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The graph of a function and its antiderivative: a praxeological analysis in the context of Mechanics of Solids for engineering

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The notion of integral is usually first introduced to engineering students in their calculus courses before later being used in their professional engineering courses. In this paper we analyse the textbooks used as main references in two engineering courses: a calculus course and a mechanics of solids course. Specifically, we compare how each textbook presents the task of sketching the graph of an antiderivative in the context of kinematics. Our results indicate that the mechanics of solids course presents this task by emphasising the notion of area and basic geometric calculations, using different notation and rationale than in the calculus course. We discuss the possible impact of these differences on the training of engineers.

Keywords: Mathematics for engineers, teaching and learning of calculus, textbook analysis, anthropological theory of the didactic, antiderivative.

Introduction

The mathematical education of engineering students is a topic of increasing concern for mathematics education researchers, university mathematics teachers and professional associations (Bingolbali, Monaghan, & Roper, 2007, p. 764). In particular, calculus is considered an important component of basic engineering curriculum, providing notions, skills, and competences that are deemed necessary for the following professional courses. Nonetheless, the difficulties that calculus courses pose for students are not restricted to engineering programs (Rasmussen, Marrongelle, & Borba, 2014), and they may become a factor leading to the abandonment of STEM degrees (Ellis, Kelton, & Rasmussen, 2014).

In the case of engineering, there is still a lack of understanding about how calculus notions are used in professional engineering courses; and by better understanding this use, some changes could be made to the content of calculus programmes. For instance, our previous research (González-Martín & Hernandes Gomes, 2017, 2018) indicates that the notions of bending moment and first moment of an area (used in civil engineering) are used in tasks that are not present in calculus courses, despite these notions being defined as integrals. Nor does their use require techniques that are explicitly derived from practices introduced in calculus courses. In the case of the notion of bending moment (González-Martín & Hernandes Gomes, 2017), some professional engineering textbook tasks require students to sketch the graph of the antiderivative of a given function. For this reason, we are currently interested in the connections between functions and antiderivatives, as well as their graphic interpretation, in professional engineering courses.

Some difficulties that students encounter in interpreting the graph of a function and the graph of its derivative are well known. For instance, Borgen and Manu (2002) identified that some students may be able to perform the necessary calculations to find the stationary points of a function, but that the

same students would not see these points as a part of the graph. In addition, Ubuz (2007) identified several difficulties students encounter in sketching the graph of the derivative of a function by looking at the graph of this function. In the same vein, some studies point out students' difficulties relating the graphs of a function and its antiderivative. Swidan and Yerushalmy (2014) found that some students try to guess the shape of the antiderivative function based on the position of the function graph, instead of correlating the y -value of the function graph with the tangent slope value of the antiderivative function graph. Finally, Marrongelle (2004) investigated how undergraduate students in an integrated calculus and physics curriculum used physics to help them solve calculus problems. The circumstantial evidence exposed in this study "supports the view that students come to understand graphs, as well as other mathematical representations, by recalling or imagining physical events" (p. 271).

Because we are interested in the training of engineering students and how they use mathematics in their professional courses, we examine whether the ability to visualise and interpret graphs of antiderivatives is required in professional engineering courses (besides the notion of bending moment) and how this relates to practices taught in calculus courses. Our research program's first step is to analyse textbooks used in these courses. In the next sections, we present the theoretical tools we used in our study, as well as our methods and main results.

Theoretical framework

To analyse the use of visualisation and the production of graphs in professional engineering textbooks, we use tools from the anthropological theory of the didactic (ATD – Chevallard, 1999). ATD considers human activities, as well as the production of knowledge, as institutionally situated; this means that knowledge about these activities and why they are important (or why we need to learn them) is also institutionally situated (Castela, 2016, p. 420).

One key element of ATD, essential in our analyses, is the notion of praxeology (in the case of the study of mathematical activity, mathematical organisation or mathematical praxeology – MO hereinafter). A praxeology $[T/\tau/\theta/\Theta]$ is formed by four elements: a type of task T to perform, a technique τ which allows the task to be completed, a rationale (technology) θ that explains and justifies the technique, and a theory Θ that includes the rationale. These elements are grouped in the practical block $[T/\tau]$ (or know-how), and the knowledge block $[\theta/\Theta]$ which describes, explains and justifies what is done. To describe mathematical knowledge (including its production, its use, and its learning), these two blocks permit an analysis of what needs to be done, how it is done, and the justifications for this. ATD distinguishes different types of MO: punctual, which are associated with a specific type of task; local, which integrate multiple punctual MOs that can be explained using the same technological rationale; and regional, which integrate local MOs that accept the same theoretical rationale (Barbé, Bosch, Espinoza, & Gascón, 2005).

Knowledge (and also praxeologies) can be used in institutions other than where it was created, which implies transpositional effects on the concerned praxeologies (Chevallard, 1999), causing some (or all) elements of the original praxeology to evolve. Although these changes may have little impact on experts, they may make it difficult for students to recognise "the same" knowledge used in another institution. Therefore, it is important to analyse the types of tasks and techniques as well

as the rationales employed (for instance, in different courses) to identify the challenges these transpositional effects pose for students. To that end, our research identifies specific local MOs present in professional courses; we analyse how calculus notions are used (practical block) and the explanations given (knowledge block) in relation to the way the notions are usually presented in calculus courses.

Methodology

As we have noted, the study presented in this paper continues our previous work examining how calculus notions are used in professional engineering courses. Our latest work focuses on future engineers' use of integrals (first, to define shear forces and bending moments, and second, to define the first moment of an area – see, respectively, González-Martín & Hernandez Gomes, 2017, 2018), which is why we are pursuing the current study along these lines. As in all our studies concerning future engineers' use of calculus notions, we worked with teachers of professional engineering courses (all of them holding engineering degrees, some of them active engineers), who guided us in identifying key notions of engineering that are based on calculus notions and helped us grasp the important elements of these notions. Our current study focuses on two courses at a Brazilian university (although the textbooks we analysed are distributed internationally): Calculus (I and II), and Mechanics of Solids for Engineers. In the latter course, students need to work with functions, interpreting their graph and constructing their antiderivative graph to solve problems related to motion. At this stage of our research, we chose to work with the reference texts for each course. Guided by our collaborating engineering teachers, we examined the following books: *Calculus*, by Stewart (2012) and *Vector Mechanics for Engineers: Statics and Dynamics* (Beer, Johnston, Mazurek, & Cornwell, 2013), an international book used in the discipline of mechanics of solids. We consider the courses to be two different institutions, since practices vary greatly between them.

At this university, calculus is taught in the first year of the engineering program over two semesters, in two courses: Calculus I and Calculus II. Integrals appear towards the end of the first course and are the main topic in the second course (the second author of this paper has taught Calculus I for 15 years and Calculus II at this same university for two years). Both courses follow the structure of Stewart (2012) fairly closely, but it is worth mentioning that the Brazilian version of this textbook is divided in two volumes: volume 1 (Calculus I - chapters 1 to 6 of the international edition) and volume 2 (Calculus II - chapters 7 to 10 of the international edition). Mechanics of Solids is taught during the third semester of the program (second year), and, as stated above, the main reference for this course is Beer et al. (2013). We identified the pages in Stewart (2012) covering the introduction of integrals, and identified the pages in Beer et al. (2013) where content related to kinematics of particles in one dimension is taught. Within these pages, we then pinpointed tasks related to sketching the graph of an antiderivative and the proposed technique(s), paying attention to the use of given definitions and properties. We also identified each task's rationales (technologies) and whether they are implicit or explicit. Therefore, for each book, the identification of common rationales allowed us to recognize local MOs, which enabled us to propose an overall organisation for the content related to sketching the graphs of antiderivatives for each book. The next section provides specific details of our analysis.

Data analysis and discussion

Organisation of Stewart (2012)

The content concerning integrals in volume 1 of the Brazilian version of Stewart (2012) is distributed among chapter 5 (integrals), 6 (applications of integration), 7 (techniques of integration), and 8 (further applications of integration). In these chapters, content is basically structured using two local MOs. The first, MO_{M1} , presents notions and results justified by the definition of integral, the use of limits, and some theorems. It informally introduces Riemann sums to define definite integrals, interpreting them as areas, and leads to the Fundamental Theorem of Calculus (FTC) and the calculation of definite integrals using Barrow's rule; this then leads to some applications of the integral (area, volume, etc.). The tasks involve the use of the sigma notation, calculating integrals using series, calculating areas, proving some properties, and so on. The second, MO_{M2} , although it requires knowledge of the notion of integral, makes use of many algebraic properties of functions. It introduces techniques for calculating indefinite integrals (immediate integration to begin with, followed by various integration techniques). Many of the techniques used in MO_{M2} are derived from MO_{M1} .

The word antiderivative appears in the book for the first time in chapter 4 to introduce its definition: "A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I ." (Stewart, 2012, p. 344). The connections between the graph of a function and its antiderivative are explained only in chapter 5, as a part of MO_{M1} , among other tasks of this MO. Section 5.3 presents the FTC, which is followed by an application to sketch the graph of the antiderivative of a function ($g(x) = \int_0^x f(t)dt$) knowing the graph of $f(x)$. The technique consists of estimating the area under the curve up to certain points (based on the technology that an integral represents an area). For instance, for the graph of Figure 1a, the value of $g(1)$ corresponds to the area of a triangle; the value of $g(2)$ corresponds to the area of the triangle and the rectangle; for $g(4)$ we can estimate that: $g(4) = g(3) + \int_3^4 f(t)dt \approx 4.3 + (-1.3) = 3.0$ (Figure 1b). Students are given five exercises to practice this technique. A second technique to validate the sketch of the graph of $g(x)$ is based on the fact that it is an antiderivative, and by estimating the slopes of tangents at different points we should get the graph of $f(x)$ (Figure 1c). Note that this technique could be seen by students as coming from another MO_{M3} (developed in the chapters on derivatives – section 4.3. *How derivatives affect the shape of a graph* – and using the relation between the sign of the derivative and the shape of the graph as technology). Tasks involving the connection between the graph of a function and its antiderivative appear spread out until chapter 7, but in all these cases the technique calls for validation using notions from derivatives exclusively (increase, decrease, maxima, and minima). We find, for instance, arguments such as: "Notice that $g(x)$ decreases when $f(x)$ is negative and increases when $f(x)$ is positive, and has its minimum value when $f(x) = 0$. So, it seems reasonable, from the graphical evidence, that g is an antiderivative of f " (Stewart, 2012, p. 409).

The number of exercises in which students are asked to relate the graphs of a function and its antiderivative is quite limited: seven exercises in section 5.3 (only five require students to sketch the graph of the antiderivative by hand), four exercises in section 5.5, five exercises in the summary of

chapter 5, four exercises in section 7.1, six exercises in section 7.2, and two exercises in section 7.4. None of them has a context of motion.

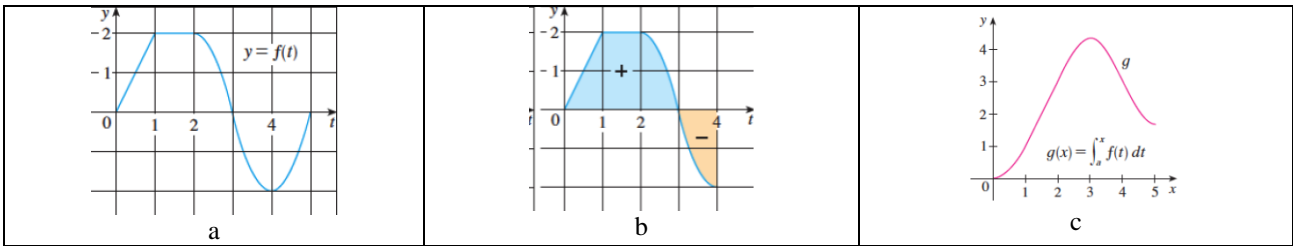


Figure 1. Solved example showing how to sketch the graph of a antiderivative (Stewart, 2012, p. 387)

Organisation of Beer et al. (2013)

In Beer et al. (2013), chapters 1 to 10 are devoted to statics. The study of dynamics begins in chapter 11, where the graphic interpretation of antiderivatives appears. This chapter deals with particles in rectilinear motion; that is, the position, velocity, and acceleration of a particle. This chapter presents an MO_{E1} with the main goal of studying kinematics problems. It uses several elements from MO_{M3} to introduce the notions of average and instantaneous velocity and acceleration. The task of sketching graphic solutions is introduced at the end of the chapter, section 11.7 (pp. 632-634), as a complimentary technique to the analytical approach. We focus our analyses on this part, comparing with the approach to solve graphic tasks in Stewart (2012). A first graphic interpretation of these elements appears on page 606, relating the graphs of a particle's position ($x = 6t^2 + t^3$), velocity, and acceleration (Figure 2). Note that the notation used in MO_{E1} is different from that used in the calculus textbook, particularly the use of x as a dependent variable (compare Figures 1 and 2).

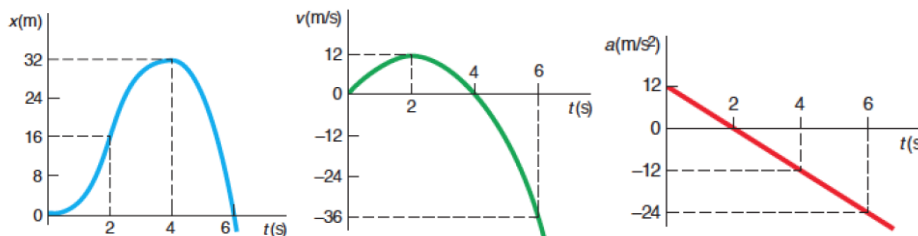


Figure 2. Graphs of position, velocity, and acceleration (Beer et al., 2013, p. 606)

On page 632 we find the type of tasks in which we are interested: the introduction of graphic techniques to solve problems involving rectilinear motion. The book states that both $v = dx/dt$ and $a = dv/dt$ have a geometrical significance and that v and a can be seen, respectively, as the slope of x and v (p. 632). This fact, sustained by elements from MO_{M3}, leads to the introduction of the backwards process to sketch the $x-t$ and $v-t$ graphs given the $a-t$ curve (Figure 3). This technique is based on elements from MO_{M1}. We note, however, that in MO_{M1}, elements related to this visual interpretation are rather marginal. There is also no physical interpretation, and different notation is used. In particular, after presenting cases where a is constant and linear, the following technique is given: “In general, if the acceleration is a polynomial of degree n in t , the velocity will be a polynomial of degree $n + 1$ and the position coordinate a polynomial of degree $n + 2$; these

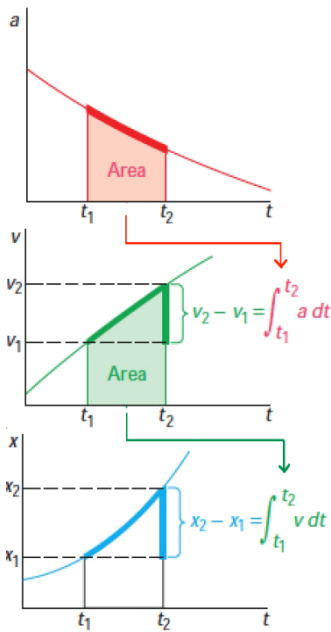


Figure 3. Backwards process
(Beer et al., 2013, p. 632)

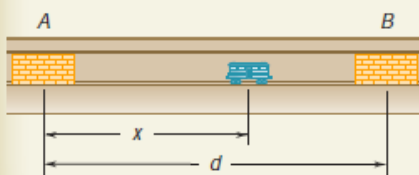
polynomials are represented by motion curves of a corresponding degree.” (Beer et al., 2013, pp. 632-633)

We note that this technique is introduced almost as a mnemotechnic rule, without any explicit connections to elements of MO_{M1} or MO_{M2} (where techniques of integration are studied). We also observe that it is written using terms from MO_{E1} , and not explicitly relating to the language used in calculus. These elements are used to solve problems such as the one in Figure 4. The application of the given technique, as well as basic geometric considerations, leads to the solution in Figure 5.

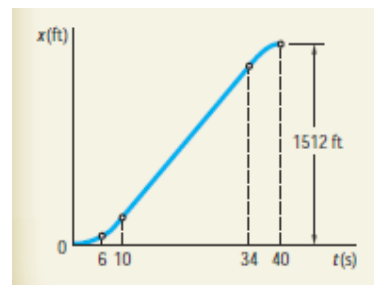
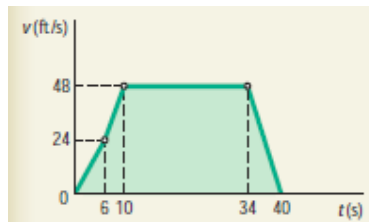
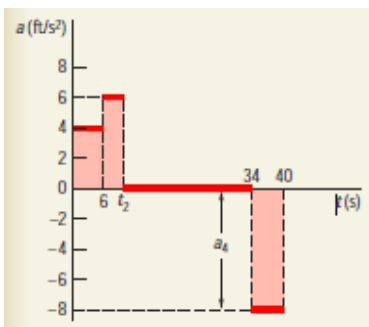
Two main observations can be made: 1) the sketching of the antiderivative is based mostly on explicit elements from MO_{E1} , which are implicitly related to elements of MO_{M1} and MO_{M3} ; 2) however, we note that the technique in MO_{E1} calls attention more directly to the analysis of the area under the curve and to geometric considerations, whereas the tasks of sketching an antiderivative graph introduced in MO_{M1} shift mostly to properties of the derivative

(from MO_{M3}). Moreover, this section of the book groups together 28 exercises for students to practice this type of task. One factor that may hinder students’ appropriation of this type of task is that the technique presented relies more heavily on basic geometric considerations, does not make explicit connections with the rationale from the calculus book, uses many interpretations based on area and kinematic ideas, and uses sensibly different notation.

SAMPLE PROBLEM 11.6



A subway car leaves station A; it gains speed at the rate of 4 ft/s^2 for 6 s and then at the rate of 6 ft/s^2 until it has reached the speed of 48 ft/s. The car maintains the same speed until it approaches station B; brakes are then applied, giving the car a constant deceleration and bringing it to a stop in 6 s. The total running time from A to B is 40 s. Draw the $a-t$, $v-t$, and $x-t$ curves, and determine the distance between stations A and B.



Final considerations

Our results indicate that there is an important rupture in the study of a similar task (sketching the graph of an antiderivative) as presented in two different textbooks used in two different courses. In the calculus book, the task is presented marginally, with a first technique practised in only five exercises and a second technique that emphasises properties of derivatives without taking into account students' known difficulties with the graphic interpretation of the derivative. On the other hand, the engineering book emphasises interpretations using the notions of area and kinematics, which seems to be a way of helping students better grasp antiderivatives graphs (Marrongelle, 2004). An important element to take into account is that the presentation of the task in the mechanics of solids book (through MO_{E1}) does not make explicit connections to the content and techniques of the calculus book (mostly, through MO_{M1} and MO_{M3}).

These findings are coherent with our earlier results concerning other engineering notions (González-Martín & Hernandez Gomes, 2017, 2018). Our previous research indicates that in professional engineering courses, although integrals are used to define certain engineering notions, tasks and techniques are presented without an explicit connection to the language and rationale of calculus, and the many techniques learned in calculus courses are not called upon to solve engineering tasks. We believe that an important element of students' difficulties in seeing the connection between calculus and their professional courses may be due to the following: although the "same" notions are used in both courses, they are used with different techniques and rationales, which may hinder students' ability to recognise these notions. Our results also indicate that a task such as the graphic sketching of an antiderivative (and its interpretation) is performed in at least two engineering courses: strength of materials (bending moments) and mechanics of solids (kinematics). More research is needed to identify whether this type of task is performed in other courses, which may justify spending more time on it in calculus courses.

Finally, we highlight the fact that the books analysed in our previous papers and in this one are of international distribution, so our analyses, although restricted to two different books, may be representative of a general situation in engineering programs. Moreover, the results regarding different topics seem to go in the same direction. We believe that more research is needed to better understand the real use of calculus notions in professional engineering courses, as well as teachers' practices and whether these practices reproduce what is presented in the reference books, to have a more precise idea of how these notions are actually introduced and used. Research on these issues will help advance an international discussion about the content traditionally taught in calculus courses and its adequacy in training engineers (especially in light of the fact many students abandon their programs due to their failing of calculus courses). Our research program intends to contribute to this discussion and our results will be the topic of future publications.

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