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Public Spending Shocks in a Liquidity-Constrained Economy*

Edouard Challe† Xavier Ragot‡

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Abstract

This paper analyses the effects of transitory increases in government spending when public debt is used as liquidity by the private sector. Aggregate shocks are introduced into a flexible-price, incomplete-market economy where heterogenous, infinitely-lived households face occasionally binding borrowing constraints and store wealth to smooth out idiosyncratic income fluctuations. Debt-financed increases in public spending facilitate self-insurance by bond holders and may crowd in private consumption. The implied higher stock of liquidity also loosens the borrowing constraints faced by firms, thereby raising labour demand and possibly the real wage. Whether private consumption and wages actually rise or fall ultimately depends on the relative strengths of the liquidity and wealth effects that arise following the shock.

Keywords: Borrowing constraints; public debt; fiscal policy shocks.

JEL codes: E21; E62.

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Introduction

This paper analyses the effects of transitory fiscal expansions when public debt is used as liquidity by the private sector. We conduct this analysis in an incomplete-market model where agents face uninsurable idiosyncratic income risk and cannot borrow against future income (i.e., markets are 'liquidity-constrained' in the terminology of Kehoe and Levine, 2001, amongst others). Non-Ricardian models of this type have on occasion been used to analyse the aggregate and welfare effects of public debt in the steady state (see Woodford, 1990; Aiyagari and McGrattan, 1998).[1] To date, there have been surprisingly few attempts at clarifying how such economies respond to aggregate fiscal *shocks*. One important contribution is Heathcote (2005), who offers a quantitative assessment of the impact effect of tax cuts. In this paper, we attempt to characterise analytically the impact and dynamic effects of public spending shocks on macroeconomic aggregates.

The spending shocks of which we analyse the effects have one significant, and realistic, feature: they are at least partly financed by government bond issues in the short run, with public debt then gradually reverting to some long-run target value thanks to future tax increases.[2] Note that whether public spending is financed by taxes or debt does not matter in complete markets, Ricardian economies with lump-sum taxation, because households' discounted disposable income flows are identical between alternative modes of government financing. Then, under reasonable assumptions about preferences and technology, the negative wealth effects associated with transitory spending shocks lead to falls in the demand for both private consumption and leisure, which in turn produces a drop in the real wage (e.g., Baxter and King, 1993).

The deficit financing of spending shocks can, however, have very different consequences

[1]Other important applications of the liquidity-constraint paradigm to macroeconomic issues include Bewley-type monetary models (e.g., Bewley, 1983; Scheinkman and Weiss, 1986), models of capital accumulation with precautionary savings (e.g., Aiyagari, 1994; Huggett, 1997), models of business cycles with heterogenous agents (e.g., Krusell and Smith, 1998) and asset-pricing models with borrowing constraints and short-sales constraints (e.g., Heaton and Lucas, 1996; Krusell and Smith, 1997).

[2]For example, Blanchard and Perotti (2002) document a limited impact response of taxes to spending shocks in the U.S., implying deficit financing in the short run. Bohn (1998) established that the U.S. debt-GDP ratio is mean-reverting thanks to the corrective action of the primary surplus.

when public debt is used as private liquidity, that is, as a store of value held by agents for precautionary, or 'self-insurance', purposes. Starting from a situation in which private liquidity is scarce (in a sense that we specify below), such policies have the side effect of increasing the stock of liquidity available in the economy, thereby facilitating self-insurance by bond holders and effectively relaxing the borrowing constraints faced by households and firms. As we show, the liquidity effects associated with rising public debt tend to foster households' private consumption demand, along with the labour demand of borrowing-constrained firms. Whether and when such liquidity effects may offset wealth effects, and thus overturn the predictions of the Ricardian model regarding the effects of spending shocks on private consumption and wages, is the central theme of this paper.

It is perhaps surprising that the actual impact of our fiscal experiment is still subject to so much empirical controversy. In particular, the application of different identification strategies to U.S. data has either supported the Ricardian prediction of a fall in private consumption and wages following an increase in public spending (Ramey and Shapiro, 1998; Ramey, 2008), or come to the opposite conclusion that both variables actually increase after the shock (e.g., Blanchard and Perotti, 2002; Perotti, 2007), which latter is consistent with the Old Keynesian model and with a version of the New Keynesian model endowed with a sufficient number of market imperfections (Gali *et al.*, 2007). Given this lack of consensus, our goal here is not to take any definitive position as to whether an adequate fiscal policy model should generate pro- or counter-cyclical responses of those variables to public spending shocks. Rather, we use our model to illustrate that both outcomes are theoretically possible (and not implausible quantitatively) depending on the relative strengths of the liquidity and wealth effects that arise following the shock. As we show, which effect actually dominates crucially depends on how quickly the fiscal rule followed by the government ensures the reversion of public debt towards its long-run target following the initial fiscal deficit. If taxes rise promptly after the increase in public spending, then public debt will not vary very much and liquidity effects will be weak; in this situation, wealth effects are likely to be dominant and private consumption and wages will fall. If, on the contrary, the slow reaction of taxes leads to a substantial growth of public debt in the short and the medium run, then liquidity effects maybe strong enough to dominate wealth effects and private consumption and wages will rise. Overall, temporary increases in public spending are all the more effective at raising output that the simultaneous response of taxes is limited.[3]

The market incompleteness-cum-borrowing constraint assumption is the only departure from the frictionless Ricardian framework considered here, the other aspects of our model remain fully standard in a stripped-down form. In contrast to several recent contributions on the effect of public spending shocks, we thus assume that the labour and goods markets are perfectly competitive, that both nominal prices and wages are fully flexible, that utility is separable over time as well as over consumption and leisure at any point in time, that all agents are utility-maximising, that there are no externalities associated with public spending, and that taxes are lump sum. In particular, our results make clear that the pro-cyclical responses of private consumption and wages after a fiscal expansion may naturally arise from the non-Ricardian nature of the model *alone*, making other familiar imperfections, or various possible combinations of them, unnecessary.[4]

Our model belongs to the growing literature on the consequences of market incompleteness and borrowing constraints for fiscal policy outcomes. Woodford (1990) derived the optimal level of steady-state public debt in a deterministic model where liquidity-constrained agents hold government bonds for precautionary purposes. This work was subsequently extended by Aiyagari and McGrattan (1998) to incorporate idiosyncratic uncertainty, and then by Floden (2001) to take into account government transfers. Heathcote (2005) introduced aggregate uncertainty about taxes into this framework, while our paper studies the effects of aggregate uncertainty about public spending (for the first time, as far as we are aware.) Methodologically, our paper is closest in spirit to Woodford's in that we construct a model

[3] This latter result is, of course, not inconsistent with some traditional Keynesian views about the effectiveness of fiscal policy (e.g., the textbook 'Keynesian cross' model). It is, however, grounded on a very different set of assumptions here.

[4] Recent fiscal policy models include Ravn *et al.* (2006), who assume imperfect competition together with habit formation over individual varieties of the consumption good, Linnemann (2006), who assumes that consumption and leisure are nonseparable while consumption is an inferior good, Linnemann and Shabert (2003), who have imperfect competition and sticky nominal prices, and Gali *et al.* (2007), who combine ad hoc 'hand to mouth' households with imperfect competition and price rigidities in both goods and labour markets. Papers analysing the effects of distortionary taxation in the neoclassical growth model include Ludvigson (1996) and Burnside *et al.* (2004), while Baxter and King (1993) consider the effects of government spending shocks when the latter generate external productivity effects.

that admits a closed-form solution wherein liquidity and wealth effects can be disentangled analytically. We are able to do so thanks to simplifying assumptions that limit agents' heterogeneity (despite the presence of uninsurable income shocks) and allow the behaviour of the model to be summarised by a small-dimensional dynamic system; while our focus is on the impact of public spending shocks here, the construction of a tractable general equilibrium model with heterogenous agents may be of interest in other contexts. Finally, Angeletos and Panousi (2007) recently analysed the effect of changes in government spending in an incomplete-market economy with idiosyncratic production risk. There are at least three important differences between their work and ours. First, they study a Ricardian economy in which there is no liquidity role for government bonds, while we precisely seek to understand how the changes in liquidity supply induced by fiscal shocks may affect aggregate outcomes. Second, they focus on permanent spending shocks (i.e., changes in the size of the government), whereas our analysis is chiefly motivated by the recent empirical puzzles pertaining to the effect of transitory fiscal shocks. Third and foremost, in their model the wealth effects associated with higher (present and future) taxes lower firms' labour demand and lead, under standard preferences, to a fall in both wages and private consumption. While such supply-side effects may arguably be at work after a permanent increase in public spending, our purpose here is to understand when and why transitory spending shocks may generate *pro-cyclical* private consumption, labour demand and wages.

Section 1 introduces our basic liquidity-constrained economy. Section 2 derives the implications of debt-financed increases in public spending shocks for the crowding-in of private consumption. Section 3 introduces borrowing-constrained entrepreneurs, allowing us to study the effect of aggregate liquidity on labour demand and the real wage. Section 4 summarises our results and discusses their robustness.

1 The model

The economy is populated by a government, as well as by a unit mass of infinitely-lived households and a large number of firms interacting in perfectly competitive goods and labour markets. The technology and preferences of our baseline model are as in Scheinkman and Weiss (1986): firms turn one unit of labour input, L_t, in one unit of the output good, Y_t, and households maximise:

$$E_t \sum_{j=0}^{\infty} \beta^j \left(u \left(c_{t+j}^i \right) - l_{t+j}^i \right), \tag{1}$$

where c_t^i and l_t^i are the consumption demand and labour supply of household i at date t, respectively, $\beta \in (0,1)$ is the subjective discount factor, and $u(c)$ is a twice continuously differentiable utility function satisfying $u'(c) > 0$, $u'(0) = \infty$, $u''(c) < 0$. In addition, we limit the curvature of $u(.)$ by assuming that $\sigma(c) \equiv -cu''(c)/u'(c) \leq 1$.[5] While our baseline model implies constant equilibrium wages ($= 1$ since $Y_t = L_t$), the production function will be modified in Section 3 so as to allow for a liquidity-sensitive labour demand curve and time-varying wages.

1.1 Households

Idiosyncratic income shocks are modelled as follows. The status of individual households in the labour market randomly switches between employment (during which they freely choose their labour supply) and unemployment (in which they are excluded from the labour market). The individual labour-income fluctuations that result are assumed to be entirely uninsurable (i.e., agents cannot issue assets contingent on their future employment status, and there are no unemployment benefits). In addition, households' asset wealth must be non-negative at all times, so that households cannot use private borrowing and lending to insulate individual consumption from idiosyncratic income fluctuations. Given these restrictions, the only way households can smooth consumption is by holding (riskless) government bonds. Household

[5] That $\sigma(c) \leq 1$ ensures that intertemporal substitution effects in consumption do indeed dominate intertemporal income effects (given a linear disutility of labour), so that the steady-state rate of interest increases with steady-state public debt.

i thus faces the following budget and non-negativity constraints:

$$c_t^i + a_t^i = a_{t-1}^i R_{t-1} + \xi_t^i l_t^i - T_t, \tag{2}$$

$$c_t^i \geq 0,\ l_t^i \geq 0,\ a_t^i \geq 0. \tag{3}$$

In equation (2), a_t^i denotes the total quantity of bonds held by household i at the end of period t, T_t is a (possibly negative) lump-sum tax collected on all households at date t, R_{t-1} is the riskless gross interest rate on bonds from date $t-1$ to date t, and ξ_t^i is an indicator variable taking on the value 1 if the household is employed at date t and 0 otherwise. Employed households have a constant probability $\pi \in (0,1)$ of falling into unemployment in the next period, and unemployed households stay so for one period only: $\text{Prob}\left(\xi_{t+1}^i = 1 \middle| \xi_t^i = 1\right) = \pi$ and $\text{Prob}\left(\xi_{t+1}^i = 0 \middle| \xi_t^i = 0\right) = 0$ (nothing substantial changes if we allow the probability of moving out of unemployment to be less than one).

Since only employed households derive income from their labour supply, household i chooses $l_t^i = 0$ whenever $\xi_t^i = 0$. On the other hand, equations (1)–(2) and the fact that the real wage is 1 imply that the intratemporal optimality condition for employed households (i.e., for whom $\xi_t^i = 1$) is l_t^i satisfying:

$$u'\left(a_{t-1}^i R_{t-1} + l_t^i - T_t - a_t^i\right) = 1 \tag{4}$$

On the other hand, since households' asset holdings must be nonnegative, the intertemporal optimality condition for household i is:

$$u'\left(a_{t-1}^i R_{t-1} + \xi_t^i l_t^i - T_t - a_t^i\right) \geq \beta R_t E_t u'\left(a_t^i R_t + \xi_{t+1}^i l_{t+1}^i - T_{t+1} - a_{t+1}^i \middle| \xi_t^i\right), \tag{5}$$

with (5) holding with strict inequality if the borrowing constraint is binding (and thus $a_t^i = 0$), and with equality otherwise (in which case $a_t^i > 0$).

In general, uninsurable income uncertainty of the kind assumed here generates a very large number of household types, due to the dependence of current decisions on the household's entire history of individual shocks, and the distribution of types must be approximated numerically (e.g., Aiyagari, 1994; Heathcote, 2005). Here we focus on a particular equilibrium with a limited number of household types and a finite-state wealth distribution, allowing us to derive the model's dynamics in closed form. We construct this equilibrium using a simple 'guess and verify' method based on two conjectures, and then derive sufficient conditions for both conjectures to hold in equilibrium once all their behavioural and market-clearing implications have been worked out. As stated in Proposition 1 further below, the sufficient conditions for both conjectures to hold are that i) public debt trend-revert towards a sufficiently low long-run target, and ii) deviations of public debt from target be of limited magnitude.

The first conjecture (**C1**) is that the borrowing constraint is always binding for unemployed households. As such, unemployed households hold no government bonds at the end of the current period (i.e., they would like to borrow, rather than save), so that we can write from (2):

$$\xi_t^i = 0 \Rightarrow c_t^i = a_{t-1}^i R_{t-1} - T_t, \tag{6}$$

where a_{t-1}^i is household i's bond holdings inherited from the previous period (when this household was employed). The second conjecture (**C2**) is that the borrowing constraint is never binding for employed households. From (1)–(2), the intratemporal optimality condition for any employed household i imposes that the marginal rate of substitution between leisure and consumption be equal to the real wage, so that we obtain:

$$\xi_t^i = 1 \Rightarrow c_t^i = u'^{-1}(1) \equiv c^e. \tag{7}$$

Any employed household stays employed in the next period with probability π and falls into unemployment with probability $1-\pi$. Conjecture **C2** implies that employed households' consumption-savings plans are interior (i.e., $a_t^i > 0$ if $\xi_t^i = 1$) and, from (1), (6) and (7), that these plans obey the following Euler equation:

$$1 = \beta \pi R_t + \beta (1-\pi) R_t E_t u'(a_t^i R_t - T_{t+1}). \tag{8}$$

The left-hand side of equation (8) is the current marginal utility of an employed household, $u'(c^e) = 1$. The first part of the right-hand side of (8) is the discounted utility of a marginal unit of savings if the household stays employed in the next period (in which case $u'(c_{t+1}^i) = u'(c^e) = 1$), while the second part is the marginal utility of the same unit when the household falls into unemployment in the next period (i.e., becomes unemployed, liquidates assets and, from equation (6), enjoys marginal utility $u'(c_{t+1}^i) = u'(a_t^i R_t - T_{t+1})$).

In equation (8), household i's current asset demand only depends on aggregate variables (R_t and T_{t+1}). The solution a_t^i to (8) is thus identical across employed households and we can write:

$$\xi_t^i = 1 \Rightarrow a_t^i = a_t \ (> 0). \tag{9}$$

Equations (6) and (9) imply that all unemployed households (labelled 'u-households' from now on) have identical consumption levels, so that their budget constraint becomes:

$$u : c_t^u = a_{t-1} R_{t-1} - T_t. \tag{10}$$

Employed households can be of two different types, depending on whether they were employed or not in the previous period. Call the former 'ee-households' and the latter 'ue-households'. In the current period, ue-households consume c^e and save a_t but enjoy no asset payoff (since they were borrowing-constrained at date $t-1$ and thus chose $a_{t-1}^i = 0$). Then, equations (2), (7) and (9) yield the labour supply of ue-households, l_t^{ue} (which is homogenous across such households) as the residual of the following equation:

$$ue : c^e + a_t = l_t^{ue} - T_t. \tag{11}$$

On the other hand, ee-households consume c^e, save a_t, and enjoy the asset payoff $a_{t-1}R_{t-1}$. This also uniquely defines their labour supply, l_t^{ee}, through the following equation:

$$ee : c^e + a_t = a_{t-1} R_{t-1} + l_t^{ee} - T_t. \tag{12}$$

To summarise, **C1** and **C2** imply that households can be of three different types only (with budget constraints (10)–(12)), while the equilibrium wealth distribution is two-state (i.e., $a_t^i = a_t > 0$ or 0). Note that it is almost sure, asymptotically, that any two randomly chosen households have different individual income histories, due to the idiosyncratic nature of unemployment shocks. Nevertheless, under our conjectures households' heterogeneity is limited by the fact that only last period's an current idiosyncratic shocks matter in determining households' types. This is because, under **C1** and **C2,** i) households falling into unemployment liquidate their asset wealth entirely, and ii) agents falling out of unemployment adjust labour supply so as to reach their target level of precautionary savings, a_t instantaneously.[6]

[6] Of course, in reality individual asset depletion and repletion following changes in labour income are gradual rather than immediate. Our focus on a tractable analysis of aggregate fiscal shocks under incomplete markets and agents' heterogeneity requires that we abstract from this inertia in individual asset adjustments. Of course, the individual wealth target itself, a_t, will vary over time following changes in public spending.

Given the assumed probabilities of changing employment status, the invariant proportions of u-, ee- and ue-households are $\Omega \equiv (1-\pi)/(2-\pi)$, $1-2\Omega$ and Ω, respectively (i.e., the proportion of employed households is $1-2\Omega+\Omega=1-\Omega$). For simplicity, we assume that the proportion of each type of household is at the invariant distribution level from $t=0$ onwards.

1.2 Government

Let G_t and T_t denote government consumption and lump-sum taxes during period t, respectively, and B_t the stock of public debt at the end of period t. The government faces the budget constraint:

$$B_{t-1}R_{t-1} + G_t = B_t + T_t. \tag{13}$$

In equation (13), we think of transitory variations in G_t as being exogenously chosen by the government, of B_t as adjusting endogenously over time depending on the primary deficit and the equilibrium interest rate, and of T_t as obeying a fiscal rule with feedback from macroeconomic and/or fiscal variables. Following the observation by Bohn (1998) that the US debt-GDP ratio is stationary, we restrict our attention to rules ensuring that public debt reverts towards its (exogenous) long-run target B at least asymptotically. Such rules, which exclude Ponzi schemes, are consistent a wide variety of feedback mechanisms, including that from public debt to primary deficit as in Bohn (1998), from output and debt to structural deficits (e.g., Gali and Perotti, 2003), as well as from public debt and public spending to taxes (e.g., Gali *et al.*, 2007). Loosely speaking, stationarity requires that the tax feedback be sufficiently strong never to allow public debt to drift away from target forever; in Section 2 we illustrate the dynamic effects of spending shocks under one of the simplest rules of this class, whereby tax revenues only react to deviations of public debt from its long-run (i.e., steady-state) target.

1.3 Market clearing

In our economy, only employed households hold government bonds. Given the asymptotic distribution of household types, the clearing of the bond, labour and goods markets requires, respectively:

$$(1-\Omega)\, a_t = B_t, \tag{14}$$

$$(1-2\Omega)\, l_t^{ee} + \Omega l_t^{ue} = L_t, \tag{15}$$

$$(1-\Omega)\, c^e + \Omega c_t^u + G_t = Y_t. \tag{16}$$

Substituting (7), (13) and (14) into the Euler equation (8), we may write the relation between the interest rate and fiscal variables as follows:

$$\beta R_t \left(\pi + (1-\pi)\, E_t u' \left((2-\pi)\left(B_{t+1} - G_{t+1} + \Omega T_{t+1}\right)\right)\right) = 1 \tag{17}$$

Note that as $\pi \to 1$ idiosyncratic uncertainty about labour income vanishes; the model then behaves like a Ricardian one and $R_t \to 1/\beta$, the gross rate of time preference.

A three-household type equilibrium is defined as sequences of individual consumption levels, $\{c^e, c_t^u\}_{t=0}^{\infty}$, individual labour supplies, $\{l_t^{ee}, l_t^{ue}\}_{t=0}^{\infty}$, individual bond holdings, $\{a_t\}_{t=0}^{\infty}$, and aggregate variables, $\{L_t, Y_t, B_t\}_{t=0}^{\infty}$ such that the optimality conditions (7)–(12) and the market clearing conditions (14)–(16) hold in every period, given the forcing sequence $\{G_t\}_{t=0}^{\infty}$ and a fiscal rule for $\{T_t\}_{t=0}^{\infty}$ that ensures the stationarity of public debt. We may now state the following existence proposition (the proof is found in Appendix A).

Proposition 1. Provided that public debt is stationary and that fluctuations around the steady state are small, then the three-household type equilibrium exists if and only if $B \in (0, B^*)$, where $B^* = \beta u'^{-1}(1) / (1 - \pi + \beta) > 0$. Along this equilibrium, $R_t < 1/\beta$ for all t.

In short, Proposition 1 indicates that our economy is liquidity-constrained if public debt is sufficiently low, in which case the equilibrium interest rate is also low (relative to that prevailing in an unconstrained economy) due to the precautionary demand for government bonds.[7] From now on we proceed under the maintained assumption that liquidity is scarce (in the sense that $B \in (0, B^*)$), and defer until the last Section the discussion of the significance of this assumption. Of course, the specific parameter restrictions imposed by the requirement that public debt be stationary depend on the particular fiscal rule followed by the government.

[7] These properties essentially parallel those obtained by Woodford (1990) with a liquidity-constrained economy where both aggregate and idiosyncratic uncertainties are shut down.

2 Liquidity and wealth effects of fiscal expansions

2.1 Aggregate and individual variables

In this section we start by showing how liquidity and wealth effects compete in determining the overall response of aggregate- and individual-level variables to public-spending shocks, and then illustrate the implied dynamic effects of such shocks under a simple fiscal rule.

Total consumption by employed households is $(1-\Omega)\,c^e$, while the total consumption of unemployed households is Ωc_t^u. Then, using (10), (13) and (14) and rearranging, total private consumption and total output can be respectively written as:

$$C_t = (1-\Omega)\,c^e + (1-\pi)\,(B_t - G_t + \Omega T_t)\,, \tag{18}$$

$$Y_t = (1-\Omega)\,c^e + (1-\pi)\,(B_t + \Omega T_t) + \pi G_t. \tag{19}$$

These static, reduced-form equations provide a first insight into how liquidity effects alter the transmission of fiscal shock relative to that at work in the Ricardian model. Imagine first the effect of a rise in public spending entirely financed by public debt, the implied increase in taxes necessary to satisfy the government's intertemporal budget constraint being left to some future periods. Private consumption does not change on impact since $\Delta T_t = 0$ and thus $\Delta B_t - \Delta G_t = 0$. However, the implied rapid growth of public debt and the delayed response of taxes may cause the quantity $B_t - G_t + \Omega T_t$ in (18) to be greater than zero over a sustained period of time starting at date $t+1$, thereby leading to a persistent crowding-in of private consumption by public consumption. Alternatively, consider the textbook Ricardian experiment of a debt-financed cut in lump-sum taxes, financed by future tax increases, with the entire path of government consumption remaining unchanged. Since $\Delta G_t = 0$ and thus $\Delta B_t = -\Delta T_t$ by assumption, we have $\Delta\,(B_t + \Omega T_t) > 0$ so that the cut raises private consumption and output on impact (recall that this experiment would be neutral under Ricardian equivalence). Finally, notice that changes in taxes and public spending that keep the primary deficit at zero (that is, $\Delta G_t = \Delta T_t$ and $\Delta B_t = 0$) affect private consumption and output in exactly the way predicted by the Ricardian model (i.e., $\Delta C_t < 0$, $\Delta Y_t > 0$): variations in the stock of public debt are thus crucial in generating the expansionary effects of fiscal shocks.

To obtain further insight into the underlying workings of these non-Ricardian effects, we need to go beyond the reduced-form equations (18)–(19) and look at household-level variables, which describe how individual consumption (i.e. the private demand side of the model) and labour supply (the supply side of the model) respond to fiscal shocks. The consumption of employed households, c^e, is not affected by fiscal shocks. Now, substitute (14) into (10) to write c_t^u as follows:

$$c_t^u = (2 - \pi) B_{t-1} R_{t-1} - T_t. \tag{20}$$

In equation (20), higher taxes lower consumption, but higher public debt raises the overall liquidation value of u-households' portfolios (i.e., the $(2 - \pi) B_{t-1} R_{t-1}$ term in (20)). Provided that the increase in public debt also persistently raises the interest rate after the spending shock (which, from (17), occurs whenever $B_t - G_t + \Omega T_t$ rises over time), the right-hand side of (20) may increase. Ultimately, whether c_t^u (and thus C_t) rises or falls thus depends on whether the liquidity effects of public debt on u-households' portfolios dominate the wealth effects of taxes following the shock.

Turning to the supply side of the model, we can substitute (7) and (14) into (11)–(12) and write labour supply by employed households as follows:

$$l_t^{ee} = c^e + (2 - \pi) (B_t - B_{t-1} R_{t-1}) + T_t, \tag{21}$$

$$l_t^{ue} = c^e + (2 - \pi) B_t + T_t. \tag{22}$$

Equations (21)–(22) show that labour supply responds to both taxes (as in the Ricardian model) and the stock of liquidity that households acquire as self-insurance against unemployment risk. ue-households, who have just moved out of unemployment and have zero beginning-of-period wealth, will seize any extra opportunity to save by raising labour supply; ee-households, who are partly self-insured when they enter the current period, adjust their labour supply depending on the new stock of government bonds available for purchase relative to the current value of their previously-accumulated portfolio. In both cases, the growth of public debt that may result from higher public spending generates liquidity effects that *strengthen* the wealth effects on labour supply.

2.2 The dynamic effects of spending shocks

Having discussed how liquidity effects may affect the response of our variables of interest to fiscal expansions, we now illustrate the dynamics of the model in the context of a specific example of a fiscal rule and a shock process. The tax rule and shock process that we consider are as follows:

$$T_t = T + \phi \left(B_t - B \right), \tag{23}$$

$$G_t = \psi G_{t-1} + \epsilon_t, \tag{24}$$

where T denotes steady-state taxes, B steady-state public debt (i.e., the long run target), $\phi > 0$ and $\psi \in (0, 1)$ are constant parameters, and ϵ_t is a innovation to public spending. Note that the qualitative properties of the model are robust to the inclusion of other feedbacks in (23) (e.g., from G_t to T_t), as well as to a lagged (rather than simultaneous) reaction of taxes to public debt; what matters for our results is the possibility that public spending shocks entail significant variations in the stock of public debt, at least in the short run.

The policy parameter ϕ in equation (23) effectively indexes the way in which fiscal expansions are financed at various horizons. If ϕ is large, taxes rise quickly following a fiscal expansion and public debt plays a relatively minor rôle in their short-run financing. Smaller values of ϕ, on the contrary, imply a muted short-run response of taxes and a more substantial rôle for public debt issuance in the short run; the ensuing rise in the stock of public debt then eventually triggers a rise in taxes in the medium run until the reversion of the public debt has been completed. In the context of (23), stationarity simply requires that ϕ be not too small –see equation (26) below. Finally, the assumption that steady-state government consumption is zero in equation (24) is made for expositional clarity and entails no loss of generality; here it implies that in the steady state tax revenues just cover interest rate payments on debt, i.e., $T = B\left(R - 1\right)$.[8]

Under (23)–(24), the dynamics of public debt can be approximated by the following process (see Appendix A for details):

$$B_t = (1 - a)B + aB_{t-1} + bG_t + cG_{t-1}, \tag{25}$$

where G_t is given by (24), $a > 0$, $b > 0$, $c < 0$ are constants that depend on the deep parameters of the model and the target debt level B, and where $\partial a / \partial \phi < 0$ (i.e., a stronger

[8] The non-Ricardian nature of the model implies that $R - 1$ may be negative if steady-state public debt, B, is sufficiently low. In this case, the steady state tax collection becomes a positive transfer of amount $-T$ (the bounds on R and the relation between R and B are detailed in Appendix A.)

tax reaction speeds up the reversion of public debt towards target). Finally, equations (13) and (18)–(19) give the equilibrium values of R_t, C_t and Y_t as functions of the endogenous state B_t and the forcing term G_t.

Since $a > 0$, stationarity of public debt requires that $a < 1$. As is shown in Appendix A, this condition is equivalent to:

$$\phi > \phi_{\min} \equiv \frac{R - 1 + (2 - \pi)\,\rho}{1 - (1 - \pi)\,\rho}, \text{ with } \rho \equiv \frac{(1 - \beta\pi R)\,\sigma\,(c^u)\,R}{1 + (1 - \pi)\,R} > 0, \tag{26}$$

and where $1 - (1 - \pi)\,\rho > 0$ and $R > 0$ is uniquely defined by the target debt level B.

To illustrate the dynamic impact of liquidity and wealth effects in our economy, we draw impulse-response functions for all relevant variables using equation (25) together with (13), (18)–(19) and (23)-(24). We use the (quarterly) parameters $\beta = 0.98$, $\pi = 0.94$ (this generates an unemployment rate of $\Omega \simeq 5.66\%$), $\psi = 0.95$, the (unique) value of B such that $R = 1.01$, and $u\,(c) = \ln c$.

Figure 1 displays the responses of our variables of interest to a public-spending shock, when $\phi = 0.2$ (the solid line) and $\phi = 1.2$ (the dotted line). It is useful to compare the debt dynamics implied by both tax rule parameters. Equation (25) gives:

$$\phi = 0.2 : B_t = 0.105B + 0.895B_{t-1} + 0.833G_t - 0.009G_{t-1};$$

$$\phi = 1.2 : B_t = 0.525B + 0.475B_{t-1} + 0.454G_t - 0.016G_{t-1}.$$

The case where $\phi = 0.2$ illustrates a situation where liquidity effects dominate wealth effects on total private consumption (except at the very moment of the shock), due to the substantial increase in public debt and the implied improvement in households' self-insurance opportunities (note that private consumption tracks public debt, and is thus far more persistent than the shock itself.). As a result, the output effect of a spending shock is large, in the sense that the spending multiplier is greater than one almost all along the adjustment path.

In contrast, wealth effects dominate when $\phi = 1.2$, due to the limited increase in public debt and the rapid reaction of taxes, resulting in a 'Ricardian' (i.e., negative) response of private consumption all along the transition path; in consequence, the government spending multiplier is always lower than one.

Holding other parameters constant, a sensitivity analysis indicates that values of ϕ between 0.2 and 1.2 cause private consumption to start falling below its steady-state level for several periods (during which public debt and implied liquidity effects are still limited), and then rise above its steady-state level for the rest of the adjustment period (after public debt has risen enough to make the liquidity effects prevalent). Since estimates of the response of taxes or the primary deficit to public debt in U.S. post-war data indicate a slow reversion of public debt and a value of ϕ closer to 0.2 than to 1.2 (see Bohn, 1998, and Gali *et al.*, 2007), plausible values of ϕ favour the dominance of liquidity effects here, at least over part of the adjustment path.

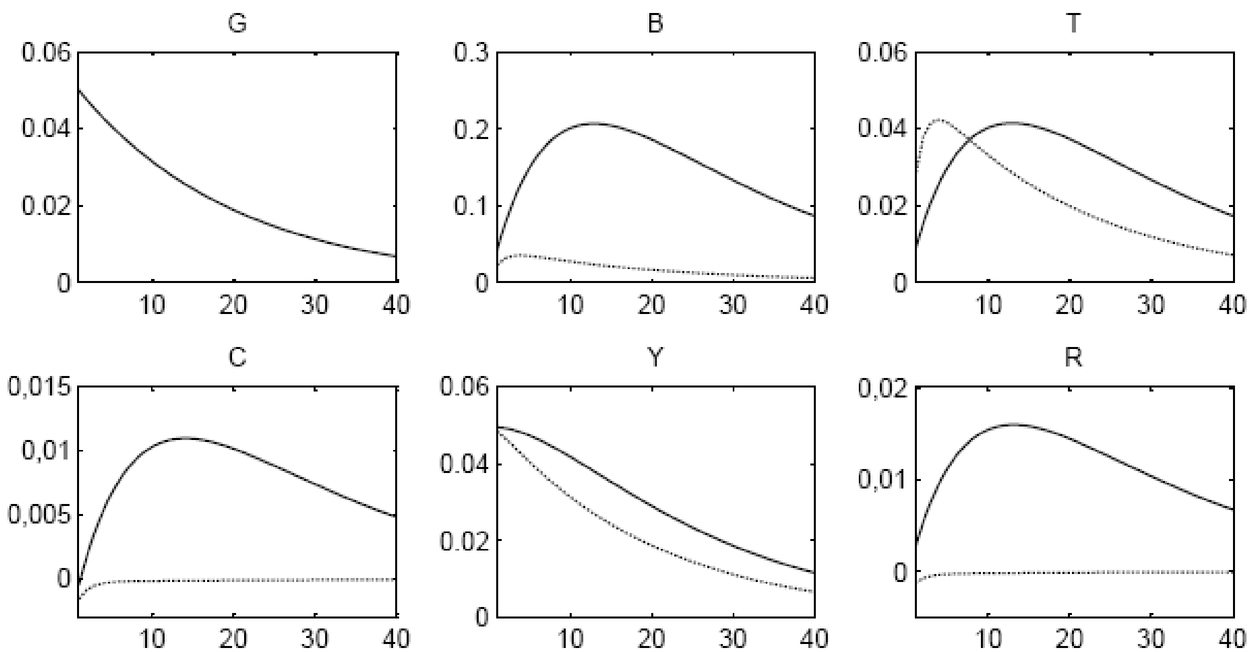

FIGURE 1. THE BASIC MODEL. The panels display the linear deviations from the steady state of public debt (B), taxes (T), private consumption (C), output (Y) and the interest rate (R) following a public spending shock (G) of 5% of steady-state output. The fiscal rule is $T_t = \phi(B_t - B)$, with $\phi = 0.2$ (solid line) and $\phi = 1.2$ (dotted line).

3 Liquidity effects on labour demand

Our analysis has thus far focused on the way in which liquidity effects may affect the labour supply and consumption demand of private agents, leaving aside their potential effects on labour demand and the equilibrium real wage. A natural way of introducing labour demand shifts into the basic model is to think of output as being produced by entrepreneurs, that is households having access to a production technology, rather than by a separate firm sector. Higher liquidity may then also relax the borrowing constraints faced by these entrepreneurs and raise their labour demand following a fiscal expansion. Here again, whether the real wage consequently rises or falls depends on whether the liquidity effects on labour demand dominate the wealth effects on labour supply following the shock.

Our entrepreneurial model is exactly the same as that in Section 2 except for one feature: employed households now have a constant probability $1-\pi$ of becoming entrepreneurs for exactly one period. Entrepreneurs have access to a production technology that yields y_{t+1}^i units of goods at date $t+1$ for $l_t^{f,i}$ units of labour hired at date t (entrepreneurs do not supply labour). The household's objective is (1) as before, and we further assume here that $u(c)=\tau\ln c$, $\tau>0$. The budget constraint of household i is now:

$$c_t^i + a_t^i + \left(1-\xi_t^i\right) w_t l_t^{f,i} = a_{t-1}^i R_{t-1} + \xi_t^i w_t l_t^i + y_t^i - T_t, \tag{27}$$

where $\xi_t^i = 1$ if the household is employed at date t and $\xi_t^i = 0$ if the household runs a firm.

Consider first the choice of an employed household. From (1) and (27), the latter chooses the level of labour supply l_t^i that satisfies:

$$u'\left(a_{t-1}^i R_{t-1} + w_t l_t^i + y_t^i - T_t - a_t^i\right) = 1. \tag{28}$$

On the other hand, employed households stay employed in the next period with probability π and become entrepreneurs with complementary probability. The optimal asset demand of an employed household i, a_t^i, must thus satisfy:

$$\begin{aligned} u'\left(a_{t-1}^i R_{t-1} + w_t l_t^i + y_t^i - T_t - a_t^i\right) \geq \pi\beta R_t E_t u'\left(a_t^i R_t + w_{t+1} l_{t+1}^i - T_{t+1} - a_{t+1}^i\right) \\ + \left(1-\pi\right)\beta R_t E_t u'(a_t^i R_t - T_{t+1} - a_{t+1}^i - w_{t+1} l_{t+1}^{f,i}), \end{aligned} \tag{29}$$

with this expression holding with strict inequality if the borrowing constraint is binding (i.e., $a_t^i = 0$), but holding with equality otherwise (i.e., $a_t^i > 0$).

Let us now turn to entrepreneurs. Their running of a firm prevents them from earning any wage income, so for them $w_t l_t^i = 0$. Since they were employed in the previous period they earn no production output, i.e., $y_t^i = 0$. Then, from (1) and (27), their optimal choices of labour demand, $l_t^{f,i}$, and asset demand, a_t^i, must satisfy, respectively,

$$u'(a_{t-1}^i R_{t-1} - T_t - a_t^i - w_t l_t^{f,i}) = \beta w_t^{-1} E_t u'(a_t^i R_t + w_{t+1} l_{t+1}^i + l_t^{f,i} - T_{t+1} - a_{t+1}^i), \qquad (30)$$

$$u'(a_{t-1}^i R_{t-1} - T_t - a_t^i - w_t l_t^{f,i}) \geq \beta R_t E_t u'(a_t^i R_t + w_{t+1} l_{t+1}^i + l_t^{f,i} - T_{t+1} - a_{t+1}^i), \qquad (31)$$

with the latter inequality holding strictly if the entrepreneur is borrowing-constrained and with equality otherwise.

Just as in Section 1, an equilibrium with a limited number of household type/asset states can be constructed by conjecturing that employed households are never constrained (i.e., they wish to save rather than borrow, so (29) holds with equality) while entrepreneurs always are (i.e., they would like to expand employment and production through borrowing but cannot do so, implying that (31) holds with strict inequality). For the sake of conciseness, we just describe the properties of this equilibrium here and then establish the sufficient conditions for its existence in Proposition 2 below.

The three types of households of the entrepreneurial model are: i) entrepreneurs (or '$f-$households'), who are currently borrowing-constrained and were employed in the previous period; ii) $ee-$households, who are currently employed after having been employed in the previous period; and iii) $fe-$households, who are currently employed after having been entrepreneurs in the previous period. Just as before, employed households are not borrowing-constrained and all choose the same consumption and asset holding levels, denoted by c_t^e and a_t (c_t^e will be time-varying here, due to changes in the real wage). We denote by c_t^f and l_t^f entrepreneurs' consumption and labour demands, respectively. The budget constraint of each type of household is now:

$$ee : c_t^e + a_t = a_{t-1} R_{t-1} + w_t l_t^{ee} - T_t, \qquad (32)$$

$$fe : c_t^e + a_t = w_t l_t^{fe} + l_{t-1}^f - T_t, \qquad (33)$$

$$f : c_t^f + w_t l_t^f = a_{t-1} R_{t-1} - T_t. \qquad (34)$$

Equation (32) is the same as (12), except for the fact that the consumption of employed households, c_t^e, is time-varying. In equation (33), fe-households earn labour income $w_t l_t^{fe}$

plus production output $y_t = l^f_{t-1}$, and this total income is used to pay for consumption, asset accumulation and taxes. Equation (34), the budget constraint of entrepreneurs, states that they entirely liquidate their stock of assets to finance consumption, taxes, and the wage bill $w_t l^f_t$ (i.e., they hold no bonds at the end of the period since they face binding borrowing constraints). From (28)–(29) and (32)–(34), the intratemporal and intertemporal optimality conditions of employed households are now, respectively:

$$w_t u'\left(c^e_t\right) = 1, \tag{35}$$

$$u'\left(c^e_t\right) = \beta R_t E_t(\pi u'\left(c^e_{t+1}\right) + (1-\pi)\, u'(c^f_{t+1})). \tag{36}$$

From (34), entrepreneurs must allocate their after-tax income, $a_{t-1}R_{t-1} - T_t$, to current consumption, c^f_t, and the wage bill, $w_t l^f_t$, taking the real wage as given. From (30) and (34), together with the fact that entrepreneurs stay so for one period only, the solution to the entrepreneur's optimal labour demand, l^f_t, must satisfy:

$$w_t u'(c^f_t) = \beta E_t u'\left(c^e_{t+1}\right). \tag{37}$$

The optimality condition (37) simply sets equal the utility fall implied by a decrease in current consumption necessary to hire an extra unit of labour to the utility gain that is expected from increasing current labour input (and thus future production) by that unit.

The market-clearing equations of the entrepreneurial model are as follows. Clearing of the market for bonds is given by equation (14) as before. Given that entrepreneurs are in proportion Ω, clearing of the labour and goods markets now requires:

$$(1-2\Omega)\, l^{ee}_t + \Omega l^{fe}_t = \Omega l^f_t, \tag{38}$$

$$(1-\Omega)\, c^e_t + \Omega c^f_t + G_t = \Omega y_t. \tag{39}$$

Finally, the government's behaviour is described by the budget constraint (13), together with our fiscal rule and shock process (23)–(24), where again ϕ must be large enough for public debt to be stationary.

A three-household type equilibrium of the entrepreneurial model is defined as sequences of individual consumption levels, $\{c^e_t, c^f_t\}^\infty_{t=0}$, individual labour supplies, $\{l^{ee}_t, l^{fe}_t\}^\infty_{t=0}$, individual bond and labour demands, $\{a_t, l^f_t\}^\infty_{t=0}$, and aggregate variables, $\{\Omega l^f_t, \Omega y_t, B_t\}^\infty_{t=0}$ such that i) only entrepreneurs face binding borrowing constraints and ii) the optimality conditions

(35)–(37) and the market clearing conditions (14) and (38)–(39) hold in every period, given the forcing sequence $\{G_t\}_{t=0}^{\infty}$ and a fiscal rule for $\{T_t\}_{t=0}^{\infty}$ ensuring the stationarity of public debt. Proposition 2, whose proof is found in Appendix B, states the conditions under which such an equilibrium exist.

Proposition 2. Provided that public debt is stationary and that fluctuations around the steady state are small, then the three-household type equilibrium of the entrepreneurial model exists if and only if $B \in (0, B^{**})$, where $B^{**} = \tau\beta^2(\beta\,(\beta + 1 - \pi)^{-1} + (1 - \pi)^{-1})$. Along this equilibrium, $R_t < 1/\beta$ for all t.

The dynamic system characterising the entrepreneurial model involves more lags and more interactions between variables than the basic model, making it difficult to compare directly the outcomes of the two specifications (the equations forming this dynamic system are described in Appendix B). For the sake of comparability, we run policy experiments with exactly the same parameter values as in the previous Section, except for π which is now set to 0.80 (this implies a share of entrepreneurs of $\Omega \simeq 17\%$).[9] Although the dynamics of the entrepreneurial model cannot be reduced to a single stochastic difference equation, it yields an expectational dynamic system that can be solved numerically for the vector of relevant variables and for the stationarity condition (the procedure is summarised in Appendix B).

Figure 2 displays the responses of fiscal and aggregate variables to a public-spending shock generated by the entrepreneurial model (note that c_t^e and l_t^f, although not represented, are tracked by w_t and Y_{t+1}, respectively). Since liquidity effects on labour demand take one period to be operative (as some employed households having increased their savings turn into entrepreneurs), wealth effects on labour supply dominate on impact whether $\phi = 0.2$ or 1.2. The ensuing increase in labour supply leads to a sharp fall in the real wage and the consumption of employed households, causing total private consumption to fall. However, when $\phi = 0.2$ liquidity effects on labour demand become dominant (in the sense of leading to higher-than-steady-state wages) for the entire adjustment path starting from one period after the shock, leading to a persistent boom in private consumption.

[9] Our empirical counterpart to the share of entrepreneurs is the number of U.S. firms, from The Census Bureau's 2002 Survey of Business Owners (23 million firms) divided by total employment by the end of the same year from the BLS Current Population Survey (136.5 millon people).

The responses of consumption and output are qualitatively similar to those in the basic model when $\phi = 1.2$ (except for the initial wiggle due to the production delay), but labour-market adjustments matter here: the strong reaction of taxes and limited growth of public both act to weaken the liquidity effects on labour demand whilst strengthening the wealth effects on labour supply. This naturally leads to a limited increase in labour demand relative to the contemporaneous increase in labour supply, and thus to a fall in the real wage and a crowding-out of private consumption by public spending. Just as in the basic model, intermediate values of ϕ (not represented here) generate a more mixed picture with dominance of either effect at different points on the transition path.

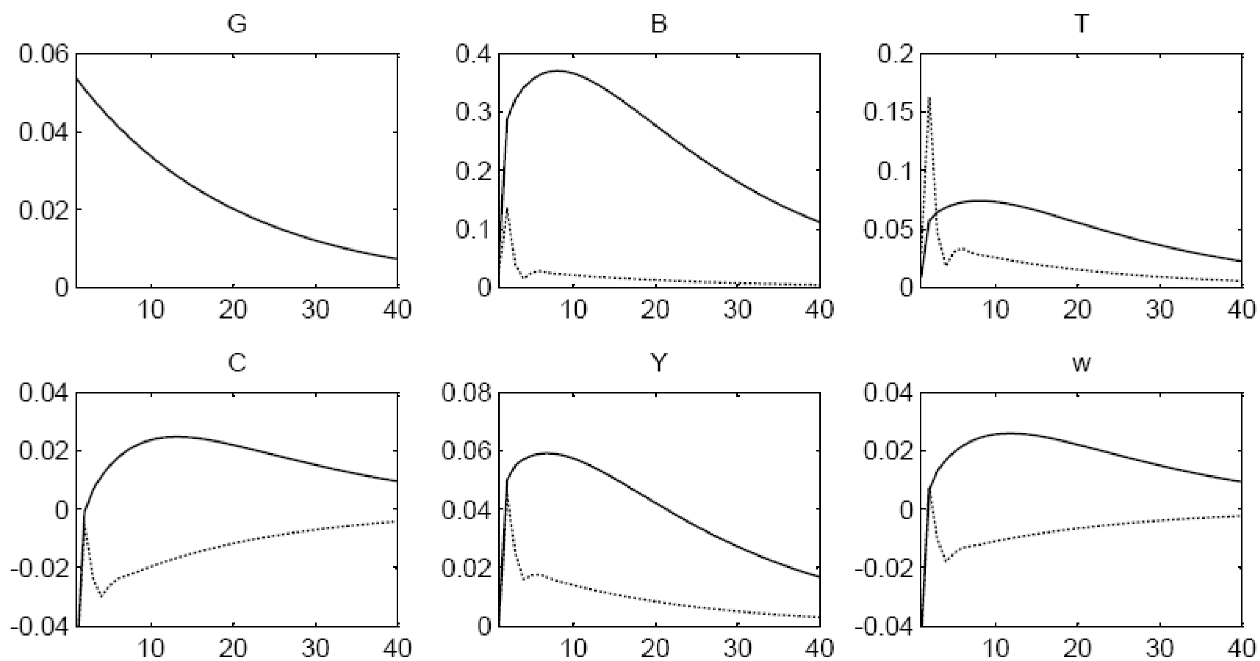

Figure 2. The entrepreneurial model. The panels display the linear deviations from the steady state of public debt (B), taxes (T), private consumption (C), output (Y) and the real wage (w) following a public spending shock (G) of 5% of steady-state output. The fiscal rule is $T_t = \phi(B_t - B)$, with $\phi = 0.2$ (solid line) and $\phi = 1.2$ (dotted line).

4 Concluding remarks

This paper has presented the predictions of a liquidity-constrained economy regarding the effects of debt-financed increases in public spending, with particular attention being paid to the effects of such shocks on private consumption and the real wage. Our main goal is to illustrate that the liquidity effects induced by temporary changes in the stock of public debt can drastically alter the predictions of the baseline Ricardian model, where changes in public spending affect aggregates only through intertemporal wealth effects. The view that the deficit financing of public spending can generate large multiplier effects, thanks to consumption crowding-in and possibly pro-cyclical wages, is often associated with the Keynesian tradition in macroeconomics; our analysis shows that such effects are consistent with a set of assumptions (i.e., incomplete markets and borrowing constraints) that differ from typical Keynesian ones (e.g., sticky prices, imperfect competition).

We explore the implications of scarce liquidity for the transmission of fiscal shocks using a stylised model that admits a closed-form solution with limited agent heterogeneity and a limited number of assets. It is thus natural to wonder whether our results would still hold in a model with more types of agents and more types of liquid assets.

Our closed-form equilibrium results from the joint property that employed households reach their target level of precautionary wealth instantaneously (an outcome of linear labour disutility), while unemployed households –or entrepreneurs– face a binding borrowing constraint and liquidate their asset portfolio from the very moment that their income falls. In a more general model with lower labour supply elasticity and slower asset liquidation, households would deplete or replete assets only gradually rather than in one go (e.g., Aiyagari and McGrattan, 1998; Heathcote, 2005), and the reactions of labour supply and consumption demand to changes in aggregate liquidity would be smoother. However, the same liquidity effects of public debt would be at work (provided that agents are effectively liquidity-constrained), resulting in non-Ricardian effects of spending shocks on private consumption, employment and wages.

Finally, models of liquidity-constrained economies all rely on the assumption that liquidity is too scarce to allow for full self-insurance, so that the decentralised outcome is socially inefficient (i.e., relative to that which would be chosen by an almighty social planner). While our model is no exception, this assumption may raise two questions here. First, are there no other means of self-insurance, like privately-issued debt instruments ('inside liquidity'), or claims to the capital stock, that may eliminate the need for government-issued liquidity? Second, even if the private supply of liquidity is too low, why should public debt itself be too low while it may be Pareto-improving to increase its stock until full self-insurance is permitted?

In our view, previous analyses provide at least partial answers to both questions. First, the supply of inside liquidity is limited by the very presence of financial constraints, and is thus likely to be in insufficient quantity in equilibrium (e.g., Holmström and Tirole, 1998; Farhi and Tirole, 2008); on the other hand, nothing guarantees that capital can provide enough liquidity in the steady state to mimic first-best outcomes, and capital investment can itself be limited by previously accumulated liquidity (Woodford, 1990).[10] Second, if public debt includes a distortionary component (as in Floden, 2001) or crowds out capital (as in Aiyagari and McGrattan, 1998), then increasing the public debt above a certain threshold may turn out to decrease, rather than increase, aggregate welfare; in this situation, the long-run level of public debt may endogenously be set by a benevolent government at a level where liquidity constraints still matter.

[10] In our model, inside liquidity can coexist with outside liquidity if the upper debt limit that households face is different from zero; our results continue to hold in this case, provided that the debt limit is sufficiently tight for the relevant households (i.e., unemployed households or entrepreneurs) to be borrowing-constrained in equilibrium. Similarly, it can be checked numerically that our results are robust to the inclusion of endogenous capital accumulation either by liquidity-constrained entrepreneurs, or by unconstrained, outside firms (provided, in the latter case, that steady-state capital is sufficiently small.)

Appendix A: The basic model

Proof of Proposition 1

If fluctuations around the steady state are sufficiently small, then **C1** and **C2** hold in every period provided that they hold in the steady state. (14) implies that $a_t > 0$ if and only if $B_t > 0$, so **C2** holds in the steady state if and only if $B > 0$. On the other hand, **C1** holds in the steady state if and only if $u'(c^u) > \beta R u'(c^e)$. From (7) and (10) we have $u'(c^e) = 1$ and $c^u = aR - T$, so the latter inequality may be written as $u'(aR - T) > \beta R$. Now, rewriting the steady-state counterpart of (8) as

$$u'(c^u) = u' - (aR - T) = (1 - \beta\pi R) / (\beta R (1 - \pi)), \tag{A1}$$

we find that the condition $u'(c^u) > \beta R u'(c^e)$ is equivalent to $R < 1/\beta$.

We now show that $R < 1/\beta$ if and only if $B < B^*$. This can be shown by first establishing that B is a continuous, strictly increasing function of R over the appropriate interval, and then by evaluating the function $B(R)$ at the point $R = 1/\beta$ to find B^*. R is given by (17). By assumption $G = 0$, implying that $T = B(R - 1)$. Thus, after some manipulations the steady-state counterpart of (17) can be written as:

$$B = \frac{1}{1 + (1 - \pi) R} u'^{-1} \left(\frac{(\beta R)^{-1} - \pi}{1 - \pi} \right) \equiv B(R), \tag{A2}$$

where $B(R)$ is positive and continuous over $(0, 1/\beta\pi)$. First, compute:

$$B'(R) = \frac{-(1 - \pi)}{(1 + (1 - \pi) R)^2} \times u'^{-1} \left(\frac{(\beta R)^{-1} - \pi}{1 - \pi} \right) + \frac{1}{1 + (1 - \pi) R} \times \frac{\partial}{\partial R} u'^{-1} \left(\frac{(\beta R)^{-1} - \pi}{1 - \pi} \right)$$

(A1) implies that $u'(c^u(R)) = ((\beta R)^{-1} - \pi) / (1 - \pi)$, so the $\partial u'^{-1}(.) / \partial R$ term above is:

$$\frac{\partial}{\partial R} u'^{-1} \left(\frac{(\beta R)^{-1} - \pi}{1 - \pi} \right) = \frac{1}{u''(c^u)} \times \frac{\partial u'(c^u)}{\partial R} = \frac{1}{u''(c^u)} \times \frac{-1}{(1 - \pi) \beta R^2}.$$

After rearranging, this allows us to rewrite $B'(R)$ as follows:

$$\begin{aligned} B'(R) &= \frac{-(1 - \pi) c^u}{(1 + (1 - \pi) R)^2} + \frac{-R^{-2}}{\beta (1 - \pi)(1 + (1 - \pi) R)} \frac{1}{u''(c^u)} \\ &= \frac{(1 - \pi) u'(c^u)}{(1 + (1 - \pi) R)^2 u''(c^u)} \left(\sigma(c^u) - \frac{1/R + 1 - \pi}{(1 - \pi)(1 - \pi\beta R)} \right) \end{aligned}$$

The term inside brackets must be negative for $B'(R)$ to be positive. Since $\sigma(c) \leq 1$ by assumption, a sufficient condition for this is that $(1/R + 1 - \pi)/(1-\pi)(1-\pi\beta R) > 1$, which is always true. Thus, $B(R)$ is continuous and strictly increasing in over $(0, 1/\beta\pi)$, while $\lim_{R\to 0} B = u'^{-1}(\infty)$ ($= 0$ by assumption) and $\lim_{R\to 1/\beta\pi} B = \beta\pi u'^{-1}(0)/(\beta\pi + 1 - \pi)$ ($\leq \infty$). Then, setting $R = 1/\beta$ in (A2) gives B^*. Note also from (A2) that $R < 1$ if $B < (2-\pi)^{-1} u'^{-1}(\beta^{-1} - \pi/(1-\pi))$.

Dynamics and stability

We use hatted variables to denote level-deviations from steady state (i.e., $\hat{X}_t = X_t - X$). First, substitute (23)–(24) into the linearised versions of (13) and (17) to obtain:

$$\hat{B}_t = \left(\frac{B}{1+\phi}\right)\hat{R}_{t-1} + \left(\frac{R}{1+\phi}\right)\hat{B}_{t-1} + \left(\frac{1}{1+\phi}\right)G_t \tag{A3}$$

$$-R^2\beta(1-\pi)(2-\pi)u''(c^u)\left((1+\phi\Omega)E_t(\hat{B}_{t+1}) - \psi G_t\right) = \hat{R}_t \tag{A4}$$

Let us rewrite (A4) as follows, making use of (A1)–(A2) and the definition of ρ in (26):

$$\left(\frac{(2-\pi)\rho}{B}\right)\left((1+\phi\Omega)E_t(\hat{B}_{t+1}) - \psi G_t\right) = \hat{R}_t \tag{A5}$$

Leading (A3) one period and taking expectations, solving (A5) for $E_t(\hat{B}_{t+1})$, and then equating the two expressions, we obtain:

$$R\hat{B}_t + \psi\left(1 - \frac{1+\phi}{1+\phi\Omega}\right)G_t = B\left(\frac{1+\phi}{(2-\pi)\rho(1+\phi\Omega)} - 1\right)\hat{R}_t$$

Now, lagging the latter equation one period, solving it for $\hat{R}_{t-1}$ and substituting the resulting expression into (A3), one finds equation (25) with coefficients:

$$a = \frac{R}{1+\phi-(2-\pi)\rho(1+\phi\Omega)},\ b = \frac{1}{1+\phi},\ c = -\frac{a\rho\psi\phi}{R(1+\phi)},$$

where R is uniquely defined by B from (A2) and the fact that $B'(R) > 0$ (see the proof of Proposition 1.)

The sign of a is related to the stationarity requirement that $|a| < 1$. If $a > 0$, then a necessary and sufficient condition for stationarity is (26) in the body of the paper, given that $\Omega = (1-\pi)/(2-\pi)$, $\sigma \in (0,1]$ and $1-(1-\pi)\rho > 0$. If, on the contrary, $a < 0$, then the necessary and sufficient condition for stationarity is $\phi < (-1 - R + (2-\pi))/\rho 1 - (1-\pi)\rho$, but the right hand side of this inequality is negative. Since this is inconsistent with $\phi > 0$, it must be the case that (26) hold, which in turns implies that $a > 0$. By implication $c < 0$, and obviously $b > 0$ since $\phi > 0$. Finally, with $a > 0$ we have that $\partial a / \partial \varphi < 0$.

Appendix B: The entrepreneurial model

Proof of Proposition 2

We must first derive the dynamic system characterising the entrepreneurial equilibrium under the joint conjecture that entrepreneurs are always borrowing-constrained while employed households never are, and then derive from the steady-state relations the range of debt levels compatible with this joint conjecture. With $u(c) = \tau \ln c$, equations (35) and (37) give:

$$c_t^e = \tau w_t, \; c_t^f = \tau w_t / \left(\beta E_t \left(w_{t+1}^{-1}\right)\right) \tag{B1}$$

Substituting (B1) into (39), the goods-market equilibrium can be written as:

$$\tau w_t + (1-\pi) \tau w_t / \left(\beta E_t \left(w_{t+1}^{-1}\right)\right) + (2-\pi) G_t = (1-\pi) l_{t-1}^f. \tag{B2}$$

Substituting (13), (14) and (B1) into the budget constraint of f-households, (34), gives:

$$\tau w_t / \left(\beta E_t \left(w_{t+1}^{-1}\right)\right) + w_t l_t^f = (2-\pi)\left(B_t - G_t + \Omega T_t\right). \tag{B3}$$

Finally, substituting (B1) into (36), the Euler equations for employed households is:

$$w_t^{-1} = \beta R_t \left(\pi E_t \left(w_{t+1}^{-1}\right) + (1-\pi) E_t \left(\beta w_{t+1}^{-1} E_{t+1}\left(w_{t+2}^{-1}\right)\right)\right) \tag{B4}$$

Since shocks are small by assumption, the dynamic system just derived is an equilibrium if, in the steady state, i) all employed households hold positive assets at the end of the current period (which, from (14), is ensured by $B > 0$), and ii) entrepreneurs are always borrowing-constrained, i.e., $u'(c^f) > \beta R u'(c^e)$. From (B1), this latter condition is equivalent to $wR < 1$. Now, the steady state counterpart of (B4) gives:

$$w = \beta^2 (1-\pi) R / (1-\beta\pi R) \tag{B5}$$

Substituting (B5) into the inequality $wR < 1$, we find that entrepreneurs are borrowing-constrained if, and only if, $R < 1/\beta$. We may now compute B^{**}, the unique upper debt level ensuring that $R \in (0, 1/\beta)$ whenever $B \in (0, B^{**})$. First, use the facts that $G = 0$ and $T = B(R-1)$ to write the steady-state counterparts of (B2) and (B3) as follows:

$$\tau w / (1-\pi) + \tau w^2/\beta = l^f$$
$$l^f = B(1 + (1-\pi) R)/w - \tau w/\beta$$

Equating the two and using (B5), we can write steady-state public debt as:

$$B(R) = \frac{\tau R}{1/R + 1 - \pi} \left(\frac{\beta^2 (1-\pi)}{1 - \beta \pi R} \right)^2 \left(\frac{1}{1-\pi} + \frac{1}{\beta} + \frac{\beta (1-\pi) R}{1 - \beta \pi R} \right) \tag{B6}$$

$B(R)$ is continuous and increasing in R over $[0, 1/\pi\beta)$, while $B(0) = 0$ and $\lim_{R \to 1/\pi\beta} B = \infty$. This uniquely defines $B^{**} = B(1/\beta)$ in Proposition 2.

Dynamics and stability

The dynamic system characterising the behaviour of the entrepreneurial model is derived as follows. First, substitute the linear counterparts of (23) and (B1) and into the linearised versions of (13) and (B2)–(B4). The latter equations then form a four-dimensional expectational dynamic system with forcing term G_t and vector of unknowns:

$$X_t = \left[\begin{array}{cccc} B_t & l_t^f & R_t & w_t \end{array} \right]'$$

This system can be solved numerically for its auto-regressive representation using standard methods once values have been assigned to all deep parameters of the model and to the target debt level B (here again the latter is chosen so as to generates a steady state value of R of 1.01, but equation (B6) rather than (A2), is used). Finally, total private consumption is $C_t = (1-\Omega) c_t^e + \Omega c_t^f$, with c_t^e and c_t^f given by (B1), and aggregate output is $Y_t = \Omega l_{t-1}^f$. For the chosen parameter configuration, the stationarity requirement becomes:

$$\phi > \phi_{\min} \simeq 0.134,$$

and is thus satisfied for both $\phi = 0.2$ and $\phi = 1.2$.

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