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Threshold Effects of the Public Capital Productivity: An International Panel Smooth Transition Approach *

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Abstract

Using a non linear panel data model we examine the threshold effects in the productivity of the public capital stocks for a panel of 21 OECD countries observed over 1965-2001. Using the so-called "augmented production function" approach, we estimate various specifications of a Panel Smooth Threshold Regression (PSTR) model recently developed by Gonzalez, Teräsvirta and Van Dijk (2004). One of our main results is the existence of strong threshold effects in the relationship between output and private and public inputs: whatever the transition mechanism specified, tests strongly reject the linearity assumption. Moreover this model allows cross-country heterogeneity and time instability of the productivity without specification of an ex-ante classification over individuals. Consequently it is possible to give estimates of productivity coefficients for both private and public capital stocks at any time and for each countries in the sample. Finally we proposed estimates of individual time varying elasticities that are much more reasonable than those previously published.

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Abstract

A partir d'un modèle non linéaire en panel, nous examinons les effets de seuils de la productivité du capital public pour 21 pays de l'OCDE sur la période 1965-2001. En reprenant l'approche en termes de fonction de production utilisée notamment par Aschauer (1989), nous estimons différentes spécifications d'un modèle de panel à seuil à transition lisse (PSTR) récemment développé par Gonzalez, Teräsvirta et Van Dijk (2004). Nous mettons ainsi en évidence la présence d'importants effets de seuil dans la relation entre le stock de capital public et la productivité des facteurs privés. Le modèle proposé permet de modéliser à la fois l'hétérogénéité et l'instabilité temporelle des effets du capital public sur la production sans pour autant poser une classification des pays ex-ante. Par conséquent, nous obtenons des estimations des élasticités de la production au capital public, variables suivant les pays et dans le temps, qui apparaissent beaucoup plus raisonnables que celles qui avaient été jusqu'à présent proposées.

- *Key Words* : Public Capital, Panel Smooth Threshold Regression Models.
- *J.E.L Classification* : C82, E22, E62.

1 Introduction

This paper provides an international comparison of the patterns of productivity of public capital in OECD countries. Our methodology is based on an augmented production function where public capital is an additional input, beside private capital and labour following Aschauer (1989). This so-called "production function approach" authorizes the derivation of the elasticity of output with respect to public capital but is not exempt of critics. At an empirical level, it is well known that studies based on national time series data and a Cobb-Douglas production function, find very large output elasticities[1]. Three main reasons are usually suggested to explain such results (see Sturm, 1998 for a survey).

The first one is the potential reverse causation from income to public capital. Several solutions have been advanced in the literature in order to circumvent this problem. One of them consists in estimating a system of simultaneous equations: one equation for the production function and another equation explaining public capital by output (Demetriades and Mamuneas, 2000). Another solution is to use an instrumental variable approach or a generalized method of moments. It is for instance the case in Finn (1993), Holtz-Eakin (1994), Baltagi and Pinnoi (1995), Ai and Cassou (1995), Otto and Voss (1998) and more recently in Calderon and Serven (2004). Finally, Canning (1999) and Canning and Bennathan (2000) argue that the use of panel estimates allows to reduce the reverse causation bias and to identify the long run production function relationship.

The second major issue raised in the literature is the non stationarity of the data used in the augmented production function. If output, private input, and public capital data all tend to grow over time, it may result in a spurious correlation between output and public capital. Indeed several empirical studies, mainly conducted on American time series (Tatom, 1991; Sturm and Haan, 1995; Crowder and Himarios, 1997) have highlighted the fact that these are not stationary and not cointegrated, *i.e.* that the total factor productivity is a non stationary process.

The last major critic, which is specific to the panel data models, concerns the cross-

[1] Hence for the US economy Aschauer gives estimates for the return on public stock varying between 60% and 80% and such values have been considered too large to be credible by Gramlich (1994)

section heterogeneity. It is well known that biases appear when parameter heterogeneities among cross-sectional units are ignored (see Hsiao, 2003; Pesaran and Smith, 1995). In a production function approach, the assumption of a common elasticity of output with respect to public and private factors is a doubtful one for international or even regional panels. However studies based on a production function approach generally specify heterogeneity only using fixed or random individual effects (Evans and Karras, 1994; Holtz-Eakin, 1994). In this regard, there is no reason to expect the cross-section homogeneity of the other production function parameters and particularly of the public capital elasticity. For example, based on an analysis of the long run relationship between infrastructure stock and per capita income, Canning and Pedroni (1999, page 8), found "*evidence for considerable heterogeneity among the key parameter estimates across countries, which suggests that directly pooling certain parameters across countries may be misleading*". Canning (1999) or Canning and Bennathan (2000) authorize a particular form of elasticity heterogeneity by splitting their sample into two groups of countries according to the observed levels of income per worker in a given year. Assuming a Cobb-Douglas, they show that infrastructure elasticities of poorer countries are small and statistically insignificant, while they remain large and significant for richer countries. However, this solution implies that sub-samples, here poor and rich countries, are specified ex-ante and exogenously determined. Moreover, an individual is not allowed to switch between groups across periods.

For these reasons, it seems that this kind of heterogeneity can be advantageously specified in terms of Panel Threshold Regression (PTR) model. This model, proposed by Hansen (1999), implies that individual observations can be divided into homogeneous classes based on the value of an observed variable. More precisely, it assumes a transition from one regime to another according to the value of a threshold variable (the income per worker for instance). In a model with two regimes, if the threshold variable is below a certain value, called the threshold parameter, the productivity is defined by one equation, and it is defined by another equation if the threshold variable exceeds the threshold parameter. However, this is not entirely satisfying and one of the main drawback of this PTR model is that it allows only for a small number of classes, *i.e.* of productivity regimes. It is highly unlikely that international or regional time varying rates of returns on public stocks can be identified in a small set of constants.

A solution, adopted in this paper, is to use a Panel Smooth Threshold Regression (PSTR) model recently developed by Gonzalez, Teräsvirta and Van Dijk (2004) and Fok, Van Dijk and Franses (2004). Two interpretations of these models are possible. On one hand the PSTR can be thought of as a regime-switching model that allows for a small number of extreme regimes associated with the extreme value of a transition function and where the transition from one regime to the other is smooth. On the other hand, the PSTR model can be said to allow for a "continuum" of regimes, each one being characterized by a different value of the transition function. The logic is then similar to that developed in the standard univariate time series STAR[2] (Smooth Transition AutoRegressive) models except for the fact that the PSTR use panel specifications (individual effects) and is non dynamic, *i.e.* without lagged endogenous variable in the explanatory variables set. In our context, the PSTR allows cross-country heterogeneity and time instability of the elasticities without specification of an ex-ante classification over individuals. Consequently it is also possible to compute estimates of productivity coefficients for both private and public capital stocks at any time and for each countries in the sample.

Besides, the use of a PSTR model is likely to improve the reliability of the estimates with respect to the first issue previously mentioned, i.e. non stationarity. Phillips and Moon (1999) note that the consequences of the non stationarity in linear panel models are not equivalent to those generally pointed out in a time series context. More precisely, if the noise can be characterized as independent across individuals then "*by pooling the cross section and time series observations we may attenuate the strong effect of the residuals in the regression while retaining the strength of the signal* [given by the explanatory variables]. *In such a case we can expect a panel-pooled regression to provide a consistent estimate of some long run regression coefficient*" (Phillips and Moon, 1999, page 58). We may expect that the same kind of result would occurred in a nonlinear context. With respect to the reverse causation problem the advantages of the PSTR model are less clear. In order to properly test for weak exogeneity, it would be necessary

[2] See Granger and Teräsvirta (1993), Teräsvirta (1998) or Van Dijk, Franses and Teräsvirta (2002) for a survey on the STAR models.

to consider a multivariate nonlinear framework as in Jansen and Teräsvirta (1996). While a single equation non-linear model may lack efficiency if weak exogeneity does not hold, it has the great advantage of avoiding the specification of additional non-linear equations and possible misspecifications that would affect estimation of all equations in the system. Besides, instrumental variable methods are not actually available in a non linear panel context. We only can observe that our estimates of individual time varying elasticities are much more reasonable than those previously published, with for example an average elasticity of 6.6% for the United States where the public investment ratio is roughly equal to 5%[3].

The paper is organized as follows. In a first section, we discuss the threshold specification of the augmented production function. For that we consider a PSTR model that allows cross-country heterogeneous and time varying elasticities of the output with respect to the private and public inputs. The choice of the threshold variable, the linearity tests and the estimation of the parameters are successively presented. In a second section we present the data and the estimates of panel linear specifications of the augmented production function. In a third section, we present the results of the linearity tests and the estimates obtained with various panel threshold models. Finally, based on these PSTR estimates, we compute individual time varying estimates of the elasticities of output with respect to the public capital stocks. A last section concludes.

[3]In a simple growth model the first (and second) best optimal ratio of public investment is equal to the elasticity of output with respect to the public capital stock.

2 Threshold Effects in the Productivity of Public Capital Stocks

The basis of our empirical approach is exactly the same as that used by many authors since the seminal paper of Aschauer (1989), and more recently by Canning (1999), Canning and Bennathan (2000) or Calderon and Serven (2004) for developing countries. It consists in estimating the parameters of an augmented production function where public capital appears as an explanatory variable. We also follow most of studies in adopting a Cobb-Douglas specification of the production function and in assuming that the public capital services are proportional to the public capital stock. For a country $i = 1,..,N$ at a time $t = 1,.,T$, we consider two specifications of this augmented production function:

$$y_{it} - k_{it} = \mu_i + \alpha \ (n_{it} - k_{it}) + \beta \ g_{it} + v_{it} \tag{1}$$

$$y_{it} - k_{it} = \mu_i + \alpha \ (n_{it} - k_{it}) + \beta \ (g_{it} - k_{it}) + v_{it} \tag{2}$$

where y_{it} is the aggregate added value, k_{it} is private capital stock, g_{it} is public capital stock and n_{it} is employment. All variables are expressed in logarithm. In these log-linear models, the parameter α denotes the elasticity of the output with respect to the labor factor whereas the parameter β denotes the elasticity of the output with respect to the public capital stock. In both specifications, the endogenous variable is the productivity of private capital stock, *i.e.* $y_t - k_t$. This normalization, used by Aschauer (1989), makes it possible to consider various assumptions on the nature of the scale returns. The first equation (1) corresponds to the assumption of private factors' constant returns to scale (PFCRS). The second equation (2) corresponds to the assumption of overall constant returns to scale (OCRS). This specification corresponds to one of Aschauers' specifications in which he obtained a public capital elasticity of 39%, *i.e.* higher than the estimated private capital elasticity (26%). Finally, many studies since the first work done by Evans and Karras (1994) have highlighted the importance of individual effects in this kind of specification when panel data are considered. Consequently, we introduce fixed individual effects μ_i in order to capture all the timeless components of the productivity of the private capital stock. Thus, the specifications given by the equations (1) and (2) correspond to the general specification used in the literature devoted to the so-called "production function approach" and based on panel data models (Evans and Karras, 1994; Holtz Eakin 1994; Munnel, 1990; Pinnoi, 1994; Baltagi and Pinnoi, 1995; Garcia-Mila and Mc Guire, 1996; Canning, 1999; Canning and Bennathan, 2000 or Calderon and Serven, 2004 etc..).

As it was previously mentioned, in this study we propose to consider exactly the same framework as that studied in this literature, except the fact that we introduce non-linearity. One justification for that can be find in the network dimension of most public investments (in roads and highways, in sanitation and sewer systems, etc.). This network dimension implies a non linearity of the marginal productivity of public capital stocks as suggested for example by Fernald (1999) for the road public capital stock in the United States. In order to take into account this specificity, a solution consists in adopting a Panel Smooth Threshold Regression (PSTR) model as proposed by Gonzalez, Teräsvirta and Van Dijk (2004) and Fok, Van Dijk and Franses (2004). Let us consider the simplest case with two extreme regimes and a single transition function. The corresponding PSTR model for the PFCRS specification is defined as:

$$\begin{aligned} y_{it} - k_{it} &= \mu_i + \alpha_0 \ (n_{it} - k_{it}) + \beta_0 \ g_{it} \\ &\quad + [\alpha_1 \ (n_{it} - k_{it}) + \beta_1 \ g_{it}] \ g\left(q_{it}; \gamma, c\right) + \varepsilon_{it} \end{aligned} \tag{3}$$

where q_{it} denotes a threshold variable and where the error ε_{it} is assumed to be $i.i.d. \left(0, \sigma^2\right)$. The transition function $g\left(q_{it}; \gamma, c\right)$ is a continuous and bounded function of the threshold variable q_{it}. Gonzalez, Teräsvirta and Van Dijk (2004), following the work of Granger and Teräsvirta (1993) for the time series STAR models, consider the following transition function:

$$g\left(q_{it}; \gamma, c\right) = \left[1 + \exp\left(-\gamma \prod_{z=1}^{m} \left(q_{it} - c_z\right)\right)\right]^{-1}, \quad \gamma > 0, \quad c_1 \leq .. \leq c_m \tag{4}$$

where $c = (c_1, .., c_m)'$ denotes a m-dimensional vector of location parameters and where γ determines the slope of the transition function. This model can be rewritten as:

$$y_{it} - k_{it} = \mu_i + \Psi_0' W_{it} + \Psi_1' \, W_{it} \, g\left(q_{it}; \gamma, c\right) + \varepsilon_{it} \tag{5}$$

where $\Psi_j = \left(\alpha_j \ \beta_j\right)'$ for $j = (0, 1)$, $W_{it} = [(n_{it} - k_{it}) \ \ g_{it}]'$ in the case of the PFCRS specification and $W_{it} = [(n_{it} - k_{it}) \ \ (g_{it} - k_{it})]'$ in the case of the OCRS specification.

In our context, the PSTR model has three main advantages. The first one is that it allows the elasticities (in particular the public capital elasticity) to vary between countries (heterogeneity issue) but also with time (stability issue). It provides a parametric approach of the cross-countries heterogeneity and the time instability of the slope coefficients of the production function. More precisely, this model allows the parameters of the production function to change smoothly as a function of the threshold variable q_{it}. For instance if the threshold variable q_{it} is different from g_{it}, the elasticity of output with respect to the public capital stock g_{it} for the i^{th} country at time t is defined by the weighted average of the parameters β_0 and β_1 obtained in the extreme regimes:

$$e_{it}^{g} = \frac{\partial y_{it}}{\partial g_{it}} = \beta_0 + \beta_1 \, g\left(q_{it}; \gamma, c\right) \qquad \forall i, \forall t \tag{6}$$

where by definition of the transition function $\beta_0 \leq e_{it}^{g} \leq \beta_0 + \beta_1$, if $\beta_1 > 0$ or $\beta_0 + \beta_1 \leq e_{it}^{g} \leq \beta_0$ if $\beta_1 < 0$, since $0 \leq g\left(q_{it}; \gamma, c\right) \leq 1$, $\forall q_{it}$. The same conclusions are valid for the elasticities of labor and private capital inputs.

The second advantage of the PSTR model is that the value of the elasticity of public capital, for a given country, at a given date, can be different from the estimated parameters for the extreme regimes, *i.e.* parameters β_0 and β_1. As illustrated by the equation (6), these parameters do not directly correspond to the public capital elasticity. The parameter β_0 corresponds to the public capital elasticity only if the transition function $(q_{it}; \gamma, c)$ tends to 0. For instance, if the threshold variable corresponds to the stock of public capital, the parameter β_0 denotes the elasticity of the public capital stock only when this stock tends to 0. The sum of the parameters β_0 and β_1 corresponds to the public capital elasticity only if the transition function $(q_{it}; \gamma, c)$ tends to 1. Between these two extremes, the elasticity e_{it}^{g} is defined as a weighted average of the parameters β_0 and β_1. Therefore, it is important to note that it is generally difficult to directly interpret the values of these parameters that correspond to extreme situations. It is generally preferable to interpret (i) the sign of these parameters which indicates an increase or a decrease of the elasticity with the value of the threshold variable and (ii) the time varying and individual elasticity of the output with respect to the public capital stock (or other factor) given the equation (6).

Finally, this model can be analyzed as a *generalization* of the Panel Threshold

Regression (PTR) model proposed by Hansen (1999) and the panel linear model with individual effects. On the Figure 1 the transition function is displayed for various values of the parameter γ in the case $m = 1$. It can be observed that when the parameter γ tends to infinity, the transition function $g(q_{it}; \gamma, c)$ tends to the indicator function $\mathbb{I}_{(q_{it} \geq c)}$. Thus, when $m = 1$ and γ tends to infinity the PSTR model gives the PTR model:

$$y_{it} - k_{it} = \mu_i + \Psi_0' W_{it} + \Psi_1' W_{it} \, \mathbb{I}_{(q_{it} \geq c)} + \varepsilon_{it} \tag{7}$$

$$\mathbb{I}_{(q_{it} \geq c)} = \begin{cases} 1 \text{ if } q_{it} \geq c \\ 0 \text{ if } q_{it} < c \end{cases} \tag{8}$$

In this case, the public capital elasticity for the country i at time t, denoted e_{it}^g, switches between two extreme values given the level of the threshold variable: $e_{it}^g = \beta_0$ if $q_{it} < c$ and $e_{it}^g = \beta_0 + \beta_1$ if $q_{it} \geq c$. When $m > 1$ and γ tends to infinity, the number of identical regimes remains two, but the function switches between zero and one at c_1, c_2, *etc...* When γ tends to zero the transition function $g(q_{it}; \gamma, c)$ is constant and the model is the standard linear model with individual effects, *i.e.* with constant and homogenous elasticities. The model corresponds to equation (1) and the public capital elasticity is simply defined by $e_{it}^g = \beta_0$, $\forall i = 1, .., N$ and $\forall t = 1, .., T$.

Insert Figure 3. Transition Function with $m = 1$ and $c = 0$.

The transition function with $m > 1$ allows different types of changes in the elasticities and a very flexible parametrization. For instance, if $m = 2$, $c_1 = c_2 = c$ and γ tends to infinity, the transition function defines a three-regime model whose outer regimes are identical and different from the mid regime. Finally, Gonzalez, Teräsvirta and Van Dijk suggest a generalization with $r+1$ extreme regimes. This generalization, called general additive PSTR model, is defined as:

$$\begin{aligned} y_{it} - k_{it} &= \mu_i + \alpha_0 \; (n_{it} - k_{it}) + \beta_0 \; g_{it} \\ &\quad + \sum_{j=1}^{r} \left[\alpha_j \; (n_{it} - k_{it}) + \beta_j \; g_{it} \right] \; g_j \left(q_{it}; \gamma_j, c_j \right) + \varepsilon_{it} \end{aligned} \tag{9}$$

or equivalently

$$y_{it} - k_{it} = \mu_i + \Psi_0' W_{it} + \sum_{j=1}^{r} \Psi_j' \, W_{it} \, g_j \left(q_{it}; \gamma_j, c_j \right) + \varepsilon_{it} \tag{10}$$

where the transition function $g_j\left(q_{it};\gamma_j,c_j\right),\forall j=1,..,r$, depends on the slope parameters γ_j and on m location parameters c_j. Even if it is possible to consider a threshold variable different or each transition function, in our application, we consider that all threshold functions share the same threshold variable. If $m=1$ for all $j=1,..r$ and if all the parameters γ_j tend to infinity, the model collapses into a $r+1$ regime PTR model. In this generalization, if the threshold variable q_{it} is different from g_{it} , the elasticity of public capital for the i^{th} country at time t is defined by the weighted average of the $r+1$ elasticities β_j obtained in the $r+1$ extreme regimes:

$$e_{it}^g=\frac{\partial y_{it}}{\partial g_{it}}=\beta_0+\sum_{j=1}^{r}\beta_j\ g_j\left(q_{it};\gamma_j,c_j\right)\quad\forall i,\forall t \tag{11}$$

The expression of the elasticity is slightly different if the threshold variable q_{it} is a function of the public capital stock. For instance, if we assume that the threshold variable is equal to the logarithm of the public capital stock per worker, *i.e.* $q_{it}=g_{it}-n_{it}$, the expression of elasticity of output with respect to the public capital stock is then defined as:

$$\begin{aligned}e_{it}^g &= \frac{\partial y_{it}}{\partial g_{it}}=\beta_0+\sum_{j=1}^{r}\beta_j\ g_j\left[q_{it};\gamma_j,c_j\right]\\ &\quad+\sum_{j=1}^{r}\left[\alpha\ \left(n_{it}-k_{it}\right)+\beta\ g_{it}\right]\ \frac{\partial g_j\left[q_{it};\gamma_j,c_j\right]}{\partial g_{it}}\quad\forall i,\forall t\end{aligned} \tag{12}$$

where

$$\begin{aligned}\frac{\partial g_j\left[q_{it};\gamma_j,c_j\right]}{\partial g_{it}} &= \left[\gamma_j\times\exp\left(-\gamma_j\prod_{z=1}^{m}\left(q_{it}-c_{jz}\right)\right)\times\frac{\partial}{\partial q_{it}}\prod_{z=1}^{m}\left(q_{it}-c_{jz}\right)\right]\\ &\quad\times\left[1+\exp\left(-\gamma_j\prod_{z=1}^{m}\left(q_{it}-c_{jz}\right)\right)\right]^{-2}\end{aligned}$$

with $c_j=\left(c_{j1},..,c_{jm}\right)'$. If $m=1$, we have:

$$\frac{\partial g_j\left(q_{it};\gamma_j,c_j\right)}{\partial g_{it}}=\frac{\gamma_j\times\exp\left[-\gamma_j\left(q_{it}-c_j\right)\right]}{\left[1+\exp\left[-\gamma_j\left(q_{it}-c_j\right)\right]\right]^2}\quad j=1,..,r \tag{13}$$

Such an expression authorizes a variety of configurations for the relationships between the time varying public capital elasticity and the level of the threshold variable (the public capital stock for instance) as we will discuss in the next part. Nevertheless, in this threshold model, there are two main problems of specification. The first one consists in choosing the threshold variable. The second concern consists in testing the number of regimes or equivalently in testing the threshold specification.

2.1 Choice of the Threshold Variable

Few technical constraints are imposed to the choice of the threshold variable. In particular, the threshold variable can not be time invariant. Therefore, this choice is mainly an economic issue. If we want to asses the idea that the public investments have a *network character*, it implies that the threshold variable should be an indicator of the completion of the main networks. It is precisely the idea raised by Gramlich (1994) or Fernald (1999): the construction of the network boosts substantially the productivity and the output, but when the construction of the network is completed, the public capital is not exceptionally productive at the margin. In this perspective, a natural candidate for the threshold variable is the existing level of available public capital stock, *i.e.* $q_{it} = g_{it}$. However, this specification obviously captures the threshold effects due to a simple country size effect. For instance, if we consider the Luxembourg and the United States in a same panel, the threshold effects associated to the variable g_{it} will mainly reflect the differences of size and not the potential network effects in public investment productivity. So, in order to avoid these size effects, we propose here to consider a model in which the marginal productivity of public capital depends on the ratio of of public capital to private capital. Finally, we consider a lagged value of this ratio as a threshold variable in order to avoid a simultaneity issue since the private capital stock is a part of the endogenous variable of our regressions. In other words, our first specification of the transition function is based on the following threshold variable:

$$\text{Model A}: \quad q_{it} = g_{i,t-1} - k_{i,t-1} \tag{14}$$

Another choice for the threshold variable consists in using the lagged level[4] of private capital per worker, *i.e.* $q_{it} = k_{i,t-1} - n_{i,t-1}$. This second specification is related to the issue of the heterogeneity of the production function between "rich" countries and

[4]We can also use a more general specification where $q_{it} = y_{i,t-d}$. In this case, the choice of the lag d can be determined by a criterion as the residual sum of squared residuals as it is generally done in the literature devoted to SETAR models in time series.

"poor" countries as suggested by Canning and Bennathan (2000). However, in contrast with Canning and Bennathan, the heterogeneity in our PSTR model is endogenous in the sense that it is the threshold variable which determine the different regimes of productivity. Moreover, a country with low productivity in the beginning of the period can have a medium or high productivity at the end of the sample period. So, a second specification is given by:

$$\text{Model B:} \quad q_{it} = k_{i,t-1} - n_{i,t-1} \tag{15}$$

In both cases, in order to limit the influence of the choice of the measure units on the estimation of the location parameters in the transition function, we consider the deviations of the variable q_{it} from the international mean as threshold variable. So, for the models A and B, the threshold variables used in the estimation are respectively defined as[5]:

$$q_{it} = (g_{i,t-1} - k_{i,t-1}) - \left(\overline{g}_{-1} - \overline{k}_{-1}\right) \tag{16}$$

$$q_{it} = (k_{i,t-1} - n_{i,t-1}) - \left(\overline{k}_{-1} - \overline{n}_{-1}\right) \tag{17}$$

where $\overline{x}_{-1} = [1/N\,(T-1)] \sum_{i=1}^{N} \sum_{t=2}^{T} x_{it-1}$ for $x_{it} = \{k_{it}; n_{it}; g_{it}\}$.

2.2 Estimation of Parameters

The estimation of the parameters of the PSTR model consists in eliminating the individual effects μ_i by removing individual-specific means and then in applying non linear least squares to the transformed model[6]. Let us consider a production function under PFCRS with one threshold function $(r = 1)$ and one location parameter c:

$$y_{it} - k_{it} = \mu_i + \Psi_0' W_{it} + \Psi_1' W_{it}\, g\,(q_{it}; \gamma, c) + \varepsilon_{it} \tag{18}$$

where $\Psi_j = \left(\alpha_j \;\, \beta_j\right)'$ for $j = \{0, 1\}$ and $W_{it} = [(n_{it} - k_{it}) \;\; g_{it}]'$. The estimation of the parameters is carried out in two steps. In the first step, the individual effects μ_i are eliminated by removing individual-specific means to the variables of the model. This first

[5] Obviously, the use of a centered threhold variable has an impact on the estimated location parameters of the transition function. However, our results show that this choice has no impact on (i) the estimated parameters, (ii) the estimated slope parameters and consequently (iii) the estimated individual elasticities of the ouput with respect to the public and private inputs.

[6] All the corresponding codes have been simultaneously developed under WinRats and Matlab 7.0 and are available upon request.

step is standard in linear models (within transformation), but it requires more careful treatment in the context of a threshold model. Let us denote $\widetilde{y}_{it} = y_{it} - k_{it} - \left(\overline{y}_i - \overline{k}_i\right)$ and $\widetilde{\varepsilon}_{it} = \varepsilon_{it} - \overline{\varepsilon}_i$. The explanatory variables must be transformed as follows. The vector W_{it} is simply transformed as $\widetilde{W_{it}} = W_{it} - \overline{W}_i$ where $\overline{W}_i = \left[\left(\overline{n}_i - \overline{k}_i\right)\ \overline{g}_i\right]'$. But the transformed explanatory variables in the second regime depends on the parameters γ and c of the transition function since:

$$\widetilde{Z}_{it}\left(\gamma, c\right) \ = W_{it}\, g\left(q_{it}; \gamma, c\right) - \overline{Z}_i\left(\gamma, c\right) \tag{19}$$

$$\overline{Z}_i\left(\gamma, c\right) = \frac{1}{T}\sum_{t=1}^{T} W_{it}\, g\left(q_{it}; \gamma, c\right) \tag{20}$$

Consequently, the matrix of transformed explanatory variables denoted $x^*_{it}\left(\gamma, c\right) = \left[\widetilde{W}'_{it} : \widetilde{Z}'_{it}\left(\gamma, c\right)\right]'$ depends on the parameters of the transition function. So, it has to be recomputed at each iteration. More precisely, given a couple (γ, c), the elasticities of the production function in the extreme regimes can be estimated by ordinary least squares, which yields:

$$\widehat{\Psi}\left(\gamma, c\right) = \left[\sum_{i=1}^{N}\sum_{t=1}^{T} x^*_{it}\left(\gamma, c\right) x^*_{it}\left(\gamma, c\right)'\right]^{-1} \left[\sum_{i=1}^{N}\sum_{t=1}^{T} x^*_{it}\left(\gamma, c\right)\ \widetilde{y}_{it}\right]$$

where $\widehat{\Psi}\left(\gamma, c\right) = \left[\widehat{\Psi}'_0\ \left(\gamma, c\right) \widehat{\Psi}'_1\left(\gamma, c\right)\right]'$ is conditional to the values $\left(\gamma, c\right)$. In a second step, conditionally to $\widehat{\Psi}\left(\gamma, c\right)$, the parameters of the transition function γ and c are estimated by NLS according to the program:

$$\left(\widehat{\gamma}, \widehat{c}\right) = \underset{\gamma \in \Gamma, c \in Q}{ArgMin} \sum_{i=1}^{N}\sum_{t=1}^{T} \left[\widetilde{y}_{it} - \widehat{\Psi}'\left(\gamma, c\right)\ \ x^*_{it}\left(\gamma, c\right)\right] \tag{21}$$

In this last step, the location parameter c is estimated by optimization on a given subset of the values of the transition variable q_{it}, .denoted Q. More generally, to ensure that a sufficient number of observations of the transition variable are available for estimation in each side of a given location parameter, the values of $min(c_{j,k})$ and $max(c_{j,k})$ are constrained by a trimming over the observations of $q^{(j)}_{it}$, *i.e.* the range for the initial $c_{j,k}$ leaves aside a certain number of smaller and greater observations of the transition variable. This constraint is particularly easy to interpret in a PTR model. In this case, it is obvious that it is undesirable for a threshold c_j to be selected which sorts too few observations into one or the other regime (Hansen, 1999). This choice of a trimming on the values of the location parameter c_j (and consequently on the slope parameters γ_j) is particularly important for the robustness of the individual estimates of the time varying and heterogeneous elasticities of the output with respect to the public and private inputs that constitute the main objective of our investigation. In time series threshold models (TAR, SETAR, STAR etc.) the choice of the subset Q is generally obtained by eliminating the smallest and largest 5%, 10% or 15% observations of the threshold variable q_{it}. However, this heuristic rule can not be directly used in panel context without being very careful. Indeed, in a panel, if we eliminate the largest 15% observations of the NT observations of the threshold variable, it may lead to withdraw one or more countries from the analysis of the transition mechanisms. For instance, lets us consider a panel with ten countries and assume that one country has very high values of q_{it} compared to the values observed for the others countries. In this case, if we eliminate 15% of the $10T$ sorted observations of q_{it}, it leads to eliminate all the observations of the country with the highest values of q_{it}. This exclusion of a country may have very strong consequences on the estimation of the transition mechanisms (location parameters and slope parameters), particularly in a panel with an important heterogeneity. In order to avoid this difficulty, we propose here to eliminate $T/2$ observations (and not a fixed proportion on the total NT observations) on the smallest and largest values of q_{it} in order to define the subset Q.

Finally, given $\widehat{\gamma}$ and $\widehat{c}$, it is possible to estimate the elasticities of the production function in the extreme regimes:

$$\widehat{\Psi}_j = \left(\widehat{\alpha}_j \ \widehat{\beta}_j\right)' = \widehat{\Psi}'_j \ (\widehat{\gamma}, \widehat{c}) \qquad j = 0, 1 \tag{22}$$

However, the convergence issue of this estimation procedure is greatly dependant upon the chosen starting values of γ and c. This is normally done by mean of a grid search, *i.e.* a selection of initial values for the slopes γ_j and the location parameters $\mathbf{c}_j$, $j = 1, \ldots, r$. Given these grids, *OLS* regressions are performed for all combinations of the initial values to estimate the corresponding β and α. The vector for which the residual sum of squares is minimum is then passed as a starting value for the realization of the second step of the estimation process described at the preceding point. Details on the choice of initial conditions can be found in Gonzalez, Teräsvirta and Van Dijk (2004).

2.3 Specification Tests

Gonzalez, Teräsvirta and Van Dijk propose a testing procedure in order (i) to test the linearity against the PSTR model and (ii) to determine the number, r, of transition functions, *i.e.* the number of extreme regimes which is equal to $r+1$. Lets us consider a model with only one location parameter $(m=1)$ and assume that the threshold variable q_{it} is known. Testing the linearity in the augmented production function under PFCRS (equation 3) can be done by testing $H_0 : \gamma = 0$ or $H_0 : \alpha_1 = \beta_1 = 0$. But in both cases, the test will be non standard since under H_0 the PSTR model contains unidentified nuisance parameters as it was the case in the Hansen's PTR model. This issue is well known in the literature devoted to the time series threshold models (Hansen, 1996). However, in the context of the PSTR model, Gonzalez, Teräsvirta and Van Dijk present an original solution similar to the solution proposed by Luukkonen, Saikkonen and Teräsvirta (1988) for time series models. It consists to replace the transition function $g\left(q_{it};\gamma,c\right)$ by its first-order Taylor expansion around $\gamma=0$ and to test an equivalent hypothesis in an auxiliary regression. If we consider the augmented production function under PFCRS (equation 1), we obtain:

$$y_{it} - k_{it} = \mu_i + \Psi_0' W_{it} + \Gamma_1' W_{it}\ q_{it} + \Gamma_2' W_{it}\ q_{it}^2 + .. + \Gamma_m' W_{it}\ q_{it}^m + \varepsilon_{it} \tag{23}$$

where $\Psi_0 = (\alpha_0\ \beta_0)'$, $W_{it} = [(n_{it} - k_{it})\ \ g_{it}]'$ and the parameter vectors Γ_i' are a multiple of the slope parameter γ (see Appendix A.1). Thus, testing the linearity against the PSTR model simply consists in testing $H_0 : \Gamma_1 = .. = \Gamma_m = 0$ in this linear panel model. If we denote SSR_0 the panel sum of squared residuals under H_0 (linear panel model with individual effects) and SSR_1 the panel sum of squared residuals under H_1 (PSTR model with two regimes), two statistics can be computed: a LM one and its F-version.

$$LM = TN\ \left(SSR_0 - SSR_1\right)/SSR_0 \tag{24}$$

$$LM_F = \left[\left(SSR_0 - SSR_1\right)/Km\right]/\left[SSR_0/\left(TN - N - mK\right)\right] \tag{25}$$

where K is the number of explanatory variables introduced in the production function (here $K = 2$). Under the null hypothesis, the LM statistic is distributed as a $\chi^2(mK)$ and the Fstatistic has an approximate $F(mK, TN - N - mK)$ distribution. In addition to these LM and LM_F tests, in this paper we propose to compute a $pseudo-LRT$ statistic[7] defined as:

$$LRT = -2\left[\log(SSR_1) - \log(SSR_0)\right] \tag{26}$$

having also a $\chi^2(mK)$ distribution under the null hypothesis.

The logic is similar when it comes to test the number of transition functions in the model or equivalently the number of extreme regimes. The idea is as follows: we use a sequential approach by testing the null hypothesis of no remaining nonlinearity in the transition function. For instance let us assume that we have rejected the linearity hypothesis. The issue is then to test whether there is one transition function ($H_0 : r = 1$) versus there is at least two transition functions ($H_1 : r = 2$). Let us assume that the model with $r = 2$ is defined as:

$$y_{it} - k_{it} = \mu_i + \Psi_0' W_{it} + \Psi_1' W_{it}\, g_1(q_{it}; \gamma_1, c_1) + \Psi_2' W_{it}\, g_2(q_{it}; \gamma_2, c_2) + \varepsilon_{it} \tag{27}$$

The logic of the test consists in replacing the second transition function by its first-order Taylor expansion around $\gamma_2 = 0$ and then in testing linear constraints on the parameters. If we use the first-order Taylor approximation of $g_2(q_{it}; \gamma_2, c_2)$, the model becomes:

$$\begin{aligned} y_{it} - k_{it} \;&=\; \mu_i + \Psi_0' W_{it} + \Psi_1' W_{it}\, g_1(q_{it}; \gamma_1, c_1) \\ &\quad + \Gamma_1' W_{it} q_{it} + \Gamma_2' W_{it} q_{it}^2 + .. + \Gamma_m' W_{it} q_{it}^m + \varepsilon_{it} \end{aligned} \tag{28}$$

and the test of no remaining nonlinearity is simply defined by $H_0 : \Gamma_1 = .. = \Gamma_m = 0$. Let us denote SSR_0 the panel sum of squared residuals under H_0, *i.e.* in a PSTR model with one transition function. Let us denote SSR_1 the sum of squared residuals of the transformed model (equation 28). As in the previous cases, three statistics LM, LM_F and pseudo LRT can be computed according to the same definitions by adjusting the

[7] Pseudo-LRT because SSR_1 is not the unconstrained residual sum of squares but the one afferent to a linearized version of the unconstrained model.

number of degrees of freedom[8]. The testing procedure is then the following. Given a PSTR model with $r = r^*$, we will test the null $H_0 : r = r^*$ against $H_1 : r = r^* + 1$. If H_0 is not rejected the procedure ends. Otherwise, the null hypothesis $H_0 : r = r^* + 1$ is tested against $H_1 : r = r^* + 2$. The testing procedure continues until the first acceptance of H_0. Given the sequential aspect of this testing procedure, at each step of the procedure the significance level must be reduced by a constant factor $\tau \in]0, 1[$ in order to avoid excessively large models. We postulate $\tau = 0.5$ as suggested by Gonzalez, Teräsvirta and Van Dijk (2004).

3 Data and Panel Linear Models

In this study, we consider a panel of 21 OECD countries over the period 1965-2001. Included countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States. As suggested by Hansen (1999), we consider a balanced panel since it is unknown if the results of estimation and testing procedures presented below extend to unbalanced panels. The data on private and public net capital stocks are drawn from Kamps (2004). Both series have been estimated according to the perpetual inventory method from the OECD series of private and public investments (Codes: IBV and IGV, OECD Analytical Database, 2002). One of the main advantages of these data is the international comparability of the definition used for the public sector. Indeed, in panel studies based on national sources the definition of the public sector underlying the investments or stocks series generally varies across countries. This lack of international comparability may have strong consequences when considering the threshold effects based on the level of the public capital stocks. On the contrary, the estimated net stocks proposed by Kamps are based on public investments series issued from a homogenous definition for all the countries of the panel[9]. It corresponds to the general government sector including the central, local and state government subsectors. Therefore, the estimated series of pub-

[8]The LM and pseudo LRT statistics have a chi-square distribution with mK degrees of freedom, whereas the F statistics has a $F\left(mK, TN - N - K\left(m + r + 1\right)\right)$ distribution.

[9]Expect for Japan and the United States, for which the stocks correspond to the public sector including the general government and nonfinancial public corporations.

lic capital stock corresponds to the government net capital stock in volume. Similarly, the estimated private capital stock corresponds to the private non residential stock in volume. Finally, the data for the real GDP are taken from the OECD Analytical Database (Code: GDPV) and the data for the total employment are issued from the OECD Economic Outlook (Code: ET). All the data, except employment, are expressed in 1995 prices and in billions of national currency units.

In order to asses the comparability of our data sets to the data sets used in previous studies, we first estimate the augmented production function in linear panel models. We present various estimates of the parameters of the production functions (1) and (2) in three specifications: (*i*) with fixed individual effects, (*ii*) with random individual effects or (*iii*) with fixed individual effects and time dummies. The results are reported in Table 1.

Insert Table 1. Public Capital Augmented Production Function.
Linear Panel Models with Fixed Effects and Year Dummies

Our results are globally similar to those reported in the literature based on a production function approach. First, the elasticity of output with respect to the public capital stock is largely significant and positive. Whatever the considered assumption on the scale returns, the estimated elasticity is particularly high and even superior to 0.30. Hence, our estimated elasticities are close to those generally reported in time series models. Indeed, since the seminal article of Aschauer (1989), many empirical studies based on this approach have yielded very high estimated elasticities on American data as well as on OECD data sets (see Gramlich, 1994 or Sturm, 1998 for a survey). The emblematic illustration of these "*stratospheric*" (Gramlich, 1994) estimated elasticities for the public capital input is the value of 0.39 obtained by Aschauer for the United States under the OCRS hypothesis. Second, the productive contributions of private factors are generally lower than the share of their respective remuneration in added value. As in Aschauer (1989), Ram and Ramsey (1989), Eisner (1994), Vijverberg and *alii.* (1997) or Sturm and De Haan (1995), the elasticity of private capital is lower than that of public capital in our OCRS specification. Third, our results are also similar to the results obtained in the literature based on panel data models. For instance, Kamps (2004) obtained an estimated elasticity of 0.31 for the public capital input and 0.72 for the labor input on the same sample when he used a group mean fully modified OLS estimator (Pedroni, 1999). Based on a smaller sample of seven OECD countries, Evans and Karras (2004) showed the importance of the introduction and the specification of the individual effects in order to capture a part of the heterogeneity of the production function. They found that when individual and time effects are introduced, the estimated elasticities for the public capital input are less important and even not significant. This is not the case in our sample. The introduction of fixed or random individual effects does not reduce the "stratospheric" estimated value of the public capital elasticity. However, we can observe that the introduction of time effects with fixed individual effects leads to a slight reduction of the estimated elasticity.

As observed in the literature, if we accept our estimates based on panel linear models as relevant, the implied annual marginal yields of public capital are then extremely high. Tatom (1991) or Gramlich (1994) calculated (starting from the elasticities estimated by Aschauer, 1989) that the annual marginal productivity of public infrastructures in the United States would range between 75% in 1970 and more than 100% in 1991. Thus, these results "*mean that one unit of government capital pays for itself in terms of higher output in a year or less, which does strike one as implausible*" (Gramlich 1994, page 1186). The issue is then to know if the introduction of threshold effects in the productivity of the public capital stock would allow estimating more reasonable rates of return on these stocks.

4 Panel Threshold Models

The first step consists in testing the log-linear specification of the production function against a specification with threshold effects. If the linearity hypothesis is rejected, it will be necessary, in a second step, to determine the number of transition functions required to capture all the non linearity of the augmented production function, or equivalently all the heterogeneity of the parameters of the production function. The results of these linearity tests and specification tests of no remaining nonlinearity are reported on Table 4. Given the definition of the threshold variable q_{it} (models A or B) and the specification of the returns to scale, four cases are considered. For each model we consider three specifications with one, two or three location parameters. For each specification, we compute the LM, $pseudo-LRT$ and LM_F statistics for the linearity tests ($H_0 : r = 0$ versus $H_1 : r = 1$) and for the tests of no remaining nonlinearity ($H_0 : r = a$ versus $H_1 : r = a + 1$). Since previous studies have documented that the F-version of the test has better size properties in small sample than the asymptotic χ^2 based statistic (Van Dijk, Teräsvirta and Franses, 2002), we only report the results of the F-version, denoted LM_F statistics[10]. The values of the statistics are reported until the first acceptance of H_0. For computational tractability, we limit our analysis to PSTR models with at most four transition functions.

Insert Table 2. LM_F Tests for Remaining Nonlinearity

The linearity tests clearly lead to the rejection of the null hypothesis of linearity of the relationships between the output and public and private inputs. The only exception is the production function considered under the assumption of PFRCS. In this case, the linearity assumption is not rejected when we consider an alternative with only one location parameter ($m = 1$). In all other cases, whatever the choice made for the threshold variable, the number of location parameters, the assumption on the returns to scale, the LM_F statistics lead to strongly reject the null $H_0 : r = 0$. The lower value of the LM_F statistic is obtained for the production function under PFCRS (model A) with two location parameters, but even in this case the value of the test statistic is largely below the critical values at standard levels. The results (not reported) are similar when we consider the other LM or $pseudo - LRT$ statistics. This first result implies that there is strong evidence that the relationship between output and the considered inputs (in particular public capital stocks) is non-linear. Our results are then compatible with the results obtained by Fernald (1999) for the road sector: for a variety of reasons, and maybe due to the network aspect of public investments in infrastructure, there is a strong non linearity in the productivity of these equipment and structures.

Thus, what could be the consequences of using a linear panel model in order to estimate the elasticity of public capital? Let us assume that one comes to estimate

[10] The values of the other statistics are available upon request.

the augmented production function with a homogenous linear model, *i.e.* a model in which the parameters of the production are assumed (wrongly) to be common to all the countries. This approach leads to ignore the heterogeneity of the productivity of the public capital and consequently to present an estimate which is, roughly speaking, a nonsensical average of heterogeneous rates of return. It is perhaps a reason why, for the same countries and for comparable linear specifications, the estimated elasticities obtained in linear panel models with fixed effects are generally lower than the ones reported in studies based on time series (Evans and Karras, 1994). Besides, a linear approach leads to ignore the potential changes of productivity regimes due to the non linearity and propose an estimate which could be analyzed, for a given country, as a nonsensical average of the different historical values of the productivity. This second source of mis-evaluation could have drastic consequences especially for industrialized countries in which the main networks of infrastructure are globally completed.

The specification tests of no remaining nonlinearity (see Table 2) lead to identify an optimal number of transition functions (or extreme regimes) in most of the cases. In all cases, the optimal number of transition functions is always inferior to the maximum number of transition functions authorized in the algorithm ($rmax = 4$). In other words, in a PSTR model, a small number of extreme regimes is sufficient to capture the non linearity of the technological relationships, or equivalently the cross-country heterogeneity and the time variability of the public and private factors productivity. Recall that a smooth transition model, even with two extreme regimes ($r = 1$), can be viewed as a model with an infinite number of intermediate regimes of productivity. The elasticities of inputs are defined at each date and for each country as weighted averages of the values obtained in the two extreme regimes. The weights depend on the value of the transition function (equation 4). So, even if $r = 1$, this model allows a "*continuum*" of elasticities (or regimes), with each one associated with a different value of the transition function $g(.)$ between 0 and 1. Thus, the choice of r is just a question of specification of the model. As previously discussed, it is one of the main advantages of this approach compared to the PTR model in which the number of regimes is fixed and limited.

Finally, in the PSTR model it is necessary to choose the number of location parameters used in the transition functions, *i.e.* the value of m. In the case of a model with at the most one transition function, Granger and Teräsvirta (1993) proposed a testing procedure which can be adapted in the case of the PSTR model for testing between $m = 1$ and $m = 2$. Except for this special case, there is no general specification test for the choice of m. However, the choice of m is not very important as long as we determine the corresponding number of transition function, denoted $r^*(m)$, which assures that there is no remaining nonlinearity in the model. As we will see, the model is so flexible that different models with different couples (m, r^*) give the same qualitative, but also quantitative, results when it comes to estimate the individual elasticities. In Table 3, for each assumed value of m we report the corresponding optimal number of transition functions deduced from the LM_F tests of remaining nonlinearity. We estimate the PSTR models for each potential specification (m, r^*), and report the number of parameters and the residual sum of squares. Given this lack of general testing procedure, we suggest here to use two standard information criteria (the Akaike and the Schwarz criteria) in order to choose a benchmark specification for each specification of the production function

Insert Table 3. Determination of the Number of Location Parameters

The selection of the model is then based on the following procedure. For each assumption on the returns to scale, we first choose the appropriate variable among the two "candidate" threshold variables (model A or model B). For that we consider the variable that gives rise to the strongest rejection of linearity whatever is the value of m (LM_F statistic for $H_0 : r = 0$, see Table 2). Obviously, the best transition variable for the OCRS specification is given by the lagged ratio of public capital stock to private capital stock $g_{i,t-1} - n_{i,t-1}$ (model A), while it is the lagged stock of private capital per worker $k_{i,t-1} - n_{i,t-1}$ (model B) when the assumption of PFCRS is considered. In a second step, based on the Schwarz and Akaike criteria, we select the optimal number of location parameters. Consequently, for the OCRS specification, we consider the model A with $m = 1$ and for the PFCRS specification we consider the model B with $m = 2$. In both cases, the model is well specified since the tests conclude to the hypothesis of no remaining nonlinearity in the relationships between GDP and private and public inputs (see Table 2). So, these choices of m lead to specifications which capture all the potential threshold effects in the productivity.

Table 4 contains the parameter estimates of the final PSTR models. Recall that the estimated parameters $\Psi_j = \left(\alpha_j, \beta_j\right)'$ cannot be directly interpreted as elasticities. As in logit or probit models, the value of the estimated parameters is not directly interpretable, but their signs can be are interpreted. For instance, let us consider the production function estimated under OCRS (third column of Table 4). In this case, the threshold variable is defined as the lagged ratio of public capital stock to private capital stock and there is only one transition function. A negative (respectively positive) parameter β_1 in the vector Ψ_1 only signifies that when the threshold variable increases, the elasticity of the public capital decreases (respectively increases). In other words, if β_1 is positive, it implies that an increase in the ratio of public capital to private capital stocks induces an increase of the public capital elasticity. This observation can be generalized in a model with more than one transition function ($r > 1$) even if things are slightly more complicated. In a model with two transition functions, if the parameter β_1 is positive and the parameter β_2 is negative, this implies that an increase of the threshold variable has two opposite effects on the elasticity. The results of these two opposite effects will depend on the value of the (i) slope parameters γ_j and (ii) the location parameters c_j. No general result can be deduced. In our PFCRS specification with $r = 3$, we can observe that the parameters β_1, β_2 and β_3 associated to the three transition function are positive and significant. Thus, when the ratio of public capital per worker increases in a country, it induces three positive effects on the marginal productivity of the public capital stocks. Similar opposite effects are observed on the private capital productivity.

Insert Table 4. Parameter Estimates for the Final PSTR Models

Finally, for all the models, at least one of the transition functions is not quite sharp. Recall that when the slope parameter tends to infinity, the transition function tends to an indicator function as in the threshold model without smooth transition. For instance, we can observe in Table 4 that the estimated slope parameter for the first transition function in the PFCRS specification is equal to 1.699. Consequently, this transition function is largely different from an indicator function. For the model under PFCRS, only one transition function (associated to the estimated slope parameter equal to 99.99.31) out of three transition functions is clearly quite sharp and equivalent to the transition function considered in a simple PTR. The same conclusion can be drawn for the PFCRS specification in which the estimated slope parameter is equal to 4.863. This point is particularly important, since it implies that the non linearity of the augmented production function can not be reduced to a limited number of regimes with different elasticities. Indeed, it is important to recall that, contrary to a PTR model, a PSTR model with a smooth transition function can be interpreted as a model which allows a "*continuum*" of regimes. This "*continuum*" of regimes is clearly required when it comes to measure the threshold effects of the public capital productivity. This result also points out the fact that the solution which consists in grouping some countries in a panel and in estimating a linear relationship between the public capital and the productivity of private inputs, or other measure of the activity, may be unsatisfactory. It is well known that this approach neglects the heterogeneity of the relationships between the countries. But for a given country, using a linear specification also leads to neglect the continuum of different productivity regimes which could be observed over the time periods. It is a strong criticism which could be directed toward the linear panel estimates in this context.

5 Individual Estimates of the Public Capital Productivity

Given the parameters estimates of the final PSTR models, it is now possible to compute, for each country of the sample and for each date, the time varying elasticity of output with respect to the public capital stock g_{it}, denoted e_{it}^{g}, $\forall i = 1, .., N$ and $\forall t = 1, .., T$. When the used threshold variable is independent of the public capital stock, these smoothed individual elasticities are given by the formula (11). In contrast, when the threshold variable is a function of the public capital stock, the formula must be adapted and is given by equation (12). The individual averages of these smoothed elasticities for public capital stock and labor as well as their variances are reported in

Tables 5 and 6. Let us consider the production function estimated under the assumption of OCRS. In the first column, the individual OLS estimated elasticity is reported with the corresponding t-statistic. In the second column, the panel Within estimated elasticity obtained in a linear specification is reported for the 21 countries of the sample Given that such a panel estimate ignores the heterogeneity of the slope parameters, the estimated elasticity is common to all the countries. In columns 3 and 4, the averages of the estimated individual smoothed elasticities are reported for the optimal specification of the PSTR model. These values correspond to the mean by country of the individual estimates:

$$\overline{e}_i^g = \frac{1}{T}\sum_{t=1}^{T} e_{it}^g = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial y_{it}}{\partial g_{it}} \qquad \forall i = 1,..,N \tag{29}$$

$$\overline{e}_i^n = \frac{1}{T}\sum_{t=1}^{T} e_{it}^n = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial y_{it}}{\partial n_{it}} \qquad \forall i = 1,..,N \tag{30}$$

where the individual time varying elasticities are given by equations (11) or (12).

Insert Table 5. Output Elasticities of the Public Capital Stocks.

Insert Table 6. Output Elasticities of the Labor Input

When OLS are used to estimate the parameters of the augmented production function country by country, we found exactly the same nonsensical values as those reported in the literature: when the estimated elasticity of the public capital stock is positive, its value is so important that it can not be considered as reasonable. The estimates are superior greater than 0.30 for Australia, Denmark, Finland, France, Germany, Norway and Switzerland. The estimated elasticity is even greater than one for Greece. For instance, if we assume that for most of these countries the public capital to output ratios range from 0.40 to 0.70 (Kamps, 2004), these results imply estimates of the marginal product of public capital that ranging from 57% to 99% per year for France or from 85% to 150% for Germany. For the United States, our estimated elasticity (0.39) is equal to that obtained by Aschauer (1989). As noticed by Gramlich, these original OLS estimates imply estimated rates of return on public capital that exceed 100 percent. While these "*stratospheric*" (Gramlich, 1994) rates of return on public capital are implausibly high, such a result is a common finding in the literature. On the contrary, the estimated coefficient on the public capital input is negative in four countries (Iceland, Ireland, Portugal and New Zealand) and the elasticity of the labor input is also negative in two other countries (Greece and Italy). Such a finding is not uncommon in the literature. It was for instance the case, in some linear specifications used by Sturm and De Haan (1995) or Vijverberg and *alii* (1997) for the United States.

The inference based on individual-country regressions is rendered difficult for many reasons (see Romp and De Haan, 2005 for a survey). Among these various reasons, the potential multicollinearity among the regressors is frequently pointed out. One way to deal with this problem is to exploit the cross sectional dimension of data and to use panel data models which allows reducing the collinearity among the explanatory variables. Indeed, the use of a linear panel model with individual fixed effects as recommended by Holtz-Eakin (1994) leads to more reasonable estimates. The estimated elasticity of output with respect to public capital (Table 5, second column), around 31%, is then more reasonable even if the estimates elasticity of the labor capital, *i.e.* 48.9%, is slightly less important than what is generally considered as valid for the industrialized countries. However, these results are obtained in a panel specification that ignores the heterogeneity of the production technology. The parameters of the augmented production function are assumed to be homogeneous. Given the individual-country regression results, it is evident that this homogeneity assumption is fallacious, even when individual effects are introduced. The standard specification tests (not reported), as proposed by Hsiao (2003), lead to strongly reject the null hypothesis of common parameters. Consequently, the within estimate corresponds to a weighted average of individual heterogeneous elasticities.

In contrast, the results derived from the various PSTR models presented in this paper are (*i*) economically reasonable, (*ii*) robust to changes in the specification of the threshold mechanisms and (*iii*) robust to the changes in the composition of the sample. Firstly, the individual estimated elasticities of output with respect to the public capital stocks and private inputs and the corresponding rates of returns are largely more reasonable than those obtained in linear OLS models. For instance, in the United States, we found an average estimated elasticity of 6.66% under the OCRS assumption (Table 4, last column). Recall that under the same assumption, Aschauer found an elasticity of 39%, and that in a simple optimal growth model, the optimal public investment ratio must be equal to this elasticity. Since the historical public investment ratio over the post war period is roughly equal to 5% in the United States, our estimate implies only a slight sub-optimality of the public investment policy in this country compared to the requirement implied by the standard OLS results. In our case, for an average ratio of government capital stock to GDP equal to $59,52\%$, the implied marginal rate of return on the public capital stock is equal to 11.19% per year. This measure is largely more reasonable than those previously mentioned. Similar results are obtained for the other OECD countries. The average estimated public capital elasticity ranges from 6.38% in New Zealand to 38.33% in Portugal. Except for Finland, Portugal, Belgium and Norway, the estimated elasticity is always smaller than 15%. For most of the major OECD countries, including Germany, France, Canada, United Kingdom, the estimated elasticities are roughly comparable to the historical public investment to GDP ratios. Besides, the estimated elasticities for the labor input are quite reasonable even if they are smaller than those obtained in linear panel models.

Secondly, whatever are the choices made in the specification of the transition function, the average estimated individual elasticities are roughly similar. Our investigations show that for a any threshold variable (model A or B), the choice on the value of m does not qualitatively affect the values of the estimated individual elasticities. For instance, under OCRS and when the threshold variable is defined as the lagged ratio of public to private capital stocks (model A), the optimal model is defined with $m = 1$ and $r = 1$. Given these specification choices, the estimated public capital elasticity for the United Stated is equal to 6.66% (Table 4). If we choose a specification with two location parameters ($m = 2$), the optimal number of transition functions is also equal to one (see Table 3). In a model with $m = 2$ and $r = 1$, the estimated elasticity for the United States is then equal to 1.90%.

Logically, these individual estimated values are not robust to a change of the threshold variable. For instance, under the OCRS assumption, if we consider a threshold variable defined as the lagged value of the stock of capital per worker (model B), the average estimated elasticity (not reported) for the United States is largely greater than our estimated value of 6.66% and is roughly equal to 0.30 whatever is the choice of m.

We can observe that these values obtained (and the values obtained for most of the countries of our sample) are roughly similar to those obtained in a linear panel with fixed individual effects. This result can be interpreted as follows. Obviously, if the transition mechanism is not well specified, *i.e.* if the threshold variable is not well chosen, the use of a PTR (or a PSTR) model implies to gather the countries according to fallacious criteria. Consequently, at each date the countries are split in a few number of randomly constituted groups and associated to different slopes parameters, according to the value of the fallacious threshold variable. So, the estimated slope parameters obtained in this context on random groups are not different from those estimated on the whole sample. Consequently, the fact that we obtain roughly the same individual estimated elasticities as those obtained in linear panel models may be interpreted as an evidence that the threshold variable is not well identified. In our context, this conclusion is reinforced by the fact that the linearity tests lead to the rejection of the model B and to the use of a threshold variable defined by the lagged ratio of public capital to private capital stock (model A, see Table 3).

Insert Figures 2 and 3. Output Elasticities of the Public Capital Stocks

On the Figures (2) and (3), the estimated elasticities e_{it}^{g} of the output with respect to the public capital stock are plotted over the period 1965-2001 for the 21 countries of our sample. For most of the countries, these elasticities are quite stable over the time period. The main exceptions are obtained for Finland, Belgium, Spain, Norway, Portugal, Sweden and Switzerland. In these countries (except for the Spain), the elasticity is decreasing over time. This decrease is clearly related to a fall of the ratio of public to private capital stocks in these countries. Indeed, it has been previously mentioned that given the negative estimated parameter β_1 associated to the transition function (see Table 4), an increase of the threshold variable induces a decrease of the public capital stock elasticity. On the contrary, in the case of the United Kingdom, the recent increase in the public investments has induced a slight increase in the public capital elasticity.

6 Conclusion

In this paper we propose an empirical evaluation of the threshold effects in the productivity of public capital stocks in OECD countries. Our assessment is based on the estimation of various threshold panel specifications of public capital augmented production functions. Our main results can be summarized in two main points. First, the relationship between the output and public capital stocks is non linear. More precisely, strong threshold effects can be identified in these relationships. This conclusion is robust to changes in the specifications of the production function and in the threshold variable. In addition, it seems that the productivity of the public capital can not be reduced to a small number of regimes and must be studied through a model allowing a "continuum" of regimes. This result reveals the importance of the heterogeneity of the economic situations of the OECD countries even in a nonlinear perspective. Second, we propose individual time varying estimates of the public capital elasticities for 21 OECD countries. These estimates, issued from a variety of PSTR models, take into account both the cross-sectional heterogeneity and the threshold effects in the production technology. These estimates confirm the influence of the public capital stocks on the productivity of other factors. But, the "*stratospheric*" estimates of the productivity sometimes reported in the empirical literature disappear when these threshold effects are controlled for.

A Appendix

A.1 First-Order Taylor Expansion of the Transition Function

Let us consider the first-order Taylor expansion around $\gamma = 0$ of the function $y_{it} - k_{it} = \mu_i + \Psi_0' W_{it} + \Psi_1' W_{it}\, g\left(q_{it};\gamma, c\right) + \varepsilon_{it}$ in the case $m = 1$. For simplicity, we consider the case in which the threshold variable q_{it} is different from the explanatory variables.

$$y_{it} - k_{it} = \mu_i + \Psi_0' W_{it} + \Psi_1' W_{it} \left(\frac{1}{2} - \frac{\gamma\, c_1}{4}\right) + \Psi_1' W_{it}\, \frac{\gamma}{4} q_{it} + \varepsilon_{it} \tag{31}$$

So, the first-order Taylor expansion depends only on q_{it} since $m = 1$ and the parameter associated to q_{it} is a multiple of the slope parameter γ. When $m = 2$, this first-order Taylor expansion is defined as:

$$\begin{aligned} y_{it} - k_{it} \quad &= \quad \mu_i + \left[\left(\Psi_0' + \frac{1}{2}\Psi_1' + \frac{\gamma\, c_1\, c_2}{4}\Psi_1'\right)\right] W_{it} \\ &\quad -\frac{\gamma}{4}\Psi_1' W_{it}\ \left(c_1 + c_2\right)\ q_{it} + \frac{\gamma}{4}\Psi_1' W_{it}\ q_{it}^2 + \varepsilon_{it} \end{aligned} \tag{32}$$

This expression depends on q_{it} and q_{it}^2, and the corresponding parameters vectors Γ_1 and Γ_2 depend on the slope parameter γ.

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Table 1. Public Capital Augmented Production Function[11]
Linear Panel Models with Fixed Effects and Year Dummies

Parameters	α	β	$cst.$	RSS
		PFCRS Specification		
Individual Fixed Effects	0.567 (27.8)	0.311 (20.6)	—	4.296
Random Individual Effects	0.552 (27.6)	0.301 (20.3)	-3.169 (-16.9)	4.300
Individual Fixed Effects and Time Effects	0.646 (30.6)	0.175 (8.92)	—	3.423
		OCRS Specification		
Individual Fixed Effects	0.489 (24.2)	0.314 (16.8)	—	4.890
Random Individual Effects	0.484 (24.3)	0.320 (17.37)	-0.334 (-5.04)	4.898
Individual Fixed Effects and Time Effects	0.705 (33.7)	0.218 (13.1)	—	3.065

Notes: The dependent variable is log GDP per unit of private capital stock. All variables are expressed in logs. The t-statistics are in parenthesis. The parameter α and β respectively denote the elasticities of output with respect to the labor and the public capital stock. The PFCRS specification corresponds to an assumption of Private Factors Constant Returns to Scale. The OCRS specification corresponds to the assumption of Overall Constant Returns to Scale.

Table 2. LM_F Tests for Remaining Nonlinearity[12]

	PFCRS Specification			
Model	Model A		Model B	
Threshold Variable	$g_{i,t-1} - k_{i,t-1}$		$k_{i,t-1} - n_{i,t-1}$	
Number of Location Parameters	$m = 1$	$m = 2$	$m = 1$	$m = 2$
$\mathrm{H}_0 : r = 0$ vs $\mathrm{H}_1 : r = 1$	0.788 (0.45)	3.552 (0.00)	24.19 (0.00)	16.73 (0.00)
$\mathrm{H}_0 : r = 1$ vs $\mathrm{H}_1 : r = 2$	—	4.419 (0.00)	5.550 (0.00)	13.25 (0.00)
$\mathrm{H}_0 : r = 2$ vs $\mathrm{H}_1 : r = 3$	—	2.883 (0.02)	3.758 (0.02)	5.101 (0.00)
$\mathrm{H}_0 : r = 3$ vs $\mathrm{H}_1 : r = 4$	—	—	—	2.971 (0.02)
$\mathrm{H}_0 : r = 4$ vs $\mathrm{H}_1 : r > 4$	—	—	—	—
	OCRS Specification			
Model	Model A		Model B	
Threshold Variable	$g_{i,t-1} - k_{i,t-1}$		$k_{i,t-1} - n_{i,t-1}$	
Number of Location Parameters	$m = 1$	$m = 2$	$m = 1$	$m = 2$
$\mathrm{H}_0 : r = 0$ vs $\mathrm{H}_1 : r = 1$	45.87 (0.00)	31.22 (0.00)	14.23 (0.00)	19.43 (0.00)
$\mathrm{H}_0 : r = 1$ vs $\mathrm{H}_1 : r = 2$	2.364 (0.09)	1.505 (0.19)	2.517 (0.08)	6.195 (0.00)
$\mathrm{H}_0 : r = 2$ vs $\mathrm{H}_1 : r = 3$	—	—	—	1.555 (0.185)
$\mathrm{H}_0 : r = 3$ vs $\mathrm{H}_1 : r = 4$	—	—	—	—
$\mathrm{H}_0 : r = 4$ vs $\mathrm{H}_1 : r > 4$	—	—	—	—

Notes: For each model (*i.e.* for each threshold variable), the testing procedure works as follows. First, test a linear model ($r = 0$) against a model with one threhosld ($r = 1$). If the null hypothesis is rejected, test the single threshold model against a double threshold model ($r = 2$). The procedure is continued until the hypothesis no additional threhold is not rejected. The corresponding LM_F statistic has an asymptotic $F\left[mK, TN - N - (r+1)\,mK\right]$ distribution under H_0, where m is the number of location parameters and K the number of explicative variables. In our specifications we have $K = 2$. The corresponding p-values are reported in parentheses.

Table 3. Determination of the Number of Location Parameters[13]

Model	PFCRS: Model A		PFCRS: Model B	
Number of Location Parameters	$m = 1$	$m = 2$	$m = 1$	$m = 2$
Optimal Number of Threshold $r^* (m)$	0	2	2	3
Residual Sum of Squares	4.296	3.360	3.135	2.805
Number of Parameters	2	12	10	17
AIC Criterion	−5.161	−5.367	−5.444	−5.527
Schwarz Criterion	−5.148	−5.293	−5.382	−5.423
Model	OCRS: Model A		OCRS: Model B	
Number of Location Parameters	$m = 1$	$m = 2$	$m = 1$	$m = 2$
Optimal Number of Threshold $r^* (m)$	1	1	1	2
Residual Sum of Squares	3.981	3.969	4.229	3.597
Number of Parameters	6	7	6	12
AIC Criterion	−5.221	−5.220	−5.160	−5.298
Schwarz Criterion	−5.184	−5.177	−5.124	−5.225

Notes: For each model (each specification), the optimal number of locations parameters used in the transitions functions can be determined as follows. For each value of m, the corresponding optimal number of thresholds, denoted $r^* (m)$, is determined according to a sequential procedure based on the LM_F statistics of the hypothesis of non remaining nonlinearity. Thus, for each couple (m, r^*), the value the RSS of the model is reported. The total number of parameters is determined by the formula $K (r^* + 1) + r^* (m + 1)$, where K denotes the number of explicative variables, *i.e.* $K = 2$ in our specifications.

Table 4. Public Capital Augmented Production Function
Parameter Estimates for the Final PSTR Models[14]

Specification	PFCRS	OCRS
Threshold Variable	Model B	Model A
(m, r^*)	$(2,3)$	$(1,1)$
Parameters $\Psi_0 = (\alpha_0\ \beta_0)$		
Labor Input Parameter α_0	2.934 (19.8)	0.346 (14.80)
Public Capital Parameter β_0	−0.345 (−9.15)	0.443 (12.59)
Parameters $\Psi_1 = (\alpha_1\ \beta_1)$		
Labor Input Parameter α_1	−1.695 (−17.4)	−0.097 (−4.905)
Public Capital Parameter β_1	0.507 (18.7)	−0.379 (−11.10)
Parameters $\Psi_2 = (\alpha_2\ \beta_2)$		
Labor Input Parameter α_2	−0.340 (−10.49)	—
Public Capital Parameter β_2	0.164 (9.26)	—
Parameters $\Psi_3 = (\alpha_3\ \beta_3)$		
Labor Input Parameter α_3	−0.086 (−7.00)	—
Public Capital Parameter β_3	0.070 (6.51)	—
Location Parameters c_j		
First Transition Function	$[0.572; 0.572]$	−0.557
Second Transition Function	$[-0.940; -0.940]$	—
Third Transition Function	$[-1.891; -1.398]$	—
Slopes Parameters γ_j	$[1.699; 6.899; 99.99]$	4.863

Notes: The Model A corresponds to the threshold variable $g_{i,t-1} - k_{i,t-1}$ and the Model B to the threshold variable $k_{i,t-1} - n_{i,t-1}$. The standard errors in parentheses are corrected for heteroskedasticity. For each model and each value of m the number of transition functions r is determined by a sequential testing procedure (see Table 2). For the j^{th} transition function, with $j = 1, ..r,$, the m estimated location parameters c_j and the corresponding estimated slope parameter γ_j are reported. The PSTR parameters can not be directly interpreted as elasticities.

Table 5. Output Elasticities of the Public Capital Stocks
Average of Individual PSTR Estimates under OCRS[15]

Model	OLS-Linear	Within	PSTR
Threshold Variable	None	None	Model A
Australia	0.439 (4.33)	0.314 (16.8)	0.136 (1.31)
Austria	0.240 (1.22)	0.314 (16.8)	0.070 (0.003)
Belgium	0.153 (7.93)	0.314 (16.8)	0.168 (0.008)
Canada	0.174 (2.22)	0.314 (16.8)	0.074 (0.008)
Denmark	0.327 (3.19)	0.314 (16.8)	0.076 (0.012)
Finland	0.677 (2.95)	0.314 (16.8)	0.237 (0.103)
France	0.399 (1.24)	0.314 (16.8)	0.074 (0.001)
Germany	0.608 (4.06)	0.314 (16.8)	0.094 (0.009)
Greece	1.052 (2.54)	0.314 (16.8)	0.085 (0.007)
Iceland	−0.031 (−0.21)	0.314 (16.8)	0.109 (0.019)
Ireland	−0.108 (−0.67)	0.314 (16.8)	0.067 (0.003)
Italy	0.166 (1.03)	0.314 (16.8)	0.101 (0.009)
Japan	0.096 (0.798)	0.314 (16.8)	0.064 (0.003)
New Zealand	−0.198 (−2.52)	0.314 (16.8)	0.063 (0.003)
Norway	0.515 (6.29)	0.314 (16.8)	0.293 (0.068)
Portugal	−0.041 (−0.24)	0.314 (16.8)	0.383 (0.055)
Spain	0.157 (1.69)	0.314 (16.8)	0.092 (0.015)
Sweden	0.244 (2.07)	0.314 (16.8)	0.105 (0.013)
Switzerland	0.778 (5.43)	0.314 (16.8)	0.144 (0.067)
United Kingdom	0.193 (3.16)	0.314 (16.8)	0.071 (0.008)
United States	0.395 (5.60)	0.314 (16.8)	0.066 (0.002)

Notes: For each country, the results of the individual-country regressions with a linear time trend (OLS-Linear) and the panel linear model (Within) are reported. The corresponding t-statistics are in parenthesis. For the PSTR models, the figures in parenthesis correspond to the standard errors of the individual estimates expressed in percent. The Model A corresponds to the threshold variable $g_{i,t-1} - k_{i,t-1}$ and the Model B to the threshold variable $k_{i,t-1} - n_{i,t-1}$.

Table 6. Output Elasticities of the Labor Input
Average of Individual PSTR Estimates under OCRS[16]

Model	OLS-Linear	Within	PSTR
Threshold Variable	None	None	Model A
Australia	1.045	0.489	0.267
	(10.10)	(24.2)	(0.33)
Austria	0.488	0.489	0.250
	(2.53)	(24.2)	(0.10)
Belgium	0.204	0.489	0.275
	(2.71)	(24.2)	(2.24)
Canada	1.117	0.489	0.251
	(11.58)	(24.2)	(0.22)
Denmark	0.965	0.489	0.252
	(9.13)	(24.2)	(0.32)
Finland	0.850	0.489	0.293
	(10.14)	(24.2)	(2.66)
France	0.183	0.489	0.251
	(1.86)	(24.2)	(0.03)
Germany	0.585	0.489	0.256
	(5.30)	(24.2)	(0.23)
Greece	−0.359	0.489	0.254
	(−1.89)	(24.2)	(0.17)
Iceland	0.561	0.489	0.260
	(4.21)	(24.2)	(0.49)
Ireland	0.842	0.489	0.249
	(14.00)	(24.2)	(0.07)
Italy	−0.013	0.489	0.258
	(−0.10)	(24.2)	(0.24)
Japan	0.173	0.489	0.248
	(2.46)	(24.2)	(0.009)
New Zealand	0.673	0.489	0.248
	(3.59)	(24.2)	(0.007)
Norway	0.638	0.489	0.307
	(14.17)	(24.2)	(1.75)
Portugal	0.332	0.489	0.330
	(1.61)	(24.2)	(1.41)
Spain	0.427	0.489	0.256
	(6.46)	(24.2)	(0.40)
Sweden	0.745	0.489	0.259
	(4.05)	(24.2)	(0.346)
Switzerland	1.141	0.489	0.269
	(8.23)	(24.2)	(1.72)
United Kingdom	1.016	0.489	0.250
	(9.93)	(24.2)	(0.20)
United States	0.802	0.489	0.249
	(6.24)	(24.2)	(0.05)

Notes: For each country, the results of the individual-country regressions with a linear time trend (OLS-Linear) and the panel linear model (Within) are reported. The corresponding t-statistics are in parenthesis. For the PSTR models, the figures in parenthesis correspond to the standard errors of the individual estimates expressed in percent. The Model A corresponds to the threshold variable $g_{i,t-1} - k_{i,t-1}$ and the Model B to the threshold variable $k_{i,t-1} - n_{i,t-1}$.

Figure 1: Transition Function with $m = 1$ and $c = 0$. Sensitivity to the Slope Parameter.

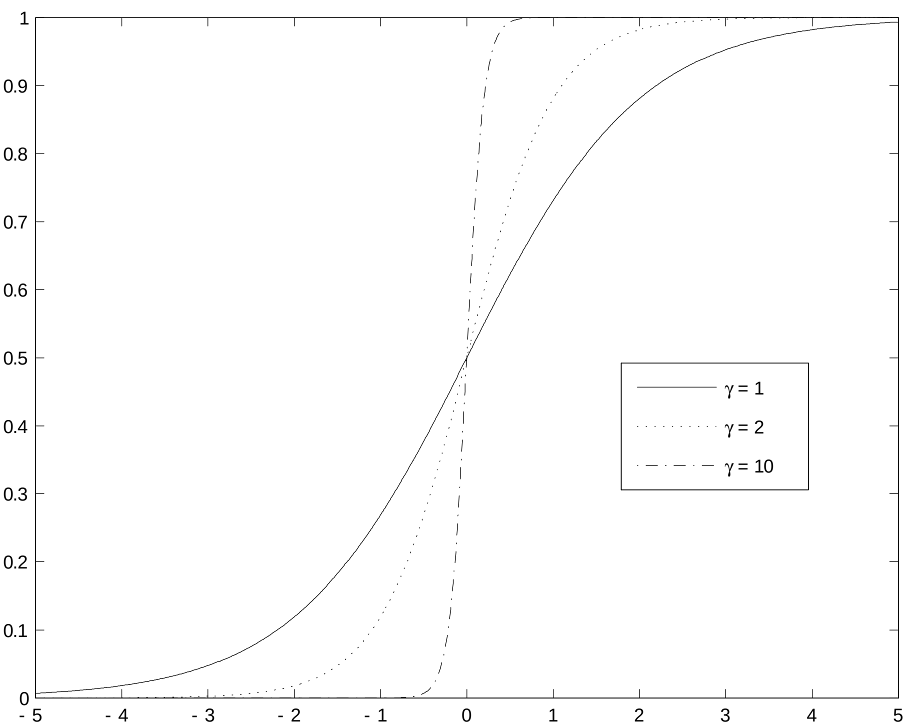

Figure 2: Output Elasticities of the Public Capital Stocks. Individual PSTR Estimates under OCRS (1965-2001)

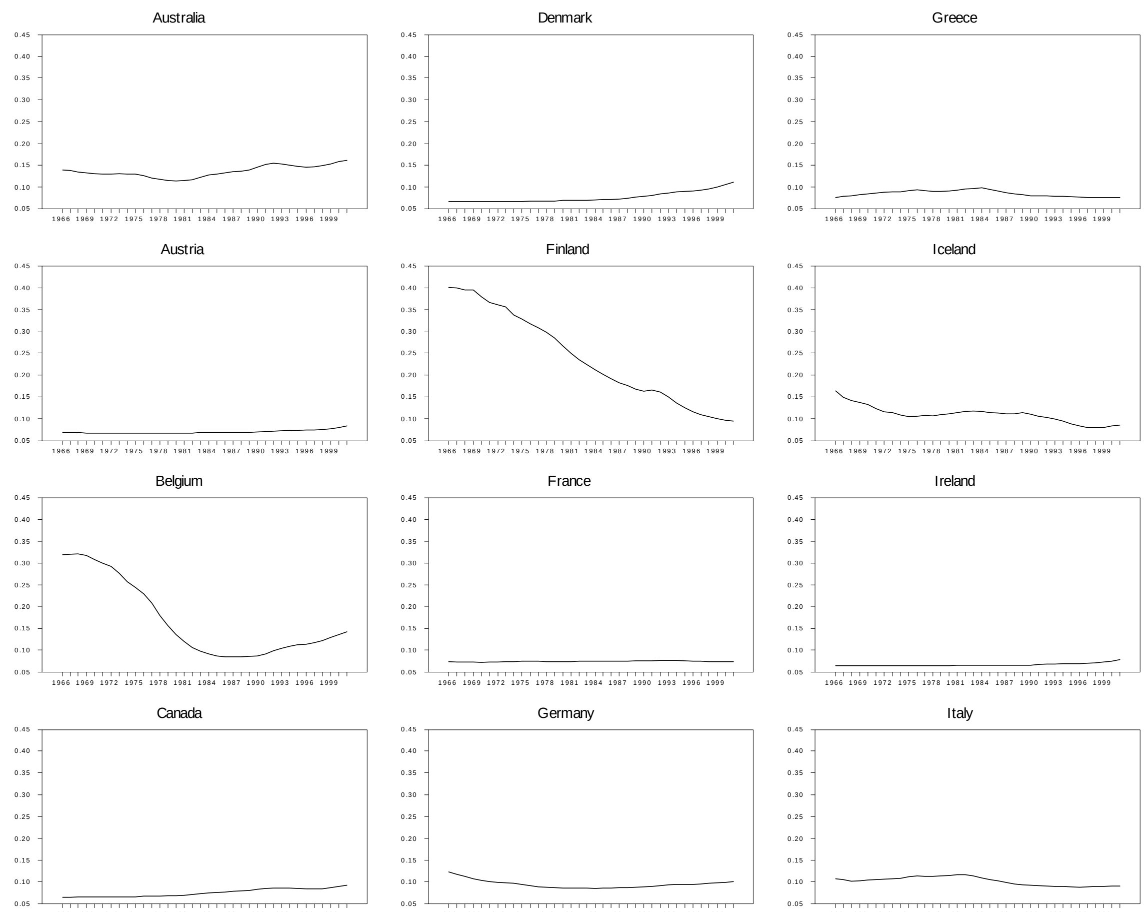

Figure 3: Output Elasticities of the Public Capital Stocks Individual PSTR Estimates under OCRS. 1965-2001 (Continued)