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Morphogenesis of spatial networks through way analysis

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1 Introduction

One may find in many physical and biological systems a spatial shape creating a network, from the streets in a city [1][2][3] to a nervation network [4], through clay desiccation patterns [5], slime mold and corals [6]. These networks share as a main characteristic their creation process : a growth mechanism through propagation, bifurcation and reticulation, mostly in the 2D plane [7]. They are the consequence of the iteration of a process through time. As such, they share characteristics that we can highlight through the right methodology and indicators of network analysis.

In this abstract, we propose through the notion of ways (the continuity of edges in a spatial network) a way to numerically extract the network from images, and an ensemble of relevant indicators to understand the historicity, the structure of the complex system. We also discuss how such analysis leads us to new approaches for a deeper understanding of growing complex network in general.

2 Description of spatial growing networks

The most obvious object to describe such networks is a simple graph, taking every cross and dead-end as vertices, and the segments as edges (from Fig.1a to 1b). However, the topology of those graphs are trivial, the degree of vertices being typically 3 for a arborescent network, 4 for a reticulated one and most of the complexity of the network is therefore lost in this simple transformation.

To keep some of the properties of our spatial networks, we consider several segments aligned with one another to be part of the same object, called a Way (Fig.1c). This object has been developed for street city networks analysis, but constructing the ways can be harder than it first seems, mainly because a criterion is needed to tell if two segments are in the same way or not. In some systems, an exogenous criterion can be given (street names, subway lines, ...) but it is not always relevant [3]. A geometric criterion based on the angle between two segments is often used [3][8][6] and a detailed discussion about this construction can be found in [1].

On cracks patterns in clay, since we have access to the structure of the network at any time, we can reconstruct the ways with a chronological criterion: for a crossing in the

final network, the two older segments that pass by are in the same way. A further work is to investigate the links between the temporal and geometrical criteria: this should give us an optimization for the limit angle in geometrical criterion.

Then, the dual graph is constructed from this ways objects, taking each way as a vertice, and making edges between two vertices if the two corresponding ways cross (Fig. **1d**). That way, we keep both a well studied mathematical structure (graph) and a lot of properties from the initial network. From a morphogenetic point of view, the growing ways are the basic bricks of our networks, and the network final complexity emerges from the growing and interactions between these ways in the 2D plane.

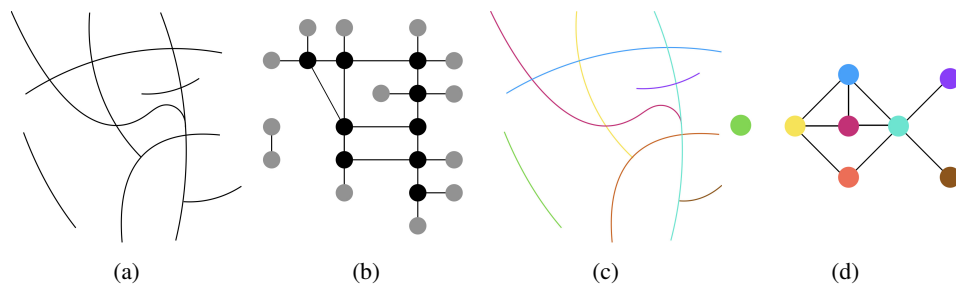


Fig. 1: Two different graph interpretation for a spatial network. **1a**: Initial spatial network. **1b**: Transformation of previous network into a simple topological graph. **1c**: Showing the "Ways" objects that compose the spatial graphs. **1d**: Dual graph of the network constructed with the ways: each vertice represents a way, each edge a cross between two ways.

3 Data extraction for quantitative analysis

We created a software called *PySkelWays* <https://github.com/DaluS/PySkelWays> that allow the extraction of ways through image analysis (Fig. **2**). The image is first binarised, then an ensemble of spatial arcs is obtained through a Voronoi skeletonisation. Unions of arcs (spatial edges) are then locally determined two by two at each intersection using propagation criteria, and ways are deduced as the ensemble of arcs two by two connected. This method is robust on a wide range of shapes, even when the width of arc is comparable to their individual length as in *Gorgonia ventalina*. This software can apply to a large variety of networks and has been tested on city streets networks, clay cracks and corals.

A module to take into account the evolution of the network for systems where dynamics can be observed is currently developed, in the context of clay dessication networks.

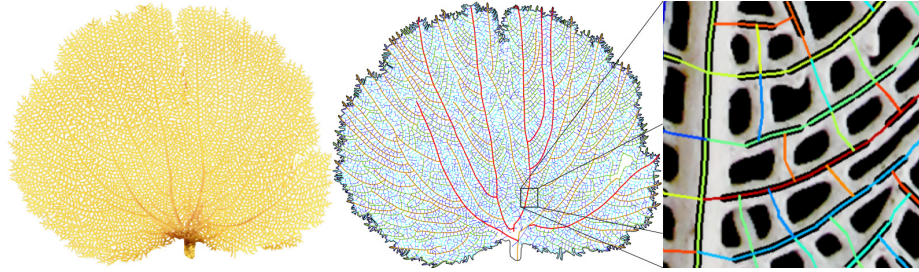


Fig. 2: Image analysis on Gorgonia Ventalina (coral): From left to right : input image, ways reconstruction (colored by number of arcs), zoom on the network (arbitrary color per way)

4 Results

From the dual graph analysis, a first result on the degree distributions can be seen through different types of networks. The degree of a node represents the number of crossing of the corresponding way. The degree distribution that we observe is typically a power-law distribution, with an exponent typically between 2 and 3 [6][8].

Since power-law can be a signature of a scale-free mechanism, we investigated the possible origin of this behavior. If one takes a square box, divides it into two equal boxes with a way, then carry on this process to the two new created box, the distribution for the degree of the ways will follow a power law with an exponent of 2 [2]. (To pass from a generation to another, the degree of a way is divided by 2 and the number of those ways is multiplied by 4). This auto-similar process with no crosses thus give a power law with a exponent of two, which gives a lower bound to the exponents we find in our networks.

From a graph generation point of view, the Barabasi-Albert model [9], where a new node is connected to several existing nodes with a probability proportional two their degrees, gives a degree distribution of a power-law with an exponent of 3. It can be related to our networks where a high-degree way is more susceptible to be a big one, thus increases the probability for a young way to end on it. However, it doesn't take the spatiality of the network into account, nor the fact that ways tend to stop growing after a time, but gives a higher bound to our exponents. The Price model [10], a directed-graph generalization of Barabasi-Albert can produce exponents from 2 to 3, but no analogy between the parameters of the Price's model and this type of spatial growth has been effectively found. Other studies for road and street networks also find an exponent between 2.2 and 2.5 [3][11]

Finally, we can add that for the length distributions some more complex distributions are observed, closer to a log-normal distribution [2]. We currently investigate the links between those two distributions, since degree and lengths tends to be correlated in our network (high degree with big lengths), but where degree is discontinuous, the length is not, which could lead to the divergence observed in the lower part of distribution.

5 Discussion and further researches

The on-going investigations of our team are on clay dessication process: after mixing clay with water, we let it dry, taking photos to access the dynamics of the crack network. We aim to understand the influence of external parameters (temperature, humidity, thickness, ...) on the final patterns. This system can be seen as a toy system to understand more complex networks where we can't control the growth.

In addition to this, we're developing a mathematical model that codes the local dynamics of ways in order to verify our assertion by comparing the structure of the network at the end of the simulation to the real ones.

If the description in ways works particularly well for system with a strong primary growth (elongation-bifurcation-reticulation) such as cracks cities and corals, system with strong secondary growth (reorganisation of the system) are harder to describe with ways. Indeed this secondary mechanism erase a part of the historical information through ways local reorganization. Still, ways are a promising element to build a descriptive model of intertwined growth shapes.

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