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Why can't we be friends? An evolutionary approach to ethno-cultural hierarchies

LUCIE LETROUIT ¹

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Abstract:

The share of ethno-cultural minorities is growing in world metropolises, thereby affecting inter-group relationships and altering ethno-cultural hierarchies. Some empirical studies have documented a positive impact of group size on group status, while some find no effect and others a negative one (i.e. a “cultural backlash”). In this paper, I propose a multi-group and multi-strategy evolutionary coordination game, which allows to explain the contrasted effects that an ethno-cultural minority’s size may have on its social status, and on the status of other minorities, hinging on the interplay between several ethno-cultural minorities’ interests. Additionally, this model (1) sheds new light on the sources of “asymmetrical homophily” between ethno-cultural groups, (2) proposes an explanation for the phenomena of fragmentation and fusion between several minorities and (3) predicts that societies with small ethno-cultural minorities will tend to converge to inegalitarian hierarchy views, which empirical studies suggest to be economically under-efficient. The model’s long-term equilibrium of ethno-cultural hierarchy views is corroborated by an exploratory empirical test based on the Ethnic Power Relations database. From the methodological point of view, this paper provides a directly applicable procedure for the theoretical resolution of parametric multi-group and multi-strategy evolutionary games, allowing for potentially very interesting applications in other socio-economic contexts.

Keywords: Evolutionary game, Ethno-cultural hierarchies, Inegalitarian norms, Economics of Minorities

JEL: C73, D02, J15

1. Introduction

Social hierarchies, defined as “organizational structures in which certain individuals or groups have greater social power and/or access to resources than others”, constitute one of the most common type of social organization, both in human and nonhuman primate groups.² These hierarchies tend to emerge rapidly from short-term social interactions, are pervasive in our daily life activities and play a determinant role in shaping our understanding of the world, our feelings, as well as our social and economic behaviors.^{3 4}

1. Researcher in Economics, Planning, Mobility and Environment, Gustave Eiffel University, Marne-La-Vallée, France, Email: lucie.letrouit@univ-eiffel.fr, Phone: +33 181668780

2. See [Mandalaywala et al. \(2019\)](#) and [Halevy et al. \(2011\)](#).

3. See [Anderson and Kilduff \(2009\)](#).

4. For example, they have been empirically shown to impact the productivity of individuals, their consumption and occupational choices, the economic opportunities they face on various markets, as well as aggregate market outcomes ([Hoff and Pandey, 2004](#), [Afridi et al., 2015](#), [Charles et al., 2009](#), [Fershtman and Weiss, 1991](#) and [Ball et al., 2001](#)).

Numerous theories have been developed by psychologists, sociologists and economists to explain the emergence of hierarchies between groups⁵ and how these hierarchies vary with group size,⁶ stressing for example the role of identity, perceived threat or evolutionary mechanisms in this matter. However, each of these theories is only able to explain either a positive effect of minority size on its status or a negative one, while empirical evidence clearly shows that both effects are frequently observed.⁷ In addition, these theories largely focus on relationships between one majority and one minority group, thereby totally abstracting from the broader ethno-cultural context in which their interactions take place. Yet, a sizable number of empirical works show that this ethno-cultural context plays a determinant role in shaping hierarchy views, by leading some minorities to compete with each other for social recognition or to ally, to assimilate with one another or split, to oppose the arrival of a new minority or to favor it.⁸ How can the complex link between minority size and status be explained in a multi-cultural society context? And, in particular, what are the mechanisms underlying "cultural backlashes", whereby an increase in a minority's size happens to be detrimental to its status?

The present paper aims at answering these questions by proposing a generalization of a baseline two-group (one majority and one minority) evolutionary model of coordination on a common hierarchy view into a three-group evolutionary model. This generalization allows for the analysis of the emergence of an ethno-cultural hierarchy in the presence of one majority and two minorities, a situation which can be expected to correctly reflect not only the case of two-minority societies but also more generally of multi-cultural societies. Although simple in nature, this generalization is methodologically challenging and brings very novel insights regarding the complex non-monotonic mechanisms that determine group statuses.

The generalized evolutionary game model proposed in this article can be summarized as follows. A society composed of three ethno-cultural groups (one majority and two minorities) is considered, in which members of the different groups progressively coordinate on common hierarchy views (i.e. their "strategies"). Each member of the society belongs to only one group and holds a personal view regarding the ethno-cultural hierarchy, i.e. regarding the social rank of the different groups in the society. At each period of time, each individual is matched with a given number of other individuals. When two individuals are matched, they interact according to a coordination game in which their respective strategies correspond to their ethno-cultural hierarchy views. If the two matched individuals belong to different groups and hold the same hierarchy views (i.e. there is coordination), they both obtain a strictly positive payoff (the same payoff for both if their statuses in the hierarchy are equal, a different one if not). On the contrary, if the two matched individuals belong to two different groups and hold different hierarchy views, both get a zero payoff. At the beginning of each period of time, one individual is given the chance to actualize his view on the ethno-cultural hierarchy. With a very large probability that tends to 1, this individual adopts his best response to the distribution of hierarchy views in the society at the previous period and, with a very small probability that tends to zero, he adopts a random

5. These theories include Terror Management Theory, Value Conflict Theory, Social Identity Theory, Realistic Group Conflict Theory, and statistical discrimination (see [Sidanius and Pratto \(2001\)](#) for a review and [Naidu et al. \(2017\)](#)).

6. For example, Intergroup Contact Theory predicts a positive link between group size and status. Social Categorization Theory, endogenous identity and Group-threat Theory predict the opposite.

7. See the references in the "Literature" section 2.

8. For instance, studying the impact of Mexican immigration to the US between 1970 and 2010, [Fouka et al. \(2020a\)](#) show that attitudes towards African Americans were improved by Mexican immigration. [Mendelsohn \(1987\)](#) shows how Jews' status in inter-war Latvia was negatively affected by their perceived similarities with former German and Russian oppressors. [Ignatiev \(2012\)](#) discusses how the long-term improvement of Irish minority's status in the United States was linked with the progressive immigration of culturally more distant groups. [Sudbury \(2001\)](#) report that multiracial blackness (African, Caribbean and Pakistani) in Britain seems to form a unified identity that Black women activists can recourse to in order to catalyze collective action. And, by contrast, [McClain and Karnig \(1990\)](#) document some level of competition in Black-Latino relations in the United States.

hierarchy view. The distribution of ethno-cultural hierarchy views in the population evolves through time according to a Markovian process that converges toward a unique stationary distribution, which depends on the relative sizes on the different ethno-cultural groups.

The model shows that an increase in a minority's size may affect its ethno-cultural status positively or negatively, depending on both minorities' sizes, as there exist interdependencies between the statuses of the different minorities. Two distinct mechanisms may lead to a negative impact. First, if the alliance of some members of two ethno-cultural groups is necessary to trigger the adoption of a more egalitarian ethno-cultural hierarchy and the members of one of these groups are more influential than the members of the other one, then the expansion of one ethno-cultural group in the alliance will only be efficient if this group is "influential" enough as compared to the other one. If not, the expansion of the group will be counterproductive. Second, if only members of the expanding group are necessary to trigger the adoption of a more egalitarian hierarchy view, then the expansion of the group may reinforce another group's willingness to favor an inegalitarian hierarchy and prevent the transition (because the expansion increased the benefits it gets from dominating the expanding group). Applying the model to a sub-population of a country's society, the model can explain the existence of "asymmetrical homophily" between two ethno-cultural groups⁹. It also sheds new light on the phenomena of fragmentation and fusion between several minorities.¹⁰ Eventually, a simple extension of the model suggests that societies with small ethno-cultural minorities will tend to converge to inegalitarian hierarchy views, which empirical research suggests to be economically under-efficient. The long-term equilibria resulting from the model are corroborated though a short exploratory empirical test, using the minority statuses reported in the Ethnic Power Relations database built by [Vogt et al. \(2015\)](#), which is publicly available on ETH Zürich' website (<https://icr.ethz.ch/data/epr/core/>).

The present paper also proposes a methodological contribution under the form of a procedure for the theoretical resolution of a parametric multi-group and multi-strategy asymmetric evolutionary game model. This procedure consists in an implementation of the steps proposed by [Young \(1993\)](#), complemented by Edmond's algorithm for the determination of long-term equilibria.¹¹ The presence of multiple groups of individuals, multiple hierarchy views and parameters for the different group sizes renders the model's resolution extremely burdensome if undertaken manually and motivates the recourse to computer programming (using the Mathematica software) in order to compute transition costs between the model's Nash equilibria depending on its parameters and to apply Edmond's algorithm. It is important to note that the computer program is not a simulation (i.e. the parameters are not fixed to any value). Instead, it solves the model analytically, depending on its parameters. To the best of my knowledge, this paper is the first to implement such a procedure as a way to analytically solve a complex evolutionary game model. The procedure could be fruitfully applied to the resolution of other parametric multi-group and multi-strategy models.

In the remainder of this paper, [Section 2](#) discusses the literature associated with the present paper. [Section 3](#) presents a simplified version of the main (three-group) evolutionary game model in the presence

9. i.e. the asymmetric willingness of members of two distinct groups to interact with each other

10. i.e. respectively when a minority breaks up into several minorities and when several minorities are progressively assimilated with one another.

11. [Young \(1993\)](#)'s methodology consists in the following steps: 1) first, all the pure Nash equilibria (i.e. distributions of hierarchy views in the society in which no individual would want to unilaterally alter his strategy) are determined, 2) then, transition costs between each pair of Nash equilibria (i.e. the number of mutations in hierarchy views necessary to trigger a transition from one Nash equilibrium to each other Nash equilibrium) are computed, depending on the model's parameters, 3) an oriented graph where nodes correspond to Nash equilibria and oriented edges' weights correspond to transition costs is built, 4) the minimum spanning tree leading to each Nash equilibrium is determined, using Edmond's algorithm, 5) the long run equilibrium then corresponds to the Nash equilibrium associated with the lowest cost minimum spanning tree.

of only two groups: one majority and one minority. Then, Section 4 provides an analysis of the general version of the model with three groups: one majority and two minorities. Section 5 shows how the model can explain the complex link between group size and group status, helps to understand the phenomenon of “asymmetric homophily”, provides predictions on the effect of a new minority’s arrival and of a minority split and allows to discuss the economic efficiency of long-term hierarchy views. Section 6 consists in an exploratory empirical test of the model’s predicted long-term equilibria. Section 7 concludes.

2. Literature

This paper is linked with several strands of the economics, sociology and psychology literature.

First, it is most closely related with the evolutionary game theory literature pertaining to the emergence of unequal, miscoordinated or sub-optimal social norms.¹² In this literature, two main papers tackle the question of unequal norms’ perpetuation through the lens of evolutionary game theory. [Naidu et al. \(2017\)](#) analyze a battle of the sexes coordination evolutionary game regarding the choice of linguistic conventions in a population composed of two classes. In this model, if the dominated population is large relative to the elite and linguistic innovations are intentional (rather than random mutations), then ambiguous and unequal linguistic norms are likely to persist in the long run. In a related paper, [Hwang et al. \(2016\)](#) explain how, in a general bipartite network linking all individuals of a population composed of an elite and a dominated group, inefficient norms may persist, if most of the dominated group’s interactions are local, while elite’s interactions are more “cosmopolitan”. In these models as well as in the rest of this branch of the literature, the focus is set on interactions between only two groups. In contrast, the present model studies the emergence of inequalitarian norms in the more complex context where multiple groups (in fact three) interact with each other, which leads to new interesting insights.

Second, this paper is linked with a very small literature dealing with the empirical measurement of the impact of a new minority’s arrival in a society on old minorities ([Hainmueller and Hangartner, 2013](#)). In [Fouka et al. \(2020b\)](#) and [Fouka et al. \(2020a\)](#), the authors provide evidence on the impact of the Great Migration (1915-1930) in the United States and of Mexican immigration in US states on the perception of other minorities by the majority group. They propose a theoretical explanation for their empirical results, relying on Social Categorization Theory. They argue that categorization of minorities into in-group or out-group is made according to a rule of maximization of a *meta-contrast ratio*, defined as the ratio of *across-category differences* over *within-category differences*. In their framework, the expansion of a minority can lead to the recategorization of another minority from out-group to in-group. This theoretical framework is interesting as it allows to account for indirect effects that minorities can have on each others’ perception by the majority group. However, its focus is on binary in-group/out-group boundaries, as perceived by the majority group only, rather than on broadly agreed upon ethno-cultural hierarchies. It thus cannot account for cumulative patterns of ethno-cultural statuses, which is a key feature of ethno-cultural hierarchies. It also sets aside the crucial role of coordination in the emergence of norms of behaviors for interactions between multiple groups. In contrast, the present model seeks to offer a more comprehensive explanation for the variety of impacts that minority sizes can have on ethno-cultural statuses, taking into account the incentives of all groups.

Thirdly, the paper relates to a large literature spanning over many fields of the social sciences that documents features of inter-group relationships, such as existing levels of discrimination, the impact of a group’s size on its status, “cultural backlash”, “asymmetrical homophily”, group split, alliances

12. See [Carvalho \(2017\)](#) and [Belloc and Bowles \(2013\)](#).

and fusions. Numerous papers attempt to estimate, in various countries,¹³ the level of discrimination against different minority groups or the social distance between a country's different ethno-cultural groups. This literature indicates a high level of consensus within groups with respect to hierarchy views (i.e. members of a same ethno-cultural group tend to widely agree on the ranking of the different minorities in the society), and a high level of consensus across groups in countries with a large majority and smaller minorities (i.e. members of different groups tend to agree on their respective rankings). It also shows that perceived cultural distances between groups as well as discrimination levels tend to be cumulative, with some minorities being considered as more distant and associated with a lower status than others. Then, a consequential number of papers documents the "cultural backlash" phenomenon, defined as the negative impact that an increase in a minority's size may have on its status, providing both economic and cultural explanations.¹⁴ Positive or null impacts of an increase in a minority's size have also been documented in a large number of other contexts,¹⁵ giving rise to the "intergroup contact theory", according to which an increase in a minority's size intensifies its contacts with the majority and thus reduces prejudice against this minority.¹⁶ The existence of "asymmetrical homophily", namely the differing levels of willingness which members of distinct groups may have to interact with each other, has been documented by [Bikmen \(2011\)](#). And numerous articles describe cases of ethno-cultural groups splitting into several groups, refusing assimilation with each other or, on the contrary, of distinct ethno-cultural groups allying or progressively merging with one another other.¹⁷ The present paper proposes a unique mechanism able to account for these diverse stylized facts.

Fourth, the present model is linked with multi-group (i.e. multi-player) and multi-strategy evolutionary game theory (i.e. the study of evolutionary games with strictly more than two groups and strictly more than two strategies per group) where some classical properties of 2×2 games, such as the risk dominance of stochastically stable Nash equilibria are not verified ([Samuelson, 1994](#)). Most models in the multi-group and multi-strategy evolutionary game theory literature are symmetric in the sense that all groups derive the same utility from a same strategy.¹⁸ However, [Young \(1993\)](#) proposes a general methodology to determine the long-term equilibria (i.e. the stochastically stable equilibria) of a $N \times M$ asymmetric evolutionary game, under a specified adaptive process, analyzing, as an illustration, a simple example of a 3×3 asymmetric evolutionary game with constant payoffs (i.e. no parameters).¹⁹ The present paper contributes to this literature by showing how Young's methodology complemented by Edmond's algorithm can allow to solve analytically and discuss a multi-group (in fact three groups) multi-strategy (in fact four strategies) evolutionary asymmetrical game model with parameters, when the computational challenge is properly addressed using computer programming.

Eventually, the paper shares some features with the literature analyzing the various economic impacts of social norms, notably on economic development, entrepreneurial activities, the organization of firms, occupational choices, cooperation, trade and political efficiency.²⁰ The model developed in

13. See [Hagendoorn \(1995\)](#), [Booth et al. \(2012\)](#), [Pager et al. \(2009\)](#), [Heath and Di Stasio \(2019\)](#) and [Zschirnt and Ruedin \(2016\)](#).

14. See [Norris and Inglehart \(2019\)](#), [Halla et al. \(2017\)](#), [Tabellini \(2020\)](#), [Fouka et al. \(2020a\)](#), [Malhotra et al. \(2013\)](#) and [Scheve and Slaughter \(2001\)](#).

15. See the corresponding references in [Schlueter and Scheepers \(2010\)](#) and also [Steinmayr \(2020\)](#), [Dustmann et al. \(2019\)](#) and [Calderon et al. \(2019\)](#) for positive effects.

16. See [Allport et al. \(1954\)](#).

17. See [Graham \(2009\)](#), [Khan \(1976\)](#), [Padilla \(1986\)](#), [Okamoto \(2003\)](#) and [Young \(2015\)](#).

18. See [Ellison \(2000\)](#), [Broom et al. \(1997\)](#), [Gokhale and Traulsen \(2014\)](#).

19. Beside these few articles, we can note that mathematicians and computer scientists have also been studying games with multiple players and multiple strategies, called "polymatrix games", but to address questions that are very different from economists', like the estimation of the probability to obtain a certain number of equilibria ([Howson Jr, 1972](#), [Eaves, 1973](#)).

20. See [Tabellini \(2010\)](#), [Greif and Tabellini \(2010\)](#), [Gorodnichenko and Roland \(2017\)](#), [Bloom et al. \(2012\)](#), [Fehr \(2009\)](#) and [Guiso et al. \(2009\)](#).

this paper contributes to this literature by suggesting that an evolutionary process of coordination on a common ethno-cultural hierarchy view often leads to the emergence of a consensus on an inegalitarian hierarchy, which reduces the overall productivity of interactions between individuals as compared to an egalitarian one.²¹

3. Two-group model

Before presenting the general model with three ethno-cultural groups, which constitutes the core contribution of the present paper, a two-group version of the model is analyzed, in order to provide a benchmark for the assessment of the general model’s results. This two-group version corresponds to a particular case of [Naidu et al. \(2017\)](#)’s model.²² It is very stylized and simple, to allow for a better understanding of the mechanisms at play and for the subsequent generalization.

3.1. Model setup

In this two-group model, a society composed of a majority ethno-cultural group A of size N_A and a minority ethno-cultural group B of size N_B is considered, with $N_A > N_B$.²³ Each individual in the society belongs to one of the two ethno-cultural groups and holds a personal view on the ethno-cultural hierarchy between these groups. Two distinct ethno-cultural hierarchy views are possible. The first one is inegalitarian and posits that group A socially dominates group B . It is denoted by $H_{A>B}$. The second one is egalitarian and considers both groups to be equal. It is denoted by $H_{A=B}$.^{24 25}

At each period of time $t = 1, 2, \dots$, each individual in the society interacts with ξ other individuals. Among these ξ individuals, the proportions of members of group A and B are assumed to reflect the relative sizes of the two groups. An individual interacting with a member of the other ethno-cultural

21. The empirical literature on the impact of unequal status on interactions between individuals indeed strongly suggests that inequality between individuals, when perceived as unfair, tends to reduce participation to public goods and to increase sabotage ([Waring and Bell, 2013](#), [Anderson et al., 2008](#), [Nishi et al., 2015](#), [Fehr, 2018](#), [Dickinson et al., 2018](#) and [Sadrieh and Verbon, 2006](#)), thereby reducing aggregate welfare. When not perceived as unfair, inequality may, however, favor collective action and increase participation to public goods ([Simpson et al., 2012](#)). In the case of ethno-cultural discrimination, unequal status is most likely to be perceived as unfair and thus to have a negative economic impact.

22. More specifically, the structure of payoffs (which are characterized through only two parameters here, θ and γ) is simplified with respect to [Naidu et al. \(2017\)](#) and intentional mutations are suppressed (only the more classical random mutations are preserved). A specification of the two-group model with intentional mutations will, however, be briefly discussed in a footnote, later on in the paper.

23. The two ethno-cultural groups are considered to be characterized by their cultural identity, made of the set of values, beliefs, world views and behavioral prescriptions shared by the group ([Akerlof and Kranton, 2000](#)), and by their phenotype.

24. To illustrate the two possible hierarchy views, consider that, if group A corresponds to the White majority in a Western country such as France or the United States and group B corresponds to the Black minority, an individual adopting hierarchy view $H_{A>B}$ believes that Blacks are inferior to Whites and an individual adopting $H_{A=B}$ believes that Blacks and Whites are equal.

25. We can note that the possibility for the minority to dominate the majority is excluded from the analysis (i.e. no hierarchy view $H_{B>A}$). However, the two-group model can quite easily be extended to allow for this additional hierarchy view. In this extension, the resolution method is similar to the one employed for the three-group model (i.e. the costs of the minimum spanning trees leading to each pure Nash equilibrium must be compared in order to determine the long term equilibrium). The results of this alternative two-group model are very similar to the baseline specification chosen in the present article. Indeed, the two same long-term equilibria are possible and the threshold of Proposition 2.2 is simply shifted: for $\frac{N_B}{N_A} < \frac{(\theta-\gamma)(2\theta-\gamma)(2\theta+\gamma)}{4\theta^3-3\gamma^2\theta+\gamma^3}$, the hierarchy view $H_{A>B}$ persists in the long term and, for $\frac{N_B}{N_A} > \frac{(\theta-\gamma)(2\theta-\gamma)(2\theta+\gamma)}{4\theta^3-3\gamma^2\theta+\gamma^3}$, $H_{A=B}$ persists in the long term. The hierarchy view $H_{B>A}$ never persists in the long term. Excluding the possibility for the minorities to dominate the majority thus does not alter fundamentally the results of the two-group model. This same assumption will be used in the three-group model, in order to keep it as simple as possible.

group gets a payoff that is dependent on the hierarchy views of both interacting parties: (i) if the two individuals hold differing hierarchy views, they both get a zero payoff, (ii) if the two individuals hold the (same) egalitarian hierarchy view, they both get a payoff equal to $\theta > 0$, (iii) if they both hold the (same) inegalitarian hierarchy view, then the member of the dominant group, A , gets a payoff of $\theta + \gamma$ and the member of the dominated group, B , gets a payoff of $\theta - \gamma$. We will assume, in the following of the paper, that $\theta > \gamma > 0$, which implies that individuals interacting with members of another ethno-cultural group always obtain a higher payoff in case of coordination (i.e. agreement on a common hierarchy view) than in case of miscoordination (i.e. disagreement regarding hierarchy views), even if coordination is on a hierarchy in which they are dominated. Payoffs between members of different groups are summarized in the following table:

		Group A	
		$H_{A>B}$	$H_{A=B}$
Group B	$H_{A>B}$	$\theta - \gamma, \theta + \gamma$	$0, 0$
	$H_{A=B}$	$0, 0$	θ, θ

An individual interacting with a member of his own ethno-cultural group, on the opposite, obtains zero payoff independently of their hierarchy views, i.e it is assumed that ethno-cultural hierarchy views do not affect interactions between the members of a same ethno-cultural group.²⁶

A typical setup in which this type of interactions can occur is at the work place in the context of a team project: two employees, belonging to two different ethno-cultural groups, are asked to collaborate on fulfilling a task. If the employee belonging to ethno-cultural group A adheres to the hierarchy view $H_{A>B}$, then he will consider it normal to take the lead in the team's organization. If the member of group B adheres to this hierarchy view, he will accept the other employees' domination and cooperation will take place, while if he does not adhere to it, he will reject it and the team work will not be fulfilled efficiently.

These assumptions on payoffs imply that members of two different ethno-cultural groups interact more efficiently if they agree on the ethno-cultural hierarchy than if not.²⁷ They translate the ability of common hierarchy views to support an efficient division of labor and to reduce conflicts (Halevy et al., 2011). Alternatively, the lower payoff obtained in case of miscoordination may also be interpreted as "cognitive dissonance" (Festinger, 1962), whereby confrontation with a differing point of view (regarding

26. This assumption is in line with Carvalho (2017), Naidu et al. (2017) and Hwang et al. (2016). It can be relaxed by assuming, instead, that if the members of a same ethno-cultural group holding a same hierarchy view interact, they get a payoff of $\delta > 0$ with $\delta < \frac{(2\theta - \gamma)\theta}{2\theta + \gamma}$, and that if they do not hold the same hierarchy view, they get a zero payoff. It essentially means that interactions between same group members are much less affected by disagreement on hierarchy views than interactions between different group members, which appears to be realistic. Under these alternative assumptions, the (qualitative) results of the two-group model are preserved. These results are detailed in a footnote after Proposition 2.2.

27. Note that the model's results are not affected by the addition of a same constant to all payoffs of a group's members. Thus, the zero payoff of each group in case of miscoordination could as well be any positive or negative constant, as long as it is below $\theta - \gamma$.

the social hierarchy) triggers discomfort and is therefore costly.^{28 29 30 31}

Denoting by $p_{l,h}$ the proportion of group l members who hold hierarchy view h during the period of time t and $V_{i,l}(h)$ the payoff extracted from an interaction by a member of group i interacting with a member of group l when there is coordination on h (as reported in the previous table), the average payoff of an individual belonging to group $i \neq j$ and holding hierarchy view h during period t is:

$$U_i(h) = \sum_{l \in \{i,j\}} \xi \frac{N_l}{N_i + N_j} p_{l,h} V_{i,l}(h) \quad (1)$$

where $\xi \frac{N_l}{N_i + N_j}$ corresponds to the number of interactions that the individual has with members of group l . For example, the payoff of a member of group A holding hierarchy view $H_{A>B}$ is $U_A(H_{A>B}) = \xi \frac{N_B}{N_A + N_B} * p_{B,H_{A>B}}(\theta + \gamma)$.

The following dynamic is assumed for the evolution of the distribution of hierarchy views in the population. At the beginning of each period of time t , one individual randomly drawn from the population gets a chance to adopt a new hierarchy view, while all other individuals keep the hierarchy views they had at $t - 1$. The randomly drawn individual adopts with probability $1 - \epsilon$ (for ϵ very small) his best response to the distribution of hierarchy views in the population at $t - 1$. This decision is myopic in the sense that the individual adopts his short-term best response without anticipating how the distribution of hierarchy views in the society will evolve in the future. With probability ϵ , the randomly drawn individual instead randomly adopts one of the (two) possible hierarchy views.³²

At each period of time t , the state of the society can thus be characterized by a $(N_A + N_B)$ -uple denoted by z_t , in which each element corresponds to the hierarchy view of a specific individual in

28. Note that it is assumed that cultural identities have no effect on the productivity of individuals. Thus, the social hierarchies that emerge are not based on the average productivity of social groups (there is no statistical discrimination).

29. Further note that interactions implying egalitarian and inegalitarian hierarchies have exactly the same productivity 2θ . If we assume, instead, that when both individuals agree on the egalitarian hierarchy $H_{A=B}$, they get a payoff of θ' , the qualitative results of the model remain unchanged, as long as $\theta' \leq \theta + \gamma$. The threshold of Proposition 2.2 is simply replaced by a new threshold that decreases with θ' so that, the higher the payoff linked with the egalitarian hierarchy view, the larger the range of $\frac{N_B}{N_A}$ for which the egalitarian hierarchy view persists in the long term. The impact of having marginally different productivities for egalitarian and inegalitarian interactions in the three-group model is also discussed in one of the extensions.

30. Also note that, in this model, it is assumed that the two interacting individuals get a zero payoff if they disagree on the hierarchy, even if the majority's member grants the minority a higher status than the minority's member does (i.e. zero payoff the member of group B when the member of group A adopts $H_{A=B}$ and the member of group B adopts $H_{A>B}$). This requirement can be alleviated by assuming, instead, that the two individuals get, in this specific case, a payoff of $\delta \in]0, \gamma[$. With this alternative assumption, all the qualitative insights of the model are preserved and the threshold specified in Proposition 2.2 is simply increased to $\frac{(\theta - \gamma)(2\theta + \gamma - \delta)}{\theta(2\theta - \gamma - \delta)}$.

31. Eventually, if γ is asymmetric, with the dominating group getting a payoff of $\theta + \gamma'$ (instead of $\theta + \gamma$), then the qualitative results of the model are unchanged and the threshold of Proposition 2.2 becomes $\frac{(\theta - \gamma)(2\theta + \gamma')}{2\theta - \gamma - \theta}$, which increases with γ' . Thus, the larger the payoff of the dominating group in the inegalitarian hierarchy, the larger the set of $\frac{N_B}{N_A}$ for which the inegalitarian hierarchy view persists in the long term.

32. Note that it is assumed that mutations are totally random. However, Bowles (2009) argues that random mutations may not be the most realistic, as they often imply that groups trigger transitions that reduce their own payoff in the long-term. An alternative specification of the model could therefore allow for individuals to have a higher probability to mutate toward a hierarchy view that is in their favor and a lower probability to mutate toward a hierarchy view that is in their disfavor. Following Naidu et al. (2017) and Hwang et al. (2016), this specification could consist in assuming a probability ϵ for an individual to mutate toward a hierarchy that is in his favor and a probability ϵ^ι to mutate toward a hierarchy that is in his disfavor (where $\iota > 1$). With this specification, it can easily be shown that, if ι is small enough (i.e. $\iota < 1 + \frac{\gamma}{\theta}$), the model's results are preserved and the threshold of Proposition 2.2 remains unchanged.

the society. The corresponding state space, $\mathcal{H}_{2g}^{N_A+N_B}$ where $\mathcal{H}_{2g} = \{H_{A>B}, H_{A=B}\}$, is clearly finite.³³ Additionally, the above described dynamic defines a Markov chain that is both irreducible and aperiodic. This Markov chain therefore admits a unique stationary distribution μ^ϵ . The stochastically stable state (or states) of the Markov chain is (or are) the one (or the ones) that has (or have) a strictly positive mass in the stationary distribution when $\epsilon \rightarrow 0$.³⁴ In practice, as the noise level ϵ becomes arbitrarily small, the adaptive process spends virtually all of the time, when $t \rightarrow \infty$, in the evolutionarily stable state(s) and, among them, in the state(s) that can be reached with the lowest transition cost (i.e. through the lowest number of ϵ -mutations).³⁵

3.2. Model resolution

Straightforwardly, the model admits two pure Nash equilibria corresponding to the full adoption of each of the two possible hierarchy views, as summarized in the following Lemma:³⁶

Lemma 2.1: *The 2×2 model admits exactly two pure Nash equilibria:*

- *An inegalitarian equilibrium, $E_{A>B}$, in which all individuals adopt the inegalitarian hierarchy view $H_{A>B}$.*
- *An egalitarian equilibrium, $E_{A=B}$, in which all individuals adopt the egalitarian hierarchy view $H_{A=B}$.*

We can now determine the transition costs between Nash equilibria, i.e. the minimum number of ϵ -mutations necessary for the society to transit from each Nash equilibrium to each other Nash equilibrium. The transition cost from equilibrium $E_{A=B}$ to $E_{A>B}$ will be denoted by $C_{E_{A=B} \rightarrow E_{A>B}}$. It is the minimum between the number of ϵ -mutations of group A members necessary to make group B members indifferent between $H_{A=B}$ and $H_{A>B}$ and the number of ϵ -mutations of group B members necessary to make group A members indifferent between these hierarchies. Indeed, once one group becomes indifferent between the initial and the final hierarchy and starts preferring the final hierarchy, its members progressively switch to the final hierarchy through best-response adjustments at no cost (i.e. with probability $1 - \epsilon \approx 1$ at each period of time). The members of the first group then also switch to the final hierarchy through best-response adjustments (because the final equilibrium is a Nash equilibrium) and the final equilibrium is reached. The group whose members mutate to the final equilibrium is called "*mutating group*" and the group whose members then prefer to switch to the final equilibrium is called

33. \mathcal{H}_{2g} means "set of possible hierarchy views in the 2-group model".

34. See [Foster and Young \(1990\)](#)

35. Also remark that this model does not necessarily assume a Darwinian process of selection on individuals themselves nor on their genes. It may instead be a Darwinian process of selection on individuals' views regarding the country's ethno-cultural hierarchy, in which the most successful views are progressively adopted by more and more individuals and spread within the population.

36. The model also admits one mixed Nash equilibrium, in which $p_{A,H1} = \frac{\theta}{2\theta - \gamma}$ and $p_{B,H1} = \frac{\theta}{2\theta + \gamma}$. However, it is a classical result in evolutionary game theory that mixed Nash equilibria in a coordination game cannot be evolutionary stable. Thus, this mixed Nash equilibrium can be set aside from the model's resolution.

“*switching group*”. The transition cost from $E_{A=B}$ to $E_{A>B}$ is therefore:³⁷

$$C_{E_{A=B} \rightarrow E_{A>B}} = \min \left(\frac{\theta N_A}{2\theta - \gamma}, \frac{\theta N_B}{2\theta + \gamma} \right) = \frac{\theta N_B}{2\theta + \gamma} \quad (4)$$

Similarly, the transition cost from equilibrium $E_{A>B}$ to $E_{A=B}$ is:

$$C_{E_{A>B} \rightarrow E_{A=B}} = \min \left(\frac{(\theta - \gamma)N_A}{2\theta - \gamma}, \frac{(\theta + \gamma)N_B}{2\theta + \gamma} \right) \quad (5)$$

We can note that transitions from the egalitarian to the inegalitarian equilibrium are always triggered by mutations of group B members, while transitions to the egalitarian equilibrium may be triggered by mutations of one group or the other, depending on the parameters.³⁸

Indeed, there exists a trade-off for the determination of the *mutating group* and of the *switching group*. On the one side, the payoff of the *switching group* in the initial equilibrium has to be small (i.e. the numerator of each fraction in the above expression of $C_{E_{A>B} \rightarrow E_{A=B}}$ has to be small), so that only few mutations are necessary to make the *switching group* indifferent between the initial and the final equilibrium (i.e. there is little “resistance” to leaving the initial equilibrium). This means that the size of the *mutating group* and the social rank of the *switching group* in the initial equilibrium have to be small. On the other side, the effect of the mutation of one member of the *mutating group* on the gap between the payoffs of the *switching group* in the initial and final equilibria has to be large (i.e. the denominator of each fraction in the above expression of $C_{E_{A>B} \rightarrow E_{A=B}}$ has to be large). In other words, mutations of the *mutating group* have to be “efficient” in altering the *switching group*’s payoff. This requires the *switching group* to enjoy a high social rank both in the initial and final equilibrium. All in all, if the minority is small enough, the first criterium dominates. If the minority is large enough, the second criterium dominates.³⁹ Comparing the expressions of the two transition costs, $C_{E_{A=B} \rightarrow E_{A>B}}$ and $C_{E_{A>B} \rightarrow E_{A=B}}$ leads to the following long-run equilibria (i.e. stochastically stable states):

Proposition 2.2: *The above presented 2-group evolutionary coordination game admits the following stochastically stable state, depending on the relative sizes of the two ethno-cultural groups:*

37. Indeed, the number of mutations from group A members necessary to make group B members prefer $H_{A>B}$ over $H_{A=B}$ is the smallest M_A such that:

$$\xi \frac{N_A - M_A}{N_A + N_B} \theta \leq \xi \frac{M_A}{N_A + N_B} (\theta - \gamma) \quad (2)$$

Similarly, the number of mutations from group B members necessary to make group A members prefer $H_{A>B}$ over $H_{A=B}$ is the smallest M_B such that:

$$\xi \frac{N_B - M_B}{N_A + N_B} \theta \leq \xi \frac{M_B}{N_A + N_B} (\theta + \gamma) \quad (3)$$

38. If the minority is small enough with respect to the majority (i.e. $\frac{N_B}{N_A} < \frac{(\theta - \gamma)(2\theta + \gamma)}{(2\theta - \gamma)(\theta + \gamma)}$), these transitions are triggered by the minority, otherwise they are triggered by members of the majority.

39. Note that the transition cost from the egalitarian equilibrium to the inegalitarian one, $C_{E_{A=B} \rightarrow E_{A>B}}$, decreases with γ , as fewer mutations become necessary for the majority to prefer the final equilibrium. In contrast, the transition cost from the inegalitarian to the egalitarian equilibrium, $C_{E_{A>B} \rightarrow E_{A=B}}$, decreases with γ if the minority is large enough (because the *switching group* is the minority and, the larger γ is, the less the minority appreciates $H_{A>B}$) and increases with it if the minority is small enough (because the *switching group* is the majority and, the larger γ is, the more the majority appreciates $H_{A=B}$).

- If $\frac{N_B}{N_A} > \frac{(\theta-\gamma)(2\theta+\gamma)}{\theta(2\theta-\gamma)}$, then $E_{A=B}$ is the unique stochastically stable state.
- If $\frac{N_B}{N_A} < \frac{(\theta-\gamma)(2\theta+\gamma)}{\theta(2\theta-\gamma)}$, then $E_{A>B}$ is the unique stochastically stable state.

Proof of Proposition 2.2: We have:

$$\begin{aligned}
C_{E_{A=B} \rightarrow E_{A>B}} < C_{E_{A>B} \rightarrow E_{A=B}} &\Leftrightarrow \frac{\theta N_B}{2\theta + \gamma} < \min \left(\frac{(\theta - \gamma)N_A}{2\theta - \gamma}, \frac{(\theta + \gamma)N_B}{2\theta + \gamma} \right) \\
&\Leftrightarrow \frac{\theta N_B}{2\theta + \gamma} < \frac{(\theta - \gamma)N_A}{2\theta - \gamma}
\end{aligned} \tag{6}$$

Applying [Young \(1993\)](#), $E_{A>B}$ (resp. $E_{A=B}$) is the stochastically stable state if and only if $C_{E_{A=B} \rightarrow E_{A>B}}$ is below $C_{E_{A>B} \rightarrow E_{A=B}}$ (resp. above). This completes the proof of the proposition.

Proposition 2.2 implies that, if the minority group B is large enough, egalitarian hierarchy views are going to spread in the society in the long term. On the contrary, if group B is small, inegalitarian hierarchy views are going to persist.^{40 41}

Note that the threshold $\frac{(\theta-\gamma)(2\theta+\gamma)}{\theta(2\theta-\gamma)}$ separating the two equilibria decreases with $\frac{\gamma}{\theta}$. Indeed, when $\frac{\gamma}{\theta}$ increases, the cost of transitions from $E_{A>B}$ to $E_{A=B}$ decreases quicker than the cost of transitions from $E_{A=B}$ to $E_{A>B}$: much fewer mutations from the majority group become necessary to make the minority indifferent between $H_{A>B}$ and $H_{A=B}$. Therefore, the minority does not need to be as large as before for the long-term hierarchy to be egalitarian. The threshold decreasing with $\frac{\gamma}{\theta}$ means that the more inegalitarian the inegalitarian hierarchy is (i.e. the larger γ is with respect to θ), the less likely it is to persist in the long term. However, if the inegalitarian hierarchy is only slightly inegalitarian, it will nearly always persist in the long term (i.e. for nearly all sizes of the minority). This feature may contribute to explain why inegalitarian hierarchies are so pervasive in human societies ([Quillian et al., 2019](#)).

The two long-term equilibria that can persist in the model can be exemplified as follows. The long-term equilibrium $E_{A>B}$ can be seen as an illustration of the strongly inegalitarian equilibrium of hierarchy views that prevailed in the 1950s' United States, which largely favored White Americans over African Americans. In contrast, the equilibrium $E_{A=B}$, can be illustrated by the close to egalitarian equilibrium of hierarchy views that exists between the Swiss-German speaking majority and the French speaking minority in today's Switzerland ([Minority Rights Group International, 2020](#)).

The inegalitarian equilibrium $E_{A>B}$ where the minority group adopts the inegalitarian hierarchy view $H_{A>B}$, in which it holds a dominated social rank, may be interpreted as a case of "internalized domination". [Sidanius and Pratto \(2001\)](#) show that internalized domination can express itself in at least three forms: (1) an asymmetrical ingroup bias, whereby members of the dominant group exhibit higher ingroup bias than members of dominated groups ([Sidanius and Pratto, 2001](#) and [Fang et al., 1998](#)), (2) ideological asymmetry, implying that legitimizing myths (from Social Dominance Theory) justify the domination exerted by the dominant group and (3) group debilitating behaviors such as "outgroup favoritism".⁴²

40. Note that, in this model, although the egalitarian hierarchy view is always risk-dominant (indeed, $\theta^2 > (\theta+\gamma)(\theta-\gamma)$), it does not always persist in the long term. This is because the two groups are not equally sized. When $N_A = N_B$, the risk-dominant hierarchy view, the egalitarian one, persists in the long term.

41. When considering an alternative specification of payoffs, where members of a same group get a payoff of $\delta > 0$ with $\delta < \frac{(2\theta-\gamma)\theta}{2\theta+\gamma}$, the results are unchanged except that the threshold for $\frac{N_B}{N_A}$ is $\frac{(\theta-\gamma)(2\theta+\gamma)-\delta(2\theta-\gamma)}{\theta(2\theta-\gamma)-\delta(2\theta+\gamma)}$.

42. Examples of "outgroup favoritism" include the cases of African Americans in the United States, Maori children in New Zealand, Black children in the Caribbean and Ethiopian Jews in Israel ([Fang et al., 1998](#), [Sachdev and Bourhis, 1987](#) and [Sidanius and Pratto, 2001](#)).

4. Three-group model

Having analyzed a simple two-group version of the model, we can now turn to the core contribution of the present paper, namely the generalization of the model to a society composed of one majority ethno-cultural group and two minorities. The presence of two minorities enriches the model significantly by allowing for a discussion of reciprocal impacts that minorities may have on each other’s social statuses. It allows to throw new light on the phenomena of “cultural backlash” and “asymmetrical homophily”, and on the impact of the immigration of a new minority and and of minority fragmentation.

4.1. Model setup

In this three-group model, the society is composed of one majority group A of size N_A and two minorities B and C of sizes N_B and N_C such that $N_A > N_B > N_C$. Each individual may adopt one of four distinct hierarchy views: (1) a *majority-dominated inegalitarian* hierarchy view $H_1 = H_{A>B=C}$ in which group A dominates groups B and C and these two minorities are equal, (2) a *minority-differentiating* hierarchy view $H_2 = H_{A=B>C}$ in which group A and B are equal and dominate group C , (3) a *cumulatively inegalitarian* hierarchy view $H_3 = H_{A>B>C}$ in which group A dominates group B which dominates group C , (4) an *egalitarian* hierarchy view $H_4 = H_{A=B=C}$ in which all groups are considered equal.^{43 44} The set of hierarchy views in the three-group model is denoted by: $\mathcal{H}_{3g} = \{H_1, H_2, H_3, H_4\}$.

As in the two-group model, at each period of time t , each individual in the population interacts with ξ other individuals belonging to the three groups in proportions that correspond to the share of the different groups in the overall population.⁴⁵ The payoffs extracted from each bilateral interaction, i.e. the $V_{i,l}(h)$ are obtained by a straightforward generalization of the two-group model. They are summarized in the following tables:⁴⁶

43. Note that, in all of the four hierarchy views considered, larger groups dominate or are equal to smaller ones. This assumption allows to limit the number of possible hierarchy views and corresponds to a large majority of cases in reality (61% of cases, see Section 6). It has been checked in the two-group model that allowing for a hierarchy view where the minority dominates the majority only marginally affects the model’s predictions. Besides, it has been verified that allowing for two additional hierarchies, $H_5 = H_{A=C>B}$ and $H_6 = H_{A>C>B}$, does not substantially alter the qualitative insights provided by the model (see Figure 15 in Appendix D). Eventually, the long-term equilibria obtained when suppressing H_3 from the set of possible hierarchy views and when suppressing both H_3 and H_2 are also discussed in Appendix D.

44. The *majority-dominated inegalitarian* hierarchy view H_1 may be exemplified by the perceived hierarchy between the majority group and the African and German minorities in Ireland, where both minorities are significantly and similarly discriminated against (see McGinnity and Lunn, 2011). The *minority-differentiating inegalitarian* hierarchy H_2 may correspond to the hierarchy between the dominant White Brazilian majority and Japanese minority, and the colored (Brown and Black) Brazilians, as while the Japanese minority enjoys a “model minority” status, the colored Brazilians face sizable discrimination (see Tsuda, 2000 and Arcand and D’hombres, 2004). The *cumulatively inegalitarian* hierarchy may be exemplified by the cumulative hierarchy between the White majority group in Portugal, the Brazilian minority (which is associated with an intermediary social rank) and the African Black minority (see de Renó Machado, 2004). Eventually, H_4 could be illustrated by the hierarchy between the Swiss-German-, the French-, and the Italian-speaking minorities in Switzerland (see Minority Rights Group International, 2020).

45. This assumption is relaxed and discussed in one of the extensions (see paragraph 11.1 in Appendix D).

46. In this specification of payoffs, it is assumed that “strong coordination” is necessary for payoffs to be non-zero, i.e. the members of two different groups must agree on the full hierarchy to be able to cooperate efficiently. In Appendix D, the results obtained when considering, alternatively, that “weak coordination” is sufficient (i.e. interactions between two groups are independent on the status of the third group), is discussed. The most realistic specification probably lies somewhere between these two extreme cases, as the status of the third group in a two-group interaction is likely to be considered of lesser importance as compared to the status of the two interacting groups (however, some interactions implying individuals from the three distinct ethno-cultural groups may require coordination of all individuals on the full ethno-cultural hierarchy to be productive).

		Group A			
		$H_{A>B=C}$	$H_{A=B>C}$	$H_{A>B>C}$	$H_{A=B=C}$
Group B	$H_{A>B=C}$	$\theta - \gamma, \theta + \gamma$	0,0	0,0	0,0
	$H_{A=B>C}$	0,0	θ, θ	0,0	0,0
	$H_{A>B>C}$	0,0	0,0	$\theta - \gamma, \theta + \gamma$	0,0
	$H_{A=B=C}$	0,0	0,0	0,0	θ, θ

		Group A			
		$H_{A>B=C}$	$H_{A=B>C}$	$H_{A>B>C}$	$H_{A=B=C}$
Group C	$H_{A>B=C}$	$\theta - \gamma, \theta + \gamma$	0,0	0,0	0,0
	$H_{A=B>C}$	0,0	$\theta - \gamma, \theta + \gamma$	0,0	0,0
	$H_{A>B>C}$	0,0	0,0	$\theta - \gamma, \theta + \gamma$	0,0
	$H_{A=B=C}$	0,0	0,0	0,0	θ, θ

		Group B			
		$H_{A>B=C}$	$H_{A=B>C}$	$H_{A>B>C}$	$H_{A=B=C}$
Group C	$H_{A>B=C}$	θ, θ	0,0	0,0	0,0
	$H_{A=B>C}$	0,0	$\theta - \gamma, \theta + \gamma$	0,0	0,0
	$H_{A>B>C}$	0,0	0,0	$\theta - \gamma, \theta + \gamma$	0,0
	$H_{A=B=C}$	0,0	0,0	0,0	θ, θ

The average payoff of an individual belonging to group i and holding hierarchy view h during a period of time t is therefore:

$$U_i(h) = \sum_{l \in \{i,j,k\}} \xi \frac{N_l p_{l,h} V_{i,l}(h)}{N_i + N_j + N_k} \quad (7)$$

where j and k index the two other ethno-cultural groups (i.e. $i \neq j \neq k$).⁴⁷

As in the two-group model also, at the beginning of each period of time, one individual is randomly drawn and has the possibility to change his hierarchy view. With probability $1 - \epsilon$, he chooses his best response to the distribution of hierarchy views in the population at $t - 1$ and, with probability ϵ , he adopts a random hierarchy view.

This dynamic defines a Markov chain with a finite state space, which is both irreducible and aperiodic and therefore admits a unique stationary distribution. Its stochastically stable state (or states), defined as in the two-group model, is the one associated with the lowest cost minimum spanning tree (see [Young, 1993](#)) in the complete oriented graph, in which nodes correspond to the four pure Nash equilibria and in which oriented edges are weighted by the transition cost between each pair of pure Nash equilibria (see [Figure 1](#) for a depiction of this graph and of an example of spanning tree associated with equilibrium

47. Note that an interesting feature of this three-group coordination model is that it is one of the few in which an agent's strategy must be used with more than one type of agents ([Neary, 2011](#)), i.e. the members of the two other ethno-cultural groups, which interests must both be taken into account.

E_2).^{48 49}

4.2. Model resolution

We clearly have the following lemma:

Lemma 3.1: *The 3-group static coordination game admits four pure Nash equilibria:*

- A “majority-dominated inegalitarian” equilibrium, E_1 , in which all individuals adopt the “majority-dominated inegalitarian” hierarchy view.
- A “minority-differentiating inegalitarian” equilibrium, E_2 , in which all individuals adopt the “minority-differentiating inegalitarian” hierarchy view.
- A “cumulatively inegalitarian” equilibrium, E_3 , in which all individuals adopt the “cumulatively inegalitarian” hierarchy view.
- An “egalitarian” equilibrium, E_4 , in which all individuals adopt the “egalitarian” hierarchy view.

Note that these four pure Nash equilibria are valued differently by the three ethno-cultural groups. The majority group A prefers to dominate both minorities (i.e. equilibria E_1 and E_3) and, as a second best, to dominate at least one minority (i.e. E_2). Thus, denoting by \succ_i the order of preferences of group i over the different pure Nash equilibria, we have $E_1 =_A E_3 \succ_A E_2 \succ_A E_4$. Minority B ranks the four pure Nash equilibria differently: $E_2 \succ_B E_4 \succ_B E_3 \succ_B E_1$. Eventually, minority C also has a different ranking: $E_4 \succ_C E_1 \succ_C E_2 =_C E_3$. As no two groups have perfectly aligned interests, each group will tend to favor transitions toward different equilibria. The long-term equilibrium will arise from these countervailing forces.

In order to determine the stochastically stable equilibrium, the following procedure proposed by Young (1993) is implemented:

1. Determination of the transition cost (i.e. the number of ϵ -mutations) necessary to go from each pure Nash equilibrium to each other Nash equilibrium.
2. Building of the oriented graph corresponding to the evolutionary game, as described above.
3. Identification of the minimum spanning tree leading to each pure Nash equilibrium (using Edmond’s algorithm which is explained hereafter).
4. Identification of the lowest cost minimum spanning tree (or trees) and its (or their) corresponding pure Nash equilibrium (or equilibria). This (or these) pure Nash equilibrium (or equilibria) is (or are) the stochastically stable state (or states), i.e. the equilibrium (or equilibria) that persists (or persist) in the long run.

Before implementing the procedure to determine the stochastically stable equilibrium, a few normalizations can be made. First, we can note that multiplying all group sizes by a same constant does not affect the model, so that the size of the majority group A can be normalized to 1 without any loss of

48. The minimum spanning tree associated with a specific pure Nash equilibrium is the oriented tree which spans over all pure Nash equilibria, which root is this specific pure Nash equilibrium and which edges are chosen so as to minimize the total cost of the tree (i.e. the sum of its edges’ weights).

49. Note that, comparably to the two-group model, the three-group model admits some mixed Nash equilibria but those are not evolutionarily stable in a coordination game.

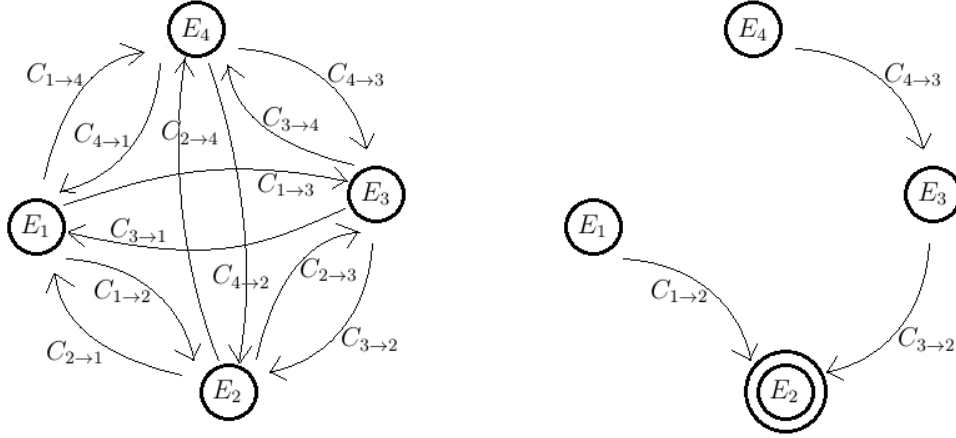


Figure 1

Note: $C_{i \rightarrow j}$ corresponds to the transition cost between E_i and E_j .

generality. Similarly, multiplying both θ and γ by a same constant leaves the model unchanged. Therefore, θ can be normalized to 1. In order to make the model more tractable, an additional assumption is made: it is assumed that γ is equal to $\frac{1}{2}$.⁵⁰ The two free parameters of the model are thus the minority sizes, N_B and N_C .

Even after those normalizations and simplification, the implementation of the resolution procedure's steps would be extremely burdensome if undertaken manually, as transition costs are piecewise functions of the two minorities' sizes, so that the minimum spanning trees leading to each equilibrium and the lowest cost minimum spanning tree or trees are also defined piecewise on numerous zones of set $\mathcal{P} \equiv \{(N_B, N_C) \in [0, 1]^2 \text{ s.t. } N_B > N_C\}$. The procedure is therefore implemented in a computer program using Mathematica, in order to analytically solve for the long-term equilibrium.⁵¹

Using the same type of reasoning as in the two-group model, it can be shown that the matrix of transition costs is the one represented in Table 1.⁵² As in the two-group model, transition costs increase with the sizes of the different groups, N_B and N_C . The *mutating* and *switching groups* corresponding to the transition between each pair of equilibria and depending on the sizes of the two minorities are depicted in Figures 10 and 11 in Appendix A.^{53 54} Figure 2 depicts a few examples of oriented graphs

50. It has been checked that variations in γ do not substantially alter the predictions of the model. See Appendix D and Figure 13.

51. The computer code is provided in Appendix E.

52. The exact procedure followed to compute the matrix of transition costs is explained in Appendix B and the corresponding Mathematica functions are produced in Appendix E.

53. The function *TransitionCostsMatrix* articulated with functions *Analysismutatinggroup* and *Analysisfirstswitchinggroup* allow to obtain these figures. They are produced in Appendix E.

54. The same trade-off as in the two-group model is at play for the choice of the *mutating* and *switching groups*: on the one hand the payoff of the *switching group* in the initial equilibrium must be as low as possible while, on the other hand, the effect of the mutation of one member of the *mutating group* on the gap between the payoffs of the first *switching group* in the initial and final equilibria must be as large as possible. As an illustration, the top-left graphs in Figure 10 and Figure 11 show that, when both minorities are small enough, the transition from equilibrium E_1 to equilibrium E_2 is triggered by mutations of members of both minorities while the first *switching group* is the majority. In this case, the two criteria in the trade-off are optimized when choosing the majority A as the *switching group*. Indeed, the majority is the group that has the lowest initial payoff (as it only interacts with two very small groups) and is also the group that benefits the most from mutations of members of the other groups (the gap between the initial and final payoff of the majority is reduced by $2\theta + 1$ at each mutation of group C members). Note that the mutation of all members of group C towards E_2 is not sufficient to trigger a transition to the final equilibrium in this case, so that mutations of group B members are also necessary. When minority B is large and minority C is small, the two criteria in the trade-off contradict each other:

corresponding to the model's transition costs between the four pure Nash equilibria for different minority sizes. In this figure, the darkness of edges corresponds to the transition cost to go from the initial equilibrium of the edge to the final one.⁵⁵ We can observe that the ordering to transition costs varies a lot with minority sizes.

	E_1	E_2	E_3	E_4
E_1	0	$\frac{1+2N_C}{5}$ if $3N_C \geq 1$ $\frac{3N_B+2N_C}{5}$ if $9N_B + 6N_C \leq 5$ $\frac{1}{3}$ otherwise	$\frac{1+2N_C}{5}$ if $3N_C \geq 1$ $\frac{N_B+N_C}{2}$ if $N_B + 2N_C \leq 1$ $\frac{1-N_C}{2}$ otherwise	$\frac{1+2N_C}{4}$ if $2N_C \geq 1$ $\frac{3(N_B+N_C)}{5}$ if $9N_B + 4N_C \leq 5$ $\frac{1+N_C}{3}$ otherwise
E_2	$\frac{1+N_B}{3}$ if $3N_C \geq 2$ $\frac{2(N_B+N_C)}{5}$ if $3N_C \leq 2N_B$ $\frac{2N_B+3N_C}{6}$ otherwise	0	$\frac{2N_B+3N_C}{6}$ if $3N_C \geq 2$ $(3N_C \geq 2N_B$ & $3N_C \leq 2$ & $(3N_B + 6N_C \geq 1$ $2N_B \leq 3N_C))$ $\frac{2(N_B+N_C)}{5}$ otherwise	$\frac{1+N_B}{3}$ if $N_B + 3N_C \geq 2$ $\frac{N_B+N_C}{2}$ otherwise
E_3	$\frac{1+N_B}{3}$ if $9N_C \geq 2 + 5N_B$ $\frac{1}{2}$ if $2N_C \leq 1$ & $N_B + N_C \geq 1$ $\frac{1+3N_C}{5}$ if $2N_C \geq 1$ & $9N_C \leq 2 + 5N_B$ $\frac{N_B+N_C}{2}$ otherwise	$\frac{1+3N_C}{6}$ if $3N_C \geq 1$ $\frac{3N_B+2N_C}{5}$ if $9N_B + 6N_C \leq 5$ $\frac{1}{3}$ otherwise	0	$\frac{1+N_B}{3}$ if $9N_C \geq 2 + 5N_B$ $\frac{1+N_C}{3}$ if $2N_C \leq 1$ & $9N_B + 4N_C \geq 5$ $\frac{3(N_B+N_C)}{5}$ if $9N_B + 4N_C \leq 5$ $\frac{1+3N_C}{5}$ otherwise
E_4	$\frac{2(N_B+N_C)}{5}$	$\frac{2N_B+N_C}{4}$ if $3N_C \leq 2N_B$ $\frac{2(N_B+N_C)}{5}$ otherwise	$\frac{2(N_B+N_C)}{5}$	0

Table 1 – Matrix of transition costs

Notes: This table depicts the transition costs for each pair of initial and final equilibria depending on the relative sizes of minorities B and C . The cell in the i -th row and j -th column corresponds to the transition cost from the initial equilibrium E_i to the final one E_j .

Before deriving the minimum spanning tree associated with each equilibrium, it is important to understand why minimum spanning trees matter in the present context where multiple pure Nash equilibria are possible. The idea is that it allows to evaluate the minimum cost necessary to make all individuals in the society adopt the final hierarchy view, whatever the initial distribution of hierarchy views in the population (distribution in which the four hierarchy views may be represented). Indeed, starting from an arbitrary initial distribution and aiming for a specific final equilibrium, it may be unnecessarily costly to try and make each individual directly adopt the final hierarchy view. The efficient way to reach this final equilibrium would be, on the contrary, to follow the minimum spanning tree. For example, let us consider the following minimum spanning tree:

while minority B has the lowest initial payoff, majority A is still the group for which the gap between the initial and the final payoff is reduced the most by mutations of the other groups' members. The first criterium dominates, so that group B is the first *switching group*, while group C and A are *mutating groups*.

55. A black color means that the transition is the least costly one in the graph, while a nearly white color means that the transition is the most costly one in the graph. Intermediary costs have various darkness levels represented in the color scale at the bottom of the figure.

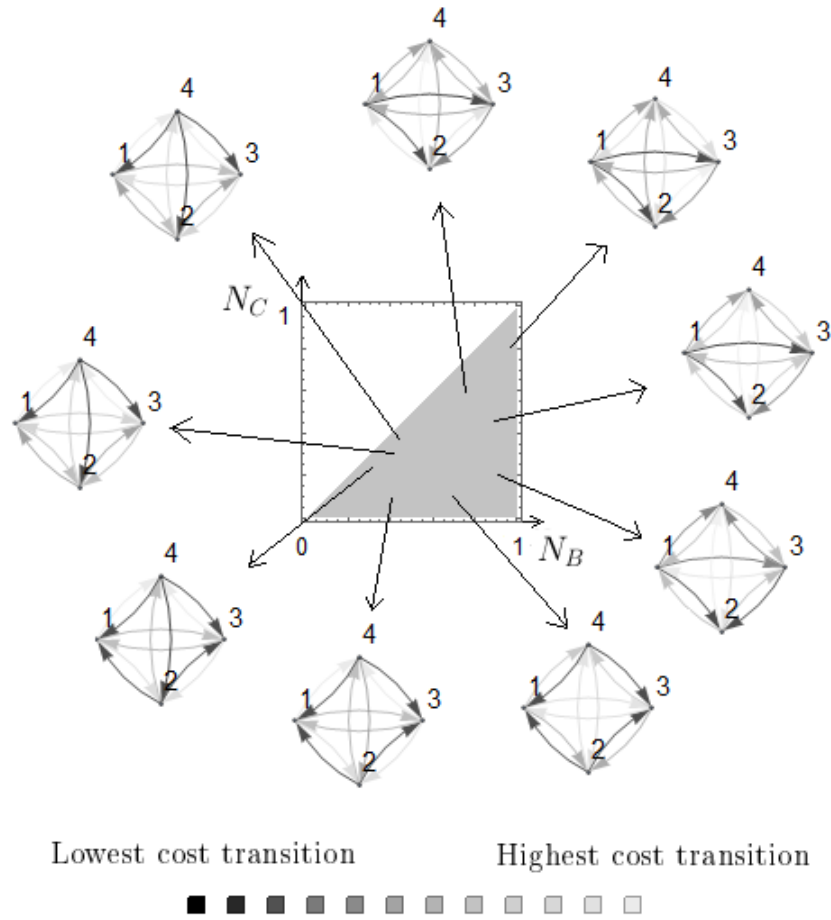
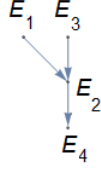


Figure 2 – Examples of mutation graphs for different sizes of the two minorities

Notes: In each example of mutation graph, the black arrows correspond to the lowest cost transitions and the very light gray arrows to the highest cost transitions. For the sake of readability, long-term equilibria E_1 , E_2 , E_3 and E_4 are simply denoted by 1, 2, 3 and 4 in the oriented graphs.



According to this minimum spanning tree, it is most efficient, in order to reach an overall agreement on hierarchy view H_4 , to first make individuals who hold hierarchy view H_1 or H_3 in the initial society adopt hierarchy view H_2 , and then to have all individuals who hold hierarchy view H_2 adopt the final hierarchy H_4 . This feature of the model is interesting as it suggests that the most efficient way to reach a given equilibrium hierarchy view is not by directly convincing individuals to adopt it, but by making them transit through intermediary hierarchy views progressively leading to the final one. For instance, if the final hierarchy view is the *egalitarian* one, it may be more efficient to make the population transit through increasingly egalitarian hierarchy views, to eventually reach the *egalitarian* one. This feature of the model appears to faithfully mirror the progressive integration of minority groups in societies, with an increasing share of the population granting minorities an intermediary social rank and then an equal rank to the majority (see, for example, the progressive recognition of the Irish minority, which used to be strongly discriminated against during the nineteenth century in the United States and progressively gained an equal footage to the White majority, [Ignatiev, 2012](#)).

The key role of minimum spanning trees in this model also implies that, for a specific equilibrium to persist in the long term, this equilibrium must be both easy to reach from any other equilibrium (possibly through transitions involving intermediary equilibria) and difficult to upset in any direction (i.e. whatever the equilibrium towards which the ϵ -mutations take place). In practice, it means that a given hierarchy view persists in the population in the long term, if it is easy enough to convince people with all other hierarchy views to adopt it and difficult enough to convince people with this given hierarchy view to adopt any other. Figure 2 provides an illustration of this double requirement. Indeed, it exemplifies the fact that the whole minimum spanning tree matters, as the equilibrium that can be reached from another equilibrium at the lowest cost is not necessarily the one that has the lowest overall minimum spanning tree: in the graph of transitions directly at the right of the highest one, E_3 and E_2 are the easiest equilibria to reach from another equilibrium (black arrows lead to each of them), but, at the same time, E_4 is hard to upset in all directions (only very light arrows lead away from it) so that it will be hard to reach E_3 and E_2 when starting at E_4 . Therefore, minimum spanning trees leading to E_3 and E_2 will be quite costly and, in the end, E_4 may be the equilibrium that persists in the long term.

At this point, Edmond’s algorithm is applied on each subdivision of \mathcal{P} that arises in the computation of the matrix of transition costs, in order to determine the minimum spanning tree associated with each equilibrium. Then, the four minimum spanning trees are compared on each subdivision of \mathcal{P} . This comparison may require even further subdivisions of set \mathcal{P} . In the end, on each of these further subdivisions, the stochastically stable equilibrium or equilibria are the ones associated with the lowest cost minimum spanning trees. Merging the subdivisions where the same equilibrium is stochastically stable the following long-term equilibria are obtained:⁵⁶

Proposition 3.2: *The above presented 3-group evolutionary coordination game admits the following stochastically stable states, depending on the sizes of the two minority ethno-cultural groups:*

⁵⁶. This result derives from the implementation of the previously described procedure. The corresponding functions (*EdmondAlgorithm*, *DeterminationLongTermEquilibriumForSpecificConstraints* and *DeterminationAllLongTermEquilibria*) are produced and explained in Appendices B and E.

- If $N_B + 6N_C \geq 5$, then E_4 is the unique stochastically stable state.
- If $N_B + 6N_C \leq 5$ and $2N_B \geq 1$ and $N_B + N_C \geq 5/6$, then E_2 is unique stochastically stable state.
- If $N_B + N_C \leq 5/6$ and $3N_C \leq 1$, then E_1 and E_3 are the two stochastically stable states.
- If $3N_C \geq 1$ and $2N_B \leq 1$, then E_3 is the unique stochastically stable state.

The stochastically stable equilibria depending on the sizes of the two minorities are represented in Figure 4 (also see Figure 3 for the often numerous minimum spanning trees corresponding to each stochastically stable equilibrium, depending on the sizes of the two minorities). As can be seen in Figure 4, the *egalitarian* hierarchy H_4 only persists when both minorities are large enough (it corresponds to the black area in this figure). Indeed, the *egalitarian* equilibrium E_4 , although not particularly easy to reach from the other equilibria in this case (i.e. arrows leaving it in Figure 2 are not particularly dark), is very hard to upset (only light arrows leaving it in Figure 2). The *minority-differentiating inegalitarian* hierarchy H_2 only persists if minority B is large enough and minority C is small enough (dark gray zone in Figure 4). In this case, transitions from E_1 or E_3 to E_2 indeed require very few mutations, as minority B can easily be convinced to switch (i.e. through very few mutations). The *cumulatively inegalitarian* hierarchy H_3 is the only one to persist if the two minorities have intermediary and quite comparable sizes (this corresponds to the intermediary gray zone in Figure 4). In this case, equilibrium E_3 is hard to upset in any direction (light gray arrows leaving it in Figure 2) and easy to reach from both the *egalitarian* equilibrium E_4 and the *minority-differentiating inegalitarian* one E_2 . E_1 and E_3 both persist in the long term if the two minorities are small enough (the nearly white zone in Figure 4). They are both easy to reach from E_2 and E_4 through mutations of the small minorities.

In order to know the probability associated with E_1 and E_3 in the long term when both minorities are small, the number of minimum spanning trees leading to each of them must be compared (see Young, 1993 for the justification). The probability of each long-term equilibrium is then the number of minimum spanning trees leading to it, divided by the total number of minimum spanning trees. As can be seen in Figure 3, whenever E_1 and E_3 are both stochastically stable, they are associated with the same number of minimum spanning trees. Therefore, they are both associated with a probability of 1/2 in the long term. The prediction of two different possible equilibria for a same value of parameters is an original feature allowed by the multiplicity of groups in this model. We will see in the final empirical tests that it is not an unrealistic prediction, but that it seems to be corroborated by the data.

Another way to look at the results of Proposition 3.2 is through the lens of fractionalization and polarization indices, as defined in Bazzi et al. (2019). In the model's setup, fractionalization is maximized when both minorities are as large as the majority. Polarization, on the other hand, is maximized when one minority is as large as the majority and the other is very small. Both indices reach their minimum, 0, when the two minorities have a zero size. Thus, it is clear that, when fractionalization is large enough, an *egalitarian* hierarchy view spreads in the population, whereas when polarization is large enough, a *minority-differentiating inegalitarian* hierarchy view is adopted by all. When both fractionalization and polarization are small enough, a *majority-dominated inegalitarian* or a *cumulatively inegalitarian* hierarchy view is adopted. Eventually, for intermediary values of both indices, the *cumulatively inegalitarian* hierarchy may persist.

With respect to Young (1993), the methodological contribution of the present paper consists in showing how Edmond's algorithm can be implemented in a Mathematica program in order to obtain minimum spanning trees, depending on the model's parameters. More precisely, Edmond's algorithm (Edmonds, 1967 and Chu and Liu, 1965) allows to identify the minimum spanning tree leading to each

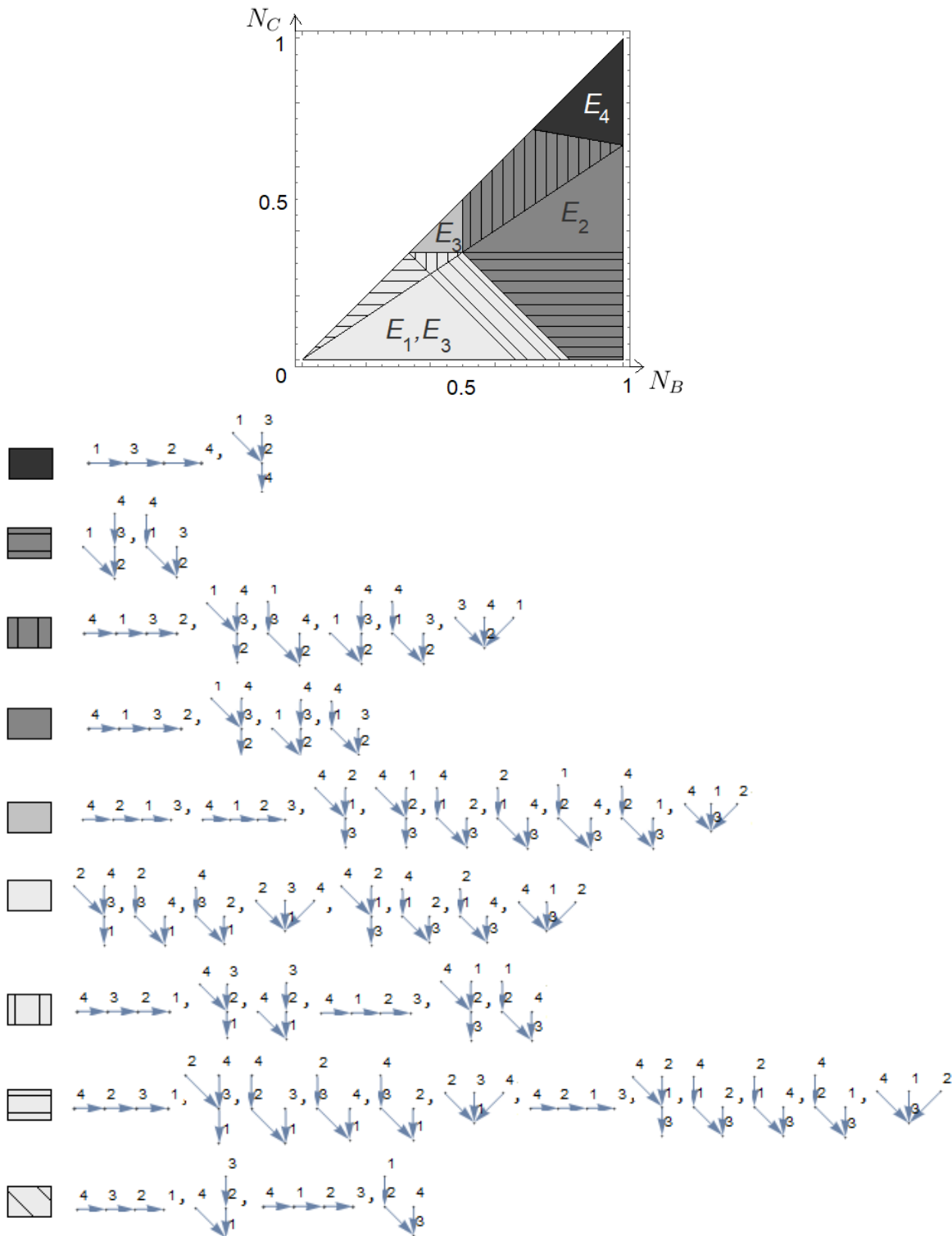


Figure 3 – All minimum spanning trees leading to each long-term equilibrium depending on the size of the two minorities

Note: For the sake of readability, equilibria E_1 , E_2 , E_3 and E_4 are simply denoted by 1, 2, 3 and 4 in the minimum spanning trees.

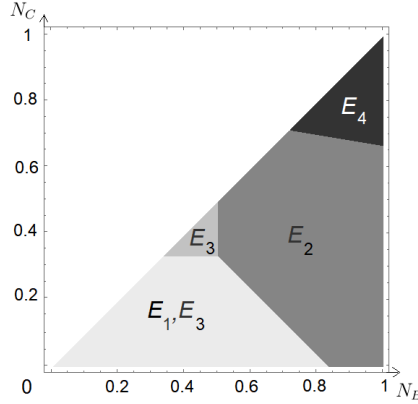


Figure 4 – Long-term equilibrium (or equilibria) depending on the two minorities' sizes, N_B and N_C .

given pure Nash equilibrium (called the “root”), proceeding as follows.⁵⁷ Starting from the complete oriented graph, a sub-graph is built, in which only the lowest cost edge leaving each equilibrium (except the root) is kept. If this sub-graph contains a cycle, a new version of the initial oriented graph is built, in which the nodes belonging to the cycle are merged into one unique node and some edges are given new weights. Then, by induction, the algorithm builds, once more, the sub-graph of this sub-graph that contains only the lowest cost edge leaving each node. It repeats the same steps until reaching a sub-graph without any cycle. It then reconstitutes the minimum spanning tree associated with the equilibrium of interest in the initial oriented graph...⁵⁸ Edmond’s algorithm relies on the comparison of the weights of all edges in the graph, which may require (for comparisons to be unambiguous) to subdivide the mathematical set \mathcal{P} into small sub-zones, on which the procedure is applied separately.⁵⁹ This algorithm is key for the theoretical resolution of the multi-group version of the model.

5. Discussion of the three-group model

5.1. Link between a minority’s size and its status

We can now analyze the model’s long-term equilibria to show that this model can explain the puzzling evidence proposed by the empirical literature, in which both cases where the growth of a minority increased its status and cases where the growth of a minority reduced its status have been reported.⁶⁰ To this aim, we can first note that a marginal increase in a group’s size has a linear (or null) effect on all transition costs. Indeed, it increases linearly (to different extents) the cost of transitions in which it is the sole *mutating group* or in which it is neither a *mutating* nor the *switching group*. It also impacts linearly the cost of transitions requiring two distinct *mutating groups*, if it is one of the two necessary *mutating groups*, positively or negatively depending on the “efficiency” of the mutations of the other *mutating group*. The expansion of a minority, however, has no effect on the cost of transitions in which it is the *switching group*. In the end, this expansion leads to a change in the ethno-cultural statuses only if the ranking of the different transition costs is sufficiently altered, so that the equilibrium

57. See Figure 9 in Appendix A to illustrate the algorithm’s steps.

58. See Appendix C for a more detailed description of the algorithm.

59. The computer program used for the implementation of Edmond’s algorithm is provided in Appendix E.

60. See the corresponding references in [Schlueter and Scheepers, 2010](#), as well as [Norris and Inglehart \(2019\)](#), [Halla et al. \(2017\)](#), [Tabellini \(2020\)](#), [Abrajano and Hajnal \(2015\)](#), [Steinmayr \(2020\)](#), [Dustmann et al. \(2019\)](#) and [Calderon et al. \(2019\)](#).

associated with the lowest cost minimum spanning tree changes. This only happens in the vicinity of the frontiers between equilibrium zones in Figure 4.

From Figure 4, it is clear that an increase in minority B 's size leads to an increase in its status when the society shifts (horizontally in the figure) from equilibrium E_3 or the two-equilibria zone E_1-E_3 to equilibrium E_2 , while it leads to a decrease in its status when the society shifts from E_2 to E_4 .⁶¹ In contrast, an increase in minority C 's size leads to an increase in its status when the society shifts (vertically in the figure) from E_2 to E_4 , but to a decrease in its status when the society shifts from E_1-E_3 to E_2 or from E_1-E_3 to E_3 . Figure 5 recapitulates these effects, with black arrows corresponding to a negative impact of an increase in a group's size on its status and white arrows corresponding to a positive impact.

The dominating mechanism at play depends on the considered shift.

The shifts in which the status of a minority is worsened by an increase in its size can be classified in two categories. In the first one, the shifts may be interpreted as an instance of "*cultural backlash*", because the increase in the minority's size leads to an increase in the status of the other minority, which reaches the highest status in the society at the expense of the expanding minority. This situation corresponds to shifts from E_1-E_3 to E_2 or from E_1-E_3 to E_3 due to an increase in group C 's size. In the second category, the shifts may be interpreted as a "*positive externality*", because the increase in the minority's size allows the second minority to reach the highest status in the society, together with the expanding minority. It corresponds to shifts from E_2 to E_4 due to an increase in group B 's size.

The mechanisms leading to these two categories of shifts are distinct but, in both cases, the ranking of some key transition costs has changed. In the "*cultural backlash*" shifts, these key transition costs are, around the frontier between E_1-E_3 and E_2 , $C_{1 \rightarrow 2}$ and $C_{3 \rightarrow 2}$, which become less costly than $C_{2 \rightarrow 1}$ and $C_{2 \rightarrow 3}$, and, around the frontier between E_1-E_3 and E_3 , $C_{1 \rightarrow 3}$, which become less costly than $C_{3 \rightarrow 2}$.⁶² The underlying mechanism is as follows. All the corresponding transitions require two *mutating groups*, one of which is minority C .⁶³ Minority C is always the most "efficient" *mutating group*, in the sense that its mutations have a large impact on the payoff of the *switching group*, because its status is always the lowest so that its impact on other groups' payoffs and, in particular, on the *switching group*'s payoff, is large. If the "efficiency" of the second *mutating group*'s mutations is much lower (i.e. it has little impact on the payoff of the *switching group*), an expansion of minority C will allow to increase a lot the average "efficiency" of the mutations required to trigger the shift while, if the "efficiency" of the second *mutating group*'s mutations is equal or only slightly lower than that of minority C , an expansion of minority C will not improve so much the average "efficiency" of the mutations required to trigger the transition.⁶⁴ The "*cultural backlash*" thus happens because minority C 's expansion has an indirect effect on the number of mutations required from the second *mutating group*. In terms of interpretation, the expansion of the minority does not benefit it because it is allied, for the obtention of more egalitarian hierarchy views, to members of a very influential ethno-cultural group, whom it is not very efficient to replace in the alliance.

In the "*positive externality*" cases (around the frontier between E_2 and E_4), the key transition cost

61. The status of a group is said to increase if the number of individuals dominating it decreases or if the number of individuals dominated by it increases.

62. This can be inferred from comparing the minimum spanning trees on both sides of the frontier of interest.

63. Two *mutating groups* are required because minority C is relatively small and thus does not suffice to trigger a transition.

64. Transitions $1 \rightarrow 2$ and $3 \rightarrow 2$ are triggered by mutations of C and A and A 's mutations are not very "efficient", while transitions $2 \rightarrow 1$ and $2 \rightarrow 3$ are triggered by mutations of C and B and B 's mutations are quite efficient. Thus, the expansion of C allows to reduce the mutation cost more in the former cases than in the latter ones. Similarly, transition $1 \rightarrow 3$ is triggered by mutations of C and A and A 's mutations are not very efficient, while transition $3 \rightarrow 2$ is triggered by mutations of C and A with A 's mutations being quite efficient. Thus, the expansion of C allows to reduce the transition cost more in the former case.

is $C_{2 \rightarrow 4}$ becoming less costly than $C_{4 \rightarrow 1}$, $C_{4 \rightarrow 2}$ and $C_{4 \rightarrow 3}$. The mechanism can be explained as follows. In all these transitions, only one *mutating group* is required. In the transition $2 \rightarrow 4$, the *switching group* is C , while in the other transitions, it is A . An expansion of group B increases the restoring force in all these transitions, but this increase is larger when C is the *switching group*, because group B holds a superior status to group C , while it is not the case when the *switching group* is A . The “*positive externality*” thus happens because group B has a larger impact on group C (the *switching group* for transition $2 \rightarrow 4$ than on group A (the *switching group* for the other transitions)). To interpret this, we may say that the expansion of the largest minority benefits more the smallest minority in the *egalitarian* hierarchy view than it benefits the majority group in the *minority-differentiating inegalitarian* hierarchy view, which leads to the adoption of the *egalitarian* hierarchy view.

Regarding the shifts in which the status of a minority is improved by an increase in its size, they happen because the expanding group is the *switching group* in the key transitions leading to the new equilibrium and not in the key transitions leading away from it. The expansion of the minority thus hampers the latter transitions but not the former ones, which leads to the persistence of the new equilibrium.⁶⁵ In other words, in these cases, the minority’s expansion allows it to hamper transitions away from its preferred hierarchy view.⁶⁶

5.2. Asymmetric homophily between groups

Considering that the model refers not to a whole society, but to a sub-population of it, the model can also be interpreted in terms of “asymmetric homophily”.⁶⁷ Indeed, we can observe that, in the zone just below the boundary between E_1 - E_3 and E_2 , group B members have an interest in attracting additional group C members into the sub-population, in order to shift upward the share of group C members in the sub-population and improve their own status (by crossing the boundary from E_1 - E_3 to E_2 in Figure 4). On the opposite, group C members would see their own status reduced if attracting additional group B members into the sub-population, as it would also change the long-term equilibrium from E_1 - E_3 to E_2 . Therefore, although group B members seek interactions with group C members, the latter will avoid it. This can be interpreted as a case of “asymmetric homophily” between groups B and C . Similarly, in the zone just below the boundary between E_2 and E_4 , group C members have an interest in bringing in additional group B members to the sub-population, while group B members would see their own status reduced if bringing in additional group C members. This produces a second case of “asymmetric homophily” in the other direction.

65. The expansion of B leads to a shift from E_1 - E_3 to E_2 because group B is the *switching group* in $1 \rightarrow 2$ and $3 \rightarrow 2$ but not $2 \rightarrow 1$ and $2 \rightarrow 3$. The expansion of B leads to a shift from E_3 to E_2 because group B is the *switching group* in $3 \rightarrow 2$ but not $2 \rightarrow 1$ and $2 \rightarrow 3$. The expansion of C leads to a shift from E_2 to E_4 because group C is the *switching group* in $2 \rightarrow 4$ but not $4 \rightarrow 1$, $4 \rightarrow 2$ and $4 \rightarrow 3$.

66. Note that the mechanisms at play in this model differ widely from those proposed in the literature to explain either positive or negative effects of a minority’s size on its status. Indeed, explanations for the “cultural backlash” in the literature are of two main natures. Some researchers (Inglehart and Norris, 2016, Fouka et al., 2020a and Golder, 2016) advance cultural arguments, generally arguing that natives feel culturally threatened by the growth of minorities adhering to different cultures and world views, which leads them to exclude minorities from their in-group (Social Categorization Theory) and/or to exacerbate their world views in the opposite direction (endogenous identity, group-threat theory). Some others (Malhotra et al., 2013, Scheve and Slaughter, 2001 and Rodrik, 2020) defend economic arguments according to which some natives feel threatened by the expansion of minorities competing with them on the job market, especially in times of economic downturns, leading them to adhere to anti-minorities theories. These concerns can be ego-tropic (individuals are pessimistic about their own situation) or socio-tropic (individuals are pessimistic about the impact of immigration on the nation’s economy as a whole). See Burns and Gimpel (2000) and Hainmueller and Hopkins (2014) on that matter.

67. See for example Bikmen (2011) for some evidence on the subject.

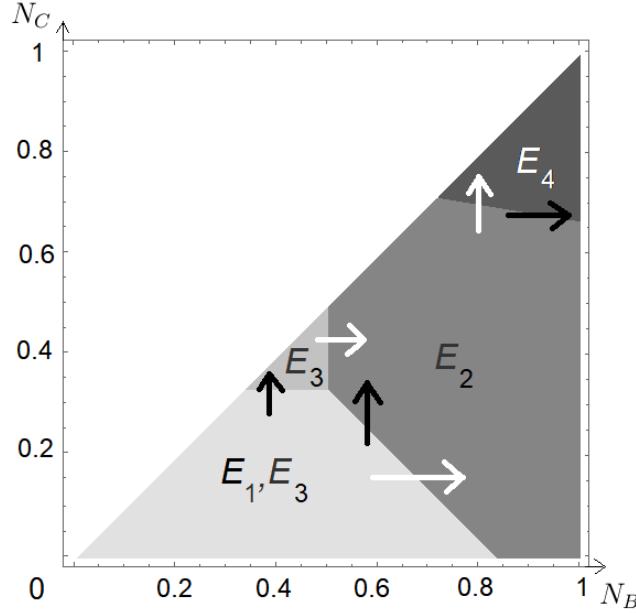


Figure 5 – Impact of a minority's size increase on its status, depending on N_B and N_C .

Note: Horizontal (resp. vertical) arrows correspond to increases in the size of minority B (resp. C). Black (resp. white) arrows correspond to cases where an increase in a minority's size leads to a decrease (resp. an increase) in this minority's status.

5.3. Comparison of the two- and three-group models

Comparing the results yielded by the two- and three-group models, it is also possible to discuss the impact on hierarchy views and on groups' utilities of the introduction of a new minority and of the splitting of one minority into two sub-minorities, when starting with a two-group setup. To allow for the comparisons of the two-group and three-group models, it is necessary to adopt the same normalizations and assumptions in both models: $\theta = 1$, $N_A = 1$ and $\gamma \approx 1/2$. With these normalizations and assumptions, the threshold of the two-group model between the egalitarian and inegalitarian equilibria becomes equal to $\frac{5}{6} \approx 0.83$.

5.3.1. Impact of the arrival of a new minority

Starting from a two-group model with majority A and minority B , the arrival of a new and smaller minority group C , keeping the sizes of the two preceding groups constant, leads the society to evolve from the long-term equilibrium of the two-group model with only groups A and B to the long-term equilibrium of the three-group modes with majority A , old minority B and the new minority C .⁶⁸ In theory, this arrival has several impacts: it embeds the statuses of group A and group B into more complex hierarchy views involving, in addition to these two groups, minority C , it intertwines the incentives of group C with those of the two previous groups (in some transitions between equilibria, the incentives of group C will be aligned with those of group A and, in others, with those of group B), and it increases the total size of minorities in the society.

The comparison of the two models is depicted in the left graph in Figure 6, where the vertical black line corresponds to the threshold on N_B in the two-group model: to the right of this line, the egalitarian equilibrium persists in the long term in the two-group model and, to its left, the inegalitarian equilibrium

68. For example, such a transition would be triggered by refugee movements or immigration.

persists. From this Figure, it clearly appears that, as minority C is hurt by the *majority-dominated inegalitarian* hierarchy view H_1 , this hierarchy view becomes scarcer with the arrival of minority C (i.e., it persists for a smaller set of values of N_B). Additionally, we can observe that, as minority C 's favorite hierarchy view is the *egalitarian* hierarchy view H_4 , this hierarchy view becomes more prevalent with the arrival of the new minority (i.e., it persists for a larger set of values of N_B). For H_2 and H_3 , the predictions are a little less straightforward. The hierarchy view H_2 , which is minority B 's favorite, persists, not only for sizes of B for which $H_{A>B}$ used to persist in the two-group model, but also for smaller ones. This is due to the fact that minority C , similarly to minority B , has little interest in the *majority-dominated inegalitarian* hierarchy view H_1 and to the fact that it is indifferent between H_3 and H_2 .⁶⁹ Eventually, the hierarchy view H_3 persists at least half of the time when both minorities are small, which is also beneficial to minority B , as it grants it an intermediary status.⁷⁰ All in all, the arrival of a new minority has either a positive or null effect on the old minority's status, while leading to a negative or null effect on the majority's status.

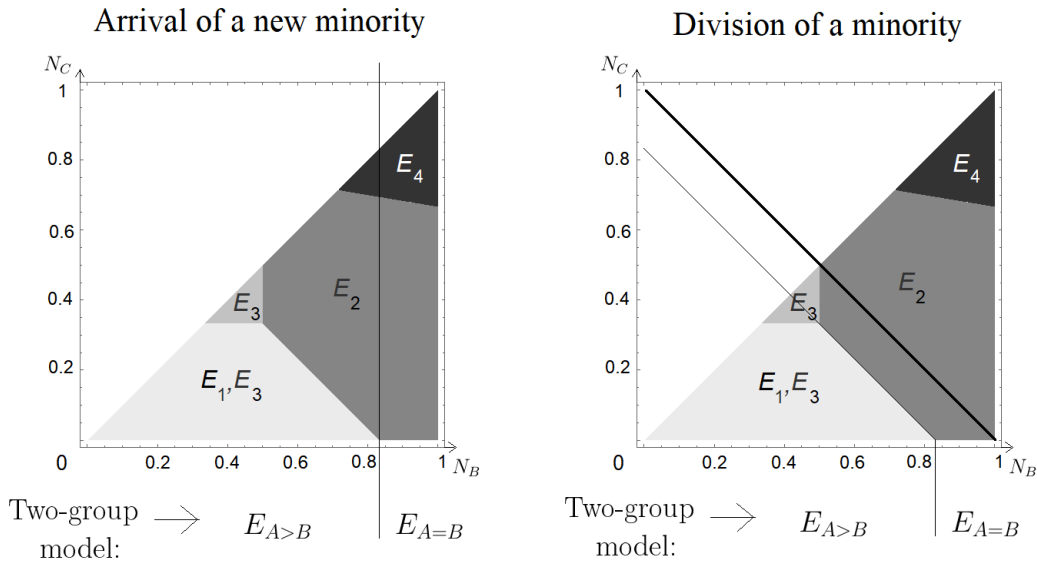


Figure 6 – Impact of the arrival of a new minority and of the division of a minority

69. This positive impact of the arrival of a new minority C on minority B 's status can explain the empirical results obtained by Fouka et al. (2020b) and Fouka et al. (2020a), in which the arrival of numerous African Americans in North US city centers (resp. of numerous Mexicans in US states) had a positive impact on the perception of European minorities (resp. of the African American minority) by the White majority.

70. More precisely, the effects of the arrival of the new minority C are as follows. If the old minority B and the majority A held an equal social rank before the arrival of the new minority (i.e. $N_B > 0.83$), then their social ranks remain equal (in both equilibria E_2 and E_4), but, if the new minority is small enough, groups A and B now dominate the new minority (i.e. equilibrium E_2 persists in the long term). If the old minority B instead used to be dominated by the majority before the arrival of the new minority (i.e. $N_B < 0.83$), then this arrival improves minority B 's status: the minority B may become equal to the majority if equilibrium E_4 or E_2 persists in the long run, or obtain an intermediary social rank if equilibrium E_3 or equilibria E_3 and E_1 persist. In contrast, the majority's status may be reduced by this arrival: if the total size of the two minorities is large enough, the majority is not the only dominant group anymore (i.e. E_2 or E_4 persists), if this total size is small enough, the majority remains the only dominant group.

5.3.2. Impact of the division of a minority

Starting, once more, from the two-group model with majority A and minority B_i (i stands for “initial”), we can now envisage the impact of the split of minority B_i into two sub-minorities, B_f and C_f . The size of group B_i in the two-group model is equal to the sum of the sizes of groups B_f and C_f in the three-group model ($N_{B_i} = N_{B_f} + N_{C_f}$). The comparison of the two models when a minority gets divided is depicted on the right graph in Figure 6. The thin oblique black line corresponds to the threshold between the two equilibria in the two-group model, translated in terms of N_{B_f} and N_{C_f} such that the total size of the new minority groups ($N_{B_f} + N_{C_f}$) equals the threshold. Comparisons of the two-group and three-group models after the split will similarly have to be made along oblique lines parallel to this one to ensure that the sizes of the two new minority groups add up to the size of the old one. The zone above the thick oblique line cannot be reached after the division of the old minority, because this old minority’s size was below the one of the majority (i.e. $N_{B_i} < N_A = 1$) so that the sizes of the two new minorities can only add up to less than 1 (i.e. $N_{B_f} + N_{C_f} < 1$). In theory, the division of a minority has several effects: it creates the possibility of new status distinctions and decouples the interests of the members of the two new minorities, while keeping the total size of minorities in the society constant.

In the right graph of Figure 6, we can observe that the division of a minority nearly always benefits the largest new minority created at the expense of the smallest one, either by allowing the largest new minority to obtain a status equal to the majority or by allowing it to obtain an intermediary status, dominating in both cases the smallest new minority. The only exception corresponds to the case where the old minority was large and the two new minorities have comparable sizes (in which case $H_{A=B_i}$ in the two-group model becomes H_3 in the three-group model), as the division leads both new minorities to be relegated to a dominated status.⁷¹

It implies that, in a society with several ethno-cultural groups, notable incentives exist that encourage the fragmentation of minorities instead of their fusion or union. Indeed, if some members of a minority decide to separate from the rest of the minority, for example by creating a new cultural or physical distinction within the minority (light vs dark Blacks for example⁷²), they can improve their social status at the expense of the rest of the minority.

The effect of the merge (or alliance) between two minorities is simply the exact opposite of minority split: instead of going from the two-group model to the three-group model, we go from the three-group model to the two-group model, thus, positive impacts become negative impacts and vice versa. Starting from the three-group model, the two minorities benefit from a merge if society is above the thin oblique line in E_3 . In this case, after merging, the unique minority obtains a status equal to the majority’s.⁷³ The model thus suggests that the matter of ethno-cultural hierarchies may give rise to alliances between minorities or free-riding depending on the ethno-cultural context, following dynamics similar to those

71. More precisely, if the old minority B_i used to have a social rank equal to the majority’s (i.e. $N_{B_i} > 0.83$, corresponding to the zone between the two oblique lines), the split of the minority is always detrimental to the smallest new minority C_f (which becomes dominated both in equilibrium E_2 and E_3) and may either benefit (if E_2 persists) or be detrimental (if E_3 persists) to the largest new minority B_f . If the old minority B_i used to be dominated (i.e. $N_{B_i} < 0.83$, corresponding to the zone below the lowest oblique line), the split improves the status of the largest new minority B_f (both if E_3 or E_3 and E_1 persist in the long term), which becomes intermediary, but hurts the smallest new minority C_f , which is now dominated both by the majority and the new minority B_f .

72. A typical example for this corresponds to African-American elite clubs before the civil rights movement, which chose to divide the “black” category into “light” and “dark black” and discriminate against the latter (Graham, 2009). Another example is provided by London inhabitants of Pakistani origin, who resist being assimilated with Indians in the “South-Asian” category (Khan, 1976).

73. Examples of merge or alliance between several minorities have often been documented: see for example the development of pan-ethnic identities among “Asians” or “Hispanics” in the United States (Padilla, 1986, Le Espiritu, 1992 and Okamoto, 2003) and the development of the Bangala and Bakongo ethnic groups in Congo (Young, 2015).

reported for other types of conflicts (Bloch, 2012).

5.4. Discussing economic impacts

We can now briefly discuss the economic conclusions that can be drawn from the model, by analyzing a very simple extension of it. So far, it has been assumed that egalitarian and inegalitarian interactions have the same level of overall productivity and all produce a total payoff of 2θ (i.e. $\theta+\theta$ or $\theta-\gamma+\theta+\gamma$). In this context, inegalitarian and egalitarian equilibria yield the same Benthamite social welfare (defined as the sum of payoffs of all individuals in the society) at each period of time, while the *egalitarian* equilibrium self-evidently brings the largest Rawlsian social welfare (defined as the minimum of payoffs of all individuals in the society).

Let us now assume that egalitarian interactions bring a payoff θ' to each interacting individual, which only very slightly differs from θ . This assumption modifies the model very marginally so that, in the long term, for θ' sufficiently close to θ , only the borders of the zones on which equilibria persist are shifted (the long-term equilibrium within each zone is preserved, as its minimum spanning tree is strictly less costly than the other Nash equilibria's).

It is not evident to theoretically anticipate whether, in reality, egalitarian or inegalitarian hierarchy views should yield the highest overall payoff and productivity. Indeed, one can expect that an inegalitarian hierarchy may allow for the easy designation of a leader in the interaction, which may improve its efficiency. But it may also lead to "stereotype threat" or a sentiment of injustice on the side of the dominated group, causing it to under-perform, while the dominant group, subject to "stereotype lift", may, on the contrary, over-perform. Furthermore, an egalitarian hierarchy may allow for a better exchange of ideas than an inegalitarian hierarchy. Despite this theoretical ambiguity, the empirical literature on the subject suggests that inequality between individuals, when perceived as unfair, tends to reduce participation to public goods and increase sabotage,⁷⁴ thereby reducing aggregate welfare. When not perceived as unfair, inequality may, however, favor collective action and increase participation to public goods (Simpson et al., 2012). In the specific case of ethno-cultural hierarchies, there appears to be little chance that an inegalitarian hierarchy would be perceived as fair by discriminated groups. Therefore, it is most probable that *egalitarian* hierarchy views are, economically speaking, the most efficient. Note that the importance of the perceived fairness of hierarchies in the determination of overall productivity may explain why dominant groups so ardently try to justify their domination through what Sidanius and Pratto (2001) call "legitimizing myths".

Coming back to the present extension of the model, considering that egalitarian interactions are marginally more productive than inegalitarian ones as suggested by the literature, i.e. $\theta' > \theta$, the *egalitarian* equilibrium E_4 becomes the socially most desirable one, both in terms of Rawlsian and Benthamite social welfare, and the *minority-differentiating inegalitarian* equilibrium E_2 becomes the second-best.⁷⁵ The model therefore predicts that interactions between ethno-cultural groups will be under-efficient if minority groups are small, in which case E_3 or E_1 persists in the long term. Most Western countries, with small minority groups and cumulatively inegalitarian ethno-cultural hierarchies, may be considered as entering into this category. The model thus suggests that the evolutionary process at play for the determination of ethno-cultural hierarchies in these countries selected economically under-efficient hierarchy views.

74. See (Waring and Bell, 2013), Anderson et al. (2008), Nishi et al. (2015), Fehr (2018), Dickinson et al. (2018) and Sadrieh and Verbon (2006).

75. If, instead, one assumes that inegalitarian interactions are marginally more productive, i.e. $\theta > \theta'$, then the society is the most productive in the *cumulatively inegalitarian* equilibrium E_3 . The second most productive equilibrium in this case is E_1 , which is reached when both minorities are small enough.

6. Empirical test

I now propose an exploratory empirical test of the model’s predictions, using the “Ethnic Power Relations” (EPR) database built by [Vogt et al. \(2015\)](#), which is publicly available on ETH Zürich’s website (<https://icr.ethz.ch/data/epr/core/>). Before starting the presentation of this test, let us remark that the model’s predictions are highly non-monotonic and depend on the two parameters, N_B and N_C , in a quite complex fashion, which is neither additive nor multiplicative. Classic empirical tests relying on OLS and its various refinements therefore don’t appear to be suitable here, which motivates the recourse to a different approach.

The EPR database used for this empirical test identifies all politically relevant ethnic groups, as well as their access to state power, in every country of the world from 1946 to 2017. It includes annual data on over 800 groups and codes the degree to which their representatives held executive-level state power—from total control of the government to overt political discrimination (1: Monopoly, 2: Dominant, 3: Senior partner, 4: Junior partner, 5: Self-exclusion, 7: Powerless, 8: Discriminated, 9: State collapse, 10: Irrelevant). It also contains a variable reporting each ethnic group’s share in the population, but this variable is rarely actualized (i.e. on average, there are only about 3 dates per country at which at least one minority size changes during the whole period of study, and these changes are often small).

As this database focuses on political power rather than directly on the perceived social status of the different ethno-cultural groups, it is not perfectly suited to test the model. However, to the best of my knowledge, this dataset is the only publicly available database containing both information on minorities’ sizes and on minorities’ relative status, while being quite comprehensive in its coverage of the different countries’ ethno-cultural groups. That’s why it was selected for this analysis.⁷⁶ We therefore have to assume, throughout the following tests, that the ethno-cultural hierarchy emerging from day-to-day interactions between members of the different ethno-cultural groups eventually translates into political power terms, as measured in the EPR database.

Starting from the raw EPR database, the following data treatments are applied. First, for each country, only years in which either the status or the size of at least one ethno-cultural group changes with respect to the previous year are kept, in order to analyze only distinct observations.⁷⁷ Then, status categories (5), (9) and (10) are removed from the data, as they are not informative on the actual social status of the group in the society. Sizes of the different groups are normalized to obtain, as in the model, a size of the largest group equal to 1.

The analysis then focuses on countries in which more than 3 ethno-cultural groups are reported⁷⁸ and, more precisely, on the 3 largest ethno-cultural groups in countries where the largest group is dominant⁷⁹ and where the second largest group has the second highest status.⁸⁰

At that point, the ethno-cultural hierarchy between the three largest groups of each country for each selected year is matched with one of the model’s stylized hierarchies. The "most intuitive" matching consists in associating H_4 to all observations in which the three group statuses are equal, H_2 to all observations where the two largest groups have the same (dominant) status, H_1 to all observations where the two minorities have the same (dominated) status and H_3 when the three statuses are distinct (the correspondence table associated with this matching is produced in the top chart of Table 2, where

76. The “Minorities At Risk” dataset ([Minorities at Risk Project, 2009](#)), due to its sole focus on minorities that are considered to be "at risk", is too selective in its coverage of ethno-cultural groups for the present empirical test and the very recent “All Minorities At Risk” dataset ([Birnie et al., 2016](#)), which is very extensive in its coverage of ethno-cultural groups, is not publicly available yet and not have a lot of historical hindsight.

77. In the raw dataset, measures of group statuses and sizes typically remain constant for many years in a row.

78. 523 country \times year observations are, at that point, present in the database.

79. 366 country \times year remain, or 70% of the 523 initially eligible country \times year observations.

80. 319 country \times year remain, or 61% of the 523 initially eligible country \times year observations.

“Most intuitive” correspondence table

Model hierarchy	Corresponding EPR hierarchies	Nb. of obs.
H_1	(2, 8, 8), (1, 8, 8), (2, 7, 7), (1, 7, 7), (3, 7, 7), (3, 8, 8), (3, 4, 4)	206
H_2	(2, 2, 4), (2, 2, 7), (3, 3, 4), (3, 3, 7), (3, 3, 8)	9
H_3	(1, 7, 8), (2, 7, 8), (3, 7, 8), (3, 4, 8), (3, 4, 7)	95
H_4	(2, 2, 2), (3, 3, 3), (4, 4, 4)	9

“Most balanced” correspondence table

Model hierarchy	Corresponding EPR hierarchies	Nb. of obs.
H_1	(2, 8, 8), (1, 8, 8), (2, 7, 7), (1, 7, 7), (3, 7, 7), (3, 8, 8)	146
H_2	(2, 2, 4), (2, 2, 7), (3, 3, 7), (3, 4, 8), (3, 4, 7), (3, 3, 8)	85
H_3	(1, 7, 8), (2, 7, 8), (3, 7, 8)	15
H_4	(2, 2, 2), (3, 3, 3), (4, 4, 4), (3, 4, 4), (3, 3, 4)	73

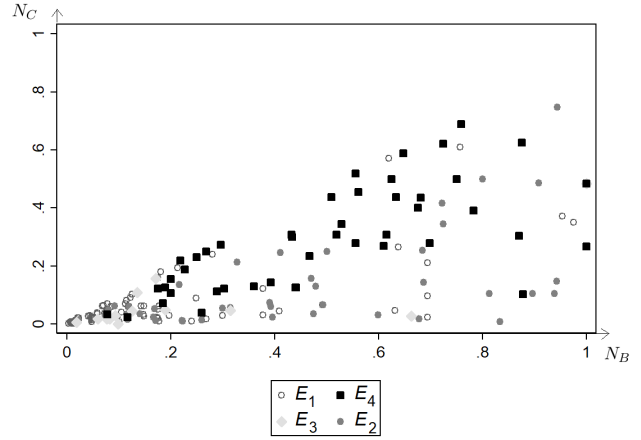
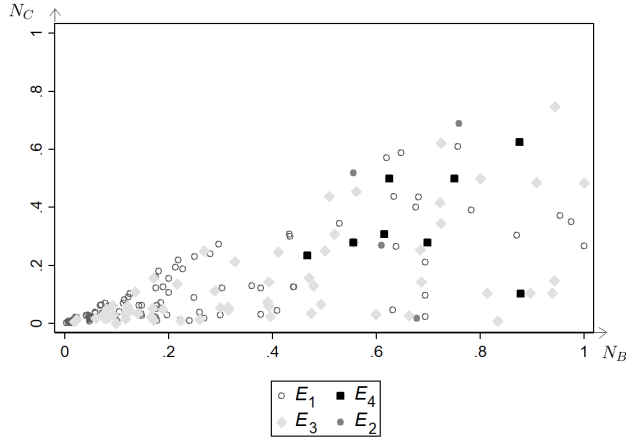
Table 2 – “Most intuitive” and “most balanced” correspondence tables between EPR data hierarchies and the model’s hierarchies

(x, y, z) is the EPR data hierarchy in which the largest group holds status x , the second largest group holds status y and the third largest group holds status z). However, this choice of correspondences leads to a very imbalanced distribution of hierarchies in the dataset (with only 9 observations of H_2 and H_4). Thus, alternative correspondence tables are also considered. The first alternative correspondence table is obtained by considering statuses 3 and 4 as equal (i.e. "Senior" and "Junior" partners are considered to be equal), which leads to a more balanced distribution of hierarchies (see the bottom chart in Table 2). We will call this correspondence table the “most balanced” correspondence table. A second alternative correspondence table, in which statuses 7 and 8 are considered to be equal instead of statuses 3 and 4 and in which hierarchies with a difference between the highest and lowest status lower than 1 are considered to be egalitarian (once more in an attempt to balance the number of observations per hierarchy type), is investigated in Appendix A (see Table 3).

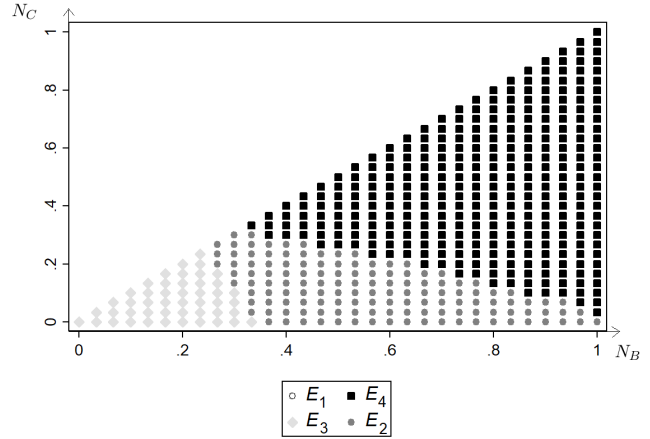
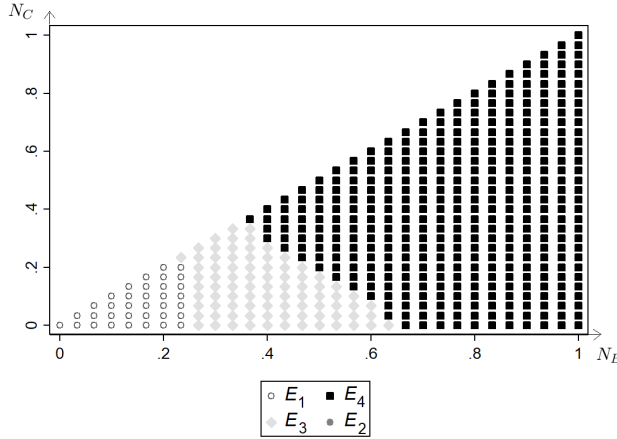
“Most intuitive” correspondence table

“Most balanced” correspondence table

Hierarchies in the EPR dataset



Best prediction hierarchy



Best and second-best prediction hierarchy

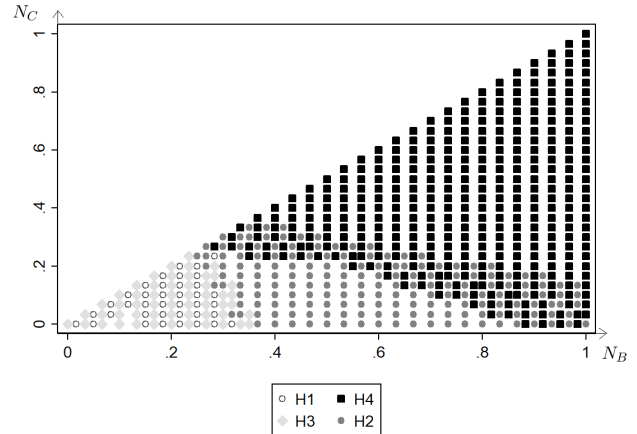
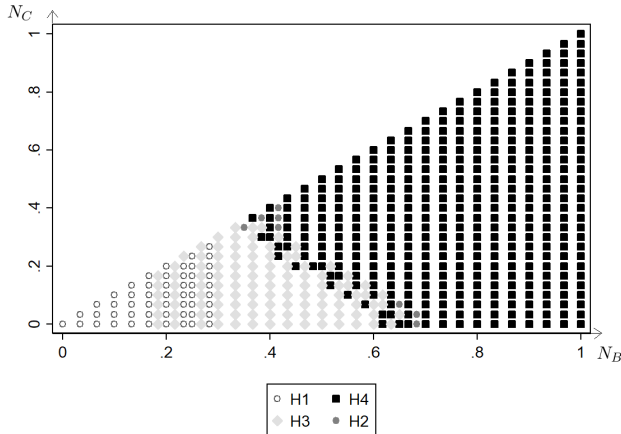


Figure 7 – Predicted hierarchy types in EPR data depending on the relative size of the second and third largest groups (with respect to the largest group’s).

Notes: Left figures: Results obtained with the “most intuitive” correspondence table. Right figures: Results obtained with the “most balanced” correspondence table. Top figures: hierarchy types observed in the EPR data according to each correspondence table. Middle figures: the best prediction hierarchy for each point of the grid. Bottom figures: the same as the middle figures with, in addition, the second-best prediction hierarchy (depicted just to the right of the best prediction if this second-best prediction is nearly as good as the best one)

The two top plots in Figure 7 depict prevailing hierarchies in the distinct observations (country \times year) of the EPR dataset, depending on minority group sizes N_B and N_C , when considering the "most intuitive" (left plot) and "most balanced" (right plot) correspondence tables.⁸¹ Then, in order to draw from these observations the predicted hierarchy views for all pairs of minority sizes, a grid is built with a mesh of 1/30 (with the same vertical and horizontal axes as in the two top plots). For each point in the grid and each hierarchy view, the average distance of the point to all observations corresponding to this hierarchy view present in the sample is computed. The point is then attributed the hierarchy view to which it has the smallest average distance (it is the predicted hierarchy view for this point).⁸²

The results obtained are depicted in the two middle graphs of Figure 7. Predicted hierarchy views are pretty consistent with the model's predictions, insofar as the *egalitarian* hierarchy view is predicted when both minorities are large, the *cumulatively inegalitarian* hierarchy view H_3 and *majority-dominated inegalitarian* hierarchy view H_1 are predicted when both minorities are small and the *minority-differentiating inegalitarian* hierarchy H_2 is predicted when minority C is small and minority B is large. We can also observe that, consistently with the model's insights, the sizes of the two minorities seem to act, to some degree, as substitutes in the determination of hierarchy views (boundaries between hierarchy views are oblique), so that the emergence of egalitarian hierarchy views requires either a large group B and a small group C or two intermediate-sized groups B and C .⁸³

Now, in order to come closer to the model's predictions, according to which multiple hierarchy views may co-exist in the long-term, a second predicted hierarchy is computed for each point of the mesh. It corresponds to the hierarchy associated with the second smallest average distance (of observed hierarchy views) to each point of the grid. If the gap between the smallest and second smallest average distance is small enough (i.e. below 0.03), both predicted hierarchies are considered to co-exist. If the gap is larger, only the hierarchy corresponding to the best prediction is kept. The results are depicted in the bottom graphs of Figure 7, which are the same as the middle ones with, in addition, the hierarchy corresponding to the second smallest average distance, depicted to the right of the best prediction. We can observe that hierarchy views H_1 and H_3 sometimes coexist when both minorities are small. Therefore, the long-term equilibrium where E_1 and E_3 have probability 1/2 seems to be quite realistic.⁸⁴

As a complementary test, the same procedure consisting in computing the best prediction on each point of a mesh of 1/30 is applied to directly predict the status of each of the two largest minorities (rather than predicting the overall hierarchy), depending on minority sizes. Focusing on statuses allows to avoid relying on a correspondence table, which choice may be criticized. Yet, it also shifts the analysis from group ranking (or ordering) to the intensity of group status differences, which are not directly featured in the model. Thus, interpreting the results becomes a little more complex. The results obtained are depicted in Figure 8, where the top-left graph displays the predicted status of the largest minority (i.e. the second largest group in the country), the top-right graph corresponds to the predicted status of the second-largest minority (i.e. the third largest group in the country) and the bottom graph is identical to the top-right one but replacing the second-largest minority by any minority

81. The corresponding plot when using the third correspondence table is the top plot of Figure 12.

82. Note that comparing average distances of each point of the grid to all observation points corresponding to each hierarchy view in order to determine which hierarchy view can be predicted is a very raw method. If the dataset contained more observations, in particular for large minority groups, it would be preferable to predict hierarchy views on the grid using a k -th-nearest neighbor algorithm (which would predict a hierarchy view at each point of the grid, based on the k nearest observations of this point) to obtain more precise empirical predictions.

83. The results obtained with the third correspondence table are depicted in Appendix A (see the middle graph of Figure 12). They show that predicted hierarchy views are reasonably robust to changes in the correspondence table.

84. The results obtained with the alternative third correspondence table are depicted in the bottom graph of Figure 12). They confirm that best and second-best predicted hierarchy views are reasonably robust to changes in the correspondence table.

ranked at least 4 in terms of group size in its country.⁸⁵ These graphs suggest, once more, a kind of substitutability between minority sizes (with mostly oblique borders between the different status zones). Additionally, they can illustrate the presence of a “cultural backlash”. Indeed, starting from a N_B comprised between 0.3 and 0.4, an increase in N_C first leads to shift group B from a predicted status of 8 to a predicted status of 4, while keeping the status of group C unchanged. Thus, an increase in N_C reduces, at first, the relative status of group C in the society, which can be interpreted as a “cultural backlash”. Further increasing N_C , however, shifts the status of group C from 8 and 4 as well, eventually countering the “cultural backlash” effect. Besides, comparing the three graphs, we can also note that, whereas the status of the largest minority monotonically increases with its size and the size of the second largest minority (i.e. it starts at 8, the lowest status, when both minorities are small, then increases to 4 and 3), it is not the case for the second-largest minority (and for smaller minorities). Indeed, the status of this second-largest minority, although always increasing with its own size, decreases and then increases with the size of the largest minority (i.e. along the N_B -axis in the bottom graph, we can observe that the status of minority C starts at 7, then decreases to the lowest status 8 before increasing to 4 and then 3). This pattern can exemplify the phenomenon of “asymmetrical homophily” discussed in the model.

All in all, these exploratory empirical tests appear to corroborate the long-term equilibria predicted in the model.⁸⁶ Some of the model’s predictions (i.e. the long-term effect of the arrival of a new minority, of the growth of a minority and of the split or fusion of minorities), however, could not be tested with the EPR dataset, due to the lack of distinct observations available for each country and of the static definition of ethno-cultural groups in the EPR dataset. It would thus be interesting, in the future, when new more detailed datasets become available, to empirically assess these other predictions. The complementary tests could be quite similar to those presented previously, replacing points corresponding to different hierarchy views with vectors linking two successive observations of hierarchy views in a same country (i.e. one at t and the other at $t + 1$ for each t). A grid could be built (with an adequately sized mesh), with tiny vectors linking vertically and horizontally its adjacent points. Then, for each tiny vector of the grid, it would be possible to compute its average distance to each type of vector observed in the dataset (a type of vector being defined as a pair of a specific initial and final hierarchy view (e.g. (H_1, H_3) or (H_2, H_3) ...) and the distance between two vectors being measured, for example, as the distance between their two initial points plus the distance between their two final points).⁸⁷ Then, this tiny vector would be associated with the type of vector with which it would have the smallest average distance. Eventually, all tiny vectors of the grid could be displayed on a graph with N_B on the x-axis and N_C on the y-axis, and compared to the model’s predictions. This test would allow for a “dynamic” evaluation of the model by studying, instead of between-countries variations of hierarchy views, within-country variations. It should allow to observe “cultural backlash” situations, as predicted by the model, when the size of the second-largest minority increases and to discuss the different possible impacts linked with variations in the sizes of minorities and the arrival of new ones.⁸⁸

85. The number of observations corresponding to the different statuses possible for the largest minority, the second largest minority and any group ranked at least fourth in terms of size in its country are summarized in Table 4 in Appendix A.

86. At the same time, these empirical tests lend support to the model’s hypothesis that members of two distinct groups need to agree not only on their relative status but also on the broader ethno-cultural hierarchy to be able to interact in the best conditions. Indeed, the long-term equilibrium obtained in Appendix D with the alternative specification of the model, in which interactions between two groups are independent of the status of the third group, differs widely from the empirical results yielded by the EPR database.

87. As already noted previously, it would be possible, in the presence of enough data points, to apply the k -th-nearest neighbors algorithm to determine predicted hierarchy views, instead of the average distance measure.

88. This methodology could also be adapted to analyze, instead of transitions in whole hierarchy views, transitions in minority statuses. To improve the test, these minority statuses could even be treated beforehand, in order to correct for

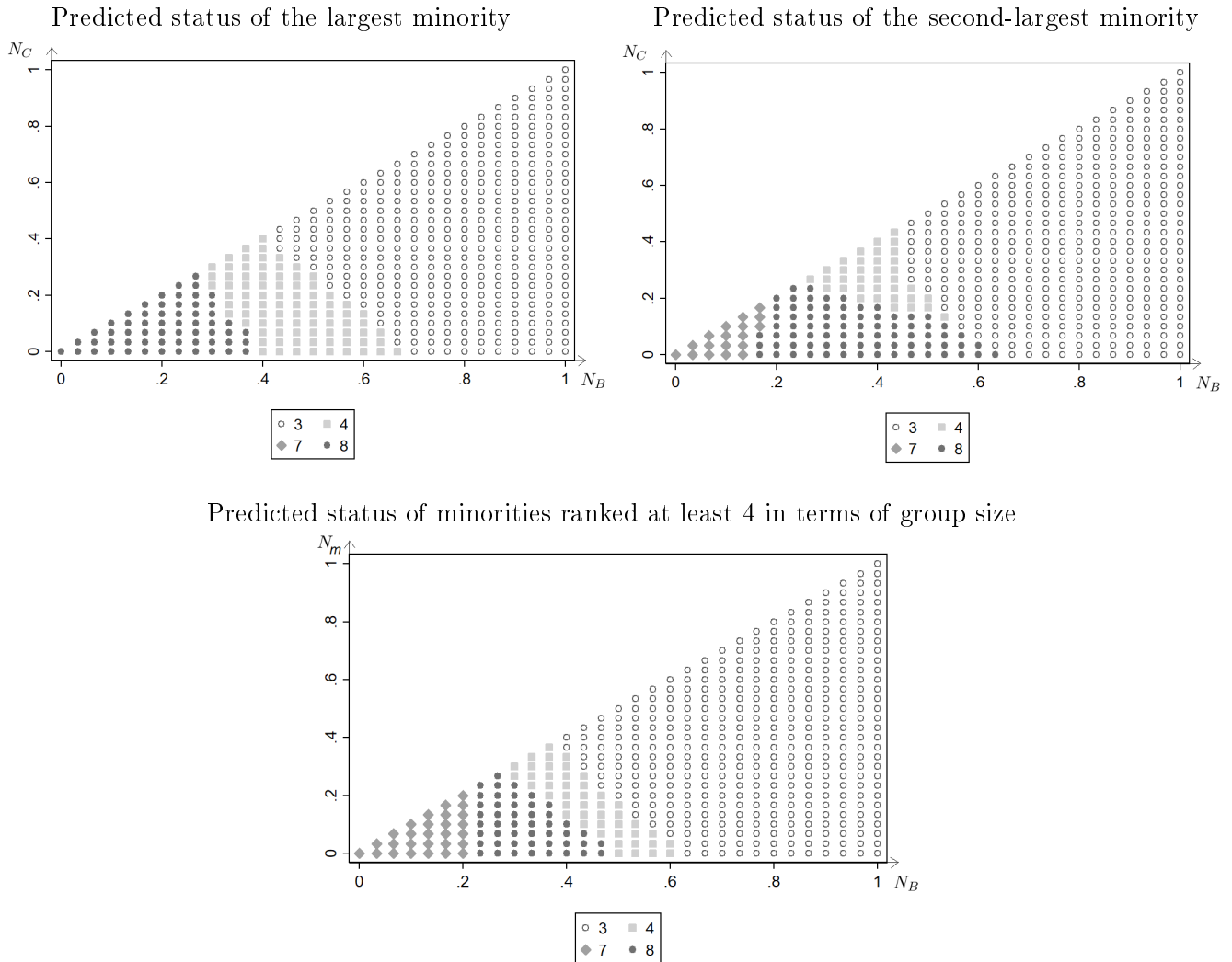


Figure 8 – Predicted statuses depending on the relative sizes of minority groups (with respect to the largest one) in EPR data.

Notes: Top-left figure: Predicted status of the largest minority for each point of the grid depending on the largest and second-largest minority's sizes. Top-right figure: Predicted status of the second-largest minority for each point of the grid depending on the largest and second-largest minority's sizes. Bottom figure: Predicted status of a minority ranked at least 4 in terms of group size in the country, depending on the relative size of the second largest group and of this minority.

7. Conclusion

While standard theories aimed at explaining unequal ethno-cultural statuses in societies are largely focused on bilateral relationships between one majority and one minority group, the present paper proposes a generalization of the evolutionary game theory approach to unequal norms into a three-group context.

This generalization allows to study how the interplay between several minorities' incentives breaks down the monotonic link between group size and status predicted in two-group models, shedding new light on the conditions leading growing minorities to experience a "cultural backlash" or, on the opposite, an improvement of their status. Indeed, the growth of a minority shifts the overall ethno-cultural balance in a country, thereby altering transition costs between the different views on the ethno-cultural hierarchy, in a way that may hurt the growing minority while benefiting the large one.

The model also provides conditions for "asymmetrical homophily" to emerge, where one minority would benefit from an increase in its interactions with another minority, while this other minority would not, due to status incentives. Comparing the two- and three-group versions, the model allows to predict that the arrival of a new and smaller minority in a country (weakly) increases the status of the old and larger minority, while (weakly) reducing the domination of the majority group. Additionally, the model shows that some members of a minority that is discriminated against may benefit from a split of this minority into smaller categories, in order to obtain a higher status for themselves after the division.

Eventually, a simple extension of the model introducing productivity differences between egalitarian and inegalitarian interactions suggests that an evolutionary process leads, in societies with small minorities, to the emergence of economically under-efficient cumulatively inegalitarian hierarchy views, instead of more efficient egalitarian ones.

An exploratory empirical test of the model's long-term equilibrium, conducted with the Ethnic Power Relation database, appears to corroborate the model's long-term equilibria of hierarchy views, depending on minority sizes, and the derived conditions for "cultural backlash".

Taking into account how the interplay between multiple ethno-cultural groups' interests shapes hierarchy views and thus the efficiency of cross-cultural interactions in a society is important nowadays and can be expected to gain in importance in the future, as world metropolises become more and more ethno-culturally diverse, multiplying cross-cultural interactions and increasing risks of ethno-cultural tensions. Further theoretical and empirical research on the dynamics of assimilation and differentiation between multiple minorities would be of great interest in order to better assess future risks in this respect.

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time trends and country fixed effects, by using a simple linear regression of the minority's status on time and country fixed effects and then analyzing the residuals obtained.

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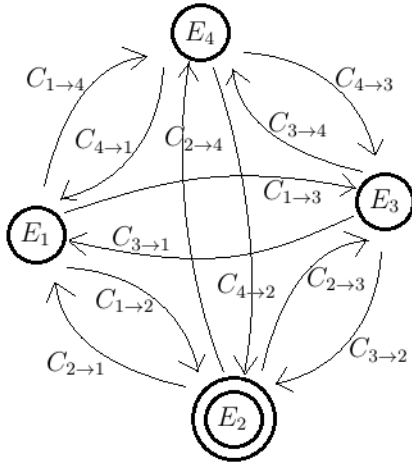
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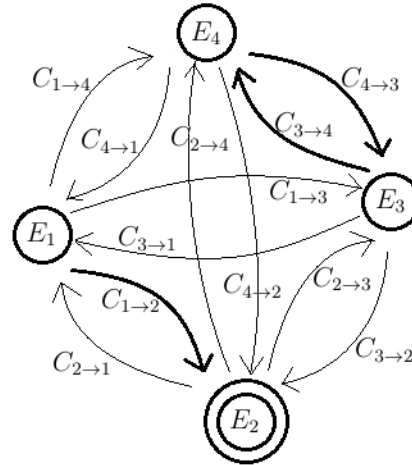
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8. Appendix A - Figures related to the three-group model

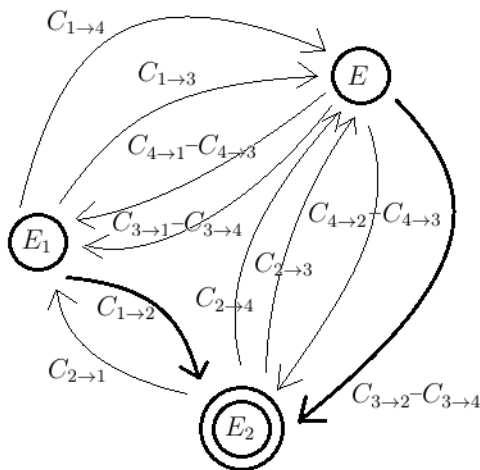
1 Graph of transition costs between Nash equilibria



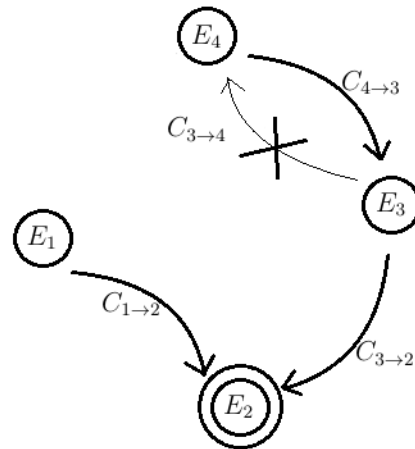
2 Lowest cost destination from each equilibrium



3 Merge of equilibria in a same cycle and reweighting



4 Reconstitution of the minimum spanning tree:



For outgoing edges of the cycle in the graph:

$$\text{New weight} = \text{Initial weight} - \text{Lowest initial destination weight}$$

Figure 9 – Steps of Edmond's algorithm to compute the minimum spanning tree leading to E_2 .

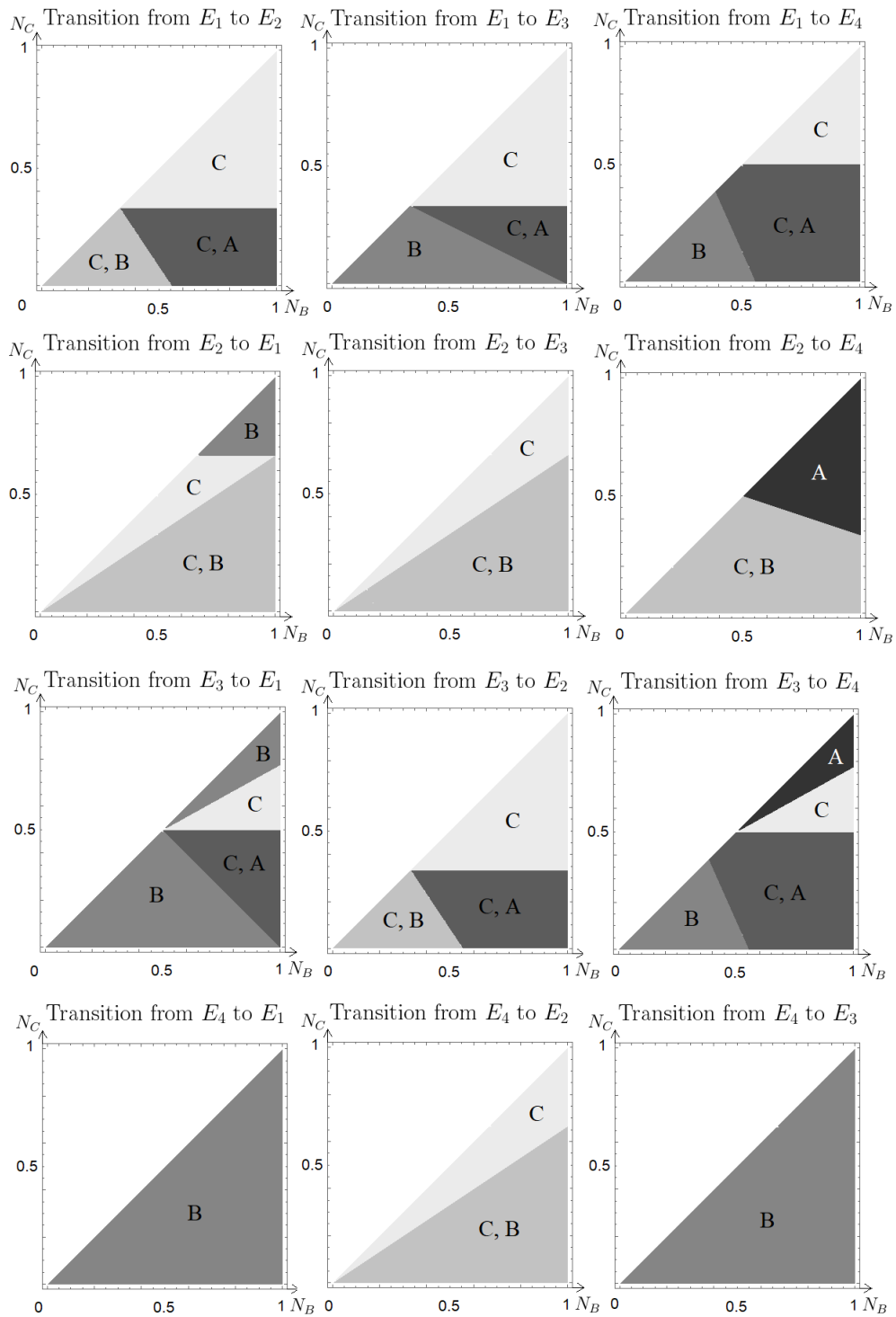


Figure 10 – *Mutating group* depending on the size of the two minorities for each transition type

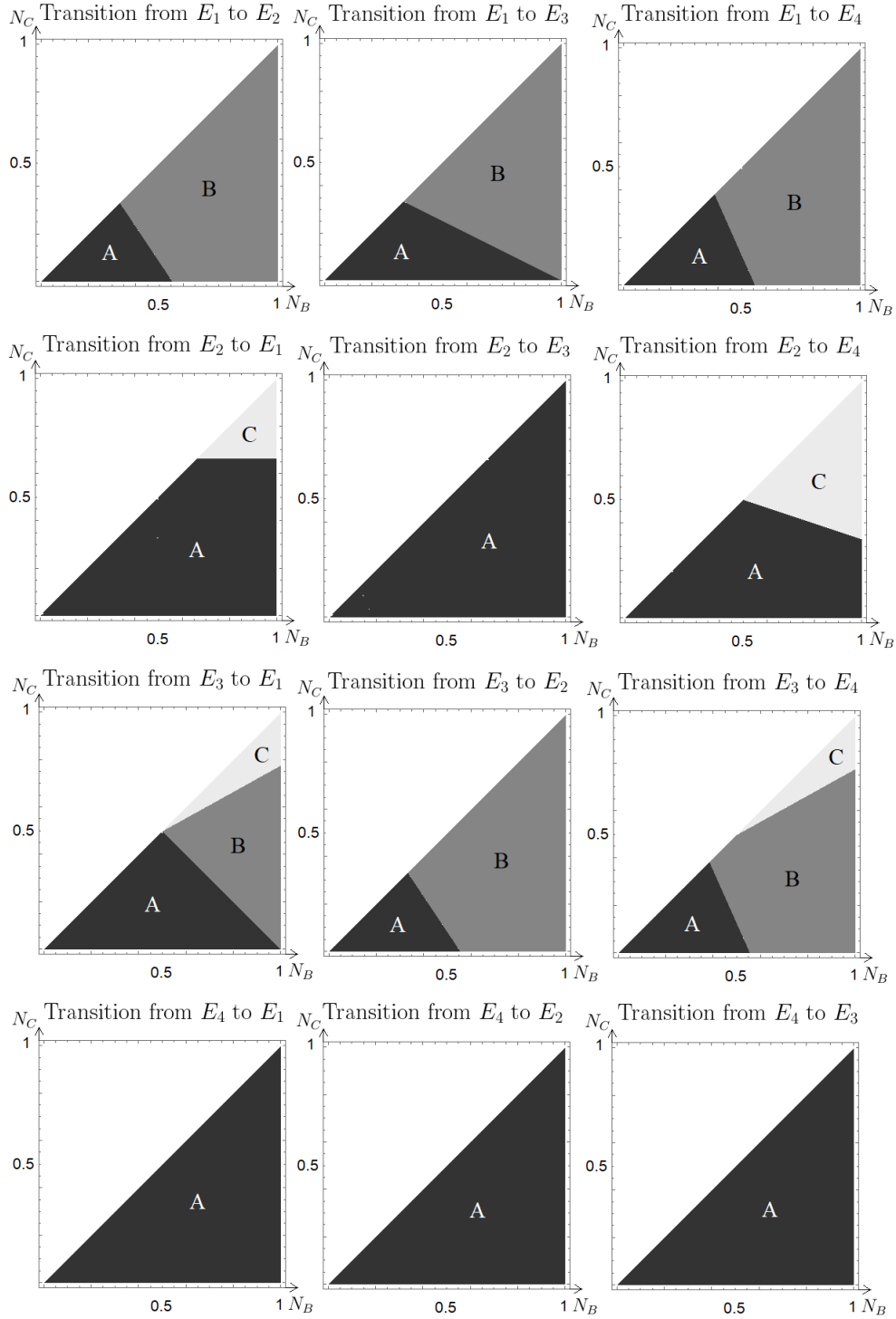


Figure 11 – First *switching group* depending on the size of the two minorities for each transition type

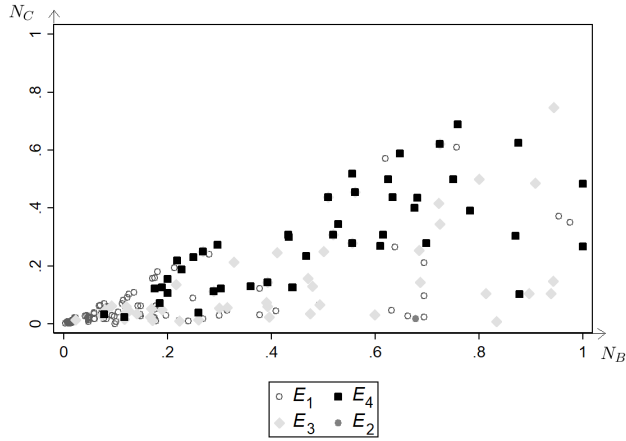
Model hierarchy	Corresponding EPR hierarchies	Nb. of obs.
H_1	(2, 8, 8), (1, 8, 8), (2, 7, 7), (1, 7, 7), (3, 7, 7), (3, 8, 8), (1, 7, 8), (2, 7, 8), (3, 7, 8)	161
H_2	(2, 2, 4), (2, 2, 7), (3, 3, 7), (3, 3, 8)	5
H_3	(3, 4, 8), (3, 4, 7)	80
H_4	(2, 2, 2), (3, 3, 3), (4, 4, 4), (3, 4, 4), (3, 3, 4)	73

Table 3 – Robustness test: Second alternative correspondence table between EPR data hierarchies and the model's hierarchies

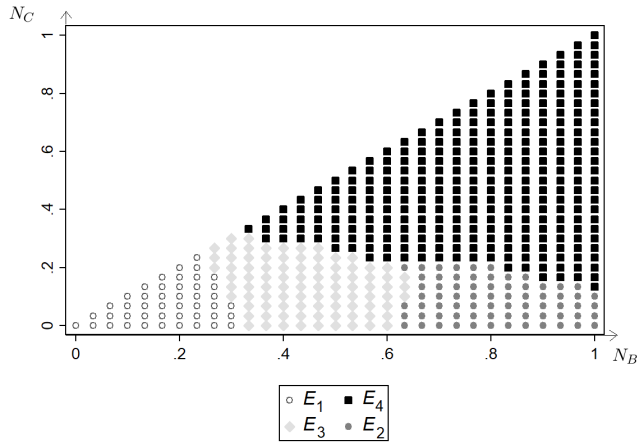
EPR status	Number of observations		
	Second largest group	Third largest group	Groups ranked at least 4
3	20	12	6
4	140	84	205
7	147	176	955
8	51	86	280

Table 4 – Number of observations (i.e. country \times year) in the EPR dataset depending on the minority size's rank in its country and on the minority's status. Column 2 (resp. 3, resp. 4): number of observations for second largest groups (resp. third largest groups, resp. groups ranked at least fourth in terms of size) in their country.

Alternative correspondence table
Hierarchies in the EPR dataset



Best prediction hierarchy



Best and second-best prediction hierarchy

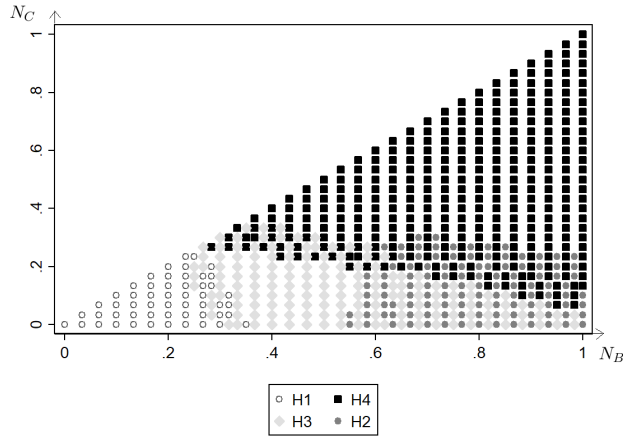


Figure 12 – Robustness test: Predicted hierarchy types in EPR data depending on the relative size of the second and third largest groups (with respect to the largest group's). Top figure: hierarchy types observed in the EPR data according to the correspondence table 3. Middle figure: the best prediction hierarchy for each point of the grid. Bottom figure: the same as the top figure with, in addition, the second-best prediction hierarchy (depicted just to the right of the best prediction if this second-best prediction is nearly as good as the best one)

9. Appendix B - Proof of Proposition 3.2

9.1. Computation of the matrix of transition costs

To compute the transition costs matrix, the following functions are implemented.⁸⁹

A first auxiliary function, *TransitionCostWithSpecificFirstSwitchingGroup*, computes, for given initial and final equilibria, a given first *switching group* and a given set of constraints on minority sizes, the transition cost. It implements the following steps:

- First, a first *mutating group* is selected among the two groups that are not the first *switching group*. To minimize the transition cost, this first *mutating group* must be the one whose mutations have the largest impact on the difference between the payoff of the first *switching group* in the initial and final equilibria (so that fewer mutations are needed to make the first *switching group* indifferent between the initial and final equilibria).
- The “naive” cost to make the first *switching group* switch is computed, using only mutations from the first *mutating group*.
- If this “naive” cost is larger than the size of the first *mutating group*, it means that having the whole first *mutating group* mutate to the final equilibrium is not sufficient to make the first *switching group* prefer the final equilibrium. In this case, the whole first *mutating group* needs to mutate, as well as part of the second *mutating group* (i.e. the remaining group that is neither the first *mutating group* nor the *switching group*). The corresponding transition cost is computed.
- If the “naive” cost is smaller than the size of the first *mutating group*, no mutations from the remaining group are required.
- Once the cost to make the first *switching group* switch has been computed, it is necessary to check whether the switch of this group to the final equilibrium is sufficient to make the two other groups switch to the final equilibrium too. For this, it is sufficient that only one of the two other groups prefers the final equilibrium because, in this case, it can switch to it through best-response adjustments and, once two whole groups have reached the final equilibrium, the third one necessarily prefers the final equilibrium too (because it is a Nash equilibrium). Therefore, it is checked whether one of the two other groups is willing to switch to the final equilibrium.
- If it is the case, the function returns the cost to make the first *switching group* switch.
- If it is not, an additional cost is computed: i.e. the number of mutations from each of the two remaining groups to the final equilibrium necessary to make the other remaining group prefer the final equilibrium. The lowest of the two possible additional costs is retained. The function returns the sum of the cost to make the first *switching group* switch and the cost to make the second group switch.

If, at any step, the function stumbles upon a comparison that remains ambiguous under the set of constraints, it raises an error and returns the ambiguous inequality.

A second auxiliary function, *TransitionCost*, computes, for given initial and final equilibria and a given set of constraints on minority sizes, the “direct” transition cost. Its steps are the following:

- It calls, for each possible first *switching group*, the previous function *DirectTransitionCostWithSpecificFirstSwitchingGroup*.
- It compares the three transition costs obtained and returns the lowest one.

If the function stumbles upon a comparison that remains ambiguous under the set of constraints, it raises an error and returns the ambiguous inequality.

⁸⁹. Note that, for each pair of equilibria (E_i, E_f) , only “direct” transition costs obtained when all mutations are directed toward the equilibrium E_f are computed, as it is clear that no minimum spanning tree would contain an “indirect” transition.

The algorithm to compute all transition costs, *TransitionCostsMatrix*, is as follows:

- For each possible pair of initial and final equilibria ($E^{(i)}, E^{(f)}$):
 - Try to compute the transition cost between them.
 - If an error is raised with a constraint on minority sizes, try to compute separately the transition cost when this constraint is met and when it is not (i.e. subdivide the set \mathcal{P}).
 - If an error is further raised with a new constraint on minority sizes, further subdivide the set \mathcal{P} , until the transition cost can be computed unambiguously on each subdivision.
- The algorithm returns the matrix of transition costs corresponding to each pair of initial and final equilibria. These transition costs are defined by part on \mathcal{P} .

9.2. Determination of the long-term equilibria

To determine the model’s long-term equilibria using the matrix of transition costs returned by *TransitionCostsMatrix*, the following functions are implemented.

A first function, *EdmondAlgorithm*, corresponds to Edmond’s algorithm, implemented recursively. It takes as arguments: a matrix of transition costs, a list of nodes (these nodes correspond to equilibria at the first step and then to cycles containing several equilibria), a table of correspondences between edges (which allows to reconstitute, at the end of the procedure, the initial graph), a root (a node that will be the root of the minimum spanning tree) and constraints on minority sizes. It returns the minimum spanning tree leading to the “root” equilibrium. The function exactly follows the steps described in Appendix C. If, at any step, the function stumbles upon a comparison that remains ambiguous under the set of constraints, it raises an error and returns the ambiguous inequality.

A second function, *DeterminationLongTermEquilibriumForSpecificConstraints*, returns, for a given subset of \mathcal{P} defined by some constraints on minority sizes (*constraints_*), the long-term equilibrium as well as the matrix of transition costs, the costs of the four minimum spanning trees (leading to each equilibrium) and the four minimum spanning trees. It proceeds as follows:

- First, a matrix of transition costs under the *constraints_* is computed. It corresponds to the values of the overall transition costs matrix on a specific subset of \mathcal{P} where all coefficients of the matrix have a unique expression instead of being defined by parts. At the same time, the table of correspondences between edges is initialized.
- Then, for each equilibrium:
 - Apply Edmond’s algorithm.
 - Compute the cost of the minimum spanning tree returned by Edmond’s algorithm.
- Compare the costs of the four minimum spanning trees and return the equilibrium (or equilibria) associated with the lowest cost minimum spanning tree.

If, at any step, the function stumbles upon a comparison that remains ambiguous under the set of constraints, it raises an error and returns the ambiguous inequality.

A third function, *DeterminationAllLongTermEquilibria*, is used to decompose the set \mathcal{P} into subsets on which the long-term equilibrium can be determined unambiguously without raising any error. It returns the list of these subsets and the list of corresponding long-term equilibria.

10. Appendix C - Chu-Liu/Edmond's algorithm

Chu-Liu/Edmonds' algorithm (Edmonds, 1967; Chu and Liu, 1965) is an algorithm from graph theory for finding a minimum spanning tree leading to a specific node r (i.e. a directed graph of minimum weight in which, for any node v different from r , there is exactly one directed path from v to r). I reproduce hereafter the description of the algorithm for the reader's information, as presented in Wikipedia (2019), except that all edges' directions have been inverted, because the present paper's model requires to find the minimum spanning tree leading to each pure Nash equilibrium, not the minimum spanning tree leaving each pure Nash equilibrium.

The algorithm takes as input a directed graph $D = \langle V, E \rangle$ where V is the set of nodes and E is the set of directed edges, a distinguished node $r \in V$ called the root, and a real-valued weight $w(e)$ for each edge $e \in E$. It returns a spanning tree A rooted at r of minimum weight, where the weight of a tree is defined to be the sum of its edge weights, $w(A) = \sum_{e \in A} w(e)$.

The algorithm has a recursive description. Let $f(D, r, w)$ denote the function which returns a spanning tree rooted at r of minimum weight. We first remove any edge from E whose origin is r . We may also replace any set of parallel edges (edges between the same pair of nodes in the same direction) by a single edge with weight equal to the minimum of the weights of these parallel edges.

Now, for each node v other than the root, find the edge outgoing from v of lowest weight (with ties broken arbitrarily). Denote the destination of this edge by $\pi(v)$. If the set of edges $P = \{(v, \pi(v)) \mid v \in V \setminus \{r\}\}$ does not contain any cycles, then $f(D, r, w) = P$.

Otherwise, P contains at least one cycle. Arbitrarily choose one of these cycles and call it C . We now define a new weighted directed graph $D' = \langle V', E' \rangle$ in which the cycle C is "contracted" into one node as follows:

The nodes of V' are the nodes of V not in C plus a new node denoted v_C .

- If (u, v) is an edge in E with $u \in C$ and $v \notin C$ (an edge going away from the cycle), then include in E' a new edge $e = (v_C, v)$, and define $w'(e) = w(u, v) - w(u, \pi(u))$.
- If (u, v) is an edge in E with $u \notin C$ and $v \in C$ (an edge going coming into the cycle), then include in E' a new edge $e = (u, v_C)$, and define $w'(e) = w(u, v)$.
- If (u, v) is an edge in E with $u \notin C$ and $v \notin C$ (an edge unrelated to the cycle), then include in E' a new edge $e = (u, v)$, and define $w'(e) = w(u, v)$.

For each edge in E' , we remember which edge in E it corresponds to.

Now find a minimum spanning tree A' of D' using a call to $f(D', r, w')$. Since A' is a spanning tree, each node has exactly one outgoing edge. Let (v_C, v) be the unique outgoing edge from v_C in A' . This edge corresponds to an edge $(u, v) \in E$ with $u \in C$. Remove the edge $(u, \pi(u))$ from C , breaking the cycle. Mark each remaining edge in C . For each edge in A' , mark its corresponding edge in E . Now we define $f(D, r, w)$ to be the set of marked edges, which form a minimum spanning tree.

Observe that $f(D, r, w)$ is defined in terms of $f(D', r, w')$, with D' having strictly fewer nodes than D . Finding $f(D, r, w)$ for a single-node graph is trivial (it is just D itself), so the recursive algorithm is guaranteed to terminate.

11. Appendix D - Extensions and alternative specifications for the three-group model

11.1. Introducing homophily

An important assumption in the three-group model analyzed in this paper is that the number of interactions of each individual with the other groups perfectly reflects the relative sizes of these groups. This hypothesis is not very realistic as it is well known that individuals tend to prefer interacting with people who are similar to them (i.e. they exhibit homophily in their choices of interactions, see [McPherson et al., 2001](#) for a review). Luckily, the model can easily be amended to allow for an exogenous homophily, as long as all groups have the same degree of homophily. Indeed, using the same definition of homophily as in [Bazzi et al. \(2019\)](#), it suffices to multiply the number of interactions with members of other groups by $\sigma \in]0, 1[$ (to make them scarcer) and to increase the number of interactions with members of one's own group by $(1 - \sigma)$ times the total size of the other groups. For example, a member of group A will now interact, at each period of time, with $\xi\sigma\frac{N_B}{N_A+N_B+N_C}$ members of group B , $\xi\sigma\frac{N_C}{N_A+N_B+N_C}$ members of group C and $\xi\frac{N_A+(1-\sigma)(N_B+N_C)}{N_A+N_B+N_C}$ members of his own group A , so that the total number of interactions per individual remains unchanged. The larger the homophily parameter σ is (while remaining below 1), the less biased individuals are in their choices of partners for interactions. These modifications of the model imply that all transition costs are multiplied by σ . Therefore, comparisons between them remain unchanged and all of the model's predictions remain valid in the presence of symmetrical homophily. The only difference with the baseline model is that transitions from one equilibrium to the other are slower, so that convergence to the long term equilibrium requires more time.

11.2. Role of the inequality parameter γ

As in the two-group model, it is easy to see that, in the three-group model, increasing γ widens the set of minority sizes for which egalitarian equilibria persist in the long run and reducing γ makes this set shrink (see [Figure 13](#) for depictions of the long-term equilibria obtained when $\gamma = 1/4$ and when $\gamma = 3/4$). Additionally, considering a simple extension of the model where one would allow γ to differ across pairs of groups so that there exist distinct γ_{AB} , γ_{AC} and γ_{BC} corresponding to the interactions between groups A and B , groups A and C and groups B and C , it is also easy to show that, for two groups i and j , the larger γ_{ij} is, the more likely it is that, in the long run, group i and j hold equal statuses.

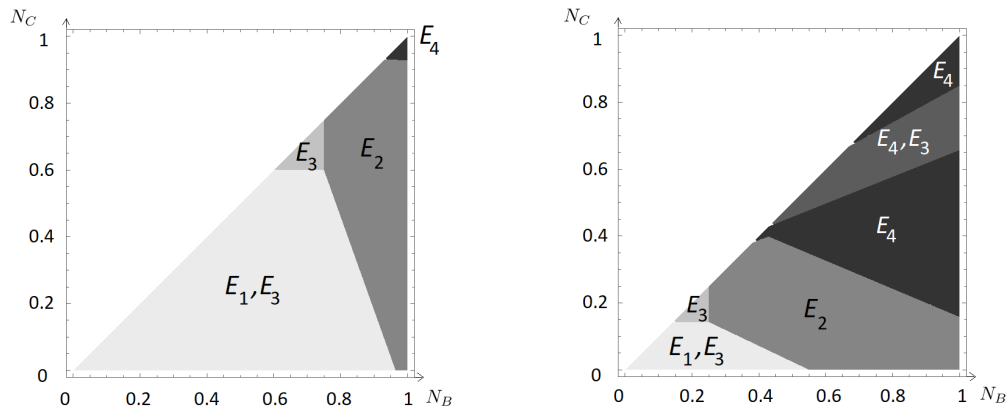


Figure 13 – Long-term equilibrium (or equilibria) depending on the two minorities' sizes, N_B and N_C , when $\gamma = 1/4$ (left graph) or $\gamma = 3/4$ (right graph).

11.3. Alternative specifications

11.3.1. When suppressing one or two hierarchy views

To appreciate the role played by the *minority-differentiating inegalitarian* hierarchy view H_2 and the *cumulatively inegalitarian* hierarchy view H_3 , the baseline three-group model's long-term equilibria can be compared to the ones that would have been obtained if the only possible hierarchy views were H_1 , H_2 and H_4 or if they were only H_1 and H_4 . In these cases, using the same resolution method, the long-term equilibria obtained are displayed in Figure 14.

Comparing the right graph of this figure with Figure 3, it can be observed that the suppression of *cumulatively inegalitarian* H_3 hierarchy views has only a mild effect on the long-term equilibria. Schematically, H_3 hierarchy views, when they are allowed, tend to replace, either partially or totally, *majority-dominated* hierarchy views H_1 , when both minorities are small. It is clear that all the baseline three-group model's qualitative results⁹⁰ are robust to the suppression of H_3 hierarchy views.⁹¹

Then, comparing the left and right graphs of Figure 14, it is clear that further suppressing H_2 hierarchy views has a much greater impact on the model's predictions. It nearly totally removes the substitutability between the two minorities' sizes⁹² and leads to predictions that are very similar to the two-group model's in the sense that the inegalitarian equilibrium E_1 persists when N_B is small and the egalitarian equilibrium E_4 when N_B is large, because the two minorities' preferences are always aligned.⁹³

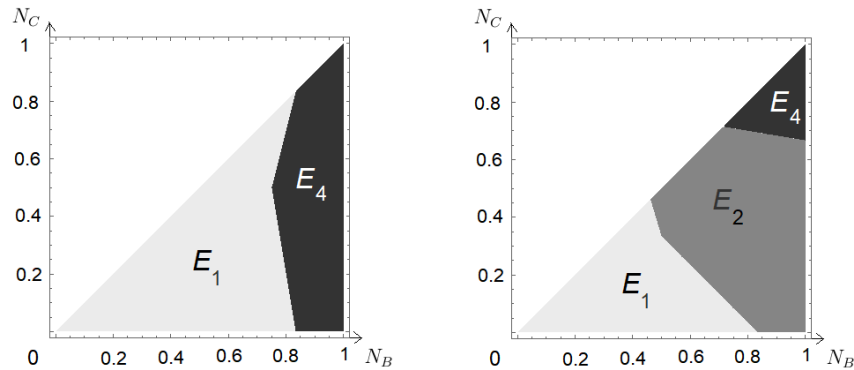


Figure 14 – Long-term equilibrium depending on the two minorities' sizes, N_B and N_C in two simplified versions of the three-group model with, in the left graph, only 2 possible hierarchy views (H_1 and H_4) and with, in the right graph, only 3 possible hierarchy views (H_1 , H_2 and H_4)

11.3.2. When adding two other hierarchy views

It is also possible, on the opposite, to add additional hierarchy views in which the smallest minority C dominates the largest minority C . This does not modify deeply the qualitative results obtained in the baseline three-group model, as evidenced by Figure 15, in particular for the minority sizes that are

90. i.e. the qualitative discussions relative to "cultural backlash", "asymmetrical homophily", the impact of the arrival of a new minority, the fusion/split of minorities and the prevalence of economically under-efficient inegalitarian hierarchy views when both minorities are small.

91. It seems important, however, to include H_3 hierarchy views in the baseline three-group model studied in this paper, as these hierarchy views appear to be the most prevalent ones in the real world.

92. The frontier between the E_1 and E_2 zones is oblique.

93. Note that the H_2 hierarchy plays a "divide-and-rule" role from the point of view of the majority, insofar as it reduces the set of minority sizes for which the egalitarian equilibrium E_4 persists in the long term, by favoring one minority over the other in the E_2 equilibrium that replaces it.

the most prevalent in reality (i.e. the third largest group is, in reality, unlikely to be nearly as large as both the majority and the second largest group).

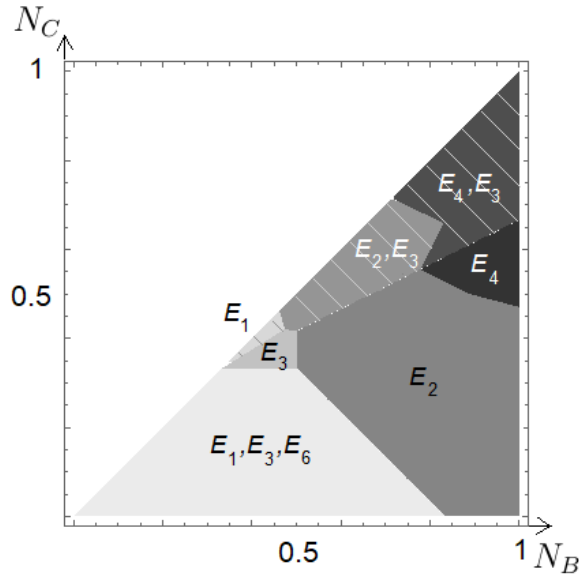


Figure 15 – Long-term equilibrium (or equilibria) depending on the two minorities' sizes, N_B and N_C , when two additional hierarchies views, $H_5 = H_{A=C>B}$ and $H_6 = H_{A>C>B}$ are introduced.

11.3.3. When interactions between two groups are independent of the status of the third group

Eventually, in the baseline three-group model, it is assumed that, when members of two groups i and j interact, their payoff is zero if they disagree in any way on the ethno-cultural hierarchy. Yet, it may be argued that a disagreement between members of two distinct groups regarding the status of a third absent group (group k different from i and j) should not affect their interaction (or should have a lesser impact than a disagreement on the relative status of the groups of the two interacting individuals). To answer this potential critique, an alternative specification of payoffs where the members of two different groups only care about their relative status (and not about the status of the absent group) is investigated in this paragraph.

The payoffs that individuals obtain from their interactions with other groups are now:

		Group A			
		$H_{A>B=C}$	$H_{A=B>C}$	$H_{A>B>C}$	$H_{A=B=C}$
Group B	$H_{A>B=C}$	$\theta - \gamma, \theta + \gamma$	0,0	$\theta - \gamma, \theta + \gamma$	0,0
	$H_{A=B>C}$	0,0	θ, θ	0,0	θ, θ
	$H_{A>B>C}$	$\theta - \gamma, \theta + \gamma$	0,0	$\theta - \gamma, \theta + \gamma$	0,0
	$H_{A=B=C}$	0,0	θ, θ	0,0	θ, θ

		Group A			
		$H_{A>B=C}$	$H_{A=B>C}$	$H_{A>B>C}$	$H_{A=B=C}$
Group C	$H_{A>B=C}$	$\theta - \gamma, \theta + \gamma$	$\theta - \gamma, \theta + \gamma$	$\theta - \gamma, \theta + \gamma$	0,0
	$H_{A=B>C}$	$\theta - \gamma, \theta + \gamma$	$\theta - \gamma, \theta + \gamma$	$\theta - \gamma, \theta + \gamma$	0,0
	$H_{A>B>C}$	$\theta - \gamma, \theta + \gamma$	$\theta - \gamma, \theta + \gamma$	$\theta - \gamma, \theta + \gamma$	0,0
	$H_{A=B=C}$	0,0	0,0	0,0	θ, θ

		Group B			
		$H_{A>B=C}$	$H_{A=B>C}$	$H_{A>B>C}$	$H_{A=B=C}$
Group C	$H_{A>B=C}$	θ, θ	0,0	0,0	θ, θ
	$H_{A=B>C}$	0,0	$\theta - \gamma, \theta + \gamma$	$\theta - \gamma, \theta + \gamma$	0,0
	$H_{A>B>C}$	0,0	$\theta - \gamma, \theta + \gamma$	$\theta - \gamma, \theta + \gamma$	0,0
	$H_{A=B=C}$	θ, θ	0,0	0,0	θ, θ

We can remark that, with this alternative specification, the model boils down to a superposition of three two-group models:

- one in which groups A and B coordinate on a common view of their relative status ($A = B$ or $A > B$)
- one in which groups A and C do the same for their relative status ($A = C$ or $A > C$)
- and one in which groups B and C do the same for their relative status ($B = C$ or $B > C$)

Thus, it can easily be shown that there are 5 possible pure Nash equilibria, which can be paralleled to those obtained in the baseline three-group model: E_1 (where $A > B = C$), E_2 (where $A = B > C$), E_3 (where $A > B > C$), E_4 (where $A = B = C$) and an additional E_5 (where $A = B$, $B = C$ and $A > C$). Note that the four first pure Nash equilibria exactly correspond to those obtained in the baseline three-group model. The only new pure Nash equilibrium, E_5 , corresponds to a case where the relative statuses on which each pair of groups coordinate, $A = B$, $B = C$ and $A > C$, are not consistent with each other, due to the disinterest of each pair of groups for the status of the third group.

The long-term equilibria depending on group sizes are obtained by superposition of the three two-group models.⁹⁴ They are depicted in the top graph of Figure 16.

As compared to the results obtained with the baseline three-group model, we can observe that, with this payoff specification, the status of a group increases monotonically with its size and decreases monotonically with the size of the other groups. In these conditions, contrarily to the baseline three-group model, there can be no “cultural backlash” nor “asymmetrical homophily”. Additionally, the arrival of a new minority does not affect the relative status of the old minority and majority groups (i.e. if they were equal before the arrival of the new minority, they remain so after the arrival of the new minority, and if they were unequal, they also remain so).⁹⁵ However, as in the baseline three-group model, there may be incentives for a minority to split into smaller minorities. Indeed, starting from a situation where there is only one majority and one minority and the minority is dominated (i.e. in bottom-right graph of Figure 16, the zone below the thin oblique line), if the minority splits into two sub-minorities, one of them can dominate the other and thus obtain an intermediary status (it is the

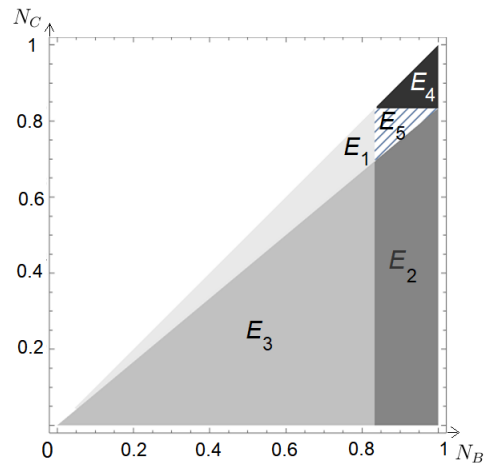
94. The same normalizations of parameters and hypotheses as in the baseline three-group model are applied: $N_A = 1$, $\theta = 1$, $\gamma = 1/2$.

95. See the bottom-left graph in Figure 16.

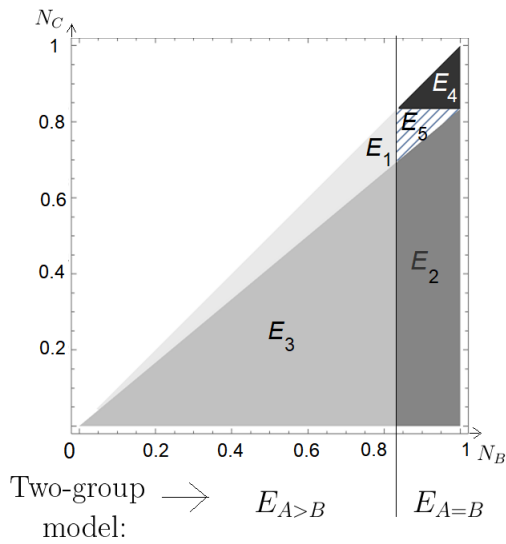
case if the long-term equilibrium is E_3 , but not if it is E_1). Eventually, as in the baseline three-group model (See subsection 5.4), we can observe that the hierarchy views that persist in the long term are most often inegalitarian, a result which persists when introducing a slight difference in the productivity of egalitarian and inegalitarian interactions. Thus, the evolutionary process also most often selects an economically under-efficient equilibrium.

While the exploratory empirical test led in Section 6 appears to lend support to the baseline three-group model, it does not seem to corroborate this alternative model's long-term equilibrium. Thus, although in practice the most realistic model would probably lie somewhere between the baseline three-group model presented in this paper and the present alternative specification, so that interactions between members of different groups would be the most efficient when agreeing on the whole ethno-cultural hierarchy and only partially efficient when agreeing only on part of it, the baseline three-group model still appears to provide a good approximation of the truth.

Long-term equilibria



Arrival of a new minority



Division of a minority

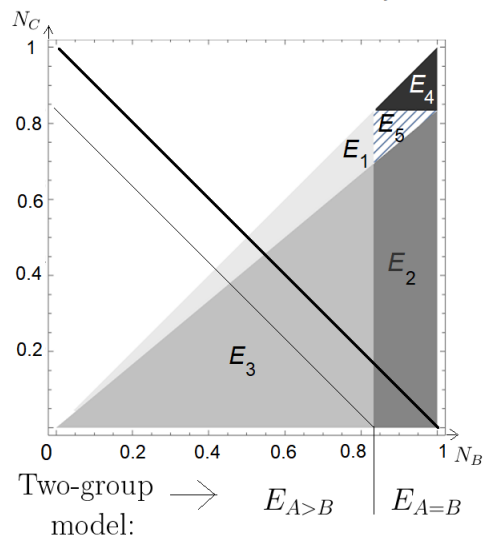


Figure 16 – Alternative specification when interactions between two groups are independent of the status of the third group. Top graph: Long-term equilibrium (or equilibria) depending on the two minorities' sizes, N_B and N_C . Bottom-left graph: Impact of the arrival of a new minority. Bottom-right graph: Impact of the split of a minority.

12. Appendix E - Mathematica functions for the resolution of the three-group model

```

(*****)
(*** Preamble ***)
(*****)

(* Size of each group *)
Size = {NA, NB, NC};

(* Payoff of group X in equilibrium Y *)
Payoff = {{(1 +  $\gamma$ ) (Size[[2]] + Size[[3]]), Size[[2]] + (1 +  $\gamma$ ) Size[[3]], (1 +  $\gamma$ ) (Size[[2]] + Size[[3]]), Size[[2]] + Size[[3]]}, {(1 -  $\gamma$ ) Size[[1]] + Size[[3]], Size[[1]] + (1 +  $\gamma$ ) Size[[3]], (1 -  $\gamma$ ) Size[[1]] + (1 +  $\gamma$ ) Size[[3]], Size[[1]] + Size[[3]]}, {(1 -  $\gamma$ ) Size[[1]] + Size[[2]], (1 -  $\gamma$ ) (Size[[1]] + Size[[2]]), (1 -  $\gamma$ ) (Size[[1]] + Size[[2]]), Size[[1]] + Size[[2]]}};

(* Baseline constraints on the parameters *)
baselineconstraints = NA  $\in$  Reals && NB  $\in$  Reals && NC  $\in$  Reals &&  $\gamma \in$  Reals &&  $\gamma < 1$  &&  $\gamma > 0$  && NB > NC && 1 > NB && NC > 0 ;
(* Payoff of group X adopting hierarchy Y when NA, nB and nC members of group A, B and C adopted the same hierarchy *)
Payoffhierarchy[{X_, Y_, nA_, nB_, nC_}] := Module[{}, {{(1 +  $\gamma$ ) (nB + nC), nB + (1 +  $\gamma$ ) nC, (1 +  $\gamma$ ) (nB + nC), nB + nC}, {(1 -  $\gamma$ ) nA + nC, nA + (1 +  $\gamma$ ) nC, (1 -  $\gamma$ ) nA + (1 +  $\gamma$ ) nC, nA + nC}, {(1 -  $\gamma$ ) nA + nB, (1 -  $\gamma$ ) (nA + nB), (1 -  $\gamma$ ) (nA + nB), nA + nB}}][X, Y] /.  $\gamma \rightarrow 1/2$ 
]

(*****)
(*** Function to count the number of mutations necessary to transit from equilibrium eqinitial_ to eqfinal_ when the first group to switch (i.e. to start preferring eqfinal_) is firstswitchinggroup_ and when the constraints_ on group sizes are imposed. The function also returns the mutating groups and the list of switching groups. ****)
(*****)

DirectTransitionCostWithSpecificFirstSwitchingGroup[{eqinitial_, eqfinal_, firstswitchinggroup_, constraints_}] :=
Module[{othergroup1, othergroup2, firstmutatinggroup, secondmutatinggroup, Costfirstswitch, possible, Intermediatepayoff, finished, Costtomakefirstmutatinggroupswitch, Costtomakesecondmutatinggroupswitch, TotalCost, aux, auxbool, auxbool2, auxboolpossible, auxboolfinished, auxboolfinished2, listmutatinggroups, listswitchinggroups},

listswitchinggroups = {firstswitchinggroup};
othergroup1 = Mod[ firstswitchinggroup , 3] + 1;
othergroup2 = Mod[firstswitchinggroup + 1, 3] + 1;
(** Choice of the first mutating group: the one that has the highest impact, when it mutates, on the difference between the payoff of firstswitchinggroup in the initial and in the final equilibrium **)
auxbool = D[Payoff[[firstswitchinggroup, eqinitial]] + Payoff[[firstswitchinggroup, eqfinal]], Size[[othergroup1]]] >= D[Payoff[[firstswitchinggroup, eqinitial]] + Payoff[[firstswitchinggroup, eqfinal]], Size[[othergroup2]]];
auxbool2 = Assuming[constraints, FullSimplify[auxbool /. {NA -> 1,  $\gamma \rightarrow 1/2$ }]];
(* If the first mutating group cannot be determined, raise an error with the undecided inequality. *)
If[ToString[auxbool2] != "True" && ToString[auxbool2] != "False", Return[Print["Error 0: "]; Print[auxbool2]; {"Error 0", auxbool2}]];

(* If it can be determined, memorize which group is the first mutating group and which is the second and store in a list of mutating groups the first mutating group (we do not know yet if the second one will be necessary). *)
firstmutatinggroup = If[auxbool2, othergroup1, othergroup2];
secondmutatinggroup = If[firstmutatinggroup == othergroup1, othergroup2, othergroup1];
listmutatinggroups = {firstmutatinggroup};

(** Naive cost to make the first switching group switch **)
Costfirstswitch = FullSimplify[Payoff[[firstswitchinggroup, eqinitial]]/D[Payoff[[firstswitchinggroup, eqinitial]] + Payoff[[firstswitchinggroup, eqfinal]], Size[[firstmutatinggroup]]];

(** Test if this first switch is possible using only the first mutating group **)
auxboolpossible = D[Payoff[[firstswitchinggroup, eqinitial]] + Payoff[[firstswitchinggroup, eqfinal]], Size[[firstmutatinggroup]]] == D[Payoff[[firstswitchinggroup, eqinitial]] + Payoff[[firstswitchinggroup, eqfinal]], Size[[secondmutatinggroup]]];
possible = Assuming[constraints, FullSimplify[(Size[[firstmutatinggroup]] >= Costfirstswitch || auxboolpossible) /. {NA -> 1,  $\gamma \rightarrow 1/2$ }]];

(* If it is not possible to decide if the boolean "possible" is True or False, raise an error with the undecided inequality *)
If[ToString[possible] != "True" && ToString[possible] != "False", Return[Print["Error 1: "]; Print[possible]; {"Error 1", possible}]];
(** If the first mutating group is not sufficient to make the first switching group switch, compute the additional cost due to the use of the second mutating group **)
Intermediatepayoff = D[Payoff[[firstswitchinggroup, eqinitial]], Size[[secondmutatinggroup]]*Size[[secondmutatinggroup]] - D[Payoff[[firstswitchinggroup, eqfinal]], Size[[firstmutatinggroup]]*Size[[firstmutatinggroup]]];

```

```

(* In this case, compute the new cost to make the first switching group switch *)
Costfirstswitch = If[possible == False, FullSimplify[Size[[firstmutatinggroup]] + Intermediatepayoff
/D[Payoff[[firstswitchinggroup, eqinitial]] + Payoff[[firstswitchinggroup, eqfinal]], Size[[secondmutatinggroup]]],
Costfirstswitch];

(* In this case still, add the second mutating group to the list of mutating groups *)
If[possible == False, listmutatinggroups = Append[listmutatinggroups, secondmutatinggroup ]];

(** Check if the switching of the first group is enough to launch a transition to the final equilibrium, i.e. will the other
groups follow and switch to the final equilibrium too?: For this to happen, one of the two other groups must be willing to
switch. Once it is done, the last group will switch, as the final equilibrium is a Nash equilibrium. Note that if the first
mutating group had to mutate entirely to make the first switching group switch, then two groups are already in the final
equilibrium so that the final equilibrium is sure to be reached **)
auxboolfinished = (Size[[firstmutatinggroup]] - Costfirstswitch)*D [Payoff[[secondmutatinggroup, eqinitial]],
Size[[firstmutatinggroup]] < Costfirstswitch*D [Payoff[[secondmutatinggroup, eqfinal]], Size[[firstmutatinggroup]] +
Size[[firstswitchinggroup]]*D [Payoff[[secondmutatinggroup, eqfinal]], Size[[firstswitchinggroup]]];
auxboolfinished2 = Size[[secondmutatinggroup]]*D [Payoff[[firstmutatinggroup, eqinitial]], Size[[secondmutatinggroup]] <
Size[[firstswitchinggroup]]*D [Payoff[[firstmutatinggroup, eqfinal]], Size[[firstswitchinggroup]]];
finished = If[ Assuming[constraints, FullSimplify[(possible == False || auxboolfinished || auxboolfinished2) /. {NA -> 1, γ
-> 1/2}], True, False];

(** If it cannot be decided if the other groups will switch, raise an error with the unresolved inequality **)
If[ToString[finished] != "False" && ToString[finished] != "True", Return[Print["Error 2 : "]; Print[finished]; {"Error 2",
Assuming[constraints, FullSimplify[(auxboolfinished || auxboolfinished2) /. {NA -> 1, γ -> 1/2}]}]];

(** If it is not enough, compare the cost of having each of the two remaining groups switch **)
If[ToString[finished] == "False",
Costtomakefirstmutatinggroupswitch = FullSimplify[(Size[[secondmutatinggroup]]*D [Payoff[[firstmutatinggroup, eqinitial]],
Size[[secondmutatinggroup]] - Size[[firstswitchinggroup]]*D [Payoff[[firstmutatinggroup, eqfinal]],
Size[[firstswitchinggroup]])/(D [Payoff[[firstmutatinggroup, eqinitial]], Size[[secondmutatinggroup]] + D
[Payoff[[firstmutatinggroup, eqfinal]], Size[[secondmutatinggroup]]]);
Costtomakesecondmutatinggroupswitch = FullSimplify[(Size[[firstmutatinggroup]] - Costfirstswitch)*D
[Payoff[[secondmutatinggroup, eqinitial]], Size[[firstmutatinggroup]] - (Costfirstswitch*D [Payoff[[secondmutatinggroup,
eqfinal]], Size[[firstmutatinggroup]]) + Size[[firstswitchinggroup]]*D [Payoff[[secondmutatinggroup, eqfinal]],
Size[[firstswitchinggroup]])/(D [Payoff[[secondmutatinggroup, eqinitial]], Size[[firstmutatinggroup]] + D
[Payoff[[secondmutatinggroup, eqfinal]], Size[[firstmutatinggroup]]]);

aux = Assuming[constraints, FullSimplify[(Costtomakefirstmutatinggroupswitch < Costtomakesecondmutatinggroupswitch) /. {NA
-> 1, γ -> 1/2}]];

(* If the comparison of the cost to make each of the two remaining groups switch cannot be decided, raise an error with the
undecided inequality *)
If[ToString[aux] != "False" && ToString[aux] != "True", Return[Print["Error 3: "]; Print[aux]; {"Error 3", aux}]];

(** Wrap up the costs **)
If[aux, TotalCost = FullSimplify[Costfirstswitch + Costtomakefirstmutatinggroupswitch];
listmutatinggroups = Append[listmutatinggroups, secondmutatinggroup ];
listswitchinggroups = Append[listswitchinggroups, firstmutatinggroup], TotalCost = FullSimplify[Costfirstswitch +
Costtomakesecondmutatinggroupswitch];
listswitchinggroups = Append[listswitchinggroups, secondmutatinggroup]
, TotalCost = Costfirstswitch ];

FullSimplify[{TotalCost, listmutatinggroups, listswitchinggroups} /. {NA -> 1, γ -> 1/2}]]

```

```

(*****)
(**** Function to compute the number of mutations necessary to transit from equilibrium eqinitial_ to eqfinal_ under the
constraints constraints_. It also returns the list of mutating groups and the list of switching groups. ****)
(*****)

DirectTransitionCost[{eqinitial_, eqfinal_, constraints_}] := Module[{a, b, c, aux},
  (** Using the previous function, compute the number of mutations necessary for each possible first switching group **)
  a = Block[{Print}, DirectTransitionCostWithSpecificFirstSwitchingGroup[{eqinitial, eqfinal, 1, constraints}]];
  If[StringContainsQ[ToString[a], "Error"], Return[a]];
  b = Block[{Print}, DirectTransitionCostWithSpecificFirstSwitchingGroup[{eqinitial, eqfinal, 2, constraints}]];
  If[StringContainsQ[ToString[b], "Error"], Return[b]];
  c = Block[{Print}, DirectTransitionCostWithSpecificFirstSwitchingGroup[{eqinitial, eqfinal, 3, constraints}]];
  If[StringContainsQ[ToString[c], "Error"], Return[c]];

  (** Compare these costs and return the cost, list of mutating groups and first switching group associated with the lowest one
  **)
  If[Assuming[constraints, FullSimplify[(a[[1]] >= b[[1]] && b[[1]] <= c[[1]]) /. {NA -> 1,  $\gamma$  -> 1/2}], Return[b]];
  If[Assuming[constraints, FullSimplify[(a[[1]] >= c[[1]] && b[[1]] >= c[[1]]) /. {NA -> 1,  $\gamma$  -> 1/2}], Return[c]];
  If[Assuming[constraints, FullSimplify[(a[[1]] <= c[[1]] && b[[1]] >= a[[1]]) /. {NA -> 1,  $\gamma$  -> 1/2}], Return[a]];

  (** If one inequality cannot be decided, raise an error with the undecided inequality **)
  aux = Assuming[constraints, FullSimplify[(a[[1]] >= b[[1]]) /. {NA -> 1,  $\gamma$  -> 1/2}]];
  If[ToString[aux] != "True" && ToString[aux] != "False", Return[Print["Error 4:"]; Print[aux]; {"Error 4", aux}]]; aux =
  Assuming[constraints, FullSimplify[(a[[1]] >= c[[1]]) /. {NA -> 1,  $\gamma$  -> 1/2}]];
  If[ToString[aux] != "True" && ToString[aux] != "False", Return[Print["Error 5:"]; Print[aux]; {"Error 5", aux}]];
  aux = Assuming[constraints, FullSimplify[(b[[1]] >= c[[1]]) /. {NA -> 1,  $\gamma$  -> 1/2}]];
  If[ToString[aux] != "True" && ToString[aux] != "False", Return[Print["Error 6:"]; Print[aux]; {"Error 6", aux}]];
]

```



```

(*****)
(***) Function to build the simplified matrix of transition costs: for each pair of initial and final equilibrium, (k,l), build
a list of all the possible expressions for the transition cost between them (i.e. TransitionCostsMatrixoneachsubzone,) and a
list of the corresponding constraints on minority sizes (i.e. Tableadditionalconstraints). (***)
(*****)

TransitionCostsMatrix[{constraints_}] := Module[{Tableadditionalconstraints, counterpos, counterposmax,
TransitionCostsMatrixoneachsubzone, countersuccessivepb, k, l, res, aux1, aux2, FirstSwitchingGroupMatrix,
MutatingGroupsMatrix},

  Tableadditionalconstraints = Table[-99, {k, 4}, {l, 4}, {i, 1, 200}];
  (** At the beginning, each table Tableadditionalconstraints[[k,l]] only contains one first cell equal to True **)
  For[k = 1, k <= 4, k++,
  For[l = 1, l <= 4, l++,
  Tableadditionalconstraints[[k, l, 1]] = True;
  ]
  ];
  counterpos = Table[1, {k, 4}, {l, 4}];
  counterposmax = Table[1, {k, 4}, {l, 4}];
  TransitionCostsMatrixoneachsubzone = Table[-99, {k, 4}, {l, 4}, {i, 1, 200}];
  FirstSwitchingGroupMatrix = Table[-99, {k, 4}, {l, 4}, {i, 1, 200}];
  MutatingGroupsMatrix = Table[-99, {k, 4}, {l, 4}, {i, 1, 200}];
  countersuccessivepb = 0;

  For[k = 1, k <= 4, k++,
  For[l = 1, l <= 4, l++,
  If[k != l,
  (** For each pair of initial and final equilibrium, (k,l), starting from the whole set P, DirectTransitionCost is called.
  If it returns an error (with a new constraint on NB and NC), then
  this constraint is added to the present cell and a new cell is added at the end of the list
  Tableadditionalconstraints[[k,l]] with the opposite constraint. Then, the function is called again on the same celle. If it
  raises an error, the same procedure as before applies. If a result is returned, it is stored in
  TransitionCostsMatrixoneachsubzone[[k,l]] at the same position as the cell presently analyzed **) While[counterpos[[k, l]] <=
  Length[Tableadditionalconstraints[[k, l]]] && ToString[Tableadditionalconstraints[[k, l, counterpos[[k, l]]]] != "-99" &&
  countersuccessivepb <= 50,
  res = (Block[{Print}, DirectTransitionCost[{k, l, constraints && Tableadditionalconstraints[[k, l, counterpos[[k,
  l]]]]]) /. {NA -> 1,  $\gamma$  -> 1/2};

  (* Case where there is an error *)
  If[StringContainsQ[ToString[res], "Error"],
  aux1 = Assuming[constraints, FullSimplify[Tableadditionalconstraints[[k, l, counterpos[[k, l]]]] && ! (res[[2]])];
  If[(! (StringContainsQ[ToString[aux1], "="))] && ToString[aux1] != "False",
  Tableadditionalconstraints[[k, l, counterposmax[[k, l]] + 1]] = aux1;
  counterposmax[[k, l]] = counterposmax[[k, l]] + 1];
  aux2 = Assuming[constraints, FullSimplify[Tableadditionalconstraints[[k, l, counterpos[[k, l]]]] && res[[2]]];
  If[(! (StringContainsQ[ToString[aux2], "="))] && ToString[aux2] != "False",
  Tableadditionalconstraints[[k, l, counterpos[[k, l]]]] = aux2;
  Tableadditionalconstraints[[k, l, counterpos[[k, l]]]] = False];
  countersuccessivepb++;

  (* Case where there is no error *)
  TransitionCostsMatrixoneachsubzone[[k, l, counterpos[[k, l]]]] = Assuming[constraints, FullSimplify[res[[1]]];
  MutatingGroupsMatrix[[k, l, counterpos[[k, l]]]] = Assuming[constraints, FullSimplify[res[[2]]];
  FirstSwitchingGroupMatrix[[k, l, counterpos[[k, l]]]] = Assuming[constraints, FullSimplify[res[[3]]];
  counterpos[[k, l]] = counterpos[[k, l]] + 1;
  countersuccessivepb = 0;
  ]
  ]
  ];
  ];
  If[countersuccessivepb >= 50, Return[Print["Too many pbs"]];
  {Tableadditionalconstraints[All, All, 1 ;; Max[counterposmax]], TransitionCostsMatrixoneachsubzone[All, All, 1 ;;
  Max[counterposmax]], MutatingGroupsMatrix[All, All, 1 ;; Max[counterposmax]], FirstSwitchingGroupMatrix[All, All, 1 ;;
  Max[counterposmax]]}
  ]

```

```
(*****
(***) Auxilliary function to the next one: it allows to verify that direct mutations are the most efficient for specific
initial equilibrium, final equilibrium and constraints on minority sizes *****)
(*****
```

```
AuxTestOptimalityDirectTransitionCosts[{onemutationcost_, constraints_, eqinitial_, eqfinal_}] := Module[{eqinter, targeteq,
bool, booltot, firstswitchinggroup, secondswitchinggroup, numeroeqinter, numerotargeteq, costfirstswitch,
lowerboundcostsecondswitch},
```

```

(* The two possible intermediary equilibria different from the initial and the final one are computed *)
eqinter = Complement[{1, 2, 3, 4}, {eqinitial, eqfinal}];
booltot = True;
(** For each possible first switching group, second switching group, intermediary equilibrium and target equilibrium (i.e.
the equilibrium towards which the second switching group switches: the final equilibrium or the second intermediary
equilibrium) compute the cost of the first switch and a lower bound on the cost of the second switch (it is the minimum between
lowerboundcostsecondswitch, lowerboundcostsecondswitch2 and lowerboundcostsecondswitch3). Then, compare the sum of the cost of
the first switch and the lower bound on the cost of the second switch with the direct transition cost and show that it is
always greater. **)
For[{firstswitchinggroup = 1, firstswitchinggroup <= 3, firstswitchinggroup++},
For[{secondswitchinggroup = 1, secondswitchinggroup <= 3, secondswitchinggroup++},
For[{numeroeqinter = 1, numeroeqinter <= 2, numeroeqinter++},
costfirstswitch = Block[{Print}, DirectTransitionCostWithSpecificFirstSwitchingGroup[{eqinitial, eqinter[[numeroeqinter]],
firstswitchinggroup, constraints}]];

If[StringContainsQ[ToString[costfirstswitch], "Error"], Return[costfirstswitch]];
costfirstswitch = costfirstswitch[[1]];
targeteq = {eqfinal, eqinter[[numeroeqinter]]};

For[{numerotargeteq = 1, numerotargeteq <= 2, numerotargeteq++},
If[{secondswitchinggroup == 1,
lowerboundcostsecondswitch = Assuming[constraints, FullSimplify[Minimize[{nB + nC, Payoffhierarchy[{1, eqinitial, 0, nBi
- nB, nCi - nC}]] == Payoffhierarchy[{1, targeteq[[numerotargeteq]], 0, nB, nC}]] == Payoffhierarchy[{1,
eqinter[[numeroeqinter]], 0, nB - nBi, nC - nCi}]] && constraints && nC >= nCi >= 0 && nB >= nBi >= nB >= 0}, {nBi, nCi,
nB, nC}]]][[1]]
];

If[{secondswitchinggroup == 2,
lowerboundcostsecondswitch = Assuming[constraints, FullSimplify[Minimize[{nA + nC, Payoffhierarchy[{2, eqinitial, nAi -
nA, 0, nCi - nC}]] == Payoffhierarchy[{2, targeteq[[numerotargeteq]], nA, 0, nC}]] == Payoffhierarchy[{2,
eqinter[[numeroeqinter]], 1 - nAi, 0, nC - nCi}]] && constraints && nC >= nCi >= nC >= 0 && 1 >= nAi >= nA >= 0}, {nAi, nCi, nA,
nC}]]][[1]]
];

If[{secondswitchinggroup == 3,
lowerboundcostsecondswitch = Assuming[constraints, FullSimplify[Minimize[{nA + nB, Payoffhierarchy[{3, eqinitial, nAi -
nA, nBi - nB, 0}]] == Payoffhierarchy[{3, targeteq[[numerotargeteq]], nA, nB, 0}]] == Payoffhierarchy[{3,
eqinter[[numeroeqinter]], 1 - nAi, nB - nBi, 0}]] && constraints && nB >= nBi >= nB >= 0 && 1 >= nAi >= nA >= 0}, {nAi, nBi, nA,
nB}]]][[1]]
];

bool = Assuming[constraints, FullSimplify[Reduce[costfirstswitch + lowerboundcostsecondswitch >= onemutationcost]];
If[ToString[bool] != "True" && ToString[bool] != "False", Return[Print["Error Test1: "]; Print[bool, ",", eqinitial,
",", eqfinal, ",", eqinter[[numeroeqinter]], ",", firstswitchinggroup, ",", secondswitchinggroup, ",", costfirstswitch, ",",
lowerboundcostsecondswitch, ",", onemutationcost, ",", constraints]; {"Error Test1", bool}]];

booltot = booltot && bool
];
]
];
booltot]

```

```
TRANSITIONMATRIX = TransitionCostsMatrix[{baselineconstraints}];
```

```

(*****
(***) Function to verify that "direct" mutations from the initial to the final equilibrium are always the most efficient way to
transit from one equilibrium to the other (i.e. mutations towards other intermediary equilibria are underefficient) ****)
(*****)

TestOptimalityDirectTransitionCosts[constraints_] := Module[{matrixdirecttransitioncosts, eqinitial, eqfinal, nbcase,
booltot, subzonestobetested, counterpos, counterposmax, aux1, aux2},
  booltot = True;
  matrixdirecttransitioncosts = TRANSITIONMATRIX;

  (** For each initial equilibrium, final equilibrium, and expression of the direct mutation cost between these two equilibria
(indexed by nbcase), try to test the property that the
  direct mutation cost is the lowest possible one, using the auxiliary function AuxTestOptimalityDirectTransitionCosts. If
this function returns an error (and a constraint on minority sizes), study the case where this new constraint is verified and
the case where it is not separately. Try again to test the property and sub-divide cases further if necessary, until the
property can be checked. **)
  For[eqinitial = 1, eqinitial <= 4, eqinitial++,
    Print["eqinitial: ", eqinitial];
    For[eqfinal = 1, eqfinal <= 4, eqfinal++,
      Print["eqfinal: ", eqfinal];
      If[eqinitial != eqfinal,
        For[nbcase = 1, nbcase <= Length[TRANSITIONMATRIX[[1, 1, 1]]], nbcase++,
          Print["nbcase: ", nbcase];
          If[ToString[TRANSITIONMATRIX[[2, eqinitial, eqfinal, nbcase]]] != "-99",
            subzonestobetested = {FullSimplify[TRANSITIONMATRIX[[1, eqinitial, eqfinal, nbcase]] && constraints];
            counterpos = 1;
            counterposmax = 1;

            While[counterpos <= counterposmax && counterposmax <= 40,
              If[ToString[subzonestobetested[[counterpos]]] == "False",
                counterpos++,
                aux1 = AuxTestOptimalityDirectTransitionCosts[{TRANSITIONMATRIX[[2, eqinitial, eqfinal, nbcase]],
subzonestobetested[[counterpos]], eqinitial, eqfinal];

                If[StringContainsQ[ToString[aux1], "Error"],
                  aux1 = FullSimplify[subzonestobetested[[counterpos]] && ! (aux1[[2]])];
                  aux2 = FullSimplify[subzonestobetested[[counterpos]] && aux1[[2]]];
                  subzonestobetested[[counterpos]] = aux1;
                  subzonestobetested = Append[subzonestobetested, aux2]; counterposmax++;
                ,
                booltot = FullSimplify[booltot && aux1];
                counterpos++;
              ]
            ]
          ]
        ]
      ]
    ];
  Print["booltot: ", booltot];
];
booltot]

```

```

(*****)
(***) Edmond algorithm function to determine, using a matrix of mutation costs, the list of Nash equilibria, a specific
equilibrium root_ and under the constraints constraints_, the minimum spanning tree which root is root_. This function is
recursive (a new graph is built at each call of Edmond algorithm, if the initial graph contains a cycle). The
InitialCorrespondingEdges_ variable allows to store, at each step of the algorithm and for each edge of the new graph, the edge
to which it corresponded in the old graph ****)
(*****)

Edmondalgorithm[{InitialTransitionCostsMatrix_, initiallistNE_, InitialCorrespondingEdges_, root_, constraints_}] :=
Module[{FunctionPi, Minimumcostfound, ListPivV, counter, GraphPivV, Cycle, VertexListCycle, Minimumspanningtree,
AuxTransitionCostsMatrix, Correspondingedgesaux, othernode, Nextminimumspanningtree, Nextcorrespondingedgesaux,
NewTransitionCostsMatrix, NewListofVertices, nodetoexpand, incomingnodetonodetoexpand, correspondingnodeinnodetoexpand,
Cyclewithoutedge, outgoingnodetonodetoexpand, listnewedgesfromnexttree, Newgraphaux, nodecorrespondingtonodetoexpand,
Graphcyclewithoutedge, listedgesinnextminimumspanningtree, i, j, bool, bool2, bool3, a, resedmond},

  (** Initialization of function pi and of the other variables **)
  FunctionPi = Table[-99, Length[InitialTransitionCostsMatrix[[1]]]];
  AuxTransitionCostsMatrix = InitialTransitionCostsMatrix;
  Correspondingedgesaux = InitialCorrespondingEdges;

  (** Test if the number of Nash equilibria is equal to 2. If yes, do nothing. If not, compute the lowest cost edge coming to
  each equilibrium in initiallistNE (except to the root) and save the initial equilibrium of this edge in FunctionPi ***)
  If[Length[initiallistNE] == 2, Null,
    For[i = 1, i <= Length[initiallistNE], i++,
      If[initiallistNE[[i]] != root,
        Minimumcostfound = Infinity;
        For[j = 1, j <= Length[initiallistNE], j++,
          bool = Assuming[constraints, Simplify[AuxTransitionCostsMatrix[[initiallistNE[[i]], initiallistNE[[j]]]] <
            Minimumcostfound]];

          If[ToString[bool] != "True" && ToString[bool] != "False", Return[Print["Error 7: "]; Print[bool]; {"Error 7", bool}]];

          If[j != i && (bool),
            Minimumcostfound = AuxTransitionCostsMatrix[[initiallistNE[[i]], initiallistNE[[j]]]];
            FunctionPi[[initiallistNE[[i]]] = initiallistNE[[j]]; Null]];
          (* Define the list ListPivV, which contains the minimum cost edge coming to each equilibrium except the root
          (ListPivV[[i,1]] corresponds to the initial equilibrium of the minimum cost edge going to ListPivV[[i,2]])*)ListPivV =
          Table[-99, {x, Length[initiallistNE] - 1}, {y, 2}];
          counter = 1;

          For[i = initiallistNE[[1]], i <= initiallistNE[[Length[initiallistNE]]], i++,
            If[i != root && MemberQ[initiallistNE, i],
              ListPivV[[counter, 1]] = FunctionPi[[i]];
              ListPivV[[counter, 2]] = i;
              counter++, Null]];
          (* ListPivV is turned into a graph in order to test whether it contains cycles *)GraphPivV = Graph[DirectedEdge @@@
            ListPivV, VertexLabels -> "Name"];
          (* Test whether there are cycles and get the first one *)
          Cycle = {};
          If[FindCycle[GraphPivV] == {}, Minimumspanningtree = GraphPivV, Cycle = FindCycle[GraphPivV][[1]];
            VertexListCycle = VertexList[Cycle];

            (** Construct the new graph replacing this cycle with a unique new node and reweighting edges properly. NewListofVertices
            contains the nodes of the new graph and NewTransitionCostsMatrix is the matrix containing the new costs of transitions **)
            (* Definition of the new list of nodes *)
            NewListofVertices = Table[-99, Length[initiallistNE] - Length[VertexListCycle] + 1];
            counter = 1;
            For[i = 1, i <= Length[initiallistNE], i++, If[MemberQ[VertexListCycle, initiallistNE[[i]]] == False,
              NewListofVertices[[counter]] = initiallistNE[[i]];
              counter++, Null]];
            NewListofVertices[[counter]] = Length[AuxTransitionCostsMatrix[[1]]] + 1;
            ];

            (** Definition of the new matrix: all the nodes belonging to the cycle in the old graph are replaced by one unique node in
            the new graph, vC, (i.e. the node in position Length[AuxTransitionCostsMatrix[[1]]]+1 of the new matrix). All edges in the old
            graph are studied one by one. When one node of the edge belongs to the cycle, this edge is replaced by a new edge (in the new
            graph), which weight is as described in Edmond's algorithm. When several edges link the same two nodes in the new graph, the
            lowest cost one is kept. **)
            NewTransitionCostsMatrix = Table[Infinity, {x, Length[AuxTransitionCostsMatrix[[1]]] + 1}, {y,
              Length[AuxTransitionCostsMatrix[[1]]] + 1}];
            For[i = 1, i <= Length[initiallistNE], i++, For[j = 1, j <= Length[initiallistNE], j++,
              (* Case where the initial node of the edge belongs to the cycle *)
              If[(MemberQ[VertexListCycle, initiallistNE[[i]]] == False) && MemberQ[VertexListCycle, initiallistNE[[j]]],
                bool2 = Assuming[constraints, Simplify[AuxTransitionCostsMatrix[[initiallistNE[[j]], initiallistNE[[i]]]] -
                  AuxTransitionCostsMatrix[[initiallistNE[[j]], FunctionPi[[initiallistNE[[j]]]]]] <
                  NewTransitionCostsMatrix[[Length[AuxTransitionCostsMatrix[[1]]] + 1, initiallistNE[[i]]]]];
                If[ToString[bool2] != "True" && ToString[bool2] != "False", Return[Print["Error 8: "]; Print[bool2]; {"Error 8",
                  bool2}]];

                If[bool2,
                  NewTransitionCostsMatrix[[Length[AuxTransitionCostsMatrix[[1]]] + 1, initiallistNE[[i]]] =

```

```

AuxTransitionCostsMatrix[[initiallistNE[[j]], initiallistNE[[i]]] - AuxTransitionCostsMatrix[[initiallistNE[[j]],
FunctionPi[[initiallistNE[[j]]]]];
Correspondingedgesaux[[initiallistNE[[i]], Length[AuxTransitionCostsMatrix[[1]]] + 1, 1]] = initiallistNE[[i]];
Correspondingedgesaux[[initiallistNE[[i]], Length[AuxTransitionCostsMatrix[[1]]] + 1, 2]] = initiallistNE[[j]];
, Null], Null];
(* Case where the final node of the edge belongs to the cycle *)
If[MemberQ[VertexListCycle, initiallistNE[[i]]] && (MemberQ[VertexListCycle, initiallistNE[[j]]] == False),

    bool3 = Assuming[constraints, Simplify[AuxTransitionCostsMatrix[[initiallistNE[[j]], initiallistNE[[i]]] <
NewTransitionCostsMatrix[[initiallistNE[[j]], Length[AuxTransitionCostsMatrix[[1]]] + 1]]];
    If[ToString[bool3] != "True" && ToString[bool3] != "False", Return[Print["Error 9: "]; Print[bool3]; {"Error 9",
bool3}]];

    If[bool3,
        NewTransitionCostsMatrix[[initiallistNE[[j]], Length[AuxTransitionCostsMatrix[[1]]] + 1]] =
AuxTransitionCostsMatrix[[initiallistNE[[j]], initiallistNE[[i]]];
        Correspondingedgesaux[[Length[AuxTransitionCostsMatrix[[1]]] + 1, initiallistNE[[j]], 1]] = initiallistNE[[i]];
        Correspondingedgesaux[[Length[AuxTransitionCostsMatrix[[1]]] + 1, initiallistNE[[j]], 2]] = initiallistNE[[j]];
        , Null];, Null];
        (* Case where neither the initial nor the final node of the edge belongs to the cycle *)
        If[(MemberQ[VertexListCycle, initiallistNE[[i]]] == False) && (MemberQ[VertexListCycle, initiallistNE[[j]]] == False) &&
i != j,
            NewTransitionCostsMatrix[[initiallistNE[[j]], initiallistNE[[i]]] = AuxTransitionCostsMatrix[[initiallistNE[[j]],
initiallistNE[[i]]];
            Correspondingedgesaux[[initiallistNE[[i]], initiallistNE[[j]], 1]] = initiallistNE[[i]];
            Correspondingedgesaux[[initiallistNE[[i]], initiallistNE[[j]], 2]] = initiallistNE[[j]];
            , Null];];];
];
(* Treatment of the case when the graph studied contains only two nodes *)
If[Length[initiallistNE] == 2, othernode = If[initiallistNE[[1]] == root, initiallistNE[[2]], initiallistNE[[1]];
Minimumspanningtree = Graph[DirectedEdge @@@ {{root, othernode}}, VertexLabels -> "Name"];, Null];
(** Returning to the case where the graph contains more than two nodes, if the new graph built contains a cycle, by
recurrence, apply Edmond's algorithm to the new graph to obtain its minimum spanning tree and then, using it, build the minimum
spanning tree of the old graph. **)
If[FindCycle[GraphPivV] != {} && Length[initiallistNE] != 2,
    resedmond = EdmondAlgorithm[NewTransitionCostsMatrix, NewListOfVertices, Correspondingedgesaux, root, constraints];

    If[StringContainsQ[ToString[resedmond], "Error"], Return[resedmond];
    (* Using the minimum spanning tree computed with Edmond's algorithm for the new graph, build the minimum spanning tree
corresponding to the old graph *)
    {Nextminimumspanningtree, Nextcorrespondingedgesaux, a} = resedmond;
    (* Find the node (i.e. nodetoexpand) of the new graph that corresponds to the cycle in the old graph. *)
    nodetoexpand = NewListOfVertices[[Length[NewListOfVertices]];
    listedgesinnextminimumspanningtree = EdgeList[Nextminimumspanningtree];
    incomingnodetonodetoexpand = -99;
    (* Find the node that is at the origin of the minimum spanning tree's edge (in the new graph) that leads to nodetoexpand *)
    For[i = 1, i <= Length[listedgesinnextminimumspanningtree], i++,
        If[listedgesinnextminimumspanningtree[[i, 2]] == nodetoexpand,
            incomingnodetonodetoexpand = listedgesinnextminimumspanningtree[[i, 1]]; Null]
        ];
    (* If there is no such node, then it must be the root of the minimum spanning tree *)
    If[incomingnodetonodetoexpand == 999999, incomingnodetonodetoexpand = root, Null];
    (* Compute the node of the old graph orrespondingnodeinnodetoexpand so that the edge (incomingnodetonodetoexpand,
correspondingnodeinnodetoexpand) of the old graph corresponds to the edge (incomingnodetonodetoexpand, nodetoexpand) in the new
graph *)
    correspondingnodeinnodetoexpand = Correspondingedgesaux[[incomingnodetonodetoexpand, nodetoexpand, 2]];

    (* Suppress, in the cycle, the minimum cost edge leading to correspondingnodeinnodetoexpand *)
    Cyclewithoutedge = EdgeDelete[Cycle, DirectedEdge[FunctionPi[[correspondingnodeinnodetoexpand]],
correspondingnodeinnodetoexpand]];
    Graphcyclewithoutedge = Graph[Cyclewithoutedge, VertexLabels -> "Name"];
    (* Build the new list of edges in the old graph's minimum spanning tree *)
    listnewedgesfromnexttree = Table[{-99, -99}, Length[listedgesinnextminimumspanningtree]];
    counter = 1;
    For[i = 1, i <= Length[listedgesinnextminimumspanningtree], i++, If[listedgesinnextminimumspanningtree[[i, 1]] ==
nodetoexpand, nodecorrespondingtonodetoexpand = Correspondingedgesaux[[nodetoexpand, listedgesinnextminimumspanningtree[[i,
2]], 1]];
        listnewedgesfromnexttree[[counter]] = {nodecorrespondingtonodetoexpand, listedgesinnextminimumspanningtree[[i, 2]]};
        counter++, If[listedgesinnextminimumspanningtree[[i, 2]] == nodetoexpand, listnewedgesfromnexttree[[counter]] =
{incomingnodetonodetoexpand, correspondingnodeinnodetoexpand};
            counter++, listnewedgesfromnexttree[[counter]] = {listedgesinnextminimumspanningtree[[i, 1]],
listedgesinnextminimumspanningtree[[i, 2]]};
            counter++];];];

    Newgraphaux = Graph[DirectedEdge @@@ listnewedgesfromnexttree, VertexLabels -> "Name"];
    (* Merge the minimum spanning tree of the new graph and the cycle of the old graph minus the suppressed edge to obtain the
minimum spanning tree of the old graph *)
    Minimumspanningtree = GraphUnion[Graphcyclewithoutedge, Newgraphaux, VertexLabels -> "Name"];
    {Minimumspanningtree, Correspondingedgesaux, 0}

```

```

(*****)
(**** Function to compute, for a given set of constraints on minority sizes, NB and NC, the lng-term equilibrium or equilibria ****)
(*****)

DeterminationLongTermEquilibriumForSpecificConstraints[{constraints_}] := Module[{i, j, k, l, a, b, Transitioncostsmatrix,
Correspondingedges, ListNE, Minimumspanningtrees, Listedgesinminimumspanningtrees, Costsminimumspanningtrees,
Minimumcostminimumspanningtree, Longtermeq, auxboolean, Nblongtermeq, auxbooleanB, resedmond},

Transitioncostsmatrix = Table[-99, {i, 1, 4}, {j, 1, 4}];
Correspondingedges = Table[-99, {i, 1, 16}, {j, 1, 16}, {k, 1, 2}];
Minimumspanningtrees = Table[-99, {i, 1, 4}];
Costsminimumspanningtrees = Table[0, {i, 1, 4}];
Listedgesinminimumspanningtrees = Table[Null, 4];
Minimumcostminimumspanningtree = Infinity;
Longtermeq = Table[Null, {i, 1, 4}];
Nblongtermeq = 0;

(** Computation of the transition cost matrix **)
For[i = 1, i <= 4, i++,
For[j = 1, j <= 4, j++,
If[j != i,
Transitioncostsmatrix[[i, j]] = DirectTransitionCost[{i, j, constraints}] /. {NA -> 1,  $\gamma$  -> 1/2};
If[StringContainsQ[ToString[Transitioncostsmatrix[[i, j]]], "Error"], Return[Transitioncostsmatrix[[i, j]]];
Transitioncostsmatrix[[i, j]] = Transitioncostsmatrix[[i, j, 1]];
Correspondingedges[[i, j, 1]] = i;
Correspondingedges[[i, j, 2]] = j;
]
]
];

(** Apply Edmond's algorithm to find the lowest cost minimum spanning tree corresponding to this matrix **)
ListNE = {1, 2, 3, 4};

(* For this, compute the minimum spanning tree for each equilibrium *)
For[k = 1, k <= 4, k++,
resedmond = Block[{Print}, Edmondalgorithm[{Transitioncostsmatrix, ListNE, Correspondingedges, k, constraints}]];
If[StringContainsQ[ToString[resedmond], "Error"], Return[resedmond]];
Minimumspanningtrees[[k]] = resedmond[[1]];
Listedgesinminimumspanningtrees[[k]] = Edgelist[Minimumspanningtrees[[k]]];
(* Compute the cost of the minimum spanning tree (for each equilibrium) *)
For[l = 1, l <= Length[Listedgesinminimumspanningtrees[[k]], l++,
a = Listedgesinminimumspanningtrees[[k]][[l, 1]];
b = Listedgesinminimumspanningtrees[[k]][[l, 2]];
Costsminimumspanningtrees[[k]] = Assuming[constraints, FullSimplify[(Costsminimumspanningtrees[[k]] +
Transitioncostsmatrix[[b, a]]) /. {NA -> 1,  $\gamma$  -> 1/2}]];
];

(* Test if the cost of this equilibrium's minimum spanning tree is equal the cost of the lowest cost minimum spanning tree
obtained so far: if yes, add this equilibrium in the list of long-term equilibria *)
auxbooleanNB = Assuming[constraints, FullSimplify[(Costsminimumspanningtrees[[k]] == Minimumcostminimumspanningtree) /. {NA
-> 1,  $\gamma$  -> 1/2}]];
If[auxbooleanNB,
Longtermeq[[Nblongtermeq + 1]] = k;
Nblongtermeq++;
];
(* Test if the cost of this equilibrium's minimum spanning tree is strictly below the cost of the lowest cost minimum
spanning tree obtained so far: if yes, save this equilibrium as the long-term equilibrium *)
auxboolean = Assuming[constraints, FullSimplify[(Costsminimumspanningtrees[[k]] < Minimumcostminimumspanningtree) /. {NA ->
1,  $\gamma$  -> 1/2}]];
If[ToString[auxboolean] != "False" && ToString[auxboolean] != "True", Return[Print["Error 10:"]; Print[auxboolean]; {"Error
10", auxboolean}]];
If[auxboolean,
Minimumcostminimumspanningtree = Costsminimumspanningtrees[[k]];
Longtermeq[[1]] = k;
Nblongtermeq = 1;
];
];
{Transitioncostsmatrix, Costsminimumspanningtrees, Longtermeq, Minimumspanningtrees}

```

```

(*****)
(***) Function (1) to split the set  $P = \{(NB, NC) \in [0, 1] \times [0, 1], NB > NC\}$  into subsets on which the long-term equilibrium (or
equilibria) can be computed unambiguously and (2) to compute the long-term equilibrium (or equilibria) on each subset ****)
(*****)

DeterminationAllLongTermEquilibria[{constraints_}] := Module[{Tableadditionalconstraints, counterpos, res, counterposmax,
Tableresultsoneachsubzone, countersuccessivepb, aux1, aux2},
  Tableadditionalconstraints = Table[-99, {i, 1, 200}];
  Tableadditionalconstraints[[1]] = True;
  counterpos = 1;
  counterposmax = 1;
  Tableresultsoneachsubzone = Table[-99, {i, 1, 200}];
  countersuccessivepb = 0;

  (** Progressively build the table Tableadditionalconstraints, in which each cell corresponds to a subset of P on which the
  long-term equilibrium can be determined: Start with a unique cell containing the value True and, then, if the function
  DeterminationLongTermEquilibriumForSpecificConstraints raises an error and returns a constraint, add this constraint to the
  present cell and build another cell at the end of the table with the opposite constraint. Then, by recurrence, try again to
  solve the present cell. If DeterminationLongTermEquilibriumForSpecificConstraints does not raise an error, save the long-term
  equilibrium obtained in table Tableresultsoneachsubzone, and start analyzing the next cell in the table **)
  While[counterpos <= Length[Tableadditionalconstraints] && ToString[Tableadditionalconstraints[[counterpos]]] != "-99" &&
countersuccessivepb <= 50,
    If[ToString[FullSimplify[constraints && Tableadditionalconstraints[[counterpos]]]] == "False",
      counterpos++,
      (* Try to run DeterminationLongTermEquilibriumForSpecificConstraints *)
      res = Block[{Print}, DeterminationLongTermEquilibriumForSpecificConstraints[{constraints &&
Tableadditionalconstraints[[counterpos]]}]];
      (* If the function returns an error (and a new constraint), add the new constraint to the present cell and build a new cell
      at the end of the table with the opposite constraint *)
      If[StringContainsQ[ToString[res], "Error"],
        aux1 = Assuming[constraints, FullSimplify[Tableadditionalconstraints[[counterpos]] && ! (res[[2]])]];
        If[(! StringContainsQ[ToString[aux1], "=="] || StringContainsQ[ToString[aux1], ">="] || StringContainsQ[ToString[aux1],
"<="] || StringContainsQ[ToString[aux1], ">"] || StringContainsQ[ToString[aux1], "<"]),
          Tableadditionalconstraints[[counterposmax + 1]] = aux1;
          counterposmax++;];
        aux2 = Assuming[constraints, FullSimplify[Tableadditionalconstraints[[counterpos]] && res[[2]]]];
        If[(! StringContainsQ[ToString[aux2], "=="] || StringContainsQ[ToString[aux2], ">="] || StringContainsQ[ToString[aux2],
"<="] || StringContainsQ[ToString[aux2], ">"] || StringContainsQ[ToString[aux2], "<"]),
          Tableadditionalconstraints[[counterpos]] = aux2;
          Tableadditionalconstraints[[counterpos]] = False;
          countersuccessivepb++;
          Tableresultsoneachsubzone[[counterpos]] = res;
          counterpos++;
          countersuccessivepb = 0;
        ]
      ]
    ];
  {Tableadditionalconstraints[[1 ;; counterposmax]], Tableresultsoneachsubzone[[1 ;; counterposmax]]}

```