \tau_i^o/p_y$ gives:

$$\bar{y}(p_y) p_y = \frac{1}{N} \sum_{i=1\dots n} \tau_i^o X_i^o. \quad (8)$$

Note that if the price of the permit is fixed, then the number of permits per potential traveller should satisfy Equation 8; or if the number of permits per potential traveller is given, then the price of the permit is determined by Equation 8. Thus, the total value of the permits distributed equals the total toll revenue. The flow on route i is given by:

$$X_i(p_y) = N \int_{\gamma} \frac{\exp[(1/\mu)(B_i - \gamma t_i(X_i) - p_y \delta_i^o + p_y \bar{y})]}{\sum_{j=1\dots n} \exp[(1/\mu)(B_j - \gamma t_j(X_j) - p_y \delta_j^o + p_y \bar{y})] + \exp[(1/\mu)(V_0(\gamma) + p_y \bar{y})]} f(\gamma) d\gamma. \quad (9)$$

Clearly the expression above is independent of the market value of the permit endowment $p_y \bar{y}$ which is not conditional on the choice. Therefore, we have:

$$X_i(p_y) = N \int_{\gamma} \frac{\exp[(1/\mu)(B_i - \gamma t_i(X_i) - \tau_i^o)]}{\sum_{j=1\dots n} \exp[(1/\mu)(B_j - \gamma t_j(X_j) - \tau_j^o)] + \exp[V_0(\gamma)/\mu]} f(\gamma) d\gamma. \quad (10)$$

The solution of the implicit system above (fixed point problem) is denoted by $\tilde{X}_i(p_y; \delta^o)$, with

$$\delta^o = \left\{ \delta_1^o = \frac{\tau_1^o}{p_y}, \dots, \delta_n^o = \frac{\tau_n^o}{p_y} \right\}. \quad (11)$$

By construction, the number of permits required to use each route is optimally chosen, so that we have $\tilde{X}_i(p_y; \delta^o) = X_i^o$. Note that the total number of permits distributed \bar{y} is arbitrary. The price of permits will adjust accordingly. The equilibrium value of the permits is (see equation (7)):

$$p_y = \frac{1}{N\bar{y}} \sum_{j=1\dots n} \tau_j^o X_j^o. \quad (12)$$

In addition, we assume that the total supply of permits is smaller than the total demand of permits at equilibrium. Otherwise, the price of permits would be zero. Note that only the total number of permits matters. The price of the permits, which equate supply and demand for permits, is equal to the total toll revenue (at the optimum), divided by the total number of permits distributed. As a consequence, the distribution of permits to the potential travellers can be arbitrary; only the total supply of permits matters. Summarizing, we get:

Lemma 3 For any given number of permits \bar{y} , there is a unique equilibrium price of permits p_y^* given by (12).

Proposition 4 The two systems, price and quantity, when optimally chosen, are equivalent.

Proof. Let $\Omega(\tau^o)$ and $\Omega(\delta^o, p_y^*)$ denote the optimum welfare attained by the price and quantity instruments respectively, where τ^o is the optimum vector of tolls, δ^o is a vector of optimum number of permits required for each route, and p_y^* the equilibrium price of permits. We wish to show that $\Omega(\tau^o) = \Omega(\delta^o, p_y^*)$.

Since τ^o is the optimum toll vector, we have

$$\Omega(\tau^o) > \Omega(\tau) \quad \forall \tau \neq \tau^o. \quad (13)$$

As shown previously, by setting $\delta = \tau^o/p_y^*$, we have, $\Omega(\delta, p_y^*) = \Omega(\tau^o)$ where p_y^* is given by Equation 12.

Now, assume that there exists $\delta = \delta^*$ and $p_y^* = \tilde{p}_y$ such that $\Omega(\delta^*, \tilde{p}_y) > \Omega(\tau^o)$. Then, setting $\tau^* = \tilde{p}_y \delta^*$, we have $\Omega(\tau^*) > \Omega(\tau^o)$. This contradicts Equation 13 and hence, the result follows. ■

Proposition 4 shows that if the supply of permits is small enough (so that the price of permits is positive), then the two systems, price and quantities are equivalent and both attain the social optimum.

STATIC NETWORK WITH VARIABLE DEMAND

In the variable demand setting, we assume that the demand (number of travellers) on a particular day follows a discrete distribution taking a value N^+ with probability q and a value N^- with probability $1 - q$, where $N^- \geq N^+$. Thus, we have two types of days (or states of nature) denoted by $+$ and $-$ and associated demands of N^+ and N^- . This may be easily extended to any number of types of days (or a continuum of types) with no methodological difficulty. It is assumed that on each day, demand (or the state of nature) is realized, and then revealed to travellers who select their route accordingly. The fixed point or equilibrium solution of flows is unique for each day and is the solution of either the first equation (when demand is N^+) or of the second equation (when demand is N^-):

$$\left\{ \begin{array}{l} X_i^+ = N^+ \int_{\gamma} \frac{\exp[(1/\mu)(B_i - \gamma t_i(X_i^+))]}{\sum_{j=1 \dots n} \exp[(1/\mu)(B_j - \gamma t_j(X_j^+))] + \exp[V_0(\gamma)/\mu]} f(\gamma) d\gamma, \\ X_i^- = N^- \int_{\gamma} \frac{\exp[(1/\mu)(B_i - \gamma t_i(X_i^-))]}{\sum_{j=1 \dots n} \exp[(1/\mu)(B_j - \gamma t_j(X_j^-))] + \exp[V_0(\gamma)/\mu]} f(\gamma) d\gamma, \quad i = 1 \dots n. \end{array} \right. \quad (14)$$

Using the implicit function theorem, it can be verified, as expected, that $X_i^+ < X_i^-$ since $N^+ < N^-$.

Price Instrument (Tolls)

Under the price instrument, the government is assumed to have a limited foresight in that it has knowledge of the demand distribution but not the specific realization of demand on a given day.

Thus, the government sets the same tolls on both days by maximizing the expected total welfare (subject to the equilibrium constraints) given by:

$$\begin{aligned} \Omega(\boldsymbol{\tau}, q) = & q\mu \sum_{j=1..N^+} \log \left[\sum_{i=1..n} \exp \left[(1/\mu) (B_i - \gamma_j t_i (X_i^+(\boldsymbol{\tau})) - \tau_i) \right] + \exp [V_0(\gamma_j)/\mu] \right] \\ & + q \sum_{i=1..n} \tau_i X_i^+(\boldsymbol{\tau}) \\ & + (1-q)\mu \sum_{j=1..N^-} \log \left[\sum_{i=1..n} \exp \left[(1/\mu) (B_i - \gamma_j t_i (X_i^-(\boldsymbol{\tau})) - \tau_i) \right] + \exp [V_0(\gamma_j)/\mu] \right] \\ & + (1-q) \sum_{i=1..n} \tau_i X_i^-(\boldsymbol{\tau}) \end{aligned} \quad (15)$$

s.t

$$\begin{cases} X_i^+ = N^+ \int_{\gamma} \frac{\exp \left[(1/\mu) (B_i - \gamma t_i (X_i^+) - \tau_i) \right]}{\sum_{j=1..n} \exp \left[(1/\mu) (B_j - \gamma t_j (X_j^+) - \tau_j) \right] + \exp [V_0(\gamma)/\mu]} f(\gamma) d\gamma, \\ X_i^- = N^- \int_{\gamma} \frac{\exp \left[(1/\mu) (B_i - \gamma t_i (X_i^-) - \tau_i) \right]}{\sum_{j=1..n} \exp \left[(1/\mu) (B_j - \gamma t_j (X_j^-) - \tau_j) \right] + \exp [V_0(\gamma)/\mu]} f(\gamma) d\gamma, \quad i = 1 \dots n. \end{cases}$$

The optimal vector of tolls and associated optimum welfare that solves the above optimization problem (Equation 15) are denoted by $\boldsymbol{\tau}^{os} = (\tau_1^{os}, \dots, \tau_n^{os})$ and $\Omega_{VP}^o \equiv \Omega(\boldsymbol{\tau}^{os}, q)$ respectively. In addition, the route flows at the optimum are denoted by $\mathbf{X}^{po+} = (X_1^{po+}, \dots, X_n^{po+})$ and $\mathbf{X}^{po-} = (X_1^{po-}, \dots, X_n^{po-})$. The welfare loss per individual for the price instrument (with respect to the fixed demand optimum) is given by:

$$\Psi(\boldsymbol{\tau}^o; q) = \frac{\Omega_F^o}{N} - \frac{\Omega_{VP}^o}{qN^- + (1-q)N^+}. \quad (16)$$

Note that to ensure a meaningful comparison, we consider the situation where $N = qN^- + (1-q)N^+$. The quantity $\Psi(\boldsymbol{\tau}^o; q)$ is positive. This is because the tolls are constrained to take the same value regardless of whether the demand is high or low.

Quantity Instrument (Mobility Permits)

In case of the mobility permit instrument, the government distributes a fixed amount of \bar{y} permits per person on each day to all potential travellers. Note that the population of potential travellers is defined by the total number of travellers when the realized demand is highest. Thus, a certain number of travellers who do not travel on the lower demand day will still possess permits which can be sold in the market the same day. The government as before is assumed to have knowledge of only the probability distribution of demand and thus, the number of permits δ_i required to use route i is the same for both days and is set by maximizing expected total welfare. However, the permit price on each day will adjust (denoted by p_y^+ and p_y^- respectively) as the supply demand equilibrium of permits in the market.

The optimal vector of permits needed, and the associated optimum welfare are denoted by $\boldsymbol{\delta}^{os} = (\delta_1^{os}, \dots, \delta_n^{os})$ and $\Omega_{VQ}^o \equiv \Omega(\boldsymbol{\delta}^{os}, q)$ respectively. In addition, the route flows at the optimum

are denoted by $\mathbf{X}^{qo+} = (X_1^{qo+}, \dots, X_n^{qo+})$ and $\mathbf{X}^{qo-} = (X_1^{qo-}, \dots, X_n^{qo-})$. These are obtained by maximizing the expected total welfare (subject to the equilibrium constraints) given by:

$$\begin{aligned} \Omega(\boldsymbol{\delta}, q) = & q \sum_{j=1..N^+} \log \left[\sum_{i=1..n} \exp \left[(1/\mu) (B_i - \gamma_j t_i (X_i^+(\boldsymbol{\delta})) - p_y^{*+} \delta_i) \right] + \exp [V_0(\gamma_j)/\mu] \right] \\ & + qp_y^{*+} \sum_{i=1..n} \delta_i X_i^+(\boldsymbol{\delta}) \\ & + (1-q) \sum_{j=1..N^-} \log \left[\sum_{i=1..n} \exp \left[(1/\mu) (B_i - \gamma_j t_i (X_i^-(\boldsymbol{\delta})) - p_y^{*-} \delta_i) \right] + \exp [V_0(\gamma_j)/\mu] \right] \\ & + (1-q)p_y^{*-} \sum_{i=1..n} \delta_i X_i^-(\boldsymbol{\delta}) \\ & s.t \\ & p_y^{*+} \geq 0; \quad p_y^{*-} \geq 0 \end{aligned} \quad (17)$$

$$\begin{cases} X_i^+ = N^+ \int_{\gamma} \frac{\exp \left[(1/\mu) (B_i - \gamma t_i (X_i^+) - p_y^{*+} \delta_i) \right]}{\sum_{j=1..n} \exp \left[(1/\mu) (B_j - \gamma t_j (X_j^+) - p_y^{*+} \delta_j) \right] + \exp [V_0(\gamma)/\mu]} f(\gamma) d\gamma, \\ X_i^- = N^- \int_{\gamma} \frac{\exp \left[(1/\mu) (B_i - \gamma t_i (X_i^-) - p_y^{*-} \delta_i) \right]}{\sum_{j=1..n} \exp \left[(1/\mu) (B_j - \gamma t_j (X_j^-) - p_y^{*-} \delta_j) \right] + \exp [V_0(\gamma)/\mu]} f(\gamma) d\gamma, \quad i = 1 \dots n, \end{cases}$$

where the prices p_y^{*+} and p_y^{*-} satisfy the following market equilibrium conditions:

$$\begin{cases} \sum_{i=1..n} \delta_i^o(p_y^{*+}) X_i^{qo+}(p_y^{*+}) = \bar{y}N^-, \\ \sum_{i=1..n} \delta_i^o(p_y^{*-}) X_i^{qo-}(p_y^{*-}) = \bar{y}N^-. \end{cases} \quad (18)$$

Recall, in the market equilibrium conditions, that we have assumed that the permits are distributed to all potential travellers, $\bar{y}N^-$ i.e. to the total number of travellers when the demand is highest. The welfare loss per individual for the quantity instrument (with respect to the fixed demand optimum) is given by:

$$\Psi(\boldsymbol{\delta}^o; q) = \frac{\Omega_F^0}{N} - \frac{\Omega_{VQ}^0}{qN^- + (1-q)N^+}, \quad (19)$$

where it is assumed that $N = qN^- + (1-q)N^+$.

Comparison

In the case of stochasticity in the system, simulation will tell us which instrument (and under what conditions) is superior. Specifically, the price control is welfare superior to the quantity control *iff*

$$\Psi(\boldsymbol{\tau}^o; q) < \Psi(\boldsymbol{\delta}^o; q) \implies \Omega_{VP}^0 > \Omega_{VQ}^0. \quad (20)$$

The reader may wonder why the two instruments are different. The reason is specific to the transportation system and our setting. In case of the price instrument, the planner fixes the tolls on each route and according to the demand level and congestion, users decide their route choice. The total number of trips per route is not fixed. In case of quantity control, the planner fixes the number of permits required for each route. The number of permits foreseen will be a balance of the optimal quantity in high and low demand conditions. In case of high demand, the quantity constraint will be binding and the price of permits increases so that those with highest willingness to pay can still get on the road. When demand for route use is low, the quantity target is too lax. There will be too many users and the price of a permit will be low. The exact nature of the price adjustment process is not described in detail in this paper, but the equilibrium price is determined. The two control mechanisms are not equivalent.

STATIC NETWORK WITH VARIABLE CAPACITY

In the variable capacity setting, we assume that the demand (number of travellers) on the two days are fixed (N), but the route capacities $\mathbf{C} = (C_1, \dots, C_n)$ vary across the two days. Thus, the route capacities take a value of $\mathbf{C}^+ = (C_1^+, \dots, C_n^+)$ with probability q and a value $\mathbf{C}^- = (C_1^-, \dots, C_n^-)$ with probability $1 - q$. In a similar manner as in the variable demand case, the fixed point or equilibrium solution of flows is unique for each day and is given by:

$$\left\{ \begin{array}{l} X_i^+ = N \int_{\gamma} \frac{\exp[(1/\mu)(B_i - \gamma t_i(X_i^+, C_i^+))]}{\sum_{j=1 \dots n} \exp[(1/\mu)(B_j - \gamma t_j(X_j^+, C_j^+))] + \exp[V_0(\gamma)/\mu]} f(\gamma) d\gamma, \\ X_i^- = N \int_{\gamma} \frac{\exp[(1/\mu)(B_i - \gamma t_i(X_i^-, C_i^-))]}{\sum_{j=1 \dots n} \exp[(1/\mu)(B_j - \gamma t_j(X_j^-, C_j^-))] + \exp[V_0(\gamma)/\mu]} f(\gamma) d\gamma, \quad i = 1 \dots n. \end{array} \right. \quad (21)$$

The optimization problems (Equations 15 and 17) can be formulated in similar fashion for the price and quantity instruments as in was in the variable demand case. Accordingly, the optimum welfare for the two instruments can be compared as per Equation 20. The details are omitted here due to a lack of space.

STATIC NETWORK WITH VARIABLE DEMAND AND CAPACITY

In the variable demand and capacity setting, we assume that the demand (number of travellers) and route capacities vary across the two days. These take a value of $\{N^+, \mathbf{C}^+\}$ with probability q and a value $\{N^-, \mathbf{C}^-\}$ with probability $1 - q$. The fixed point or equilibrium solution of flows is once again unique for each day and is given by:

$$\left\{ \begin{array}{l} X_i^+ = N^+ \int_{\gamma} \frac{\exp[(1/\mu)(B_i - \gamma t_i(X_i^+, C_i^+))]}{\sum_{j=1 \dots n} \exp[(1/\mu)(B_j - \gamma t_j(X_j^+, C_j^+))] + \exp[V_0(\gamma)/\mu]} f(\gamma) d\gamma, \\ X_i^- = N^- \int_{\gamma} \frac{\exp[(1/\mu)(B_i - \gamma t_i(X_i^-, C_i^-))]}{\sum_{j=1 \dots n} \exp[(1/\mu)(B_j - \gamma t_j(X_j^-, C_j^-))] + \exp[V_0(\gamma)/\mu]} f(\gamma) d\gamma, \quad i = 1 \dots n. \end{array} \right. \quad (22)$$

As before, the optimization problems (Equations 15 and 17) can be formulated for the price and quantity instruments and the optimum welfare for the two instruments can be compared as per

Equation 20. Note that on the supply side, in addition to the route capacity, the free flow travel times may also be varied across days with no methodological difficulties.

EXPERIMENTS AND DISCUSSION

This section reports results from a set of experiments conducted to compare the price (toll) and quantity control (mobility permit) instruments. The objectives of the numerical experiments are to identify the superior instrument, quantify the range of welfare differences between the two and the effect of various supply and demand factors. A simple transportation network is considered for the experiments consisting of a single origin-destination pair and six alternatives, five alternative routes (car) and one public transport alternative. The supply and demand characteristics of the transportation system are as described in Sections 3 and 4. The experimental design and inputs are first discussed. This is followed by a discussion of the results.

Experimental Design and Inputs

Three distinct settings are considered for the experiments that differ in the nature of stochasticity in the system: (1) Variable demand, (2) Variable demand and capacity, and (3) Variable demand, capacity and free flow times. Under variable demand, two levels are considered for the route capacities, high and low (the two scenarios are referred to as VD-HC and VD-LC which stand for variable demand high capacity and variable demand low capacity respectively). Note that in these two scenarios the route capacity does not vary across the good and bad days.

Finally, in the last scenario, demand, capacity and free flow times all vary across the two days (referred to as VD/C/FF). The capacity and free flow times are assumed to have a high negative correlation ($\rho = -0.8$). Thus, in total, four scenarios are considered in the experiments (VD-HC, VD-LC, VD/C and VD/C/FF).

For each of the four scenarios, the following inputs are systematically varied: 1) Demand Level (N^+ , N^-), 2) Logit scale parameter (μ), 3) BPR congestion coefficient (β), and 4) Probability of low demand day occurring (q). The various experiment parameters are summarized in Table 1 where $U(a, b)$ refers to a uniform distribution between a and b . A total of 324 instances were tested.

TABLE 1 Experiment Factors and Factor Levels

Factor	Levels			
Scenarios	VD-LC	VD-HC	VD/C	VD/C/FF
Mu	1	1.5	2	
Beta	U (2,2.25)	U(4,4.25)	U(6,6.25)	
q	0.4	0.5	0.6	
$N^+;N^-$	10000;15000	10400;15600	11200;16800	

With regard to the fixed inputs, the free flow travel times are randomly generated between 10 and 17.5 minutes (which assuming a free flow speed of 60 km/hr correspond to a trip length of 10-15 km and are typical of urban transportation networks). The benefit B_i is randomly generated between 12.5\$ and 17.5\$ and the route capacities between 1500 – 2000 vehicles/hour. The demand levels are fixed to ensure that the average ratio of congested to free flow system travel time (in the absence of tolls/permits) is in the range 1.25 – 2.5.

TABLE 2 Fixed Inputs

Parameter	Values					
	R1	R2	R3	R4	R5	PT
B (\$)	15.3	16.2	12.9	16.3	13.8	14.5
Alpha	0.171	0.163	0.159	0.158	0.168	-
Scenarios: VD-HC and VD-LC						
Free flow time (min)	14.5	10.1	10.4	14.9	13.0	16.21
Capacity (veh/hr) High	1729	1520	1580	1688	1535	∞
Capacity (veh/hr) Low	1504	1322	1374	1468	1335	∞
Scenario: VD/C						
Free flow time (min)	14.5	10.1	10.4	14.9	13.0	16.21
Capacity (veh/hr) +	1729	1520	1580	1688	1535	∞
Capacity (veh/hr) -	1504	1322	1374	1468	1335	∞
Scenario: VD/C/FF						
Free flow time (min) +	14.5	10.1	10.4	14.9	13.0	16.21
Free flow time (min) -	13.5	12.1	11.0	16.6	13.1	16.21
Capacity (veh/hr) -	1847	1726	1657	1559	1794	∞
Capacity (veh/hr) +	1529	1869	1587	1753	1804	∞
$\bar{\gamma}$ (\$ per minute)	0.33					
$\sigma\gamma$ (\$ per minute)	0.066					
$\bar{y}N^-$	15000					

The inputs for all four scenarios are summarized in Table 2. Note that for the VD-HC and VD-LC scenarios, all parameters except route capacity are the same.

The performance measures used to evaluate the toll and mobility permit systems are first, logsum value (LS), consumer surplus (CS) and welfare (W). Note that in the toll system, the logsum value is equal to the consumer surplus and the welfare is the sum of the consumer surplus and toll revenue. In the mobility permit system, the welfare is equal to the consumer surplus which is the sum of the logsum value and the market value of the total permit endowment because the mobility permits are received for free.

In case of the price instrument, the performance measures for each test instance are computed by solving the optimization problem in Equation 17 as a bilevel optimization problem in MATLAB. The *fminunc* routine is used for the upper level (the sequential quadratic programming algorithm is applied which is known to work well for non-convex problems, [14]) and the *lsqnonlin* routine is used for the lower level equilibrium problem. For each test case ten different randomly generated initial toll vectors were considered and in all 324 test instances the ten different initial solutions were found to converge to the same optimum.

In case of the quantity instrument, arbitrary permit prices (for the good and bad days) are first assumed and the optimization problem in Equation 15 is solved in similar fashion using the *fminunc* and *lsqnonlin* MATLAB routines. Based on the resulting demand of permits on the two days, the permit prices are now adjusted using a simple bisection method and the optimization problem is resolved. This procedure is performed iteratively until the supply demand equilibrium of permits is satisfied (Equation 18). As before, ten randomly generated starting points (or permits needed, δ) are considered for each run of the bilevel optimization program and in all cases, the same optimum was obtained.

Results

The results from the numerical simulations are organized into two sub sections. The first examines the welfare components for the two instruments across the various test cases. The second examines equity considerations using the gini coefficient.

Welfare Components

The results of the experiments are given in Table 3 which provides summary statistics (across the 324 test cases) for the differences in welfare components between: (1) Variable Price system (VP hereafter; this refers to the price instrument under variable demand/supply) and No Toll system (NT hereafter; this refers to the system with no tolls or permits under variable demand/supply), Variable Quantity system (VQ hereafter; this refers to the mobility permit instrument under variable demand/supply) and NT system, (2) VP system and Fixed Optimum (FO hereafter; this refers to the price or quantity instrument under fixed demand/supply which are identical), VQ system and FO, and (3) VP and VQ systems. The individual performance measures as noted earlier are the logsum value (LS), consumer surplus (CS) and total welfare (W). All numbers are in dollar amounts which are computed for one hour per day.

TABLE 3 Summary Statistics: Welfare Components

Statistic	VP-NT (\$)		VQ-NT (\$)		VP-FO (\$)		VQ-FO (\$)		VQ-VP (\$)		
	LS	W	LS	W	LS	W	LS	W	LS	W	CS
Mean	-11235.2	10320.3	-9831.9	10493.1	-539.4	-943.6	863.9	-770.7	1403.3	172.8	21728.3
Median	-11100.2	10080.3	-9699.0	10235.4	-439.0	-846.7	598.8	-727.2	1074.5	201.8	21595.0
Minimum	-17837.0	5317.9	-15476.4	4511.8	-1680.4	-2314.4	-392.9	-1849.5	62.9	-808.3	15136.8
Maximum	-5174.2	18541.9	-4959.4	19003.6	4.7	-309.0	5142.8	-239.4	6394.6	1006.7	30608.4
25th Percentile	-13784.8	8469.0	-11712.5	8425.2	-752.1	-1161.1	204.5	-973.7	533.1	0.0	18525.4
75th Percentile	-8649.4	11966.6	-7871.7	12183.9	-249.0	-625.4	1282.1	-530.8	1908.6	346.8	24343.3
% Scenarios >0	0	100	0	100	0.66	0	79.67	0	100	74.75	100

The results indicate that in a stochastic environment, the mobility permit instrument is slightly better in welfare terms than the tolls instrument. The total welfare of the VQ system is higher than the VP system in 74.75% of the test instances. The difference in welfare between the two instruments (VQ-VP) ranges between -808.3\$ and 1006.7\$ with a mean of 172.8\$ and median of 201.8\$ (penultimate column in Table 3). The welfare gains are of the order of 1 to 5% of the toll revenues. Note also that the magnitude of the difference is significant in relation to the difference between the VP and FO systems (mean of -943\$) or VQ and FO systems (-770.7\$). As expected, the welfare of the VQ and VP systems is always less than the corresponding welfare of the deterministic optimum solution which reflects the cost of stochasticity.

The consumer surplus is also always superior for the mobility permit system and the difference is substantial, ranging between 15137\$ and 30608\$ with a mean difference of 21728.3\$. The reason is obvious: permits are received for free which leads to a large additional consumer surplus. The distributions of difference in consumer surplus and total welfare are given in Figure 2.

Effect of Logit Scale Parameter μ

An analysis of variance of the difference in consumer surplus between the price and quantity control mechanisms across the three levels of the logit scale parameter indicates a statistically significant effect at a confidence level of $\alpha = 0.01$. The largest improvements in consumer surplus

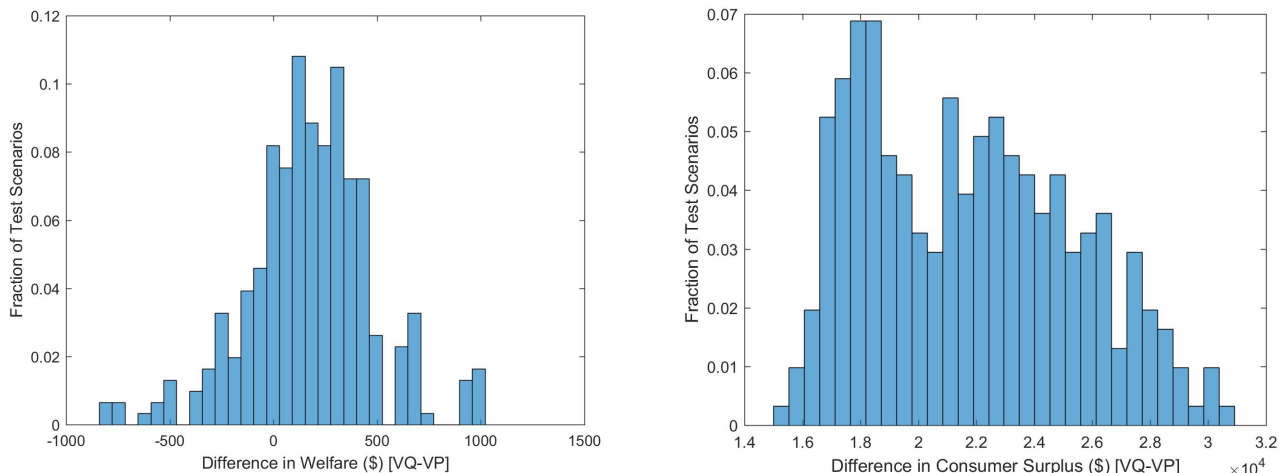


FIGURE 2 Distribution of difference in (a) Total Welfare and (b) Consumer Surplus

(average improvement of 25850\$) are observed for a value of $\mu = 2$ compared to an average improvement of 17897\$ for $\mu = 1$. Thus, as the variance of the logit error term increases, or the extent of stochasticity in route choices increases, the difference in consumer surplus between the two instruments increases. The effect of the logit scale parameter on difference in consumer surplus is shown in the box plot in Figure 3.²

In contrast, the logit scale parameter does not have a statistically significant effect (at $\alpha = 0.01$) on the difference in total welfare between the quantity and price systems. The average difference increases from 131\$ at $\mu = 1$ to 181\$ at $\mu = 1.5$ and 211\$ at $\mu = 2$. However, the percentage of scenarios where the quantity control system is superior decreases from 82% at $\mu = 1$ to 74.5% at $\mu = 1.5$ and 66% at $\mu = 2$. The effect of the logit scale parameter on difference in total welfare is shown in the box plot in Figure 3.

Effect of BPR Congestion Coefficient β

The BPR congestion coefficient β has a statistically significant effect (at a confidence level of $\alpha = 0.01$) on the difference in consumer surplus between the price and quantity control systems. The largest improvements in consumer surplus (average improvement of 23000\$) are observed for high values of $\beta = 6 - 6.25$ compared to an average improvement of 20190\$ for $\beta = 2 - 2.25$. The consumer surplus advantage of a permit system increases because the higher level of congestion requires higher permit prices and therefore a higher value of the permits received for free. The effect of β on percentage difference in consumer surplus is shown in the box plot in Figure 4.

The BPR congestion coefficient β also has a statistically significant effect (at $\alpha = 0.01$) on the difference in total welfare between the price and quantity control systems. The average difference varies from -6.5 \$ at $\beta = 2$ to 110\$ at $\beta = 4$ and 386\$ at $\beta = 6$. Further, the percentage of scenarios where the quantity control system is superior increases from 48% at $\beta = 2$ to 74% at $\beta = 4$ and 97% at $\beta = 6$. The effect of β on the percentage difference in total welfare is shown

²For a given x-axis parameter value, the edges of the box indicate the 25th and 75th percentiles and the red line indicates the median of the performance measure (on the y-axis) across the respective scenarios. The dotted line stretches from the minimum to maximum value.

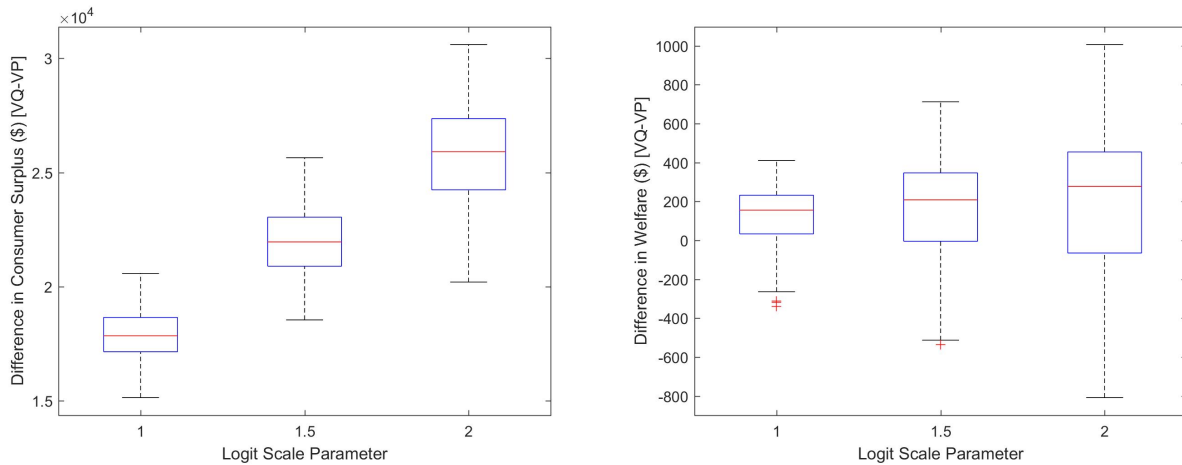


FIGURE 3 Effect of Logit scale parameter on Difference in (a) CS and (b) W

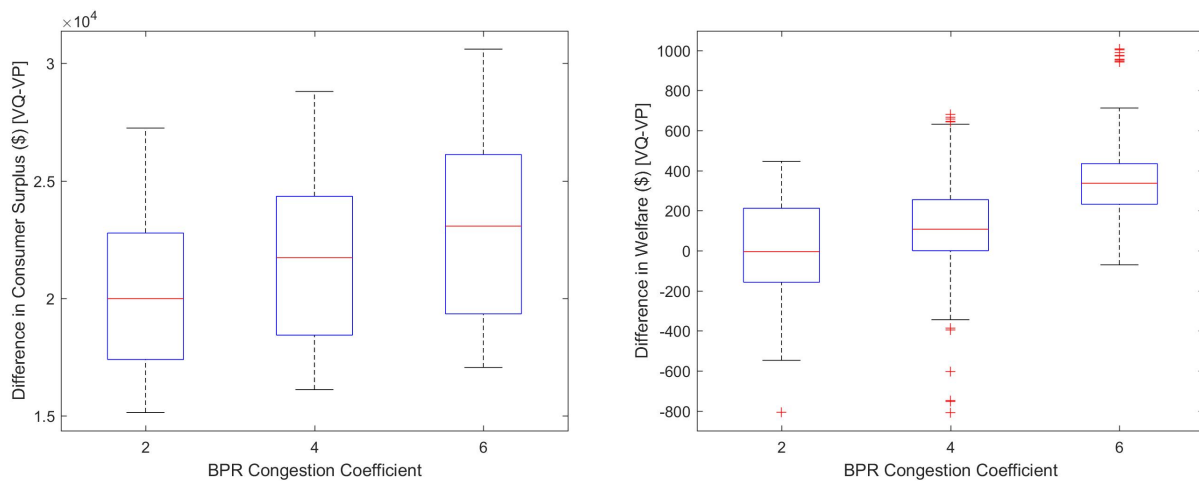


FIGURE 4 Effect of β on Difference in (a) CS and (b) W

in the box plot in Figure 4 where it can be seen that the quantity control mechanism is superior in terms of total welfare typically when congestion effects are more severe (high value of $\beta = 6$).

Effect of Demand Level

The demand level has a statistically significant effect (at a confidence level of $\alpha = 0.01$) on the difference in consumer surplus between the price and quantity control systems. The reason is again the higher price of permits that is needed when demand is higher. The largest improvements in consumer surplus (average improvement of 22990\$) are observed for the high demand level.

The demand level also has a statistically significant effect (at $\alpha = 0.1$) on the difference in total welfare between the price quantity control systems. The mean difference in welfare increases from 150\$ at demand level 1 to 205 \$ at demand level 3. Further, the percentage of scenarios where the quantity mechanism is superior increases from 70% at demand level 1 to 80% at demand level 3. This further corroborates the conjecture that when the extent of congestion is more severe, the quantity instrument is typically superior.

Effect of Scenario Type (Nature of Stochasticity)

The effect of the scenario type on the difference in consumer surplus and welfare are examined in this section.

The results indicate that the largest improvements in welfare (average of 465\$) are obtained in the scenario with variable demand and variable capacity (Scenario 3), where there is a good day with low demand and high capacity and a bad day with high demand and low capacity. In this scenario, the quantity control mechanism is superior in 99% of the test cases. This is no surprise as this scenario combines the effects of high demand and low capacity. In contrast, the smallest differences in welfare (average of -43\$) are observed in scenario 4 where the demand, capacity and free flow times are all variable but there is a smaller distinction between the two days since the route capacities and free flow times are independently drawn from a lognormal distribution. Thus, the day with high demand may also have higher capacities and lower free flow travel times on some routes compared to the day with low demand. In this scenario, the quantity mechanism is superior in only 44% of the test cases. In addition, for the variable demand case, the number of test cases where the quantity mechanism is superior is 82% for the scenario with lower route capacities (Scenario 2) compared to 67% for the scenario with higher capacities (Scenario 1).

In summary, the quantity instrument is found to be slightly superior to the price instrument typically when the demand levels are high, which leads to greater congestion, when congestion effects are more severe (higher values of the BPR coefficient β) and when the degree of stochasticity (day to day variability in system supply/demand) is higher.

Equity

The Gini coefficient is used to examine equity considerations relating to the quantity and price instruments. We refer the reader to [15] for a detailed discussion of measures of inequality in the transport and mobility context. The computation of the Gini coefficient is first described followed by a discussion of results.

Consider the population of N ($k = 1 \dots N$) drivers arranged in increasing order of their value of time (VOT_k). Let $x = k/N$ and let $LS(k)$ denote the logsum value in \$ of individual k . Define,

$$g(x) = \frac{\sum_{j=1}^{xN} LS(j)}{\sum_{j=1}^N LS(j)}. \quad (23)$$

The gini coefficient is computed as,

$$\frac{\left| 0.5 - \int_0^1 g(x) dx \right|}{0.5}. \quad (24)$$

The Gini coefficient is a measure of equity. Total equity implies that it is equal to zero. The larger it is, the more inequitable is the policy. Often Gini coefficients are computed with respect to income. Here, we introduce a measure based on the *LogSum*, which is a welfare measure.

The summary statistics for the percentage difference in the Gini coefficient between the price control and no toll/permit (NT) system [$100 * (GC_{VP} - GC_{NT})/GC_{NT}$], quantity control and NT system [$100 * (GC_{VQ} - GC_{NT})/GC_{NT}$], and quantity control and price control systems [$100 * (GC_{VQ} - GC_{VP})/GC_{VP}$] are given in Table 4.

TABLE 4 Summary Statistics: Percentage Difference in Gini Coefficient

Statistic	$100 * \frac{GC_{VQ} - GC_{NT}}{GC_{NT}}$	$100 * \frac{GC_{VP} - GC_{NT}}{GC_{NT}}$	$100 * \frac{GC_{VQ} - GC_{VP}}{GC_{VP}}$
Mean	-12.19	-11.24	-1.07
Median	-12.16	-11.36	-0.99
Minimum	-19.91	-19.03	-2.58
Maximum	-4.92	-3.40	-0.12
25th Percentile	-14.68	-13.64	-1.45
75th Percentile	-9.79	-8.85	-0.66
% Scenarios >0	0.00	0.00	0.00

The results indicate that the Gini coefficient for the quantity control and price control systems are always superior (smaller) to the NT system. This is an important finding as this implies that introducing a price or quantity control system always slightly favours the poor when the toll revenues or permits are redistributed equally to all potential drivers. The main reason is that the low value of time drivers receive a relatively larger share of the toll revenues or can sell part of their free permits for a high price to the high value of time drivers.

Further, the Gini coefficient for the quantity control system is smaller than the price control system in all the tested scenarios (average of -1.07% and median of -0.99%). This implies that the quantity instrument is more equitable than the price instrument. Since acceptability is a major concern for road pricing, our result shows a second advantage for quantity regulation (versus price regulation).

A main difference between permits and tolls is that the government can change the endowment of permits in order to improve equity. Specific distributions of mobility permits does not change the total welfare, but does influence the equity issues; we conjecture that any distribution of burdens can be replicated with an appropriate distribution of mobility permits.

CONCLUSIONS

This paper has developed an algorithm to compute the optimal permit and toll systems when road demand and road capacity are ex ante uncertain and where controls have to be decided ex ante. The efficiency properties of the optimized permits and toll systems have been compared numerically for a traffic system with parallel highways and a public transport alternative. The permit system was found to be on average more efficient than the toll system. This advantage was more pronounced when the traffic system was faced with more congestion. In absolute terms, the advantage is small as it is limited to a maximum of 5% of toll revenues.

The comparison of permits and prices was explained intuitively in the linear case for one type of externality by Weitzman. However, the situation considered here is far more complex. De Palma and Lindsey (2016) [16] have started to develop some intuition mainly in the case of a single route, and provide conditions when mobility permits are superior to tolls. The additional complexity addressed in this study, that is, heterogeneity, mode choice and multiple routes, make the analysis considerably more complicated, and intuition remains very hard to develop.

The findings from this study are important but need to be put in perspective in four respects. First, this is a numerical comparison for a given road transport system. It was submitted to many sensitivity tests but the overall design of the road system may still matter. Second, the design of the permit and toll systems matters. One of the restrictions imposed on the toll system was that

the toll is fixed ex ante. Third, transaction costs and second order effects of the use of the revenues may still tilt the efficiency balance towards tolling systems. Finally, the public acceptability of permit systems appear better than toll systems but acceptability is not guaranteed as they entail large permit price variations and there is no obvious way to distribute the mobility permits.

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