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Generation and generation tasks in mathematics didactics

Marianne Schäfer¹ and Rita Borromeo Ferri²

¹University of Kassel, Germany; mschaefer@mathematik.uni-kassel.de

²University of Kassel, Germany; borromeo@mathematik.uni-kassel.de

As part of the PRONET project of the University of Kassel¹, the further development of teacher training through the "implementation of a coherent professionalisation concept" is being investigated. The aim of the subproject is to combine mathematical-didactic knowledge with pedagogically relevant and cognitive-psychologically anchored concepts, including the "desirable difficulties". The generation effect is one of the four best-known "desirable difficulties" and aims to ensure that knowledge is not merely predetermined but is generated to a certain extent from one's own knowledge. In order to achieve this, we have developed for the first time generation tasks on mathematical-didactic topics of a basic lecture for mathematics teachers at their study beginning. The generation tasks and first results are part of this paper and are complemented by the basic theory of "desirable complications".

Generation effect, mathematics didactics, desirable difficulties, mathematical-didactic knowledge, teacher education

Theoretical Background

The focus of this contribution lies on the generation and is reflected approach by the university students. For this reason Generation and generation tasks in mathematics didactics training of future teachers are illustrated more detailed.

Desirable difficulties of learning

One possible encouragement of sustainable knowledge building is the concept of 'desirable difficulties' (Bjork, 1994, 2011; Lipowsky et al., 2015). The term "desirable difficulties" is a cognitive psychological concept and a didactic procedure that makes learning more difficult in the short term, but sustainable with the goal to a better retention of the learning content (ibid.). Desirable difficulties include such popular techniques as the testing effect, the distributed learning, the generating effect, and the interleaving practice (Bjork & Bjork, 2011).

Generation effect

The generating effect appears, when the information is generated from own knowledge (e.g. Salmecka & Graf, 1978). The focus of our research is on the generation effect. This effect has been recognized, reproduced and characterized as extremely stable within controlled learning environments and with simple learning content (e.g. word pairs) (Slamecka & Graf, 1978; Bertsch

¹ The project is financed as part of the PRONET project from the "Quality Initiative for Teacher Education" programme of the Federal Ministry of Education and Research in Germany.

et al., 2007). Chen et al. (2015) have shown that the generation effect is more distinctive if very less is dictated by the task itself or by solution examples and teacher interference. They also found that this increase and stabilisation of the generation effect depends on the type of task. While learners benefited more from the generation in simple tasks, the learning outcomes in complex tasks through guided learning were higher (ibid.). In the field of mathematics, the generation effect is only within the framework of multiplication tasks (cf. McNamara & Healy, 2000) but is not researched with reference to mathematical-didactic knowledge. It becomes clear from the previous studies that generation tasks can extend beyond word pair formation, but that the complexity should not be increased immeasurably, otherwise there is a danger that there will be no more lasting learning effect.

Generation and generation tasks in mathematics didactics training of future teachers

Generation in mathematical didactics means therefore to stimulate the cognitive thought processes of the students and to have new information generated from themselves by means of suitable generation tasks. The self-dependent generation of knowledge takes place within the framework of mathematics didactic generation, for example by linking to previous experiences, for example by generating examples. Another possibility is the creation of individual memory aids in which students use their personal thinking structure to memorize new knowledge. In particular, it should be noted that the generation tasks developed include the basic, recurring contents of mathematics didactics on the one hand and are not too complex in terms of the task and the focus of content on the other. In addition, it must be taken into account that the prospective teachers can build on their own knowledge, although the "new knowledge" cannot be presented to them in advance, otherwise there is no generation effect. Therefore, the above-mentioned lecture contents were chosen which are simple enough to dispense with illustrated examples and complex enough to serve as a basis for further study and structural (mathematical-didactic) expertise (Borromeo Ferri & Schäfer, 2017).

Research questions

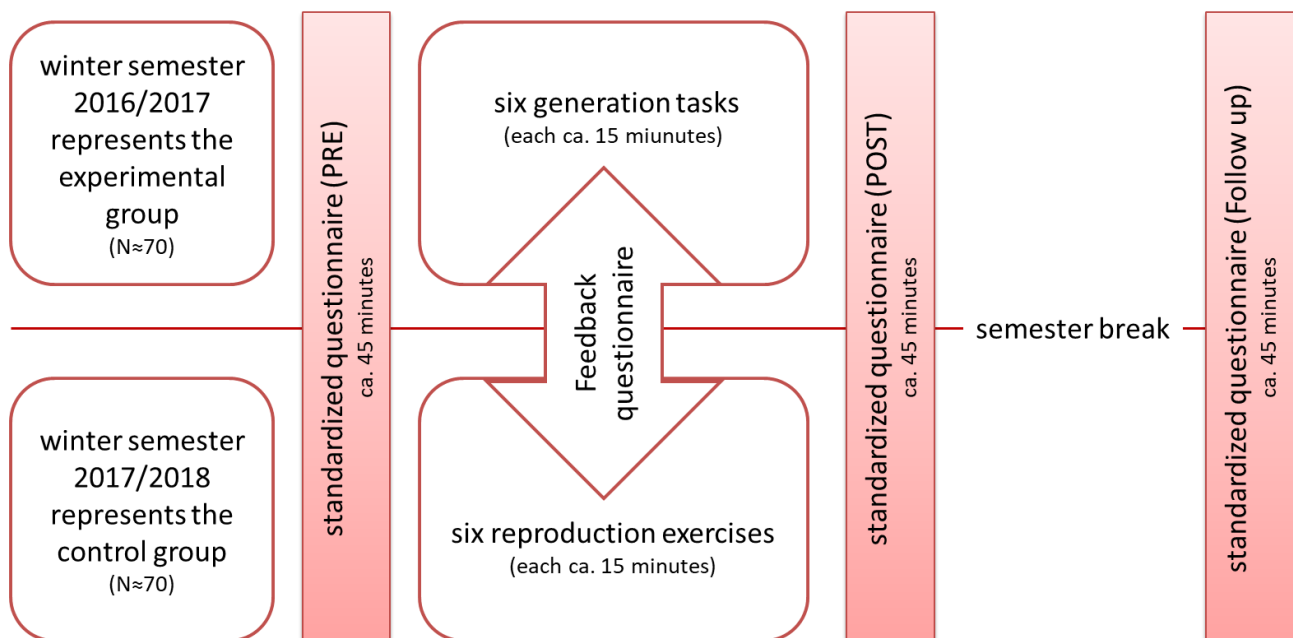
On the basis of the theoretical background to the generation effect, it was shown that the specific generation can be regarded as a research gap in mathematics didactics. It is to be examined whether and how the generation effect is shown by the use of generation tasks with a mathematical-didactic focus and how effectively the knowledge thus acquired is consolidated among the students. An all-encompassing answer to this main question is not yet possible at this point in time, as the statistical data could not yet be fully evaluated. Therefore, in this paper we limit ourselves to demonstrating what marked generation tasks and highlight the following two research questions:

1. Are there any differences between the generation and reproduction cohort regarding the perception of the learning process?
2. Are there any differences between the generation and reproduction cohort regarding the perception of the tasks?

Methodology

The study is conceived as a prospective and hypothesis-generating comparison of two cohorts attending the mathematics didactic basic lecture at the University of Kassel, Germany. The cohort of the winter semester 2016/2017 represents the experimental group and works on the generation tasks. The control group is represented by the cohort of the winter semester 2017/2018 and works on the reproduction tasks. The teacher training students attend the lecture for the first time and are on average in the 1st to 3rd semester. The semester differences are due to the fact that the lecture is attended by various teacher training courses (secondary school, grammar school and vocational school). In order to ensure the comparability of the two cohorts, both the lecture contents and their sequence are held identically by the same professor in both years. Likewise, all questionnaires, feedback forms, the subject areas of the tasks used and the survey times are identical. The study design is graphically illustrated below (Fig. 1).

Figure 1: study design



Due to the fact that this paper has put the emphasis on generation tasks, an insight into what generation tasks are will now be given. The aim of these tasks, as explained above, is to move students of teaching professions to generate their own knowledge out of themselves in a mathematics didactic basic lecture. What does it mean to generate knowledge and how should this be done? Producing knowledge out of them means nothing other than that the teacher training students open up new fields of mathematics didactics for them with little input. For example, the general mathematical competences will be addressed within the framework of the lecture. This is followed by a generation task, which calls for a memory aid to be considered in order to memorize these six competencies. The desirable difficulty for the teacher training students now consists in using a useful memory aid for them without knowing the deeper theoretical contents of the competences. The theoretical contents of the mathematical competences are discussed after the

generation task. In this approach, the main difference to reproduction tasks also becomes clear, since these are always carried out after a theoretical input and usually allow this to be summarised. An exemplary comparison of generation and reproduction tasks in the field of general mathematical competences is given in Table 1. A total of six grading tasks have been developed and applied for use in the mathematics didactic basic lecture. These deal with basic mathematical-didactic content on the topics (1) mathematical educational standards, (2) general mathematical competences, (3) mathematical competence: mathematical modelling and the modelling cycle, (4) mathematical competences: solving problems mathematically, (5) mathematical thinking styles and (6) psychology of mathematics.

Table 1: Example Generation and Reproduction Task²

generation task
<p>The general mathematical competences of the lower secondary level are:</p> <ul style="list-style-type: none"> • argue mathematically • solve problems mathematically • mathematical modelling • use mathematical representations • to deal with mathematics symbolically/formally/technically • communicate mathematically <p>Create a personal memory aid to memorize general mathematical skills.</p> <p>Explain (briefly) your procedure and briefly justify your choice of memory aid.</p>
reproduction task
<p>Julia, Stefan, Sarah, Carsten and Nadine take the school bus to school every morning. Since they get on at the first stop, they always manage to sit in the five seats of the last row. One day Julia remarks: "The school year takes about 210 days. Is it possible that we five always sit in the last row in different ways?"</p> <p>What is the guiding idea behind the task? Which mathematical competence(s) are relevant to answer the question?</p>

In order to answer the research questions underlying this paper, we have developed, used and evaluated a feedback questionnaire that students of teaching professions can use with regard to their perception of the learning process and their perception of the tasks questioned. The perception of

² Translation of the German original by the authors.

the learning process questions whether there has been in-depth information processing or whether students have perceived it as in-depth information. This scale comprises four items, an item example is here: "I have succeeded in developing my own thoughts and ideas on this subject". The perception of the tasks questions to what extent the students come to the conclusion that the respective task was meaningful, helpful, stimulating or similar. This scale comprises five items, an item example for this is "The task used in the pre-presentation illustrates the benefit of the learning content". The two perceptions were recorded using a 6-stage, end-point-based scale. The scale ranges from the negative endpoint "Does not apply at all", which is rated one in the evaluation, to the positive endpoint "Fully applies", which is rated six. In order to analyse the differences, a grouping variable was first created that distinguishes between the cohorts and their tasks in generation tasks and non-generation tasks (reproduction tasks). Subsequently, a scale was introduced for each perception and each task, so that in the end 12 scales were created. Six scales for the perception of the learning process and six scales for the perception of the tasks. These 12 scales were checked for their one-dimensionality by means of a main component analysis (PCA), which turned out to be given for all of them. The scales for the perception of learning process show a reliability of $0.85 \leq \alpha \leq 0.90$. The scales for the perception of the tasks show a reliability of $0.76 \leq \alpha \leq 0.87$. All items can be described as selective in their respective scales. Due to the sample size of $N = 117$ student teachers, the verification of the normal distribution has been recorded. For this reason, a T-test for independent samples is performed to investigate the differences and Cohan's δ is calculated manually to determine the effect strength³.

Results

1. Are there any differences between the generation and reproduction cohort regarding the perception of the learning process?

When the mean values and the standard deviation are considered for the first time, the first differences can be seen purely descriptively on all task scales with the exception of the scale for the fourth task. These descriptive mean differences are checked by means of a T-test for independent samples. It turned out that two of the six task scales for the perception of the learning are a significant difference in the mean value in favor of the cohort that worked with the generation tasks. This significant mean difference is reflected in the tasks three and five. Task three is the generation task that deals with the mathematical competence of mathematical modeling and the modeling cycle. This is a medium effect ($\delta = 0.63$). Task five contains the three levels of representation according to Bruner (enactive, iconic, symbolic) (Brunner et al., 1971). This is an almost medium effect ($\delta = 0.48$).

Table 2 below gives an overview of all mean values, standard deviations, sample distribution, significance level and effect strength for the respective task scales one to six.

³ Cohan's d is calculated using the online calculation tool: <https://www.psychometrica.de/effektstaerke.html> and to the rehearsal again with <http://www.soerenwallrodt.de/index.php>.

Table 2: table of results for the perception of the learning process

task number	perception of the learning			level of significance	Effect strength Cohan's δ	
		<i>N</i>	<i>MW</i>	<i>SD</i>		
1	GA	73	4,03	0,86	p=0.204	
	NGA	35	4,25	0,81		
2	GA	65	4,19	0,86	p=0.720	
	NGA	35	4,25	0,72		
3	GA	73	4,63	0,68	p=0.003	0.63
	NGA	35	4,16	0,90		
4	GA	65	4,06	1,05	p=0.961	
	NGA	35	4,07	0,82		
5	GA	65	4,40	0,75	p=0.023	0.48
	NGA	35	3,99	0,98		
6	GA	66	4,10	0,76	p=0.086	
	NGA	35	3,81	0,82		

GA = generation tasks and NGA = non-generation tasks

2. Are there any differences between the generation and reproduction cohort regarding the perception of the tasks?

The second question dealt with the question of whether differences in the perception of the tasks of the two cohorts were recognizable. Here, too, the first mean value differences could be seen purely descriptively on all six scales. A T-test for independent groups was also performed to test the significance of the differences. It was found that the performance of the cohorts in two tasks differed significantly. These significant differences were seen in the scales for tasks three and four. Task four dealt thematically with the graduated aid according to Zech (1996). It is striking that the significant effect was only in the case of task three in favor of the cohort that worked with the generation tasks. This is a medium effect ($\delta = 0.52$). In the case of task four, the effect in favor of the not yet generation group is significantly positive, although this is only weak ($\delta = 0.38$).

Table 3 below gives an overview of all mean values, standard deviations, sample distribution, significance level and effect strength for the respective task scales one to six.

Table 3: table of results for the perception of the tasks

task number	perception of the tasks				level of significance	Effect strength Cohan's δ
		<i>N</i>	<i>MW</i>	<i>SD</i>		
1	GA NGA	73 35	3,92 4,29	1,08 0,77	p=0.078	
2	GA NGA	65 35	3,91 4,31	1,10 0,94	p=0.071	
3	GA NGA	73 35	4,67 4,25	0,80 0,92	p=0.014	0.52
4	GA NGA	65 35	3,89 4,30	1,21 0,82	p=0.045	0.38
5	GA NGA	65 35	4,43 4,11	0,99 0,76	p=0.106	
6	GA NGA	66 35	3,86 3,52	1,06 0,86	p=0.103	

GA = generation tasks and NGA = non-generation tasks

Conclusion

The differences determined in the course of the T-test provide initial indications of a possible effect of the generation tasks, as these are assessed by the students as being much more meaningful. They acknowledge the in-depth information processing and the value of the transfer and assess it positively. As part of a further study, we also analyse the extent to which students have gone through a "generational process". In order to be able to work this out, the generation tasks and interviews are evaluated with regard to the solution process and reflection. The first interview and task analyses showed that the experimental cohorts were able to reproduce the contents of the generation tasks better than the control cohorts were able to reproduce the contents of the reproduction tasks. Furthermore, an initial analysis has shown that no appreciable proportion of self-reflection is discernible in the context of reproductive tasks. In contrast, there are different approaches to reflection in the solutions of the generational tasks. On the one hand, individual solution steps are reflected and corrected if necessary. On the other hand, the individual participants reflect the entire solution process. In particular, the reflexion-in-the-action (Korthagen et al., 2002) is very pronounced in the generation tasks. One possible reason may be that the generation tasks address deeper cognitive structures, similar to problem-solving tasks, and thus their contents are anchored more sustainably in the memory. So far, however, the results are purely indicative, as only a few interviews and task solutions of the teacher training students have been evaluated so far. For this reason, it is necessary to deepen the analyses in the upcoming research process. For this purpose it is planned to develop a detailed coding manual for the analysis of the generation tasks and interviews on the one hand and to evaluate the questionnaire and the knowledge tests on the other hand. Further evaluations then compare the cohorts and include other factors such as choice of

subjects, motivation (MOLAMA), learning strategies, mathematical thinking, study satisfaction or mathematical convictions.

References

- Bertsch, S., Pesta, B. J., Wiscott, R., & A., M. M. (2007). The generation effect: A meta-analytic review. *Memory & Cognition* (35), pp. 201-210.
- Bjork, E. L., & Bjork, R. A. (2011). Making things harder on yourself, but in a good way: Creating desirable difficulties to enhance learning. In M. A. Gernsbacher, R. W. Pew, L. M. Hough, & J. R. Pomerantz, *Psychology and the real world: Essays illustrating fundamental contributions to society* (pp. 56-64). New York: Worth Publishers.
- Bjork, R. A. (1994). Memory and metamemory considerations in the training of human beings. In J. Metcalfe, & A. Shimamura, *Metakognition: Knowing about Knowing*. (pp. 189-192). Cambridge: MA:MIT Press.
- Borromeo Ferri, R., & Schäfer, M. (2017). Aufbau von nachhaltig gelerntem mathematikdidaktischen Professionswissen im Studium durch die Integration von Generierungsaufgaben. In U. Kortenkamp, & A. Kuzle, *Beiträge zum Mathematikunterricht 2017* (pp. 825-828). Münster: WTM.
- Bruner, J. S., Oliver, R. S., & Greenfield, P. M. (1971). Studien zur kognitiven Entwicklung. Stuttgart: Kohlhammer.
- Chen, O., Kalyuga, S., & Sweller, J. (2015). The Worked Example Effect, the Generation Effect, and Element Interactivity. *Journal of Educational Psychology*, Vol. 107, No. 3, pp. 689-704.
- Korthagen, F., Kessels, J., Koster, B., Lagewerf, B., & Wubbles, T. (2002). Schulentwicklung und Lehrerbildung. Reflexion der Lehrtätigkeit. *Übersetzt von Wolfgang Meyer*. Hamburg: EB-Verlag.
- Lipowsky, F., Richter, T., Borromeo Ferr, R., Ebersbach, M., & Hänze, M. (2015). *Schulpädagogik Heute*. Retrieved November 25, 2015, from Wünschenswerte Erschwernisse beim Lernen: www.schulpaedagogik-heute.de/conimg/Archiv/SH_11/06_01.pdf
- McNamara, D. S., & Healy, A. F. (2000). A Procedural Explanation of the Generation Effect for Simple and Difficult Multiplication Problems and Answers. *Journal of Memory and Language* (43), pp. 652-679.
- Salmecka, N. J. (1978). „The Generation Effect: Delineation of a Phenomenon.“. *Journal of Experimental Psychology: Human, Learning and Memory*, Vol. 4, No. 6, pp. 592-604.
- Zech, F. (1996). Grundkurs Mathematikdidaktik. Weinheim: Beltz Verlag.