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ABSTRACT

We study price personalization in a two period duopoly with horizontally differentiated products. In the second period, a firm has collected detailed information on its old customers, using it to engage in price personalization. Customers, when returning to buy, may choose to incur a cost in order to access the standard offer of their previous provider in addition to its personalized offer and the standard offer of its rival. The analysis confirms that firms' second period profits are boosted when consumers are active in this sense (being equal to perfect price discrimination ones when initial market hares do not differ too much) but it reveals that this advantage is dissipated and possibly over-dissipated by the resulting fierce first-period competition *for the market*. Two-period aggregate profits are smaller with active customers provided the consumers are naive and/or the firms patient enough. Consumers' access to both personalized and standard firms' offers which benefit the oligopolists in mature markets may plausibly hurt them in emergent ones. The equilibrium is shown not to depend on the level of the cost as long as it is below some critical value.

KEYWORDS : Behavior-Base price discrimination; active customers; identity management

JEL CLASSIFICATION : D43, L13

I. Introduction

The increasing ability of firms to collect huge amounts of consumers information is now very well-documented as well as the way they can analyse and use this information to produce very accurate customer profiling.[1] There are more or less sophisticated ways of collecting data. Supermarkets and other large sellers for instance simply use loyalty cards to obtain very precise informations on their customers. Many firms now employ chief data officers to manage this increasingly important part of their activities.[2] A large evidence exists[3] that the information which is collected and treated in this way enables the collecting firms to use coupons, promotions, personalized discounts vouchers and post-sale services in order to engage in price discrimination based on consumers' willingness-to-pay. Popular examples include the travel agency Orbitz proposing different offers to customers according as they were Mac or PC users, the Princeton Review's 'strategy of levying different prices based on zip code resulted in Asians being twice as likely to be charged a higher price' (Mohammed 2017) or white men getting better deals on cars.[4]

There have been for at least two decades a plethora of papers (see for instance Thisse and Vives 1988; Fudenberg and Tirole 2000; Choe, King, and Matsushima 2018) which have shown that the acquisition of information on consumers, enabling subsequent price discrimination, may be detrimental to competing firms by exacerbating price competition. This paradoxical effect of being better informed has been called 'the curse of knowledge' (Laussel, Long, and Resende 2020). In these papers, consumers are 'passive', since they can't avoid to be recognized by their previous suppliers, and have no other choice that either to accept the personalized offers[5] they receive or to stay out of the market.

Consumers are however increasingly conscious that firms currently collect and use their personal

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[1]See for instance Max Freedman "How Businesses Are Collecting Data (And What They're Doing With It)", June 17, 2020, https://www.businessnewsdaily.com/10625-businesses-collecting-data.html.

[2]See for instance https://www.forrester.com/report/Chief+Data+ Officers+Evolve+Your+Teams+To+Accelerate+Impact+From+Data+Insights/-/E-RES163256#.

[3]See Ezrachi and Stucke (2016). Mohammed (2017), Wallheimer (2018)., Mikians et al. (2012), (Mikians et al. 2013). See also Shaw and Vulkan (2012) for an empirical investigation.

[4]https://www.nytimes.com/1990/12/13/us/white-men-get-better-deals-on-cars-study-finds.html?pagewanted=all.

[5]Prices may be more or less finely targeted (See Liu and Serfes 2004) with in the limit fully personalized prices.

informations to target them offers which may often be less advantageous for instance than those officially reserved to new customers (the prices for first-year subscriptions are very often much lower than for regular ones). Accordingly they may find profitable to become 'active', i.e. to use different 'identity management'[6] devices to access offers which the firms would like to reserve to others. Current antivirus software may for instance be used to block and/or erase cookies. Chen, Choe, and Matsushima (2020) speak of ex ante identity management when firms are prevented from collecting information on customers and of ex post identity management when it allows old customers to access the firms' offers targeted at new customers. In the first case, the consumers do not receive personalized offers from their previous provider, in the second one they do receive them but can also, at a cost, access the firm's standard offer. In the second case, speaking of ex post identity management may be unduly restrictive. Of course, consumers may use different credit cards or different accounts with different names in order to avoid to be recognized when they return buying, so as to benefit for instance of promotional offers officially targeted to new customers, or they can buy anonymously at brick and mortar shops. But they can also complain to the firm in order to get access to its uniform price. Of course, in both cases, consumers willing to stay or to become anonymous have to spend some time and/or some money in order to do so. In any case, it is convenient to speak of active customers to refer to those who incur some cost to access the firms' standard prices, the other ones being referred to as passive customers.[7]

This paper tries to investigate whether the existence of active customers leads to a reversal of the 'curse of knowledge' and to what extent that depends on the level of costs. Chen, Choe, and Matsushima (2020) have shown that such a reversal occurs in a one-period duopoly model with costless ex post identity management. In this framework, they indeed show in the full market-coverage case[8] that when the firms' market shares do not differ too much, there exists a perfect price discrimination equilibrium. The intuition why is that the firms then refrain from setting uniform prices which would allow them to attract some of their rival's targeted customers[9] lest their own active targeted customers would choose to buy at these prices rather than at the personalized prices designed for them. The present model shares with Chen et al. paper what they call the *ex post* identity management assumption but differs from it in two important respects. The first one is that accessing to uniform prices is here *costly* The second is that we consider a *two-period* behaviour-based price discrimination model in which the firms first ignore the consumers' preferences but happen to learn them in the first period so that they can, in the second period, propose them personalized prices.

We focus mainly on the case when the privacy cost is not too high relative to the gross utility of the good, namely when the sum of the privacy cost and the (linear) transportation cost is smaller than the gross utility from consuming one unit of the good. In that case, we show that the second-period equilibrium analysis is the same as when the privacy cost is zero, i.e. the situation considered in Chen, Choe, and Matsushima (2020).[10] There are two types of second-period equilibria. When the two firms have similar initial market shares, the equilibrium is one of bilateral perfect price discrimination where the consumers are left with no surplus, with consequence that the first-period demand functions are the static ones. When one firm has a relatively small initial market share, it finds more profitable to poach some of its (big) rival's old customers by offering them an attractive uniform price, though its own initial customers prefer themselves to buy at this price that at the personalized prices which they are offered. We show that the model has a symmetrical unique pure strategy first-period Nash equilibrium when the ratio between the gross utility and the linear transportation cost is below a critical value. The equilibrium values of first-period prices and two-period profits

[6]See Acquisti, 2008.
[7]Chen, Choe, and Matsushima (2020) call this cost a privacy cost. We can more generally speak of an "access cost".
[8]This is the case when every consumer is recognized and receives a personalized offer by one or the other firm.
[9]In their framework, the "targeted" consumers of a firm are the consumers whose it knows the exact preferences.
[10]There are some, inessential, modelling differences however, since we use a standard Hotelling model with quadratic transportation costs whereas they use the "brand loyalty" model of Fudenberg and Tirole (2000).

are shown to be independent of the value of the privacy cost. At this equilibrium, where firms have identical initial market shares, second-period profits are greater than at the static equilibrium and at the equilibrium with only passive customers, confirming the result in Chen, Choe, and Matsushima (2020). However, the two-period equilibrium discounted profits are smaller than the profits from the repetition of the static equilibrium, showing that second-period extra profits are more than dissipated by a fiercer first-period price competition *for the market*. Even more interestingly, they are smaller than in the case when all customers are passive (Choe, King, and Matsushima 2018) when the consumers are sufficiently naive and/or the firms patient enough.

Section II reviews related literature. Section III sets out the model. Section IV characterizessecond-period equilibrium. Section V studies first-period price competition. Section V is devoted to payoff comparisons. Section VII concludes.

II. Related literature

The present paper is related to several overlapping strands of literature.

The first one has studied competitive personalized pricing in a duopoly setting. Many papers (see for instance Thisse and Vives 1988; Chen and Iyer 2002, Choe, King, and Matsushima 2018) have argued that there is indeed in this context a 'curse of knowledge' since having an information on passive consumers[11] allows the competing firms to propose them personalized prices, leading however to lower firms equilibrium profits. This occurs via a very intuitive competition effect: instead of competing in uniform prices over a set of heterogeneous customers, firms compete separately *à la Bertrand* for each type of (homogeneous) customers.

A second, more specialized, strand of literature (the behaviour-based price discrimination -BBPD- literature) has considered the case when firms acquire their information on consumers through an initial interaction with them: intertemporal price discrimination practices are based on customers' purchase history. In the seminal works by Chen (1997), Villas-Boas 1999, and Fudenberg and Tirole (2000), the information collected by the firms on their old customers is very crude and only enables subsequent third-degree price discrimination between old and new customers. More recently (see Choe, King, and Matsushima 2018; Garella, Laussel, and Resende 2021), the firms learn through first-period interaction the *exact* preferences of their previous customers, allowing second-period first-degree price discrimination. In this literature, until very recently, consumers were supposed purely passive, in the sense that old customers cannot avoid to be recognized by their initial provider and have no other choice than between buying from it at the personalized price they are offered or at its rival's uniform price. This is known as the no 'leakage' assumption: old customers cannot buy at the uniform price which their initial supplier targets at new customers.

A third strand of literature deals with privacy[12] in an oligopolistic framework,[13] i.e. with the possibility that consumers protect actively their privacy in order either to avoid to be recognized when returning to buy at their previous supplier. Two papers in particular are somewhat related to the present one. Montes, Sand-Zantman, and Valletti (2019), consider a static duopoly model in which the firms may learn the consumers' preferences by buying this information from a monopolistic data broker but consumers may avoid to be recognized by paying a privacy cost. They consider ex ante identity management since the consumers who pay to be anonymous do not receive personalized offers. Chen, Choe, and Matsushima (2020), deal also with a static model in which the firms target each an exogenous segment of consumers,[14] to whom they may propose to buy at personalized prices while setting an uniform price for the other consumers whom they don't target, and compare the case where consumers are passive to the one where they are active, in the sense of 'ex post identity management', i.e. they can incur a cost to buy at the uniform price. They show that the

[11]It is not necessary for this result that each firm be informed on the preferences of *all* customers as it is assumed by Thisse and Vives.
[12]See the survey by Acquisti, Taylor, and Wagman (2016), on the economics of privacy.
[13]In the monopoly case see Belleflamme and Vergote (2016).
[14]The market is not necessarily covered and segments may also possibly overlap.

presence of active consumers is likely to benefit firms by dampening competition. When a firm's targeted customers can buy at its uniform price rather at the personalized ones, a smaller uniform price may allow a firm to poach more rival's targeted customers but this is at the cost of having targeted customers preferring to buy at this price.[15]

The present paper draws on the three strands of literature. Like for instance in Thisse and Vives (1988), the firms exercise first-order price discrimination. Like in BBPD models, this is only possible over their old customers whose preferences they learn through first-period interaction. Finally it is assumed that consumers may access to their previous provider's uniform price by incurring a cost.

The closest paper is Chen, Choe, and Matsushima (2020). Indeed, our second-period equilibrium is close to their one-period static one in the case where the market is fully covered and all customers are active. Our second-period analysis in this case is more general since (i) we don't assume as they do a specific value of the gross utility parameter[16] and (ii) more importantly, we suppose non-zero access cost which the consumers decide to incur or not to incur whereas they consider the case where all consumers are active. We are able to show the same qualitative results obtain as in the zero access cost provided this cost is below a critical value which is the difference between the gross utility and the linear transportation cost. Moreover, the second-period model is embedded in a two-period BBPD model so that the second-period market shares rather than being exogenous are the outcome of first-period price competition. This allows us to show that, if our analysis confirms that active consumers are good for second-period equilibrium firms' profits, extra profits are at least partly wiped out by first-period competition, leading in some interesting cases to smaller two-period equilibrium profits than with purely passive customers. On the other hand, when comparing aggregate two-period equilibrium profits with their value at the repeated static equilibrium we obtain the same global outcome as Fudenberg and Tirole (2000), that the latter are greater than the former, but the per period outcomes are exactly opposite: in their paper second period profits are smaller than their static equilibrium value but, through a competition *dampening* effect, first-period ones are greater.

III. The model

Two competing firms 1 and 2 are located at the two extremes of the Hotelling line. Their marginal costs are normalized to 0. We consider customers who are uniformly distributed on the unit segment. The gross utility of a type x-consumer from buying one unit of good $i = 1, 2,$ equals $v - t(x - x_i)^2$, where $x_1 = 0$ and $x_2 = 1$. So transportation costs are quadratic and t is the transportation cost parameter. In the following the ratio $V = v/t$ between the maximum gross utility and the transportation cost will play an important role. In particular the assumption below will be useful.

Assumption 1: This assumption is necessary and sufficient to ensure that the static duopolists cover the whole market. At the static Hotelling equilibrium with full coverage, it is well-known that one obtains equilibrium prices p^{Hot} equal to t.[17] The minimum net utility of a type x-customer, under linear transportation costs, obtains at $x = 1/2$ and equals $v - \frac{t}{4} - p^{Hot} = t(V - \frac{5}{4})$. Chen, Choe, and Matsushima (2020), make a stronger assumption that a static monopolist would just cover the market, which in this model would require that $V = 3$.

In the first period, the two firms do not know the customers' preferences. They simultaneously select uniform prices p_1 and p_1 which are commonly observable. Given p_1 and p_2, the customer of type $z(p_1, p_2)$ is indifferent between buying good 1 or good 2 in the first period. The customers whose types are smaller than z will therefore buy good 1 whereas the customers such that type is larger than z will buy good 2 in the first period.

[15] A related paper is Belleflamme, Lam, and Vergote (2020). In an extension of their paper they consider the case when the firm must publish their uniform price, something akin to "ex-post price discrimination". This is however different from the present paper since buying at the uniform price does not follow from customers' choices and is free.

[16] They take it equal to the value which ensures that a non-discriminating monopolist would just cover the market.

[17] The same equilibrium obtains whether transportation costs are linear or quadratic.

At the beginning of the second period, each firm has learned the preferences of its first-period customers but continues to ignore those of its rival customers. The timing of the second-period game is as follows. First the two firms simultaneously select the uniform (standard) prices P_1 and P_2 of their products which are commonly observable. Then they choose the personalized prices $s_1(x)$ and $s_2(x)$ at which they propose to their old customers to buy their product. This sequentiality assumption is standard in the literature (see for instance Choe, King, and Matsushima 2018; or Chen, Choe, and Matsushima 2020 among others) and corresponds to the observation that the choice of standard product prices is less frequent and made at a higher level of management. After observing the prices P_i, $s_i(x)$ and P_j, $i,j=1,2$, $i\neq j$, a Firm i's type $x-$ previous customer chooses to pay or not to pay an access cost $c\geq 0$. If she incurs this cost, she may choose one of three options: to buy i's product at the uniform price, to buy j's product at the standard price or to buy i's product at the personalized price targeted to her. If she chooses not to incur the access cost, she can only select one of the two last options. We of course assume that she chooses to incur the cost c if this gives her the highest net utility.

In the following we shall assume that the access cost is not too high, namely that it is smaller than the difference between the gross utility v and the unit transportation cost t.

Assumption 2 This assumption means that there always exists a positive price at which *all* consumers are willing to buy a firm's product even when having to incur the access cost in addition to the price and the transportation cost.

Finally the firms' and consumers' discount factors are respectively denoted δ and β. Both are assumed positive and smaller than 1.

IV. Second period equilibrium

Remember that the second-period game is a two-stage one. In the first stage the two firms choose their uniform (standard) prices P_1 and P_2. Then in the second stage they define the personalized price schedules $s_1(x)$ and $s_2(x)$ which they propose to their respective old customers. Given its rival's standard price P_2, the best Firm $1's$ personalized price schedule is $s_1(x)=\min\{\max\{0,P_2+t(1-2x)\},v-tx^2\}$. Indeed (i) a customer will never pay a personalized price $v_1(x)>v-tx^2$ which would leave her with a negative surplus and (ii) $P_2+t(1-2x)$ is the maximum price at which Firm 1 can secure a profitable sale to a type $x-$ customer but it has to be non-negative since it does not want to sell at a loss. By a similar argument, given its rival's standard price P_1, the best Firm $2's$ personalized price schedule is $s_2(x)=\min\{\max\{0,P_1+t(2x-1)\},v-t(1-x)^2\}$.

Given the first-period market shares and the expected optimal second stage personalized price schedules, what are the first stage optimal uniform prices?

Lemma 1 below shows that, under Assumption 2, there are only two relevant options for Firm i: either to set a high enough uniform price $P_i\geq v-c$ such that no customer is willing to buy at this price and to sell at personalized prices to all its old customers or to set a low enough uniform price P_i such that all its old customers and some rival's old customers buy at this price.

Lemma 1 *Under Assumption 2, there is no uniform price $P_i\geq 0$at which Firm imay possibly sell its product to some of its rival's old customers, whom it poaches, while selling it at personalized prices to some of its own old customers..*

Proof. See Appendix. ■

The best option for the firm is obviously the one which gives here the highest profits. Accordingly, under Assumption 2, there are only two types of second-period equilibria: bilateral *perfect price discrimination equilibria* where both firms choose uniform prices $P_1=P_2=v-c$ and subsequently $v_i(x)=v-t(x-x_i)^2$ where $x_1=0$, $x_2=1$, and unilateral *perfect price discrimination cum poaching* equilibria where the big firm i chooses $P_i=v-c$ and subsequently proposes personalized prices $\min\{\max\{0,P_j+t|2x-1|\},v-t(x-x_i)^2\}$ to its old customers while the small firm j chooses some

$P_j < v - c$ and serves at this price its old customers and some of its rival's old customers whom it poaches.

At a bilateral PPD equilibrium the firms' second-period profits are as follows:

$$\pi_1^{PPD}(z) = \int_0^z (v - tx^2)dx = t(Vz - \frac{1}{3}z^3). \quad (1)$$

$$\pi_2^{PPD}(z) = \int_z^1 (v - t(1-x)^2)dx$$
$$= t(V(1-z) - \frac{1}{3}(1-z)^3). \quad (2)$$

Remember that z is the consumer's type which in period 1 is indifferent between buying at Firm 1 and buying at Firm 2.

Let us now consider a unilateral PPD by Firm 2 plus one way poaching equilibrium by Firm 1 such that: (i) Firm 1 (the poaching firm) sets a uniform price $P_1 < v - c$ and (ii) Firm 2 selects a uniform price $P_2 = v - c$, ensuring that none of its old customers (and by the way none of the rival's old customers) chooses this standard offer, and offers to each x of its old customers a personalized price $s_2(x) = \min\{\max\{0, P_1 + t(2x-1)\}, v - t(1-x)^2\}$ which is a best reply to P_1. As shown in the proof of Lemma 1, in any equilibrium where there is some actual poaching by Firm 1, the optimal value of $P_1 = t/2$, with Firm 1 serving all customers in $[0, 1/4]$ while Firm 2 offers a price equal to zero to its old customers in $[z, 1/4]$ and a personalized price $s_2(x) = \min\{t(2x - \frac{1}{2}), v - t(1-x)^2\}$ to its old customers belonging to $[1/4, 1]$. The Firms' profits are then

$$\pi_1^{Po1}(z) = t/8, \quad (3)$$

$$\pi_2^{Po1}(z) = 9t/16 \text{ if } V \geq 3/2,$$

$$= V - \frac{13}{48} - (2V-1)\sqrt{V - \frac{1}{2}} \text{if} V \leq 3/2. \quad (4)$$

Notice that $t(2x - \frac{1}{2}) \leq (\text{resp. } \geq)\ v - t(1-x)^2$ if $x \leq$ (resp. $\geq$) $\sqrt{V - \frac{1}{2}}$. When $V \geq 3/2$, $s_2(x) = t(2x - \frac{1}{2})$ for all $x \in [1/4, 1]$.

The equilibrium with PPD by Firm 1 and poaching by Firm 2 is described symmetrically with profits:

$$\pi_1^{Po2}(z) = 9t/16, \text{ if } V \geq 3/2, \quad (5)$$

$$= V - \frac{13}{48} - (2V-1)\sqrt{V - \frac{1}{2}} \text{if} V \leq 3/2. \quad (6)$$

$$\pi_2^{Po1}(z) = t/8.$$

Let $\underline{z}$ be the unique solution $\in [0, 1]$ of the equation $\frac{1}{8} = (Vz - \frac{1}{3}z^3)$ and $\overline{z} = 1 - \underline{z}$.[18] $\underline{z}$ is the first-period market share at which Firm 1, when the small firm, is indifferent between poaching Firm 2 and exercising first-order price discrimination on its old customers. $\overline{z}$ is defined symmetrically.

Lemma 2 *Bilateral Perfect price discrimination (PPD) is a second-period equilibrium iff* $z \in [\underline{z}, \overline{z}]$.

Proof. See Appendix. ■

The intuition for Lemma 2 is rather simple. A deviation from the PPD strategy has a cost for the deviating firm which are the benefits foregone on former customers: they then pay a uniform price which is smaller than the personalized price which would wholly extract their surplus. The small firm's benefit from deviation is to be able to poach some of its rival's old customers. The greatest the firm's initial market share the greatest are the benefits foregone and the smallest the profits from poaching the rival's customers. Accordingly, perfect price discrimination by both firms is a second-period equilibrium iff the firms' initial market shares are close enough.

It is worthwhile to understand why PPD is a second-period equilibrium when customers are active whereas it is not generally considered as such with passive customers. This is related to the sequentiality of moves in the second-period game where it is assumed that the duopolists first choose the standard, uniform, prices of their products and then define the personalized prices offered to their old customers. The smaller firm is always able to successfully protect its own turf whatever its rival's

[18] Notice that given Assumption 1, this equation has one and only solution $\in [0, 1]$. Moreover it is easy to check that $\underline{z} < 1/4$;.

standard price so that, when all customers are passive, the bigger one is indifferent between all non-negative standard (poaching) prices. A 'Bertrand competition' argument (Choe, King, and Matsushima 2018), and/or an equivalent 'refinement' argument (Chen, Choe, and Matsushima 2020) selects the equilibrium where the big firm chooses a price equal to its marginal cost. With active customers, the big firm is no more indifferent with respect to the price of its standard product since, if it is not high enough, its old customers prefer to reject the personalized offers: by choosing a dissuasive standard price it can, on the contrary, exercise PPD on its previous customers.

Lemma 3 *(i) Unilateral PPD by Firm 2 and one-way poaching by Firm 1 is a second-period equilibrium iff* $z < \underline{z}$;

(ii) Unilateral PPD by Firm 2 and one-way poaching by Firm 1 is a second-period equilibrium iff $z > \overline{z}$.

Proof. See Appendix. ■

In Figure 1 below we picture the areas in the (V, z)-space corresponding to the three possible second-period equilibria.

From Lemmas 2 and 3, and Equation (1) to (5) , the profit of the two firms are discontinuous functions of z. Let us for instance consider $\pi_1(z)$. It equals $t/8$ when $z \in 0, \underset{0.3em\ \$-\$}{z})$, $vz - \frac{1}{3}tz^3$, and is thus strictly increasing in z, when $z \in \left[\underset{0.3em\ \$-\$}{z}, \overline{z}\right]$ and $\min\left\{9t/16, V - \frac{13}{48} - (2V-1)\sqrt{V-\frac{1}{2}}\right\}$ when

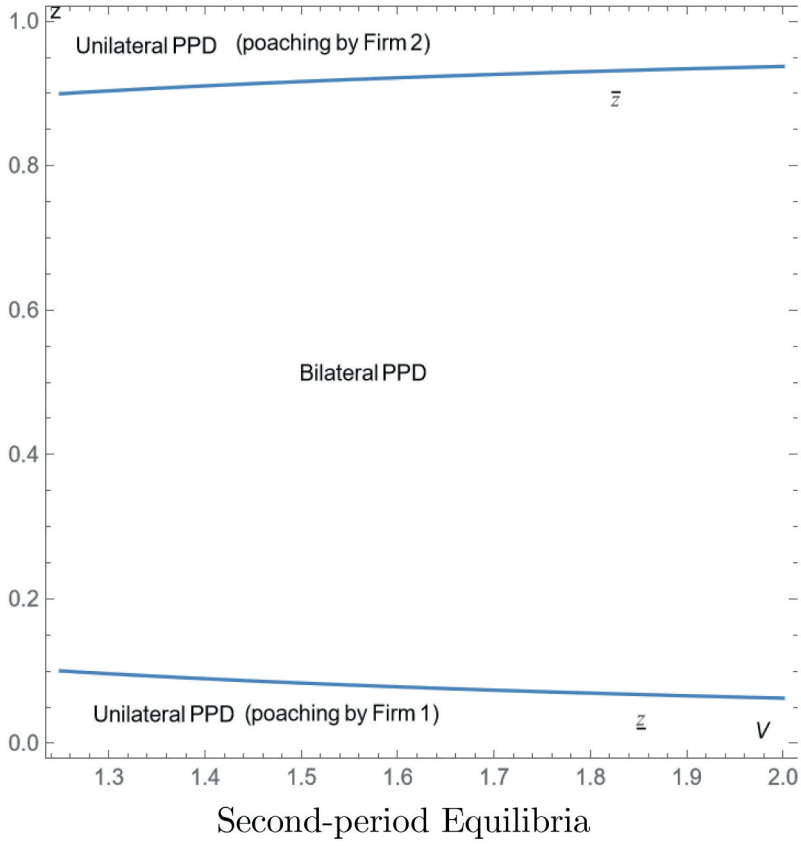
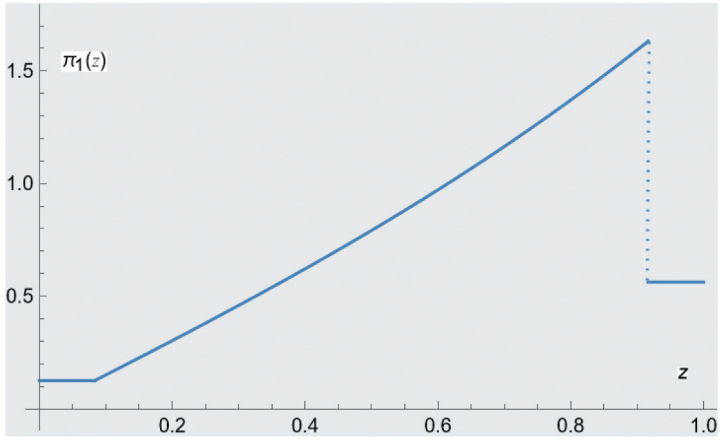


Figure 1. Second-period equilibria.



Firm 1's second-period profits as function of initial market share

Figure 2. Firm 1's second-period profits as function of initial market share,

$z \in (\bar{z}, 1]$. This function is continuous by construction at $z = \underset{0.3em\ \$\ -\$}{z}$ but discontinuous at $z = \bar{z}$ where Firm 1's profits jump down as z further increases. The reason is that, at this point, a further increase in Firm 1's initial market share induces Firm 2 to change its strategy from $P_2 = v$ to $P_2 = t/2 < v$ in order to poach Firm 1's old customers. This is pictured in Figure 2 below for $t = 1$ and $V = 3/2$.

V. First-Period equilibrium

Let us begin by determining the first-period demand functions. In the case when $z \in [\underline{z}, \overline{z}]$, the customers' second-period net utility is zero whatever the good they buy. Accordingly they only compare first period utilities and the first-period demand function for good 1 is the static demand function:

$$z = \frac{1}{2} + \frac{p_2 - p_1}{2t}. \tag{7}$$

The first demand function for good 2 is obviously $1 - z$.

When $z < \underline{z}$, the marginal customer in the first period (i.e. the $z-$ type one) is then indifferent between staying with Firm 1 for the two periods and choosing Firm 2 in the first period and switch to Firm 1 only in the second one. In both cases it buys in the second period at Firm 1's uniform price $P_1 = t/2$. So, once again, the comparison is between first-period utilities and the demand function is given by (7). The same conclusion obtains when $z > \overline{z}$.

That the first-period demand function be the static one is a noticeable difference with existing BBPD models. For instance, in Fudenberg and Tirole (2000), the price sensitivity of the first period demand function is smaller than the one of the static demand function as long as consumers are not completely naive.[19] In Choe, King, and Matsushima (2018), or in Laussel and Resende (2021), the reverse occurs, under the same condition, along with a competition *increasing* effect.

Proposition 1 *Iff* $V \leq \overline{V} = \frac{24+4\sqrt{3}\sqrt{12-\delta}+3\delta}{12\delta}$, *there is a unique, symmetric, first-period Nash equilibrium such that*

$$p_i^* = \frac{t}{4}(4 + \delta - 4\delta V), \quad i = 1, 2, \tag{8}$$

$$\Pi_i^* = t\frac{6+\delta}{12}, i = 1, 2.$$

At this equilibrium, both firms exercise second-period perfect price discrimination and their second-period equilibrium profits equal

$$\pi_i^* = t\left(\frac{V}{2} - \frac{1}{24}\right), i = 1, 2. \tag{9}$$

Proof. See Appendix. ■

[19]This gives rise to a *competition dampening effect* which partially compensates the effect of the decrease of second period profits on two-period ones.

Notice that no pure strategy first-period Nash equilibrium exists for very great values of V ($=v/t$) such that $V>\overline{V}$. For such values, the first-period candidate equilibrium prices would be low, so that, given the negative candidate equilibrium price of its rival, it is profitable for each firm to deviate towards a higher price, in order to obtain greater first-period profits, or simply to limit first-period losses, even if it means having a small initial market and poaching its rival's old customers in the second period.

A striking feature of the equilibrium characterized in Proposition 1 is that the firms' two-period discounted profits do not depend on V. This may look surprising since any increase in V results in greater second-period profits (the greater the customers' reservation value for the good the greater the profits from perfect price discrimination). However, this increase in second-period profits is exactly wiped out by the resulting decrease in the first-period equilibrium prices: a greater initial market share being rationally expected to be more profitable in the second period, the firms compete more fiercely in the first period. This phenomenon is reminiscent of what happens when two duopolists who compete in prices receive in addition exogenous fixed receipts proportional their volume of sales, whether these receipts be government subsidies or, in the case of newspapers, advertising receipts.[20]

VI. Pay-Off comparisons

There are two benchmarks against which to usefully evaluate the equilibrium profits, social welfare and consumers surplus: the static Hotelling equilibrium and the BBPD model with purely passive customers. In each case, we shall see that it is worthwhile to consider not only the two-period payoffs but also, separately, the first-period and second-period payoffs.

At the static Hotelling equilibrium, the firms' equilibrium payoffs $\pi_i^{Hot} = t/2,\ i=1,2$, the social welfare equals

$$w^{Hot} = \int_0^{1/2}(v-tx^2)dx + \int_{1/2}^{1}(v-t(1-x)^2)dx = v-\frac{1}{12}t.$$

The consumers' aggregate surplus is therefore the difference

$$cs^{Hot} = w^{Hot} - 2\pi_i^{Hot} = v-\frac{13}{12}t.$$

(i) *The two-period discounted equilibrium profits are smaller than the profits from the repetition of the static equilibrium;*
(ii) *The social welfare is the same, and takes its maximum value, in both periods in the two cases;*
(iii) *Aggregate consumers' surplus is smaller in the second period but greater in the first one than at the static equilibrium.*

Proposition 2 *(i) The second-period equilibrium profits are greater than their static counterpart while the first-period equilibrium profits are smaller that their static counterpart;*

Proof. See Appendix. ■

Proposition 2 shows that with (potentially) active customers the effects of BBPD on firms and consumers payoffs are very contrasted according to the period which one considers. Since the aggregate social welfare is unchanged in both periods, the effects of BBPD are purely distributional. In the case of a mature market, when firms are installed ones with inherited market shares (second period), they are better off when they have been able to learn their old customers' preferences than when they ignore them. (Old) Consumers on the contrary are worse-off. When one considers a new market, the first-period firms profits are smaller than at the static equilibrium with no information collection, meaning that consumers surplus is greater. The two effects follow from fiercer price competition for new customers. Notice that while two-period profits are both smaller in this model and in Fudenberg and Tirole (2000), than at the

[20] On the latter issue see for instance Gabszewicz, Laussel, and Sonnac (2005).

repetition of the static equilibrium, this comes from very different, if not opposite, sources. In Fudenberg and Tirole, second-period profits are lower than at the Hotelling equilibrium but the first-period price elasticity of demand is smaller so that first-period profits are greater, without however compensating the second-period shortfall. Notice that there is in FT no competition for the market effect since, at equilibrium, second-period profits are not affected by market size. In the present model, on the contrary, second-period profits are greater than at the Hotelling equilibrium but this gain is more than *dissipated* by first-period competition for the market.[21] On the other hand, differently from FT, the price-elasticity of demand is the same as in the static equilibrium.

The second benchmark is the BBPD model with purely passive customers such as studied by Choe, King, and Matsushima (2018). In this model there are two mirror first-period equilibria. Let us denote β the consumers' common discount factor (which may differ from δ).[22]

(i) *The social welfare is greater in both periods when consumers are active;*
(ii) *The two-period discounted equilibrium aggregate profits are smaller with active customers than with passive ones if consumers are naive and/or firms are patient enough.*

Proposition 3 *(i) The second-period aggregate equilibrium profits are greater with active customers than with passive ones while the contrary holds true for first-period profits;*

Proof. See Appendix. ■

From Proposition 3, the possibility for consumers to access their previous provider's uniform price is clearly beneficial to installed firms, confirming the results in Chen, Choe, and Matsushima (2020): second-period aggregate equilibrium profits are greater. These benefits are eroded by price competition: first-period aggregate equilibrium profits are then smaller. The possibility that two-period discounted aggregate profits are smaller when consumers are active appears when consumers are naive (i.e. β is small) and/or firms are patient (δ is great). The opportunity for consumers to buy at the uniform price may accordingly hurt the firms in emergent markets.

Figure 3 above shows that there is an area (in blue) where two-period profits are smaller when customers are active. Points in this area are interesting because they correspond to the empirically relevant situations where the consumers discount heavily future payoffs while firms give them much more weight. This obtains despite the fact that, in all cases, second-period profits are greater when consumers are active, a point which has been convincingly made by Chen, Choe, and Matsushima (2020). We have shown that, in a two-period model first-period price competition may plausibly reverse this result. The clue is that, when customers are active, the firms value more their initial market shares and compete accordingly more fiercely for the market in the first period.

VII. Extensions and robustness

Ex ante identity management

Following Chen, Choe, and Matsushima (2020), we have assumed, according to their terminology, that consumers are able to hide their identity *ex post*, once they happen to know the second-period uniform price of their first-period suppliers. This assumption is crucial in this model as well as in Chen et al, since the fact that their old customers may choose to buy at the uniform price instead that at the personalized prices is what leads the firms to set in the second period a dissuasive uniform price.[23] Would only *ex ante* identity management be possible, the second-period equilibrium outcome would never imply perfect price discrimination by one or both firms. However there does not seem to be any good reason why gaining access to uniform prices would be impossible or entail prohibitive costs so that only ex ante identity would be available. So what is interesting may rather be to consider the case where 'ex post identity

[21]This effect appears since second-period profits are strictly increasing in the firm's market share.
[22]β does not play any role in **our** model.
[23]In the case when the initial market shares are not too different.

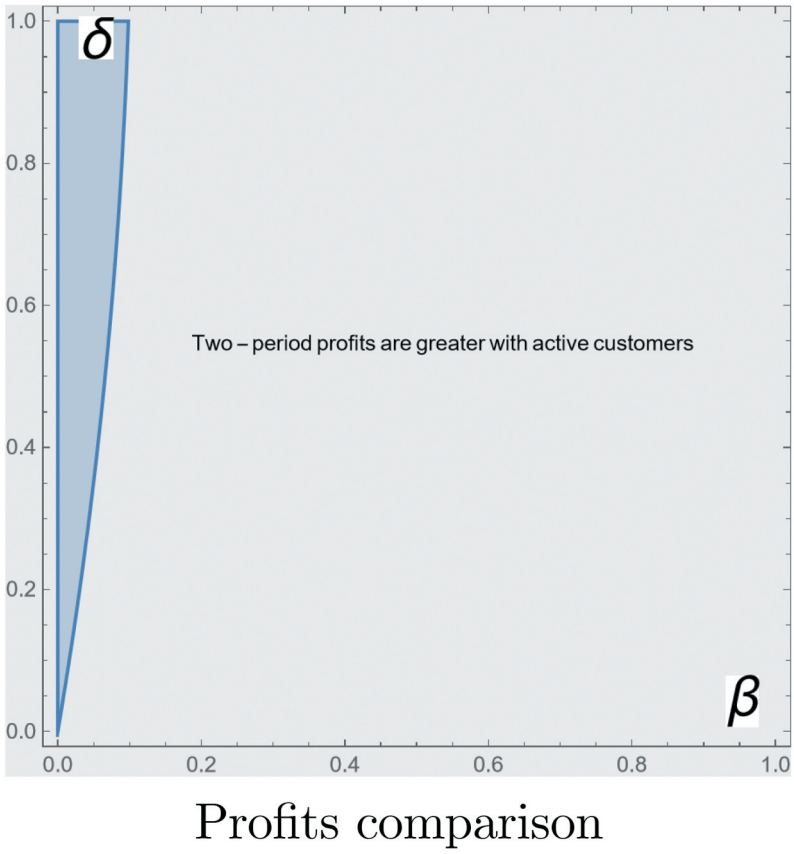


Figure 3. Profits comparison.

management' is possible at cost $c>0$ in an initial stage of the game, say at the beginning of period 1, while ex ante identity management is available at some cost $m>0$.

A first result is straightforward. If ex ante identity management is more costly than 'expost identity management', i.e. if $m \geq \beta c$ (what happens if consumers are naive enough), there is no equilibrium with active ex ante identity management. Suppose on the contrary that some type x-customers bear a cost m initially in order to buy in the second period Firm i's good at the uniform price P_i. Independently of x and P_i, she would clearly be better off not hiding her identity in the first period but doing that only in the second period. Indeed the same desired outcome is obtained at a smaller cost.

Suppose then that ex ante identity management is less costly, i.e; $m<\beta c$. Conditional on no customer choosing to pay the ex ante privacy cost m, the continuation equilibrium when z is close enough to 1/2, is obviously the one described in Proposition 1 above in which the two firms select in the second period a dissuasive uniform price $v-c$ and consumers do not hide their identity. Given that all other customers choose not to hide initially their identity, any consumer rationally expects that the second-period uniform prices will be dissuasive and is accordingly better off not incurring the ex ante privacy cost. This is clearly a Nash equilibrium between consumers in the initial stage of the game: there is always an equilibrium where consumers do not access their initial provider's uniform price. The equilibrium described in Proposition 1 is always part of equilibrium of the more general game where customers may hide their identity from the start.

New customers

In previous sections, we assumed that there are no new consumers in the second period so that the firms know the preferences of all their old passive consumers . The consequence of this assumption is that when figuring out what uniform price to offer in the second period a firm considers only the

consequences this may have on old customers (a low price may induce them to pay the access cost in order to buy advantageously at this price) and on rival's old customers (which may possibly be poached if the firm is small). The existence of a fraction μ of new customers (because, for instance, a fraction of old customers die and are replaced by an equivalent number of new ones) may change the picture. Since the firms try to attract new customers, they may be dissuaded to set the very high prices which in our main analysis result in second-period perfect price discrimination equilibria. This is likely to result in second-period equilibria which are closer to the perfect price discrimination one[24] when the proportion of new customers is small and closer to the Hotelling equilibrium when this proportion is important. We indeed show in Appendix that for small enough values of μ, if z belongs to a non-void interval $[\underline{z}, \overline{z}]$ which includes $1/2$, the second-period equilibrium still implies prices $P_i = v - c,\ i = 1, 2,$ i.e. (surviving) old customers never hide their identity and receive personalized offers which extract all their surplus. This establishes that the existence of new customers does not change the qualitative properties of equilibrium, provided their proportion is not too important.

VIII. Conclusion

In this paper we have analysed a behaviour-based price discrimination model in a duopoly setting when (i) the firms learn in the first period the exact willingness to pay of their customers and are able to propose them in the second period to buy at personalized prices but (ii) the old customers are active in the sense that they can incur an access cost in order to buy at their first-period provider standard (uniform) price. Our analysis has confirmed that, when first-period market shares are close enough, the existence of active customers allows both firms to exercise perfect price discrimination in the second period and thus to achieve greater profits than when all customers are passive. In *mature markets*, firms clearly benefit from the consumers' opportunity to hide their identity when they return buying (ex post identity management). However we have also shown that, in a two-period model, these extra profits are dissipated, and sometimes over-dissipated, by the induced fiercer first-period price competition *for the market* between the duopolists. Whereas the equilibrium second-period profits are greater than static equilibrium profits, the firms two-period discounted profits are always smaller than the profits which would follow from the repetition of the static equilibrium and they are smaller than in the case of passive customers when consumers are naive and/or firms are patient. So the effect of 'ex post identity management' in *emerging markets* may well be, under reasonable conditions, to hurt the firms. Not surprisingly, the effects on consumers' expected surplus of the possibility of 'ex post identity management' are the opposite: negative in mature markets and possibly positive in emerging ones. The effects on social welfare are on the contrary straightforward: it is greater in both periods when consumers are active so that policies reducing consumers' access cost to uniform prices (for instance imposing publication of lists of prices) are clearly welfare-enhancing. We have shown that these results are robust (i) with respect to the introduction of positive access costs, provided they are not very high, (ii) with respect to the possibly that consumers may hide their identity from the start and (iii) with respect to the existence of a small enough proportion of new customers.

A worthwhile, but not easy, extension would be to consider a more general game where the firms would be initially able to commit not to set personalized prices in period 2. We conjecture that it would be a subgame-perfect Nash equilibrium of this more general game that both firms don't commit to use only uniform prices.[25]

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[24] For the case of z not too far from 1/2.

[25] The argument used in the proof in Proposition 1 for ruling out a price p_1 such Firm 1 poaches Firm 2's consumers and does not offer personalized prices may be transposed for the case where Firm 1 commits not to use personalized prices (while Firm 2 does not make such commitment).

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References

Acquisti, A. 2008. "Identity Management, Privacy, and Price Discrimination." *IEEE Security & Privacy* 6 (2): 46–50. doi:10.1109/MSP.2008.35.

Acquisti, A., C. Taylor, and L. Wagman. 2016. "The Economics of Privacy." *Journal of Economic Literature* 54 (2): 442–492. doi:10.1257/jel.54.2.442.

Belleflamme, P., and W. Vergote. 2016. "Monopoly Price Discrimination and Privacy: The Hidden Cost of Hiding." *Economics Letters* 149: 141–144. doi:10.1016/j.econlet.2016.10.027.

Belleflamme, P., W. M. W. Lam, and W. Vergote. 2020. "Competitive Imperfect Price Discrimination and Market Power." *Marketing Science* 39 (5): 996–1015. doi:10.1287/mksc.2020.1234.

Chen, Y. 1997. "Paying Customers to Switch." *Journal of Economics & Management Strategy* 6 (4): 877–897. doi:10.1162/105864097567291.

Chen, Y., and G. Iyer. 2002. "Research Note Consumer Addressability and Customized Pricing." *Marketing Science* 21 (2): 197–208. doi:10.1287/mksc.21.2.197.153.

Chen, Z., C. Choe, and N. Matsushima. 2020. *Competitive Personalized Pricing*. Management Science 66 (9): 4003-4023.

Choe, C., S. King, and N. Matsushima. 2018. "Pricing with Cookies: Behavior-Based Price Discrimination and Spatial Competition." *Management Science* 64 (12): 5669–5687. doi:10.1287/mnsc.2017.2873.

Ezrachi, A., and M. E. Stucke. 2016. *Virtual Competition*. Cambridge: Harvard University Press.

Fudenberg, D., and J. Tirole. 2000. "Customer Poaching and Brand Switching." *The Rand Journal of Economics* 31 (4): 634–657. doi:10.2307/2696352.

Gabszewicz, J. J., D. Laussel, and N. Sonnac. 2005. "Does Advertising Lower the Price of Newspapers to Consumers? a Theoretical Appraisal." *Economics Letters* 87 (1): 127–134. doi:10.1016/j.econlet.2004.06.024.

Garella, P. G., D. Laussel, and J. Resende. 2021. "Behavior Based Price Personalization Under Vertical Product Differentiation." *International Journal of Industrial Organization* 76: 102717. doi:10.1016/j.ijindorg.2021.102717.

Laussel, D., N. V. Long, and J. Resende. 2020. "The Curse of Knowledge: Having Access to Customer Information Can Reduce Monopoly Profits." *The Rand Journal of Economics* 51 (3): 650–675. doi:10.1111/1756-2171.12336.

Laussel, D., and J. Resende. 2021. "Profit Enhancing Product Personalization: A Behavior Based Discrimination Model, Mimeo."

Liu, Q., and K. Serfes. 2004. "Quality of Information and Oligopolistic Price Discrimination." *Journal of Economics & Management Strategy* 13 (4): 671–702. doi:10.1111/j.1430-9134.2004.00028.x.

Mikians, J., L. Gyarmati, V. Erramilli, and N. Laoutaris 2012, October. Detecting Price and Search Discrimination on the Internet. Proceedings of the 11th ACM workshop on hot topics in networks (pp. 79–84), Redmond Washington.

Mikians, J., L. Gyarmati, V. Erramilli, and N. Laoutaris 2013, December. Crowd-Assisted Search for Price Discrimination in E-Commerce: First Results. In Proceedings of the ninth ACM conference on Emerging networking experiments and technologies (pp. 1–6), Santa Barbara California USA.

Mohammed. 2017. "How Retailers Use Personalized Prices to Test What You'Re Willing to Pay." Harvard Business Review, https://hbr.org/2017/10/how-retailers-use-personalized-prices-to-test-what-youre-willing-to-pay

Montes, R., W. Sand-Zantman, and T. Valletti. 2019. "The Value of Personal Information in Online Markets with Endogenous Privacy." *Management Science* 65 (3): 1342–1362. doi:10.1287/mnsc.2017.2989.

Shaw, I., and N. Vulkan. 2012. "Competitive Personalized Pricing: An Experimental Investigation." Review of Economic Studies (Submitted).

Thisse, J. F., and X. Vives. 1988. "On the Strategic Choice of Spatial Price Policy." *The American Economic Review* 78: 122–137.

Villas-Boas, J. M. 1999. "Dynamic Competition with Customer Recognition." *The Rand Journal of Economics* 604–631. doi:10.2307/2556067.

Wallheimer. 2018. "Are You Ready for Personalized Pricing?." Chicago Booth Review, https://review.chicagobooth.edu/marketing/2018/article/are-you-ready-personalized-pricing

APPENDIX

Proof of Lemma 1

This is obvious when i is the big firm since, as we already saw, it can always protect its own turf successfully. So let's consider the case when i is the small firm, say $i = 1$ (so that $z \leq 1/2$). Firm 2's best reply to a uniform price P_1 is $s_2(x) = \max\{0, P_1 + t(2x-1)\}$ so that Firm 1 can poach some rival's first-period customers iff it chooses a price P_1 such that $P_1 < t(1-2z)$. On the other hand, Firm 1 's old type $x-$ customers choose to buy at the personalized prices, which leave them with zero surplus, iff incurring the access cost to buy at the uniform price would leave them with a negative net utility, *i.e.*, $v - tx^2 - c - P_1 < 0$. There is accordingly a non-void subset of such consumers iff $P_1 > v - c - tz^2$. For the two conditions to hold simultaneously at some P_1, it then must be that $v - c - tz^2 < t(1-2z)$. Given Assumption 2, this inequality cannot hold at any $z \in [0,1]$[26] the RHS of this inequality is strictly negative so that there is no positive value of z at which the two conditions may simultaneously hold. +

Proof of Lemma 2

Notice first that if $z = 1/2$, no firm can benefit by deviating from the PPD equilibrium since it is unable to poach any rival's customer. Suppose then without loss of generality that Firm 1 is the small firm, i.e. $z < 1/2$. Firm 2 cannot poach a Firm 1's old customer and accordingly cannot benefit by deviating to a uniform price lower than v.

Suppose now that the small Firm 1 (i.e; $z < 1/2$) deviates to a price $P_1' < v - c$. Firm 2's best reply is to offer a personalized price schedule $\max\{0, P_1' + t(2x-1)\}$ to its old customers $\in [z.1]$. Remember that it has already selected a uniform price $P_2 = v - c$, so that no Firm 1's old customer is willing to buy its product. The only deviations to consider are to prices P_1' which allow Firm 1 to poach some Firm 2's old customers, i.e. such that $P_1' < t(1-2z)$, since other deviations are clearly dominated by the equilibrium strategy. For such prices the demand for good 1 equals $\widehat{x} = \frac{1}{2} - \frac{P_1'}{2t} > z$ (Firm 1 poaches some rival's customers) and Firm 1's (deviation) profit is $\left(\frac{1}{2} - \frac{P_1'}{2t}\right)P_1'$, which is a concave function of P_1'. This function either takes its maximum (equal to $t/8$) at $P_1' = t/2$ (interior solution) iff $z < 1/4$ or is increasing in P_1' for all $P_1' \in 0, t(1-2z))$ if $z \geq 1/4$. Only the former case is a potentially profitable deviation. It is indeed profitable iff $t/8 > vz - \frac{1}{3}tz^3$, if $z < \underline{z}$.[27] Accordingly $z \geq \underline{z}$ is a necessary condition for a deviation not to be profitable. It is also sufficient since

(i) when $z \in \underline{z}, 1/4)$ the best deviation is to the interior solution $P_1' = t/2$ but is dominated by the equilibrium strategy and (ii) when $z \in 1/4, 1/2]$ the best deviation is towards a uniform price such that Firm 1 does not poach any rival's old customer, what is obviously dominated by perfect price discrimination over Firm 1's old customers.

A similar argument shows that $z \leq z$ is necessary and sufficient to ensure that Firm 2 cannot profitably deviate from the equilibrium path. +

Proof of Lemma 3

The proof is simply the mirror of the proof of Lemma 1 above. Let us indeed consider without loss of generality the case when Firm 1 is the poaching firm. Firm 2 has clearly no incentive to deviate. Firm 1 could possibly deviate by choosing $P_1 = v - c$ and subsequently offering a personalized price $v - tx^2$ to its old customers. This would result into profits $vz - \frac{1}{3}tz^3$ which are smaller than $t/8$ iff $z < \underline{z}$. Notice that if this condition holds, $z < 1/4$, so that there is actual equilibrium poaching. +

Proof of Proposition 1

(i) Let us first show that $\frac{t}{4}(4 + \delta - 4\delta V)$ corresponds to a local maximum of $\Pi_1(p_1, \frac{t}{4}(4 + \delta - 4\delta V))$. For couples (p_1, p_2) such that $z(p_1, p_2) \in [\underline{z}, \overline{z}]$, $\Pi_1 = p_1 z + \delta(vz - \frac{t}{3}z^3)$. This function is concave in p_1 and from the FOC with respect to p_1 we obtain that it has a maximum at $p_1 = \frac{t}{4}(4 + \delta - 4\delta V)$. A symmetric argument shows that $\frac{t}{4}(4 + \delta - 4\delta V)$ corresponds to a local maximum of $\Pi_2(\frac{t}{4}(4 + \delta - 4\delta V), p_2)$ wrt p_2.

(ii) Let us now show that this also corresponds to a global maximum.

Consider first a deviation towards a value of p_1 such that $0 \leq z(p_1, \frac{t}{4}(4 + \delta - 4\delta V)) < \underline{z}$. Over this range of values, $\Pi_1 = p_1 z + \delta\frac{t}{8}$, which is a concave function of p_1. Its unconstrained maximum obtains at $p_1 = \frac{t}{8}(8 + \delta - 4V\delta)$ with corresponding profits

$$\Pi_1 = \frac{t}{128}\left(64 + \delta\left(32 + \delta + 16V^2\delta - 8V(8+\delta)\right)\right);$$

This is smaller than the candidate equilibrium value $t\frac{6+\delta}{12}$ iff $V \leq \overline{V}$.

Consider now a deviation towards a value of p_1 such that $1 \geq z(p_1, \frac{t}{4}(4 + \delta - 4\delta V)) > \overline{z}$, where $\Pi_1 = p_1 z + \delta\pi_1^{Po2}(z)$ where z is given by Equation (7) and $\pi_1^{Po2}(z)$ follows from Equation (5) . This profit function is increasing in p_1 over the interval and has an upward jump at p_1 such that $z(p_1, \frac{t}{4}(4 + \delta - 4\delta V)) = \overline{z}$ so that profits are always greater at the symmetric equilibrium.

A symmetric argument shows that $\frac{t}{4}(4 + \delta - 4\delta V)$ corresponds to a global maximum of $\Pi_2(\frac{t}{4}(4 + \delta - 4\delta V), p_2)$.

[26] Notice that, denoting $f(z) = (v - c - t) - tz^2 + 2tz$, $f(z)$ is maximum at $z = 1$ and given Assumption 2, we have $f(0) > 0$ and $f(1) > 0$. It follows that $f(z) > 0$ over [0, 1].

[27] Notice that $\underline{z} < 1/4$ given Assumption 1.

(iii) Let us now show that this equilibrium is unique. Any alternative equilibrium should be such either that $z<\underline{z}$ or $z>\overline{z}$. Let us consider the second case without loss of generality. We should observe that $p_1<p_2$. Now on one hand p_1 should maximize $\Pi_1=p_1z+\delta\pi_1^{Po2}(z)$, so that $p_1=\frac{t+p_2}{2}$. On the other hand, p_2 should maximize $\Pi_2=p_2(1-z)+\delta\pi_2^{Po2}(z)$ so that $p_2=\frac{t+p_1}{2}$. Accordingly we should observe that $p_1=p_2=t$ and then $z=1/2<\overline{z}$, a contradiction. +

Proof of Proposition 2

(i) In second period $\pi_i^*=t\left(\frac{V}{2}-\frac{1}{24}\right)>\pi_i^{Hot}=\ t/2$ (this is equivalent to $V>13/12$, which always holds true under Assumption 1). First-period equilibrium profits in our model are equal to $\frac{t}{8}(4+\delta-4\delta V)$. They are smaller than $t/2$ iff $V>1/4$, what is always true under Assumption 1.

(ii) The difference between two-period discounted equilibrium profits and the profits from the repetition of the static equilibrium is $t\frac{6+\delta}{12}-\frac{t}{2}(1+\delta)=-\frac{5}{12}t\delta<0$.

(iii) Given that the market is fully covered in both cases, welfare depends only on the market shares, and is maximum when the firms share equally the market, what is the case at this model equilibrium as well as at the static equilibrium.

(iv) In each period and each case, aggregate consumers' surplus equals the social welfare minus the aggregate firms' profits. Then the result follows directly from (i) and (iii). +

Proof of Proposition 3

(i) The first point is straightforward since the second-period profits in Choe et al. are known to be smaller than the profits at the static equilibrium (see Proposition 2, page 9) while the latter have been shown in Proposition 2 to be smaller than at the present model equilibrium.

In Choe et al., considering without loss of generality the case where Firm 1 is the small firm, $z=\frac{12-6\beta+\delta}{2(12-6\beta+\delta)}$, $\pi_1^*=\frac{t}{8}(14z-4z^2)$, $\pi_2^*=\frac{t}{16}(3-2z)^2$. Substituting for z its equilibrium value, we obtain the small and big firm's equilibrium second-period profits as functions of t,β and δ. On the other hand, the two-period discounted profits in the Choe, King, and Matsushima (2018), model are, in our notations

$$\Pi_S^*=t\frac{36(2-\beta)^3+24\delta(2-\beta)^2+(2-\beta)\delta^2+\delta^3}{4(12-6\beta+\delta)^2},\quad(10)$$

$$\Pi_B^*=t\frac{36(2-\beta)^3+36\delta(2-\beta)^2+3(2-\beta)\delta^2-2\delta^3}{4(12-6\beta+\delta)^2}.$$

To obtain the first-period equilibrium profits for firm $X=S,B,$[28] one simply has to compute $\Pi_X^*-\delta\pi_X^*$. Accordingly aggregate first-period profits equal

$$t\frac{18(2-\beta)^3-3\left(\delta(2-\beta)^2+8(-2+\beta)\delta^2-\delta^3\right)}{(12-6\beta+\delta)^2}.\quad(11)$$

In the present model first-period profits are decreasing in V so that there exist a critical value of V

$$\widehat{V}=\frac{1-\beta}{\delta}+\frac{108(2-\beta)^2-176\delta(2-\beta)-23\delta^2}{4\delta(12-6\beta+\delta)^2},$$

such that for all $V>\widehat{V}$ the first-period profits are smaller than (11). It is easy to see that $\widehat{V}<5/4$, so that, given Assumption 1, first-period profits are always smaller when consumers are active.

(i) Compare the equilibrium profits in Proposition 1 with (10). The difference depends only on the values of β and δ. It is straightforward to see that profits are always greater in the case of active customers.

(ii) The result is straightforward since in the case of passive customers firms have different market shares in each period whereas in the present model these market shares are equal, minimizing transportation costs and thus maximizing welfare. +

New Customers

Suppose that there is in the second period a proportion $\mu\in[0,1]$ of new customers and consider for the sake of computational simplicity the case of *linear* rather than quadratic transportation costs.[29]

Let us show that there exists a range of values $[0,\mu^*]$ of μ such that $P_i=v-c$ is an equilibrium of the second period game provided that z belongs to an interval $[\underline{z},\overline{z}]$ which includes 1/2. Let us consider Firm 1 without loss of generality and notice that Lemma 1 still holds.[30] Firm 1's profits are the following function of its price:

$$\pi_1=\mu\left(\frac{1}{2}+\frac{v-c-P_1}{2t}\right)P_1+(1-\mu)\int_0^z(v-tx)dx\ \text{if},$$

$$=\mu\left(\frac{1}{2}+\frac{v-c-P_1}{2t}\right)P_1+(1-\mu)\left[\int_{\frac{v-c-P_1}{t}}^z(v-tx)dx+P_1\frac{v-c-P_1}{t}\right]\text{if}P_1\in[v-c-tz,v-c],$$

[28] S stands for small, B for big.
[29] The main model is easily solved in this case, leaving qualitatively unchanged the equilibrium properties. Results are available on demand.
[30] The proof is only slightly different.

$$= \mu\left(\frac{1}{2} + \frac{v - c - P_1}{2t}\right)P_1 + (1-\mu)[P_1 z] \text{if} P_1 \in t(1-2z), v - c - tz],$$

$$= \mu\left(\frac{1}{2} + \frac{v - c - P_1}{2t}\right)P_1 + (1-\mu)\left(\frac{1}{2} + \frac{-P_1}{2t}\right)P_1 \text{if} P_1 \leq t(1-2z).$$

The last case is only relevant when Firm 1 is the small firm, i.e. when $z < 1/2$. It turns out that

(i) Given Assumption 2, $\pi_1'(P_1) < 0$ for all $P_1 \in v - c; v]$,
(ii) $\pi_1'(P_1) \geq 0$ for all $P_1 \in v - c - tz; v - c]$, iff $\mu \in 0, \frac{2c}{v+c-t}]$,
(iii) $\pi_1'(P_1) \geq 0$ for all $P_1 \in t(1-2z), v - c - tz]$, iff $\mu \in \left[0, \frac{2tz}{v-c-t}\right]$;
(iv) $\pi_1(P_1)$ is a concave function of P_1 for $P_1 \leq t(1-2z)$. It takes its maximum over this range either at $P_1 = \frac{1}{2}(t + \mu(v-c))$ if $z \leq \frac{1}{4} - \mu\frac{v-c}{4t}$ (unconstrained maximum) or at $P_1 = t(1-2z)$ if $z \geq \frac{1}{4} - \mu\frac{v-c}{4t}$ (corner).

When Firm 1 is the big firm, case (iv) above is irrelevant and for low enough values of μ, if $\mu \in 0, \min\{\frac{2c}{v+c-t}, \frac{2tz}{v-c-t}\}]$, Firm 1 's best reply to $P_2 = v - c$ is clearly $P_1 = v - c$.

When Firm 1 is the small firm and $\mu \in 0, \min\{\frac{2c}{v+c-t}, \frac{2tz}{v-c-t}\}]$, the only profitable deviation is towards a price $P_1 < t(1-2z)$ where it poaches some of its rival's old customers (see (iv) above). Comparing profits in that case to profits at the candidate equilibrium, one obtains that a deviation is not profitable[31] iff $z \geq \underline{z}$ where, denoting $C = c/t$,

$$\underline{z} = V - \frac{1}{2}\sqrt{4V^2 - \frac{(1-\mu(V-C))^2}{1-\mu}}.$$

From Assumption 2, $V > 1 + C$. We have $\underline{z} < \frac{1}{4} - \mu\frac{v-c}{4t} < 1/2$ iff $V > \frac{(5-\mu)(1+\mu C)}{8-3\mu-\mu^2}$, what holds always true under Assumption 2. By a a similar argument as in the proof of Lemma 2, $z \geq \underline{z}$ is a necessary and sufficient condition for a deviation by (small) Firm 1 to be unprofitable.

A similar argument shows that there exists $\overline{z}$, where $\underline{z} + \overline{z} = 1$, such that $z \geq \overline{z}$ is a necessary and sufficient condition for a deviation by Firm 2 be unprofitable.

[31] The argument is basically the same as in the proof of Lemma 2.