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Turning dilate from point tool into part of an instrument: an example of a preservice mathematics teacher working on a dynamic geometry system

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The aim of this paper is to investigate a preservice mathematics teacher's instrumental genesis of 'dilate from point' tool in a dynamic geometry system. Data is collected through a series of task-based interviews including seven specific construction tasks regarding the use of dilate from point tool with a Turkish preservice mathematics teacher. Video-recorded interviews and screen recorder software productions were triangulated and analysed within the theoretical lens of instrumental genesis. According to the findings, the participant firstly explored the properties of the dilation transformation and its relation to other geometrical concepts and transformations. Secondly, the participant developed instrumented schemes related to the dilate from point tool for applying appropriate strategies in the construction tasks.

Keywords: Preservice mathematics teachers, Dynamic geometry system, Construction tasks, Instrumental genesis.

Introduction

Geometric similarity is regarded as a crucial concept in most school geometry curriculum. Likewise, in Turkish secondary school mathematics (MoNE, 2018a; MoNE, 2018b) this concept has its place focusing on making connections between measurement of angles and side lengths of two polygons through reasoning about the concepts of ratio and proportion. Considering the similarity related terms and concepts, the importance of dilation and scale factor has been emphasised in the related literature (e.g. Cox, 2013) since the deep understanding of these concepts do not only enable learners to solve similarity problems but also help them to involve in ratio-related problems. However, these concepts do not take place in the Turkish mathematics curriculums as a geometrical transformation. Instead, the focus is on investigating similar polygons in different sizes by focusing on the ratio between side lengths through paper folding tasks or drawing activities on a plotting paper (MoNE, 2018a). Also, tasks based on the basic proportionality theorem (MoNE, 2018b) are addressed without mentioning the concepts of scale factor or centre of dilation. Starting from this point, we focus on teachers learning of dilation as a geometrical transformation. This is particularly crucial in teacher education as it is essential for preservice mathematics teachers to have a wide and deep understanding of geometric similarity before their actual teaching in classrooms.

Focusing on these concepts, dynamic geometry systems (DGSs) in particular measurement and dragging facilities are considered as having a critical role to support learners' reasoning processes on the ratio and proportional relationships between two figures (Denton, 2017). In this regard, we focus on a preservice mathematics teacher learning dilation and scale factor concepts in a DGS as Simsek and Clark-Wilson (2018) in their review stated that there is a need to know about how dynamic technology tools shape teachers knowledge on geometric similarity related concepts. Considering the geometrical transformation tools in a DGS, it has become apparent that dilate from point tool is an unfamiliar tool for Turkish preservice mathematics teachers in its practical use as well as in its mathematical content. Along this direction, we focus on encouraging a preservice mathematics teacher to explore dilate from

point tool as a geometrical transformation tool in which she first had a difficulty in making the mathematical meaning of the tool.

Theoretical Framework

Instrumental genesis offering a model to describe the process of learning to use technological tools is chosen as a theoretical perspective in this study (Artigue, 2002). This is derived from the instrumental approach issuing from cognitive ergonomics (Vérillon & Rabardel, 1995) pointing out the difference between the terms ‘artefact’ and ‘instrument’ and the fact that using a tool is a dialectic between the user and the instrument. In this approach, the process through which an artefact becomes part of an instrument is called instrumental genesis ‘involving the construction of personal schemes or, more generally, the appropriation of social pre-existing schemes’ (Artigue, 2002, p. 250). The instrument is used in a psychological sense, which is developed by the user through mental schemes in order to use it for specific tasks. Instrumental genesis consists of two interrelated processes *instrumentalisation* and *instrumentation*. The former is directed towards the artefact and concerns the way in which the artefact is shaped by the user. The latter is directed towards the user and concerns ‘the emergence and evolution of schemes of a subject for the execution of a given task’ (Guin & Trouche, 2002, p. 205).

For the aim of this study, both instrumentalisation and instrumentation processes are of crucial importance. For the instrumentalisation process, we will focus on a stage of discovery of dilate from point tool and a process of differentiation of this tool itself in which instrumentation of the other DGS tools can emerge (Guin & Trouche, 2002). For the instrumentation process, we will focus on instrumented schemes for working on dilation and scale factor related tasks in a DGS. Hence, the research question addressed in this study is: How does instrumental genesis process of a preservice mathematics teacher occur in utilisation of dilate from point tool while solving enlargement/reduction and scale factor related construction tasks in a DGS?

Research Context and Methods

This paper is part of an ongoing research project that is designed as a case study focusing on instrumental genesis of preservice mathematics teachers while they commence GeoGebra. For the sake of page constraints, this particular study reports the case of one preservice mathematics teacher, Selin’s (pseudonym) appropriation of dilate from point tool in GeoGebra while working on construction tasks. She enrolled in a secondary mathematics teacher education program at a state university in Turkey. She took several courses (e.g. abstract and discrete mathematics, calculus, geometry and linear algebra) before the interviews were completed. She was selected mainly because she had no previous experience with GeoGebra in particular while solving construction tasks. Also, we considered her performance on geometry course since she achieved A from geometry for the grades F (fail) to A (excellent). In addition, her communication skills especially in terms of expressing her thinking clearly has been taken into account.

Data was collected from three task-based interviews, during which the participant talked/explained her solutions while working on a mathematical task. These interviews were recorded through a video-camera filming the participant’s working environment, which included a laptop with GeoGebra installed in front of her. Screen recorder software was also used to capture the techniques she employed during the tasks in detail. All the collected data were triangulated and analysed through thematic analysis techniques. Codes and themes were assigned by researchers according to the participant’s solutions for each task, and the tools

utilised by the participant and her reasoning behind it with reference to the used techniques during her solutions.

The Tasks

For exploring the process of instrumental genesis of dilate from point tool, we prepared seven different construction tasks. Since the participant initially had no insight into how dilate from point tool worked, the first task consisted of the analysis of this tool. The interviewer showed Selin the icon of dilate from point tool in GeoGebra and the task was proposed as: explore this tool and explain its mathematical feature. Then she was asked three different construction tasks focusing on enlargement/reduction concepts without the use of this tool:

- Dilate a circle by scale factors of 2 and 3 (without using dilate from point tool),
- Dilate a triangle by scale factors of 2 and 3 (without using dilate from point and reflect object in point tools),
- Dilate the given QRS triangle from the centre point T by the scale factor of $-1/3$ (without using dilate from point tool).

By this way the participant was encouraged to think about possible ways to obtain dilation transformation by using other GeoGebra tools. These three tasks were proposed in order for her to become familiar with the use of dilate from point tool and elaborate its mathematical meaning, hence the focus was on the instrumentalisation process. Finally in order to explore the participants' instrumented schemes for this specific tool, three more tasks were asked:

- Construct the centre of gravity point in a given triangle (without using line construction tools),
- Divide a line segment into two parts by a certain ratio (without using parallel line construction tool),
- Construct a trapezoid with a given triangle (without using parallel line construction tool)

During these tasks, the participant was asked to develop strategies without the use of some tools in order to encourage her to think indirect ways of constructing a figure which aimed to give deeper insight into the extent of the participant's knowledge.

Findings

Initial analysis of the 'dilate from point' tool

Task 1: Explore the dilate from point tool and explain its mathematical feature

Selin started her analysis by selecting the tool, respectively clicking on point *A* (as object) and point *B* (as centre of dilation), and entered the number 5 as a scale factor. As a result, she assumed that the distance between the two points (*A* and *B*) on the screen would be five (the image of the point *A* was not seen on the screen). After making use of distance or length tool, she found the distance *AB* 3.1 (Figure 1) and refuted her initial conjecture. She then said: "I would like to say that there is a certain proportion between points but I don't see a third point so I am confused".



Figure 1

Selin applied the dilate from point tool again for C as object and D as centre of dilation. Then she used the zoom out tool to look for a third point, to her, which was possible to emerge on the screen as the image of C (Figure 2). When she found the point C' (image of the original point) on the screen, she said: ‘aha, now it is OK’.

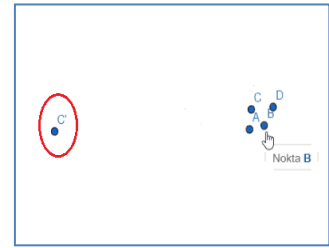


Figure 2

She then chose a triangle (as object) and a point (as centre of dilation), and entered 5 as a scale factor.

She stated that the tool constructed a bigger triangle which was constructed by the ratio that was entered as 5.

She also worked on the two triangles and explained the similarity between them based on the basic proportionality theorem (Figure 3).

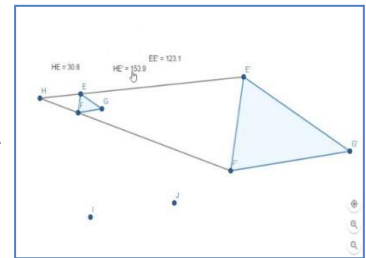


Figure 3

Selin then applied the dilate from point tool on various other figures (a triangle and a circle) and examined the relationships between the original figures and their images. She suggested that the scale factor affected each point of the original figures and made an assumption that the original shapes and their images were similar.

In order to support her assumption, she used the distance or length and area tools and justified it with proportional relational facts. Based on this, she explained the similarity of figures and ratios.

In the second step of the analysis process, she examined the simple fractions and negative numbers as scale factors. After examining the new shapes that appear as a result of her operations, she stated that for fractions the smaller versions of the original shape occurred as its images, whereas for negative numbers, the similar versions of the original shape occurred at the opposite side of the centre of dilation.

Elaboration of the ‘dilation’ by the use of other types of transformations

Task 2: Dilate a circle by scale factors 2 and 3 respectively without the use of dilate from point tool.

During this task, she, by following the steps below:

- drew a circle with the centre P through point Q and chose a point called R as centre of dilation,
- reflected the point R in the point Q and also in the point P by using *reflect object in point* tool,
- obtained two new points R' and R_1' ,
- constructed the new circle with these points by using *circle with centre through point* tool, which was a dilation image of the original circle by scale factor 2 (Figure 4),
- reflected the point Q in the point R' and reflected the point P in the point R_1' for the dilation image by scale factor 3 (Figure 4).

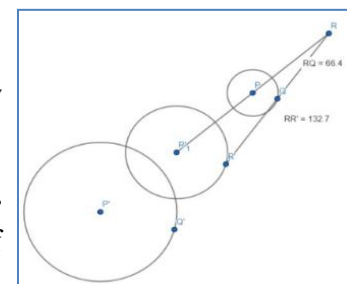


Figure 4

The reasoning behind this instrumented action was notions of extending a segment by projecting it by its own length.

Task 3: Dilate a given triangle by scale factors 2 and 3 respectively without the use of dilate from point or reflect object in point tools.

At this step, Selin focused on the use of translate by vector tool. She, by following the steps below:

- drew a vector from the centre of dilation V to one of the vertices of the given triangle,
- translated the triangle by the vector (Figure 5).

After this operation, she noticed that while there was a ratio of 2 between V and one vertex of the constructed image, this ratio was not preserved for the other two vertices of the image and she concluded: “I now translated the whole triangle by the same vector”. Then, she, by changing her operations as follows:

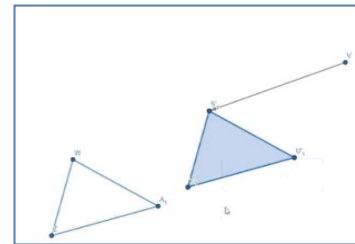


Figure 5

- translated each vertices of the original triangle by vector (similar with the use of ‘reflect object in point’ in the previous instrumented action),
- drew three different vectors from the centre of dilation V to the three vertices of the given triangle and translated each vertex by the corresponding vector,
- obtained three vertices of the dilated image of the original triangle by scale factor 2 (Figure 6),
- did the same operation for scale factor 3.

The reasoning was again related to extending a segment by projecting it by its own length.

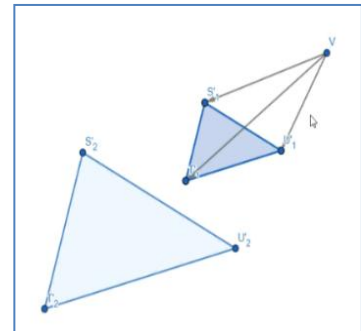


Figure 6

Task 4: Dilate the given QRS triangle with a centre of dilation T by a scale factor of $-1/3$ without using the dilate from point tool.

Selin initially said: “the image should be on the right side of the dilation centre and needed to be smaller than the object. How can I do that? For instance, the image of point Q needs to be somewhere on the line QT (collinear) and its distance from the centre should be one third of the line QT . So then if I divide QT line segment into three parts and reflect the point in the dilation centre I think that I could obtain its image by $-1/3$ ”.

In this sense, her initial aim was to divide line segment QT into three equal parts. At this point she, by following the steps below:

- drew two lines (QT and RQ),
- constructed three equal line segments on the line RQ through constructing three new points called R' , Q' and R'' by using the reflect object in point tool (e.g. reflected R in Q , then reflected Q in R' , and reflected R' in Q'),
- drew the line segment TR'' ,
- constructed parallel lines passing other two points (R' and Q') to the line segment TR'' ,
- constructed three equal line segments on the line segment QT based on the basic proportionality theorem (Figure 7),
- specified the point U on the line segment QT , which was the intersection point of the parallel line passing Q' and QT (Figure 8),

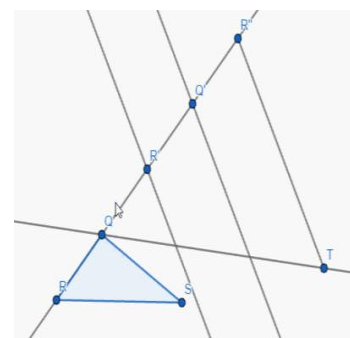


Figure 7

- applied the same division method on the other two lines (RT and ST),
- constructed the vertices of a new triangle (UZW) which was the image of QRS triangle by a scale factor of $1/3$,
- reflected the points U , Z , and W in the centre of dilation T and constructed $U'Z'W'$ triangle, which was the reduction image of QRS triangle by a scale factor of $-1/3$ (Figure 8).

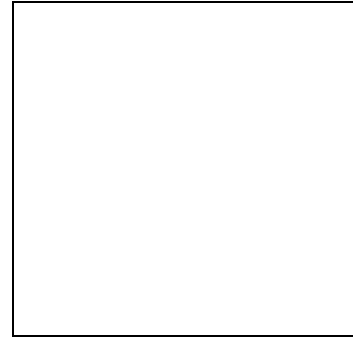


Figure 8

In this task, she developed an instrumented scheme in which she related the reduction with division of line segments and negative scale factors with reflect object in point.

Construction of the centre of gravity point in a given triangle

Task 5: Construct the centre of gravity point in a triangle without the use of line tools.

In the solution process, she, by following the steps below:

- firstly constructed the midpoints (D_1 , E_1 , F_1) of the sides of a triangle ($A_1B_1C_1$) with the use of ‘midpoint or centre’ tool (Figure 9),
- and then reasoned on the ratio between the parts of median that separated by the gravity point.

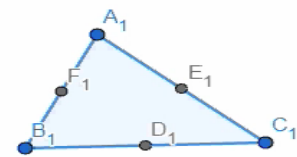


Figure 9

In order to obtain the ratio between the parts as $1/3$, she decided to use dilate from point tool and thought about what value should be entered in the scale factor window by considering the procedural features of the tool. As a result, she chose B_1 as object and E_1 as the centre of dilation. She said: “since the image should be on the left side of centre of dilation, the scale factor should be positive and since the ratio is $1/3$, the scale factor should be $1/3$ ”. Therefore, she put the value $1/3$ as a scale factor and completed the task.

Division of a line segment into two parts by a certain ratio

Task 6: Selin was given two different line segments (called RS and UV). The line segment RS was divided into two parts from the point T (Figure 10). The task was to divide the line segment UV into two parts with the same ratio of $|RT|/|TS|$ without the use of parallel lines.

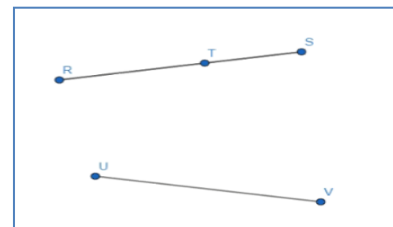


Figure 10

She started considering the ratio concept and the use of dilate from point tool. She, by following the steps below:

- constructed a line segment F_1G_1 which was equal length to the ratio of $|RS| / |TS|$ with the use of segment with given length tool (Figure 11),
- used dilate from point tool by choosing U as object, V as centre of dilation and the distance F_1G_1 as a scale factor.

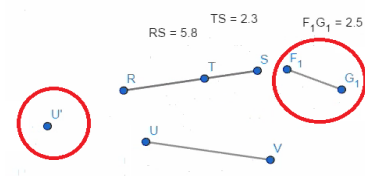


Figure 11

However, she noticed that the point, which she wanted to construct, appeared outside of the UV (Figure 11).

Then she said: “the image should be between object and the centre of dilation therefore I need to use a fraction as a scale factor”. Finally she changed the scale factor as $1/\text{distance}F_1G_1$ and repeated the operation again and finished her construction. She concluded that she could also use $\text{distance}TS/\text{distance}RS$ as a scale factor which would have done the same operation.

Construction of a trapezoid with a given triangle as part of the trapezoid

Selin was firstly asked to examine the properties of the given quadrilateral in Figure 12 ($M_1N_1L_1K_1$) focusing on its variants and invariants through dragging. At this step, she stated that the given quadrilateral was a trapezoid for the reason that two opposed sides were always parallel within the quadrilateral. After her explanations, she was ensured to remember the diagonal based definition of trapezoid.

Task 7: A triangle ($P_1Q_1R_1$) was given to her and she was asked to reason on how a trapezoid can be constructed if the given triangle was formed by the diagonals of the trapezoid (Figure 12) without the use of parallel line tool.

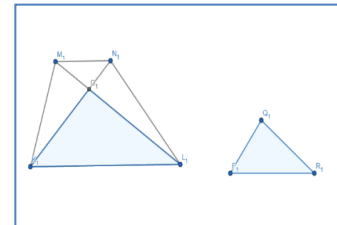


Figure 12

Selin focused on using dilate from point tool by considering the fact that the diagonals of a trapezoid divide each other into two parts by the same ratio. Then, she, by following the steps below:

- extended one of edges of the triangle by using the line tool and marked a random point on this line as third vertex of the trapezoid (point S_1) (Figure 13),
- examined the ratio of the distances of the opposed vertices (S_1 and R_1) from the intersection point of the diagonals Q_1 by using the length measurement tool (Figure 13),
- selected the dilate from point tool and chose Q_1 as object, P_1 as the centre of dilation, and distance S_1R_1 /distance Q_1R_1 as a scale factor,
- constructed U_1 as the fourth vertex of the trapezoid (Figure 14) and completed the task by using the segment tool.

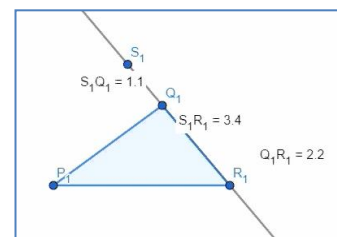


Figure 13

As a second solution to the same task, she dilated the given triangle ($P_1Q_1R_1$) as a whole, by choosing the given triangle as object, the intersection point of the diagonals Q_1 as a centre of dilation and $-$ distance S_1Q_1 /distance Q_1R_1 as a scale factor.

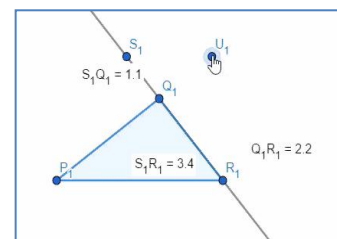


Figure 14

Discussion and Conclusion

This study focuses on instrumental genesis of a preservice mathematics teacher's use of dilate from point tool working on enlargement/reduction and scale factor related construction tasks in GeoGebra. For the initial analysis and elaboration of the tool, the participant discovered and understood the use of dilate from point tool particularly by engaging in thinking of the use of other tools to perform dilation. After that, she developed a number of instrumented schemes regarding the dilate from point tool while working on construction tasks related to proportional relationships.

During the initial analysis and elaboration of the tool, on the one hand, she differentiated the properties of the dilation from other geometric transformations (e.g enlarging the given triangle by the use of translate by vector). On the other hand, she made connections between the properties of dilation and other transformations, which enabled her to develop a deeper understanding of dilation concept. For instance, in the example of dilating the given triangle by a scale factor of $-1/3$, it was seen that she mainly engaged in geometric thinking and used other concepts/transformations (e.g. division of line segments for reduction, the use of reflect object in point for negative scale factors) to complete the task. By this way, she extended and supported her understanding of the dilate from point tool regarding its underlying mathematical meaning and its practical use.

Considering the instrumentation process of the dilate from point tool, it was observed that she developed different schemes for positive/negative scale factors as well as for integer/fractional scale factors. For the former, she referred to the direction of the image from the centre of dilation (i.e. when the value of scale factor is negative, the dilation takes place in the opposite direction from the center of dilation). For the latter, she considered the size of the image whether it would be bigger or smaller than the original object and also the proportional relationships between the distances of the points of object and its image. In addition, she developed schemes related to the concept of dilation as a geometrical transformation in which centre point and scale factor are considered as parameters. These schemes about dilation and scale factors in a DGS indicated also some connections between algebraic and geometric strategies of the participant (Cox, 2013).

To conclude, this study focused on the learning process of dilation transformation in a DGS in the context of the instrumental genesis of the dilate from point tool. The results of this research indicated that the instrumental genesis process for a DGS tool provided scaffolding in the construction and understanding of the dilation and scale factor related concepts. However, this study examined one specific preservice mathematics teacher's solution processes of different construction tasks, which needs to be borne in mind.

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