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Edmond Baranes, Marion Podesta, Jean-Christophe Poudou

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Edmond BARANES[†], Marion PODESTA[‡], and Jean-Christophe POUDOU[§]

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Abstract

We study the incentives to collude when firms use mixed bundling or independent pricing strategies for the sale of two components of a composite good. The main finding is that collusion is less sustainable under mixed bundling, because this increases the profitability of deviations from the collusive path. The result is robust to extensions with an endogenous choice of the mode of competition (with bundling or independent pricing) and to competition in quantities. These results offer a novel argument against a *per se* rule concerning bundling in antitrust policy.

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[†]LAMETA, University of Montpellier.

[‡]ART-Dev, University of Perpignan Via Domitia

[§]LAMETA, University of Montpellier, Faculty of Economics, Espace Richter, av. Raymond Dugrand, CS 79606 34960 Montpellier cedex 2. `jean-christophe.poudou@umontpellier.fr`

1 Introduction

For several years, an increasing number of industries have proposed to package together multiple goods and/or services. In telecommunications for example, many different types of packages include bundling of minutes (e.g., contracts with a fee for a given number of minutes of calls), bundling of services (e.g., SMS and voice), and bundling of complementary products (handsets and service contracts). In energy markets, firms typically propose packages composed of energy and services.[1] In the banking and financial service sector, packaged and tied offers (e.g. banking services, extension of credit, insurance, individual pension schemes) are also very common. Recently the focus of antitrust analysis has been on the anticompetitive effects of bundling, because the banking package service pricing is complex, quantity undefined, and sets of offers are difficult to compare between providers.[2] Indeed, when purchasing bundled products, consumers may find it difficult to terminate their contracts, especially due to high closing costs involved under bundling. In the telecommunication or energy sectors, consumers also may have difficulties comparing offers and prices. When bundling offers a way to obfuscate effective prices, it may be seen as a strategy that can harm consumers. These examples suggest that, when firms use bundling practices, they may, simultaneously, be motivated to engage in strategic behaviors in order to exert their market power.

There is a growing debate on the treatment of bundling and the associated market power. As revealed by European data, industries that practice bundling (medias, telecoms, banks, energy) are also more exposed to collective dominance allegations by competition authorities.[3] Because bundling can be viewed as a tool to reduce competitive pressure for firms and to exploit significant market power, it is interesting to analyze how bundling practices could potentially affect incentives for firms to collude. We show that collusion is less sustainable when firms use mixed bundling strategies than when they adopt independent pricing. The main results show that mixed bundling strategy hinders collusion.

Bundling has been widely discussed in economics and marketing literature. In general, it refers to the practice of selling two or more goods together at a unique price.[4] The economic literature on bundling isolates several effects starting from its role as a price discrimination device.[5] Bundling allows a firm to sort consumers according to their willingness to pay as

[1] In France, for example, dominant operators propose gas and electricity packages to residential consumers, such as "Dolce Vita" for Gaz de France-Suez (now Engie) and "Bleu Ciel" for Electricité de France. In the United Kingdom, E.ON and Atlantic Electric & Gas both offer Dual Fuel products.

[2] In 2014, The European Council adopted a directive that promotes the comparability of fees related to payment accounts, payment account switching and access to payment accounts with basic features

[3] Petit and Neyrinck (2013) show that, since 2002, 47% of collective dominance cases treated by EU concern these industrial sectors.

[4] When a firm sells its goods both separately and bundled in a package, it follows a mixed bundling strategy. When a firm commits to supply only the bundle, it chooses a pure bundling strategy.

[5] In general, price discrimination of degree I allows a monopolist to extract all consumer surplus. Price discrimination of degree II is usually associated with non-linear pricing (see Sürücü, 2016). Price discrimination of degree

analyzed by Adams and Yellen (1976) for the case of a two-product monopoly. In an analysis dealing with specific cases, these authors show that mixed bundling is generally the optimal strategy.[6]

A more recent literature has analyzed the entry deterrence role of bundling. Whinston (1990), Nalebuff (2004), and Peitz (2008) emphasize that a monopolist in a primary market can deter the entry of a competitor in a secondary market through a commitment to a bundling strategy.[7] The aim of this literature is to show that bundling can have anti-competitive effects which operate through aggressive pricing of the monopolist with its bundling strategy: this deters entry of the rival and can lead to monopolization of the secondary market, with negative consequences for consumers (see also Choi and Stefanadis, 2001, and Carlton and Waldman, 2002).[8] However, aggressive pricing creates positive gains for consumers that can be counterbalanced only under specific conditions,[9] and can induce also the rivals to adopt bundling strategies, with more complex consequences. Anderson and Leruth (1993) analyze bundling in a complementary goods duopoly and distinguish independent pricing as the dominant strategy in the commitment case. Economides (1993), using the same framework, shows that firms follow mixed bundling strategies in the Nash equilibrium. However, these firms earn lower profits than they would if they had adopted an independent pricing strategy. Reisinger (2006) finds the same results when consumers' reservation values are negatively correlated (because bundling reduces consumer heterogeneity and makes price competition more aggressive).[10] Armstrong and Vickers (2010) examine a unit-demand model in which consumers may buy one product from one firm and another product from another firm, in accordance with nonlinear pricing. They show that bundling generally acts to reduce profit and welfare, and boost consumer surplus,[11] but they assume that there is an intrinsic extra shopping cost when consumers have to purchase each good in a different location. Thanassoulis (2007) finds that if buyers incur firm specific costs or have shop specific tastes, competitive mixed bundling lowers consumer surplus overall and raises profits. Reisinger (2006) shows that mixed bundling can reduce prices but also consumers' surplus because some consumers cannot buy their preferred products. Finally, Granier and Podesta (2010) show that bundling motivates horizontally-differentiated firms to merge, which also deteriorates consumer surplus.

III is based on linear pricing (see Braouezec, 2016).

[6]Schmalensee (1984) shows that these results are robust to a bivariate normal distribution. McAfee *et al.* (1989) generalize the results to almost all distributions. Under general distribution of consumer preferences, Chen and Riordan (2013) extend the profitability of mixed bundling to a negative dependence, independence, or limited positive dependence between consumers' values for the two products.

[7]In Nalebuff (2004), pure bundling is optimal even without any commitment.

[8]For a survey on the theories of bundling as an anticompetitive device see Motta (2004).

[9]With endogenous entry in the secondary market (Etro, 2011), bundling is adopted as an aggressive strategy which tends to increase welfare without consumer harm.

[10]Two effects are created by bundling: the well-known "sorting effect" and the "business-stealing effect," which results from bundle-to-bundle competition.

[11]In contrast, when consumers buy all their products from one firm (the one-stop shopping model), nonlinear pricing leads to higher profit and welfare but often to lower consumers surplus, compared with linear pricing.

Different factors influence the level of competitiveness in different industries. Bernheim and Whinston (1990) examine conditions in which multimarket contact facilitates collusion; if identical firms with the same constant returns-to-scale technologies produce homogeneous goods, multimarket contact does not sustain collusive commitments. However, few works have considered the relationship between collusion and pricing strategies. Dana and Fong (2011) focus on bundling and collusion in an infinitely-repeated oligopoly price game and show that intertemporal bundling facilitates collusion. More precisely, they highlight that with long-term contracts, tacit collusion is sustainable for a wider range of discount factors and market structures. Spector (2007) shows that leverage-based theoretical arguments are persistent when collusion is considered; tying a good produced monopolistically with a complementary good may be a profitable strategy because it facilitates collusion in the tied market. Another general aspect that influences deeply the sustainability of collusion is the way collusive agreements are achieved in the industry. Price agreements have been mainly considered in the literature but quantity ones (i.e. quotas or market shares) are also of importance. Deneckere (1983, 1984) has shown that with constant marginal costs, it is easier to sustain collusion under Bertrand duopoly only if the products are very close substitutes. We will show that this last argument is still robust when mixed-bundling is considered.

Our focus is on the firms' incentives to collude when they use a mixed bundling strategy. We use a standard linear demand system as in Economides and Salop (1992) and Economides (1993), with two firms that produce two complementary goods, and we start from a price competition setting where mixed bundling is a Nash equilibrium of the pricing game. Then, we analyze the possibility for firms to sustain collusive prices in a repeated game. We conclude that collusion is less sustainable under mixed bundling than under independent pricing one.

The above results are obtained using an exogenous assignment of pricing devices to firms. In order to check its robustness, we study two extensions of our basic model. First, we allow for endogenous choices between mixed bundling and independent pricing. The ability of firms to determine their pricing devices (in an irreversible or flexible way) gives some credible arguments to the hindering effect of mixed bundling. Second, we allow for quantity competition (i.e., a softer competition setting): the hindering effect of mixed bundling is even much stronger.

The section **??** presents the basic model and introduces the game of collusion under independent pricing and mixed bundling strategies. We compare the sustainability of collusion between both independent pricing and mixed bundling, assuming it to be exogenously adopted by firms. In the section **??** we extend the result analyzing firms' optimal decisions among independent pricing and mixed bundling schemes. In the section **??** we explore how our basic result works under quantity competition. The final section concludes. All proofs appear in the Appendix.

2 The Model

We use a model with a standard linear demand system as in Economides and Salop (1992) and Economides (1993), in which two firms denoted 1 and 2 produce two complementary goods that are components of a composite good (i.e. a system). The composite good is described by two ranked subscripts ij with $i,j=1,2$ that refer to the firm's origin. This means that the composite good ij includes a unit of the *first component* sold by firm $i=1,2$ and one unit of the *second component* sold by firm $j=1,2$. Components are strict complements within each composite, but composites are assumed substitutable. More precisely, we consider a representative consumer for which preferences on both components are represented by a quadratic quasi-linear utility function where Y is the quantity of numéraire:

$$u(\mathbf{X})=\alpha\sum\nolimits_{i,j=1}^{2}X_{ij}-\frac{1}{2}\beta\sum\nolimits_{i,j=1}^{2}\left(X_{ij}\right)^{2}-\gamma\sum\nolimits_{i,j=1,i\neq j}^{2}X_{ij}\,X_{ji}+Y \tag{1}$$

where $\mathbf{X}=(X_{11},X_{12},X_{21},X_{22})$ is the vector of all system quantities consumed, that is X_{ij} is a system good composed by units of the first component purchased to the firm i and units of the second component purchased to the firm j. Parameters α,β,γ are positive reals with $\beta>\gamma$. Given income E, the net utility is given by $U(\mathbf{X})=u(\mathbf{X})-\sum_{i,j=1}^{2}s_{ij}\,X_{ij}+E$ where s_{ij} is the unit price of each composite good ij. Maximization of $U(\mathbf{X})$ with respect to $\mathbf{X}$, gives the necessary and sufficient conditions:[12]

$$\begin{cases} s_{ij}=\alpha-\beta X_{ij}-\gamma(X_{ii}+X_{ji}+X_{jj}) \\ s_{ii}=\alpha-\beta X_{ii}-\gamma(X_{ij}+X_{ji}+X_{jj}) \end{cases} \text{for all } i,j=1,2,i\neq j. \tag{2}$$

Letting $\kappa=(\beta-\gamma)(\beta+3\gamma)>0$; $\hat{a}=\alpha(\beta-\gamma)/\kappa>0$; $b=(\beta+2\gamma)/\kappa>0$; and $c=\gamma/\kappa>0$, and putting $X_{ij}=D_{ij}$, this involves a linear demand structure given by:

$$\begin{cases} D_{ij}=\hat{a}-bs_{ij}+c(s_{ii}+s_{ji}+s_{jj}) \\ D_{ii}=\hat{a}-bs_{ii}+c(s_{ij}+s_{ji}+s_{jj}) \end{cases} \text{for all } i,j=1,2,i\neq j. \tag{3}$$

The parameter $\hat{a}$ represents the maximum level of demand for each system if prices were zero. For convenience we denote $a=\hat{a}/c$. The parameter b describes how demand for a given composite product falls as its own price increases, and the parameter c reflects the cross-price elasticity of demand across systems. Note that the ratio $x=b/c$ is related to the relative degree of substitutability between composites, which is high when x is low because c is large relative to b. To insure a strictly positive demand for all firms we assume an upper bound on substitutability assuming $x>\xi\equiv\frac{13+\sqrt{73}}{6}\simeq 3.59$ as discussed below. In our duopoly setting, this means that monopolization is excluded.

A given firm can sell the set of products individually at prices $\{\underline{p}^{i},\bar{p}^{i}\}$, using separate sales or independent pricing strategy (hereafter abbreviated to *IP*): this means that the first component is offered as a price $\underline{p}^{i}$ and the second at $\bar{p}^{i}$ for each unit purchased. Otherwise, the firm

[12]Sufficiency is obtained by concavity of U i.e. $\beta>\gamma$.

can propose both products separately *and* in a bundle $\{\underline{p}^i, \bar{p}^i, r^i\}$, using what is called a mixed bundling strategy (hereafter abbreviated to *MB*): then r^i is the unit price of the entire composite good sold as a bundle. If firms do not adopt a mixed bundling strategy, the price of the combination of one unit of each component is the sum of their individual sale prices, $s_{ii} = \underline{p}^i + \bar{p}^i$. However, if firms use a mixed bundling strategy, the price of the combination is a bundle price, $s_{ii} = r^i$. We assume the production costs are equal to zero and firms are profit maximizers. All profit functions in all market configurations will be derived in the next subsections.

In the following sub-sections we will just compare the sustainability of tacit collusion under repeated competition with independent pricing or mixed bundling (later on we will allow firms to choose their pricing strategy).

2.1 Competitive Pricing Strategies

When firms $i = 1,2$ follow *independent pricing*, they can sell their complementary goods independently at two separate prices $\{\underline{p}^i, \bar{p}^i\}$, $i = 1,2$. Explicitly, demand prices are given by $s_{ii} = \underline{p}^i + \bar{p}^i; s_{ij} = \underline{p}^i + \bar{p}^j$ for all $i, j = 1,2$. As the demand system is given by (**??**), the profit functions for both firms are:

$$\Pi^i_{IP} = \underline{p}^i (D_{ii} + D_{ij}) + \underline{p}^i (D_{ii} + D_{ji}) \qquad \text{for all } i, j = 1,2, i \neq j$$

At a non-cooperative price equilibrium each firm seeks to maximize Π^i_{IP} with respect to its own IP prices $\left(\underline{p}^i, \bar{p}^i\right)$ and it is characterized by the first-order conditions, for $i = 1,2$:

$$\begin{aligned} \frac{\partial \Pi^i_{IP}}{\partial \underline{p}^i} &= 0 \Leftrightarrow ac - 2(b-c)\underline{p}^i - (b-3c)\bar{p}^i + 2c\underline{p}^j - \frac{1}{2}(b-3c)\bar{p}^j = 0 \\ \frac{\partial \Pi^i_{IP}}{\partial \bar{p}^i} &= 0 \Leftrightarrow ac - 2(b-c)\bar{p}^i - (b-3c)\underline{p}^i + 2c\bar{p}^j - \frac{1}{2}(b-3c)\underline{p}^j = 0 \end{aligned}$$

Solving this linear system determines the best replies of the firm i when IP applies, that is (using $x = b/c$):

$$\left\{ \begin{array}{l} \underline{\mathbf{p}}^i\left(\underline{p}^j, \bar{p}^j\right) = \frac{a}{3x-5} + \frac{1}{2}\frac{x+1}{3x-5}\underline{p}^j - \frac{x-3}{3x-5}\bar{p}^j \\ \bar{\mathbf{p}}^i\left(\underline{p}^j, \bar{p}^j\right) = \frac{a}{3x-5} + \frac{1}{2}\frac{x+1}{3x-5}\bar{p}^j - \frac{x-3}{3x-5}\underline{p}^j \end{array} \right. \tag{4}$$

By definition, a competitive price (interior) equilibrium $\left(\underline{p}^i, \bar{p}^i, \underline{p}^j, \bar{p}^j\right)$ is a fixed point of the best reply functions such that $\underline{p}^i = \underline{\mathbf{p}}^i\left(\underline{p}^j, \bar{p}^j\right)$ and $\bar{p}^i = \bar{\mathbf{p}}^i\left(\underline{p}^j, \bar{p}^j\right)$ for all $i, j = 1,2, i \neq j$. By linearity and symmetry of these best replies, there is a unique and symmetric competitive price equilibrium when *IP* applies (Economides, 1993):

$$\underline{p}^n_{IP} = \bar{p}^n_{IP} = \frac{2a}{7x-17} \tag{5}$$

where the superscript n indicates the competitive outcome. Note that equilibrium demands are all identical and equal to $D_{ij}^{IP} = ac\frac{3x-5}{7x-17}$ for all $i, j = 1,2$, which is always positive under our assumption $x > \xi$.

Consider now the case in which both firms use *mixed bundling*, so that they can also supply a bundle composed of the first component good and the second at a unique price r^i. Therefore, under a mixed bundling strategy, each firm chooses three prices $\{\underline{p}^i, \bar{p}^i, r^i\}$ with $i = 1,2$. Thus, the final prices of the composites are given by $s_{ii} = r^i$, $s_{ij} = \underline{p}^i + \bar{p}^j$ for $i, j = 1,2$. The profit functions of the two firms are:

$$\Pi_{MB}^i = r^i D_{ii} + \underline{p}^i D_{ij} + \underline{p}^i D_{ji} \qquad \text{for all } i, j = 1,2, i \neq j$$

and maximizing Π_{MB}^i with respect to $\left(\underline{p}^i, \bar{p}^i, r^i\right)$, leads to the first order conditions:

$$\begin{aligned}
\frac{\partial \Pi_{MB}^i}{\partial r^i} &= 0 \Leftrightarrow ac - 2br^i + 2c\left(\underline{p}^i + \bar{p}^i\right) + c\left(r^j + \underline{p}^j + \bar{p}^j\right) = 0 \\
\frac{\partial \Pi_{MB}^i}{\partial \underline{p}^i} &= 0 \Leftrightarrow ac - 2b\underline{p}^i + 2c\left(r^i + \bar{p}^i\right) + c\left(r^j + \underline{p}^j\right) - b\bar{p}^j = 0 \\
\frac{\partial \Pi_{MB}^i}{\partial \bar{p}^i} &= 0 \Leftrightarrow ac - 2b\bar{p}^i + 2c\left(r^i + \underline{p}^i\right) + c\left(r^j + \bar{p}^j\right) - b\underline{p}^j = 0
\end{aligned}$$

Solving this linear system gives the best replies of the firm i when MB applies, that is:

$$\begin{cases} \mathbf{r}^i\left(r^j, \underline{p}^j, \bar{p}^j\right) = \frac{a}{2(x-2)} + \frac{1}{2(x-2)}r^j \\ \underline{\mathbf{p}}^i\left(r^j, \underline{p}^j, \bar{p}^j\right) = \frac{a}{2(x-2)} + \frac{1}{2(x-2)}r^j - \frac{1}{2}\bar{p}^j \\ \bar{\mathbf{p}}^i\left(r^j, \underline{p}^j, \bar{p}^j\right) = \frac{a}{2(x-2)} + \frac{1}{2(x-2)}r^j - \frac{1}{2}\underline{p}^j \end{cases} \tag{6}$$

The fixed point of these best reply functions yields the unique and competitive price equilibrium with:

$$r_{MB}^n = \frac{a}{2x-5}$$

and

$$\underline{p}_{MB}^n = \bar{p}_{MB}^n = \frac{2a}{3(2x-5)} \tag{7}$$

Note that equilibrium demands are equal to $D_{ii}^{MB} = \frac{ac}{3}\frac{3x-4}{2x-5}$ for $i = 1,2$ and $D_{ij}^{MB} = \frac{ac}{3}$ for $i \neq j = 1,2$, which are all positive when $x > \xi$.

Calculating the profits in both regimes, we can summarize the equilibria as follows:

Proposition 1 (Economides, 1993)**.** *Under independent pricing, the competitive equilibrium prices and profits are:*

$$p_{IP}^n = \frac{2a}{7x-17} \text{ and } \Pi_{IP}^n = \frac{8(3x-5)a^2c}{(7x-17)^2}$$

Under mixed bundling, the equilibrium prices and profits are:

$$p_{MB}^n = \frac{2a}{3(2x-5)}\text{ ; } r_{MB}^n = \frac{a}{2x-5}, \text{ and } \Pi_{MB}^n = \frac{(17x-32)a^2c}{9(2x-5)^2}$$

According to intuition, mixed bundling allows firms to offer a discount for the composite good when proposed as a bundle, since $r^n_{MB} < \underline{p}^n_{MB} + \bar{p}^n_{MB}$. This form of aggressive pricing is done to conquer consumers but it leads to a reduction in profits since $\Pi^n_{IP} > \Pi^n_{MB}$, as already noticed by Economides (1993). In both cases, however, firms would prefer to collude and increase their prices. In the next subsections, we explore the conditions under which such a collusion is sustainable in an infinitely repeated game under either independent pricing or mixed bundling.

2.2 Collusion Agreements

Firms may be able to sustain collusive prices in the long run. We assume that collusion is based on the highest joint profit-maximizing outcome. We use a superscript c to indicate the collusive outcome. With *independent pricing*, the collusive prices of both firms for their complementary goods are denoted as $p^i = \underline{p}^c$ and $\bar{p}^i = \bar{p}^c$, and the profit function is the following:

$$\pi_{IP} = \underline{p}^c(D_{ii} + D_{ij} + D_{jj} + D_{ji}) + \bar{p}^c(D_{ii} + D_{ji} + D_{jj} + D_{ij})$$

The FOCs are symmetric and given by:

$$\begin{aligned} \frac{\partial \pi_{IP}}{\partial \underline{p}^c} &= 0 \Leftrightarrow ac + \underline{p}^c(12c - 4b) = 0 \\ \frac{\partial \pi_{IP}}{\partial \bar{p}^c} &= 0 \Leftrightarrow ac + \bar{p}^c(12c - 4b) = 0 \end{aligned}$$

Solving this system leads to:

Proposition 2. *Collusive prices and industry profit are as follows:*

$$p^c = \frac{a}{4(x-3)} \text{ and } \pi^c = \frac{a^2 c}{x-3}. \tag{8}$$

Indeed, due to symmetry, the collusive profits are equally shared, such that $\Pi^{iC}_{IP} = \Pi^c = \frac{1}{2}\pi^c$. The collusive prices and profits are obviously higher than those emerging under price competition. All demands equal $D^c_{ij} = \frac{ac}{2}$ when the collusive agreement applies. As obvious under our symmetric assumptions, the results in Proposition **??** hold also when *mixed bundling* is available. In the model that we use, when firms are able to collude, prices and profits with an independent pricing strategy and with a mixed bundling strategy are equal. Indeed, writing the joint profit when MB applies shows that:

$$\pi_{MB} = \underline{p}^c(D_{ij} + D_{ji}) + \bar{p}^c(D_{ji} + D_{ij}) + r^c\left(D_{ii} + D_{jj}\right).$$

One can see that $\pi_{MB} = \pi_{IP}$ provided the bundle price is the sum of the goods sold individually (i.e. $r^c = \underline{p}^c + \bar{p}^c$). The introduction of the package in a situation in which firms can coordinate on prices has no strategic effect. Hence unambiguously, one can state that $\Pi^{iC}_{MB} = \Pi^c$ for $i = 1, 2$. This knife-edge property will be very useful in analyzing the sustainability of collusion as it neutralizes the effect of pricing devices on the collusive agreement.

2.3 Deviation Outcomes

Deviation outcomes correspond to situations where firms consider unilateral changes in their price levels from the collusive agreement studied above. Without loss of generality, we assume that firm i undercuts its prices (results are exactly the same if deviation is made by firm j) and we use a superscript d to indicate deviations outcomes. Using *independent pricing* devices to deviate, firm i chooses its prices $\underline{p}^i = \underline{p}^d$ and $\bar{p}^i = \bar{p}^d$ by anticipating that firm j carries on the collusive agreement $\{\underline{p}^j = \underline{p}^c; \bar{p}^j = \bar{p}^c\}$ with $i, j = 1,2$ and $i \neq j$; therefore, the deviation profit is:

$$\Pi^d_{IP} = \underline{p}^d (D_{ii} + D_{ij}) + \bar{p}^d (D_{ii} + D_{ji}).$$

As we have already determined the best replies of a firm i in (**??**) when IP applies, the solution of the deviation problem is simply:

$$\begin{cases} \underline{p}^d_{IP} = \underline{\mathbf{p}}^i \left(\underline{p}^c, \bar{p}^c\right) \\ \bar{p}^d_{IP} = \bar{\mathbf{p}}^i \left(\underline{p}^c, \bar{p}^c\right) \end{cases}$$

Substituting the collusive price p^c found in Proposition **??**, one can directly derive the IP deviation prices:

$$\underline{p}^d_{IP} = \bar{p}^d_{IP} = \frac{(7x-17)\,a}{8(3x-5)(x-3)}$$

Due to price cuts involved in deviation outcomes, it may be checked that the rival demand $\left(D_{jj}\right)$ is not lowered too much and becomes zero. Indeed, in this case we verify that $D_{jj} = \frac{ac(3x^2-13x+8)}{2(3x-5)(x-3)} > 0$ if and only if $x > \frac{13+\sqrt{73}}{6} \equiv \xi$.

We turn now to the case of *mixed bundling*. As in the previous paragraph, we assume that firm i undercuts its prices in response to the collusive agreement from firm j and chooses its deviation prices $\underline{p}^i = \underline{p}^d$, $\bar{p}^i = \bar{p}^d$, and $r^i = r^d$, by anticipating that the rival carries on the collusive agreement $\{\underline{p}^j = \underline{p}^c; \bar{p}^j = \bar{p}^c; r^j = \underline{p}^c + \bar{p}^c\}$, with $i, j = 1,2$ and $i \neq j$. The deviation profit of firm i then is given by:

$$\Pi^d_{MB} = r^d (D_{ii}) + \underline{p}^d (D_{ij}) + \bar{p}^d (D_{ji})$$

Again from the best replies of a firm i in (**??**) when MB applies, the solution of the deviation problem can be derived as:

$$\begin{cases} r^d_{MB} = \mathbf{r}^i \left(\underline{p}^c + \bar{p}^c, \underline{p}^c, \bar{p}^c\right) \\ \underline{p}^d_{MB} = \underline{\mathbf{p}}^i \left(\underline{p}^c + \bar{p}^c, \underline{p}^c, \bar{p}^c\right) \\ \bar{p}^d_{MB} = \bar{\mathbf{p}}^i \left(\underline{p}^c + \bar{p}^c, \underline{p}^c, \bar{p}^c\right) \end{cases}$$

By adequate substitution of the collusive prices in the previous system, we can state the following result:

Proposition 3. *The optimal prices and profits in the deviation strategy implies:*
(i) with independent pricing:

$$\underline{p}^d_{IP} = \bar{p}^d_{IP} = \frac{(7x-17)\,a}{8(3x-5)(x-3)},$$

$$\Pi_{IP}^{d} = \frac{(7x-17)^2 a^2 c}{32(3x-5)(x-3)^2}. \tag{9}$$

(ii) with mixed bundling:

$$\underline{p}_{MB}^{d} = \bar{p}_{MB}^{d} = \frac{(3x-8)\,a}{8(x-2)(x-3)} \text{ and } r_{MB}^{d} = \frac{(2x-5)\,a}{4(x-2)(x-3)},$$

$$\Pi_{MB}^{d} = \frac{\left(17x^2-87x+112\right)a^2 c}{32(x-3)^2(x-2)}. \tag{10}$$

We state now a useful comparative result between the different outcomes:

Lemma 1. *Profits are ranked as follows:*

$$\Pi_{IP}^{n} > \Pi_{MB}^{n} \quad \text{and} \quad \Pi_{IP}^{d} < \Pi_{MB}^{d}$$

This lemma indicates first that mixed bundling strategies create a prisoner's dilemma, as shown by Economides (1993): in a static game where firms can independently choose between independent pricing and mixed bundling, the Nash equilibrium involves mixed bundling even if both firms would prefer to commit to independent pricing strategies. Mixed bundling intensifies competition between firms, because more instruments are available to react to rival price undercutting. However, in deviation regimes; mixed bundling is always weakly dominating, because it allows firms to undercut their prices from three markets rather than two, as with independent pricing strategies. Indeed, from Propositions **??** and **??** one can see that $p_{MB}^{d} - p_{MB}^{n} > p_{IP}^{d} - p_{IP}^{n}$ and $r_{MB}^{d} - r_{MB}^{n} > 2\left(p_{IP}^{d} - p_{IP}^{n}\right)$: price cuts due to deviation[13] are more relevant with mixed bundling than with independent pricing.

Now we turn to analyze the sustainability of collusion.

2.4 Critical Discount Factors and Sustainability of Collusion

Following the trend of literature on tacit collusion initiated by Friedman (1971), we consider an infinitely repeated stage game where firms maximize the present discounted value of future profits using a common discount factor $\delta > 0$. In such a setting, collusion can be sustained in the long run as a subgame-perfect equilibrium, provided that the discount factor is sufficiently large.[14] Collusion emerges when firms predict that any attempt to undercut the collusive prices will be followed by tough retaliation from competitors. Retaliations are commonly assumed

[13] As a matter of fact, a price cut from the collusive agreement p^c implies a price spread $p^c - p^d$ lower than $p^c - p^n$, the one between collusion and competition. The difference between these gaps actually writes $p^d - p^n$.

[14] It is well known, of course, that collusive agreements other than the joint profit maximizing one can be achieved as a subgame perfect equilibrium. Explicitly, we focus on the highest profit equilibrium sustained by competitive (i.e. Nash equilibrium off-the-equilibrium-path) punishments.

to be Nash-reversion trigger strategies. Here we consider that firms go back to the competitive strategies according to pricing devices assigned. Because retaliation arises in the future, whereas deviations generate immediate profits, the ability to collude depends on the relative importance of current profits compared with future profits in the firm's objective.

As well known, the ability to collude is sustainable when the discount factor is above a critical level δ_k for $k = IP, MB$. To determine this cut-off, notice that firm i would not deviate from the collusive agreement if:

$$\frac{1}{1-\delta}\Pi^c \geq \Pi_k^d + \frac{\delta}{1-\delta}\Pi_k^n \Rightarrow \delta \geq \delta_k \equiv \frac{\Pi_k^d - \Pi^c}{\Pi_k^d - \Pi_k^n}. \tag{11}$$

With an *independent pricing* strategy, the ability to collude is sustainable when:

$$\delta \geq \delta_{IP} \equiv \frac{\Pi_{IP}^d - \Pi^c}{\Pi_{IP}^d - \Pi_{IP}^n}.$$

As well known, δ_{IP} is the ratio of the gain of deviation $G = \Pi_{IP}^d - \Pi^c$ and the cost of punishment $C = \Pi_{IP}^d - \Pi_{IP}^n$. Using the expressions for profits from Propositions **??**, **??** and **??**, one can form the critical discount factor as:

$$\delta_{IP} = \frac{(7x-17)^2}{97x^2 - 462x + 529}. \tag{12}$$

Studying this critical discount factor shows that it decreases from $x = \xi$ to $x = 7$ and it increases after. Indeed, its limit value for $x \to \infty$ is $\frac{49}{97} > \frac{1}{2}$, and its derivative can be written as:

$$\frac{\partial \delta_{IP}}{\partial x} = \frac{64\,(x-7)\,\delta_{IP}^2}{(7x-17)^3},$$

which has the same sign as $x-7$. Therefore, at $x = 7$ the critical discount factor reaches a minimal limit value[15] of $\delta_{IP} = \frac{1}{2}$ such that $\Pi_{IP}^d = \Pi^c = \Pi_{IP}^n$. For this particular value of the relative degree of substitutability between composite goods, the inner complementary effect of components within each composite good is exactly balanced by the substitutability between composites. As a result, markets behave as independent markets: competitive, collusive and deviation outcomes are equivalent in terms of equilibrium prices and profits.[16] For all other admissible values of x, $\Pi_{IP}^d > \Pi^c > \Pi_{IP}^n$ obviously. However, for lower values of x, the effect of substitutability between composite goods is stronger than the inner complementary effect of

[15] Using L'Hôpital's rule twice as $\Pi_{IP}^d - \Pi^c = \Pi_{IP}^d - \Pi_{IP}^n = 0$ for $x = 7$ and $\frac{d\left(\Pi_{IP}^d - \Pi^c\right)}{dx} = \frac{d\left(\Pi_{IP}^d - \Pi_{IP}^n\right)}{dx} = 0$ it follows that, when $x = 7$,

$$\frac{\Pi_{IP}^d - \Pi^c}{\Pi_{IP}^d - \Pi_{IP}^n} = \frac{\frac{d^2\left(\Pi_{IP}^d - \Pi^c\right)}{dx^2}}{\frac{d^2\left(\Pi_{IP}^d - \Pi^c\right)}{dx^2}} = \frac{1}{2}.$$

[16] For $x = 7$, for all outcomes, prices are all equal to $a/16$.

components and a *decrease* in x (i.e. more substitutability) yields the intensification of price competition across the industry for firms supplying substitutable goods. Then deviations result in important price cuts from the collusive agreement. This leads to a more important increase in the rate of gain from deviation, i.e. $\frac{\Delta G}{G}$, than the increase in the relative cost of punishment, i.e. $\frac{\Delta C}{C}$: as a result the critical discount factor increases when x decreases below $x = 7$. Finally, for $x > 7$, the effect of substitutability between composite goods is weaker than the inner complementary effect of components. Then an *increase* in x (i.e. more complementarity) means again an intensification of price competition, but for firms that supply complementary goods: this leads to a relative increase in the gains of deviation,[17] therefore the critical discount factor increases with x in this region.

With a *mixed bundling* strategy, the firm i would not deviate from the collusive agreement in a mixed bundling setting if:

$$\delta \geq \delta_{MB} \equiv \frac{\Pi^d_{MB} - \Pi^c}{\Pi^d_{MB} - \Pi^n_{MB}}.$$

Calculating the critical level of the discount factor with mixed bundling leads to:

$$\delta_{MB} = \frac{9(x^2 - 7x + 16)(2x - 5)^2}{68x^4 - 816x^3 + 3901x^2 - 8439x + 6768}. \tag{13}$$

As for the independent pricing configuration, one can see that δ_{MB} is also U-shaped with respect to x. It is decreasing for x smaller than $\underline{\xi} \simeq 4.86 > \xi$ and increasing thereafter. Its limit value for $x \to \infty$ is $\frac{9}{17} > \frac{49}{47}$. The derivative can be calculated as:

$$\frac{\partial \delta_{MB}}{\partial x} = \frac{32K(x)\,\delta^2_{MB}}{9(x^2 - 7x + 16)^2(2x - 5)^3},$$

which as the same sign as the expression $K(x) = x^4 - 8x^3 + 8x^2 + 68x - 159$, where $K(x)$ is an increasing polynomial function of x (for $x \geq \xi$) such that $K(\underline{\xi}) = 0$.[18] Same arguments as developed above hold: as x increases, a relative change occurs in the competition setting from substitutable goods towards complementary goods. Hence, when goods are relatively substitutable, the cost rate of punishment is greater than the rate of gain from deviation and the critical discount factor increases accordingly. The reverse occurs when goods become relatively complementary.

Both critical discount factors under *IP* and *MB* regimes are depicted in Figure **??**. We can now compare them and derive the *main result* on the sustainability of tacit collusion when firms

[17] Indeed with complements, an anticommons issue arises: competing firms set higher prices than a single monopolist.

[18] Indeed $K(x)$ is continuous and strictly increasing from $x = \xi$ to ∞ with $K(\xi) = -15.817$ to $\lim_{x\to\infty} K(x) = \infty$. Invoking the intermediate value theorem, it exists a value $\underline{\xi}$ such that $K(\underline{\xi}) = 0$. By approximation, it can be found that $\underline{\xi} \simeq 4.86$.

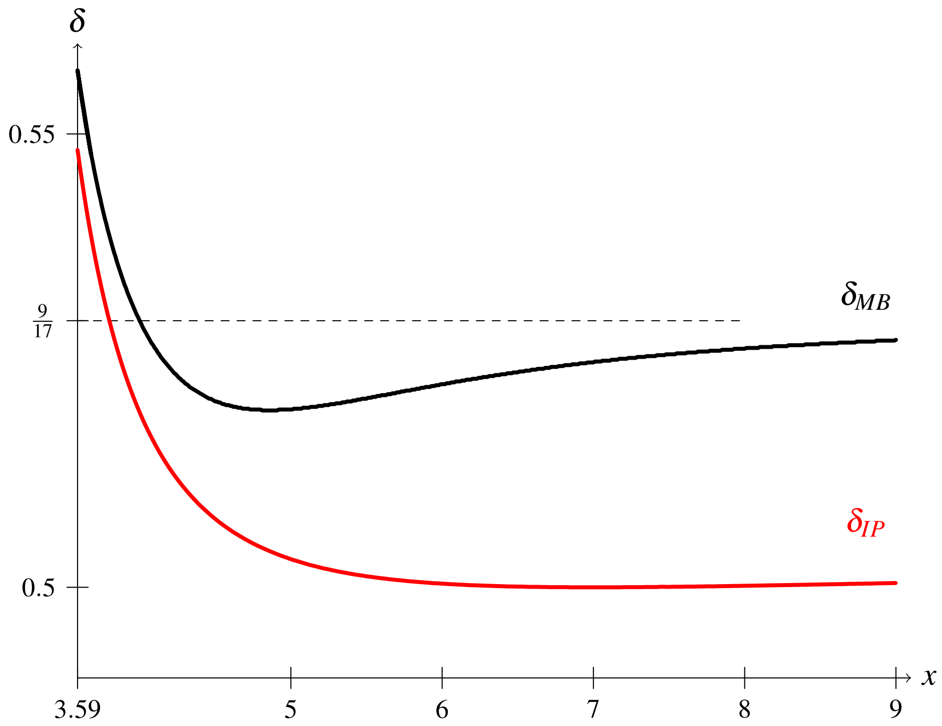

Figure 1: Critical factors as a function of x

could commit to use either mixed bundling or independent pricing strategies. To do this, the next proposition compares the critical discount factors in both alternative pricing devices obtaining an unambiguous result:

Proposition 4. *Collusion is less likely when firms adopt mixed bundling than when they choose independent pricing strategies:*

$$\delta_{IP} < \delta_{MB}.$$

Proposition **??** states that the critical discount factor is higher when mixed bundling is adopted by all firms rather than independent pricing. As a result, collusion is hindered by bundling. To realize the intuition for this result, notice that there are two opposite effects on the critical discount factor, due to the deviation of a firm. On the one hand, if a firm deviates from the collusive path, it enjoys a static gain from its deviation. As Lemma **??** shows, this gain is larger under a mixed bundling strategy because there is more flexibility in exploiting the deviation, $\Pi^d_{MB} - \Pi^c > \Pi^d_{IP} - \Pi^c$. From this point of view, the short-run benefits from the deviation using a mixed bundling strategy tend to *increase* the related critical factor with respect to the independent pricing strategy, which generally discourages price collusion. On the other hand, Nash reversion implies a larger cost under mixed bundling compared to independent pricing because the losses from retaliation occur on sales of each independent good and also of the bundle, so that $\Pi^d_{MB} - \Pi^n_{MB} > \Pi^d_{IP} - \Pi^n_{IP}$, as Lemma **??** shows. From this point of view, using mixed bundling implies higher punishment losses than using independent pricing and these long-run losses tend to *decrease* the critical discount factor above which collusion is sustainable. Consequently, there is a trade-off between short-run gains and long-run losses, and, in our model, using a mixed bundling strategy implies that the gain increase dominates the punishment increase because, as noted in Lemma **??**, mixed bundling is more effective and profitable when it is used as an instrument to deviate. This result aligns with Bernheim and Whinston's (1990) finding: bundling could be considered as a device for multimarket contacts.

The existing literature has established a fairly strong case against mixed bundling, both in terms of exclusionary effects and in terms of discriminatory effects that are harmful to consumers. The main policy implication of Proposition **??** is that, because mixed bundling strategies make it harder to sustain collusion, they can be beneficial for consumers. As a result, a legal ban on bundling makes tacit collusion more likely. When we consider the incentive to collude in price competition, a pro-competitive effect of bundling emerges.

The result of Proposition **??** compared the sustainability of tacit collusion when all firms are exogenously assumed to use mixed bundling or independent pricing strategies. In the next section we extend the model allowing firms to choose optimally how to collude, how to deviate and how to punish themselves if they can choose amongst mixed bundling or independent pricing devices.

3 Endogenous Pricing Devices

In order to explore further the anticollusive effects of mixed bundling practices, we assume now that firms can choose the pricing strategy (independent pricing or bundling) to sustain tacit collusion. We then consider two alternative configurations. First, we consider that for each firm an *ex ante* irreversible decision is made simultaneously about what pricing device to set, IP or MB, after which the standard collusive price supergame is played. Second, we allow for a complete flexibility, that is in each period each firm makes a choice on the pricing strategy.

3.1 Irreversible choices

In this subsection, we consider a timing of events that differs from that in the basic model. We drop the exogeneity assumption about the device used by the firms, and allow for an initial irreversible decision between independent pricing and mixed bundling. Hence, in the first stage, each firm chooses a device strategy $\mu_i \in M_i = \{IP, MB\}$ on which it commits during the subsequent infinite stages.

From the first stage point of view, let us define the intertemporal payoff of a firm $i = 1,2$ that opts for a given pricing device strategy $\mu_i \in M_i$. Once a pricing device has been chosen by each firm, if collusion is sustainable in the industry, then the intertemporal payoff of firm i will be the corresponding discounted sum of future profits it earns when it colludes. That is, $\frac{1}{1-\delta}\Pi_i^c(\mu_i,\mu_{-i})$ where $\Pi_i^c(\mu_i,\mu_{-i})$ represents the current flow of collusion profits when firm i has chosen $\mu_i \in M_i$ and its rivals $\mu_{-i} \in M_{-i}$.

We already know from our previous analysis that $\Pi_i^c(IP,IP) = \Pi_i^c(MB,MB) = \frac{1}{2}\pi^c$ is given in Eq. (**??**). The intertemporal profit level $\frac{1}{1-\delta}\Pi_i^c(\mu_i,\mu_{-i})$ is only achievable if, for each firm, the sustainability of the corresponding collusive agreement is respected, i.e., for $i = 1,2$:

$$\frac{1}{1-\delta}\Pi_i^c(\mu_i,\mu_{-i}) \geq \Pi_i^d(\mu_i,\mu_{-i}) + \frac{\delta}{1-\delta}\Pi_i^n(\mu_i,\mu_{-i}).$$

That is if

$$\delta \geq \delta_i(\mu_i,\mu_{-i}) = \frac{\Pi_i^d(\mu_i,\mu_{-i}) - \Pi_i^c(\mu_i,\mu_{-i})}{\Pi_i^d(\mu_i,\mu_{-i}) - \Pi_i^n(\mu_i,\mu_{-i})},\ \forall i \in \{1,2\} \tag{14}$$

One can see that (due to symmetry between firms) this defines four relevant critical discount factors: $\delta_i(IP,IP), \delta_i(MB,MB)$, $\delta_i(IP,MB)$ and $\delta_i(MB,IP)$. Of course the first two have been already specified and studied as $\delta_i(IP,IP) = \delta_{IP}$ and $\delta_i(MB,MB) = \delta_{MB}$ given in Eq. (**??**) and (**??**). The two remaining critical discount factors correspond to non-symmetric choices of pricing devices and will be described in the Lemma **??** below. However, even if collusion is sustainable one cannot exclude that a firm prefers *not* to collude and then opt for competition. This could be the case if, for a given profile of pricing devices, $\Pi_i^c(\mu_i,\mu_{-i}) < \Pi_i^n(\mu_i,\mu_{-i})$.[19] In this case, the intertemporal payoff of firm i will be $\frac{1}{1-\delta}\Pi^n(\mu_i,\mu_{-i})$. Finally, if, for a given discount factor level, collusion is not sustainable - that is we have $\delta < \delta_i(\mu_i,\mu_{-i})$ for firm i - collusion profits cannot be achieved and competition takes place for ever. As a result, in the first stage, the global payoff of each firm can be summed up as follows:

$$\mathbf{V}(\mu_i,\mu_{-i}) = \frac{1}{1-\delta}\begin{cases} \max\{\Pi^c(\mu_i,\mu_{-i}),\Pi^n(\mu_i,\mu_{-i})\} & \text{if } \delta \geq \delta^*(\mu_1,\mu_2) \\ \Pi^n(\mu_i,\mu_{-i}) & \quad\ \ \delta < \delta^*(\mu_1,\mu_2) \end{cases}$$

where $\delta^*(\mu_1,\mu_2) = \max\{\delta_1(\mu_1,\mu_2), \delta_2(\mu_2,\mu_1)\}$.

Lemma 2. *When irreversible choices of pricing strategies are allowed, critical discount factors are ranked as follows:*

$$\delta^*(IP,IP) < \delta^*(MB,MB) < \delta^*(MB,IP) = \delta^*(IP,MB)$$

The result in Lemma **??** shows that collusion is *even* less likely when one firm adopts mixed bundling and the other firm adopts independent pricing. More generally, collusion is even less likely when firms can choose their pricing strategies non-cooperatively. The intuition is that the cost of punishment is weaker, as competitive profit is higher when a firm adopts mixed bundling

[19] We already know from Lemma **??** that this is not the case for symmetric choices $\mu = (IP,IP)$ and $\mu = (MB,MB)$.

while its rival has chosen independent pricing. This hinders the incentives to collude and we can argue that firms would benefit from not committing to mixed bundling in order to sustain tacit collusion.

Our next step is to determine the subgame perfect equilibria of the whole game when, in the initial stage, firms choose (μ_1, μ_2) and then play the collusion game. As the objective function $\mathbf{V}(\mu_i, \mu_{-i})$ sums up the optimal intertemporal payoff when a subgame perfect equilibrium (i.e., competitive or collusive for all periods) is played at the start of the competition stage, one must have to find Nash equilibria (in pure strategies) (μ_1^*, μ_2^*) of the reduced form game played at the beginning of the initial stage, such that $\forall i \in \{1,2\}, \mathbf{V}\left(\mu_i^*, \mu_{-i}^*\right) \geq \mathbf{V}\left(\mu_i, \mu_{-i}^*\right)$ for all $\mu_i \in M_i = \{IP, MB\}$. The following proposition does not fully describe those equilibria (see the Appendix **??** for more details) but states that mixed bundling may be always chosen as an equilibrium.

Proposition 5. *For any level of the discount factor δ in the industry, the profile $\mu = (MB, MB)$ is always a Nash equilibrium of the pricing device (first-stage) game. However, whenever $\delta_{IP} < \delta \leq \delta_{MB}$, it exists a level $\tilde{\xi}$ of the relative degree of substitutability x, above which a prisoner's dilemma problem appears: firms are better off when they both commit to independent pricing.*

When firms can choose their pricing device non-cooperatively, mixed bundling is always a profile on which they can commit to. However, this *ex ante* commitment may be detrimental for them in terms of the sustainability of collusion. Indeed when $\delta_{IP} < \delta \leq \delta_{MB}$, collusion is not sustainable at the Nash equilibrium (MB, MB), but it is when using (IP, IP), which is also a Nash equilibrium for relatively substitutable composite goods (i.e. $x \leq \tilde{\xi}$). There are two Nash equilibria, the independent pricing equilibrium to collude $\mu = (IP, IP)$ and the mixed bundling equilibrium to compete $\mu = (MB, MB)$. As a result, mixed bundling is then a Pareto-dominated Nash equilibrium: firms would be better off by not choosing this device and by coordinating on *IP*. For less substitutable composites ($x > \tilde{\xi}$), mixed bundling is the unique Nash equilibrium: firms cannot avoid to chose the Pareto-dominated profile *MB*. Again this reinforces our main result given in Proposition **??**: firms may noncooperatively choose mixed bundling that hinders collusion among them.

3.2 Flexible choices

In this section we extend the analysis of the endogenous choices of pricing strategies involved in the intertemporal collusion game. We investigate what are the optimal collusive, competitive and deviation schemes when firms can choose in *any period* either mixed bundling or independent pricing devices - that is to say, choices are completely flexible. Thus we consider a timing of events where *at each date,* firms simultaneously choose the pricing devices they use if they compete, if they collude and if they deviate from a given collusive agreement.

As a result, firms now have to determine a pricing device strategy $\mu_i^h(s) \in M_i = \{IP, MB\}$

for each date s starting from a given time t for all possible market outcomes $h = \{n,c,d\}$ and a corresponding price vector that maximizes their intertemporal discounted profit. Hence the sustainability of the path of collusive agreements from a date t is achieved if:

$$\sum_{s=t}^{\infty} \delta^s \Pi_i^c \left(\mu_i^c(s), \mu_{-i}^c(s)\right) \geq \Pi_i^d \left(\mu_i^d(t), \mu_{-i}^c(t)\right) + \sum_{s=t+1}^{\infty} \delta^s \Pi_i^n \left(\mu_i^n(s), \mu_{-i}^n(s)\right).$$

As the basic game is time-invariant, we immediately drop the time argument and consider time-invariant strategies $\mu_i^h \in M_i = \{IP, MB\}, \forall h = \{n,c,d\}$ and for $i = 1,2$, in order to write the above sustainability constraint as:

$$\frac{1}{1-\delta} \Pi_i^c \left(\mu_i^c, \mu_{-i}^c\right) \geq \Pi_i^d \left(\mu_i^d, \mu_{-i}^c\right) + \frac{\delta}{1-\delta} \Pi_i^n \left(\mu_i^n, \mu_{-i}^n\right).$$

As usual, it implies that:

$$\delta \geq \delta_i (\mathbf{m}_i, \mathbf{m}_{-i}) = \frac{\Pi_i^d \left(\mu_i^d, \mu_{-i}^c\right) - \Pi_i^c \left(\mu_i^c, \mu_{-i}^c\right)}{\Pi_i^d \left(\mu_i^d, \mu_{-i}^c\right) - \Pi_i^n \left(\mu_i^n, \mu_{-i}^n\right)}, \ \forall i \in \{1,2\} \tag{15}$$

where $\mathbf{m}_i = (\mu_i^c, \mu_i^d, \mu_i^n)$ for $i = 1,2$. Consequently, optimal device decisions will consist of profiles $(\mathbf{m}_1^*, \mathbf{m}_2^*)$, such that for each firm the optimal collusive scheme maximizes the current joint-profit, the optimal deviation scheme maximizes the firm's profit given a collusive scheme for the rival, and the competitive scheme corresponds to a Nash equilibrium for these device decisions.[20] This will allow us to determine and analyze the critical discount factor $\delta^* (\mathbf{m}_1^*, \mathbf{m}_2^*) = \max\{\delta_1 (\mathbf{m}_1^*, \mathbf{m}_2^*), \delta_2 (\mathbf{m}_2^*, \mathbf{m}_1^*)\}$ that is relevant when choices about pricing devices are flexible.

First, notice that, in this new setting, Lemma **??** helps us to determine the optimal collusive and competitive schemes. Due to the symmetry of our model, the collusive agreement is invariant to the device decision between IP or MB, hence one can set $\mu_i^{c*} = \{MB\}, \forall i = 1,2$.[21] Concerning the competitive outcome, as shown by Economides (1993), the mixed bundling outcome is the Nash equilibrium of the pricing scheme static game ($\mu_i^{n*} = \{MB\}, \forall i = 1,2$), even if it is less profitable than independent pricing (i.e., a prisoner's dilemma configuration). In the collusion dynamic game, this implies that MB will be chosen as the optimal punishment pricing device, as it yields a higher punishment cost.

The issue of deviation schemes is less straightforward. In the standard version of the collusion game we analyzed in Section **??**, when both firms follow independent pricing strategies, one firm is assumed to deviate from each of the component markets using independent pricing schemes only. In the same way, if both firms follow mixed bundling strategies, they deviate

[20] More precisely for $i = 1,2 : \mu_i^{d*} \in \arg\max_{\mu_i^d} \Pi_i^c \left(\mu_i^d, \mu_{-i}^c\right), \forall \mu_{-i}^c \in \{IP, MB\}$; $\mu_i^{n*} \in \arg\max_{\mu_i^n} \Pi_i^n \left(\mu_i^n, \mu_{-i}^{n*}\right)$ and $(\mu_1^{c*}, \mu_2^{c*}) \in \arg\max_{(\mu_1^c, \mu_2^c)} \Pi_1^c (\mu_1^c, \mu_2^c) + \Pi_2^c (\mu_2^c, \mu_1^c)$.

[21] Indeed, it is also shown in Appendix **??** that $\Pi_i^c (IP, MB) = \Pi_i^c (IP, MB)$.

from each of the component markets and the bundle market. Now, focusing on optimal deviations, we consider a more general pattern of deviations. Firms are assumed to deviate from any market (first component market and/or second and/or package market) - that is, they can shift to any pricing device. For instance, if tacit collusion is sustained using an independent pricing device, the deviating firm can either choose to offer a different price in the first or second component market only or different prices in both the first and second component markets, or even in the package market. The levels of equilibrium prices and profits of firm i, for all these possible deviations are relegated in Appendix **??**, from which we can derive the following Lemma.

Lemma 3. *Mixed bundling deviations are optimal, whatever the collusive agreement is.*

Whenever independent pricing or mixed bundling is chosen by or imposed upon competitors, mixed bundling deviations always dominate in terms of deviation profits earned, i.e. $\mu_i^{d*} = \{MB\}, \forall i = 1,2$. The generic intuition of this result is that a full *mixed bundling deviation* allows the firm to have a more flexible tool for adjusting its pricing strategy; a greater discrimination practice is permitted through the use of this deviation scheme. With these results, we can compare a firm's incentives to collude when it adopts an independent pricing strategy and when it can offer a bundle with a mixed bundling strategy considering optimal deviations.

Because mixed bundling is always the optimal deviation device but also is more likely to be chosen in order to find a collusive agreement or to compete, the critical factor with flexible pricing devices has the same definition and level as the one in the basic model in Section **??**. That is, $\delta^*(\mathbf{m}_1^*, \mathbf{m}_2^*) = \delta_{MB}$. However, if firms would commit to use independent pricing in collusion and competition, the optimal deviation would consist in using mixed bundling still, we denote the corresponding critical factor $\delta_{IP}^* = \delta^*(\hat{\mathbf{m}}_1, \hat{\mathbf{m}}_2)$ where $\hat{\mathbf{m}}_i = (IP, MB, IP), \forall i = 1,2$. Comparing these different thresholds leads to Proposition **??**.

Proposition 6. *Mixed bundling is an optimal flexible choice of pricing devices at each date and the critical discount factors are ranked as follows:*

$$\delta_{IP} < \delta_{MB} < \delta_{IP}^*.$$

The main result in Proposition **??** is that the result stated in Proposition **??** is robust when we relax the *ex ante* exogeneity assumption. Indeed firms prefer the mixed bundling device in each setting (i.e., collusion, competition, and deviation) and at each date, but this leads to a higher or equal level for the critical discount factor ($\delta^*(\mathbf{m}_1^*, \mathbf{m}_2^*) = \delta_{MB} > \delta_{IP}$). That is, collusion is less sustainable when firms can adapt their pricing device strategies continually. This effect looks like a "dynamic prisoner's dilemma" in the collusion game: in terms of sustainability of collusion, firms should be better off committing to the IP device. However, deviating using mixed bundling strategies is more profitable when retaliations are based on independent pricing schemes than when they are based on mixed bundling, as $\delta^*_{IP} > \delta_{MB}$. Hence, mixed bundling may not hinder collusion as effectively as in the case of independent pricing when optimal deviations flexible schemes are considered. The intuition underlying this result is that the competitive effect of bundling is stronger when it is used against rivals that adopt independent pricing strategies. When firms use independent price strategies and optimal deviations in mixed bundling, the gain of deviation is very high, for two reasons. First, the deviation in mixed bundling gives a third instrument to the firm that deviates and thus allows it to capture more consumers' demand and surplus. Second, price deviations are more aggressive, so the static gain of optimal deviation in an independent price regime –that is, the difference between the collusion profit and the mixed bundling deviation profit– is very high.

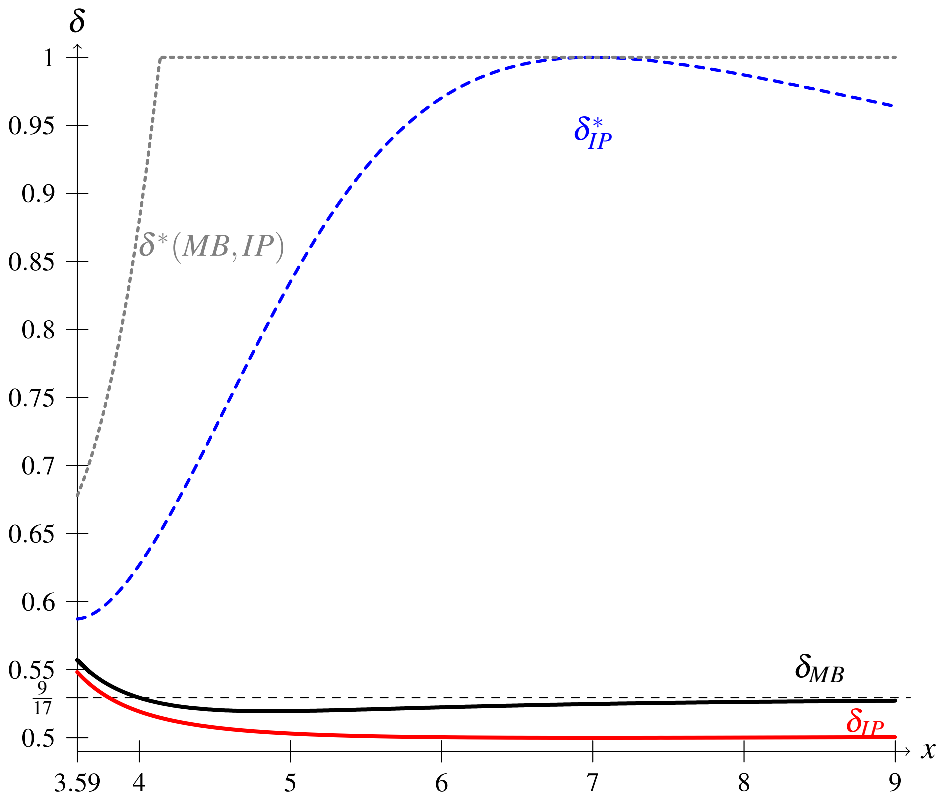

Figure 2: Critical factors as a function of x: Exogenous *vs.* flexible devices

In Figure **??**, we depict the different critical factors involved in Proposition **??** and **??** and lemma **??**.

4 Quantity decisions

In this section we provide an extension of the analysis by considering the effect of the mode of competition between firms. We want to analyze how the result in Proposition **??** is robust to a change in the strategic variable used within the commercial practice of bundling. For this we extend the framework of Martin (1999), which studies strategic bundling using a standard Cournot oligopoly model,[22] to the multiproduct duopoly setting we analyzed so far. Then, we consider that firms compete in quantities instead of prices. Such an analysis in quantities can be motivated by antitrust cases in Europe. The French Competition Authority has been concerned about anticompetitive agreements that distorted market competition and could have been harmful for consumers. For instance, in 2005, the Competition Authority observed, that from 2000 onwards, three telecom operators' sharing of informations enabled them to monitor the separate agreement they had reached on the development of their respective market shares. It was also found that, between 2000 and 2002, these three operators entered into an agreement aimed at stabilizing the development of their respective shares of the market.

In the model, from the consumers' point of view, the optimization problem remains the same as in Section **??** but now optimal consumption levels are analyzed using the (linear) inverse demands given in the system (**??**). The inverse demand price of the composite good ij is therefore:

$$\begin{cases} P_{ij} = \alpha - \beta X_{ij} - \gamma(X_{ii} + X_{ji} + X_{jj}) \\ P_{ii} = \alpha - \beta X_{ii} - \gamma(X_{ij} + X_{ji} + X_{jj}) \end{cases} \text{for all } i, j = 1, 2, i \neq j.$$

where α, β, and γ are the positive coefficients of the utility function given for expression (**??**). Using, the previous parameters a, b, c, x, they also imply $\alpha = a\frac{a}{x-3}$, $\beta = \frac{x-2}{c(1+x)(x-3)}$, and $\gamma = \frac{x}{c(1+x)(x-3)}$, and they are all positive coefficients for any $x > \xi$.

In the quantity decisions setting, a given firm i can use an *independent product sales* scheme,[23] supplying $\underline{q}^i$ and $\overline{q}^i$ or a mixed bundling quantity strategy, such as $\{\underline{q}^i, \overline{q}^i, y^i\}$. If firms choose independent product sales, they supply $\underline{q}^i$ units of the first component and $\overline{q}^i$ units of the second component, and the combination of each one is merely the sum of the two components sold individually, $X_{ii} = \underline{q}^i + \overline{q}^i$. However, if firms follow a *mixed bundling quantity strategy*, they keep on supplying $\underline{q}^i$ units of the first component and $\overline{q}^i$ units of the second component, but also offer the combination of each one combined in y^i units of bundles, such that $X_{ii} = y^i$, where the bundle is made of fixed but arbitrary proportions of each components.[24] As in Section **??**, we assume that, in a first stage, a given selling device, independent sales or a mixed bundling is assigned exogenously to both firms. In subsequent stages, firms compete repeatedly in quantity according to the selling device assigned and decide whether to sustain a tacit collusion using quantities agreements.

Concerning competition, firms can follow two selling strategies: on the one hand, assuming

[22] He shows that bundling can allow a firm with a monopoly in one market to exercise greater market power in other markets, which strategically disadvantages rivals in those markets. These leverage theory arguments also remain valid when the monopolist firm offers a variable proportion of the monopolized product in the bundle.

[23] Considering the quantity setting, independent pricing is not an appropriate way to describe the situation when firms do not offer their goods in a bundle. We prefer to refer to it is an "independent sale".

[24] Martin (1999) assumes a fixed proportion of the second component to one unit of the first component. This way to normalize the composition of a bundle does not play any role at the equilibrium when mixed bundling is available.

that a given firm uses an independent sales strategy, it can supply its complementary goods independently in quantities $\{\underline{q}^i, \overline{q}^i\}$, such that the profit function is as follows:

$$\hat{\Pi}^n_{IP} = \max_{\underline{q}^i, \overline{q}^i} P_{ii} \left(\underline{q}^i + \overline{q}^i\right) + P_{ij}\, \underline{q}^i + P_{ji}\, \overline{q}^i, \quad \text{with } i, j = 1, 2. \tag{16}$$

On the other hand, when a firm i follows a mixed bundling strategy, it can also supply y^i bundles composed of both (normalized) components. Therefore, with a mixed bundling quantity strategy, the firm chooses three quantities $\{\underline{q}^i, \overline{q}^i, y^i\}$ and the corresponding profit function is:

$$\hat{\Pi}^n_{MB} = \max_{\underline{q}^i, \overline{q}^i, y^i} P_{ii}\, y^i + P_{ij}\, \underline{q}^i + P_{ji}\, \overline{q}^i, \quad \text{with } i, j = 1, 2. \tag{17}$$

For the collusion quantity game, obviously we find that the collusive agreement leads to the same results in terms of profits as with price competition. Firms earn the same level of profit if they follow independent sales or mixed bundling quantity strategies: there is no strategic effect due to the bundle offer (again this is due to the knife-edge property of the model). As a result, quantity decisions lead to identical consumptions as in Proposition **??**, that is $X_{ij} = D^c_{ij} = \frac{ac}{2}$ so each firm supplies[25] $q^c = \frac{ac}{4}$ of each component or $y^c = 2q^c$ as a bundle, and profits remain equal to $\Pi^c = \frac{1}{2}\pi^c$.

Concerning deviations, if one firm deviates from the collusive path by increasing its supplies over the collusive quantities, the results are somewhat modified. Without loss of generality, we assume that firm i increases its quantities (the results are exactly the same if the deviation comes from firm j). Firm i chooses its quantities $\underline{q}^i = \underline{q}^d$ and $\overline{q}^i = \overline{q}^d$ by anticipating that firm j carries on the collusive agreement $\{\underline{q}^j = \underline{q}^c; \overline{q}^j = \overline{q}^c\}$. Thus

$$\hat{\Pi}^d_{IP} = \max_{\underline{q}^d, \overline{q}^d} (P_{ii} + P_{ij})\, \underline{q}^d + (P_{ii} + P_{ji})\, \overline{q}^d \text{ with } i, j = 1, 2.$$

Let us now consider the case where firms follow a mixed bundling quantity strategy. If one firm (firm i, for example) deviates from the collusive agreement by increasing its quantities (again the results are the same with firm j), then firm i chooses its quantities $\underline{q}^i = \underline{q}^d; \overline{q}^i = \overline{q}^d$ and $y^i = y^d$ by anticipating that firm j stays in the collusive agreement $\{\underline{q}^j = \underline{q}^c; \overline{q}^j = \overline{q}^c; y^j = y^c\}$. The profit then is given by:

$$\hat{\Pi}^d_{MB} = \max_{\underline{q}^d, \overline{q}^d, y^d} (P_{ii})\, y^d + (P_{ij})\, \underline{q}^d + (P_{ji})\, \overline{q}^d,$$

All equilibrium prices and profits for competition, collusion, and deviation configurations are given in Appendix **??**.

Now we consider that firms can compete repeatedly, so they are able to sustain collusive quantities, and they can decide to deviate from the collusive path. As explained in Section **??**,

[25] Details can be found in Appendix **??**.

the ability to collude using quantity decisions is reflected by the critical discount factor $\hat{\delta}^{IP}$ defined $\hat{\delta}_{IP} = \frac{\hat{\Pi}^d_{IP} - \Pi^c}{\hat{\Pi}^d_{IP} - \hat{\Pi}^n_{IP}}$ if they use independent sales. Calculating this critical discount factor, we find:

$$\hat{\delta}_{IP} = \frac{(7x+3)^2}{97x^2 + 74x - 7}. \tag{18}$$

As shown in Appendix **??**, this factor is a strictly decreasing function of x with a limit value of $\frac{49}{97}$.

If they use mixed bundling quantities, the critical discount factor is defined by $\hat{\delta}_{MB} = \frac{\hat{\Pi}^d_{MB} - \Pi^c}{\hat{\Pi}^d_{MB} - \hat{\Pi}^n_{MB}}$ and writes:

$$\hat{\delta}_{MB} = 9\frac{\left(x^2 + 3x + 6\right)(2x+1)^2}{68x^4 + 27x^3 + 637x^2 + 451x + 54}. \tag{19}$$

In Appendix **??**, we show $\hat{\delta}_{MB}(x)$ is U-shaped and reaches a minimum value for $x = \bar{\xi} > \xi$. However, due to scaling, this cannot be seen distinctly on Figure **??** as the slope of $\hat{\delta}_{MB}(x)$ is nearly zero.

Thus, we can state a result that compares the sustainability of tacit collusion for both competition contexts, prices and quantities, and both commercial devices, independent sales and mixed bundling (hatted variables are related to quantity strategies). Let us define two thresholds $\hat{\xi} \simeq 4.02$ and $\hat{\xi}' \simeq 3.93$ for the degree of substitutability between composites goods. As shown in the Appendix, we can state the following result:

Proposition 7. *(i) With mixed bundling, collusion is more difficult to sustain than with independent products, regardless the mode of competition (i.e. Cournot or Bertrand), that is:*

$$\delta_{IP} < \delta_{MB} \text{ and } \hat{\delta}_{IP} < \hat{\delta}_{MB}.$$

(ii) Quantity collusion is less likely than price collusion, if the degree between degree of substitutability between composites goods is low, regardless the pricing or sale strategy (i.e. IP or MB), that is:

$$\begin{aligned} \delta_{MB} &< \hat{\delta}_{MB} \text{ if } x \geq \hat{\xi} \\ \delta_{IP} &< \hat{\delta}_{IP} \text{ if } x \geq \hat{\xi}' \end{aligned}$$

Figure **??** depicts the different critical factors involved in Proposition **??**.

The first result in Proposition **??** shows that mixed bundling strategies hinder collusion also when Cournot competition takes place. In some sense it reinforces again the result given of Proposition **??**. Again mixed bundling is a more effective tool to deviate than to compete.

The second point of the Proposition is reminiscent of the results in Deneckere (1983, 1984): it shows that the relative degree of substitutability affects the sustainability of collusion according to the mode of competition, quantity or price. If the relative degree of substitutability is high

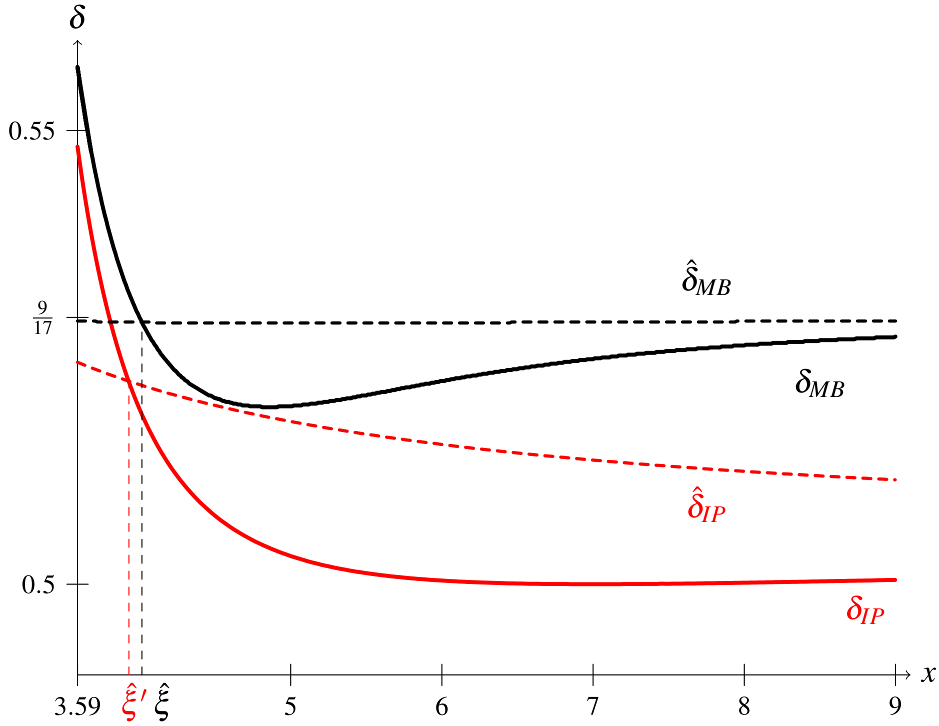

Figure 3: Critical factors as a function of x: Prices *vs.* quantities

(x is low), Bertrand competition is less collusive than Cournot competition whatever the pricing or sale device used. In this case, competition is less fierce when firms use quantity strategies rather than price strategies and deviations in quantities are less profitable than those in price. Our second result shows that the ability of firms to use bundling practices does not alter this fundamentally.

5 Conclusion

Bundling is widespread in various sectors. According to the European Commission, bundling should be used with caution, because it usually deteriorates consumer surplus. The existing law and economics literature have established a fairly strong case against mixed bundling, both in terms of exclusionary effects and in terms of discriminatory effects which are harmful to consumers. Without necessarily overturning this consensus, we point out that mixed bundling can be a (low) impediment to collusion. We have focused on firms' incentives to collude with a mixed bundling strategy. Economides (1993) has already shown that the mixed bundling strategy is a Nash equilibrium of the game in a static framework, but if firms compete repeatedly, they may be able to sustain collusive prices. Because bundling reduces competition between firms, we focus on its ability to affect the incentive for firms to collude. This study shows that collusion is harder for companies to sustain when mixed bundling strategies than when pricing independently their products. Thus, mixed bundling may hinder collusion. When the firm practices a mixed bundling strategy, it experiences two opposite effects on the discount factor when one firm deviates from the collusive agreement. There is a trade-off between the short-run gains and long-run losses; in our model with a mixed bundling strategy, the losses are lower than the gains. These results are based on the assumption that the pricing device is given. However, our results are strengthened if firms have the ability to determine their pricing device. Thus, our results offer a good argument for allowing firms to use bundling practices, regardless of the conventional belief that they can be anticompetitive. Surplus losses that may arise as a result of the choice of mixed bundling strategies by competitors can be overwhelmed by some potential rewards for consumers when the incentives to collude get diminished. Moreover, the competitive mode (i.e., Cournot or Bertrand) does not matter: collusion is even less likely with quantity strategies and mixed bundling compared to pricing strategies. The model has important implications for *competitive policy*: a legal ban on bundling might make tacit collusion more likely.

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6 Appendix

6.1 Proof of Lemma ??.

The inequality $\Pi^n_{IP} > \Pi^n_{MB}$ is proved in Economides (1993) for $x > 3.24$ so this is true in our context as $x > \xi > 3.24$. Straightforwardly from Equations (**??**) and (**??**) we can see that $\Pi^d_{MB} - \Pi^d_{IP} = \frac{a^2c}{16}\frac{x+1}{(x-2)(3x-5)} > 0$ if $x > \xi$. ■

6.2 Proof of Proposition ??.

From Equations (**??**) and (**??**) in the text, we study both critical factors as functions of x for $x \geq \xi > 3$. However, without loss of generality, we conduct a broader (and simpler) study for all values of $x > 3$:

$$\delta_{IP}(x) = \frac{(7x-17)^2}{97x^2-462x+529} \text{ and } \delta_{MB}(x) = \frac{9(x^2-7x+16)(2x-5)^2}{68x^4-816x^3+3901x^2-8439x+6768} \tag{20}$$

where $\delta_{IP}(3) = \delta_{MB}(3) = 1$ and $\lim_{x\to\infty}\delta_{MB}(x) = \frac{9}{16} > \lim_{x\to\infty}\delta_{IP}(x) = \frac{49}{97} > \frac{1}{2}$.
The difference $\Delta(x) = \delta_{MB}(x) - \delta_{IP}(x)$ can be written as a function of x, $\Delta(x) = \frac{32(1+x)(x-3)^2A(x)}{B(x)\,C(x)}$, where $A(x)$, $B(x)$ and $C(x)$ are given by

$$\begin{aligned} A(x) &= 5x^3-49x^2+163x-179 \;\; ; B(x) = 97x^2-462x+529 \\ C(x) &= 68x^4-816x^3+3901x^2-8439x+6768 \end{aligned}$$

Studying these polynomial functions reveals that they are all positive functions of x for $x \geq 3$, so as a result, $\Delta(x) > 0$. Furthermore, $A'(x)$ is convex for $x > 3$ and reaches a local minimum for $x = \frac{49}{15} \simeq 3.26$, such that $A'(x) \geq A'(\frac{49}{15}) = \frac{3292}{675} > 0$ and $A(x)$ is strictly increasing for $x > 3$. Because $A(3) = 4$, $A(x) \geq 4 > 0$. Considering $B(x)$, it equals zero for $x = \frac{32}{97}\sqrt{2} + \frac{231}{97} = 2.85$ and, as a convex polynomial function, $B(x) \geq B(3) = 16 > 0$, so $B(x) > 0$ for all $x > 3$. Finally $C''(x)$ is clearly a positive function for $x > 3$, and because $C'(3) = 279 > 0$, $C(x)$ is increasing from $C(3) = 36$ to ∞, such that $C(x) > 0$. ■

6.3 Proof of Lemma ??.

When all firms choose the same pricing device that is $\mu_1 = \mu_2 \in \{IP, MB\}$ then the critical discount factor for each firm is identical $\delta_1(IP,IP) = \delta_2(IP,IP) = \delta_{IP}$ and $\delta_1(MB,MB) = \delta_2(MB,MB) = \delta_{MB}$ given in Eq. (**??**) and (**??**). Hence for the industry we have $\delta^*(IP,IP) = \delta_{IP}$ and $\delta^*(MB,MB) = \delta_{MB}$, so their ranking is given in Proposition **??**, this proves the first part of the inequality. In order to complete the proof of this Lemma, we have to find critical discount factors for asymmetric choices. Due to symmetry of firms, without loss of generality, let us consider that firm 1 is choosing IP and firm 2 is choosing MB i.e. $\mu_1 = IP$ and $\mu_2 = MB$. Then we have to find competitive, collusive and deviation outcomes of this asymmetric case. For instance, consider the competition setting, when firm 1 follows IP i.e. $\mu_1 = \{IP\}$, it sells goods independently at two separate prices $\{\underline{p}^1, \bar{p}^1\}$ and when firm 2 follows MB i.e. $\mu_2 = \{MB\}$, it sells goods at prices $\{\underline{p}^2, \bar{p}^2, r^2\}$, so demand prices are given by $s_{12} = \underline{p}^1 + \bar{p}^2$; $s_{21} = \underline{p}^2 + \bar{p}^1$; $s_{11} = \underline{p}^1 + \bar{p}^1$ and $s_{22} = r^2$. The equilibrium profit functions are then as follows:

$$\Pi_1(IP,MB) = \underline{p}^1(D_{11}+D_{12}) + \bar{p}^1(D_{11}+D_{21}) \text{ and } \Pi_2(MB,IP) = r^2(D_{22}) + \underline{p}^2(D_{21}) + \bar{p}^2(D_{12}).$$

Maximizing Π_1 w.r.t. $\{\underline{p}^1, \bar{p}^1\}$ and Π_2 w.r.t.$\{\underline{p}^2, \bar{p}^2, r^2\}$ gives the unique Nash equilibrium (in prices) and profits:

$$\begin{aligned}
\underline{p}^{1n}_{IP} &= \bar{p}^{1n}_{IP} = \frac{3a(x-1)}{\varphi} \text{ ; } \underline{p}^{2n}_{MB} = \bar{p}^{2n}_{MB} = \frac{a(4x-3)}{\varphi} \text{ and } r^{2n}_{MB} = \frac{a(11x-9)}{2\varphi} \\
\Pi^n_1(IP,MB) &= 18a^2c\frac{(x-1)^2(3x-5)}{\varphi^2} \text{ and } \Pi^n_2(MB,IP) = 3a^2c\frac{83x^3-290x^2+299x-96}{4\varphi^2}
\end{aligned}$$

with $\varphi = 11x^2 - 37x + 24 > 0$ for $x > 3$. Symmetrically, we would find that $\Pi^n_2(IP,MB) = \Pi^n_1(IP,MB)$ and $\Pi^n_1(MB,IP) = \Pi^n_2(MB,IP)$. Moreover, one can easily see that $\Pi^n_2(MB,IP) - \Pi^n_1(IP,MB) = \frac{3a^2c}{4}(x+1)/\varphi > 0$. In the same vein, one can determine that the collusive outcome is unchanged compare to respective *IP* and *MB* prices found in the basic version of the model and reported in Eq. (**??**). Again this is due to the knife-edge property of the linear model, so $\Pi^c_1(IP,MB) = \Pi^c_2(MB,IP) = \Pi^c$. Finally studying deviations from the unchanged collusive agreement, we find that

$$\underline{p}^{1d}_{IP} = \bar{p}^{1d}_{IP} = \frac{a}{8}\frac{7x-17}{(x-3)(3x-5)} \text{ ; } \underline{p}^{2d}_{MB} = \bar{p}^{2d}_{MB} = \frac{a}{8}\frac{3x-5}{(x-3)(x-2)} \text{ and } r^{2d}_{MB} = \frac{a}{4}\frac{2x-5}{(x-3)(x-2)},$$

and profits are such that $\Pi^d_1(IP,MB) = \Pi^d_{IP}$ and $\Pi^d_2(MB,IP) = \Pi^d_{MB}$ given respectively in Eqs. (**??**) and (**??**). Symmetrically $\Pi^d_2(IP,MB) = \Pi^d_{IP}$ and $\Pi^d_1(MB,IP) = \Pi^d_{MB}$.
Therefore from (**??**), we have:

$$\delta_1(IP,MB) = \frac{\Pi^d_{IP} - \Pi^c}{\Pi^d_{IP} - \Pi^n_1(IP,MB)} \text{ and } \delta_2(MB,IP) = \frac{\Pi^d_{MB} - \Pi^c}{\Pi^d_{MB} - \Pi^n_2(MB,IP)} \tag{21}$$

From the Proof of Lemma **??**, we know that $\Pi^d_{MB} - \Pi^d_{IP} = \frac{a^2}{16c}(1+)x/[(x-2)(3x-5)] > 0$ for $x > 3$ and we have just seen that $\Pi^n_2(MB,IP) - \Pi^n_1(IP,MB) = \frac{3a^2c}{4}(1+x)/\varphi > 0$. Therefore by difference, we have $\Delta^\Pi = [\Pi^d_{IP} - \Pi^n_1(IP,MB)] - [\Pi^d_{MB} - \Pi^n_2(MB,IP)]$ that writes:

$$\Delta^\Pi = \frac{a^2c}{16}\frac{(1+x)\left(25x^2-95x+96\right)}{(x-2)(3x-5)\varphi} > 0$$

as $25x^2 - 95x + 96$ is a convex polynomial in x that reaches its minimal value, $\frac{23}{4}$, for $x = 1.9$. As a result we have both $\Pi^d_{MB} - \Pi^c > \Pi^d_{IP} - \Pi^c$ and $\Delta^\Pi > 0$, which leads to $\delta_1(IP,MB) < \delta_2(MB,IP)$, so $\delta^*(IP,MB) = \delta_2(MB,IP)$. Symmetrically, we would find that $\delta^*(MB,IP) = \delta_1(MB,IP)$. Finally, from the definitions of critical factors $\delta^*(IP,MB)$ and $\delta^*(MB,MB) = \delta_{MB}$, one can see they only differ from the value of the competitive profit. Then one can form $\Pi^n_2(MB,IP) - \Pi^n_{MB} = \frac{a^2c}{36}(1+x)D(x)/[\varphi(2x-5)]^2 > 0$ where $D(x) = 736x^4 - 6036x^3 + 17765x^2 - 21759x + 8928 > 0$, since $D''(x)$ is clearly a positive function for $x > 3$, and because $D'(3) = 1347 > 0$, $D(x)$ is increasing from $D(3) = 180$ to ∞. This proves that $\delta^*(IP,MB) > \delta_{MB}$. ■

6.4 Proof of Proposition ??.

Before finding the Nash-equilibria of the pricing device game we need to compare some profit levels. First, we claim that for all $i = 1,2 : \Pi^n_{MB} > \Pi^n_i(IP,MB)$ and $\Pi^c \gtreqless \Pi^n_i(MB,IP)$ if $x \lesseqgtr \tilde{\xi} \simeq 4.137$. Indeed the difference $\Pi^c - \Pi^n_i(IP,MB) = \frac{a^2c}{2}E(x)/[(2x-5)\varphi^2]$ with $\varphi = 11x^2 - 37x + 24$ and where $E(x) = 113x^4 - 975x^3 + 3058x^2 - 4044x + 1818 > 0$, since $E''(x)$ is clearly a positive function for $x > \xi$, and because $E'(3) = 183 > 0$, $E(x)$ is increasing from $E(3) = 36$ to ∞; point *(i)* is proved. Now the difference $\Pi^c - \Pi^n_i(MB,IP) = \frac{a^2c}{4}F(x)/[(x-3)\varphi^2]$ where $F(x) = 288 - 7x^4 - 11x^3 + 287x^2 - 573x$. Here $F''(x) = 574 - 84x^2 - 66x$ is concave in x and has a positive zero for $x = \frac{\sqrt{3}}{84}\sqrt{16435} - \frac{11}{28} \simeq 2.506$, then it is negative and $F'(x)$ is decreasing for $x > \xi$. As $F'(\xi) = -233.16 < 0$, it implies that $F'(x) < 0$ for all $x > \xi$. Then as $F(\xi) = 258.142 > 0$ and $F(5) = -1152 < 0$, by the intermediate value theorem, we know that it exists a value of x such that $F(x) = 0$ and by approximation, we find that is is merely $\tilde{\xi}$. This proves point *(ii)*. Using Lemma **??**, these results also imply that $\Pi^c > \Pi^n_{IP} > \Pi^n_{MB} > \Pi^n_i(IP,MB)$. Now from Lemma **??**, we know that $\delta_{IP} < \delta_{MB} < \delta^*(MB,IP) = \delta^*(IP,MB)$, hence four configurations occurs for the first stage game. **1.** If $\delta^*(MB,IP) < \delta \leq 1$ then $\delta > \delta_i(\mu_i,\mu_{-i})$ for all $i = 1,2$ and collusion is sustainable and optimal for all pricing devices, therefore they are all Nash-equilibria of the first stage game. However this case does not occur if $x > \tilde{\xi}$ as $\delta^*(MB,IP) \geq 1$. **2.** If $\delta_{MB} < \delta \leq \delta^*(MB,IP)$ then collusion is only sustainable in case of perfect coordination about pricing devices i.e. if $\mu_1 = \mu_2$. Indeed, the strategic form of the first stage game can be depicted by the following classical payoff-matrix (up to the multiplicative factor $\frac{1}{1-\delta}$):

		Firm 2	
	(μ_1,μ_2)	IP	MB
Firm 1	IP	Π^c, Π^c	$\Pi^n_1(IP,MB), \Pi^n_2(MB,IP)$
	MB	$\Pi^n_1(MB,IP), \Pi^n_2(IP,MB)$	Π^c, Π^c

where best replies are $\mu^r_i(MB) = MB$ and $\mu^r_i(IP) = IP$ for $x \leq \tilde{\xi}$ but $\mu^r_i(IP) = MB$ for $x > \tilde{\xi}$. Then for all $x > \xi$, the profile $\mu = (MB,MB)$ is a Nash-equilibrium (followed by MB collusive prices infinitely) and $\mu = (IP,IP)$ is a Nash-equilibrium only for $x \in]\xi, \tilde{\xi}]$. **3.** If $\delta_{IP} < \delta \leq \delta_{MB}$ then the only change in the pricing device game is that payoffs are (Π^n_{MB}, Π^n_{MB}) for the strategy profile $\mu = (MB,MB)$, that is collusion is not sustainable in (MB,MB). Hence Nash-equilibria of the first stage remains the same but $\mu = (MB,MB)$ is followed by competition infinitely. Hence, when $x \leq \tilde{\xi}$, the (IP,IP) equilibrium Pareto dominates the (MB,MB) ones but when $x > \tilde{\xi}$, a prisoner's dilemma situation appears: firms are better off when they both commit *not* to use MB to compete but IP to collude. **4.** If $\delta \leq \delta_{IP}$ then collusion is non sustainable whatever the pricing profile chosen and competition takes place infinitely and the result in Economides (1993) follows $\mu = (MB,MB)$ is the unique dominant strategy equilibrium. ■

6.5 Proof of Lemma ??.

We consider optimal deviations from independent pricing collusive agreements, as they are already described in Subsection **??**.

•Optimal deviations from collusion with IP. 1. We assume that firm i chooses to deviate from only one market (results are the same if firm i deviates from the other market), it adjusts its price, for example, in the first component market, $\underline{p}^i = \underline{p}^d$, and keeps its other price at the collusion level, $\bar{p}^i = \bar{p}^c$, anticipating that firm j carries on the collusive agreement $\{\underline{p}^j = \underline{p}^c; \bar{p}^j = \bar{p}^c\}$ with $i,j = 1,2$ and $i \neq j$. The profit of firm i is then $\Pi^{d_1}_{IP} = \max_{\underline{p}^d} \underline{p}^d(D_{ii} + D_{ij}) + \bar{p}^c(D_{ii} + D_{ji})$,and $\underline{p}^{d_1} = \arg\max_{\underline{p}^d} \Pi^{d_1}_{IP}$, then at the equilibrium, the price and profit of firm i are given by:

$$\underline{p}^{d_1} = \frac{a}{16}\frac{5x-11}{(x-3)(x-1)}; \; \Pi^{d_1}_{IP} = \frac{ac}{128}\frac{65x^2 - 270x + 241}{(x-3)^2(x-1)}. \tag{22}$$

2. In this case we assume that firm i chooses to deviate by proposing an extra bundle, it chooses prices $\underline{p}^i = \underline{p}^c$, $\bar{p}^i = \bar{p}^c$ and $r^i = r^d$, while firm j carries on the collusive agreement $\{\underline{p}^j = \underline{p}^c; \bar{p}^j = \bar{p}^c\}$ with $i,j = 1,2$ and $i \neq j$. The profit of firm i is given by $\Pi^{d_2}_{IP} = \max_{r^d} \underline{p}^c(D_{ij}) + \bar{p}^c(D_{ji}) + r^d(D_{ii})$,such that $r^{d_2} = \arg\max_{r^d} \Pi^{d_2}_{IP}$. At the equilibrium we have:

$$r^{d_2} = \frac{a}{2}\frac{x-1}{x(x-3)};\ \Pi^{d_2}_{IP} = \frac{a^2c}{4}\frac{2x^2-6x+1}{x(x-3)^2}. \tag{23}$$

3. If firm i chooses to deviate from one market, for example the first component market and the bundle market, it chooses its prices $\underline{p}^i = \underline{p}^d$, $\bar{p}^i = \bar{p}^c$, and $r^i = r^d$, with the anticipation that firm j will carry on the collusive deal $\{\underline{p}^j = \underline{p}^c; \bar{p}^j = \bar{p}^c\}$ with $i, j = 1,2$ and $i \neq j$. The profit of firm i is $\Pi^{d_3}_{IP} = \max_{\underline{p}^d, r^d} \underline{p}^d(D_{ij}) + \bar{p}^c(D_{ji}) + r^d(D_{ii})$,so at the equilibrium:

$$\underline{p}^{d_3} = \frac{3a}{8}\frac{x-2}{(x-3)(x-1)}; r^{d_3} = \frac{a}{8}\frac{4x-7}{(x-3)(x-1)};\ \text{and}\ \Pi^{d_3}_{IP} = \frac{3a^2c}{64}\frac{11x^2-45x+40}{(x-1)(x-3)^2}. \tag{24}$$

4. If firm i deviates from independent pricing strategy to a mixed bundling strategy, it chooses its prices $\underline{p}^i = \underline{p}^d$, $\bar{p}^i = \bar{p}^d$ and $r^i = r^d$, and firm j observes the collusive agreement $\{\underline{p}^j = \underline{p}^c; \bar{p}^j = \bar{p}^c\}$ with $i, j = 1,2$ and $i \neq j$. This last configuration[26] can be viewed as a *full MB deviation*. The profit of firm i is given by $\Pi^{d_4}_{IP} = \max_{\underline{p}^d, \bar{p}^d, r^d}\ \underline{p}^d(D_{ij}) + \bar{p}^d(D_{ji}) + r^d(D_{ii})$ and the solution is the same as in Proposition **??** for MB:

$$\underline{p}^{d_4} = \bar{p}^{d_4} = \underline{p}^d_{MB} = \bar{p}^d_{MB}\ ;\ r^{d_4} = r^d_{MB}\ ; \Pi^{d_4}_{IP} = \Pi^d_{MB}. \tag{25}$$

5. Finally the solution given in Proposition **??** for IP is still valid, that is when firm i deviates from an IP strategy to an IP strategy. In that case, profit is given by (**??**).

The optimal deviation across all possible deviation outcomes, namely, (**??**), (**??**), (**??**), (**??**) and (**??**), is now proved to be the deviation (**??**). Indeed first $\Pi^d_{MB} - \Pi^d_{IP} > 0$ from Lemma **??**. Second, using simple algebraic calculus, we can show that for $x > 3$, $\Pi^d_{MB} - \Pi^{d_1}_{IP} = \frac{a^2c}{128}\frac{(x+1)(3x^2-19x+34)}{(x-3)^2(x-2)(x-1)} > 0$; $\Pi^d_{MB} - \Pi^{d_2}_{IP} = \frac{a^2c}{32}\frac{(x+1)(x-4)^2}{x(x-3)^2(x-2)} > 0$; and $\Pi^d_{MB} - \Pi^{d_3}_{IP} = \frac{a^2c}{64}\frac{(x+1)(x-4)^2}{(x-3)^2(x-2)(x-1)} \geq 0$. However, $\Pi^{d_2}_{IP}$ can be lower than Π^d_{IP} if $x > 11.53$. Thus, we rank the set of deviation profits as $\Pi^d_{MB} > \Pi^{d_3}_{IP} > \Pi^{d_2}_{IP} \gtrless \Pi^d_{IP} > \Pi^{d_1}_{IP}$. This shows that $\Pi^d_{MB} > \max\{\Pi^d_{IP}, \Pi^{d_1}_{IP}, \Pi^{d_2}_{IP}, \Pi^{d_3}_{IP}\}$. Consequently, when both firms follow an independent pricing strategy, the optimal deviation is to deviate using a mixed bundling strategy.

•**Optimal deviations from collusion with MB.** Now we turn to optimal deviations from mixed bundling collusive agreements. If both firms follow mixed bundling strategies, we can study the optimal deviation using a systematic approach similar to the previous case. That is, if a given firm deviates from only one market (e.g. for the first component), it will adjust its market price $\underline{p}^i = \underline{p}^d$ and keep its prices on the other market and the bundle in accordance with the mixed-bundling collusion level, $\bar{p}^i = \bar{p}^c$ and $r^i = r^c$, while the other firm carries on the collusive agreement $\{\underline{p}^j = \underline{p}^c; \bar{p}^j = \bar{p}^c; r^j = r^c\}$.
1. If firm i deviates from only one component, either the first component or the second one (here we suppose the first one), profit is as follows $\Pi^{d_1}_{MB} = \underline{p}^{d_1}D_{ij} + \bar{p}^cD_{ji} + r^cD_{ii}$At the equilibrium, price and profit of the firm i are given by:

$$\underline{p}^{d_1} = \frac{3a}{8}\frac{x-1}{x(x-3)};\ \Pi^{d_1}_{MB} = \frac{3a^2c}{64}\frac{11x-1}{x(x-3)}. \tag{26}$$

2. If firm i deviates from only the bundle and independent prices remain at the collusive level, profit is given by $\Pi^{d_2}_{MB} = \underline{p}^cD_{ij} + \bar{p}^cD_{ji} + r^{d_2}D_{ii}$. At the equilibrium, price and profit of firm i are given by:

$$r^{d_2} = \frac{a}{2}\frac{x-1}{x(x-3)};\ \Pi^{d_2}_{MB} = \frac{a^2c}{4}\frac{2x^2-6x+1}{x(x-3)^2}. \tag{27}$$

3. If firm i deviates from one component (either the first or the second component, and here we suppose the first one) and also the bundle, profit is given by $\Pi^{d_3}_{MB} = \underline{p}^{d_3}D_{ij} + \bar{p}^cD_{ji} + r^{d_3}D_{ii}$. At the equilibrium, prices and profit of

[26]We could also consider the case in which firm i can deviate by proposing only the bundle, such that it chooses its bundle price $r^i = r^d$, assuming that the firm j stays with the collusive deal $\{\underline{p}^j = \underline{p}^c; \bar{p}^j = \bar{p}^c\}$ with $i, j = 1,2$ and $i \neq j$. In this case, firm i changes its selling strategy. This outcome does not change the result in Lemma **??**.

firm i are given by:

$$\underline{p}^{d_3}=\frac{3a}{8}\frac{x-2}{(x-3)(x-1)};\ r^{d_3}=\frac{a}{8}\frac{4x-7}{(x-3)(x-1)};\ \Pi_{MB}^{d_3}=\frac{3a^2c}{64}\frac{11x^2-45x+40}{(x-3)^2(x-1)}. \tag{28}$$

4. If firm i deviates from a mixed bundling strategy to an independent pricing one (i.e., from the first and the second components), profit is given by $\Pi_{MB}^{d_4}=\underline{p}^{d_4}D_{ij}+\overline{p}^{d_4}D_{ji}+r^cD_{ii}$. At the equilibrium, the prices and profit of firm i are the same as in Proposition **??** for IP:

$$\underline{p}^{d_4}=\overline{p}^{d_4}=\underline{p}_{IP}^{d}=\bar{p}_{IP}^{d}\ \ ;\Pi_{MB}^{d_4}=\Pi_{IP}^{d}. \tag{29}$$

5. Finally the solution given in Proposition **??** for MB is still possible, that is when firm i deviates from an MB strategy to an MB strategy. In that case, profit is given by (**??**).

The optimal deviation across all possible deviation outcomes, namely, (**??**), (**??**), (**??**), (**??**), and (**??**), is the deviation (**??**), Indeed $\Pi_{MB}^{d}-\Pi_{MB}^{d_1}=\frac{a^2c}{64}\frac{(x+1)\left(x^2-7x+18\right)}{x(x-3)^2(x-2)}>0$**;** $\Pi_{MB}^{d}-\Pi_{MB}^{d_2}=\frac{a^2c}{32}\frac{(x+1)(x-4)^2}{(x-3)^2(x-2)}>0$; $\Pi_{MB}^{d}-\Pi_{MB}^{d_3}=\frac{a^2c}{64}\frac{(x+1)(x-4)^2}{(x-3)^2(x-2)(x-1)}>0$; and $\Pi_{MB}^{d}-\Pi_{IP}^{d}>0$ from Lemma **??**. Therefore, we can rank all profits levels: $\Pi_{MB}^{d}>\Pi_{MB}^{d_3}>\Pi_{MB}^{d_2}\gtrless\Pi_{IP}^{d}\gtrless\Pi_{MB}^{d_1}$. Again in this case, $\Pi_{MB}^{d_2}$ can be lower than Π_{IP}^{d} if $x>11.53$, and lower than $\Pi_{MB}^{d_1}$ if $x>5$. So we have shown that $\Pi_{MB}^{d}>\max\{\Pi_{MB}^{d_1},\Pi_{MB}^{d_2},\Pi_{MB}^{d_3},\Pi_{MB}^{d_4}\}$. When both firms follow a mixed bundling strategy, the optimal deviation is to deviate from the mixed bundling strategy too (i.e., from the first component, the second one and the bundle). ■

6.6 Proof of Proposition ??.

To prove Proposition **??**, we first aim to solve the programs stated in footnote **??** to find optimal flexible pricing devices for collusion, competition and deviation.

Considering the collusion outcome, we showed in Proposition **??** that both devices are equivalent. So $\mu_i^{c*}=\{MB\},\forall i=1,2$ is optimal (among others). Considering competition, Economides (1993) has shown that mixed bundling is the Nash equilibrium of the pricing device static game that is $\mu_i^{n*}=\{MB\},\forall i=1,2$. Considering deviation, Lemma **??** shows that $\mu_i^{d*}=\{MB\},\forall i=1,2$. So one can conclude that $\mathbf{m}_i^*=(MB,MB,MB)$ is an optimal flexible choice of pricing device. Actually there are multiple equilibria, the others one are such that for firm $j=1,2:\mathbf{m}_j^*=(\mu,MB,MB)$ with $\mu=\{IP\}$ to collude but they lead to the same level of the critical discount factor. Consequently $\delta^*(\mathbf{m}_1^*,\mathbf{m}_2^*)=\delta_{MB}(x)$ as given in (**??**).

Second, we consider the sub-optimal profile $\hat{\mathbf{m}}_i=(IP,MB,IP)$ which corresponds to the optimal MB deviation when both firms follow IP pricing to collude and compete. The related critical discount factor is $\delta^*(\hat{\mathbf{m}}_1,\hat{\mathbf{m}}_2)$ and is defined by:

$$\delta^*(\hat{\mathbf{m}}_1,\hat{\mathbf{m}}_2)=\frac{\Pi_{MB}^{d}-\Pi^c}{\Pi_{MB}^{d}-\Pi_{IP}^{n}}=\frac{\left(x^2-7x+16\right)(7x-17)^2}{65x^4-885x^3+4739x^2-11095x+9328}. \tag{30}$$

As a function of x, we denote $\delta_{IP}^*(x)$ this factor, where $\delta_{IP}^*(3)=1$ and $\lim_{x\to\infty}\delta_{IP}^*(x)=\frac{49}{65}$. From Equations (**??**) and (**??**), we can form the difference $\Delta^o(x)=\delta_{MB}(x)-\delta_{IP}^*(x)$ and show that $\Delta^o(x)=-\frac{32A^o(x)}{B^o(x)C^o(x)}$ where expressions $A^o(x)$, $B^o(x)$ and $C^o(x)$ are given by:

$$\begin{aligned}A^o(x)&=(x^2-7x+16)(x-2)(x+1)(31x^2-177x+248)(x-3)^2\\B^o(x)&=68x^4-816x^3+3901x^2-8439x+6768\\C^o(x)&=65x^4-885x^3+4739x^2-11095x+9328\end{aligned}$$

When we examine these polynomial functions, one can see that they are all positive functions of x whenever $x>\xi$. Indeed, $A^o(x)$ is negative for $3<x<177/62+(1/62)\sqrt{577}\simeq 3,24$, and positive when $x>3,24$ with $A^o(3)=0$. Considering $B^{o\prime\prime}(x)$ is clearly a positive function for $x>3$, and because $B^{o\prime}(3)=279>0$, $B^o(x)$ is increasing from $B^o(3)=36$ to infinity, such that $B^o(x)>0$. Finally, $C^o(x)$ is also a positive function for all $x>0$, as $C^{o\prime}(3)=464>0$, $C^o(x)$ is increasing from $C^o(3)=64$ to infinity, we can conclude that $C^o(x)>0$. Hence $\Delta^o(x)<0$ and from Proposition **??**, we can state the result: $\delta_{IP}(x)<\delta_{MB}(x)<\delta_{IP}^*(x)$. ■

6.7 Proof of Proposition ??.

To prove this proposition we need to derive competitive, collusion, and deviation outcomes from Cournot competition.

•*Competitive outcomes*.We establish the Nash-Cournot equilibrium by seeking the quantities firms choose to maximize profits, as given by Equations (**??**) and (**??**).

1. When firms use *independent sales* strategies, it is characterized by the first-order conditions, for $i = 1,2$:

$$\begin{aligned} \frac{\partial \hat{\Pi}^i_{IP}}{\partial \underline{q}^i} &= 0 \Leftrightarrow ac - \frac{2}{(x+1)}\left[(x-1)\underline{q}^i + \underline{q}^j\right] - \bar{q}^i - \frac{1}{2}\bar{q}^j = 0 \\ \frac{\partial \Pi^i_{IP}}{\partial \bar{q}^i} &= 0 \Leftrightarrow ac - \frac{2}{(x+1)}\left[(x-1)\underline{q}^i + \underline{q}^j\right] - \underline{q}^i - \frac{1}{2}\bar{q}^j = 0 \end{aligned}$$

Solving this linear system determines the quantity best replies of the firm i when IP applies, that is

$$\left\{ \begin{array}{l} \underline{\mathbf{q}}^i\left(\underline{q}^j, \bar{q}^j\right) = ac\frac{x+1}{3x-1} - \frac{x+1}{3x-1}\underline{q}^j + \frac{1}{2}\frac{x-3}{3x-1}\bar{q}^j \\ \bar{\mathbf{q}}^i\left(\underline{q}^j, \bar{q}^j\right) = ac\frac{x+1}{3x-1} - \frac{x+1}{3x-1}\bar{q}^j + \frac{1}{2}\frac{x-3}{3x-1}\underline{q}^j \end{array} \right. \tag{31}$$

Intersecting these best replies leads to the equilibrium quantities and profits:

$$\underline{q}^i = \overline{q}^i = q^n_{IP} = 2ac\frac{x+1}{7x+3},\ \hat{\Pi}^i_{IP} = \hat{\Pi}^n_{IP} = 8a^2c\frac{(x+1)(3x-1)}{(x-3)(7x+3)^2} \quad \text{with } i = 1,2.$$

2. When firms use *mixed bundling* quantity strategies, the first-order conditions, for $i = 1,2$ write

$$\begin{aligned} \frac{\partial \hat{\Pi}^i_{MB}}{\partial \underline{q}^i} &= 0 \Leftrightarrow ac - \frac{x-2}{x+1}\left[2\underline{q}^i + \bar{q}^j\right] - \frac{1}{x+1}\left[2\bar{q}^i + \underline{q}^j\right] - \frac{1+\eta}{x+1}\left[2y^i + y^j\right] = 0 \\ \frac{\partial \Pi^i_{MB}}{\partial \bar{q}^i} &= 0 \Leftrightarrow ac - \frac{x-2}{x+1}\left[2\bar{q}^i + \underline{q}^j\right] - \frac{1}{x+1}\left[2\underline{q}^i + \bar{q}^j\right] - \frac{1+\eta}{x+1}\left[2y^i + y^j\right] = 0 \\ \frac{\partial \Pi^i_{MB}}{\partial y^i} &= 0 \Leftrightarrow ac - \frac{1+\eta}{x+1}\left[(x-2)y^i + y^j\right] - \frac{1}{x+1}\left[2\underline{q}^i + 2\bar{q}^i + \underline{q}^j + \bar{q}^j\right] = 0 \end{aligned}$$

Solving this linear system determines the quantity best replies of the firm i when MB applies, that is

$$\left\{ \begin{array}{l} \underline{\mathbf{q}}^i\left(\underline{q}^j, \bar{q}^j, y^j\right) = ac\frac{x+1}{2x} - \frac{1}{2}\bar{q}^j - \frac{1}{2}\frac{1+\eta}{x}y^j \\ \bar{\mathbf{q}}^i\left(\underline{q}^j, \bar{q}^j, y^j\right) = ac\frac{x+1}{2x} - \frac{1}{2}\underline{q}^j - \frac{1}{2}\frac{1+\eta}{x}y^j \\ \mathbf{y}^i\left(\underline{q}^j, \bar{q}^j, y^j\right) = ac\frac{x+1}{2x(1+\eta)} - \frac{1}{2x}y^j \end{array} \right. \tag{32}$$

Then the equilibrium quantities and profits write:

$$\begin{aligned} \underline{q}^i &= \bar{q}^i = q^n_{MB} = \frac{2}{3}\frac{ac(x+1)}{(2x+1)};\ y^i = y^n_{MB} = \frac{ac(x+1)}{2x+1}; \\ \hat{\Pi}^i_{MB} &= \hat{\Pi}^n_{MB} = \frac{a^2c}{9}\frac{(x+1)(17x-2)}{(x-3)(2x+1)^2} \quad \text{with } i = 1,2. \end{aligned}$$

One can easily see that $q^n_{MB} \geq q^n_{IP}$ and $y^n_{MB} \leq 2q^n_{IP}$ and $\hat{\Pi}^n_{MB} \geq \hat{\Pi}^i_{IP}$.

•*Collusive outcomes*. With a independent sales or mixed bundling strategies, when firms coordinate on a collusive equilibrium, the joint profit writes $\pi = \max_{\underline{q}^1, \bar{q}^2,} P_1\ X_{11} + P_{12}\left(\underline{q}^1 + \overline{q}^2\right) + P_{21}\ \left(\underline{q}^2 + \overline{q}^1\right) + P_{22}\ X_{22}$ where X_{ii} writes $X_{ii} = \underline{q}^i + \overline{q}^i$ or $X_{ii} = y^i$ indifferently. Then optimal quantities and profits are as follows:

$$\underline{q}^i = \overline{q}^i = q^c = \frac{1}{4}ac\ \ ; y^i = y^c = 2q^c \quad \text{and } \hat{\Pi}^c = \frac{1}{2}\frac{a^2c}{x-3} \quad \text{with } i = 1,2.$$

•*Deviation outcomes*. **1.** Suppose that both firms follow an independent sales strategy. If firm i adjusts its quantities and chooses $\underline{q}^i$ and $\overline{q}^i$ by anticipating that firm j carries on the collusive agreement $\{\underline{q}^j = q^c; \overline{q}^j = q^c\}$. Hence, at the equilibrium, firm plays its best replies represented by (??), i.e. $\underline{\mathbf{q}}^i(q^c,q^c)$ and $\bar{\mathbf{q}}^i(q^c,q^c)$ which leads to $q^d_{IP} = \frac{ac}{8}\frac{7x+3}{3x-1}$. Profit for deviating is given by:

$$\hat{\Pi}^d_{IP} = \frac{ac}{32}\frac{(7x+3)^2}{(x-3)(x+1)(3x-1)}. \tag{33}$$

2. Consider that both firms follow a mixed bundling quantity strategy. If firm i deviates from the collusive agreement, it chooses its quantities $\underline{q}^i = \underline{q}^d$, $\overline{q}^i = \overline{q}^d$, and $y^i = y^d$, anticipating that firm j keeps sticking to the collusive agreement $\{\underline{q}^j = \underline{q}^c; \overline{q}^j = \overline{q}^c; y^j = y^c\}$. Hence, at the equilibrium, firm i plays its best replies represented by (??), i.e. $\underline{\mathbf{q}}^i(q^c,q^c,y^c)$, $\bar{\mathbf{q}}^i(q^c,q^c,y^c)$ and $\mathbf{y}^i(q^c,q^c,y^c)$ which leads to these equilibrium quantities and profits of both firms:

$$\begin{aligned}
\underline{q}^i &= \overline{q}^i = q^d_{MB} = \frac{ac}{8}\frac{3x+2}{x};\ y^i = y^d_{MB} = \frac{ac}{4}\frac{2x+1}{(1+\eta)x} \text{ with } i=1,2.\\
\hat{\Pi}^d_{MB} &= \frac{a^2c}{32}\frac{17x^2+19x+6}{x(1+x)(x-3)}
\end{aligned} \tag{34}$$

•Proposition **??**. *(a)* Now we can form the critical discount factors using all profit levels computed so far.

$$\hat{\delta}_{IP}(x) = \frac{\hat{\Pi}^d_{IP} - \hat{\Pi}^c}{\hat{\Pi}^d_{IP} - \hat{\Pi}^n_{IP}} = \frac{(7x+3)^2}{97x^2+74x-7}$$

where $97x^2+74x-7>0$ for all $x>3$, and

$$\hat{\delta}_{MB}(x) = \frac{\hat{\Pi}^d_{MB} - \hat{\Pi}^c}{\hat{\Pi}^d_{MB} - \hat{\Pi}^n_{MB}} = 9\frac{\left(x^2+3x+6\right)(2x+1)^2}{\hat{A}(x)}$$

where $\hat{A}(x) = 68x^4+27x^3+637x^2+451x+54>0$ for all $x>0$. This corresponds to Equations (??) and (??) in the text. Here we see that $\hat{\delta}_{IP}(3) = \hat{\delta}_{MB}(3) = \frac{9}{17}$; and $\lim_{x\to\infty}\hat{\delta}_{MB}(x) = \frac{9}{17} > \lim_{x\to\infty}\hat{\delta}_{IP}(x) = \frac{49}{97} > \frac{1}{2}$. Moreover one can see that

$$\hat{\delta}'_{IP}(x) = -64\hat{\delta}_{IP}(x)\frac{(x+5)}{(7x+3)\left(97x^2+74x-7\right)} < 0$$

then $\hat{\delta}_{IP}(x)$ is strictly decreasing to $\frac{49}{97}$ and

$$\hat{\delta}'_{MB}(x) = 32\hat{\delta}_{MB}(x)\frac{\hat{B}(x)}{(3x+x^2+6)(2x+1)\hat{A}(x)}$$

where $\hat{B}(x) = x^4-16x^2-36x-39$. Studying $\hat{B}(x)$ shows that $\hat{B}'(x) = 4x\left(x^2-8\right)-36$ which is positive for all $x\geq\xi$. Hence, $\hat{B}(x)$ is an increasing function of x for $x>\xi$. As $\hat{B}(\xi) = -\frac{1}{62}\left(22889+1271\sqrt{73}\right)<0$ and $\lim_{x\to+\infty}\hat{B}(x) = +\infty$, it exists a value $\bar{\xi}$ of x such that $\hat{B}(\bar{\xi})=0$ and $\hat{B}(\xi)>0$ above $\bar{\xi}$. By approximation, one can find that $\bar{\xi}\simeq 4.98$. Hence $\hat{\delta}_{MB}(x)$ is U-shaped and reach a minimum value for $x=\bar{\xi}$.

We now compare all these critical discount factors with those with pricing strategies.
(b) The difference $\hat{\Delta}(x) = \hat{\delta}_{MB}(x) - \hat{\delta}_{IP}(x)$ writes $\hat{\Delta}(x) = \frac{32\,(x-3)(x+1)^2\hat{C}(x)}{\hat{D}(x)\,\hat{A}(x)}$, where $\hat{C}(x)$ and $\hat{D}(x)$ are given by:

$$\hat{C}(x) = 5x^3+19x^2+27x \quad\text{and } \hat{D}(x) = 97x^2+74x-7$$

A glance at these polynomial functions reveals that they are all positive functions of x for $x\geq\xi$, such that $\hat{\Delta}(x)>0$.
(c) The difference $\bar{\Delta}(x) = \hat{\delta}_{IP}(x) - \delta_{IP}(x)$ writes

$$\bar{\Delta}(x) = \frac{128\,(x-1)(7x^2-14x-53)}{\left(97x^2+74x-7\right)\left(97x^2-462x+529\right)}$$

where $97x^2-462x+529>0$ for all $x>3$. Then $\bar{\Delta}(x)$ has the sign of of the convex function of $x: 7x^2-14x-53$ which is zero for $x = \frac{2}{7}\sqrt{7}\sqrt{15}+1\simeq 3.93$. Hence, denoting $\hat{\xi}'$ this value, we have $\bar{\Delta}(x)<0$ for $x<\hat{\xi}'$ and

$\bar{\Delta}(x) \geq 0$ for $x \geq \hat{\xi}'$.
(d) Using Equation (**??**) in the text, the difference $\tilde{\Delta}(x) = \hat{\delta}_{MB}(x) - \delta_{MB}(x)$ is given by $\tilde{\Delta}(x) = -\frac{576(x-1)\,\tilde{A}(x)}{\tilde{B}(x)\,\hat{A}(x)}$, where $\tilde{A}(x)$ and $\tilde{B}(x)$ are given by:

$$\tilde{A}(x) = 7x^4 - 28x^3 + 8x^2 + 40x - 297 \quad \text{and} \quad \tilde{B}(x) = 68x^4 - 816x^3 + 3901x^2 - 8439x + 6768$$

First, $\tilde{A}''(x) = 84x^2 - 168x + 16$ is a positive function for $x > \xi$, so $\tilde{A}'(x)$ is an increasing function for $x > \xi$ with $\tilde{A}'(\xi) = \frac{1}{27}\left(4424 + 464\sqrt{73}\right) \simeq 310.68$, hence $\tilde{A}'(x) > 0$ for $x > \xi$ and $\tilde{A}'(x)$ is an increasing function of x. As $\tilde{A}(\xi) = \frac{1}{162}\left(-36863 + 867\sqrt{73}\right) \simeq -181.82$ then it exists a value $\hat{\xi} > \xi$ of x such that $\tilde{A}(\hat{\xi}) = 0$ and positive above. More precisely $\hat{\xi} = 1 + \frac{1}{7}\sqrt{119 + 7\sqrt{2179}} \simeq 4.016$. Second, the function $\tilde{B}'''(x)$ is an increasing function for $x > 3$, because it is zero for $x = 3$, and then $\tilde{B}''(x)$ is an increasing function for $x > \xi$ with $\tilde{B}''(\xi) = \frac{1}{3}\left(8038 - 680\sqrt{73}\right) \simeq 742.69 > 0$, such that $\tilde{B}'(x)$ is a positive function, with $\tilde{B}'(\xi) \simeq 605.5 > 0$. Finally, $\tilde{B}(x)$ is strictly increasing for $x > \xi$ from $\tilde{B}(\xi) \simeq 289$ to infinity. As a result $\tilde{\Delta}(x) \geq 0$ if $x \geq \hat{\xi}$ and $\tilde{\Delta}(x) < 0$ elsewhere.
(iii) From Proposition **??** we know that $\delta_{IP} < \delta_{MB}$, and from previous points *(a)* to *(d)*, we can write the following ranking: $\max\{\delta_{IP}, \hat{\delta}_{IP}, \hat{\delta}_{MB}\} < \delta_{MB}$ for $x > \hat{\xi}$ and $\max\{\delta_{IP}, \hat{\delta}_{IP}, \delta_{MB}\} \leq \hat{\delta}_{MB}$ for $x \leq \hat{\xi}$. ■