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# Reputation Effects in Repeated Audits, with Application to Insurance Fraud Deterrence

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## Abstract

In principal-agent problems, the repetition of interactions in a dynamic setting may alter the equilibrium outcomes. In insurance fraud, the frequency of auditor-auditee interactions is higher when there is collusion between policyholders and service providers (e.g., car repairers, health care providers...). The same service provider usually handles claims filed by many policyholders affiliated to the same insurer, and thus the insurer-service provider interactions are repeated with reputation effects. We analyze this issue in a repeated game where the insurer may potentially face a dishonest service provider who colludes with policyholders. The insurer has beliefs about the type (honest or dishonest) of the service provider and she may verify the truthfulness of the claim through costly audits. The reputation of the service provider corresponds to these beliefs and changes over time, and misbehaving deteriorates this reputation. In the end, it may lead to a breach of contract and thus represents a threat that may deter from defrauding. We show that, at early periods, the insurer audits agents who would not be monitored in a static setting because their reputation is good enough. Corresponding dishonest agents who slipped under the radar and have an initially good reputation do not defraud systematically at early periods, as opposed to the instantaneous game. In addition, auditing efforts for medium reputations are lower as dishonest agents want to preserve the possibility of defrauding later. Both aspects corresponds to a reputation-based deterrence mechanism, where the fear of deteriorating one's reputation acts as a discipline device for dishonest service providers.

**Keywords:** Insurance fraud, Optimal auditing, Reputation, Learning, Deterrence.

**JEL:** C73, D82, D83, G22.

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# 1 Introduction

Many principal-agent relationships have a repeated nature, and from period to period, the principal may improve its knowledge of some hidden information of the agent. This situation arises in many problems of fraud inspection such as, for instance, the relationship between a firm and a subcontractor, or between a tax authority and a tax payer. In this paper, we consider the case of insurance fraud where insurers have repeated relationships with service providers who may be honest or dishonest.

Insurance fraud is a fundamental and complex problem for insurance markets. It is fundamental because it heavily distorts the risk-sharing mechanisms upon which insurance contracts rely, and without which insurance markets might collapse. It is also complex because auditing is motivated by mechanisms whose roles overlap and are hard to disentangle. Is an audit revealing no fraud synonymous of failure? Or does it just mean it completely deters fraud? How can information acquisition still motivate an audit if in the end deterrence is too strong to learn anything valuable?

The purpose of this paper is to combine both deterrence and informational aspects of auditing in a single framework to analyze their interactions. To do so, we model a repeated game between an insurer and a (honest or dishonest) auditee. The repeated character of interactions is particularly relevant when there is collusion between policyholders and service providers (SPs). As a single SP (e.g., car repairer or health service provider) may serve several policyholders of the same insurer, he interacts with the latter on a frequent basis. An SP acts as an expert who certifies<sup>1</sup> the claim, without which a fraud cannot occur, making an SP's complicity a necessary condition for cheating. Honest types may only certify invalid claims by mistake. Dishonest types on the other hand may defraud willingly by voluntarily certifying an illegitimate claim. Auditing is only profitable when fraudulent activity is large enough, and dishonest types rationally balance the risk of being caught with the potential gain from fraud. This is true even in a static setting when deterrence effects rely on the auditee's reaction to the threat of an audit. Informational effects come into play once interactions are repeated: based on the previous outcomes,

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<sup>1</sup>i.e., that the policyholder has received a service and made a payment in return.

the insurer has beliefs about the auditee's type, i.e., a reputation, on which the auditing decision depends.

Our main result shows that this combination exhibits a reputation-based deterrence effect that is two-fold: first, the set of auditees is larger at earlier periods and includes auditees whose reputation is good enough not to be audited in a static setting. Second, the insurer needs to deploy less auditing efforts at earlier periods to deter auditees who would have been audited at later ones. Interestingly, this is the result of the interaction of two mechanisms, corresponding respectively to the deterrence and to the learning effects. With reputation effects, an auditee balances the intertemporal costs and benefits of cheating in the present: an audit today still threatens him as before, but being caught may also hinder the future opportunities of defrauding because the reputation worsens. The second mechanism of the reputation-based deterrence effects relies in the discriminatory power of auditing. Inspection allows the insurer to improve her information and to better audit in the future periods, or even exclude a dishonest auditee from his affiliation network.

These aspects relate to several strands of the academic literature. First and foremost, this article contributes to the literature on optimal auditing for insurance fraud.<sup>2</sup> Following the work of [Townsend \(1979\)](#) and [Gale & Hellwig \(1985\)](#) on the Costly State Verification approach, it considers the problem of conducting an audit when the latter is costly. It tackles questions related to the relevance of random auditing, thus expanding the analysis of [Mookherjee & Png \(1989\)](#) by conditioning the levels of auditing on the reputation of auditees. Extending the idea of Costly State Falsification developed by [Crocker & Morgan \(1998\)](#), it exhibits a form of opportunity cost for the auditee to not defraud in order to mislead the auditor into believing he is not dishonest. More specifically, our work relates to the analysis of the different roles of auditing, besides recovering illegitimate payments. [Dionne et al. \(2008\)](#) focus on auditing as a deterrence tool in a static Stackelberg game, and show that some claims should still be audited in spite of being individually unprofitable to audit. In [Aboutajdine & Picard \(2018\)](#) and [Aboutajdine](#)

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<sup>2</sup>[Picard \(2013\)](#) offers a comprehensive review of this literature.

& Picard (2019), we consider an alternative situation where there is no deterrence<sup>3</sup> but the auditor can learn about the auditee's type in a multiperiod framework. The present article addresses the problem of combining both deterrence and information in a single framework.

Second, an important feature of our model, namely being a repeated game with monitoring, relates our work to the inspection games literature (see Avenhaus et al. (2002) for a literature review). Originally applied to arms control and disarmament, these models have also been used for accounting and auditing problems in economics. Borch (1990) evokes the potential application of inspection games to the economics of insurance, specifically to ex-ante moral hazard problems where the insured has to take special measures to avoid accidents. In our setting, the problem is one of ex-post moral hazard (submitting a claim, valid or invalid, after the occurrence of a potential accident). Inspection games have also been applied to tax evasion settings. Rubinstein (1979) examines a case where an audit might unjustly target an honest auditee and shows that, while a one-shot game gives no choice but to set a large penalty, a repeated game allows for a more lenient policy that also induces the auditee to comply. Greenberg (1984) proposes an auditing scheme where individuals are separated into different groups with different auditing levels. Auditees move between groups depending on the previous cheating outcome. Another example of repeated game where cheating history is given by Reinganum & Wilde (1986). They apply the sequential equilibrium concept in a model where income reporting is a source of information for the tax inspector. More recently, Varas et al. (2018) analyze a firm's reputation for quality problem, with an inspector who can conduct costly inspections. Notably, in their model, monitoring plays both an incentive and an informational role. Finally, the existence of an optimal information acquisition dimension to the insurer's problem relates it to the exploration/exploitation dilemma. It places our work in the continuity of the strategic experimentation literature and bandit problems (see Bergemann & Välimäki (2008)). In Aboutajdine & Picard (2019), we show that the optimal auditing with learning but no deterrence problem is akin to a regular one-armed bandit

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<sup>3</sup>i.e., the type dependent behavior of auditees is given and not affected by the risk of being spotted.

problem where playing the risky arm corresponds to auditing, and playing the safe arm to not auditing. This is because the auditees are non-strategic and honest and dishonest auditees always submit a claim. In this paper, we relax both these assumptions. The underlying bandit problem becomes far more complex as it involves information collection even when no action is taken, which is related to the restless bandit literature (introduced by Whittle (1988)). It also involves strategic arms whose rewards depend on the player’s strategy (e.g., Braverman et al. (2017)). Both topics have been rarely tackled in the academic literature, especially in the case of strategic armed bandits.

The rest of the paper is as follows: Section 2 presents the model, Section 3 characterizes our main results and Section 4 shows how our problem is related to bandit problems. Section 5 concludes. All proofs are in the Appendix.

## 2 Model

### 2.1 Notations and main assumptions

We consider the relationship between an insurer (she) and a service provider (SP, he) during  $T$  periods indexed by  $t = 1, \dots, T$ . The SP is in touch with policyholders and he certifies their claims (i.e., he asserts that he delivered the service required by the event described in the claim, at the price reported by the policyholder). The SP may certify at most one claim per period to the insurer (certifying more claims would signal that he behaves in a fraudulent way). The SP may be honest or dishonest, i.e. of type  $H$  or  $D$ , respectively, and the insurer has beliefs about the  $SP$ ’s type.

Claims are valid or invalid, and invalid claims may have been filed involuntarily (by error) or voluntarily (by fraud). A claim is involuntary invalid when the policyholder wrongly but honestly thought that the loss at the origin of the claim should be covered by the insurance policy, and the service provider does not observe any wrongdoing. Claims are voluntary invalid when the policyholder and the SP collude: the policyholder files a claim certified by the SP in order to trigger the payment of an insurance indemnity not included in the contractual coverage (e.g., falsifying a claim by inventing an event that did

not occur, or misrepresenting the circumstances of this event). Honest SPs only certify invalid claims involuntarily, while dishonest SPs may certify invalid claims knowing that they are invalid. In short, insurance fraud takes the form of invalid claims that are willfully transmitted to the insurer by policyholders, with the complicity of dishonest service providers.

Let  $p_0$  and  $p_H$  be the probability for an SP to certify a valid claim or to certify involuntarily an invalid claim, whatever the SP's type, with  $p_0 + p_H < 1$ . Let  $y_t \in [0, 1]$  the probability for a dishonest SP to wilfully certify an invalid claim at period  $t$  when he has no claim to truthfully certify, which occurs with probability  $1 - p_0 - p_H$ . In what follows,  $y_t$  will be referred to as the fraud probability of a type  $D$  SP, which corresponds to the voluntary certification of an invalid claim with probability  $y_t(1 - p_0 - p_H)$  when the SP is of type  $D$ . The insurer may audit a claim, which costs  $c$  to her, and we denote  $x_t \in [0, 1]$  the probability of auditing a claim filed at period  $t$ . Audit reveals whether the claim is valid or not. Usually, in the case of an invalid claim, audit does not allow the insurer to establish whether this was intentional or unintentional, i.e., whether this was fraud or simply an error. In both cases, no indemnity is paid to the policyholder. When a fraudulent invalid claim is audited, the insurer may detect fraud unambiguously with probability  $\alpha \in [0, 1)$ .

*Remark 1.* In a formally equivalent way, our model may also be interpreted as corresponding to the relationship between a firm and a sub-contractor supplying spare parts, assembled by the firm into a finished product. Each year, the firm expresses its needs under the form of new product specifications (e.g., additional safety or quality criteria), and the sub-contractor answers by transmitting a forecast report about expected additional costs induced by the change in product specifications. In this interpretation of the model,  $p_0$  is the probability that the change in specifications actually induces unavoidable additional costs. However, with probability  $p_H$  the additional expected costs reported by the sub-contractor simply reflect a misunderstanding of the firm's requirements, or the fact that it is unaware of costless ways to comply with the firm's requirement (e.g., by using different inputs or by subcontracting itself with another firm for some parts of the

product to be supplied). Such misunderstandings may be cleared by the firm through an audit of the sub-contractor's report. However, the announced additional costs may also result from the willingness of a dishonest sub-contractor to falsify its true cost, possibly through a cost-accounting manipulation, which may be detected with probability  $\alpha$  if the forecast report is audited.

We wish to focus attention on the case where proving fraud is difficult, possibly impossible, and thus we postulate that  $\alpha$  is small, possibly zero. More precisely, we assume

$$0 \leq \alpha < \frac{p_0}{p_0 + p_H}. \quad (1)$$

Each claim corresponds to a contractual insurance indemnity  $I$ , while defrauding costs  $\omega$  to a dishonest SP. We may interpret  $\omega$  as the expenses incurred by the SP and by the policyholder to falsify the claim (i.e., to prevent the insurer to immediately detect that the claim is invalid without any audit). Hence  $I - \omega$  is the collusive gain in case of undetected fraud, to be shared between the dishonest *SP* and the policyholder. For simplicity, we assume that collusion takes the form of a take-or-leave it offer made by the SP to the policyholder, with the whole collusive surplus  $I - \omega$  being allocated to the SP, but this is an unimportant assumption in our analysis. More importantly, we assume:

$$c < (1 - p_0)I, \quad (2)$$

$$(p_0 + p_H)c > p_H I. \quad (3)$$

According to assumption (2), if the insurer knows with certainty that the SP is of type *D* and that he systematically defrauds (i.e.,  $y = 1$ ), then it is profitable to audit the claim, because the audit cost  $c$  is lower than the expected recouped indemnity  $(1 - p_0)I$ . Conversely, according to assumption (3), if the insurer knows with certainty that the SP is of type *H*, then it is not worthwhile to audit the claim, because the audit cost  $c$  is larger than the expected recouped indemnity  $p_H I / (p_0 + p_H)$  associated with involuntary errors.

The insurer offers a multi-period contract to the SP at the beginning of period 1,



whereby the SP is allowed to act as an insurance intermediary from  $t = 1$  to  $t = T$ , which consists in certifying claims filed by policyholders. At period 1, the insurer has initial beliefs  $\pi_1 \in (0, 1)$  about the SP's type:  $\pi_1$  is the probability that the SP is of type  $D$ , and thus  $1 - \pi_1$  is the probability that he is of type  $H$ .<sup>4</sup> At each period  $t = 2, \dots, T$ , the insurer has updated beliefs  $\pi_t$  deduced from  $\pi_{t-1}$  and from the scenario that occurred at period  $t - 1$ . A breach of contract takes place at the beginning of period  $t$  only if the insurer can prove unambiguously that the SP is dishonest.<sup>5</sup> In such a case, another contract is offered to a new SP for the remaining periods  $t, t + 1, \dots, T$ , with insurer's beliefs  $\bar{\pi} \in (0, 1)$  about this new SP.

For any fraudulent claim filed at period  $t$ , the expected profit of the dishonest SP involved in the collusion is the difference between his gain  $I - \omega$  if there is no audit, and his loss  $\omega$  in the case of an audit, weighted by the probabilities  $1 - x_t$  and  $x_t$ , respectively. Since a type  $D$  SP defrauds with probability  $y_t(1 - p_0 - p_H)$ . We may denote

$$\Pi(x_t, y_t) = y_t(1 - p_0 - p_H)[(1 - x_t)(I - \omega) - x_t\omega], \quad (4)$$

his expected profit drawn from fraud at period  $t$ .

We also denote

$$C(x_t, y_t, \pi_t) = I[p_0 + K(\pi_t y_t)] - x_t[IK(\pi_t y_t) - c(p_0 + K(\pi_t y_t))] \quad (5)$$

the expected cost of the insurer at period  $t$ , where

$$K(\pi_t y_t) = p_H + \pi_t y_t(1 - p_0 - p_H) \quad (6)$$

is the probability of an invalid claim at period  $t$  when the SP is of type  $D$  with probability  $\pi_t$ . This probability is the sum of  $p_H$ , which is the probability of an invalid

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<sup>4</sup>This may correspond to the fraction of dishonest individuals in a population, in which the SP is randomly drawn. Initial beliefs may also be affected by business references or by other signals informative about the SP's honesty.

<sup>5</sup>A breach of contract without evidence of dishonesty would lead to bad faith penalty imposed by a court.

claim transmitted by error, and of  $\pi_t y_t (1 - p_0 - p_H)$  which is the probability of a fraudulent claim. The expected insurer's cost  $C(x_t, y_t, \pi_t)$  is the difference between the expected insurance indemnity  $I[p_0 + K(\pi_t y_t)]$  and the expected net proceeds from audit  $x_t [IK(\pi_t y_t) - c(p_0 + K(\pi_t y_t))]$ .

Five scenarios may occur at each each period  $t = 1, \dots, T$ , denoted  $s_t \in S$  with

$$S = \{AF, AI, AV, NA, NC\}.$$

These scenarios are detailed in Table 1. The objective of the insurer is to minimize her discounted expected cost, and the objective of a type  $D$  SP is to maximize his discounted expected profit from fraud. We denote  $\delta \in (0, 1]$  the discounting factor, common to the insurer and to the SP.

Scenario	Details	Auditor's Cost
AF	A claim is filed, audit reveals an invalid claim, fraud is proved	$c$
AI	A claim is filed, audit reveals an invalid claim, no fraud is proved	$c$
AV	A claim is filed, audit reveals a valid claim	$I + c$
NA	A claim is filed, no audit is performed	$I$
NC	No claim is filed	$0$

Table 1: Possible scenarios at a given period

## 2.2 Definition of an equilibrium

Let  $h_t = (s_1, s_2, \dots, s_{t-1}) \in S^t$  be the history of the relationship between the insurer and the SP before period  $t$ , with  $t \geq 1$ . This history affects the expected profit of a type  $D$  SP and the expected cost of the insurer only through its effect on beliefs  $\pi_t$ . We make the following assumptions about strategies and beliefs:

1. Strategies are Markovian: they define audit and fraud decisions at period  $t$  as functions of the insurer's beliefs at the same period, and they are denoted  $x_t(\pi_t)$  and  $y_t(\pi_t)$  for all  $t = 1, \dots, T$ .

2. Beliefs  $\pi_t$  depend on previous beliefs  $\pi_{t-1}$  and on the scenario  $s_{t-1}$  that occurred at the previous period, and we will write them as:  $\pi_t = \pi_t(\pi_{t-1}, s_{t-1})$  for all  $t = 2, \dots, T$ .

**Definition 1.** A Markov Subgame Perfect Bayesian Equilibrium (in short, an Equilibrium) is characterized by strategies  $x_t^*(\cdot) : [0, 1] \rightarrow [0, 1]$  and  $y_t^*(\cdot) : [0, 1] \rightarrow [0, 1]$  for all  $t = 1, \dots, T$  defining actions  $x_t^*(\pi_t), y_t^*(\pi_t)$  as functions of beliefs, and by belief updating functions  $\pi_t^*(\cdot) : [0, 1] \times S \rightarrow [0, 1]$  for all  $t = 2, \dots, T$  defining updated beliefs  $\pi_t^*(\pi_{t-1}, s_{t-1})$  as functions of previous beliefs and previous period scenario, such that:

1. For all  $t = 1, \dots, T$  and all belief  $\pi_t \in [0, 1]$ ,  $x_t^*(\pi_t)$  minimizes

$$C_t(x_t, y_t^*(\pi_t), \pi_t) = C_t(x_t, y_t^*(\pi_t), \pi_t) + \sum_{\theta=1}^{T-t} \delta^\theta \mathbb{E}_{\tilde{\pi}_{t+\theta}} \left[ C(x_{t+\theta}^*(\tilde{\pi}_{t+\theta}), y_{t+\theta}^*(\tilde{\pi}_{t+\theta}), \tilde{\pi}_{t+\theta}) \right],$$

w.r.t.  $x_t \in [0, 1]$ , where, for all  $\theta = 1, \dots, T - t$ ,  $\tilde{\pi}_{t+\theta}$  denotes the random beliefs deduced from strategies  $x_t, x_{t+1}^*(\cdot), \dots, x_T^*(\cdot)$  and  $y_t^*(\cdot), y_{t+1}^*(\cdot), \dots, y_T^*(\cdot)$ , and from the belief updating functions  $\pi_{t+1}^*(\cdot), \dots, \pi_T^*(\cdot)$ .<sup>6</sup>

2. For all  $t = 1, \dots, T$  and all beliefs  $\pi_t \in [0, 1]$ ,  $y_t^*(\pi_t)$  maximizes

$$\Pi_t(x_t^*(\pi_t), y_t, \pi_t) = \Pi_t(x_t^*(\pi_t), y_t, \pi_t) + \sum_{\theta=1}^{T-t} \delta^\theta \mathbb{E}_{\tilde{\pi}_{t+\theta}} \left[ \Pi(x_{t+\theta}^*(\tilde{\pi}_{t+\theta}), y_{t+\theta}^*(\tilde{\pi}_{t+\theta}), \tilde{\pi}_{t+\theta}) \right],$$

w.r.t.  $y_t \in [0, 1]$ , where, for all  $\theta = 1, \dots, T - t$ ,  $\tilde{\pi}_{t+\theta}$  denote the random beliefs deduced from strategies  $x_t^*(\cdot), x_{t+1}^*(\cdot), \dots, x_T^*(\cdot)$  and  $y_t, y_{t+1}^*(\cdot), \dots, y_T^*(\cdot)$ , and from the belief updating functions  $\pi_{t+1}^*(\cdot), \dots, \pi_T^*(\cdot)$ .

3. For all  $t = 1, \dots, T$  and all  $(\pi_{t-1}, s_{t-1})$  that can be reached with positive probability on the equilibrium path, the belief updating functions  $\pi_t^*(\pi_{t-1}, s_{t-1})$  are deduced from strategies  $x_{t-1}^*(\pi_{t-1}), y_{t-1}^*(\pi_{t-1})$  through Bayes Law when  $s_{t-1} \neq AF$ , and  $\pi_t^*(\pi_{t-1}, s_{t-1}) = \bar{\pi}$  when  $s_{t-1} = AF$ .

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<sup>6</sup>In other words,  $\tilde{\pi}_{t+\theta} = \pi_{t+\theta}^*(\tilde{\pi}_{t+\theta-1}, \tilde{s}_{t+\theta-1})$  for  $\theta = 1, \dots, T - t$ , where  $\tilde{s}_{t+\theta-1}$  is distributed in  $S$  with probabilities induced by strategies  $x_{t+\theta-1}^*(\tilde{\pi}_{t+\theta}), y_{t+\theta-1}^*(\tilde{\pi}_{t+\theta})$  when  $\theta = 2, \dots, T - 1$  and by strategies  $x_t, y_t^*(\pi_t)$  when  $\theta = 1$ . Similarly for  $\tilde{\pi}_{t+\theta}$ .

Hereafter, for all  $t = 1, \dots, T$ , the optimized cost and payoff are defined as functions of  $\pi_t$  alone:

$$C_t^*(\pi_t) = C_t(x_t^*(\pi_t), y_t^*(\pi_t), \pi_t) \quad \text{and} \quad \Pi_t^*(\pi_t) = \Pi_t(x_t^*(\pi_t), y_t^*(\pi_t), \pi_t).$$

For simplicity of comparison, we can also define their average discounted counterparts as

$$\bar{C}_t^*(\pi_t) = \frac{1}{\sum_{\theta=0}^{T-t} \delta^\theta} C_t^*(\pi_t) \quad \text{and} \quad \bar{\Pi}_t^*(\pi_t) = \frac{1}{\sum_{\theta=0}^{T-t} \delta^\theta} \Pi_t^*(\pi_t).$$

### 2.3 Instantaneous game

Let us start with the one-period model, i.e.,  $T = 1$  without repeated interaction between insurer and SP. We refer to this case as the *instantaneous game*. Its equilibrium is characterized in Proposition 1.

**Proposition 1.** *An equilibrium of the instantaneous game (i.e., when  $T = 1$ ) is given, for an initial belief  $\pi$  at the beginning of the period, by  $x_1^*(\pi) = \hat{x}(\pi)$ ,  $y_1^*(\pi) = \hat{y}(\pi)$ , where*

$$\begin{aligned} \hat{x}(\pi) &= \hat{x} \quad \text{and} \quad \hat{y}(\pi) = \frac{\hat{\pi}}{\pi} \quad \text{if } \pi > \hat{\pi}, \\ \hat{x}(\pi) &= 0 \quad \text{and} \quad \hat{y}(\pi) = 1 \quad \text{if } \pi \leq \hat{\pi}, \end{aligned}$$

where

$$\hat{\pi} = \frac{(p_0 + p_H)c - p_H I}{(1 - p_0 - p_H)(I - c)} \in (0, 1), \quad (7)$$

$$\hat{x} = \frac{I - \omega}{I} \in (0, 1). \quad (8)$$

Proposition 1 is very intuitive.  $\hat{\pi}$  is the threshold such that, when  $\pi < \hat{\pi}$ , there is not enough incentives for triggering an audit, and a type  $D$  SP always defraud, i.e.,  $x_1^*(\pi) = 0$  and  $y_1^*(\pi) = 1$ . When  $\pi > \hat{\pi}$ , a type  $D$  SP should defraud with probability less than one, for otherwise the insurer would systematically trigger an audit. In that case, the fraud

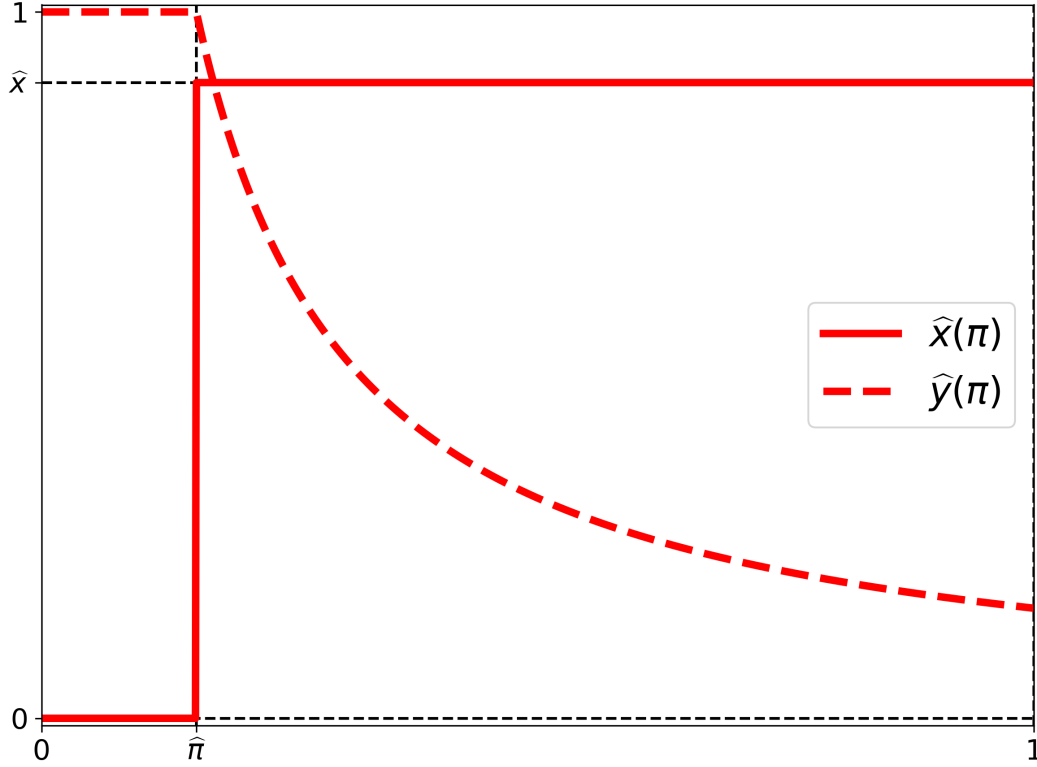


Figure 1: Instantaneous Equilibrium Auditing and Fraud

probability  $y_1^*(\pi) = \hat{y}(\pi) = \hat{\pi}/\pi \in (0, 1)$  makes the insurer indifferent between auditing and not auditing. Symmetrically, the equilibrium audit probability  $x_1^*(\pi) = \hat{x} \in (0, 1)$  makes the type  $D$  SP indifferent defrauding and not defrauding. The equilibrium is thus in mixed strategies when  $\pi > \hat{\pi}$ .

A simple calculation shows that the equilibrium cost of the insurer may be written as

$$\hat{C}(\pi) = C(\hat{x}(\pi), \hat{y}(\pi), \pi) = \begin{cases} IJ(\pi) & \text{if } \pi < \hat{\pi}, \\ p_0 I^2 / (I - c) & \text{if } \pi \geq \hat{\pi}, \end{cases} \quad (9)$$

where

$$J(\pi) = p_0 + p_H + \pi(1 - p_0 - p_H)$$

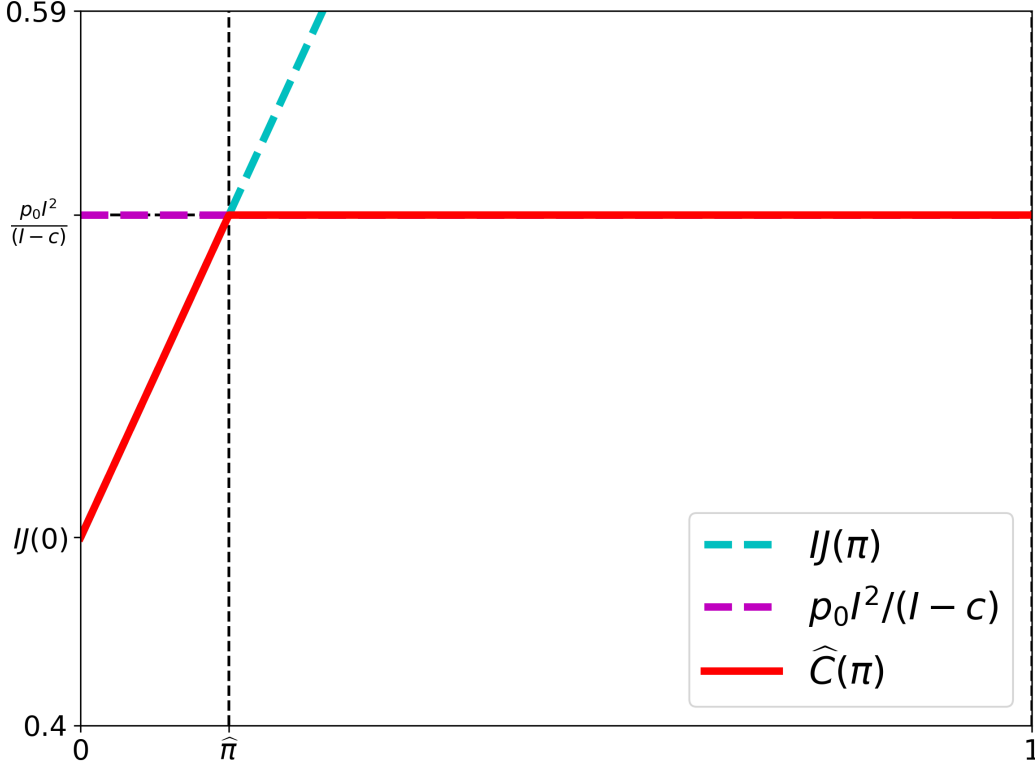


Figure 2: Instantaneous Equilibrium Cost

or equivalently

$$\hat{C}(\pi) = C(\hat{x}(\pi), \hat{y}(\pi), \pi) = \inf\{IJ(\pi), p_0 I^2 / (I - c)\}. \quad (10)$$

Hence, when  $\pi < \hat{\pi}$ , there is no audit and the expected costs increases linearly with the probability of a type  $D$  SP. When  $\pi \geq \hat{\pi}$ , the reputation of the SP is bad enough for auditing with positive probability to be worthwhile, and the equilibrium expected cost reaches an upper bound  $p_0 I^2 / (I - c)$ .

### 3 Reputation-based Deterrence

In the following we are interested in multiperiod settings where  $T \geq 2$ . In Section 3.1, we explain how the insurer refines his beliefs through Bayesian updating and how this may induce players to deviate from the myopic strategies. We also define two types of reputation-based deterrence effects. In Section 3.2, we solve the game for two periods and

show that only one type of reputation-based deterrence occurs, unless fraud can be proved unambiguously. Section 3 shows that, with three periods, both types of reputation-based deterrence take place, regardless of the possibility to unambiguously prove fraud. Finally, Section 3.4 gives sufficient conditions for the first type of reputation-based deterrence effect to happen for more than three periods.

### 3.1 Learning and the inter-temporal threat of an audit

At the end of period  $t$ , whether a claim was submitted or not and the outcome of a potential audit allow the insurer to refine his beliefs about the SP's type.

If a claim was submitted and an audit reveals unambiguously that the claim was fraudulent, the contract between the SP and the insurer is breached. In that case, another contract is offered for period  $t + 1$  to a new SP. Since the insurer has beliefs  $\bar{\pi}$  about the type of this new SP, we have

$$\pi_{t+1} = \bar{\pi} \text{ if } s_t = AF.$$

Using Bayes law yields the updated beliefs for all other scenarios for initial beliefs  $\pi_t$  and strategy  $y_t \equiv y_t(\pi_t)$ . Simple calculations yield

$$\pi_{t+1} = \varphi_{AF}(\pi_t, y_t) = \bar{\pi} \quad \text{when } s_t = AF,$$

$$\pi_{t+1} = \varphi_{AV}(\pi_t, y_t) = \pi_t \quad \text{when } s_t = AV,$$

$$\pi_{t+1} = \varphi_{AI}(\pi_t, y_t) = \frac{\pi_t[p_H + y_t(1 - \alpha)(1 - p_0 - p_H)]}{p_H + \pi_t y_t(1 - \alpha)(1 - p_0 - p_H)} \quad \text{when } s_t = AI,$$

$$\pi_{t+1} = \varphi_{NA}(\pi_t, y_t) = \frac{\pi_t[p_0 + p_H + y_t(1 - p_0 - p_H)]}{p_0 + p_H + \pi_t y_t(1 - p_0 - p_H)} \quad \text{when } s_t = NA,$$

$$\pi_{t+1} = \varphi_{NC}(\pi_t, y_t) = \frac{\pi_t(1 - y_t)}{1 - \pi_t y_t} \quad \text{when } s_t = NC.$$

Functions  $\varphi_{AI}(\pi_t, y_t)$ ,  $\varphi_{NA}(\pi_t, y_t)$  are increasing and concave w.r.t.  $\pi_t$ , and  $\varphi_{NC}(\pi_t, y_t)$  is increasing and convex w.r.t.  $\pi_t$ , with

$$\varphi_{NC}(\pi_t, y_t) < \pi_t < \varphi_{NA}(\pi_t, y_t) < \varphi_{AI}(\pi_t, y_t), \quad (11)$$

for all  $\pi_t \in (0, 1]$ ,  $y_t \in [0, 1]$ , where the last inequality results from assumption (2).

Figure 3 plots the updating functions for low fraud (Figure 3a,  $y_t = 0.4$ ) and high fraud (Figure 3b,  $y_t = 0.8$ ). The higher the fraud probability, the more different the updating functions, and the more information obtained from updating. It also shows how an SP can influence his reputation. By defrauding more, his reputation is damaged more when caught defrauding. In other words, there is a trade-off for the SP between immediate gains from fraud, and long-term gains from preserving his reputation.

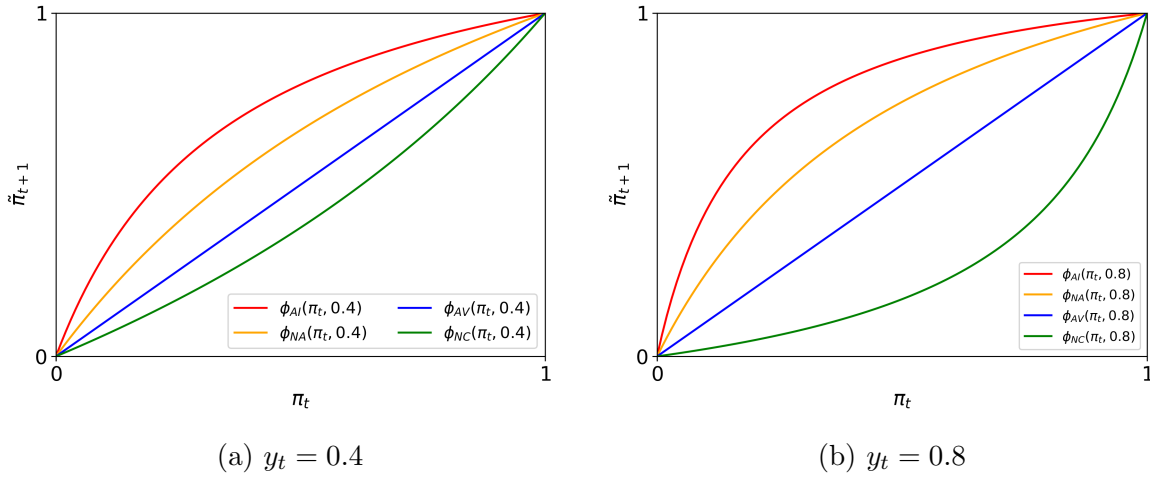


Figure 3: Updating functions for  $(p_0 = 0.2, p_H = 0.1)$

**Definition 2** (Reputation-based fraud deterrence). In a multiperiod setting (i.e.,  $T \geq 2$ ), an equilibrium may exhibit a reputation-based fraud deterrence effect in the extensive and/or in the intensive margins.

An equilibrium at period  $t$  exhibits a reputation-based fraud deterrence effect in the extensive margin when there exists an interval  $(a, b]$ , with  $a < b \leq \hat{\pi}$ , such that  $x_t^*(\pi_t) > 0$  and/or  $y_t^*(\pi_t) < 1$  when  $\pi_t \in (a, b]$ .

An equilibrium at period  $t$  exhibits a reputation-based fraud deterrence effect in the intensive margin when there exists an interval  $[d, e)$ , with  $\hat{\pi} \leq d < e$ , such that



$x_t^*(\pi_t) < \hat{x}$  and/or  $y_t^*(\pi_t) < \hat{y}(\pi_t)$  when  $\pi_t \in [d, e)$ .

In words, an equilibrium displays a reputation-based fraud deterrence effect in the extensive margin when, at period  $t \in \{1, \dots, T - 1\}$ , the insurer audits claims with positive probability and/or the type  $D$  SP defrauds with probability less than one, under beliefs  $\pi_t \in (0, \hat{\pi})$  for which there would be systematic fraud without audit at the equilibrium of the instantaneous game. In this case, auditing increases in the extensive margin as SPs with priors below the instantaneous profitability threshold  $\hat{\pi}$  enter the auditing pool. Alternatively, fraud decreases in the extensive margin as SPs with priors below the instantaneous threshold  $\hat{\pi}$  exit the systematic fraud pool.

As can be expected from the characterization of the instantaneous game equilibrium, when  $\pi_t < \hat{\pi}$ , auditing claims cannot be profitable to the insurer if her calculation is only based on the auditing cost at the same period. However, auditing also allows the insurer to improve her information about the SP's type, which may be useful to her in the next period. The threat of being more precisely identified as a type  $D$  at the beginning of the following period reduces the propensity of a dishonest SP to defraud, hence the possibility of an equilibrium where there is auditing and no-systematic fraud because of this learning process.

An equilibrium displays a reputation-based fraud deterrence effect in the intensive margin when, at period  $t \in \{1, \dots, T - 1\}$ , the insurer audits claims with positive but lower probability than the instantaneous auditing probability, and/or the type  $D$  SP defrauds with a probability lower than the instantaneous defrauding probability. In this case, auditing decreases in the intensive margin as the insurer needs to put in less auditing efforts to make a type  $D$  indifferent. Alternatively, fraud decreases in the intensive margin as a type  $D$  needs to defraud less to make the auditor indifferent.

Since optimal auditing and optimal fraud depend on the reputation  $\pi_t$ , the evolution of beliefs through learning may change the optimal inter-temporal auditing and defrauding strategies. Hereafter, we will primarily focus on how the set of beliefs  $\pi_t$  with a pure equilibrium  $(x^*(\pi_t), y^*(\pi_t)) = (0, 1)$  evolves in time, i.e., on the first type of reputation-based deterrence. In that perspective, let us define the optimal thresholds  $(\pi_t^*)_{t \in \{1, \dots, T\}}$  as

follows.

**Definition 3.** For any  $t \in \{1, \dots, T\}$ , let

$$\pi_t^* = \sup \{ \pi \in [0, 1] \mid \forall \mu \in (0, \pi), (x_t^*(\mu), y_t^*(\mu)) = (0, 1) \} \quad (12)$$

$\pi_t^*$  is the belief threshold under which there is no deterrence, i.e., the insurer does not audit and type  $D$  SPs always defraud. In particular, at time  $T$ ,  $\pi_T^* = \hat{\pi}$ . This is a direct consequence of the fact that at the last period  $T$ , there is no subsequent period through which learning in  $T$  might influence optimal policies, and the equilibrium is the instantaneous one. In addition, the existence of reputation-based deterrence in the extensive margin at time  $t$  is equivalent to having  $\pi_t^* < \hat{\pi}$ .

### 3.2 $T = 2$ : Reputation-based deterrence in the intensive margin only

To establish the possibility of such a learning-based deterrence effect, we will first consider a two-period model, i.e., the case where  $T = 2$ . Period 2 is the last period, and thus the equilibrium of the instantaneous game described in Proposition 1 is also an equilibrium of at period 2. In other words, period 2 equilibrium strategies are

$$\begin{aligned} x_2^*(\pi_2) &= \hat{x}(\pi_2), \\ y_2^*(\pi_2) &= \hat{y}(\pi_2), \end{aligned}$$

where functions  $\hat{x}(\cdot)$  and  $\hat{y}(\cdot)$  correspond to the definition given in Proposition 1.

In addition to the current period myopic cost, the insurer takes into account the future cost that depends on the outcome of the current period, and on the corresponding updating transition. Let  $y_1^*(\pi_1)$  be the period 1 equilibrium fraud and define the period 1 thresholds  $\pi_1^{NA}$  and  $\pi_1^{NC}$  as

$$\varphi_{NA}(\pi_1^{NA}, y_1^*(\pi_1^{NA})) = \hat{\pi} \quad \varphi_{NC}(\pi_1^{NC}, y_1^*(\pi_1^{NC})) = \hat{\pi}.$$

We have

$$\pi_1^{NA} < \hat{\pi} < \pi_1^{NC}.$$

### 3.2.1 Case where unambiguously proving fraud is impossible ( $\alpha = 0$ )

**Proposition 2.** *When  $T = 2$  and  $\alpha = 0$ , the  $t = 1$  equilibrium exhibits no reputation-based fraud deterrence effect in the extensive margin as  $\pi_1^* = \hat{\pi}$ . In addition, for all priors, equilibrium fraud is the instantaneous fraud  $y_1^* = \hat{y}(\pi_1)$ . However, the auditor makes less auditing efforts for intermediate reputations  $\pi_1 \in (\hat{\pi}, \pi_1^{NC})$ , as  $x_1^*(\pi_1) = \hat{x}$  on this interval, with*

$$\hat{x} = \frac{(I - \omega)(1 - \delta(1 - p_0 - p_H))}{I} = (1 - \delta(1 - p_0 - p_H))\hat{x} < \hat{x}.$$

*Thus, we have reputation-based fraud deterrence effects in the intensive margin.*

Figures 4 represent the optimal audit, fraud and average discounted cost as functions of initial belief  $\pi = \pi_1$ . Figure 4a shows that there is no reputation-based deterrence in the extensive margin in the penultimate period as  $\pi_{T-1}^* = \pi_1^* = \hat{\pi}$ . On the opposite, reputation-based fraud deterrence in the intensive margin occurs, as  $[d, e) = [\hat{\pi}, \pi_1^{NC})$ . Indeed, the payoff of a type  $D$  has two components, a current period one, equivalent to the myopic payoff, and a subsequent period one, equal to  $\delta(1 - y_1)(1 - p_0 - p_H)$ . The derivative of the current period component is equal to 0 for  $x_1 = \hat{x}$  from Proposition 1. Thus the total derivative is equal to  $-\delta(1 - p_0 - p_H)$ , independent of  $x_1$ . Consequently, for  $x_1 = \hat{x}$ , type  $D$ 's best response is  $y_1 = 0$ , and the myopic equilibrium cannot be an equilibrium at  $t = 1$  in  $(\hat{\pi}, \pi_1^{NC})$ . Figure 4b illustrates the fact that the optimal fraud level at equilibrium does not change. Figure 4c illustrates the equilibrium optimal average discounted cost and shows the decrease in cost thanks to the learning effect. Intuitively, this decrease occurs for intermediate reputations  $(\pi_1^{NA}, \pi_1^{NC})$  for which uncertainty reduction is the most effective.

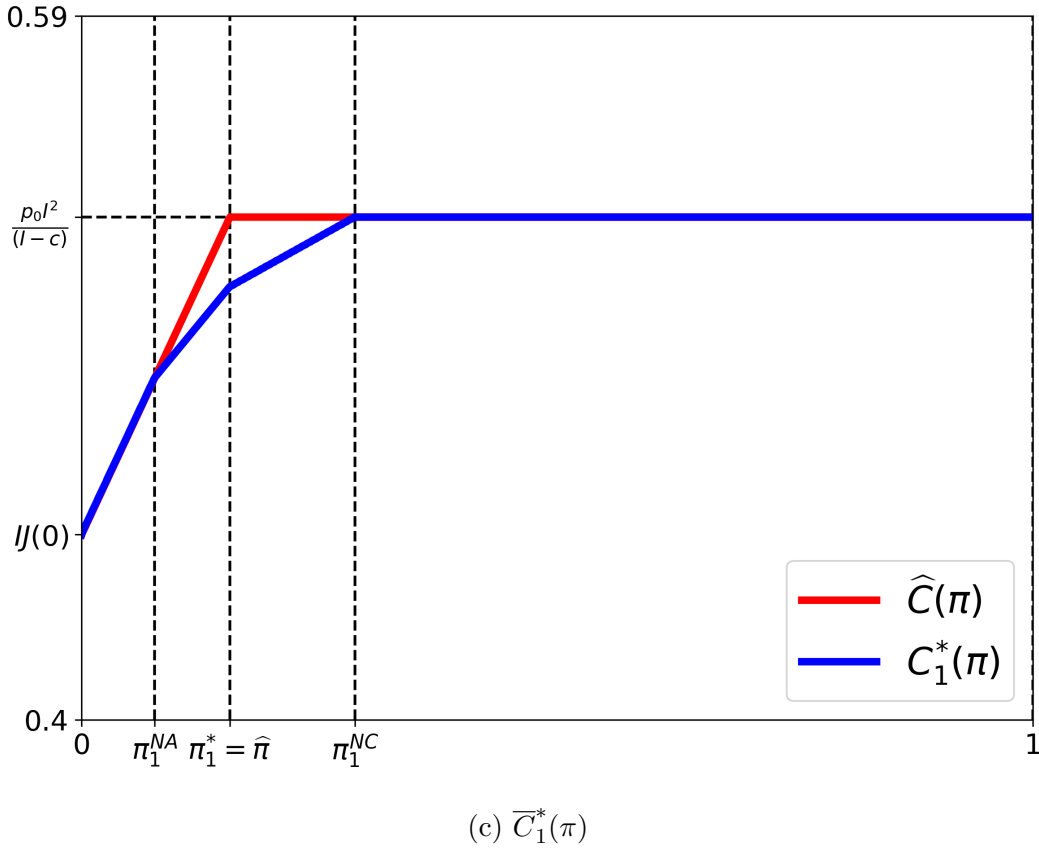
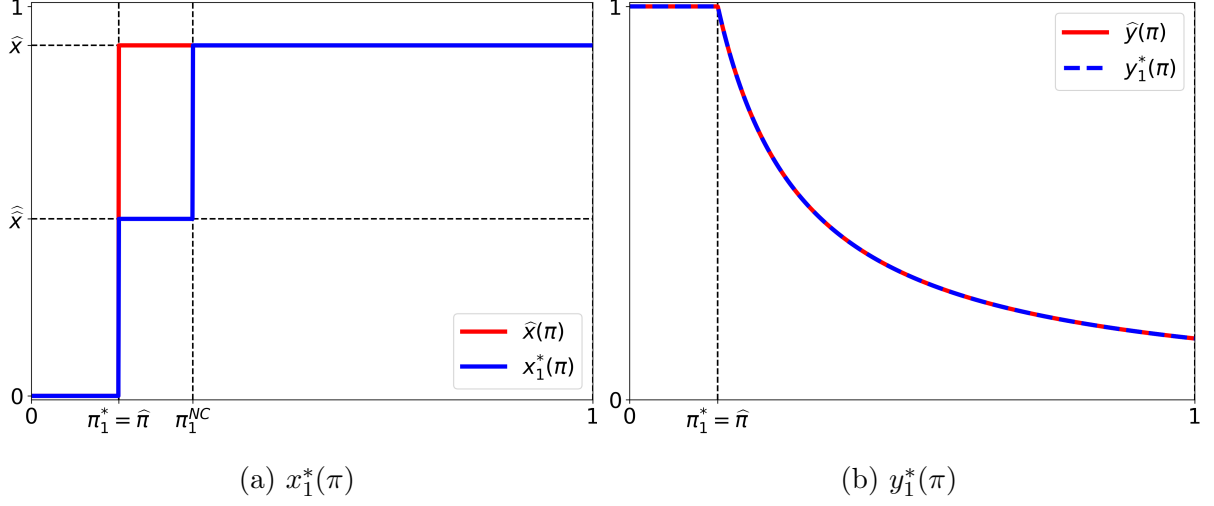


Figure 4: Audit, fraud and average discounted cost at  $(t, T) = (1, 2)$

### 3.2.2 Case where it may be possible to unambiguously prove fraud ( $\alpha > 0$ )

Before considering a  $T = 3$  setting, we focus on the case where fraud can be proven unambiguously, i.e.,  $\alpha > 0$ .

**Proposition 3.** *When  $T=2$ , the equilibrium exhibits a reputation-based fraud deterrence effect in the extensive margin when  $\alpha > 0$  and  $\bar{\pi} < \hat{\pi}$ .*

In other words, the proof of Proposition 3 shows that there is an interval  $(\pi_1^\alpha, \hat{\pi})$  where  $y_1^*(\pi) < 1$  and  $x_1^*(\pi) > 0$ , meaning auditing to trigger deterrence effects is an equilibrium strategy, while there would have been no equilibrium auditing in this same interval in a purely instantaneous setting. This is a very intuitive result: when there is a positive probability to unambiguously detect fraud, the possibility to replace the defrauder by another SP with better reputation provides an additional incentive to audit claims at period 1. This creates a credible threat for type D SPs, and consequently fraud is reduced and auditing occurs with positive probability for some  $\pi_1$  smaller than  $\hat{\pi}$ .

### 3.3 $T = 3$ : Reputation-based deterrence in the extensive and the intensive margins

As for Section 3.2, it will be useful to define some thresholds. First, because of the notation, the first period from the  $T = 2$  case becomes the second one, and the previous thresholds are now denoted  $\pi_2^{NA}$  and  $\pi_2^{NC}$ . Let  $y_1^*(\pi_1)$  be the period 1 equilibrium fraud when  $T = 3$  and define the period 1 thresholds  $\pi_1^{NA}$  and  $\pi_1^{NC}$  as

$$\varphi_{NA}(\pi_1^{NA}, y_1^*(\pi_1^{NA})) = \pi_2^{NA} \quad \varphi_{NC}(\pi_1^{NC}, y_1^*(\pi_1^{NC})) = \pi_1^{NC}.$$

We have

$$\pi_1^{NA} < \pi_2^{NA} < \hat{\pi} < \pi_2^{NC} < \pi_1^{NC}.$$

**Proposition 4.** *Assume  $\bar{\pi} < \hat{\pi}$ , and*

$$p_0 + p_H < \frac{1 - \hat{\pi}}{2 - \hat{\pi}}, \tag{13}$$

$$1 - \sqrt{\frac{c}{I}} < \frac{I - c}{2I - c}. \tag{14}$$

*Then, for  $T = 3$ , the  $t = 1$  equilibrium exhibits a reputation-based fraud deterrence effect in the extensive margin regardless of  $\alpha$  as  $\pi_1^* < \hat{\pi}$ .*

Condition (13) is a sufficient condition for the proposition to hold. It is made to make the proof easier, but it is not necessary. It may be interpreted as an assumption about the importance of fraud: the probability that a type  $D$  is in a position to defraud is  $1 - p_0 - p_H$ , and according to (13), this probability should be larger than  $1/(2 - \hat{\pi})$ . (14) is a condition on  $I$  and  $c$  under which (2), (3) and (13) are compatible. Contrary to Proposition 3, it is valid also when  $\alpha = 0$ .

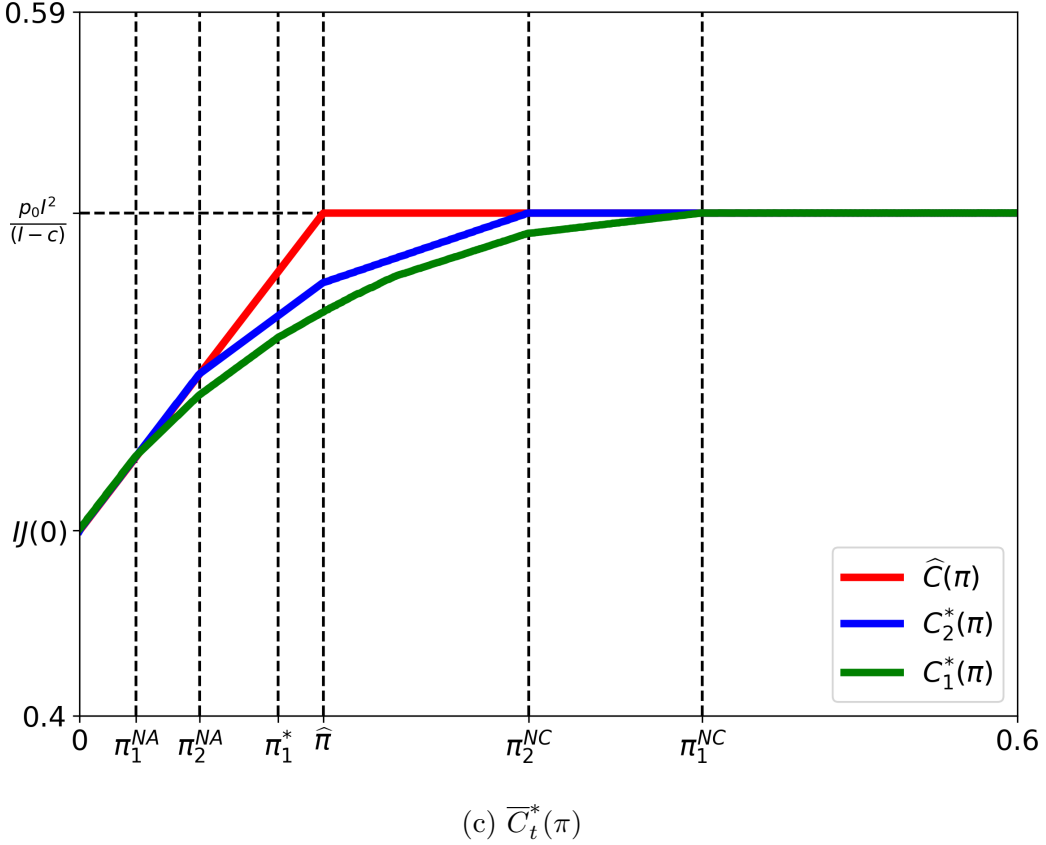
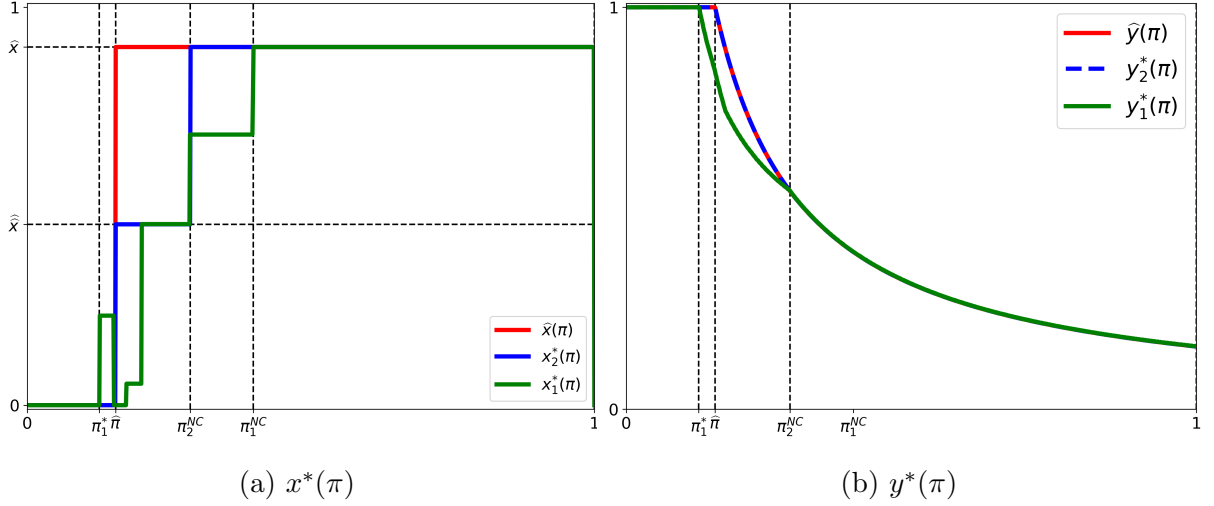


Figure 5: Audit, fraud and average discounted cost at  $(t, T) = (1, 3)$

Figures 5 represent simulation<sup>7</sup> results for the optimal audit, fraud and average discounted cost at the first period when there are three periods in total. Figure 5a shows that there is some reputation-based deterrence in the first period as  $\pi_{T-2}^* = \pi_1^* < \hat{\pi}$ . Figure 5b shows that the optimal fraud level decreases in comparison to the instantaneous benchmark on the interval  $(\pi_1^*, \pi_2^{NC})$  as a result of this reputation-based deterrence effect. Figure 5c illustrates the equilibrium optimal average discounted cost and shows the decrease in cost thanks to the accumulation of learning in time. Notably, the set of beliefs for which a decrease occurs is larger in the first period (green curve) than in the second/penultimate period.

Therefore, both types of reputation-based deterrence effects come into play. First, for priors in the interval  $[\hat{\pi}, \pi_1^{NC})$ , auditing is lower than in the myopic equilibrium. More specifically, in the interval  $[\hat{\pi}, \pi_2^{NC})$ , fraud is also lower than in the myopic equilibrium. Both situations correspond to reputation-based deterrence in the intensive margin. Strikingly, the simulation results indicate that the mixed equilibrium auditing may drop to 0 above  $\hat{\pi}$ . Second, for priors in the interval  $(\pi_{t+1}^*, \hat{\pi}]$ , there is a mixed equilibrium with some fraud and non systematic deterrence. This is a reputation-based deterrence effect in the extensive margin, whereby priors  $(\pi_{t+1}^*, \hat{\pi}]$  are included in the auditing set while they are not in the myopic equilibrium.

The proof relies on a very intuitive approach, where we look, for a prior  $\pi_{T-2}^0 < \hat{\pi}$  and close enough to  $\hat{\pi}$ , at a difference in costs  $\xi(\pi_{T-2}^0)$ . This quantity is the difference between the auditor's cost under myopic strategies  $(x_{T-2}, y_{T-2} = (0, 1))$  and the cost the auditor deviates to systematic auditing  $((x_{T-2}, y_{T-2}) = (1, 1))$ .

$$\xi(\pi_{T-2}^0) = \Delta C + \delta \Delta \mathbb{E} \Psi_{T-1} + \delta^2 \Delta \mathbb{E} \Psi_T, \quad \text{where } \pi_{T-2}^0 \longrightarrow \hat{\pi}^-.$$

The first term is the difference between both strategies in the instantaneous costs, the second the proceeds from  $T-1$  and the last one the proceeds from  $T$ . Figure 6 plots each component to compare them in the vicinity of  $\hat{\pi}$ . The only term that is positive, i.e., for

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<sup>7</sup>For  $(p_0, p_H, c, I) = (0.3, 0.15, 0.44, 1)$ . Assumptions (2),(3),(13) and (14) are compatible for these values.

which a deviation towards systematic auditing is unprofitable, is the current period term. The two other terms are, as shown in the proof, negative. Remembering the definition of  $\hat{\pi}$  in equation (7), this is the prior that equalizes auditing and not auditing instantaneous costs. Therefore,  $\Delta C$  tends to 0 when the prior gets closer to  $\hat{\pi}$ . However this is not sufficient to trigger auditing below  $\hat{\pi}$ :  $\Delta\mathbb{E}\Psi_{T-1}$ , the difference in costs between auditing and not auditing in the intermediary period  $T - 1$  is smaller in absolute value than  $\Delta C$  (see Figure 6b). The  $T - 1$  term, as a term that account for information gains, is negative, but also tends towards 0 when approaching  $\hat{\pi}$ . This is why there is no reputation-based deterrence effect below  $\hat{\pi}$  at  $T = 2$ . Finally, including a third period adds the  $\Delta\mathbb{E}\Psi_T$  term to the balance. This term is negative, but does not tend to 0 at  $\hat{\pi}$  (see green line in Figure 6a).

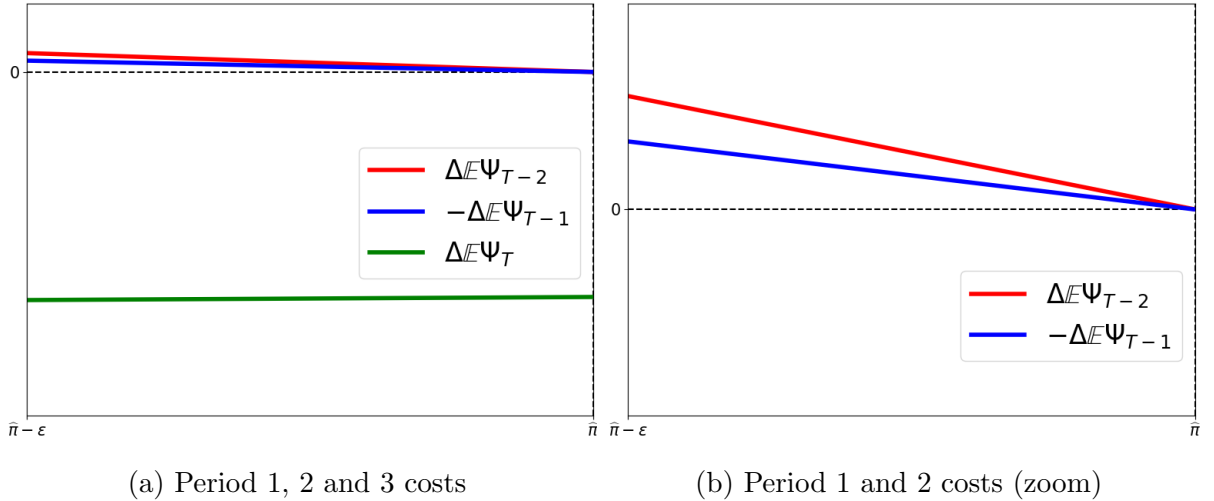


Figure 6: Multi-period costs in the vicinity of  $\hat{\pi}$

Let us again think about the situation under scrutiny through the lens of deviation towards total auditing, i.e.,  $x = 1$ . Figure 7 plots the period 1 average discounted cost under the second period equilibrium actions  $(x_2^*, y_2^*)$ . It also plots the period 1 average discounted cost under a deviation from the previous action consisting in systematically auditing. We see that this deviation becomes profitable above  $\pi_1^*$ , hence the reputation-based deterrence effect. Note also the discontinuity of the blue curve, i.e., under a myopic equilibrium, at the prior  $\hat{\pi}$ . This is because auditing, which means resorting with probability  $x_1^* > 0$  to transitions  $\varphi_{AI}$  and  $\varphi_{AV}$  instead of  $\varphi_{NA}$ , becomes profitable at soon as



$\pi > \pi_1^*$ , but does not come into play until  $\hat{\pi}$ . When it does all of a sudden, the related cost saving makes the cost curve discontinuous.

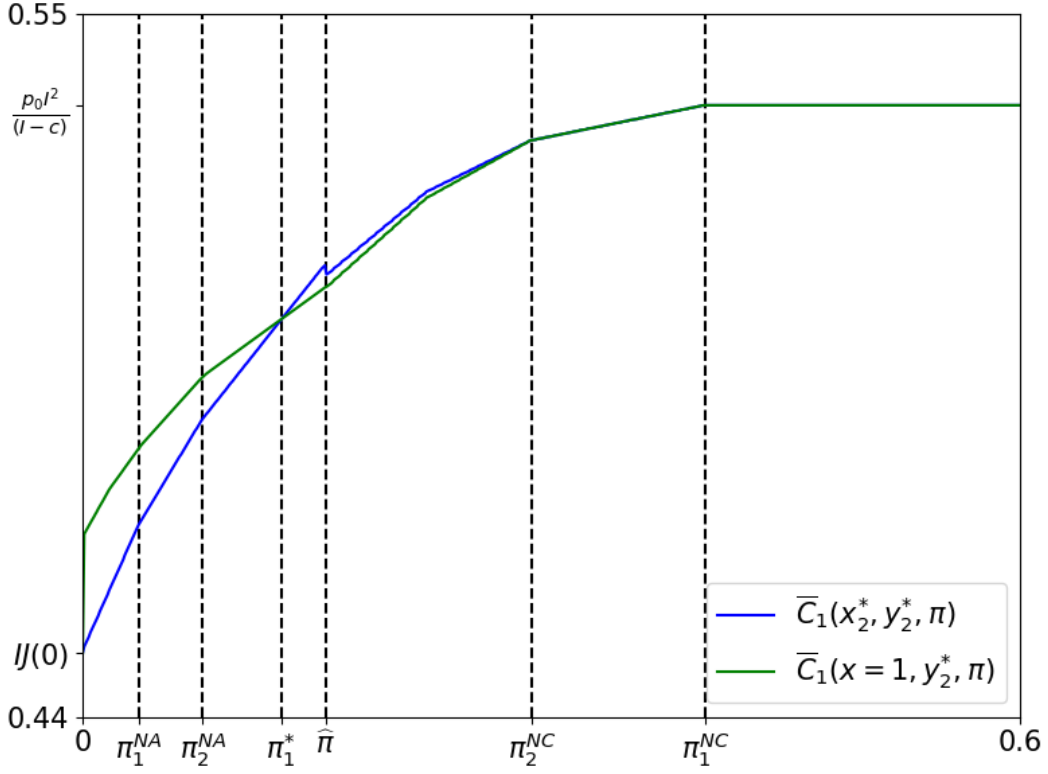


Figure 7:  $\bar{C}_1(x_1, y_1, \pi)$  for  $T = 3$

### 3.4 Multiperiod setting

In a multiperiod setting where  $T \geq 3$ , Proposition 5 shows that if the horizon is far enough and the insurer is patient enough, reputation-based fraud deterrence in the extensive margin takes place.

**Proposition 5.** *For an arbitrary number of periods  $T$ , there is reputation-based deterrence if the remaining relationship is long enough, i.e., for  $t$  such that*

$$(1 - p_0 - p_H) \frac{\delta(1 - \delta^{T-t})}{1 - \delta} > 1,$$

*which is true for  $T - t$  large enough, provided the auditor is patient enough*

$$\delta > \frac{1}{2 - p_0 - p_H}.$$

In words, for  $\delta$  large enough, a type D SP whose reputation is just below the threshold  $\hat{\pi}$  will have an incentive to deviate from the instantaneous equilibrium  $\hat{y}(\hat{\pi} - \epsilon) = 1$  if  $T - t$  is large enough. Intuitively, if the auditor never audits in the neighborhood just below  $\hat{\pi}$  and expects a type D to systematically defraud, then not observing a claim leads him to believe once and for all that the scrutinized auditee is of type H. In this context, by deviating, an auditee of type D will masquerade as a type H for the remaining of the relationship and defraud systematically with no fear of being audited. These future accumulated profits are increasing with the number of remaining periods and the discounting factor, which explains the existence of thresholds in the proposition. In other words, the threat of a future audit deters fraud at the beginning of the relationship.

## 4 Auditing with Deterrence and Learning Effects and the Restless Bandit Problem

In this section, we provide a short description of the relationship between our auditing problem and the restless bandit problem. We leave a formal resolution of the problem for future work. We also set aside the strategic arms aspect.

A fundamental assumption in the regular bandit literature is that the reward associated to each arm only evolves if the arm is chosen. In [Aboutajdine & Picard \(2019\)](#), this was true because not auditing had an updating function equal to the identity function. In this paper, not auditing corresponds to the updating functions  $\varphi_{NA}$  and  $\varphi_{NC}$ , which means that an arm (an SP) not played (audited) still sees the associated belief evolve. This is due to the difference in claim submission between types. A type  $D$  submits valid and mistakenly invalid claims with the same probability as a type  $H$ , but the type  $D$  also voluntarily submits invalid claims, which means he submits more claims on average than the type  $H$ . Therefore, even without auditing, observing a claim indicates that an SP is more likely to be of type  $D$ .

This is an important aspect as we can no longer rely on an optimal stopping formulation of the problem. Even if the belief drops below the threshold  $\pi_t^*$ , auditing does not stop and may resume if the belief goes back above the threshold through non auditing

based updating.

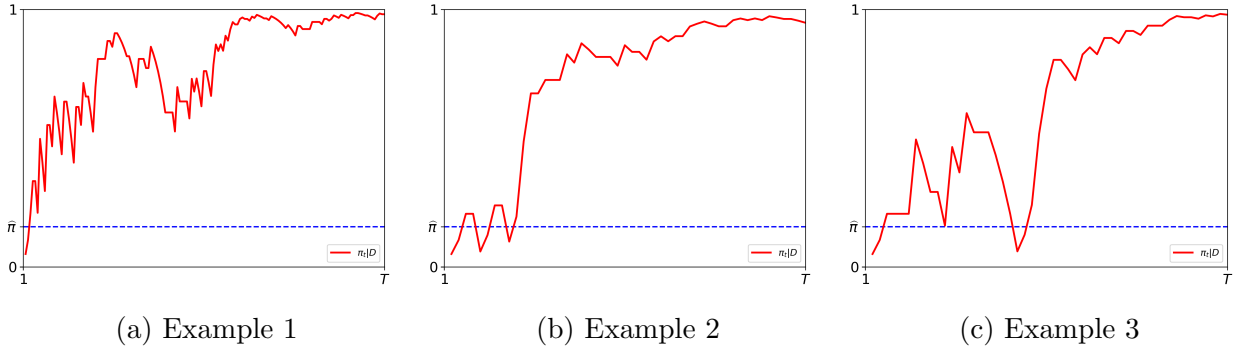


Figure 8: Belief trajectories for a type  $D$  under myopic strategies

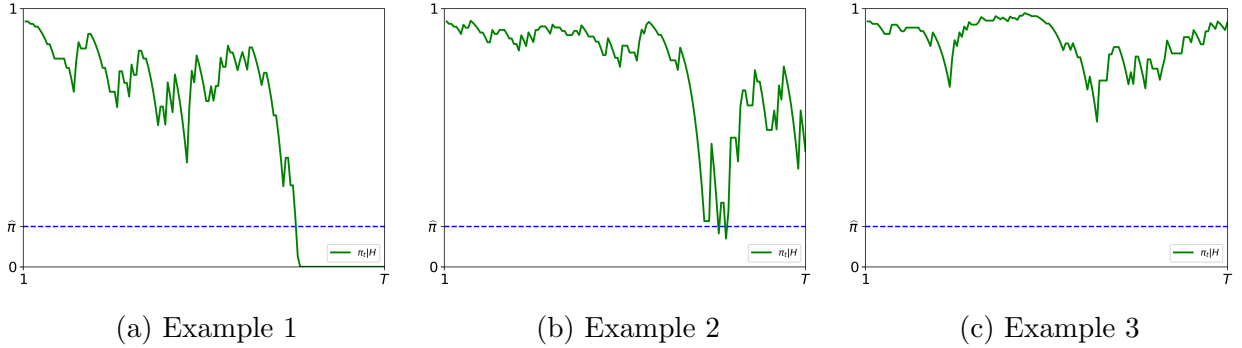


Figure 9: Belief trajectories for a type  $H$  under myopic strategies

Figures 8 and 9 show simulations of the trajectories of beliefs for a type  $D$  and a type  $H$  under myopic strategies at all periods, but with updating. Although not realistic, as an insurer who updates beliefs cannot plausibly use myopic strategies, these examples still illustrate the ideas above. In Figure 8a, the belief converges to the true one, even after starting below the threshold. In addition, beliefs in Figures 8b and 8c do cross the threshold more than once before converging to the true belief. In the case of a type  $H$ , once he reaches the zone where no auditing is conducted below the threshold, it is sufficient for him to not claim once to see his belief become equal to 0, the true belief. However, as shown by the random draw in Figure 9b, if a claim is submitted right after entering the no auditing zone, the SP's belief remains strictly above 0. In addition, these trajectories are obtained under myopic strategies when type  $D$  SPs and the insurer do not adapt their behavior earlier in the relationship. Therefore, there is no guarantee that the true beliefs 0 and 1 will be the only stationary points asymptotically.

Therefore, our dynamic auditing problem with deterrence and learning is related to a complex class of bandit problems where the reward distribution of non-played arms still evolves and arms adapt to the player's strategy.

## 5 Conclusion

This paper has analyzed the insurance fraud problem when the auditor faces strategic auditees with a reputation. The objectives of auditing are threefold: recovering illegitimate claims, deterring fraud and learning about auditees' types, and their effects overlap. The main result of our paper is that the combination of information learning by auditing and strategic behavior of auditees produces a reputation-based deterrence effect as a result of their interaction.

An important but ambitious extension of our analysis would be to approach it as a bandit problem, but, as discussed in the last section of the paper, the problem is particularly complex. One possibility would be to decompose the problem and start with an intermediary setting. Going back to the model of [Aboutajdine & Picard \(2019\)](#), one could relax only the assumption of having both types submit exactly one claim per period, and keep auditees non-strategic. Then, our problem would be a restless bandit problem, without strategic arms.

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# A Proofs

## A.1 Proof of Proposition 1

*Proof.* Under an audit strategy  $x_1(\cdot)$ , the best response  $y_1(\pi_1)$  of a type  $D$  SP under insurer's belief  $\pi_1$  is obtained by maximizing  $\Pi(x_1(\pi_1), y_1, \pi_1)$  w.r.t.  $y_1 \in [0, 1]$ , which gives

$$y_1(\pi_1) = 1 \quad \text{if} \quad (I - \omega)[1 - x_1(\pi_1)] - \omega x_1(\pi_1) > 0, \quad (15)$$

$$y_1(\pi_1) \in [0, 1] \quad \text{if} \quad (I - \omega)[1 - x_1(\pi_1)] - \omega x_1(\pi_1) = 0, \quad (16)$$

$$y_1(\pi_1) = 0 \quad \text{if} \quad (I - \omega)[1 - x_1(\pi_1)] - \omega x_1(\pi_1) < 0. \quad (17)$$

Under a fraud strategy  $y_1(\cdot)$ , the best response  $x_1(\pi_1)$  of the insurer under belief  $\pi_1$  is obtained by minimizing  $C(x_1, y_1(\pi_1), \pi_1)$  w.r.t.  $x_1 \in [0, 1]$ , which gives

$$x_1(\pi_1) = 1$$

$$\text{if } c[p_0 + p_H + \pi_1 y_1(\pi_1)(1 - p_0 - p_H)] - I[p_H + \pi_1 y_1(\pi_1)(1 - p_0 - p_H)] < 0 \quad (18)$$

$$x_1(\pi_1) \in [0, 1]$$

$$\text{if } c[p_0 + p_H + \pi_1 y_1(\pi_1)(1 - p_0 - p_H)] - I[p_H + \pi_1 y_1(\pi_1)(1 - p_0 - p_H)] = 0, \quad (19)$$

$$x_1(\pi_1) = 0$$

$$\text{if } c[p_0 + p_H + \pi_1 y_1(\pi_1)(1 - p_0 - p_H)] - I[p_H + \pi_1 y_1(\pi_1)(1 - p_0 - p_H)] > 0. \quad (20)$$

If  $x_1(\pi_1) = 1$ , then (17) gives  $y_1(\pi_1) = 0$ . Using (3) and (20) yields  $x_1(\pi_1) = 0$ , hence a contradiction. Thus, we have  $x_1(\pi_1) < 1$ . In other words, systematic auditing cannot be an equilibrium outcome.

If  $x_1(\pi_1) \in (0, 1)$ , then (16) gives  $y_1(\pi_1) = \hat{y}(\pi_1)$ , with  $\hat{y}(\pi_1) \in (0, 1)$  if  $\pi_1 > \hat{\pi}$ , and  $\hat{y}(\pi_1) = 1$  if  $\pi_1 \leq \hat{\pi}$ . Using (15) and (16) gives  $x_1(\pi_1) = \hat{x} \in (0, 1)$  when  $\hat{y}(\pi_1) \in (0, 1)$ , i.e., when  $\pi_1 > \hat{\pi}$ , and  $x_1(\pi_1) \leq \hat{x}$  when  $\hat{y}(\pi_1) = 1$ , i.e., when  $\pi_1 = \hat{\pi}$ . Hence, there exists

a mixed equilibrium where no player audits or defrauds with certainty, such that

$$x_1^*(\pi_1) = \hat{x} \in (0, 1) \text{ and } y_1^*(\pi_1) = \hat{y}(\pi_1) \in (0, 1) \text{ if } \pi_1 > \hat{\pi},$$

Finally, if  $x_1(\pi_1) = 0$ , then (15) gives  $y_1(\pi_1) = 1$ , and (20) is satisfied if  $\pi_1 \leq \hat{\pi}$ . Hence there exists an equilibrium such that

$$x_1^*(\pi_1) = 0 \text{ and } y_1^*(\pi_1) = 1 \text{ if } \pi_1 \leq \hat{\pi}.$$

Patching up these cases yields the characterization provided in Proposition 1. □

## A.2 Joint proof of Propositions 2 and 3

*Proof.* Assume  $\bar{\pi} < \hat{\pi}$ . Let  $\pi_{AI}(\pi_1, y_1)$ ,  $\pi_{NA}(\pi_1, y_1)$  and  $\pi_{NC}(\pi_1, y_1)$  defined by

$$\varphi_{AI}(\pi_{AI}, y_1) = \hat{\pi},$$

$$\varphi_{NA}(\pi_{NA}, y_1) = \hat{\pi},$$

$$\varphi_{NC}(\pi_{NC}, y_1) = \hat{\pi},$$

with

$$\pi_{AI}(\pi_1, y_1) < \pi_{NA}(\pi_1, y_1) < \hat{\pi} < \pi_{NC}(\pi_1, y_1),$$

for all  $\pi_1, y_1$ .

Consider period 1 equilibrium strategies  $x_1^*(\pi_1)$  and  $y_1^*(\pi_1)$ . Let us denote  $K^*(\pi_1) \equiv K(\pi_1 y_1^*(\pi_1))$  and

$$\Delta = \frac{p_0 I^2}{(I - c)} - IJ(\bar{\pi}),$$

with  $\Delta > 0$  from  $\bar{\pi} < \hat{\pi}$ .

According to initial beliefs  $\pi_1$ , five possible cases have to be distinguished.



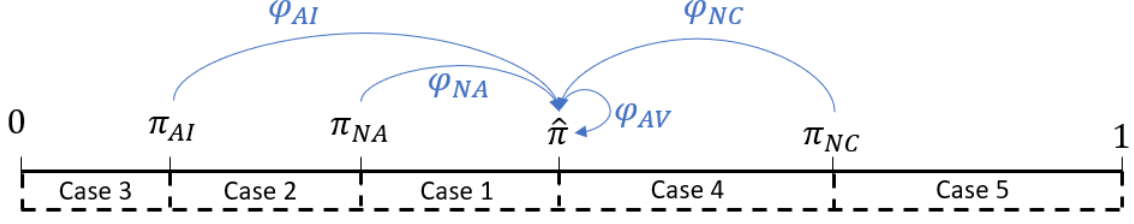


Figure 10: Proof cases

**Case 1** :  $\pi_1$  is such that  $\pi_{NA}(\pi_1, y_1^*(\pi_1)) < \pi_1 < \hat{\pi}$ .

In the case of an audit when a claim is filed, we have

$$\pi_2 = \pi_1 < \hat{\pi} \quad \text{with prob. } \frac{p_0}{p_0 + K^*(\pi_1)}, \quad (21)$$

$$\pi_2 = \bar{\pi} < \hat{\pi} \quad \text{with prob. } \frac{\alpha \pi_1 y_1^*(\pi_1)(1 - p_0 - p_H)}{p_0 + K^*(\pi_1)}, \quad (22)$$

$$\pi_2 = \varphi_{AI}(\pi_1, y_1^*(\pi_1)) > \hat{\pi} \quad \text{with prob. } \frac{K^*(\pi_1) - \alpha \pi_1 y_1^*(\pi_1)(1 - p_0 - p_H)}{p_0 + K^*(\pi_1)}. \quad (23)$$

If there is no audit, we have

$$\pi_2 = \varphi_{NA}(\pi_1, y_1^*(\pi_1)) > \hat{\pi}. \quad (24)$$

The insurer chooses  $x_1$  in  $[0, 1]$  in order to minimize

$$\begin{aligned} & C(x_1, y_1^*(\pi_1), \pi_1) + \delta \mathbb{E}_{\tilde{\pi}_2} [C(x_2^*(\tilde{\pi}_2), y_2^*(\tilde{\pi}_2), \tilde{\pi}_2)] \\ & = C(x_1, y_1^*(\pi_1), \pi_1) + \delta \mathbb{E}_{\tilde{\pi}_2} C(\hat{x}(\tilde{\pi}_2), \hat{y}(\tilde{\pi}_2), \tilde{\pi}_2). \end{aligned}$$

Using (9) and (21)-(24) shows that  $x_1$  minimizes

$$\begin{aligned}
& I + x_1 \left[ c - I \frac{K^*(\pi_1)}{p_0 + K^*(\pi_1)} \right] + \delta x_1 \left[ \frac{p_0}{p_0 + K^*(\pi_1)} IJ(\pi_1) \right. \\
& \left. + \frac{\alpha \pi_1 y_1^*(\pi_1)(1 - p_0 - p_H)}{p_0 + K^*(\pi_1)} IJ(\bar{\pi}) + \frac{K^*(\pi_1) - \alpha \pi_1 y_1^*(\pi_1)(1 - p_0 - p_H)}{p_0 + K^*(\pi_1)} \times \frac{p_0 I^2}{I - c} \right] \\
& + \delta(1 - x_1) \frac{p_0 I^2}{I - c},
\end{aligned}$$

in  $[0, 1]$ . Since the function to be minimized is linear w.r.t.  $x_1$ , we deduce

$$x_1 \begin{cases} = 1 \\ \in [0, 1] \\ = 0 \end{cases} \quad \text{if } [p_0 + K^*(\pi_1)]c - IK^*(\pi_1) + \delta p_0 \left[ J(\pi_1)I - \frac{p_0 I^2}{I - c} \right]$$

$$\begin{aligned}
& < 0 \\
& + \delta \alpha \pi_1 y_1^*(\pi_1)(1 - p_0 - p_H) \left[ IJ(\bar{\pi}) - \frac{p_0 I^2}{I - c} \right] = 0 \quad . \\
& > 0
\end{aligned}$$

This gives

$$x_1 \begin{cases} = 1 & > \\ \in [0, 1] & \text{if } y_1^*(\pi_1) = f(\pi_1) \\ = 0 & < \end{cases} \quad (25)$$

where

$$f(\pi_1) = \frac{k}{\pi_1} [\hat{\pi} - \eta(\hat{\pi} - \pi_1)], \quad (26)$$

and

$$\begin{aligned}
k &= \frac{I - c}{I - c + \delta \alpha \Delta} \in (0, 1], \\
\eta &= \frac{\delta p_0 I}{I - c} \in (0, 1)
\end{aligned}$$

with  $f'(\pi_1) < 0$  and  $f(\hat{\pi}) < 1$  if  $\alpha > 0$  and  $f(\hat{\pi}) = 1$  if  $\alpha = 0$ , because  $k < 1$  when  $\alpha > 0$

and  $k = 1$  when  $\alpha = 0$ . Note also that  $f(\pi_{**}) = 1$  with

$$\pi_{**} = \hat{\pi} \frac{I - c - \delta p_0 I}{I - c - \delta p_0 I + \delta \alpha \Delta} = \hat{\pi} \frac{k(1 - \eta)}{k(1 - \eta) + 1 - k}, \quad (27)$$

and  $\pi_{**} < \hat{\pi}$  if  $\alpha > 0$ , and  $\pi_{**} = \hat{\pi}$  if  $\alpha = 0$ .

Consider now the optimal choice of a type  $D$  SP (in the case where he has the opportunity to certify a fraudulent claim, which occurs with probability  $1 - p_0 - p_H$ ). Let  $\mathbb{E}_F \left[ \Pi(\hat{x}(\tilde{\pi}_2), \hat{y}(\tilde{\pi}_2), \tilde{\pi}_2) \right]$  and  $\mathbb{E}_N \left[ \Pi(\hat{x}(\tilde{\pi}_2), \hat{y}(\tilde{\pi}_2), \tilde{\pi}_2) \right]$  be the period 2 expected fraud profit of a type  $D$  SP in case of fraud or no-fraud at period 1 respectively. Here  $\tilde{\pi}_2$  denotes period 2 beliefs in each case (fraud  $F$  and no fraud  $N$ ), and these beliefs depend on initial beliefs  $\pi_1$ , on the type  $D$  SP strategy  $y_1^*(\pi_1)$ , and on  $s_1$ . For beliefs and equilibrium strategies such that  $\pi_{NA}(\pi_1, y_1^*(\pi_1)) < \pi_1 < \hat{\pi}$ , if the SP defrauds, we have

$$\tilde{\pi}_2 = \begin{cases} \varphi_{AI}(\pi_1, y_1^*(\pi_1)) > \hat{\pi} \text{ in the case of an audit without} \\ \text{unambiguous proof of fraud,} \\ \varphi_{NA}(\pi_1, y_1^*(\pi_1)) > \hat{\pi} \text{ if there is no audit.} \end{cases}$$

In both case, the SP's expected profit at period 2 is zero. This is also the case, if an audit provides an unambiguous proof of fraud. Hence, we have

$$\mathbb{E}_F \left[ \Pi(\hat{x}(\tilde{\pi}_2), \hat{y}(\tilde{\pi}_2), \tilde{\pi}_2) \right] = 0.$$

If the type  $D$  SP does not defraud, no claim is filed and thus, we have

$$\tilde{\pi}_2 = \varphi_{NC}(\pi_1, y_1^*(\pi_1)) < \hat{\pi},$$

which gives

$$\mathbb{E}_N \left[ \Pi(\hat{x}(\tilde{\pi}_2), \hat{y}(\tilde{\pi}_2), \tilde{\pi}_2) \right] = (1 - p_0 - p_H)(I - \omega).$$

Hence, at period 1, the type  $D$  best response is given by maximizing

$$y_1 \left\{ [1 - x_1^*(\pi_1)]I - x_1^*(\pi_1)\omega + \delta \mathbb{E}_F \left[ \Pi(\hat{x}(\tilde{\pi}_2), \hat{y}(\tilde{\pi}_2), \tilde{\pi}_2) \right] - \delta \mathbb{E}_N \left[ \Pi(\hat{x}(\tilde{\pi}_2), \hat{y}(\tilde{\pi}_2), \tilde{\pi}_2) \right] \right\},$$

w.r.t.  $y_1$  in  $[0, 1]$ , which gives

$$y_1 \begin{cases} = 1 & < 0 \\ \in [0, 1] & \text{if } [1 - x_1^*(\pi_1)](I - \omega) - x_1^*(\pi_1)\omega - \delta(1 - p_0 - p_H)(I - \omega) = 0, \\ = 0 & > 0 \end{cases}$$

or equivalently

$$y_1 \begin{cases} = 1 & < \\ \in [0, 1] & \text{if } x_1^*(\pi_1) = x_{**}, \\ = 0 & > \end{cases} \quad (28)$$

with

$$x_{**} = \frac{(1 - \delta(1 - p_0 - p_H))(I - \omega)}{I} \in (0, 1). \quad (29)$$

Suppose  $x_1^*(\pi_1) = 1$ . Then (25) gives  $y_1^*(\pi_1) \geq f(\pi_1) > 0$ , and (39) implies  $x_1^*(\pi_1) \leq x^{**} < 1$ , hence a contradiction. If  $x_1^*(\pi_1) = 0$ , then (25) gives  $y_1^*(\pi_1) \leq f(\pi_1) < 1$ , and (39) gives  $x_1^*(\pi_1) \geq x^{**} > 0$ , which once again is a contradiction. Hence, we have  $x_1^*(\pi_1) \in (0, 1)$ , and (25) implies  $y_1^*(\pi_1) = f(\pi_1)$ , with  $\pi_{**} \leq \pi_1 < \hat{\pi}$ . When  $\pi_{**} < \pi_1 < \hat{\pi}$ , we have  $y_1^*(\pi_1) = f(\pi_1) \in (0, 1)$ , and (39) gives  $x_1^*(\pi_1) = x_{**} \in (0, 1)$ . When  $\pi_1 = \pi_{**}$ , we have  $y_1^*(\pi_1) = f(\pi_1) = 1$ , and (39) gives  $x_1^*(\pi_1) \leq x_{**}$ .

Such an equilibrium exists if  $\varphi_{NA}(\pi_1, y_1^*(\pi_1)) < \pi_1 < \hat{\pi}$  or, equivalently, if  $\varphi_{NA}(\pi_1, y_1^*(\pi_1)) > \hat{\pi}$  and  $\pi_1 < \hat{\pi}$ . This is equivalent to

$$\pi_{NA}^* < \pi_1 < \hat{\pi},$$

where

$$\pi_{NA}^* = \hat{\pi} \left[ 1 - \frac{k(1 - \hat{\pi})(1 - p_0 - p_H)}{p_0 + p_H + k\eta(1 - \hat{\pi})((1 - p_0 - p_H))} \right] < \hat{\pi}.$$

Hence, an equilibrium exists with  $x_1^*(\pi_1) = x_{**} \in (0, 1)$  and  $y_1^*(\pi_1) = f(\pi_1) \in (0, 1)$  if  $\sup\{\pi_{NA}^*, \pi_{**}\} < \pi_1 < \hat{\pi}$ , which requires  $\alpha > 0$ . Everything else given, we have  $\pi_{NA}^* < \pi_{**}$ , when  $k$  is small enough, which corresponds to  $\alpha$  small. If  $\alpha = 0$  and  $x_1^*(\pi_1) > 0$ , we would have  $y_1^*(\pi_1) \geq f(\pi_1) > 0$  if  $\pi_1 < \hat{\pi}$ , hence a contradiction. Thus, we have  $x_1^*(\pi_1) = 0$  and  $y_1^*(\pi_1) = 1$  when  $\alpha = 0$ .

**Case 2** :  $\pi_1$  is such that  $\pi_{AI}(\pi_1, y_1^*(\pi_1)) < \pi_1 < \pi_{NA}(\pi_1, y_1^*(\pi_1))$ .

In the case of an audit when a claim is filed, we have

$$\pi_2 = \pi_1 < \hat{\pi} \text{ with prob. } \frac{p_0}{p_0 + K^*(\pi_1)}, \quad (30)$$

$$\pi_2 = \bar{\pi} < \hat{\pi} \text{ with prob. } \frac{\alpha\pi_1 y_1^*(\pi_1)(1 - p_0 - p_H)}{p_0 + K^*(\pi_1)}, \quad (31)$$

$$\begin{aligned} \pi_2 &= \varphi_{AI}(\pi_1, y_1^*(\pi_1)) > \hat{\pi} \\ &\text{with prob. } \frac{K^*(\pi_1) - \alpha\pi_1 y_1^*(\pi_1)(1 - p_0 - p_H)}{p_0 + K^*(\pi_1)}. \end{aligned} \quad (32)$$

If there is no audit, we have

$$\pi_2 = \varphi_{NA}(\pi_1, y_1^*(\pi_1)) < \hat{\pi}. \quad (33)$$

In this case, the insurer chooses  $x_1$  in  $[0, 1]$  in order to minimize

$$\begin{aligned} &I + x_1 \left[ c - I \frac{K^*(\pi_1)}{p_0 + K^*(\pi_1)} \right] + \delta x_1 \left[ \frac{p_0}{p_0 + K^*(\pi_1)} IJ(\pi_1) \right. \\ &+ \left. \frac{\alpha\pi_1 y_1^*(\pi_1)(1 - p_0 - p_H)}{p_0 + K^*(\pi_1)} IJ(\bar{\pi}) + \frac{K^*(\pi_1) - \alpha\pi_1 y_1^*(\pi_1)(1 - p_0 - p_H)}{p_0 + K^*(\pi_1)} \times \frac{p_0 I^2}{I - c} \right] \\ &\delta(1 - x_1) IJ(\varphi_{NA}(\pi_1, y_1^*(\pi_1))), \end{aligned}$$

in  $[0, 1]$ . We deduce

$$x_1 \begin{cases} = 1 \\ \in [0, 1] \\ = 0 \end{cases} \quad \text{if } [p_0 + K^*(\pi_1)]c - IK^*(\pi_1) + \delta I[p_H(1 - p_0 - p_H)]$$

$$\begin{aligned} &< 0 \\ &-K^*(\pi_1) \frac{I(1 - p_0) - c}{I - c} - \alpha \Delta \pi_1 y_1^*(\pi_1) (1 - p_0 - p_H) = 0 \quad . \\ &> 0 \end{aligned}$$

This gives after simplification

$$x_1 \begin{cases} = 1 & > \\ \in [0, 1] & \text{if } y_1^*(\pi_1) = g(\pi_1) \\ = 0 & < \end{cases} \quad (34)$$

where

$$g(\pi_1) = \frac{k}{\pi_1} \left[ \frac{p_0 \hat{\pi} + \eta p_H (\hat{\pi} - \pi_1)}{p_0 + k\eta(1 - p_0 - p_H)(1 - \hat{\pi})} \right], \quad (35)$$

with  $g'(\pi_1) < 0$ . Note that

$$f(\pi_{NA}^*) = g(\pi_{NA}^*) = \frac{k\hat{\pi}(p_0 + p_H)}{p_0 + p_H - k(1 - \eta)(1 - \hat{\pi})(1 - p_0 - p_H)},$$

which corresponds to the continuity between cases 1 and 2 at  $\pi_1 = \pi_{NA}^*$ . Reasoning as in Case 1 shows that when  $\alpha > 0$  there exists an equilibrium in mixed strategies  $x_1^*(\pi_1) = x_{***} \in (0, 1)$  and  $y_1^*(\pi_1) = g(\pi_1) \in (0, 1)$  if  $\sup\{\pi_{AI}^*, \pi_{***}\} < \pi_1 < \pi_{NA}^*$  and  $\pi_{NA}^* < \pi_{**}$ , where  $\pi_{AI}^*$  and  $x_{***}$  are defined by  $\pi_{AI}(\pi_1, y_1^*(\pi_1)) = \hat{\pi}$ , or equivalently  $\varphi_{AI}(\pi_1, y_1^*(\pi_1)) > \hat{\pi}$ , and by  $g(\pi_{***}) = 1$ , respectively.  $x_{***}$  is the audit probability that makes the type  $D$  SP indifferent between defrauding or not-defrauding.

**Case 3** :  $\pi_1$  is such that  $\pi_1 < \pi_{AI}(\pi_1, y_1)$ .

Straight forward calculation shows that  $x_1^*(\pi_1) = 0$  and  $y_1^*(\pi_1) = 1$  in that case. This case holds when  $\pi_1 < \sup\{\pi_{AI}^*, \pi_{***}\}$ .

**Case 4** :  $\hat{\pi}$  is such that  $\hat{\pi} < \pi_1 < \pi_{NC}(\pi_1, y_1^*(\pi_1))$ .

When  $\alpha = 0$ , in the case of an audit when a claim is filed, we have

$$\pi_2 = \pi_1 > \hat{\pi} \text{ with prob. } \frac{p_0}{p_0 + K^*(\pi_1)}, \quad (36)$$

$$\pi_2 = \varphi_{AI}(\pi_1, y_1^*(\pi_1)) > \hat{\pi} \text{ with prob. } \frac{K^*(\pi_1)}{p_0 + K^*(\pi_1)}. \quad (37)$$

If there is no audit, we have

$$\pi_2 = \varphi_{NA}(\pi_1, y_1^*(\pi_1)) > \hat{\pi}. \quad (38)$$

In this case the type  $D$  chooses  $y_1$  in  $[0, 1]$  in order to maximize

$$y_1 \left\{ [1 - x_1^*(\pi_1)]I - x_1^*(\pi_1)\omega + \delta \mathbb{E}_F \left[ \Pi(\hat{x}(\tilde{\pi}_2), \hat{y}(\tilde{\pi}_2), \tilde{\pi}_2) \right] - \delta \mathbb{E}_N \left[ \Pi(\hat{x}(\tilde{\pi}_2), \hat{y}(\tilde{\pi}_2), \tilde{\pi}_2) \right] \right\},$$

w.r.t.  $y_1$  in  $[0, 1]$ , which gives

$$y_1 \begin{cases} = 1 & < 0 \\ \in [0, 1] & \text{if } [1 - x_1^*(\pi_1)](I - \omega) - x_1^*(\pi_1)\omega - \delta(1 - p_0 - p_H)(I - \omega) = 0, \\ = 0 & > 0 \end{cases}$$

or equivalently

$$y_1 \begin{cases} = 1 & < \\ \in [0, 1] & \text{if } x_1^*(\pi_1) = \hat{x}, \\ = 0 & > \end{cases} \quad (39)$$

with

$$\hat{x} = \frac{(I - \omega)(1 - \delta(1 - p_0 - p_H))}{I} = (1 - \delta(1 - p_0 - p_H))\hat{x} \in (0, 1). \quad (40)$$

Consider now the optimal choice of the insurer. She has to minimize

$$C(x_1, y_1^*(\pi_1), \pi_1) + \delta \mathbb{E}_{\tilde{\pi}_2} [C(x_2^*(\tilde{\pi}_2), y_2^*(\tilde{\pi}_2), \tilde{\pi}_2)] = C(x_1, y_1^*(\pi_1), \pi_1) + \delta \left[ \frac{p_0 I^2}{I - c} \right].$$

The second period cost is independent of  $x_1$ . Indeed, all outcomes  $NA, AV$  and  $AI$  lead

to posteriors larger than  $\hat{\pi}$ . Therefore, the insurer is made indifferent for the myopic fraud level  $\hat{y}(\pi_1) = \frac{\hat{\pi}}{\pi_1} \in (0, 1)$ .

For  $\alpha = 0$ , the equilibrium in Case 4 is then given by

$$x_1^*(\pi_1) = \frac{(I - \omega)(1 - \delta(1 - p_0 - p_H))}{I} < \hat{x} \quad \text{and} \quad y_1(\pi_1) = \frac{\hat{\pi}}{\pi_1}.$$

**Case 5** :  $\hat{\pi}$  is such that  $\pi_1 > \pi_{NC}(\pi_1, y_1^*(\pi_1))$ .

In this case, all priors in period 2 obtained from  $\pi_1$  through updating are larger than  $\hat{\pi}$ . Then the expectation of period 2 cost is known and independent of  $x_1$

$$\mathbb{E}_{\tilde{\pi}_2} [C(x_2^*(\tilde{\pi}_2), y_2^*(\tilde{\pi}_2), \tilde{\pi}_2)] = \frac{p_0 I^2}{I - c}.$$

The type  $D$  has to maximize

$$y_1 \left\{ [1 - x_1^*(\pi_1)]I - x_1^*(\pi_1)\omega + \delta \mathbb{E}_F [\Pi(\hat{x}(\tilde{\pi}_2), \hat{y}(\tilde{\pi}_2), \tilde{\pi}_2)] - \delta \mathbb{E}_N [\Pi(\hat{x}(\tilde{\pi}_2), \hat{y}(\tilde{\pi}_2), \tilde{\pi}_2)] \right\},$$

where the period 2 terms are all equal to zero since all updated beliefs are larger than  $\hat{\pi}$ . Therefore, the period 1 equilibrium audit and fraud are the myopic ones, for beliefs larger than  $\pi_{NC}(\pi_1, y_1^*(\pi_1))$

$$x_1^*(\pi_1) = \hat{x} = \frac{I - \omega}{I} \quad \text{and} \quad y_1^*(\pi_1) = \hat{y}(\pi_1) = \frac{\hat{\pi}}{\pi_1}.$$

□

### A.3 Proof of Proposition 4

*Proof.* Let  $T = 3$ . We already know that the proposition is valid when  $\alpha > 0$ , and thus we may restrict the proof to the case  $\alpha = 0$ . Let

$$\Psi(\pi) = \inf\{IJ(\pi), p_0 I^2 / (I - c)\}.$$



$\Psi(\pi)$  is the expected cost of the insurer at the equilibrium of the instantaneous game, under beliefs  $\pi$ . Hence,

$$C(x_T^*(\pi_T), y_T^*(\pi_T), \pi_T) = \Psi(\pi_T),$$

for all  $\pi_T$  in  $[0, 1]$ , since  $T$  is the last period of the game. Furthermore, when  $\alpha = 0$ , the equilibrium strategies of period  $T - 1$  also coincide with those of the instantaneous game, with

$$C(x_{T-1}^*(\pi_{T-1}), y_{T-1}^*(\pi_{T-1}), \pi_{T-1}) = \Psi(\pi_{T-1}),$$

for all  $\pi_{T-1}$  in  $[0, 1]$ .

Assume

$$x_{T-2}^*(\pi_{T-2}) = \hat{x}(\pi_{T-2}), \quad (41)$$

$$y_{T-2}^*(\pi_{T-2}) = \hat{y}(\pi_{T-2}), \quad (42)$$

for all  $\pi_{T-2} \in [0, 1]$ . Let  $\pi_{T-2}^0 \in (0, \hat{\pi})$ ,  $\pi_{T-2}^0$  close to  $\hat{\pi}$ . (41) and (42) imply

$$x_{T-2}^*(\pi_{T-2}^0) = 0,$$

$$y_{T-2}^*(\pi_{T-2}^0) = 1.$$

Consider a deviation of the insurer's strategy from  $x_{T-2}^*(\pi_{T-2}^0) = 0$  to  $x_{T-2}(\pi_{T-2}^0) = 1$ . Let  $\Psi_{T-1}^N$  and  $\Psi_{T-1}^A$  denote the insurer's expected cost at period  $T - 1$  for beliefs  $\pi_{T-2}^0$ , if she does not deviate (**N**o audit) and if she deviates (**A**udit) at period  $T - 2$ .

*Consider first the case where the insurer does not deviate from her equilibrium strategy*  $x_{T-2}^*(\pi_{T-2}^0) = 0$ . We have

$$\pi_{T-1} = 0 \text{ with prob. } (1 - \pi_{T-2}^0)(1 - p_0 - p_H), \quad (43)$$

$$\pi_{T-1} = \varphi_{NA}(\pi_{T-2}^0, 1) > \hat{\pi} \text{ with prob. } 1 - (1 - \pi_{T-2}^0)(1 - p_0 - p_H), \quad (44)$$

and thus

$$\begin{aligned}\mathbb{E}\Psi_{T-1}^N &= I(p_0 + p_H)(1 - \pi_{T-2}^0)(1 - p_0 - p_H) \\ &\quad + \frac{p_0 I^2}{I - c} [1 - (1 - \pi_{T-2}^0)(1 - p_0 - p_H)],\end{aligned}$$

where the expected value of the insurer's cost is conditional on period  $T - 2$  belief  $\pi_{T-2}^0$ .

When the insurer deviates from her equilibrium strategy to  $x_{T-2}(\pi_{T-2}^0) = 1$ , we have

$$\pi_{T-1} = 0 \text{ with prob. } (1 - \pi_{T-2}^0)(1 - p_0 - p_H), \quad (45)$$

$$\pi_{T-1} = \pi_{T-2}^0 \text{ with prob. } p_0, \quad (46)$$

$$\pi_{T-1} = \varphi_{AI}(\pi_{T-2}^0, 1) > \hat{\pi} \text{ with prob. } 1 - (1 - \pi_{T-2}^0)(1 - p_0 - p_H) - p_0, \quad (47)$$

and thus

$$\begin{aligned}\mathbb{E}\Psi_{T-1}^A &= I(p_0 + p_H)(1 - \pi_{T-2}^0)(1 - p_0 - p_H) \\ &\quad + I p_0 [p_0 + p_H + \pi_{T-2}^0](1 - p_0 - p_H) \\ &\quad + \frac{p_0 I^2}{I - c} [1 - (1 - \pi_{T-2}^0)(1 - p_0 - p_H) - p_0].\end{aligned}$$

We deduce

$$\Delta \mathbb{E}\Psi_{T-1} \equiv \mathbb{E}\Psi_{T-1}^A - \mathbb{E}\Psi_{T-1}^N = -p_0 I (1 - p_0 - p_H) (\hat{\pi} - \pi_{T-2}^0). \quad (48)$$

Consider now the strategies played at period  $T$  following either an audit or no audit at period  $T - 2$ , and denote  $\Psi_T^A$  and  $\Psi_T^N$  the corresponding expected costs.

*Suppose first that no audit has been performed at period  $T - 2$ . Consider the two cases that may occur.*

If  $\pi_{T-1} = 0$ , we have  $\pi_T = 0$ , and thus

$$\mathbb{E}[\Psi_T^N(\pi_T) \mid \pi_{T-1} = 0] = I(p_0 + p_H). \quad (49)$$

If  $\pi_{T-1} = \varphi_{NA}(\pi_{T-2}^0, 1)$ , we have

$$\pi_T \geq \varphi_{s_{T-1}}(\varphi_{NA}(\pi_{T-2}^0, 1), \widehat{y}(\varphi_{NA}(\pi_{T-2}^0, 1))), \quad (50)$$

in all scenarios  $s_{T-1}$  that may occur at period  $T - 1$ . Note that

$$\varphi_{NA}(\widehat{\pi}, 1) = \frac{\widehat{\pi}}{p_0 + p_H + \widehat{\pi}(1 - p_0 - p_H)}$$

and thus

$$\begin{aligned} \varphi_{s_{T-1}}(\varphi_{NA}(\widehat{\pi}, 1), \widehat{y}(\varphi_{NA}(\widehat{\pi}, 1))) &= \varphi_{s_{T-1}}\left(\varphi_{NA}(\widehat{\pi}, 1), \frac{\widehat{\pi}}{\varphi_{NA}(\widehat{\pi}, 1)}\right) \\ &= \widehat{\pi} \frac{1 - p_0 - p_H}{\widehat{\pi} + (1 - \widehat{\pi})(p_0 + p_H)} \\ &> \widehat{\pi}, \end{aligned}$$

where the inequality results from (13). Consequently, we have

$$\varphi_{s_{T-1}}(\varphi_{NA}(\pi_{T-2}^0, 1), \widehat{y}(\varphi_{NA}(\pi_{T-2}^0, 1))) > \widehat{\pi}, \quad (51)$$

when  $\pi_{T-2}^0$  is close to  $\widehat{\pi}$ . Hence, (44) and (45) show that  $\pi_T > \widehat{\pi}$  when  $\pi_{T-2}^0$  is close to  $\widehat{\pi}$ , in all possible scenarios following  $\pi_{T-1} = \varphi_{NA}(\pi_{T-2}^0, 1)$ .

Consequently, we have

$$\mathbb{E}[\Psi_T^N(\pi_T) \mid \pi_{T-1} = \varphi_{NA}(\pi_{T-2}^0, 1)] = \frac{p_0 I^2}{I - c}, \quad (52)$$

and using (43),(44),(49) and (52) gives

$$\begin{aligned} \mathbb{E}\Psi_T^N &= (1 - \pi_{T-2}^0)(1 - p_0 - p_H)(p_0 + p_H)I \\ &\quad + [1 - (1 - \pi_{T-2}^0)(1 - p_0 - p_H)] \frac{p_0 I^2}{I - c}, \end{aligned}$$

where the expected value of the insurer's cost at period  $T$  is conditional on beliefs  $\pi_{T-2}^0$ .

*Suppose now that an audit has been performed at period  $T - 2$ . Consider the three*

cases that may occur.

If  $\pi_{T-1} = 0$ , as in the previous case we have  $\pi_T = 0$ , and

$$\mathbb{E}[\Psi_T^N(\pi_T) \mid \pi_{T-1} = 0] = I(p_0 + p_H). \quad (53)$$

If  $\pi_{T-1} = \pi_{T-2}^0$ , then using  $x_{T-1}^*(\pi_{T-2}^0) = \hat{x}(\pi_{T-2}^0) = 0$  and  $y_{T-1}^*(\pi_{T-2}^0) = \hat{y}(\pi_{T-2}^0) = 1$  gives

$$\begin{aligned} \pi_T &= 0 \text{ with prob. } (1 - \pi_{T-2}^0)(1 - p_0 - p_H), \\ \pi_T &= \varphi_{NA}(\pi_{T-2}^0, 1) > \hat{\pi} \text{ with prob. } 1 - (1 - \pi_{T-2}^0)(1 - p_0 - p_H). \end{aligned}$$

If  $\pi_{T-1} = \varphi_{AI}(\pi_{T-2}^0, 1)$ , we have  $\Delta$

$$\pi_T \geq \varphi_{st}(\varphi_{AI}(\pi_{T-2}^0, 1), \hat{y}(\varphi_{AI}(\pi_{T-2}^0, 1))), \quad (54)$$

in all scenarios  $s_{T-1}$  that may occur at period  $T - 1$ . Since  $\varphi_{AI}(\pi_{T-2}^0, 1) > \varphi_{NA}(\pi_{T-2}^0, 1)$ , using (13) yields  $\pi_T > \hat{\pi}$  in all scenarios that may occur at period  $T - 1$ , following  $\pi_{T-1} = \varphi_{AI}(\pi_{T-2}^0, 1)$ . Hence, we have

$$\begin{aligned} \mathbb{E}\Psi_T^A &= (1 - \pi_{T-2}^0)(1 - p_0 - p_H)(p_0 + p_H)I \\ &\quad + p_0 \left[ (1 - \pi_{T-2}^0)(1 - p_0 - p_H)(p_0 + p_H)I \right. \\ &\quad \left. + [1 - (1 - \pi_{T-2}^0)(1 - p_0 - p_H)] \frac{p_0 I^2}{I - c} \right] \\ &\quad + [1 - (1 - \pi_{T-2}^0)(1 - p_0 - p_H)] \frac{p_0 I^2}{I - c}. \end{aligned}$$

We obtain

$$\begin{aligned} \Delta \mathbb{E}\Psi_T &\equiv \mathbb{E}\Psi_T^A - \mathbb{E}\Psi_T^N = p_0(1 - \pi_{T-2}^0)(1 - p_0 - p_H) \\ &\quad \times \left[ (p_0 + p_H)I - \frac{p_0 I^2}{I - c} \right] \\ &= -p_0 I \hat{\pi} (1 - p_0 - p_H)(1 - \pi_{T-2}^0). \end{aligned} \quad (55)$$

Finally, a deviation from  $x_{T-2}^*(\pi_{T-2}^0) = 0$  to  $x_{T-2}(\pi_{T-2}^0) = 1$  induces a change  $\Delta C$  in insurance cost at period  $T - 2$  defined by

$$\begin{aligned}\Delta C &= C(1, 1, \pi_{T-2}^0) - C(0, 1, \pi_{T-2}^0) \\ &= cp_0 - (I - c)[p_h + (1 - p_0 - p_H)\pi_{T-2}^0],\end{aligned}$$

where  $cp_0$  is the expected audit cost for valid claims and  $(I - c)[p_h + (1 - p_0 - p_H)\pi_{T-2}^0]$  is the expected value of recouped indemnities, net of audit cost, for detected invalid claims.

Let

$$\xi(\pi_{T-2}^0) = \Delta C + \delta\Delta\mathbb{E}\Psi_{T-1} + \delta^2\Delta\mathbb{E}\Psi_T,$$

the discounted expected variation in the insurer's cost after the deviation. We have  $\Delta C \rightarrow 0$  and  $\Delta\mathbb{E}\Psi_{T-1} \rightarrow 0$  when  $\pi_{T-2}^0 \rightarrow \hat{\pi}$ , and thus  $\xi(\pi_{T-2}^0) \rightarrow -\delta^2 p_0 I \hat{\pi} (1 - p_0 - p_H)(1 - \hat{\pi}) < 0$  when  $\pi_{T-2}^0 \rightarrow \hat{\pi}$ , which shows that the deviation is profitable to the insurer when  $\pi_{T-2}^0$  is close to  $\hat{\pi}$ .  $\square$

## A.4 Proof of Proposition 5

Assume there exists  $t \geq 1$  such that  $y_{t,T}^*(\pi_t) = 1$  for  $\pi_t \in [\hat{\pi} - \epsilon, \hat{\pi}] \equiv \mathcal{A}$  where  $\epsilon > 0$ . Assume an SP of type  $D$  with reputation  $\pi_t$  has no legitimate claim to channel and either defrauds with certainty ( $y_{t,T} = 1$ ) or does nothing ( $y_{t,T} = 0$ ). If the SP deviates and  $y_{t,T} = 0$ , then, by Bayes law,  $\pi_{t+1} = 0$  and he will be able to defraud in all subsequent periods without being audited (since  $x_{t,T}^* = 0$  at any time  $t$ ). Denoting  $\mathbb{E}_F[V(\tilde{\pi}_{t+1})|\pi_t]$  and  $\mathbb{E}_N[V(\tilde{\pi}_{t+1})|\pi_t]$  the time  $t$  discounted expected gain over periods  $t, t + 1, \dots, T$  of a type  $D$  with reputation  $\pi_t$  who respectively defrauds (F,  $y_{t,T} = 1$ ) or does not defraud (N,  $y_{t,T} = 0$ ), for  $\pi_t \in [\hat{\pi} - \epsilon, \hat{\pi}]$ , we then have

$$\begin{aligned}
\mathbb{E}_N[V(\tilde{\pi}_{t+1})|\pi_t] &= (I - \omega)(1 - p_0 - p_H) \sum_{i=1}^{T-t} \delta^i \\
&= (I - \omega)(1 - p_0 - p_H) \frac{\delta(1 - \delta^{T-t})}{1 - \delta},
\end{aligned} \tag{56}$$

If the type  $D$  SP chooses to defraud with  $y_{t,T} = 1$ , then  $\pi_{t+1} = \varphi_{NA}(\pi_t, 1)$  with probability  $(1 - x_{t,T}^*(\pi_{t,T}))$  and  $\pi_{t+1} = \varphi_{AI}(\pi_t, 1)$  with probability  $(x_{t,T}^*(\pi_{t,T}))$ . Therefore  $\pi_{t+1} > \hat{\pi}$  in both cases for  $\epsilon$  small enough. Then, at period  $t + 1$ , the equilibrium is a mixed one and the SP of type  $D$  is indifferent between defrauding or not. So the optimal expected gain at time  $t + 1$  is obtained by looking at total fraud  $y_{t,T} = 1$ . In particular, if he defrauds with certainty when given the chance at all subsequent periods, his priors  $\pi_{t+i}$  will always increase and stay above  $\hat{\pi}$ . Therefore, the SP no longer has any expected gain starting from period  $t + 1$  if he defrauds with certainty at period  $t$ . Then

$$\begin{aligned}
\mathbb{E}_N[V(\tilde{\pi}_{t+1})|\pi_{t,T}] &= (I - \omega)(1 - x_{t,T}^*(\pi_{t,T})) - \omega x_{t,T}^*(\pi_t) \\
&\leq (I - \omega),
\end{aligned} \tag{57}$$

Equations (56) and (57) imply that a sufficient condition for

$$\mathbb{E}_N[V(\tilde{\pi}_{t+1})|\pi_t] > \mathbb{E}_F[V(\tilde{\pi}_{t+1})|\pi_t],$$

is given by

$$(1 - p_0 - p_H) \frac{\delta(1 - \delta^{T-t})}{1 - \delta} > 1,$$

which is true for  $T - t$  large enough if

$$\delta > \frac{1}{2 - p_0 - p_H}.$$