Description of the linguistic expressions of fractions

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ABSTRACT: This paper proposes a typology of fractional numbers. A few fractional numbers can be expressed by suppletive (non-systematic) forms, whereas analytical (systematic) linguistic patterns of formation produce "bi-dimensional" numerical forms which refer to both the numerator and the denominator (double argument), or "mono-dimensional" forms which refer to only one of these numbers (single argument). Moreover, a fraction in a partitive expression can be an indivisible semantic unit or may, on the contrary, have a noun or a measure word inserted between its constituents.

KEY WORDS: Fractions; Numerals; Grammatical number; Partitive expressions; Divisible semantic units; Language planning.

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1. DESCRIBING FRACTIONAL NUMBERS IN LANGUAGES

Large-scale cross-linguistic surveys were completed regarding the expression of integers, however much less was accomplished on the linguistic expression of fractional numbers. For instance, Greenberg (1978, 2000) dealt with the expression of integers; he mentioned instances of some fraction names used within the expressions of some integers – e.g., *halv* {1/2}[1] in the construction of some cardinals in Danish – however did not address the general expression of fractions, which was not his subject.

[1] The following Leipzig Glossing Rules' abbreviations are used throughout this paper: ABL: ablative; ACC: accusative; ACT: active; ADJ: adjective; ART: article; CARD: cardinal; COL: collective; DAT: dative; DECL: declarative; DEF: definite; DET: determiner; DU: dual; F: feminine; FRAC: fractional; FUT: future; GEN: genitive; IND: indicative; M: masculine; MID: middle voice; MW: measure word; NEG: negation; NOM: nominative; OBJ: object; ORD: ordinal; PART: particle; PASS: passive; PERF: perfective; PL: plural; POSS: possessive; PRS: present; PST: past; PTCP: participle; SG: singular; the numbers 1, 2 and 3 followed by a grammatical indication (e.g., OBJ, SG or PL) not restricted to numerals: respectively 1st, 2nd and 3rd person

The most inclusive source for the general history of fractions today is still Benoit, Chemla & Ritter's *History of Fractions, Fractions of History*, it focused on the *concept* of fractional numbers in ancient civilizations and only incidentally collected data on their linguistic expression. In fact, the linguistic information is often irretrievable from the symbolic notations of numbers – this means we do not know how they were read and we are not even sure they reflected spoken words (Ritter 2001) – making the available written material at times inadequate for a linguistic study. To illustrate this phenomenon, let us consider our modern notation 100 for *one hundred* {1}{100} in English with *one* {1} and *cent* {100} in Modern French with no {1}. This gives a simple illustration of how the numerical symbols can hide the difference between various linguistic patterns[2]. A more complex situation is found with the late Egyptians, who manipulated what we conceptualize as unit-fractions[3] and wrote them in hieratic with a sign which meant *part* placed on top of a sign denoting the denominator, however we are not sure of the morpho-syntactic construction of the *linguistic* expressions corresponding to these notations[4].

Benoit et al. (1992) made us understand a probable reason why fractional numbers were seldom-considered in the study of natural languages. It is because, ultimately in this field, the specifically linguistic questions seem secondary to the conceptual frameworks. In other words, the emergence of linguistic expressions for fractions was driven much more by conceptual developments among restricted groups of technically trained people – and of what (Haugen 1983) called the *corpus planning*[5] they would initiate – than by any *natural* or *normal* language evolution, be that phonetic, morphosyntactic or lexical.

Nevertheless, in this paper, we can offer a typology which enables to describe and classify the linguistic expressions for fractions. This is achieved according to way the numerical information – the values of numerator and denominator – is expressed within fractional numbers, regardless of the intricacy of the morpho-syntactic means used to build these expressions, or the

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pronoun; an integer *n* followed by a grammatical indication related to numerals (e.g., CARD, FRAC, ORD, etc.): a numeral that expresses the integer *n*.
These standard abbreviations are augmented with: {*n*}: an unanalysable (or unanalysed) form for the integer *n*; {*n*/*d*}: a suppletive form for the fraction *n*/*d*.

[2] Following Huddleston & Pullum (2002), one needs to differentiate *numerals* which denote linguistic expressions (e.g., English '*one hundred*' or French '*cent*') versus *numbers* which are the numerical meaning or content (e.g., the quantity 100). Please note that 19th century French had '*un cent*' {1}{100}.

[3] Egyptian calculations with fractions resulted in unit-fractions, while non-unit fractions were expressed as sums of unit-fractions (any fraction can indeed be expressed as the sum of unitary fractions.); e.g., 4/7 was conveyed as the sum of 1/2 and 1/14 both written in hieratic (Gardiner 1957: 196). The only known exceptions were the use of special signs for the two non-unit fractions 2/3 and 3/4.

[4] Sethe (1916: 84–86) relied on Coptic sources and conjectured a genitive construction starting with '*r*' (*part*) and linked by a preposition to a cardinal which expressed a denominator; e.g., 1/5 would have been expressed as '*part of five*'.

[5] Haugen distinguished four steps: *selection of norm* (which is societal and somehow exterior to the language); *codification of the norm*; *implementation of function* (includes the activities of writers and institutions); and *elaboration of function* (involves the production of a linguistic corpus complying with the norm).

partitive expressions including them. Actually, putting aside *most* of the morpho-syntactic features in the examples I gathered was essentially how I could arrive at my typology.

As should be expected, the majority of our evidence, either formal or linguistic, come from secondary literature and technical texts, e.g., mathematics, architecture, geography, hydraulics.

I tried to wider the range of languages I could account for. But I faced the lack of data in the languages of cultures with no strong mathematical traditions. These cultures did not develop linguistic means to express arithmetical matters they were not interested in, or the influence of the people who cared for these issues was not strong enough to pervade the whole society, nor the general language which is documented today. Be that as it may, these sociolinguistic aspects are outside the scope of this paper.

Producing a large-scale survey, or providing an organized language corpus, seems, at best, hardly feasible in the current state of the literature. Even the linguistic data base WALS Online[6] does not consider fractions, and this is because monographies on individual languages usually do not do it either. As a matter of fact, we hope our contribution will initiate enough interest for researchers to systematically gather the relevant information on these matters.

Our typology proposes to categorize any expression of a fractional number as either a *suppletive* form (non-systematic) or an *analytic* form (systematic). The set of suppletive forms in a given language maps a finite set of fractions. The analytic form can split into *mono-dimensional* (systematic, single argument) and *bi-dimensional* (systematic, double argument), and they can define limited sets or general series of fractions.

The simplicity of this typology might reflect a case of what biologists call *convergent evolution*, that is the independent evolution of similar features in species of different periods or epochs in time, where similar forms or functions were created that were not present in the last common ancestor of those species. In the field of linguistics, we would talk of similarities in the patterns of expression which are not the result of a genetic relation nor of a process of borrowing, but emerged separately to answer the same needs.

In order to exemplify each type of expression, this paper reorganized, according to the typology, the data which initially enabled us to induce relevant descriptive criteria. One side-result was the possibility to deduce and assess some capabilities of each type of expression.

[6] Dryer, Matthew S. & Haspelmath, Martin (eds.) 2013. The World Atlas of Language Structures Online. Leipzig: Max Planck Institute for Evolutionary Anthropology: http://wals.info [accessed on 2022.03.23].

2. SUPPLETIVE FORMS

Suppletive forms can be used to express a few particular fractions, they are not obviously derived from cardinal forms, at least from a synchronic view-point.

Let us see some suppletive forms for 1/2:

	Suppletive forms for the fraction 1/2	The cardinal 2
English	*one-half*	*two*
Latin (Maher and Makowski 2001)	*semis*	*duo*
Standard Arabic (Schulz et al. 2000: 111, 214)	*niṣf*[7] (ADJ)	*iṯnāni* (ADJ.M)
Thai (Smyth 2002: 178)	*khrueng*[8]	*song*
Tibetan (Wang Zhijing 1994: 109)	*phye-ka*[9]	*gnyis*
Contemporary Standard Chinese	*bàn* or *yī bàn*[10]	*èr* (cardinal or ordinal) *liǎng* (cardinal only)

In terms of their capacities in quantification or partitive phrases, Tibetan *phye-ka* and Thai *khrueng* are juxtaposed with a noun or measure word in the same manner as names of integers. Arabic *niṣf* requires nouns to bear the definite article as shown in (1). English *one half* can make use of a preposition in (2) forming a usual partitive expression, however the juxtaposition *one half the population* is possible also, perhaps more colloquial.

(1) *niṣf* *aš-šaʿb*
{1/2}.ADJ ART.DEF-population
'half of the population'

(2) *one half* *of* *the* *population*
{1}{1/2} *of* ART.DEF *population*

Latin (Maher & Makowski 2001)[11] had quite an impressive list of suppletive forms for 1/2, 1/3, 2/3, 1/4, 3/4, 1/6, 5/6, 1/8, 1/12. Then, the names of the multiples of 1/12 up to 11/12, and the names of 1/24, 1/36, 1/72, 1/144 and 1/288 (all fractions of 1/12) can be viewed as

[7] The transliteration DIN 31635 is used for Arabic.
[8] ISO 11940-2 is used in this paper to transcribe Thai.
[9] The Wylie transliteration is used for Tibetan.
[10] The *Hànyǔ Pīnyīn* transliteration is used for Chinese characters. According to the rules of this transcription, the digit [1] is always Romanized *yī* with a first tone mark regardless of the actual tone in Contemporary Chinese. This tone depends on that of the following syllable and *yī bàn* can be pronounced *yí bàn*.
[11] (Maher & Makowski 2001) account for what is available in Classical Latin sources.

suppletive forms themselves, or can be analysed with the fraction name *uncia* {1/12} (the cardinal 12.CARD was *duodecim*). For example, in (3), taken from a passage about arithmetical education in Rome, we can see the suppletive forms *semis* {1/2}, *triens* {1/3} and *uncia* {1/12}, while 5/12 can be construed with 5.CARD and {1/12}:

(3) Si de quinc-unce
if of 5.CARD-{1/12}.ABL.SG
'If from 5/12

remo-t-a est uncia,
remove-PTCP.PERF.PASS-F.NOM.SG be.IND.PRS.3SG {1/12}.NOM.SG
is taken 1/12,

quid supera-t? […] Triens. […]
what remain-IND.PRS.3SG […] {1/3}[NOM.SG] […]
what is left?' […] '1/3.' […]

Redi-t uncia,
return-IND.PRS.ACT.3SG {1/12}.NOM.SG
'Now 1/12 comes back,

quid fi-t? Semis.
what be done-IND.PRS.ACT.3SG {1/2}[NOM.SG]
what is obtained? 1/2.'
(Horace [1st c. BCE], *Ars poetica* [*The art of poetry*], verses 327–330)

By the way, the two terms *uncia* and *scripulum*, besides their numerical values, were also – or *originally* – used in metrological scales: *uncia*[12] was the suppletive form {1/12}, a unit of area (1/12 of 1 *iugerium*), a monetary unit and a unit of weight (1/12 of 1 *as*); *scripulum* was {1/288} and also a unit of area (1/288 of 1 *iugerium*). However, according to (Maher and Makowski

[12] The Latin word *uncia*, through two different processes of phonological changes, has given the English length unit *inch* (1/12 of a *foot*) and the mass unit *ounce* (it has at times been defined as 1/16 or 1/12 of a *pound*).

2001), in example (3) the two words *uncia* and *scripulum* are to be understood as fraction names, not as submultiples within a metrological scale[13].

Contemporary Chinese has a suppletive term *bàn* for 1/2 which cannot be analysed from the names *èr* and *liǎng* of the integer 2.

Classical Chinese had suppletive forms for 1/2, 1/3 and 2/3, respectively *bàn* (half), *shǎobàn* (the smaller half) and *tàibàn* (the larger half), both derived from *bàn* {1/2}; they are shown in the examples (4) and (5). These suppletive forms of fractions can be used in the same manner as integers with nouns and measure words[14].

(4) *yī* *bàn* *chéng* *yī,* *bàn* *yě,*
1.CARD {1/2} multiply 1.CARD {1/2} DECL
'1/2 times 1 is 1/2,

chéng *bàn* *sì* *fēn* *yī* *yě.*
multiply {1/2} 4.CARD part 1.CARD DECL
times 1/2 is 1/4.'
(*Suàn shù shū*[15], strip 3)

(5) *shǎobàn* *chéng* *tàibàn,* *jiǔ* *fēn* *èr* *yě.*
{1/3} multiply {2/3} 9.CARD part 2.CARD DECL
'1/3 times 2/3 is 2/9.'
(*Suàn shù shū*, strip 8)

Some texts[16] also contain *ruòbàn* (the weak half) for 1/4 and *qiángbàn* (the strong half) for 3/4, and the term *zhōngbàn* (the middle half) for 1/2. These terms – but {1/2} – could be replaced by analytical forms made with 3.CARD *fēn* and 4.CARD *fēn*. Most of these suppletive forms were lost in Contemporary Chinese: only *bàn* {1/2} and *yī bàn* i.e. 1.CARD {1/2} remain

[13] Another case of *fluidity* between fraction names and submultiples of a metrological scale can be observed in Egyptian (Ritter 2003). This lexical interchangeability can be understood in terms of the conceptual continuum between subunits and fractions, see Ritter (1992, 2001, 2003).
[14] One can see Anicotte (2015 a, 2015 b, 2017, 2019 a, 2019 b) about the fraction numbers in Chinese.
[15] The *Suàn shù shū* [*A Book on the Reckonings with Counting Rods*] is an excavated text dated to the beginning of the 2nd century BCE. When we wrote this paper, two other long mathematical texts were available: *Suàn shù* [*Reckoning Procedures*] also dated to the beginning of the 2nd century BCE, and *Shù* [*Numbers*] dated to the 3rd century BCE. They contain hundreds of fractions expressed in natural language, with no symbolic notation.
[16] The term *ruòbàn* for 1/4 is found in the first chapter of the *Xiàhóu Yáng suàn jīng* [*Xiahou Yang's Mathematical Manual*] (oldest version 1084). The terms *qiángbàn* for 3/4 and *zhōngbàn* for 1/2 are found in the *Shù shū jiǔ zhāng* [*Mathematical Treatise in Nine Sections*] (1247) (Libbrecht 1973: 70–71).

for 1/2. Moreover, the regular analytical form 2.CARD *fēn zhī* 1.CARD is also available now. The term *tàibàn* (or *dàbàn*) remains as an approximate number meaning *most*, no longer as an exact number.

3. ANALYTICAL FORMS

Using non-analytical expressions appears viable for a finite list of fractions, but a *generic* linguistic pattern capable of expressing any fractional number would be analytic and account for both the numerator and the denominator as we conceptualize them today.

This requirement may suggest that expressions involving two integer names should cover all situations. But this is not the case. Besides the numerical phrases which I call *bi-dimensional* because they do refer to both the denominators and the numerators, we also encounter *mono-dimensional* phrases referring only to their denominators or their numerators.

The bi-dimensional patterns are the most general ones, in the sense that they can potentially express any fraction; or at least any *proper fractions* (smaller than 1, e.g., 1/3). Besides, we found that the expression of so-called *improper fractions* (greater than 1, e.g., 4/3) can rely on the same pattern or not, they can also be expressed as *mixed numbers* (the sum of an integer and a proper fraction, e.g., 1+1/3 instead of 4/3).

3-1 *Mono-dimensional forms*

Some analytical mono-dimensional patterns can express potentially infinite sets of fractions. For example, unit-fractions in Latin, Sanskrit, and in Classical Chinese, are mono-dimensional forms which mention the denominator only, while our numerator 1 is not stated[17].

The unit-fractions (let us say 1/*d* with the numerator 1 and the denominator *d*) in Classical Chinese would usually take the form *d*.CARD *fēn* stating the cardinal *d*, and *fēn*, a term whose original meaning is *part* (noun and verb), forming compounds like 4.CARD *fēn* in (6). I call 4.CARD *fēn* a *mono-dimensiona*l phrase because only the numeral 4.CARD is expressed. The numerator's name 1.CARD was not compulsory and was usually omitted when the fraction name occurred as a factor in a multiplication. However, it was possible to use it, typically when communicating the result of an operation. This is the case in (6) with the *bi-dimensional* phrase 16.CARD *fēn* 1.CARD.

[17] Benoit et al. (1992) point out that the modern concept of a fraction, that is to say an object of the form *p*/*q* where *p* and *q* are integers, is a late-comer to history, and that almost all ancient literate societies only conceived of fractions in the form now called *unitary*, that is, with numerator 1 – or more precisely, conceptualized without numerator.

(6) *sì* *fēn* *chéng* *sì* *fēn* *shí liù* *fēn* *yī*
4.CARD part multiply 4.CARD part 16.CARD part 1.CARD
‘1/4 times 1/4 are 1/16’
(*Suàn shù shū*, strip 9)

Moreover, these Chinese mono-dimensional forms are used in the same manner as integers and suppletive terms for fractions before measure words making quantification phrases ‘*d*.CARD *fēn* MW’[18].

As for Latin, in *De architectura*, Vitruvius used ordinals associated with the word *pars* (part) to express unit-fractions. On one occasion given in (7), this pattern was even used to express 1/8 instead of using the suppletive form *sescuncia*.

(7) *octav-a* pars
8.ORD-NOM.F part[NOM.SG.F]
‘1/8’
(Vitruvius [1st c. BCE], *De architectura* [*On architecture*], 1.6.9)

Pliny the Elder made great use of this analytical expression of unit-fractions:

(8) Longissim-a die-i spati-a
longest-NOM.PL day-GEN.SG extent-NOM.PL
The longest extensions of the day are

hor-arum aequinoctial-ium quindecim addi-t-a
hour-GEN.PL equinoctial-GEN.PL 15.CARD add-PTCP.PERF.PASS.NOM.PL
15 equinoctial hours added of

non-a parte un-ius hor-ae
9.ORD-ABL.SG.F part.ABL.S.F 1.CARD-GEN.SG hour-GEN.SG
1/9 of an hour

[18] Anicotte (2017, 2019 b)

aut,	ut	Nigidio	plac-u-it,	quint-a.[19]
or	as	Nigidius.ABL.SG	please-PERF.3SG.ACT	5.ORD-ABL.SG.F

or 1/5 according to Nigidius.'
(Pliny the Elder [1st c. CE], *Naturalis Historia* [*Natural history*], 6.39)

The word *pars* would even be dropped when the context was clear enough to indicate that fractions were involved, as it is with *quinta* [*parte*] in (8).

Frontinus used the same pattern, and *pars* would be dropped occasionally, as for example once in (9):

(9)

Est	autem	digit-us	ut	conveni-t
be.IND.PRS.3SG	however	digit-NOM.SG.M	as	suit-IND.PRS.3SG

'However, by convention, the digit is

sexta decima	pars	ped-is,
16.ORD.NOM.F	part[NOM.SG.F]	foot-GEN.SG

1/16 of a foot,

uncia	duodecim-a.
inch.NOM.F	12.ORD-NOM.F

the inch [is] 1/12 [of a foot].' (Frontinus, *De aquae ductu urbis Romae* [*Water management of the city of Rome*] [cerca 98 CE], 1.24)

Sanskrit[20] names for unit-fractions were also mono-dimensional compounds formed with the nouns *bhāga-* or *aṃśa-*, both meaning *part*, and added to either the cardinal form as in (10) and (11) or the ordinal form[21] of the denominator's name as in (12).

[19] In the corpus available online or in printed editions, *quindecim* and *nona* in (8) are in fact respectively written XV and IX, but *quinta* is written in full letters, not as V, this is said for the sake of precision but is of no consequence for this paper.
[20] Datta & Singh (1935: 185–186), Filliozat & Mazars (1987), Mazars (1992). The expression of fractions has most likely evolved during the history of Sanskrit and other Indic languages, however available instances are too scarce for us to perceive the changes.
[21] Or rather a form derived from the ordinal according to the grammarian Pāṇini who commented that the term used in fraction names could show phonetic alterations with respect to the original ordinal form (Filliozat & Mazars 1987); Pāṇini is reputed to have been active around the 5th century BCE.

(10)	*sapta*	*bhāga*	(11)	*pañcadaśa*	*bhāga*	(12)	*pañcama*	*bhāga*
	7.CARD	part		15.CARD	part		5.ORD	part
	‘1/7’			‘1/15’			‘1/5’	

Besides names for unit-fractions (numerator 1), we wondered if there exited mono-dimensional names for fractions of a given denominator. We know only of one Chinese finite mono-dimensional series of terms which can express the tenths with the phrases *n*.CARD *chéng* (*n* being an integer from 1 to 9) which state only the numerator, while the denominator 10 is understood by a linguistic convention applicable to this particular pattern[22]. This vernacular form occurs only in non-technical contexts (e.g., to convey the likelihood of an event).

3-2 *Case of expression of the numerator with non-lexical means*

One pattern in Arabic proved enlightening as to what we could – and should – count as an expression of the numerator.

Arabic names of denominators 3 through 10 are specific words built on the basis of a vocalic shift on the cardinal forms (Schulz et al. 2000: 214). For example the word 5.FRAC.SG for 1/5 in (14) may seem to be a mono-dimensional form derived from the cardinal 5.CARD given in (13).

(13)	*ḥamas*		(14)	*ḥumus*
	5.CARD			5.FRAC.SG
	‘5’			‘1/5’

The name of 3/5 in (16) is obviously a bi-dimensional term made with the adjectival form of the cardinal for 3 and the plural form of 5.FRAC, that is to say the numerator 3 and the denominator 5 are expressed lexically.

(15)	*ḥumusain*		(16)	*ṯālṯat*	*ʾaḥmās*
	5.FRAC.DU			3.ADJ	5.FRAC.PL
	‘2/5’			‘3/5’	

Now, in the name 5.FRAC.DU of 2/5 in (15), the value 2 of the numerator can be readily retrieved from the dual form, and is therefore a bi-dimensional form. And then, we realise that

[22] Second entry for *chéng* in the *Xiandai Hanyu guifan cidian* [Dictionary of Contemporary Chinese], 2010, Beijing: Waiyu jiaoxue yu yanjiu chubanshe, p.163.

in 5.FRAC.SG the value of the numerator 1 is explicitly expressed by the singular form. That is to say, this Arabic 5.FRAC.SG is also a bi-dimensional term, dissimilar to mono-dimensional expressions of unitary fractions, exposed in the previous section, and where *grammatical number* was not present in the expressions, or not even a feature of the language in Chinese[23].

3-3 *Bi-dimensional forms*

First of all, we should be careful that not all bi-dimensional numerical phrases have to do with fractions.

When we see, or hear, the phrase '*five minus one*', we retrieve two numbers in an operation, but there is no fraction involved. The situation is similar with '*five parts minus one (part)*' which concerns not one fraction, but two.

Some bi-dimensional phares express *proportions*, e.g., 'a 40-60 split' in English or its Chinese equivalents '4 6 *kāi*' and '4 6 *fēn chéng*'. We may consider that they underly fractions (40% and 60% for the English phrase, 4/10 and 6/10 for the Chinese one), but the denominators 100 or 10 are not *expressed*.

All these phrases should be kept outside in the present study which concerns the expression of the numerator and the denominator of a fraction.

Now let us illustrate the morpho-syntactic variety of the bi-dimensional phrases for fractional numbers, that is to say the nature of the two numerical items, and the way they are joined together.

The denominator can be cardinals (denoted by CARD) as in Chinese, etc., ordinals (ORD) as in English, French, Latin, Sanskrit, etc., or have a special form (FRAC) as Arabic in the examples (14)–(16), and also in German and Swedish which respectively possess the specific suffixes *-tel* and *-del* to produce denominator names[24]. Here is an example in Swedish:

(17) två femtedel-ar
2.CARD 5.FRAC-PL[25]
'2/5'

[23] Xu Dan (2012: 1-19).
[24] These forms are invariable in German and can take the plural mark *-ar* in Swedish. Attention however to German irregular form *Drittel* (3.FRAC) and variant *Siebtel* (7.FRAC) for *Siebentel* (7.CARD-*tel*).
[25] The suffix *-del* is used to form fraction numerals, therefore *femtedelar* in (17) could be formally construed as *femte-del-ar* and glossed 5.CARD-*del*-PL, but this would hide the specificity of the pattern.

The numerator can be in first position. This is case in English, French, German, Spanish, Swedish, Thai[26], etc. And, it was exemplified above for Arabic with the example (16) showing an adjectival form of the numerator's name preceding the plural form of the special numeral expressing the denominator. Below, we will see more examples with fraction in Modern Greek, Hebrew, Latin, and proper fractions in Sanskrit.

But the numerator may as well be in second position. This is exemplified, in this paper, by Chinese, Ancient Greek, Japhug, Tibetan, and improper fractions in Sanskrit.[27]

A generic pattern for proper fractions in Latin can be seen in (18) and (19) cited by (Maher and Makowski 2001). It relies on the cardinal form of the numerator followed by the ordinal form of the denominator, which takes a plural mark, reflecting the pattern of quantification phrases.

(18) quadrat-us tribus quartis decumis suis
square-NOM.SG 3.CARD.ABL 14.ORD.ABL 3.POSS.ABL.PL
'the square is larger than the round by 3/14 of its own size'

rotund-o maior est
round-ABL.SG larger[NOM.SG] be.IND.PRS.3SG
(*De aquae ductu urbis Romae* [*Water management of the city of Rome*], 1.24)

(19) rotund-us tribus undecumis suis
round-NOM.SG 3.CARD.ABL 11.ORD.ABL 3.POSS.ABL.PL
'the round is smaller than the square by 3/11 of its own size'

quadrat-o minor est
square-ABL.SG smaller[NOM.SG] be.IND.PRS.3SG
(*De aquae ductu urbis Romae* [*Water management of the city of Rome*], 1.24)

Fraction names in Modern Hebrew take their roots in Mishnaic Hebrew (1st to 4th centuries CE). The fraction name for 2/5 in (20) can be construed as a noun phrase with the name of the numerator and the name of the denominator, in this order. They are linked using the

[26] Smyth (2002: 177–199): The bi-dimensional expressions are "numerator's name + *nai* + denominator's name" built with the preposition *nai* (in, of).
[27] This order is also found in Japanese (Martin 1975: 767) and Korean (Martin 1992: 188).

so-called *construct* pattern (based on the genitive form of the first constituent) and is to be understood as the genitival *two of five*:

(20) *štei* *ḫamišiot*
2.CARD.F.GEN 5.CARD.F.PL
‘2/5

The Modern Greek fraction name for 2/5 in (21) can be interpreted as a quantification phrase, with the denominator’s name playing the role of the quantified item and bearing a plural marker.

(21) *dyo* *pempt-a*
2.CARD 5.CARD-NOM.PL
‘2/5’

Sanskrit names for the proper fractions 3/8 and 2/7 are shown in (22) and (23), and they put the numerator in first position:

(22) *tri* *aṣṭama*
3.CARD 8.ORD
‘3/8’
(*Śulbasūtras*[28] [Datta & Singh 1935: 186])

(23) *dvi* *saptama*
2.CARD 7.ORD
‘2/7’
(*Śulbasūtras* [Datta & Singh 1935: 186])

However Sanskrit fractions are not limited to proper fractions (smaller than 1). They can also be improper fractions (greater than 1) like 5/4 in (24), showing the name for 5/4 with the numerator put in final position, formed with the suffix *-ka* added to the cardinal 5, and meaning a collection of 5 items.

[28] The *Śulbasūtras* is a collection of transmitted texts generally dated to the period between the 9th and 2nd century BCE; dates are tentative because the transmission of the corpus was only oral for centuries.

(24) *catur*[29] *pañca-ka*
{1/4} 5.CARD-COL
‘5/4’ (literally: ‘a fivesome of fourths’ according to [Filliozat & Mazars 1987])
(*Bakhśalī manuscript*[30])

Other examples of expressions with the denominator in first position are found in Ancient Greek[31] in (25) which shows a noun phrase expressing 2/5. It relied on the juxtaposition of the genitive form 5.CARD.GEN of the denominator’s name followed by the noun *moira* (part), itself quantified by the numerator’s name 2.CARD.

(25) *Lakedaimon-iōv* […] *Peloponnes-ou*
Lakedaimon-ADJ.GEN.PL […] Peloponnes-GEN.SG
The *Lacedaemons* […] of the Peloponnese
‘The Lacedaimons […] occupy two fifths of the Peloponnese’

t-ōn	*pente*	*t-as*	*dyo*	*moir-as*
ART.DEF-GEN.PL	5.CARD	ART.DEF-ACC.F.PL	2.CARD	part-ACC.F.PL
of the five		the two parts		

nem-ontai
occupy-PRS.IND.MID.3PL
occupy
(Thucydides [5th c. BCE], *The History of the Peloponnesian War*, 1-1-10)

The Tibetan bi-dimensional pattern for fractions is explained by Goldstein et al. (1991: 200) and Wang Zhijing (1994: 108–109). The syllable *cha*[32] is placed after the denominator’s name, and the compound “denominator’s name-*cha*” is inserted before the numerator’s name as

[29] The final *r* in *catur* (a word for 1/4) could alternatively be written as its allophone *ḥ* forming the word *catuḥpañcka*.
[30] The *Bakhśalī manuscript* is a mathematical text on pieces of birch bark which was excavated in 1881, it was arguably written around the 7th century C.E. in a late or hybrid variety of Sanskrit (Hayashi 1995).
[31] Waanders (1992) implies that very few instances of bi-dimensional expressions for fractions are known in Ancient Greek.
[32] In Tibetan, *fraction* is said *cha-grangs* i.e. ‘number of parts’ from *cha* ‘part’ and *grangs* ‘number’. The Tibetan-Chinese dictionary *Zàng Hàn dà cí diǎn* (Beijing: Mínzú chūbǎnshè [The Ethnic Publishing House], 1993, vol.1: 772) states that *cha* is a noun meaning *part*, and can also be a measure word meaning *pair*. It is used in the expression of fractions and also appears as a suffix in some nouns formed from other nouns.

shown in (26). A pattern which respects the order "*noun* + *numeral*" of Tibetan's quantification phrases, assuming we construe the compound suffixed with *-cha* as a nominal form and the numerator's name as the quantifier:

(26) *lgna* *cha* *gnys*
5.card *cha* 2.CARD
'2/5'
(Goldstein et al. 1991: 200)

Japhug (Jacques 2008, 2021) is a Tibeto-Burman language of the rGyalrong branch (not a Tibetic language) spoken in Sichuan Province, China. The Japhug bi-dimensional expression of 1/3 in (27) relies on the words tɯ-tɯcɯr (part) and ŋgɯ (interior):

(27) χsɯ tɯcɯr ɣɯ ɯ ŋgɯ tɯ tɯcɯr
3.CARD part GEN 3SG.POSS interior 1.CARD part
'1/3' (expressed as '*one part within three parts*')
(Jacques [personal communication by e-mail, March 2014])

In Classical Chinese, the bi-dimensional pattern stated the denominator and the numerator in this order; see examples (28)–(33):

(28) *jiǔ* *fēn* *èr*
9.CARD part 2.CARD
'2/9'
(*Suàn shù shū*, strip 8)

(29) *qī* *fēn* *zhū* *liù*
7.CARD part *zhū* 6.CARD
'6/7 *zhū*' (*zhū* is a unit of weight)
(*Suàn shù shū*, strip 28)

When a measure word was involved, it was put directly after *fēn* as above in (29) and below in (31) and (33). The genitive morpheme *zhī* could optionally be inserted in the bi-dimensional construction, as can be seen in (30), (31) and (33):

(30) *gè* *shòu* *sān shí* *fēn* *zhī* *èr shí sān*
each get 30.CARD part GEN 23.CARD
'each gets 23/30' (about the sharing of a sum of money)
(*Suàn shù shū*, strip 26)

(31) *jīn* *qī* *fēn* *zhū* *zhī* *sān*
gold 7.CARD part *zhū* GEN 3.CARD
'3/7 *zhū* of gold' (*zhū* is a unit of weigh)
(*Suàn shù shū*, strip 30)

No improper fraction would be mentioned. However, there were mixed numbers (sums of an integer and a proper fraction); as in (32), the sum of 12 and 11/72 with no measure word expressed by juxtaposition, and in (33) with a measure word. Sometimes, a conjunction *yòu* can be found between the integer and the proper fraction.

(32) *shí èr* *qī shí èr* *fēn* *shí yī*
12.CARD 72.CARD part 11.CARD
'12 11/72'
(*Suàn shù shū*, strip 36)

(33) *zòng* *yī* *bù* *liù* *fēn* *bù* *zhī* *yī*
length 1.CARD *bù* 6.CARD part *bù* GEN 1.CARD
'a length of 1 *bù* 1/6 *bù*' (*bù* is a unit of length)
(*Suàn shù shū*, strip 121)

In the *Jiǔ zhāng suàn shù* [*Nine Chapters on the Mathematical Art*][33], the item *zhī* is used systematically in bi-dimensional fraction names. This form was also used in the *Shù shū jiǔ zhāng* [*Mathematical Treatise in Nine Sections*][34] (1247) and it was transmitted to Korea and Japan[35].

[33] We only have a transmitted version. One can refer to Chemla & Guo (2004: 71–97) for a detailed history of the manuscript.
[34] Qin Jiushao used a symbolic notation to write down numbers involved in operations; however, all the numbers were repeated in the text using the linguistic numeration.
[35] The pattern "denominator's name + *bun no* + numerator's name" is found in Japanese (Martin 1975: 767) with *bun*, the Sino-Japanese reading of Chinese *fēn*, and *no* the Japanese determination linker. In

However, when no measure word was involved, the pattern "denominator's name + *zhī* + numerator's name" (without *fēn*) was also found in both non-mathematical and mathematical texts[36]. Besides, there were scarce alternative bi-dimensional expressions of fractions other than the sequences with obligatory *fēn* and optional *zhī*, they were found in both excavated and transmitted texts, whether mathematical or not[37]. For instance, *qǔ* (take) could be found instead of *zhī*.

This variety of different forms at a given time means that the attempts to standardize the pattern of expression remained in the realm of technical texts written in a quasi-formal language by literati who devoted their work to mathematics, while the general corpuses would exhibit free formation.

In today's Chinese, the phrases "denominator's name + *fēn zhī* + numerator's name" contain the sequence *fēn zhī* and are indivisible: measure words are placed after these fraction names and not between their constituents "denominator's name + *fēn*" and "*zhī* + numerator's name"[38]. However, one should note that free formation – or conservatism – can still be seen in Sinitic languages with, for example, *zhī* non-compulsory in Cantonese fraction names (Matthews & Yip 2011: 453).

We end here this accumulation of various morpho-syntactic situations, at times within one language. As we wrote at the beginning of the paper, this variety was what we had to face when we started to deal with this subject, and we used it to induce our typology of fraction expressions.

3-4 *Bi-dimensional forms as divisible or indivisible semantic units*

Let us now focus on one particular characteristic of bi-dimensional fraction names when they are put into a partitive expression: they can perform as indivisible semantic units or, on the contrary, the sequence of their constituents can be split by nouns or measure words.

This issue arose with the examples (29), (31) and (33) above where the constituents of the bi-dimensional phrases for fractional numbers in Classical Chinese could be separated, and a

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Korean there are "denominator's name + *pun uy/ci* + numerator's name" with *uy* the indigenous Korean genitive linker or *ci* its Sino-Korean counterpart (Martin 1992: 188).

[36] Notably by Li Zhizao in his *Tóng wén suàn zhǐ* [*Arithmetic Guidance of the Combined Learning*] (1613) and Wu Jiashan in his *Suàn xué èr shí yī zhǒng* [*Twenty-one Books of Arithmetic*] (1863) only to cite modern mathematical texts.

[37] They were listed, in Chinese, by Hu Changqing (1996), Zhang Shoujun (1997), Fang Wenyi (1995, 2009), Da Zhengyue & Yin Shunmin (2008). One can see Anicotte (2015 b), in English, for a study on this issue.

[38] It is hard to assess the detailed process of the change which occurred in the context of the abandonment of Classical Chinese, and of educational reforms. Exploring the evolution of fraction names would be in itself an object of research, the part played by institutions of standardization also.

measure word would be placed just after the sequence "denominator's name + *fēn*", and therefore before the numerator's name.

On the contrary, the English partitive expressions in (34) and the Modern Greek one in (35) both involve fraction names which are indivisible units connected to the quantified item with a genitive construction (a preposition in English and the genitive case in Greek).

(34) two fifth-s of a litre of water
2.CARD 5.ORD-PL of ART-SG litre of water

(35) *t-a* *dyo* *pempt-a* *t-on* *Ellen-on*
ART.DEF-NOM.PL 2.CARD 5.CARD-NOM.PL ART.DEF-GEN.PL Greek-GEN.PL
'2/5 of the Greeks'

Modern Standard Arabic partitive expressions in (36) and (37) also involve indivisible fraction names; an article is required on the noun.

(36) *ḥumusay* *aš-ša ʿb*
5.FRAC.DU.ADJ ART.DEF-people
'2/5 of the people'

(37) *ṯālṯat* *ʾaḥmās* *aš-ša ʿb*
3.CARD.ADJ 5.FRAC.PL ART.DEF-people
'3/5 of the people'

In Contemporary Chinese, the sequences "denominator's name + *fēn zhī* + numerator's name" are also indivisible semantic units. They can occur before or after what they quantify, directly juxtaposed or linked with the determination particle *de*. Examples of juxtaposition are given in (38) with a measure word and in (39) with a noun:

(38) *wǔ* *fēn zhī* *èr* *shēng*
5.CARD *fēn zhī* 2.CARD litre
'2/5 [of a] litre'

(39) *wǔ* *fēn zhī* *èr* *rénkǒu*
5.CARD *fēn zhī* 2.CARD population
'2/5 [of the] population'

Examples of the use of the determination particle *de* are given in (40) and (41). The order "fraction name + *de* + noun" in (40) leaves the noun in the head position while the order "noun + *de* + fraction name" in (41) puts the fraction in the head position, slightly changing the focus of the noun phrase.

(40) *wǔ* *fēn zhī* *èr* *de* *rénkǒu*
5.CARD *fēn zhī* 2.CARD DET population
'2/5 of the population'

(41) *rénkǒu* *de* *wǔ* *fēn zhī* *èr*
population DET 5.CARD *fēn zhī* 2.CARD
'2/5 of the population'

The Contemporary Chinese indivisible bi-dimensional expressions for fraction names can be used in the same manner as integers when they occur in quantification phrases.

4. CONCLUSION

The words or phrases expressing fractions in one language can be indigenous or take their origin in contacts and loans from a literary language. They can be standardized or freely made up on the spot, using a great variety of morpho-syntactic means.

Some languages rely on the ordinal form or other special form of the denominator; while cardinals and a given structure or order for the constituents can suffice in other languages. The name of the numerator can be in first position (this is the case in Arabic, English, French, German, Modern Greek, Hebrew, Latin, Spanish, Swedish, Thai, etc.), or in second position (e.g., in Chinese, Ancient Greek, Japanese, Japhug, Korean, Tibetan, etc.), while Sanskrit had instances of both orders. The phrases expressing fractions can *at times* reflects the patterns of quantification phrases with adjectival or genitive constructions.

Our typology puts this diversity aside, and classifies fractional numbers according to the way the numerical information (numerator and denominator) is conveyed. We ended up with suppletive forms vs analytical forms (further divided into mono- or bi-dimensional).

In a given language, a finite list of fractions can be expressed with suppletive forms. Analytic mono-dimensional phrases stating only the denominator or the numerator can also be observed, this scheme is restricted to a few series of fractions; for example, the infinite sets of unit fractions 1/*n* in Sanskrit, Latin, and Classical Chinese. Finally, the most generic patterns to express fractional numbers are analytical bi-dimensional phrases accounting, in lexical or morpho-syntactical ways, for both the numerator and the denominator; these bi-dimensional patterns of formation can potentially express any fraction of two integers.

The cross-linguistic variety of the partitive expressions with fractions is due to the cross-linguistic diversity of the partitive expressions themselves, but actually this was not relevant to this paper, except for the fact it is only when they become part of a partitive expression that it becomes apparent whether bi-dimensional phrases for fractional numbers are indivisible semantic units or not. Parenthetically, it should be noted that mono-dimensional expressions, as well as suppletive forms, do not work in the same manner as numerals in quantitative phrases in all languages. This is why, whenever enough data is accessible, not only the details of the formation of fractional numbers, but also their syntactic insertion in partitive expressions or quantification phrases deserve to accounted for.

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