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Tertiary education in the digital age

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We address the relationship between digital technologies and tertiary education. For that purpose, we first consider some conceptualisations of the idea of “digital resource” from different points of view, and the evolution of the presence of digital technologies in our lives. Then, we wonder whether digital resources play a particular role in the university, compared to primary or secondary education. We also consider some affordances and constraints of their use. Afterwards, we explore how the idea of “didactic paradigm” provides a framework for the analysis of different possible uses of digital resources. Finally, we report on some experiences about the use of digital technologies motivated by the appearance of the COVID-19.

Keywords: digital resource, tertiary education, mathematics education.

INTRODUCTION

When the panel started to be planned, by December 2019, we could not imagine how trendy the topic of the panel was going to be now, along the year 2020. The aspect of potentials of digital means to organise teaching and learning in a different way have gained increasing attention. Resources and teaching environments that allow avoiding physical presence at universities during the COVID-19 pandemic were (and still are) extensively needed and used. The impact of the pandemic in education and the role played by digital resources to overcome the difficulties and to face the challenges will be one of the issues we will address here. But not the only one.

Digital resources provide both teachers and students with a whole world of possibilities, and so many questions arise concerning their use in the teaching of mathematics at the tertiary level. We have tried to consider some important aspects of this work.

We start the first section by considering the question of what a digital resource is. This is an important question, because the way we conceptualise it strongly relies on the didactic paradigm we assume, and deeply affects the way we analyse different uses of digital materials. Next, we will explain that, as a matter of fact, digital resources seem to be more used at tertiary education than they are at primary or secondary education. We will also provide possible reasons to explain it.

Digital resources enlarge the collection of possibilities for the teachers to present the contents of their courses and for students to engage in these contents. Some of these resources can be provided by the educational institutions, and some others are used by

students at their own initiative. However, these resources are not always exempt from problems in their use, either for technical reasons (experienced by teachers and/or students) or for issues concerning the quality of the resources themselves. In relation to this, we will address which are the affordances and constraints of the use of digital resources.

In the context of education, digital resources do not exist by themselves, unrelated to anything else. They are rather placed in the frame of a *didactic paradigm*. In other words, when an educational institution suggests the use of a certain digital device, or when a certain student decides to use it, there is always an underlying set of educational ends and an underlying epistemological model. Both the educational ends and the epistemological model deeply affect what kind of digital resources are to be used, how, and to what purpose. Therefore, we will also make some considerations about digital materials in connection with didactic paradigms.

Finally, as we are not only researchers but also teachers, we will as well consider how the COVID-19 crisis has given rise to the intervention of digital resources in our own practice, and what difficulties or revelations have appeared related to this.

At the end, we present the main ideas in a conclusive section.

WHAT IS A DIGITAL RESOURCE?

We can distinguish between digital resources from a technological and mathematics education point of view. From a technological point of view, the term “digital resource” refers to any resource that is in a digitalised form. Digital resources may include hardware technologies (e.g., a calculator, laptop, or mobile phone), and educational technologies that can be divided into two categories: first, pedagogical software technologies such as Wolfram Alpha, GeoGebra, Stack, or Numbas; and second, generic software technologies that cover a variety of mathematical topics such as Khan Academy, online math video lectures, Massive Open Online Courses (MOOC), Facebook resources, etc.

From the point of view of mathematics education, the term “digital resource” may be conceptualised using the instrumental and documentational approach to didactics (Trouche, 2004; Trouche, Gueudet, & Pepin, 2018). The key notions of the instrumental approach are “instrumentation” and “instrumentalisation”, and the transformation of digital artefacts into instruments. The instrumental approach is most often used with a student perspective.

The documentational approach mainly takes a teacher perspective, but it can also be used to explore students’ use of resources (Hillesund, 2020). This approach to didactics is considered as a further development of the instrumental approach with some more key notions such as document and resource (both digital and non-digital resources). Moreover, with the documentational approach, there is a distinction between educational technologies and digital curriculum resources.

Educational technologies at the tertiary level are studied for a long time (see, e.g., section 5 in the ICMI study dedicated to the teaching and learning of mathematics at the university level, Holton et al., 2001). In recent research, the interest in digital curriculum resources and their use by students has developed. Concerning the conceptualisation of digital curriculum resources, we refer to Pepin, Choppin, Ruthven and Sinclair (2017, p. 647), who contend that:

It is the attention to sequencing—of grade- or age-level learning topics (all or part of, or of content associated with a particular course of study (e.g., algebra)—so as to cover (all or part of) a curriculum specification, which differentiates Digital Curriculum Resources from other types of digital instructional tools or educational software programmes.

Some studies choose a quantitative approach to this issue. They study what resources are used by students, and for what purposes. Stadler, Bengmark, Thunberg and Winberg (2013) observe that during the secondary-tertiary transition, students in Sweden increasingly use Internet-based resources. This observation can depend on the national context: indeed, in the UK, Anastasakis, Robinson and Lerman (2017) note that students mostly use the resources provided by the institution, with exam-related goals. Also, in the UK, Inglis, Palipana, Trenholm and Ward (2011) investigate the use of three kinds of (optional) resources by students: live lectures, online lectures, Mathematics Support Centres. Interestingly, only a minority of students use more than one resource. The authors conclude that students need guidance for blending different resources.

Recent studies use the documental approach (Gueudet, Pepin & Trouche, 2012) to investigate the use of resources by students at the university level. This theoretical approach is associated with case studies. Kanwal (2018) studies cases of engineering students working with a learning management system. She observes that the form of the assessment influences their use of resources and concludes that the tasks proposed in a digital curriculum resource must be carefully designed, to lead students to the expected mathematical activity. This coincides with results obtained by Gueudet and Pepin (2018), who observe that some resources are misused by students. The rules of the didactic contract, concerning the use of resources, remain mostly implicit and the actual use by students does not correspond to the use expected by teachers. Pepin and Kock (2019) note that in different courses (Calculus vs Linear Algebra), different kinds of resources are proposed by teachers. Students use institutional resources when they are aligned with examinations. Otherwise, they search themselves for resources, in particular human and social resources.

In terms of the evolution of digital resources in mathematics education, there is a trend. In the beginning, there was a prevalence of visualisation tools, e-learning 2.0, blended and mobile learning, e-assessment systems, programming languages. Later, there seems to be a preference for resources with advanced functionalities such as e-learning 3.0, multi-touch technologies, embodied learning technologies, artificial intelligence-based tutoring tools with feedback, and, most importantly, technologies that connect mathematics education to computational thinking and artificial intelligence, and

educational *Internet of Things* aimed at connecting and integrating digital resources into people's everyday life as a guiding principle (Ashton, 2009). Today, students can connect computers, laptops, tablets, and smartphones in mathematics classrooms. Moreover, people can now connect smartwatches, smart devices, cars, and other devices that collect and transfer mathematical data.

More specifically, educational institutions are moving from the early Internet of Things of smart connections to a new phase, one of invisible integration, which results in the disappearance of digital resources in the vision of *ubiquitous computing* formulated by Weiser (1991, pp. 94), who pointed out that

the most profound technologies are those that disappear. They weave themselves into the fabric of everyday life until they are indistinguishable from it.

In other words, digital resources disappear in a manner that they are in the mode of being the philosopher Heidegger called the *ready-to-handedness* of tools (Heidegger, 1953). This means that the word “digital” starts disappearing from educational terminology, which sooner or later will result in a “post-digital” education (Pandrić, 2018). Are we entering the age of post-digital mathematics education, also partly due to the acceleration of digital teaching in this pandemic period?

PRESENCE OF DIGITAL RESOURCES TO TERTIARY EDUCATION

As a matter of fact, the relevance of digital resources to tertiary education rests, firstly, on the degree and extent to which the resources are used (by mathematicians, mathematics educators, non-specialists such as engineers and biologists, students and teachers from different disciplines), and, secondly, on the integration of digital resources into educational settings. Given these considerations, in practice, it seems that digital resources are more present in tertiary education than they are in primary or secondary education.

Indeed, there seems to be an extensive use and integration of digital resources at the tertiary level across all subjects, combining face-to-face and distance learning, frequently in relation to flipped classroom methods (Pinto & Leite, 2020). In many countries, both teachers and students are now using visualisation and simulation tools, computer-based assessment systems, programming languages to acquire computational thinking skills for mathematical explorations and investigations.

Let us examine some examples of uses of digital resources at the university.

From the point of view of MatRIC, the Centre for Research, Innovation and Coordination of Mathematics Teaching, which is a learning community working for excellence in teaching mathematics in Norwegian universities, digital resources are very relevant for the study processes at tertiary level. Many MatRIC-driven activities at the University of Agder and other universities in Norway reveal the relevance of digital resources for modelling activities, simulation, visualisation, and assessment, etc. Likewise, the new activities aiming at digital mathematics teaching show the relevance of digital resources for inquiry-based mathematics education and online

mathematics teaching and learning. Moreover, MatRIC has developed *Drop-in*, an additional digital resource that offers extra help and guidance to support students who are working with challenging mathematical tasks.

The use of programming language for mathematical investigations at university has been studied for several years in the context of the MICA course (Mathematics Integrated with Computers and Applications, see, e.g. Buteau & Muller 2010) at Brock University in Canada. In this course, mathematics majors and future mathematics teachers learn and use programming for mathematical investigations “like mathematicians”. A programming language falls within the artefact definition given by Rabardel (1995). Nevertheless, compared, for example, with Digital Geometry Systems, it is clearly of a different nature. For this reason, studying instrumental geneses linked with the use of a programming language when solving mathematical problems can lead to identifying new kinds of schemes, and deepen our understanding of the relations between computer science knowledge and mathematical knowledge. Along with their use of the programming language, students develop an instrument. This instrument associates the programming artefact with different kinds of schemes, in particular, what we call “p+m-schemes” where knowledge about programming and about mathematics are strongly associated (see Guedet, Buteau, Muller, Mgonbelo & Sacristan, 2020).

Also, the relevance of digital resources can be seen in mathematics education for non-specialists. Indeed, in many mathematics courses, mathematical objects are not clearly linked to objects in the “real world”, are not used to create models of “real systems”. Rather, there seems to be an emphasis on understanding the behaviour of those objects regardless of the properties of the “exterior world”. However, in mathematics courses of other disciplines (e.g. engineering, biology, etc.), mathematics is expected to provide, via modelling, useful information about certain systems appearing in nature, in real-life. Digital resources for engineering students and other non-specialists seem to be of help in this task. Engineers on the workplace use computers and software, and their studies have to prepare them for this use. Nevertheless, engineering students sometimes use technology as a black box, allowing them to obtain a solution without understanding the mathematics behind (e.g. Kanwal, 2020). Recent research has advanced our understanding of this complex issue. Drawing on the concept of “techno-mathematical literacies” (Kent, Bakker, Hoyles & Noss, 2005), defined as combinations of mathematical, Internet of Things and workplace-specific competencies, van der Waal, Bakker and Drijvers (2017) identified seven categories of techno-mathematical literacies for working engineers. Drawing on this work, they implemented and evaluated inquiry-based teaching where engineering students were invited to present and comment on their use of software (van der Waal, Bakker & Drijvers, 2019). This use was then collectively discussed. The authors evidenced that this kind of teaching can support the development of techno-mathematical literacies for future engineers.

A possible explanation for the increasing use of digital resources at the tertiary level is that university mathematics students are given more responsibility for learning than in school mathematics (Hillesund, 2020). This also entails that, while universities provide students with digital resources, it is up to them to decide how to use the resources. Indeed, on tertiary level students are usually expected to have, to a certain extent, the competence to work on their own, since the scheduling of tertiary education mostly just dedicates a relatively minor percentage of the total required learning time to supervision by teaching staff. Depending on a variety of contextual factors, students can use other digital resources that are freely available online, some of them are related and other unrelated to the ones used in the university courses across many disciplines. Among those factors, we can find technicalities of the resource, familiarity with the resource, availability of time and human resources (teachers, peers, etc.), exam situations and mandatory tasks, etc. External digital resources beyond the ones used at the university may include video resources, e-books, simulation and visualisation tools, games, videos used in flipped classrooms, MOOCs, collaborative distance learning environments, etc. University students can make their own decisions as to whether to use external digital resources and to what extent and purpose, particularly with examinations and compulsory assignments in mind (Hillesund, 2020). Thus, it is sometimes required to find and work with adequate learning materials, but also, to some degree, to distinguish between good and bad materials that can be found and to take responsibility for learning outcomes, and these requirements are increasing with the ascending educational level. Thus, in comparison to primary or secondary education, the potential for individual use of digital learning materials and the self-reliant use of those materials appears to be higher at the tertiary educational level.

AFFORDANCES AND CONSTRAINTS OF THE USE OF DIGITAL RESOURCES

Digital learning resources can be used to enable students to follow new learning trajectories and to change the way students engage in the learning of mathematics (Sacristán et al., 2010).

The flexibility digital resources provide concerning the pace, order and organisation of learning can allow students for developing their own, individually preferred learning routines, strategies and schedules according to their personal needs (Gold, Fleischmann, Mai, Biehler & Kempen, in press).

Digital learning materials can be used to support understanding certain mathematical concepts (in the sense of Tall & Vinner, 1981) by offering a variety of different representations, or by providing detailed feedback. Software environments like GeoGebra (Hohenwarter & Jones, 2007), STACK (Sangwin, 2003) and many others are available, and guide students through the challenging process of change of representational register (Duval, 2006), using dynamic illustrations and the opportunity to actively construct mathematical objects digitally, for example in geometry. There also exist elaborated learning environments and teaching concepts where these

technical means are implemented in tertiary education (Kinnear, 2019; Biehler, Fleischmann & Gold, 2018).

Evaluations of the students' learning behaviour when using digital resources are pointing in the direction that, when students work independently with a comprehensive digital mathematics bridging course material, they mainly concentrate on solving tasks rather than working through theory (Fleischmann, Kempen, Biehler, Gold & Mai, 2019). However, thanks to their technical format, digital materials can here support the (mental) linkage between theory and application by providing quick access to the relevant passages and offering detailed feedback on the solutions entered by the student.

Having said that, teaching mathematics in a technology-based environment rests on a combination of several factors: the characteristics of the digital resource, teachers' digital competencies, students' mathematical knowledge background and digital skills, the subject curriculum, the topic to be taught, the discipline (mathematics, engineering, biology, etc.), the learning goal, and most importantly the affordances and constraints that emerge in mathematics educational contexts at the tertiary level.

Therefore, the following question seems appropriate: what affordances and constraints emerge from the interactions between users (teachers/students) and digital resources in mathematics educational contexts?

The types of affordances and constraints that emerge from the interaction between users and digital resources at various levels depending on many factors highlighted above. More specifically, affordances result from the characteristics of the resources and the way users interact with them in an educational context. In other words, affordances involve both the knowledge background of the users and the features of the resources (Hadjerrouit, 2020). Thus, affordances and constraints are not intrinsic properties of the digital resource or users (teachers, students), but rather properties of the whole conglomerate formed by the digital resource, the teachers, and the students, all together.

The affordance issue has two didactical consequences. Firstly, using digital resources for teaching mathematics require good technological, didactical skills and mathematical competencies to foster the emergence of affordances and minimise constraints. Secondly, teachers need to develop a reflective attitude towards the use of digital resources and consider both affordances and constraints in designing mathematical tasks.

In the paragraph above, we had in mind digital resources typically provided to the students by the teaching institution, like a certain programming language or a specific applet suggested by the teacher to represent mathematical objects or to make symbolic calculations. But, of course, those are not the only kind of digital tools. The variety of digital learning resources used at current tertiary level goes from complete online courses, which are available for students, focusing mostly on the study entrance phase (Biehler, Fleischmann, Gold & Mai, 2017; Kinnear, 2019), over online platforms used

for communication between students and teachers by most universities, up to a broad and constantly growing collection of medial resources that can be found online with public access. These online materials include forums where mathematics is discussed informally, websites that offer calculations and visualisations, such as Wolfram Alpha, and online videos uploaded mainly on YouTube. In particular, these online videos gained increasing popularity within students (Acuña-Soto, Liern & Pérez-Gladish, 2020), but the fact that the quality concerning contents is sometimes questionable and, like for most online resources, outside of the control of teaching staff at universities, also contains a risk for the education of students using these media. One can identify high potential in the offers of digital means, and the fact that it becomes technically easier to create and provide new materials online leads to the highly desirable opportunity that also unconventional approaches are followed and find their audience. On the other hand, there are also risks and challenges associated with educational use of digital resources. Suitable learning materials must be identified, and reliable criteria for the quality of these materials must be at hand (Hadjerrouit, 2010). Moreover, working with some medial formats, such as videos, can lead to an “illusion of understanding” that might come with the consumption of these materials, that does not necessarily enable students to think and solve problems themselves (Schwartz, 2013).

Essential for the teaching and learning of mathematics is the communication between the teachers and the students, and digital resources can constitute valuable means to support this communication process. While the platforms used by many universities offer channels to provide *learning materials*, to ask questions (possibly also anonymously, which might lower the barrier to do so for some students), also the exchange of *feedback* on the teaching and the learning can be supported by digital environments to the profit for teachers and learners. Data provided by the digital learning environment concerning the work behaviour of the students can help to adjust the teaching (Reinholz, Bradfield & Apkarian, 2019). In the other direction, studies show that students appreciate getting digital feedback on their work via an oral commentary that is recorded in the form of video-based feedback (Robinson, Loch & Croft, 2015; Grove & Good, 2020).

DIGITAL RESOURCES IN CONNECTION TO SPECIFIC DIDACTIC PARADIGMS

It would be interesting to consider whether the inclusion of digital resources is an actual innovation rather than just a variation of the means (Lindmeier, 2018). Moreover, it could be clarifying to regard digital resources in education (and, actually, also about analogue resources, like handbooks, blackboards, chalks, notebooks, etc.), as means to achieve certain ends in the framework of an explicit *didactic paradigm* (Gascón & Nicolás, 2019, 2020, 2021) for the teaching of mathematics. The idea of *didactic paradigm* (assumed by an educational institution) comprises both the assumed *educational purposes* and the assumed *epistemological model* of mathematics. The educational ends are the answers to the question “Which is the purpose of teaching

mathematics?”. The epistemological model is the answer to the question “What is to know mathematics?”, which is closely related to the question “What is to teach mathematics?”. Only under the premises stipulated by a given didactic paradigm we can provide arguments for one or another use of a given digital resource.

For instance, if, according to our epistemological model of mathematics, students have to do a considerable amount of empirical work to construct mathematical knowledge (for instance, discovering by themselves counterexamples to their conjectures), like in the Theory of Didactic Situations (Brousseau, 1997), then some software (to represent functions, to deal with statistics, etc.) would be very helpful to support this work.

One of the possible purposes of teaching mathematics (or teaching, in general) could be to promote a receptive attitude towards posing and answering objective questions about the world in a rational way. This is what in the anthropological theory of the didactic (Chevallard, 2006) has been called the *paradigm of questioning the world*. The typical means proposed by that theory to design and describe study processes within that paradigm are the so-called *study and research paths*. Those are “paths” that start with a meaningful *generating question*, to which the students are supposed to provide a “suitable” answer. Normally, one of the few clauses of the didactic contract in those study and research paths is that the teacher is not going to say any possible answer(s), and students are allowed to do whatever to find an answer. And, of course, they have to show some kind of argument to defend the suitability of that answer. If the students use the freedom provided by the didactic contract in the study and research paths, it is quite reasonable to suppose that students are going to use digital resources (search engines, applets, programming languages, etc.) to look for an answer to the generating question. Thus, in the paradigm of questioning the world, digital technologies are implicitly regarded as normal ready-made objects to be used along a path from a question to a corresponding answer.

For instance, if one of the *purposes of teaching mathematics* is to get the students kind of familiar with the mathematical activity, like in the Inquiry-Based Learning approach, then one could use some specific software ‘for mathematicians’, and also allow the students to look for solutions to their problems on the Internet, as the mathematicians do themselves. Digital resources today have become more sophisticated and have the potential to be more than tools to perform tasks faster than by hand and paper-pencil techniques. Digital resources are equipped with interactive graphical user interfaces, making students able to participate more actively with the help of different forms of feedback. Hence, from a didactical point of view, it appears that there is a shift from teacher-centred to a more student-centred pedagogical approach to mathematics education. A good example is mathematics flipped classrooms, which, according to some versions, it takes a student-centred approach to learn at the university level, as studied in (Fredriksen, 2020) with a group of engineering students. The shift consists of moving away from a teacher-centred model, where the teacher is the main source of instruction towards a student-centred approach, where in-class time is used for exploring topics gained from out-of-class video

watching, creating rich learning opportunities among students. This is just one example of the impact of current digital resources on instruction. Other examples of student-centred approaches using advanced digital resources are linked to e-assessment systems with formative feedback such as Numbas and Stack, programming languages with interactive user interfaces such as Python and MATLAB, visualisation and simulation tools such as SimReal (Hadjerrouit, 2020; Hadjerrouit & Gautestad, 2019).

Of course, one of the purposes of many degrees at university is to prepare students for certain professions. As some of those professions make extensive use of digital resources (for instance, engineering), their study becomes an essential goal in these degrees. Some study and research paths have been designed and implemented with future engineers (e.g. Florensa, Bosch, Gascon and Mata, 2016, Quéré, 2019), and so they use a generating question anchored in working engineers' practices. For example, for future chemistry engineers, Quéré (2019) proposed a study and research path in statistics starting with the question: "In the pharmaceutical industry, how do you make sure that the product (medicine) meets the dosage on the package?". The students developed sub-questions linked with statistical tests and studied these questions, using in particular statistical software. While the use of educational technologies in study and research paths addressed to future engineers is not a central aim, it is frequently present and plays an important role. Indeed, there are indications that task design for digital resources will be crucial in technological-based mathematics courses at the tertiary level (Leung & Baccaglioni, 2017). Task design is important due to the challenges faced by many engineering students in considering mathematics as detached from real-life applications (Fredriksen, 2020). Designing realistic mathematical tasks will thus become crucial in mathematics education across many disciplines.

COVID-19 AND DIGITAL RESOURCES

This last part of the paper will be devoted to reporting on the teaching experiences of two of the four authors during the 2020 lockdown. Needless to say, those reports are not intended to be scientific conclusions. Nevertheless, they can still be of interests, as testimonies of an era in which the implementation of digital resources is being accelerated due to the COVID-19 crisis. In (Clark-Wilson, Robutti & Thomas, 2020) the reader can find further interesting reflection on teaching with technology during the COVID-19 period, mainly concerned with secondary school.

Due to the lockdown of NTNU in the middle of March 2020, Yael Fleischmann had to switch from a traditional attendance-based lecture (with about 120 participants, on Euclidian and hyperbolic geometry) in the middle of the Norwegian spring semester to a teaching format that was completely realised online. Using the screencast software "Explain everything", she decided to record her lectures and upload them for the students twice per week. The software provides a digital "whiteboard" where one can write and record a voice-over explanation simultaneously. Additionally, she arranged real-time online meetings with the students, using the universities digital learning- and communication-platform "Blackboard", where students could ask questions and discuss tasks with the lecturer (Yael). As a consequence of this shift of format of the

lectures, she noticed that students, even if there were several channels to do this, hesitated to ask questions. As the lecturer, Yael also missed the feedback of the students concerning their level of understanding for the contents she was explaining, as she usually had gotten during the live lectures. Yael also noticed that speaking to a tablet computer instead of an audience influenced the formerly quite lively atmosphere of the lecture heavily. To tackle these challenges, she decided to include something that she called “sound mystery” into the lectures. That was a sound, for example, the music theme of the computer game “Tetris”, that was played sometimes during the recorded lecture, and students were asked to identify the sound and send her the answer. The motivation for this unconventional step was to motivate and provoke reactions of the students, and hereby to lower their barrier to get in touch with her as the lecturer. Indeed, this worked well and together with the answer to the “sound mystery”, students started to send questions concerning the contents of the lecture and also feedback on the format and style, which was very helpful for her as the lecturer. From the times and numbers of reactions, it was also possible to estimate by how many students and when the videos were watched (which is data that the universities’ platform cannot provide to the lecturer). Part of the feedback on the lecture was that students expressed very different needs concerning the new learning situation during the lockdown. While some students appreciated the flexibility given by the opportunity to watch the videos at any time and pace, others expressed their need for a given time schedule to stay motivated and disciplined. Yael tried to address this by providing videos of approximately the length of the former live lectures twice per week shortly before the former lecture hours. Students also expressed a high level of insecurity, in particular regarding the exam that was planned for this course and had to be taken as a digital home exam in consequence of the pandemic circumstances. Here, it was essential to communicate also minor decisions and developments concerning organisational and administrative details very frequently. It must also be said that the development of an exam that was supposed to be written by the students at home, with all possible (online-) sources and communication channels available for the students during the examination time, was a particularly hard challenge, and also the grading of an exam where different students did make use of these unconventional opportunities to dramatically different extents was demanding. It yet remains as an open question of what kind of digital resources could possibly be developed and used in the future to allow appropriate assessment under circumstances that do not allow physical presence.

During the first lockdown in France, Ghislaine Gueudet taught to prospective secondary school mathematics teachers, and to students in educational research. Both kinds of students were engaged in Master degrees; the groups were between 15 and 35 students. Most activities Ghislaine proposed were asynchronous: she offered resources and tasks on a Moodle platform, the students uploaded their work, she corrected it and sent it back. She also organised some synchronous activities but noticed it was very

difficult for some students to have access to the video-conference platform. Finally, the best solution to communicate with them was by using cell phones and WhatsApp.

The most important lesson Ghislaine learned was linked with the observation of the difficult situation of some students. The research in mathematics education already addresses for a long time the issue of equity. Concerning the use of digital resources, Forgasz, Vale and Ursini (2010) noted that while the issue of access was important, some research has evidenced that digital resources can contribute to offer equitable learning opportunities. The study of equity issues at the university level is very active. For example, Adiredja and Andrews-Larson (2017) describe the evolution of research towards an increased interest in socio-political issues. They present in particular research that addresses the impact of social discourses and institutional contexts on the negotiations of power and identity in postsecondary mathematics.

Nevertheless, the research they cite in their synthesis does not consider the difficulties raised or the opportunities created by the use of digital resources at the tertiary level. In this time of crisis, maybe the most important question concerning digital resources at the tertiary level is: "How should digital resources be used at tertiary level for fostering equity?"

CONCLUSIONS

There are different conceptualisations of the notion of *digital resource*. Here, beyond the technical one, we have examined those provided by the instrumental and the documentational approach. We have also pointed out that digital resources are considered each timeless noteworthy orthopaedic tools to help the study, as they are becoming more and more transparent and integrated into the normal current life of students.

Sometimes the use made by students of digital resources is essentially rejected, or looked with suspicion, or at least not taken into account, by some didactic paradigms. For instance, some regard the teacher and the notebook (and perhaps some web pages specifically designed for the corresponding course, as if they were *digital notebooks*) as the only sources of information for students. This seems to be the case more frequently in primary and secondary education. In tertiary education students are given more autonomy and more responsibility of their own learning, and, at this point, digital resources (those provided by the university, or any others) seem to be a usual way to support their study.

However, even if the use of digital technologies enlarges both teachers' and students' affordances, it also presents certain constraints, either due to technical reasons (for instance, the complexity of a certain applet) or to the reliability of the service provided by the technology itself. Anyway, those affordances and constraints are never due just to the digital materials or the teacher or the student, but rather to the indivisible system formed by these three components. Not to mention the importance of the content to be studied and the way it is related to the digital resources at stake.

Typically, digital resources do not provide by themselves new ends of education. Instead, they enlarge the collection of means to achieve those ends. Indeed, digital technology offers new possibilities for the design and management of study processes, both for teachers and students. Those new possibilities concern many aspects of study processes: representation of mathematical objects, feedback in the resolution of tasks, etc.

There are didactic paradigms that incorporate digital resources themselves like objects of study (for instance, in degrees devoted to the preparation for a profession in which those resources are of typical use), or even objects of study and, at the same time, means for the achievement of further ends (like in the case of a programming language for the design of algorithms that use some theorems in a course on Numerical Analysis).

Also, concerning certain didactic paradigms, even when a digital resource is not the object of study, it can be considered as an already-made object that students can use at any time along the study process. Actually, digital materials are very likely to be used by students if they have to carry out research in order to provide an answer to a question, and they are allowed to use any means. This is supposed to be the case in many didactic paradigms laying under the broad label of *inquiry-based learning*.

The COVID-19 crisis has forced the imposition of online teaching. This change of scenario has revealed sometimes the economic and social inequality that exists in a single group of students. This is a reality which underlies educational institutions, and that may affect students' commitment to their own education. Also, a teaching proposal completely interfered by digital technologies, with no physical presence, seems to experience defective communication between teachers and students. Moreover, a sudden and unexpected switch to a distance learning regime also entails deep problems in assessments, and not only those concerning cheating prevention.

As we have seen, tertiary education in the digital age is full of challenges and complex issues, some of which have been pointed out and taken under consideration in this work.

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