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## To cite this version:

Dominique Raynaud. Linear perspective in Masaccio's Trinity fresco: Demonstration or selfpersuasion?. Nuncius, 2003, 18 (1), pp.331-344. halshs-00005538

HAL Id: halshs-00005538
https://shs.hal.science/halshs-00005538
Submitted on 14 Nov 2005

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# Linear perspective in Masaccio's Trinity fresco: Demonstration or self-persuasion? 

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#### Abstract

Summary: The present paper discusses some new findings on the topic of Masaccio's Trinity fresco perspective reconstruction. Some scholars have tried to reduce the fresco's anomalies with the help of photogrammetrical reproductions and computer analysis. On this basis, it has been argued that Masaccio used the normal case of costruzione legittima. It is very likely that Masaccio took the lines of the plane joining the abaci of the capitals as a guide for drawing the vanishing lines of the coffered vault. However, in doing so, he followed an erroneous method of construction disregarded till now. The aberrations thus discovered strongly support the view that Masaccio's Trinity fresco is not a model of linear perspective. The thesis that Masaccio designed the fresco with the help of a ground plan and elevations is dubious, and the search for the distance point is condemned to remain an unending problem.


keywords: Linear perspective, Masaccio's Trinity, critical analysis.

Riassunto: Questo articlo discute nuovi risultati sul tema della ricostruzione prospettica della Trinità di Masaccio. Alcuni hanno cercato di ridurre le anomalie dell'affresco con l'aiuto di riproduzioni fotogrammetriche e analisi per computer. Su questa base, hanno proposto che Masaccio utilizzò il caso usuale della costruzione legittima. È molto probabile che Masaccio abbia preso le linee del piano unendo gli abachi dei capitelli come guida per disegnare le linee di fuga della volta a cassonetti. Così facendo, Masaccio seguì un metodo falso, trascurati fino ad allora. Le aberrazioni così scoperte accreditano l'idea che la Trinità di Masaccio non è un caso esemplare di prospettiva lineare. La tesi secondo la quale Masaccio avrebbe disegnato l'affresco con pianta ed alzati è dubbia e la ricerca del punto di distanza è condannato a rimanere un problema senza soluzione.

Parole-chiave: Prospettiva lineare, Trinità, Masaccio, analisi critica.

## 1. Introduction

The Trinity fresco at Santa Maria Novella in Florence, painted by Masaccio around 14251428, is a fresco of large dimensions ( 667 by 317 cm ) that allows one to make perspective tests in very good conditions. As a result, it has been the subject of a wide range of publica-
tions in art and perspective techniques history ${ }^{1}$. Many historians of art, like Panofsky ${ }^{2}$ and others, assumed the perspective correctness of the Trinity fresco-probably following Vasari's Vite. John White says for instance: "The foreshortening of the architecture, in accordance with the principles of artificial perspective, is accurate both in the diminution of the coffering and in the single vanishing point which lies slightly below the plane on which the donors kneel ${ }^{1 "}$. However, the last accurate research on the geometrical aspects of the fresco led to somewhat unexpected conclusions. Field et al. ${ }^{4}$ noticed many geometrical anomalies, in direct contrast to previous art historians' judgment.

## 2. RECENT OUTCOMES

[^0]After the date of this key publication, several studies have appeared ${ }^{5}$. In the present paper we will not discuss every one of them, but only the ones that differ most one from the other. ${ }^{6}$

Hoffmann's communication to the 4th ILabHS is slightly adapted from his previous article. This work offers a remarkable effort to analyse and confront all the major publications concerned with the geometrical analysis of the fresco. The author then presents his own work. At the very beginning, in line with Field et al., Hoffmann gives an account of some errors of construction (for instance, the vanishing lines do not precisely meet in a single point). But the major problem with the fresco is much more serious: all the authors having tried a reconstruction do not agree on the position of the distance point. They locate it at a distance varying from 210.5 cm to 894.2 cm , which is properly baseless according to the rules of linear perspective.

Hoffmann presents the problem in these terms: "There are only two explanations for such a chaotic scientific situation: [either] the fresco has not been constructed with the help of linear perspective or the methods of analyzing the perspective are not worth anything" ${ }^{17}$. He

[^1]then opts for the hypothesis that reconstruction methods are faulty, and develops an accurate analysis of the Trinity fresco based on photogrammetrical reproductions and computer analysis. Using a four-point reconstruction, he obtains a quite good correspondence between the sinopia's lines and the theoretical scheme. Fig. 1 shows the four-point reconstruction superimposed on the photogrammetrical reproduction ${ }^{8}$. The conclusion reached with the help of new technologies is clear-cut: "All in all, there is enough reason to believe that Masaccio constructed the perspective of the Trinity according to the normal case of the costruzione legitti$m a^{\prime \prime}$. Thus Hoffmann returns to the traditional judgement of correctness, in clear contradiction with the statement of Field et al. What's the basis for such a discrepancy of view? The present article aims to make an attempt to answer the question.

Hoffmann maintains in fact a threefold thesis:

1) We have to rethink and propose a new scheme for the vault ribs construction.
2) With this method, we reach the result that Masaccio's used costruzione legittima.
3) Using plans and elevations, we may calculate the viewing distance ( 452 cm ).

Let us consider Hoffmann's thesis without being too critical. On the one hand, he considers that the fresco has to be studied geometrically ${ }^{10}$ and tries to show that Masaccio took the lines of a lower plane (joining the abaci of the capitals) as a guide for drawing the vanishing lines of the coffered vault. On the other hand, Hoffmann understands by costruzione legitti-

[^2]$m a^{11}$ either the "viewing beam method" (Alberti) or the "distance point method" (Piero della Francesca). It is common knowledge that both of them lead to the same results, but he makes some attempts in order to show that the second method fits better with Masaccio's perspective scheme. Let us now examine precisely each one of these points.

## 3. The vault ribs construction

In this bounded context, the main contradiction between Hoffmann and Field et al. concerns the construction method as such. In Fig. 2, we just have traced Hoffmann's diagram, adding a new lettering for the convenience of the discussion. Lines $A F, J F, K F \ldots B F$ (images of orthogonals) converge at the vanishing point $F$. Lines $A B, A_{1} B_{1}, A_{2} B_{2} \ldots C^{\prime} D^{\prime}$ (images of transversals) show a perfect foreshortening, for the diagonal $B C^{\prime}$ covers all the intersections of orthogonals with transversals.

This perspective scheme clearly conflicts with the previous one gathered from in situ measurements of the arch divisions:
"[Masaccio] seems to have changed his mind about the importance of this simplification [i.e. equal arcs]. For the lowest division [...] gives an arc that subtends about $30^{\circ}$ at the centre of the circle. The three remaining arcs, which are equal to one another, each subtend about $20^{\circ} .{ }^{12}$

In Hoffmann's reconstruction, the lenght of the arcs $A J^{\prime}, J^{\prime} K^{\prime}, K^{\prime} L^{\prime}, L^{\prime} M^{\prime}$ depends on the perspective reduction of the square $A B C^{\prime} D^{\prime}$ inner divisions. It is then possible to calculate the values of the angles subtending those arcs in function of the perspective parameters. Consider the orthogonal coordinate system ( $\mathrm{O}, x, y$ ). F being the point of coordinates $(0,0), M$-i.e. the

[^3]centre of the circle of the front arch-the point of coordinates $(311.2,0)$, and $R$ the radius of the circle $R=105.8$, the equation of the circle is:
$$
(x-311.2)^{2}+y^{2}=105.8^{2}
$$

On the other hand, given that $A, J, K, L, M$ are equidistants points, equations of straight lines $F A, F J, F K \ldots$ are easy to find. For instance, $F J$ is as $y=0.255 x$.

The searched points $J^{\prime}, K^{\prime}, L^{\prime} \ldots$ being both on the circle and on straight lines, their coordinates are solutions of a system of two equations. The point $J^{\prime}\left(x_{J^{\prime}}, y_{J^{\prime}}\right)$ has to fit:

$$
\left\{\begin{array}{l}
(x-311.2)^{2}+y^{2}=105.8^{2} \\
y=0.255 x
\end{array}\right.
$$

Replacing $y$ in the first equation, we obtain:

$$
(x-311.2)^{2}+(0.255 x)^{2}=105.8^{2}
$$

Then:
$1.065 x^{2}-622.4 x+85651.8=0$
The positive solution of this second degree equation is:
$x_{J^{\prime}}=362.6$
The second equation gives:
$y_{J^{\prime}}=92.5$
Consequently:
$J^{\prime}(362.6,92.5)$
We may get the coordinates of the other points by the same method:

| $A$ | $(311.2$, | $105.8)$ |
| :--- | ---: | ---: |
| $J^{\prime}$ | $(362.9$, | $92.5)$ |
| $K^{\prime}$ | $(393.2$, | $66.8)$ |
| $L^{\prime}$ | $(411.5$, | $33.7)$ |
| $M^{\prime}$ | $(417.0$, | $0)$ |

Knowing the coordinates of the points $A, J^{\prime}, K^{\prime} \ldots$ of the vault, we calculate the lenght of chords $A J^{\prime}, J^{\prime} K^{\prime} \ldots$ by the formula of euclidean distance. For instance, we get from the coordinates of $A$ and $J^{\prime}$ that:

$$
d_{A J^{\prime}}=\sqrt{\left(x_{A}-x_{J^{\prime}}\right)^{2}+\left(y_{A}-y_{J^{\prime}}\right)^{2}}=53.1 \mathrm{~cm}
$$

Given the formula of a chord, we have, in the case of $\alpha=\angle A M J^{\prime}$ :

$$
\alpha=2 \sin ^{-1}\left(\frac{d_{A J^{\prime}}}{2 R}\right)=29^{\circ} 04^{\prime}
$$

We thus fix precisely the angles subtended by all chords, according to Hoffmann's scheme. Let us now present the results in a tabular form:

## Field et al. Hoffmann

| $\alpha$ | $\angle A M J^{\prime}$ | $=$ | $\sim 30^{\circ}$ |
| :--- | :--- | :--- | :--- |
|  |  | $29^{\circ} 04^{\prime}$ |  |
| $\beta$ | $\angle J^{\prime} M K^{\prime}$ | $=$ | $\sim 20^{\circ}$ |
| $\gamma$ | $\angle K^{\prime} M L^{\prime}$ | $=$ | $\sim 20^{\circ} 46^{\circ}$ |
| $\delta$ | $\angle L^{\prime} M M^{\prime}$ | $=$ | $\sim 20^{\circ}$ |
|  |  | $10^{\circ} 36^{\circ}$ |  |
|  |  |  |  |

We observe that the calculated angular values according to Hoffmann's scheme match quite well with the angular values independently measured by Field et al. on the fresco. Given that the maximum difference between theoretical and observed values is of one degree and a half ${ }^{13}$, and that Field et al. speak only in terms of "about $30^{\circ} \ldots$ about $20^{\circ{ }^{\circ 14}}$, we have every reason to think that Hoffmann is right when maintaining that Masaccio took the lines FA, FJ, $F K . . . F B$ to set the vault ribs up ${ }^{15}$. That is the best part in Hoffmann's analysis.

[^4]
## 4. Masaccio's use of costruzione legittima

Subsequently, Hoffmann moves towards the conclusion that Massacio used the normal case of the costruzione legittima in designing the fresco. Nevertheless, the application of this perspective scheme remains highly questionable.

In order for the diagonal $B C^{\prime}$ on Fig. 2 to make sense, the two sets of lines $A B, A_{l} B_{l}$, $A_{2} B_{2} \ldots$ and $J^{\prime} F, K^{\prime} F, L^{\prime} F \ldots$ should be located on the very same plane. This is not the case. On that point, Hoffmann's thesis lacks consistency, because he confuses the operations done on the horizontal plane and on the cylinder. He says:

[^5]Suppose first that transversals $A B, A_{1} B_{1}, A_{2} B_{2} \ldots C^{\prime} D^{\prime}$ are placed on the horizontal plane $A B C^{\prime} D^{\prime}$ while orthogonals $J^{\prime} F, K^{\prime} F, L^{\prime} F \ldots P^{\prime} F$ belong to the barrel vault cylinder. In that context, the diagonal $B C^{\prime}$ has neither precise geometrical meaning nor spatial location.

Suppose now the diagonal $B C^{\prime}$ to have a geometrical signification. We then need to reduce lines $J^{\prime} F, K^{\prime} F \ldots P^{\prime} F$ into $J F, K F \ldots P F$ of the horizontal plane $A B C^{\prime} D^{\prime}$. In this context, the lines $J F, K F \ldots P F$ are baseless, because they are not visible in the fresco. We should notice here that transversal and orthogonal lines of the coffered vault are the only visible lines of the fresco that can inform us about Masaccio's method of foreshortening ${ }^{17}$. Being absent on both the painting and the sinopia, one can draw them anywhere necessary in order to prove the perspective correctness. The reconstruction thus falls into an assumption-not a demonstration. Therefore, each alternative leads to incongruities.

The curvilinear trapezoids that Hoffmann claims to be the coffers of the barrel vault are ${ }^{16}$ HOFFMANN, The Trinity of Masaccio, op. cit., p. 6.
${ }^{17}$ Except the lines drawn from the abaci of the capitals. But these ones run in a somewhat erratic manner so that right abacus and left abacus provides us with a distance point varying from simple to double (see section 5 below).
disposed in a way that roughly defies either the common architectural patterns or the rules of linear perspective.

On the one hand, on can assume the Trinity to be a correct perspective. The coffers of the vault then appear to be of variable width. The circle being of perimeter $C$ and radius $R=$ 105.8 cm , and given the angular values: $\alpha=29^{\circ} 04^{\prime}, \beta=21^{\circ} 46^{\prime}, \gamma=20^{\circ} 36^{\prime}, \delta=18^{\circ} 34^{\prime}$, we may deduce the lenght of all corresponding arcs by:

$$
X Y=\frac{\alpha}{2 \pi} C, \quad \alpha=\angle X M Y
$$

Taken on the front arch, the measures are:

```
AJ' = 53.6 cm
J'K
K'L'}=38.0\textrm{cm
L'M'}=34.3\textrm{cm
```

But this pattern is somewhat unfounded with regard to the models of Renaissance architecture -and, beyond, of Roman and neo-classical architecture. The coffers that we can know through the history of architecture are always of a regular shape and almost always square ${ }^{18}$.

Let us suppose, on the other hand, that Masaccio's Trinity represents a regular coffering (we repeat: there is not a single example of variable rectangular coffers). Such a possibility indicates a serious error of construction or-to say the least-a high degree of "independence" in relation to the usual proceedings of perspective. This gap is probably the point that best invalidates the idea that Masaccio designed perspective with the help of plans

[^6]and elevations ${ }^{19}$. If he did so, he had never given a variable width to the coffers of the barrel vault. There is, in fact, only one method (known from Euclid's Elements, Book III, 30) to draw the longitudinal division of the vault in correct perspective:
"Dividing a semicircular arc into eight equal parts, or a quadrant into four, is a very simple mathematical task, merely involving repeated bissections. Having divided his semicircle, or quadrant, Masaccio could then have joined the points marking the divisions to the centric point, thus obtaining the centre lines of the longitudinal ribs. ${ }^{120}$

As we can see, on the comparative Fig. 3, the above construction is quite different from the one assumed by Hoffmann. The fact still remains that Masaccio did not use this basic-and nonetheless correct-method. Indeed, in correct perspective, the angles $A M J^{\prime \prime}$, $J^{\prime \prime} M K^{\prime \prime}, K^{\prime \prime} M L^{\prime \prime} \ldots P^{\prime \prime} M B$ are all equal to $\frac{\pi}{8}=22^{\circ} 30^{\prime}$ :

## Hoffmann Exact values

| $\alpha$ | $\angle A M J^{\prime}$ | $=$ | $29^{\circ} 04^{\prime}$ |
| :--- | :--- | :--- | :--- |

$C$ being the perimeter of the circle corresponding to the front arch, we may calculate the length of an arc $X Y$ in correct perspective:

$$
X Y=\frac{1}{16} C=41.5 \mathrm{~cm}
$$

The maximum difference between Hoffmann's scheme and exact values is reached for the outermost points $J^{\prime} J^{\prime \prime}$ and $P^{\prime} P^{\prime \prime}$ for which we have: $J^{\prime \prime} J^{\prime}=P^{\prime} P^{\prime \prime}=12.1 \mathrm{~cm}$. This metric dif${ }^{19}$ HOFFMANN, The Trinity of Masaccio, op. cit., pp. 11-12, admits that Masaccio used plans when creating this perspective: "It is fairly obvious that one of those plans had to be the ground plan [...] To this [Masaccio] added either the frontal elevation or the side elevation. But there is reason to believe that Masaccio actually used both the frontal elevation and the side elevation". Nothing is obvious here, except the need for founding this assumption.
${ }^{20}$ FIELD et al., The perspective scheme, op. cit., p. 49.
ference is much too important to be considered an accidental error: the method as such is wrong. Therefore, by ruling the coffers of the vault by the inner divisions of the square $A B C^{\prime} D^{\prime}$, Masaccio took an erroneous shortcut to the problem, in a way amazingly much more complicated than the method he actually had to apply. And that is, in fact, the conclusion we are in position to deduce from Hoffmann's interesting analysis.

## 5. The determination of the viewing distance

After he had become convinced that Masaccio used costruzione legittima, Hoffmann takes a new view on the difficult problem of the viewing distance determination. Let us put aside the fact that the use of ground plan and elevations is not at all established in Masaccio's work: this assumption is necessary for fixing any distance point. The pattern of the coffered vault seems to be the only way to calculate the viewing distance of the fresco ${ }^{21}$. According to Hoffmann's modelling, the distance point would have be exactly at 452 cm of the picture plane. This intermediate solution-remember that the previous determinations covered a range from 210.5 cm to 894.2 cm -still remains dubious, because it is based on the line $B C^{\prime}$, that is the most feeble mark of the demonstration ${ }^{22}$ (see section 4 above).

There is in fact only one geometrical method to determine the distance point from the visible lines of the fresco. Consider all the points defining the curvilinear trapezoids of the barrel vault (Fig. 4). In order to replace line $B C^{\prime}$ by a consistent diagonal, we have first to transfer all the points $A_{n}, J_{n}^{\prime}, K_{n}^{\prime}, L_{n}^{\prime} \ldots$-whatever the indices-onto a horizontal plane tangent to the vault cylinder by the segment $M^{\prime} M_{l}^{23}$. For the sake of clarity, just consider the points $A, J^{\prime}$,

[^7]$K^{\prime}, L^{\prime}, M^{\prime}$ belonging to the frontal arch. We know the width of all the coffers-that is to say, the lenght of all the consecutive arcs $X Y$ of the arch. We need now to unroll the circle onto the horizontal tangent to the circle $A M^{\prime} B$ by the point $M^{\prime}$. To the source-points $J^{\prime}, K^{\prime}, L^{\prime} \ldots$ correspond the image-points $J^{\prime \prime \prime}, K^{\prime \prime \prime}, L^{\prime \prime \prime} \ldots$ Nonetheless, the above arc values equal the ones of the corresponding straight segments of the new diagram. So we have:
\[

$$
\begin{aligned}
& \overline{A^{\prime \prime} J^{\prime \prime \prime}}=53.6 \mathrm{~cm} \\
& \overline{J^{\prime \prime} K^{\prime \prime \prime}}=40.2 \mathrm{~cm} \\
& \overline{\overline{K^{\prime \prime} L^{\prime \prime \prime}}}=38.0 \mathrm{~cm} \\
& \overline{L^{\prime \prime \prime} M^{\prime}}=34.3 \mathrm{~cm}
\end{aligned}
$$
\]

All those points generate the orthogonals $F A^{\prime \prime \prime}, F J^{\prime \prime \prime}, F K^{\prime \prime \prime} \ldots$ while the arcs summits $M^{\prime}$, $M_{1}, M_{2} \ldots$ fix the height of the transversals $A^{\prime \prime \prime} B^{\prime \prime \prime}, A_{1}^{\prime \prime \prime} B_{1}^{\prime \prime \prime}, A_{2}^{\prime \prime \prime} B_{2}^{\prime \prime} \ldots$ We can draw the grid resulting from the intersection of the two set of lines, and trace the diagonals of all squares. So doing, we immediatly notice that the network of diagonals is a convex one, Fig. 4. The lines $A_{2}^{\prime \prime \prime} K^{\prime \prime \prime}, A_{3}^{\prime \prime \prime} L^{\prime \prime \prime} \ldots$ are not straight lines but broken lines, thereby not allowing us to fix any distance point.

If the only available method for determining the viewing distance fails, the problem consequently has no solution. This supplies us with another reason to think that, once this perspective problem being solved, Masaccio's fresco cannot be considered as a model of a linear perspective. And this is why, when Hoffmann says:

[^8]we have to choose, not the second, but the first alternative.

[^9]
## 5. Conclusions

The sum of arguments expounded in this article strongly supports the view that Masaccio designed the vaulted space of the Trinity in a somewhat empirical way. As Hoffmann has noticed, it is very likely that Masaccio took the lines of the horizontal plane joining the abaci of the capitals as a guide for drawing the vanishing lines of the coffered vault. But, so doing, he followed an erroneous method of perspective construction. This painting of course conveys a very convincing sense of depht, but it is not a linear perspective-for the artistic illusion of depth works with little mathematical rigour. Consequently, it is not necessary to assume that the painter handled ground plan and elevations. Moreover, the search for the distance point is condemned to remain an unending problem, because the distance point is definable only if the rules of linear perspective are applied à la lettre.

The present observations contribute further-and at times go over and beyond-the results of Field, Lunardi and Settle. In contrast, some of the views defended by Hoffmann overevaluate Trinity's perspective correctness. It seemed important to me to draw attention to this fact before the rush for high technology takes a step forward once again.

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[^0]:    ${ }^{7}$ Before 1989, the main works on the perspective construction were: Guido J. KERN, Das Dreifaltigkeitsfresko von S. Maria Novella. Eine perspektivisch-architektur geschichtliche Studie, «Jahrbuch der königlich preussischen Kunstsammlungen », 24, 1913, pp. 36-58; Piero SANPAOLESI, Brunelleschi, Milano, Edizioni per il Club del Libro, 1962; Horst W. JANSON, Ground plan and elevation of Masaccio's Trinity fresco, in Douglas FRASER et al., eds., Essays in the History of Art presented to Rudolf Wittkower, London, Phaidon, 1967, pp. 8388; Joseph POLZER, The anatomy of Masaccio's Holy Trinity, « Jahrbuch der Berliner Museen », 13, 1971, pp. 18-59. Further references are given in the article by FIELD, LUNARDI, SETTLE, quoted note 4.
    ${ }^{2}$ Erwin PANOFSKY, Die Perspektive als "symbolische Form", « Vorträge der Bibliothek Warburg », 4, 19241925, pp. 258-331.
    ${ }^{3}$ John WHITE, The Birth and rebirth of pictorial space, London, Faber and Faber, 1967, p. 139.
    ${ }^{4}$ Judith V. FIELD, Roberto LUNARDI, Thomas B. SETTLE, The perspective scheme of Masaccio's Trinity fresco, «Nuncius», 4 (2), 1989, pp. 31-118. They say, p. 34: "The very success of the Trinity fresco in presenting space that seems as real as the figures that inhabit it may explain why so many scholars have taken the perspective scheme for granted". This critical view has been summarised on many occasions. Judith V. FIELD, The invention of infinity: Mathematics and art in the Renaissance, Oxford, Oxford University Press, 1997, p. 72, says: "There are serious departures from mathematical correctness in Masaccio's Trinity fresco". She also enhanced this statement in the 4th ILabHS in Florence: "The lines of the edges of the ribs do not meet so neatly [...] So the ribs do not really provide strong evidence for Masaccio having understood the properties of what Alberti, writing about ten years later, was to call the 'centric point'", Judith V. FIELD, What mathematical analysis can tell us about a fifteenth-century picture? «Art, science and techniques of drafting in the Renaissance, 4th ILabHS », working paper, Florence, 24 May-1 June 2001.

[^1]:    ${ }^{5}$ Martin KEMP, The science of art. Optical themes in Western art from Brunelleschi to Seurat, London / New Haven, Yale University Press, 1990, draws up an inventory of at least six errors but supports the idea of a manipulation of ground plan and elevations. A sophisticated system has been proposed by Jane A. AIKEN, The perspective construction of Masaccio's Trinity fresco and medieval astronomical graphics, « Artibus et Historiae », 31, 1995, pp. 171-187. She postulates that Masaccio obtained the diminution of the vault ribs with the help of an astrolabe and stereographic projection. Nevertheless, it is highly questionable how much the orthographic and stereographic projections of the astronomers were "readily available sources to Masaccio and Brunelleschi", p. 173. The length and complexity of the procedure proves an obvious lack of proportion between means and ends, so that one could wonder whether so sophisticated a technique has ever been used. See also: Volker HOFFMANN, Masaccios Trinitätsfresko: Die perspektivkonstruktion und ihr Entwurfsverfahren, « Mitteilungen des Kunsthistorischen Institutes in Florenz », 40, 1996, pp. 42-77; Rona GOFFEN, ed., Masaccio's Trinity, Cambridge, Cambridge University Press, 1998.
    ${ }^{6}$ That is to say, on the one hand, FIELD et al., The perspective scheme of Masaccio's Trinity fresco, op. cit., and, on the other hand, HOFFMANN, Masaccios Trinitätsfresko, op. cit., Idem, The Trinity of Masaccio: perspective construction-isometric transformation-coordinate system, «Art, science and techniques of drafting in the Renaissance, 4th ILabHS », working paper, Florence, 24 May-1 June 2001.
    ${ }^{7}$ Volker HOFFMANN, Brunelleschi's invention of linear perspective: The fixation and simulation of the optical view, «Art, science and techniques of drafting in the Renaissance, 4th ILabHS », working paper, Florence, 24 May-1 June 2001, s.p. [p. 2].

[^2]:    ${ }^{\overline{8}}$ After having unsuccessfully tested a three-point reconstruction (Entwurf / Dreipunkte-Rekonstruktion) the author experiments a four-point reconstruction (Ausführung / Vierpunkte-Rekonstruktion) that corresponds to a superimposition of the photogrammetry and of the perspective drawing, according to a fixed congruence of the following points: F (vanishing point), A and B (foreground ionic capitals abaci's top corners) and $\mathrm{C}^{\prime}$ (background left ionic capital abacus' top corner), HOFFMANN, Masaccios Trinitätsfresko, op. cit., p. 45.
    ${ }^{9}$ HOFFMANN, The Trinity of Masaccio, op. cit., p. 8, italics mine.
    ${ }^{10}$ "As the construction of linear perspective first and foremost centers around questions of projective geometry $[\ldots]$ the demonstration will have to be of a geometrical nature", HOFFMANN, The Trinity of Masaccio, op. cit., p. 2. Some scholars have expressed doubts on the ability of painters to use such mathematical procedures, but let us again follow the author as far as possible.

[^3]:    ${ }^{11}$ We will set aside some minor problems, as the semantic difficulty appearing when one applies to an early quattrocento painting the word costruzione legittima, that was used for the first time by Pietro Accolti in 1625. Other scholars have written at great length on this anachronism. See FIELD, The invention of infinity, op. cit., and Pietro ROCCASECCA, Il "modo optimo" di Leon Battista Alberti, «Studi di Storia dell'Arte », 4, 1993, pp. 245-262.
    ${ }^{12}$ FIELD et al., The perspective scheme, op. cit., pp. 49-50.

[^4]:    ${ }^{\frac{1}{13}}$ The mean difference between the four angular values, reported in situ, represents 22 mm on the arch line.
    ${ }^{14}$ FIELD et al., The perspective scheme, op. cit., pp. 49-50.
    ${ }^{15}$ So much so that we no longer need an "artistic" explanation for the difference between the angle $A M J^{\prime}$ and the three other ones: "The best explanation for the 'incorrect' positioning of the outermost ribs would seem to lie in a consideration of the surface geometry of Masaccio's picture. As painted, the ribs link Christ's hands with the volutes of the columns. Moreover, their closeness to the receding edges of the front abaci allows the eye to run easily along these lines, whereas the short receding edge might otherwise have been rather lost against the pattern of the vault", FIELD et al., The perspective scheme, op. cit., pp. 50-51. Masaccio could have followed the simple pattern enhanced by Hoffmann.

[^5]:    "The points created by the intersection of the lines running parallel to $A B$ with the vertical $[M F]$ become the centres of semicircles, the endpoints of which rest on $A C^{\prime}$ and $B D^{\prime}$. Semicircles and orthogonals create curvilinear trapezoids: these are the coffers of the barrel vault."16

[^6]:    ${ }^{18}$ Listed here are some examples of such coffered vaults. Type I (square-coffered barrel vault): Thermae in Rome, Nympheaum of Cicero's Villa in Formia, S. Andrea in Mantova by Alberti, S. Pietro in the Vatican by Bramante and Maderno, Gesù in Rome by Della Porta. Type II (flat ceiling with square coffering): Basilica of the Palace in Treves, S. Maria Maggiore in Rome. Type III (dome with square coffering): Pantheon in Rome, $S$. Maria in Carpitelli in Rome by De Rossi, Library project by Durand. Type IV (other Baroque geometrical patterns): S. Carlo ai Catinari in Rome (circles), S. Andrea al Quirinale in Rome by Bernini (hexagons), S. Carlo alle Quattro Fontane in Rome by Borromini (hexagons, circles, ovals and crosses).

[^7]:    ${ }^{21}$ FIELD et al., The perspective scheme, op. cit., pp. 42-43, Appendix 6 , have yet tried to determine the viewing distance by another method, using the vanishing lines of the abaci of the capitals. But they could not reach any viable result, because there is too deep a discrepancy between the distance obtained from the upper front abacus ( 594.4 cm ) and from the lower back abacus ( 345.7 cm ).
    ${ }^{22}$ HOFFMANN, Masaccios Trinitätsfresko, op. cit., Plates 1-6.
    ${ }^{23}$ Fig. 4 has been just traced from the previous photogrammetrical four-point reconstruction (Fig. 1) so avoiding the image of the fresco to be distorted.

[^8]:    "There are only two explanations for such a chaotic scientific situation: [either] the fresco has not been constructed with the help of linear perspective or the methods of analyzing the perspective are not worth anything" ${ }^{.24}$

[^9]:    ${ }^{24}$ HOFFMANN, Brunelleschi's invention of linear perspective, op. cit., s.p. [p. 2].

