



**HAL**  
open science

## School choice : in the context of differentiation by ability and income

Saïd Hanchane, Tarek Mostafa

► **To cite this version:**

Saïd Hanchane, Tarek Mostafa. School choice : in the context of differentiation by ability and income. 2006. halshs-00009533v1

**HAL Id: halshs-00009533**

**<https://shs.hal.science/halshs-00009533v1>**

Preprint submitted on 9 Mar 2006 (v1), last revised 14 May 2007 (v3)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# School Choice: In the context of differentiation by ability and income

HANCHANE Saïd\* ; MOSTAFA Tarek\*\*

**Abstract:** In this paper, we analyze the equilibrium on the market for schooling where both public and private schools coexist and where individuals are differentiated by income and ability. We study the distribution of students across sectors while examining the conditions for the existence of a majority voting equilibrium in the context of non single peaked preferences. Finally, we examine the existence of a hierarchy of school qualities, as a consequence of the discriminating pricing strategy used by private schools to internalize the effect of peer groups.

**JEL Classification:** I20, I21, H52.

**Keywords:** Education market, majority voting equilibrium, peer group effect, pricing discrimination, educational opportunity.

---

\* Laboratory of Sociology and Labor Economics, CNRS. Université de la méditerranée (Aix Marseille II).  
35 avenue Jules Ferry, 13626 Aix en Provence, France. E-mail : [said.hanchane@univmed.fr](mailto:said.hanchane@univmed.fr)  
Tel. 0033-6-21401247. Fax : 0033-4-42267937

\*\* Laboratory of Sociology and Labor Economics, CNRS. Université de la méditerranée (Aix Marseille II).  
35 avenue Jules Ferry, 13626 Aix en Provence, France. E-mail : [tarekmostafa82@hotmail.com](mailto:tarekmostafa82@hotmail.com).  
Tel. 0033-6-21401247. Fax : 0033-4-42267937

Scarce are the studies that have analyzed school choice between public and private sectors, we identify Joseph Stiglitz (1974)<sup>1</sup>, Clotfelter (1976)<sup>2</sup>, Sonstelie (1979)<sup>3</sup>, Glomm et Ravikumar (1998)<sup>4</sup> and Epple et Romano (1998)<sup>5</sup>.

Stiglitz analyzes voting equilibrium in the context of non single peaked preferences; he concludes that this particularity of individual behavior may prevent the existence of majority voting equilibrium determined by the median income voter. Clotfelter examines the effect of school racial composition as a factor of school choice following the integration policies in the United States. Sonstelie introduces the quality of education while explaining choice through family surplus in both sectors. Glomm and Ravikumar construct a majority voting equilibrium determined by the individual with median income in the context of non single peaked preferences. All these studies have two characteristics in common, first they consider a population where individuals are only differentiated by income; second they consider a market where the private sector is passive, in other words individuals demand education continuously in function of their disposable income without considering the effects of supply and private school pricing strategies. Conversely, Epple and Romano consider an economy where individuals are differentiated by income and ability; the private sector is active and private schools maximize their profit, this strategic behavior leads to the existence of a hierarchy of school qualities.

In this paper, we carry on the work made by Glomm, Ravikumar, Epple and Romano. Our model differ in a number of ways, in comparison to the study of Glomm and Ravikumar, we consider an economy where individuals are differentiated by both income and ability. We present the necessary conditions for the existence of a majority

---

<sup>1</sup> J. Stiglitz. (1974), "The demand for education in public and private school systems". *Journal of public economics* Vol 3 No 4 (349-385).

<sup>2</sup> C. Clotfelter. 1976, "School desegregation, Tipping and private school enrollment" *The Journal of human resources* Vol 11 No 1 (28-50).

<sup>3</sup> J. Sonstelie. 1979, "Public school quality and private school enrollments". *National Tax Journal* (343-353).

<sup>4</sup> G. Glomm and B. Ravikumar. 1998, "opting out of publicly provided services: A majority voting Equilibrium" *Social Choice and Welfare* Vol 15 (187-199).

<sup>5</sup> D. Epple and R. Romano. 1998, "Competition between public and private schools, Vouchers, and peer group effects" *The American Economic Review* Vol 88 No 1 (33-62).

voting equilibrium in the context of non single peaked preferences. We analyze the distribution of students across sectors and schools, while presenting two equity related problems. First, private schools will operate to the detriment of public schools by siphoning high ability, high income students. Second, inequalities of educational opportunity exist when a fraction of the population does not have an equal access to educational achievement, given its allocation of income and ability.

In our model, similarly to Epple and Romano, the private sector is active. Private schools will act strategically in order to maximize individual utility by applying a discriminating pricing strategy which depends on ability and income levels. Peer group effect is included as a component of school quality; this factor is the corner stone of the discriminating pricing strategy. The inclusion of peer group effect is consistent with the theoretical literature on this subject, Summers and Wolfe (1977)<sup>6</sup>, Henderson (1978)<sup>7</sup>, Sorensen and Hallinan (1986)<sup>8</sup> and Kulik (1992)<sup>9</sup> have provided evidence that such an effect exists.

Our equilibrium will be characterized by the existence of an indifference point between the two sectors determined by a couple of income and ability. The distribution of students across sectors and schools, given the discriminating pricing strategy will happen to the detriment of public schools. Individuals with high ability and income levels will be in the private sector; individuals with high ability and low income levels will be in the private sector and will receive financial aides; individuals with high income and low ability levels will be in the private sector and will pay tuition premia; individuals with low ability and income levels will be in the public sector. The existence of a majority voting equilibrium will be directly dependent on individual behavior and preferences.

---

<sup>6</sup> A. Summers and B. Wolfe. 1977, "Do schools make a difference?" *American Economic Review* Vol 67 No 4 (639-652).

<sup>7</sup> V. Henderson, P. Mieszkowski and Y. Sauvageau. 1978, "Peer group effects in educational production functions" *Journal of Public Economics* Vol 10 No 1 (97-106).

<sup>8</sup> A. Sorensen and M. Hallinan. 1986, "Effects of ability grouping on growth in academic achievement" *American Educational Research Journal* Vol 24 No 4 (519-542).

<sup>9</sup> J. Kulik and C. Kulik 1992, "Effects of ability groupings on secondary school students: A Meta analysis of evaluation findings" *American Education Research Journal* Vol 19 No 3 (415-428).

Section I presents the model, equilibrium conditions follow in section II, section III examines the distribution of students across sectors, the conditions for a majority voting equilibrium and the existence of school quality hierarchy, section IV concludes. An appendix contains mathematical details.

## **I- The model**

In our model, we consider an economy populated by a continuum of agents; the number of agent is largely superior to that of schools. Agents have identical preferences over private consumption; they are differentiated by their income and ability levels, an individual  $i$  (with  $i = 1, 2, 3, 4, \dots, n$ ) has an income  $y_i$  and an ability  $b_i$ . Income and ability are distributed in the population according to  $f(b, y)$  which is positive, and continuous on its support  $S \equiv (0, b_{\max}) \times (0, y_{\max})$ .

Individual utility is assumed to be a function of consumption goods other than education and school quality. It is denoted  $U(c_i, q_i)$ , and is continuous and increasing in both arguments. Education is present in the utility function as unites of quality; component  $c_i$  is not related to education.

School quality is dependant on financial resources invested per student, as well as scholastic achievement of the student.

Scholastic achievement of a student depends on his own ability and on the mean ability of his school, denoted  $\theta_j$ .  $j$  is an index denoting a particular school, with  $j = 1, 2, 3, \dots, n$ . Scholastic achievement is denoted  $a = a(\theta_j, b_i)$ .  $\theta_j$  Reflects the peer group effect, previous studies have showed that higher achievement levels tend to positively influence that of a student with lower achievement level.

Education production cost depends only on the number of students enrolled in a particular school, it is denoted  $C(k) = V(k) + F$  Avec  $V' > 0$  et  $V'' < 0$ .  $k$  is the number of student in this school;  $F$  is a constant reflecting a fixed cost when no student are enrolled. The existence of economies of scale in the production of educational services is realistic; they will prevent the existence of an infinite number of private schools. In our model we suppose that the number of student types is largely superior to the number of schools.

In our model, each private school will retain a fraction of the student population that has chosen the private sector, by applying tuition and admission policy permitting the maximization of each individual's utility. The pricing strategies will be characterized by a meritocracy system.

In the private sector, students with high ability and low income will profit from a tuition reduction, conversely, students with low ability and high income will pay tuition premia. The difference in pricing strategy is due to the existence of peer group effect. This effect presents a positive externality for students with ability levels inferior to mean ability of the school; it also presents a negative externality for students with ability levels superior to mean ability of the school.

This discriminating pricing strategy is justified in the context of internalizing the externality resulting from peer group effect. Students with ability levels superior to mean ability of the school will ameliorate the peer group effect, thus schools will benefit from their presence, so it is normal to expect a tuition reduction for those students. Students with ability levels inferior to mean ability of the school will benefit from peer group effect in two ways; first they will be associated with a school that has a higher mean ability than their own, second they will ameliorate their ability levels by profiting from peers abilities.

It is important to note that ability amelioration is due only to peer group effect; there will be no supplementary educational investments in students with low ability levels, as a consequence private school expenditure per student will be the same for all students in a particular school.

Individuals in the private sector can always choose public sector schools, conversely individuals obliged to enroll in the public sector given their allocation of income and ability do not have the opportunity to enroll in private schools.

### **The public Sector:**

In our model, public sector school quality is not homogenous between schools; conversely to the hypothesis retained by Epple and Romano, Glomm and Ravikumar, for whom the public sector is formed of a unique very large public school.

Public schools are income tax financed; all individuals pay taxes dedicated to financing public education even if they do not use public schools. Public sector resources are determined by  $tY$  ; and the expenditure per student is determined by  $\frac{tY}{N}$  . Public sector schools do not receive tuition, thus  $p_{iju} = 0$  .

With,  $t$ : The tax rate determined through majority voting.

$Y$ : Total income.

$N$ : The number of students in the public sector.  $N \in [0, P]$  .

$P$ : The number of students in the economy.

$u$  is an index denoting a particular public school.

The quality of education in the public sector for an individual  $i$  enrolled in a school  $j$  is written as follows:

$$q_{iju} = q\left[\frac{tY}{N}, a(\theta_{ju}, b_i)\right]$$

Expenditure and achievement are transformed into unites of quality following a Cob-Dougllass function of both factors:

$q_{iju} = A \left( \frac{tY}{N} \right)^\gamma a^\beta$  With  $\gamma$  the part of quality due to public expenditures, and  $\beta$  the part of quality due to scholastic achievement. With,  $\gamma \in [0,1]$   $\beta \in [0,1]$  and  $\gamma + \beta = 1$ . A is a constant.

In this model,  $\frac{tY}{N}$  is homogenous for all schools and students. However scholastic achievement is different between schools and between students, this difference can be attributed to technical characteristics of the school: teachers' qualification, school environment...

One might think that given the free access to public schools, students will choose the public school with the highest quality. In reality, this is not true; students will be subject to a geographical constraint resulting from their families' residential choice. In this case a student can have a choice between a limited number of public schools, thus public school quality is not homogenous.

We denote  $\alpha_{ju}(b, y)$  as the number of students of type  $(b, y)$  who are enrolled in a particular public school.  $\sum \alpha_{ju} = N$

Individual utility in the public sector is of the following form:

$$U_{iu} = U[y_i(1-t), q_{iju}]$$

Indirect utility can be written as:

$$W_{iu} = W(t, y_i, Y, N, \theta_{ju}, b_i)$$



In the public sector, utility maximization is simple to solve. A student will allocate his disposable income (after the deduction of taxes dedicated to public school finance) to private consumption.

### **The private sector:**

In the private sector, schools are tuition financed. Parents pay a positive tuition equal to  $p_{ijr}$ . Private school resources can be determined by  $\sum p_{ijr}$ . These resources are not entirely distributed over students; per student expenditure is determined by dividing the production cost of education by the number of students in the school. The difference between resources and expenditure represent private school's profit. This positive profit will encourage new schools to enter the market until it vanishes (in a competitive market).

Expenditure per student is given by: 
$$\frac{C(k_{jr})}{\alpha_{jr}(b_i, y_i)} = \frac{V(k_{jr}) + F}{\alpha_{jr}(b_i, y_i)}$$

$r$  is an index denoting a particular private school.

$\alpha_{jr}(b, y) = k_{jr}$  is the number of students of type  $(b, y)$  enrolled in a particular private school.  $\sum \alpha_{jr}(b, y) = P - N$ .  $P - N$  is the number of students in the private sector.

The quality of education in the private sector for an individual  $i$  and a school  $j$  is written as follows:

$$q_{ijr} = q \left[ \frac{C_{jr}}{\alpha_{jr}(b_i, y_i)}, a(\theta_{jr}, b_i) \right]$$

It should be noted that private school quality is positively correlated with individual income; when income increases, the private quality that can be purchased is much higher. When an individual compares quality in both sectors; his ability level has no influence because it is the same in all schools.

Expenditure and achievement are transformed into unites of quality following a Cob-Douglass function of both factors:

$$q_{ijr} = A \left( \frac{C_{jr}}{\alpha_{jr}(b_i, y_i)} \right)^\gamma a^\beta \quad \text{With, } \gamma \in [0,1], \beta \in [0,1] \text{ and } \gamma + \beta = 1. A \text{ is a constant.}$$

Private sector quality is heterogeneous between schools in function of both arguments. Private school expenditure is homogenous for students of the same school, and heterogeneous between schools. Scholastic achievement is heterogeneous between schools as well as students of the same school.

Private sector pricing strategies are determined in correlation with an individual i income and ability levels. Individuals with high ability and low income will receive tuition reductions; conversely individuals with high income and low ability will pay tuition premia. These pricing strategies are the corner stone of our redistribution system; the former group will be subsidized by the latter.

Individual utility in the private sector is of the following form:

$$U_{ir} = U[y_i(1-t) - p_{ijr}, q_{ijr}]$$

Individuals are going to choose an optimal combination between consumption goods and school quality in order to maximize their utility.

$$\text{Max } U_{ir} = U[y_i(1-t) - p_{ijr}, q_{ijr}]$$

$$\text{Subject to } p_{ijr} = y_i(1-t) - c_i$$

Indirect utility can be written as:  $W_{ir} = W(t, y_i, \theta_{jr}, b_i)$ , it represents the only solution to this optimization.

Private schools are profit making organizations; they maximize their profit as utility takers by conditioning admission and tuition according to ability levels. “They attract students by offering admission at a tuition yielding at least the maximum utility that can

be obtained elsewhere”<sup>10</sup>. This behavior is similar to that of private clubs with non anonymous crowding<sup>11</sup>. It should be noted that individual ability is perfectly observable by private schools.

Private schools have to maximize the following function:

$$Max\pi_{jr} = \iint_s [p_{ijr}(b, y)\alpha_{jr}(b, y)f(b, y)dbdy] - V(k_j) - F \quad (1)$$

Subject to:

$$\alpha_{jr}(b, y) \in [0, P - N] \quad \forall(b, y) \quad (1a)$$

$$U_r = U[y_i(1-t) - p_{ijr}(b, y), q_{ijr}] \geq MaxU[y_i(1-t) - p_{ijr}(b, y), q_{ijr}] \quad \forall(b, y) \quad (1b)$$

$$k_{jr} = \iint_s \alpha_{jr}(b, y)f(b, y)dbdy \quad (1c)$$

$$\theta_{jr} = \frac{1}{k_{jr}} \iint_s b\alpha_{jr}(b, y)f(b, y)dbdy \quad (1d)$$

Constraint (1c) determines the number of students enrolled in a particular private school. Constraint (1d) determines mean ability in this school which also represents peer group effect. Constraint (1b) represents the utility taking assumption.

## **II- Equilibrium conditions**

Three types of conditions are necessary for the existence of equilibrium:

### **A- The conditions for private sector equilibrium are:**

- Individual utility maximization:

$$U^*(b, y) = MaxU[y_i(1-t) - p_{ijr}, q_{ijr}]$$

- Private schools' profit maximization:

$$[\theta_{jr}, k_{jr}, p_{ijr}(b, y), \alpha_{jr}(b, y)] \quad \text{Will satisfy equation (1).}$$

<sup>10</sup> See Epple and Romano 1998. P38.

<sup>11</sup> S. Scotchmer and M. Wooders 1987. "Competitive equilibrium and the core in club economics with non anonymous crowding". *Journal of Public Economics*, Vol34 No 2. (159-173).

- New entries are expected so long as private schools expect to make a profit; the private sector equilibrium will be defined by.

$$\pi_{jr} = 0 \quad j=1, 2, 3, 4, \dots, n. \quad \text{new entries are no longer profitable.}$$

### **B- The conditions for public sector equilibrium are:**

- Public schools are not tuition financed:

$$p_{iju}(b, y) = 0$$

- $\alpha_{ju}(b, y) \in [0, N] \quad \forall(b, y)$

- The number of students in a public school is given by:

$$k_{ju} = \iint_s \alpha_{ju}(b, y) f(b, y) db dy$$

- Public school's mean ability is given by :

$$\theta_{ju} = \frac{1}{k_{ju}} \iint_s b \alpha_{ju}(b, y) f(b, y) db dy$$

### **C- All students in the age for schooling are enrolled in one of the two sectors:**

$$\sum \alpha_{ju}(b, y) + \sum \alpha_{jr}(b, y) = N + (P - N) = P \quad \forall(b, y)$$

This condition represents market clearance; public schools are preferred to no schooling.

## **III- Theoretical results:**

### **A- Private schools' profit maximization results.**

The optimal function combining tuition and individual utility is given by:

$$U_{ir}[y_i(1-t) - p_{ijr}^*, q_{ijr}] = U^*(b, y) = W_{ir}(t, y_i, \theta_{jr}, b_i) \quad \forall(b, y) \quad (2)$$

The optimal level of tuition is:

$$\alpha_{ju}(b, y) \in [0, N] \quad (3)$$

$$\alpha_{jr}(b, y) \in [0, P - N]$$

$$p_{ijr}^*(b_i, y_i, \theta_{jr}) = V'(k_{jr}) + \mu_{jr}(\theta_{jr} - b_i) \quad \forall(b, y) \quad \text{Et} \quad i=1, 2, 3, \dots, n$$

The Lagrangian multiplier is:

$$\mu_{jr} = \frac{1}{k_{jr}} \iint_s \left[ \frac{\partial p_{ijr}^*(b_i, y_i, \theta_{jr})}{\partial \theta_{jr}} \alpha_{jr}(b, y) f(b, y) db dy \right] \quad i=1, 2, 3, \dots, n \quad (4)^{12}$$

In our hypothesis we have considered that private school quality depends on school expenditure per student and scholastic achievement. In our pricing function  $p_{ijr}^*$  is the price of this quality. This price is formed of two parts; fraction  $V'(k_{jr})$  represents a homogenous price for all students resulting from the production cost of education, fraction  $\mu_{jr}(\theta_{jr} - b_i)$  represents the price resulting from the difference between own ability and mean ability (peer group effect).

Students with ability levels lower than school mean ability  $b_i < \theta_{jr}$  will have to pay a tuition premia equal to  $\mu_{jr}(\theta_{jr} - b_i) > 0$ . Students with ability levels higher than school mean ability  $b_i > \theta_{jr}$  will receive tuition discount equal to  $\mu_{jr}(\theta_{jr} - b_i) < 0$ .

From a student point of view;  $p_{ijr}^*$  is the reservation price. If an individual is not capable of paying that price he will not be admitted in this particular school. This price also represents the marginal cost of admitting a supplementary student in the school. For individuals with ability levels below mean ability,  $\mu_{jr}(\theta_{jr} - b_i) > 0$  is a positive cost; conversely for students with ability levels above mean ability this  $\mu_{jr}(\theta_{jr} - b_i) < 0$  becomes a profit. We denote  $MC_j = p_{ijr}^* = V'(k_{jr}) + \eta_{jr}(\theta_{jr} - b_i)$  the marginal cost of accepting an individual  $i$  in a school  $j$ . Thus an individual wishing to be enrolled in a school  $j$ , must be able to pay  $p_{ijr} \geq MC_j$ .

It is of a great importance to note that the price of private education can be negative. For students with ability largely superior to school mean ability  $\theta_{jr}$ ; we

---

<sup>12</sup> Results (3) and (4) are obtained by forming a Lagrangian function to take account of o constraints (1c) and (1d), then by optimizing over  $\alpha_{jr}$ . Mathematical details are to be found in the appendix.

have  $\mu_{jr}(\theta_{jr} - b_i) < 0$  and superior to  $V'(k_{jr})$ . In this case  $V'(k_{jr}) + \mu_{jr}(\theta_{jr} - b_i) < 0$ . This negative price represents a financial aide to highly performing students whose permit an important amelioration of peer group effect. This pricing strategy represents the corner stone of our meritocracy system.

### B- Majority voting results.

Proofs of propositions and lemmas are to be found in the appendix.

Notation remark: If  $(b', y')$  is better than  $(\hat{b}, \hat{y})$ . It is denoted  $(b', y') > (\hat{b}, \hat{y})$ . This means that  $b' > \hat{b}$  and  $y' > \hat{y}$ , or  $b' > \hat{b}$  and  $y' = \hat{y}$ , or  $b' = \hat{b}$  and  $y' > \hat{y}$ , or  $b'$  is weakly inferior to  $\hat{b}$  while  $y'$  is largely superior to  $\hat{y}$ , or finally  $b'$  is largely superior to  $\hat{b}$  while  $y'$  is weakly inferior to  $\hat{y}$ . With  $(b', y')$  and  $(\hat{b}, \hat{y})$  two couples of ability and income.

**Proposition 1:** we suppose that  $\lim_{c \rightarrow \infty} U(c, q) = 0 \quad \forall q > 0$ .

Given  $t \in [0, 1]$ ,  $N \in (0, P)$  and  $Y \in \mathfrak{R}^+$ . There exists a unique couple  $(\hat{b}, \hat{y})$  with  $\hat{b} > 0$  and  $\hat{y} > 0$  such that  $W_{iu} = W(t, y_i, Y, N, \theta_{ju}, b_i) \geq W_{ir} = W(t, y_i, \theta_{jr}, b_i)$  if and only if  $y_i < \hat{y}$  and  $b_i < \hat{b}$ .

In this proposition, we ignore extreme cases. For  $t=0$ , we have  $N=0$ , nobody will choose the public sector. Conversely for  $t=1$ , we have  $N=1$ , everybody will choose the public sector. An important remark to take into account is that the positioning of  $(\hat{b}, \hat{y})$  will depend on income and ability distribution.

**Lemma 1:** as in proposition 1 we suppose that  $\lim_{c \rightarrow \infty} U(c, q) = 0 \quad \forall q > 0$ .

Given  $t \in [0,1]$  ,  $N \in (0, P)$  and  $Y \in \mathfrak{R}^+$  . (i) Individuals with  $y_i < \hat{y}$  and  $b_i < \hat{b}$  will be enrolled in the public sector. (ii) Individuals with  $y_i > \hat{y}$  and  $b_i > \hat{b}$  will be enrolled in the private sector. (iii) Individual with  $y_i < \hat{y}$  and  $b_i > \hat{b}$  will be enrolled in the private sector and will receive tuition discounts. (iiii) Individuals with  $y_i > \hat{y}$  and  $b_i < \hat{b}$  will be enrolled in the private sector and will pay tuition premia.

(i) And (ii) represent the outcome of proposition 1. The proof of (iii) and (iiii) depends on a number of considerations; first on the positive correlation between income and private school quality; second on the fact that  $\lim_{c \rightarrow \infty} U(c, q) = 0$ , in other words when consumption increases and becomes largely superior to school quality (when most of individual income is dedicated to that consumption), individual utility tends to decrease. For a balanced growth of utility, both components must grow or at least school quality must grow with a small decrease of consumption.

**Lemma 2:** in this lemma we establish the properties of the couple  $(\hat{b}, \hat{y})$  relatively to N, Y and t. (i) For  $N \in (0, P)$ , the couple  $(\hat{b}, \hat{y})$  is decreasing in N. (ii) For  $Y \in \mathfrak{R}^+$  , the couple  $(\hat{b}, \hat{y})$  is increasing in Y. (iii) For  $t \in [0,1]$ , the couple  $(\hat{b}, \hat{y})$  is increasing in t.

Proofs for (i) and (ii) are based on the fact that private school quality is not related to N and Y. These factors will only influence school choice through public school quality. Proof for (iii) is based on the fact that t will have the same influence on consumption in both sectors. t also will have an effect on public school quality without affecting that of private schools.

When we say  $(\hat{b}, \hat{y})$  increases, this means that both  $\hat{b}$  and  $\hat{y}$  increase, or one of them increases while the other remains the same, or one of them largely increases while the other weakly decreases. When we say  $(\hat{b}, \hat{y})$  decreases, it is exactly the reverse.

**Proposition 2:** for  $t \in [0,1]$  and  $Y \in \mathfrak{R}^+$ . There exists a unique  $N^*$  which determines the number of students in the public sector as a result of majority voting. The number of individuals with  $y_i < \hat{y}$  and  $b_i < \hat{b}$  must be exactly equal to  $N^*$ .

So far we have considered an exogenous tax rate; in the following sections the tax rate becomes endogenous through majority voting. For an individual  $i$ , the most preferred tax rate is denoted as:

$$t^*(b_i, y_i) = \text{Arg max} \{ \max[W_{iu}(t, y_i, Y, N, \theta_{ju}, b_i), W_{ir}(t, y_i, \theta_{jr}, b_i)] \}$$

This tax rate will maximize individual utility in the sector offering the highest indirect utility.

We denote  $t_{iu}(b_i, y_i)$  the tax rate that maximizes utility in the public sector for an individual  $i$  with a couple  $(b_i, y_i)$ .  $t_{iu}(b_i, y_i) = \arg \max W_{iu}(t, y_i, Y, N, \theta_{ju}, b_i)$ .

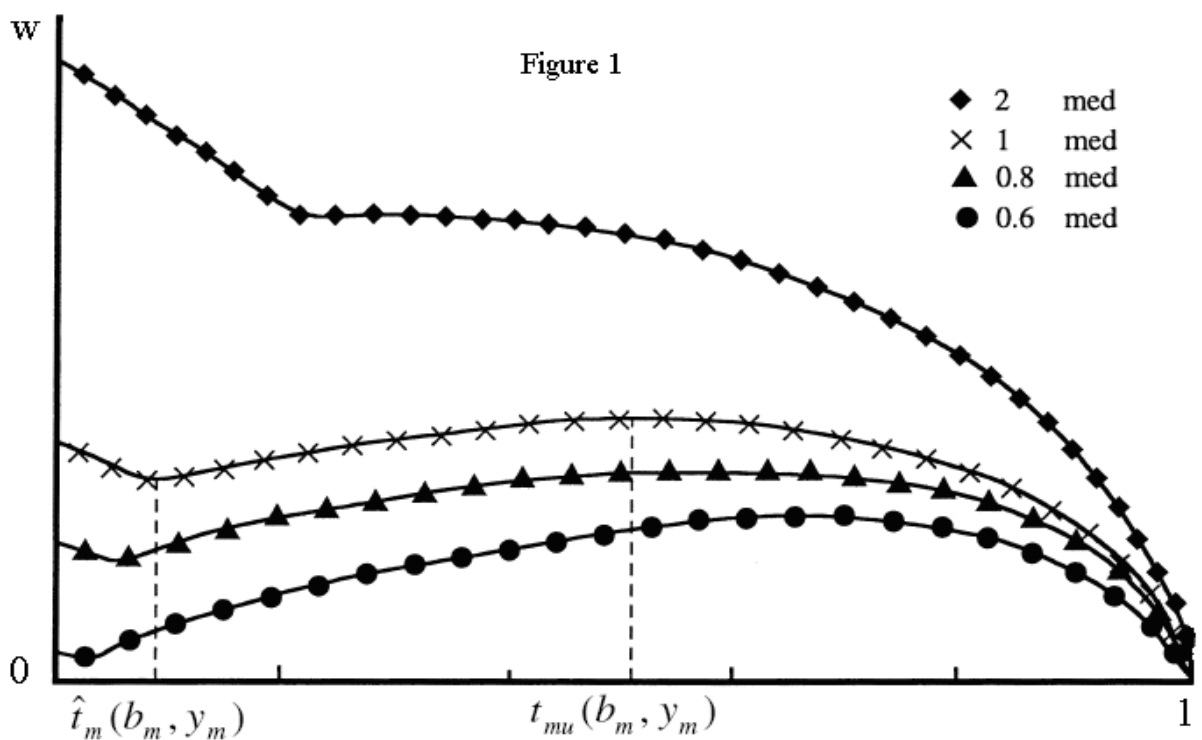
We denote  $\hat{t}_i(b_i, y_i)$  the tax rate for which an individual will be indifferent between the two sectors. In this case  $W_{iu}(t, y_i, Y, N, \theta_{ju}, b_i) = W_{ir}(t, y_i, \theta_{jr}, b_i)$ . It is clear that for each couple of ability and income, there exists a critical tax rate which determines the state of indifference. For low levels of  $t$ , the private sector is preferred; for high level of  $t$ , the public sector is preferred. While passing from an extreme to the other, there exists a tipping point of preferences determined by the tax rate  $\hat{t}_i(b_i, y_i)$ .

Joseph Stiglitz (1974) has presented the fact that preferences over tax rates are not single peaked. For low tax rates, public school quality is low and individuals prefer private schools. A marginal increase of the tax rate will not increase public school quality that much, but will reduce consumption; private schools will still be preferred and utility will be reduced. An important increase of the tax rate will induce an important increase of public school quality; at  $\hat{t}_i(b_i, y_i)$  individuals will be indifferent between the two sectors; from this point on, an increase in tax rate will reverse tendencies and the public sector will be preferred, utility is increasing in  $t$ . When the tax rate reaches  $t_{iu}(b_i, y_i)$  utility is



maximal in the public sector; from this point on utility is decreasing in  $t$ . Thus we find that utilities may have two peaks, one in each sector. This is represented in figure 1 quoted from Glomm and Ravikumar.

For extreme cases where individuals have low ability and income levels; they can not choose private schools and thus they are obliged to enroll in public ones; they have single peaked preferences. Conversely, individuals with high income and ability levels will choose private schools (See Lemma 1); they have single peaked preferences.



**Lemma 3:** The indifference tax rate  $\hat{t}_i(b_i, y_i)$  is increasing in the couple  $(b_i, y_i)$  over the interval  $(\hat{b}, b_{\max}) \times (\hat{y}, y_{\max})$ .

In this part of our analyses we study the existence of a majority voting equilibrium determined by the individual with the mean couple of income and ability that we denote  $(b_m, y_m)$ . We establish the necessary conditions for this existence.

We denote  $t_{mu}(b_m, y_m)$  the tax rate which maximizes the utility of this individual;  $\hat{t}_m(b_m, y_m)$  the tax rate for which the individual is indifferent between public and private sectors. We denote  $N_m$  the number of students evaluated under  $t_{mu}(b_m, y_m)$ .

The individual with the median couple of income and ability separates the population into two fractions of 50%. In order to demonstrate that a majority voting equilibrium exists and is determined by this individual; we have to demonstrate that there does not exist a tax rate that is preferred to  $t_{mu}(b_m, y_m)$  by more than 50% of the population. It is clear that for the individual with the median couple of income and ability;  $t_{mu}(b_m, y_m)$  is preferred. Thus, it will be sufficient to demonstrate that the fraction of the population with  $(b, y) > (b_m, y_m)$  or  $(b, y) < (b_m, y_m)$  prefer the tax rate  $t_{mu}(b_m, y_m)$  to prove the existence of a majority voting equilibrium determined by this individual.

Two conditions are necessary for the existence of a majority voting equilibrium determined by the individual with the median couple of income and ability:

1-  $t_{iu}(b_i, y_i)$  is decreasing in the couple  $(b_i, y_i)$ . For two individuals having  $(b_2, y_2) > (b_1, y_1)$  we have  $t_{1u}(b_1, y_1) > t_{2u}(b_2, y_2)$ .

2-  $W_{mr}[0, y_m, \theta_{jr} b_m] < W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N_m, \theta_{ju}, b_m]$ , this means that the individual with the median couple of income and ability prefer public education financed at a tax rate  $t_{mu}(b_m, y_m)$  to private education when the public sector does not exist.

Given individual behavior represented in figure 1; we are going to study the existence of majority voting equilibrium over 3 intervals:  $[0, \hat{t}_m(b_m, y_m)]$ ,  $[\hat{t}_m(b_m, y_m), t_{mu}(b_m, y_m)]$  et  $[t_{mu}(b_m, y_m), 1]$ .

**Lemma 4:** over the interval  $[t_{mu}(b_m, y_m), 1]$ , there does not exist a tax rate which is preferred to  $t_{mu}(b_m, y_m)$  by more than 50% of the population.

**Lemma 5:** over the interval  $[\hat{t}_m(b_m, y_m), t_{mu}(b_m, y_m)]$ , there does not exist a tax rate which is preferred to  $t_{mu}(b_m, y_m)$  by more than 50% of the population.

An important property used in the proofs of lemma 4 and 5 is based on the fact that public sector utility is increasing over the interval  $[\hat{t}_i(b_i, y_i), t_{iu}(b_i, y_i)]$  and decreasing over the interval  $[t_{iu}(b_i, y_i), 1]$  in relation to the tax rate.

**Lemma 6:**

(i) If  $W_{mr}[0, y_m, \theta_{jr}, b_m] < W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N, \theta_{ju}, b_m]$

Then  $W_{1r}[0, y_1, \theta_{jr}, b_1] < W_{1u}[t_{mu}(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1]$  For  $(b_1, y_1) < (b_m, y_m)$ .

(ii) If  $W_{mr}[0, y_m, \theta_{jr}, b_m] > W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N, \theta_{ju}, b_m]$

Then  $W_{1r}[0, y_1, \theta_{jr}, b_1] > W_{1u}[t_{mu}(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1]$  For  $(b_1, y_1) > (b_m, y_m)$ .

Lemma 6 means that if the individual with the mean couple of income and ability does prefer (or does not prefer) the positive tax rate  $t_{mu}(b_m, y_m)$  over zero; the fraction of the population having an inferior (or superior) couple of income and ability will have the same behavior as this individual.

**Proposition 3:** if  $W_{mr}[0, y_m, \theta_{jr}, b_m] < W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N, \theta_{ju}, b_m]$  is true, then the couple  $(t_m, N_m)$  is a majority voting equilibrium.

Conversely, if  $W_{mr}[0, y_m, \theta_{jr}, b_m] > W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N, \theta_{ju}, b_m]$ . All individuals with a couple of income and ability superior to  $(b_m, y_m)$  will prefer a zero tax rate because  $W_{1r}[0, y_1, \theta_{jr}, b_1] > W_{1u}[t_{mu}(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1]$ . The majority voting equilibrium is determined by the couple  $(0,0)$ .

If the first equilibrium condition is not satisfied, then a majority voting equilibrium may not exist. If the second condition is not satisfied then (0,0) is a majority voting equilibrium.

### **C- Private market equilibrium and the existence of a hierarchy of quality.**

In their analyses, Epple and Romano have demonstrated the existence of a hierarchy of school quality in the private sector. They have argued that if two schools have the same level of quality then they can increase their profit by modifying the student composition of the school. This strategy will allow a profit increase following a differentiation of quality<sup>13</sup>.

In our model, equilibrium in both sectors is characterized by the existence of a hierarchy of peer group effect.

$$\theta_{1r} < \theta_{2r} < \dots < \theta_{nr} \quad \text{For the private sector.}$$

$$\theta_{1u} < \theta_{2u} < \dots < \theta_{nu} \quad \text{For the public sector.}$$

This hierarchy of peer group effect leads to that of school quality in the public sector,  $q_{i1u} < q_{i2u} < \dots < q_{inu}$ , because public expenditure per student is equal in all public schools.

In private sector schools, we have considered that quality is dependent on school expenditure per student and scholastic achievement. School expenditure per student is different between private schools; this may cause our hierarchy to deviate from that of Epple and Romano.

---

<sup>13</sup> See Epple and Romano 1998, proposition 1.

However it should be noted that the structure of educational production cost will make private school expenditure per student  $\frac{C_{jr}}{\alpha_{jr}(b_i, y_i)}$  very similar between schools, the sole difference will be attributed to the fixed cost F. If two schools have the same quality; given the similarity of educational production costs, these schools can increase their profit by increasing or decreasing the educational quality they offer through a modification of peer group effect. This quality differentiation will be made by changing the student composition of the school. Similarly to Epple and Romano, we will obtain a private school hierarchy. This hierarchy exists when the effect of school expenditure per pupil on quality differentiation is limited, and when this differentiation is made through peer group effect.

In the private sector the hierarchy of peer group effect  $\theta_{1r} < \theta_{2r} < \dots < \theta_{nr}$  leads to that of school qualities  $q_{i1r} < q_{i2r} < \dots < q_{inr}$ . As the quality of private schools is always superior to that of public schools; we can write

$$q_{i1u} < q_{i2u} < \dots < q_{inu} < q_{i1r} < q_{i2r} < \dots < q_{inr}.$$

The presence of a quality hierarchy in the private sector will be at the origin of two equity related problems. First, private schools will operate to the detriment of public schools by siphoning higher ability and higher income students. Second, students with low ability and income will always be obliged to enroll in the public sector; this may reveal the existence of a problem of inequality of educational opportunities.

#### **IV- Conclusion**

In this paper, we have constructed an equilibrium on the market for education where individuals are differentiated by their ability and income. We have studied the distribution of students across sectors while mentioning the fact that private schools will capture high income high ability students to the detriment of the public sector. We have presented the necessary conditions for the existence of a majority voting equilibrium

determined by the individual with the median couple of income and ability. Finally, we have analyzed the presence of a hierarchy of quality in both sectors. Peer group effect was integrated to our analysis as the corner stone to our private discriminating pricing strategy.

According to our results, a fraction of the student population will never have the opportunity to access a more performing education, given its allocation of income and ability. It would be interesting for future studies to analyze equality of educational opportunity while considering peer group effect and private schools' strategic behavior as major factors of school choice.

## Appendix

### Optimization in a private school:

Private schools profit maximization:

$$MAX \pi_{jr} = \iint_s [p_{ijr}(b, y) \alpha_{jr}(b, y) f(b, y) dbdy] - V(k_{jr}) - F$$

Subject to:

$$k_{jr} = \iint_s \alpha_{jr}(b, y) f(b, y) dbdy \quad (1c)$$

$$\theta_{jr} = \frac{1}{k_{jr}} \iint_s b \alpha_{jr}(b, y) f(b, y) dbdy \quad (1d)$$

The two constraints can be integrated in a single one:

$$\theta_{jr} \iint_s \alpha_{jr}(b, y) f(b, y) dbdy = \iint_s b_i \alpha_{jr}(b, y) f(b, y) dbdy$$

$$\theta_{jr} \iint_s \alpha_{jr}(b, y) f(b, y) dbdy - \iint_s b_i \alpha_{jr}(b, y) f(b, y) dbdy = 0$$

The Lagrangian function is of the following form:

$$\int_s [p_{ijr}(b, y)\alpha_{jr}(b, y)f(b, y)dbdy] - V(k_{jr}) - F - \mu_{jr} \left[ \theta_{jr} \int_s \alpha_{jr}(b, y)f(b, y)dbdy - \int_s b_i \alpha_{jr}(b, y)f(b, y)dbdy \right]$$

With  $\mu_{jr}$  The Lagrangian multiplier.

Optimization is made through the partial derivation of the Lagrangian function over  $\alpha_{jr}(b, y)$ , results are the following:

$$p_{ijr}^* - V'(k_{jr}) - \mu_{jr}(\theta_{jr} - b_i) = 0$$

$$p_{ijr}^* = V'(k_{jr}) + \mu_{jr}(\theta_{jr} - b_i)$$

$$\mu_{jr} = \frac{1}{k_{jr}} \int_s \left[ \frac{\partial p_{ijr}^*(b_i, y_i, \theta_{jr})}{\partial \theta_{jr}} \alpha_{jr}(b, y)f(b, y)dbdy \right] \quad i=1, 2, 3, \dots, n$$

### Proof of proposition 1:

The public sector is chosen when:

$$W_{iu} = W(t, y_i, Y, N, \theta_{ju}, b_i) \geq W_{ir} = W(t, y_i, \theta_{jr}, b_i)$$

The couple  $(\hat{b}, \hat{y})$  must satisfy the following equation:

$$U_{iu}[(1-t)y_i, q_{iju}] = U_{ir}^*[(1-t)y_i - p_{ijr}^*, q_{ijr}^*] = U_{ir}^*(c_{ir}^*, q_{ijr}^*)$$

It should be noted that private school quality is always superior to that of public schools. This is consistent with the reality; if public schools have better qualities than private schools, rational individuals will choose free public schools and the private sector will not exist. A private school with a quality lower than that of a public school will be eliminated from the market by the simple functioning of competition. Thus we can write:

$$q_{iju} < q_{ijr} \quad \text{With } j \in N$$

In order to demonstrate the existence of  $(\hat{b}, \hat{y})$  we analyze two extreme cases.

**First case:**

For very low income levels, individuals can not pay private school tuition for the school with the lowest quality. We can write  $(1-t)y_i < p_{ijr}^*$ . For very low abilities, individuals can not benefit from a financial aide or a tuition discount. We have  $p_{ijr}^* > 0$ . As a consequence we can write  $c_{ir}^* = (1-t)y_i - p_{ijr}^* < 0$ .

Since consumption under the private sector becomes negative; these individuals can not pay the reservation price in the private sector, and no private school accepts to admit them. Thus they are obliged to choose the private sector  $U_{ir}^* = 0 < U_{iu}$ .

**Second Case:**

For very high incomes, individuals can pay private school tuition. We can write  $(1-t)y_i > p_{ijr}^*$ . For very high abilities, individuals can obtain financial aide (scholarships). We have  $p_{ijr}^* < 0$ . As a consequence we can write  $c_{ir}^* = (1-t)y_i - p_{ijr}^* > (1-t)y_i > 0$ .

Since individual consumption in the private sector becomes superior to that in the public sector, and given the superiority of quality in the former sector.  $U_{ir}^* > U_{iu}$ . Individuals will choose the private sector.

We have seen that for low levels of income and ability, the public sector is preferred; conversely for high levels of income and ability the private sector is preferred. When ability and income levels increase (going from one extreme to the other) it is clear that there exists a point for which preferences will be reversed. This point is characterized by a couple  $(\hat{b}, \hat{y})$  for which individuals will be indifferent between the two sectors.



Proof of Lemma 1:

(i) and (ii) are the outcome of proposition 1.  $(\hat{b}, \hat{y})$  is the indifference point between the two sectors. For individuals with  $y_i < \hat{y}$  and  $b_i < \hat{b}$  the public sector is preferred. For individuals with  $y_i > \hat{y}$  and  $b_i > \hat{b}$  the private sector is preferred.

For (iii) and (iii):

The couple  $(\hat{b}, \hat{y})$  must solve the following function:

$$U_{iu}[(1-t)y_i, q_{iju}] = U_{ir}^*[(1-t)y_i - p_{ijr}^*, q_{ijr}^*]$$

$q_{ijr}^*$  is positively correlated to  $y_i$ . When  $y_i$  increases, the individual purchasing power increases. Thus an individual will have a better reservation price and the quality he can obtain is higher.

**The right hand side of the equation is linear in y and b:**

If b increases  $\Rightarrow p_{ijr}^*$  decreases  $\Rightarrow [(1-t)y_i - p_{ijr}^*]$  increases  $\Rightarrow U_{ir}^*$  increases.

If b increases  $\Rightarrow q_{ijr}^*$  increases  $\Rightarrow U_{ir}^*$  increases.

Thus If b increases,  $U_{ir}^*$  increases in both arguments.

If y increases  $\Rightarrow [(1-t)y_i - p_{ijr}^*]$  increases  $\Rightarrow U_{ir}^*$  increases.

If y increases  $\Rightarrow q_{ijr}^*$  increases  $\Rightarrow U_{ir}^*$  increases.

Thus If y increases,  $U_{ir}^*$  increases in both arguments.

**The left hand side of the equation is strictly concave in y and b:**

If b increases  $\Rightarrow [(1-t)y_i]$  and  $U_{iu}$  remain the same.

If b increases  $\Rightarrow q_{iju}$  increases  $\Rightarrow U_{iu}$  increases.

For an income  $y = \hat{y}$ . If  $b$  increases,  $U_{iu}$  increases (for marginal growth in  $b$ , with  $b < \hat{b}$ ), but decreases for important growth of  $b$  ( $b > \hat{b}$ ).  $\lim_{b \rightarrow \infty} U_{iu} = 0$ .

Explanation: when  $b$  increases,  $U_{ir}^*$  increases in both arguments, while  $U_{iu}$  increases in  $q_{iju}$ . When  $b$  becomes important ( $b > \hat{b}$ ), public school quality increases but remains inferior to that of private schools while consumption remains the same. At this point, if the student is still enrolled in the public sector he will not benefit from his higher abilities. These abilities allow him to enroll in the private sector and to obtain a tuition discount. In this case educational quality will be much higher, and consumption will be higher or weakly lower than that of the public sector. Finally,  $U_{iu}$  increases for marginal growth of  $b$  and decreases when  $b$  becomes important. When  $b \rightarrow \infty$  we have  $U_{iu} \rightarrow 0$ .

If  $y$  increases  $\Rightarrow [(1-t)y_i]$  increases  $\Rightarrow U_{iu}$  increases.

If  $y$  increases  $\Rightarrow q_{iju}$  and  $U_{iu}$  remain the same.

$q_{iju}$  depends of the total income  $Y$ , individual income does not have any influence over total income.

For an ability  $b = \hat{b}$ , if  $y$  increases,  $U_{iu}$  increases (for marginal growth of  $y$ , with  $y < \hat{y}$ ), but decreases for important growth of  $y$  ( $y > \hat{y}$ ).  $\lim_{y \rightarrow \infty} U_{iu} = 0$ .

Explanation: when  $y$  increases,  $U_{ir}^*$  increases in both arguments, while  $U_{iu}$  increases in  $[(1-t)y_i]$ . When  $y$  becomes important ( $y > \hat{y}$ ), consumption under the public sector becomes important while public school quality remains the same. This quality becomes largely lower than that he can purchase in the private sector. At this point, if the student is still enrolled in the public sector he will not benefit from his higher income. His utility will decrease as  $\lim_{c \rightarrow \infty} U(c, q) = 0$ . Finally,  $U_{iu}$  increases for marginal growth of  $y$  and decreases when  $y$  becomes important. When  $y \rightarrow \infty$  we have  $U_{iu} \rightarrow 0$ .

By combining the two effects, we have  $\lim_{(b,y) \rightarrow \infty} U_{iu} = 0$ . Individual utility under the public school increases in  $(b_i, y_i)$  until it reaches its maximum when  $(b_i, y_i) = (\hat{b}, \hat{y})$ . Then it starts to decrease.

For two couples  $(b', y') < (b, y)$ . If  $(b, y) < (\hat{b}, \hat{y})$ , individual utility under the public sector for the individual with  $(b, y)$  is superior to that of the individual with  $(b', y')$ . If  $(b, y) > (\hat{b}, \hat{y})$ , individual utility under the public sector for the individual with  $(b, y)$  is inferior to that of the individual with  $(b', y')$ .

For two couples  $(b', y') < (b, y)$ . Individual utility under the private sector for the individual with  $(b, y)$  is superior to that of the individual with  $(b', y')$ .

It should be noted that individuals with  $(b, y) < (\hat{b}, \hat{y})$  can only choose public sector schools; while individuals with  $(b, y) > (\hat{b}, \hat{y})$  have the choice between public and private schools, even if individual utility is much higher in the private sector.

(iii) and (iiii) represent the outcome of previous explanations:

(iii) an individual with  $y_i < \hat{y}$  and  $b_i > \hat{b}$  can choose a private school which offers a tuition discount that allows his income to get closer to  $\hat{y}$ . In this case he will have a higher or weakly lower consumption than in the public sector but a largely better quality of education. (iiii) an individual with  $y_i > \hat{y}$  and  $b_i < \hat{b}$  can choose a private school that allows his ability to get closer and even surpass  $\hat{b}$  given his income level. If the individual remains in the public sector; his utility will decrease as consumption becomes important and educational quality remains the same ( $\lim_{c \rightarrow \infty} U(c, q) = 0$ ).

Proof of Lemma 2:

(i) For  $N \in (0, P)$ , the couple  $(\hat{b}, \hat{y})$  is decreasing in N. An increase of N represents a congestion effect of public schools; public expenditure per pupil  $\frac{tY}{N}$  decreases as N increases. As a consequence public school quality decreases. Students on the margin between sectors will opt out of the public sector to join the private one. In this case  $(\hat{b}, \hat{y})$  decreases.

(ii) For  $Y \in \mathfrak{R}^+$ , the couple  $(\hat{b}, \hat{y})$  is increasing of Y. An increase of Y implies a growth of public school quality following a growth of public expenditure per pupil  $\frac{tY}{N}$ . Students on the margin between sectors will opt out of the private sector to join the public one. In this case  $(\hat{b}, \hat{y})$  increases.

(iii) For  $t \in [0,1]$ , the couple  $(\hat{b}, \hat{y})$  is increasing of t. On one hand, an increase of t implies a decrease of consumption in both sectors, thus the effect of t on consumption as a factor of individual utility is neutral. On the other hand an increase of t induces a growth of public expenditure per pupil. As a consequence public school quality increases. Students on the margin between sectors will opt out of the private sector to join the public one. In this case  $(\hat{b}, \hat{y})$  increases.

Proof of proposition 2:

In Lemma 2 we have presented the fact the public school quality is decreasing in N. Conversely, private school quality is not dependent on N.

$$U_{iu}[(1-t)y_i, q_{iju}] = U_{ir}^*[(1-t)y_i - p_{ijr}^*, q_{ijr}^*]$$

The left hand side of the equation is decreasing in N; while the right hand side of the equation is independent of N. For N very low, public school quality is very high. For N very high, public school quality is very low.

While passing from one extreme to the other. The equality or in other words the intersection between the decreasing and the steady function determines the unique equilibrium number of students  $N^*$ .

Proof of Lemma 3:

We consider two individuals, the first has a couple  $(b_1, y_1)$  and the second has a couple  $(b_2, y_2)$ . We consider that  $(b_1, y_1) > (b_2, y_2) > (\hat{b}, \hat{y})$ .

We suppose that  $\hat{t}_1(b_1, y_1) < \hat{t}_2(b_2, y_2)$

For the individual with  $(b_1, y_1)$ , we have:

$$W_{1u}[\hat{t}_1(b_1, y_1), y_1, Y, N, \theta_{ju}, b_1] = W_{1r}[\hat{t}_1(b_1, y_1), y_1, \theta_{jr}, b_1] \quad (a)$$

For the individual with  $(b_2, y_2)$ , we have:

$$W_{2u}[\hat{t}_2(b_2, y_2), y_2, Y, N, \theta_{ju}, b_2] = W_{2r}[\hat{t}_2(b_2, y_2), y_2, \theta_{jr}, b_2]$$

According to figure 1 and the explanation of individual behavior, for each  $t < \hat{t}_2(b_2, y_2)$ , we have:

$$W_{2u}[t, y_2, Y, N, \theta_{ju}, b_2] < W_{2r}[t, y_2, \theta_{jr}, b_2]$$

As we have  $\hat{t}_1(b_1, y_1) < \hat{t}_2(b_2, y_2)$ , we can write:

$$W_{2u}[\hat{t}_1(b_1, y_1), y_2, Y, N, \theta_{ju}, b_2] < W_{2r}[\hat{t}_1(b_1, y_1), y_2, \theta_{jr}, b_2] \quad (b)$$

As we have  $(b_1, y_1) > (b_2, y_2)$ , (See proof of lemma 1) we can write:

$$W_{2r}[\hat{t}_1(b_1, y_1), y_2, Y, N, \theta_{ju}, b_2] < W_{1r}[\hat{t}_1(b_1, y_1), y_1, \theta_{jr}, b_1] \quad (c)$$

From (a), (b), and (c) we can write:

$$W_{2u}[\hat{t}_1(b_1, y_1), y_2, Y, N, \theta_{ju}, b_2] < W_{1u}[\hat{t}_1(b_1, y_1), y_1, Y, N, \theta_{ju}, b_1] \quad (d)$$

In lemma 1, we have seen that for  $(b_1, y_1) > (b_2, y_2)$  with  $(b_1, y_1) > (b_2, y_2) > (\hat{b}, \hat{y})$ , we have:

$W_{2u}[\hat{t}_1(b_1, y_1), y_2, Y, N, \theta_{ju}, b_2] > W_{1u}[\hat{t}_1(b_1, y_1), y_1, Y, N, \theta_{ju}, b_1]$ . This is a contradiction with (d).

In this case, our supposition that  $\hat{t}_1(b_1, y_1) < \hat{t}_2(b_2, y_2)$  is wrong. This implies that the reverse is true  $\hat{t}_1(b_1, y_1) > \hat{t}_2(b_2, y_2)$ . As a consequence the indifference tax rate  $\hat{t}_i(b_i, y_i)$  is increasing in the couple  $(b_i, y_i)$  over the interval  $(\hat{b}, b_{\max}) \times (\hat{y}, y_{\max})$ .

The case where  $(b_1, y_1) < (\hat{b}, \hat{y})$  or  $(b_2, y_2) < (\hat{b}, \hat{y})$ , does not have any influence, individuals are obliged to choose the public sector and they do not have an indifference point between sectors.

Proof of Lemma 4: For  $t \in [t_{mu}(b_m, y_m), 1] \Rightarrow t > t_{mu}(b_m, y_m)$ .

We consider an individual 1, he has  $(b_1, y_1) > (b_m, y_m)$ .

The first condition of the existence of a majority voting equilibrium determined by the individual with the median couple of income and ability implies:

$(b_1, y_1) > (b_m, y_m) \Rightarrow t_{mu}(b_m, y_m) > t_{1u}(b_1, y_1)$ . We have two situations to analyze:

1- For an individual with  $(b_1, y_1)$ , we suppose that  $\hat{t}_1(b_1, y_1) < t_{1u}(b_1, y_1)$

From figure 1, this individual will be in the public sector for  $t \in [t_{mu}(b_m, y_m), 1]$ . We can write:  $\hat{t}_1(b_1, y_1) < t_{1u}(b_1, y_1) < t_{mu}(b_m, y_m) < t \in [t_{mu}(b_m, y_m), 1]$

As public sector's utility is decreasing in  $t$  over the interval  $[t_{1u}(b_1, y_1), 1]$ .

The individual will prefer  $t_{mu}(b_m, y_m)$  to any other tax rate  $t > t_{mu}(b_m, y_m)$ .

In this case, we have:  $W_{1u}[t_{mu}(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1] > W_{1u}[t, y_1, Y, N_m, \theta_{ju}, b_1]$

2- For an individual with  $(b_1, y_1)$  we suppose that  $\hat{t}_1(b_1, y_1) > t_{1u}(b_1, y_1)$ .

From figure 1, this individual will be in the private sector  $\forall t$ .

As private sector's utility is decreasing in  $t \forall t$ . The individual will prefer the lowest tax rate over the interval, thus he chooses  $t_{mu}(b_m, y_m)$ .

In this case, we have:  $W_{1r}[t_{mu}(b_m, y_m), y_1, \theta_{jr}, b_1] < W_{1r}[t, y_1, \theta_{jr}, b_1]$

Over the interval  $[t_{mu}(b_m, y_m), 1]$ , there does not exist a tax rate different than  $t_{mu}(b_m, y_m)$  which represents a majority voting equilibrium.

Proof of Lemma 5: For  $t \in [\hat{t}_m(b_m, y_m), t_{mu}(b_m, y_m)]$ , we consider an individual 1, he has  $(b_1, y_1) < (b_m, y_m)$ .

$t \in [\hat{t}_m(b_m, y_m), t_{mu}(b_m, y_m)] \Rightarrow t < t_{mu}(b_m, y_m)$

As public sector's utility is increasing in  $t$  over this interval. The individual will choose the highest tax rate over this interval, thus he chooses  $t_{mu}(b_m, y_m)$ .

In this case, we have:  $W_{1u}[t_{mu}(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1] > W_{1u}[t, y_1, Y, N_m, \theta_{ju}, b_1]$

Over the interval  $[\hat{t}_m(b_m, y_m), t_{mu}(b_m, y_m)]$ , there does not exist a tax rate different than  $t_{mu}(b_m, y_m)$  which represents a majority voting equilibrium.

Proof of Lemma 6:

(i) For  $(b_1, y_1) < (b_m, y_m)$  we can write:

$W_{mu}[t_m(b_m, y_m), y_m, Y, N_m, \theta_{ju}, b_m] < W_{1u}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1]$

With  $(\hat{b}, \hat{y}) < (b_1, y_1) < (b_m, y_m)$ .

If  $W_{mr}[0, y_m, \theta_{jr}, b_m] < W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N_m, \theta_{ju}, b_m]$

And since  $W_{1r}[0, y_1, \theta_{jr}, b_1] < W_{mr}[0, y_m, \theta_{jr}, b_m]$  as  $(b_1, y_1) < (b_m, y_m)$ .

We can write:

$W_{1r}[0, y_1, \theta_{jr}, b_1] < W_{mr}[0, y_m, \theta_{jr}, b_m] < W_{mu}[t_{mu}(b_m, y_m), y_m, Y, N_m, \theta_{ju}, b_m]$

$< W_{1u}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1]$

Thus:  $W_{1r}[0, y_1, \theta_{jr}, b_1] < W_{1u}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1]$ .

(ii) For  $(b_1, y_1) > (b_m, y_m)$  we can write:

$$W_{mu}[t_m(b_m, y_m), y_m, Y, N_m, \theta_{ju}, b_m] > W_{1u}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1]$$

With  $(\hat{b}, \hat{y}) < (b_1, y_1) < (b_m, y_m)$ .

$$\text{If } W_{mr}[0, y_m, \theta_{jr}, b_m] > W_{mu}[t_m(b_m, y_m), y_m, Y, N_m, \theta_{ju}, b_m]$$

And since  $W_{1r}[0, y_1, \theta_{jr}, b_1] > W_{mr}[0, y_m, \theta_{jr}, b_m]$  as  $(b_1, y_1) > (b_m, y_m)$ .

We can write:

$$W_{1r}[0, y_1, \theta_{jr}, b_1] > W_{mr}[0, y_m, \theta_{jr}, b_m] > W_{mu}[t_m(b_m, y_m), y_m, Y, N_m, \theta_{ju}, b_m]$$

$$> W_{1u}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1]$$

Thus:  $W_{1r}[0, y_1, \theta_{jr}, b_1] > W_{1u}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1]$ .

The case where  $(\hat{b}, \hat{y}) > (b_1, y_1)$  and  $(\hat{b}, \hat{y}) > (b_m, y_m)$ , has no influence since individual 1 and individual m have no choice between the two sectors. They are obliged to enroll in public schools.

### Proof of proposition 3:

In this proposition, we have to demonstrate that over the interval  $[0, \hat{t}_m(b_m, y_m)]$ , there does not exist a tax rate which is preferred to  $t_{mu}(b_m, y_m)$  by more than 50% of the population. For an individual with  $(b_1, y_1) < (b_m, y_m)$  we have two situations to analyze.

The interval is divided into two  $[0, \hat{t}_1(b_1, y_1)]$  and  $[\hat{t}_1(b_1, y_1), \hat{t}_m(b_m, y_m)]$ .

1- For  $t \in [0, \hat{t}_1(b_1, y_1)] \Rightarrow t < t_{mu}(b_m, y_m)$ .

Since  $W_{1r}[t, y_1, \theta_{jr}, b_1] < W_{1r}[0, y_1, \theta_{jr}, b_1]$ . (As  $W_{1r}$  is decreasing in t).

And since  $W_{1r}[0, y_1, \theta_{jr}, b_1] < W_{1u}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1]$ . (From the first part of Lemma 6).



We can write:  $W_{1r}[t, y_1, \theta_{jr}, b_1] < W_{1u}[t_m(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1]$

Thus the tax rate  $t_{mu}(b_m, y_m)$  is preferred to any other tax rate over the interval  $[0, \hat{t}_1(b_1, y_1)]$ .

2- For  $t \in [\hat{t}_1(b_1, y_1), \hat{t}_m(b_m, y_m)] \Rightarrow t < t_{mu}(b_m, y_m)$ .

Over this interval, public sector's utility is increasing in  $t$ . The individual chooses the highest tax rate  $t_{mu}(b_m, y_m)$ .

$$W_{1u}[t_{mu}(b_m, y_m), y_1, Y, N_m, \theta_{ju}, b_1] > W_{1u}[t, y_1, Y, N_m, \theta_{ju}, b_1]$$

In conclusion we find that over the three intervals  $[0, \hat{t}_m(b_m, y_m)]$ ,

$[\hat{t}_m(b_m, y_m), t_{mu}(b_m, y_m)]$  and  $[t_{mu}(b_m, y_m), 1]$ . There does not exist any tax rate that is preferred to  $t_{mu}(b_m, y_m)$  by more than 50% of the population. Thus the couple  $(t_m, N_m)$  is a majority voting equilibrium.

## References

- Clotfelter C. 1976. School desegregation, Tipping and private school enrollment. *The Journal of human resources* Vol 11 No 1 (28-50).
- Epple D. and R. Romano, 1998. Competition between public and private schools, Vouchers, and peer group effects. *The American Economic Review* Vol 88 No 1 (33-62).
- Glomm G. and B. Ravikumar, 1998. Opting out of publicly provided services: A majority voting Equilibrium. *Social Choice and Welfare* Vol 15 (187-199).
- Henderson V; P. Mieszkowski, and Y. Sauvageau, 1978. Peer group effects in educational production functions. *Journal of Public Economics* Vol 10 No 1 (97-106).
- Kenny L. 1982. Economies of scale in schooling. *Economics of Education Review* Vol 2 No1 (1-24).
- Kulik J. and C. Kulik, 1992. Effects of ability groupings on secondary school students: A Meta analysis of evaluation findings. *American Education Research Journal* Vol 19 No 3 (415-428).
- Scotchmer S. and M. Wooders, 1987. Competitive equilibrium and the core in club economics with non anonymous crowding. *Journal of Public Economics*, Vol34 No 2. (159-173).
- Sonstelie J. 1979. Public school quality and private school enrollments. *National Tax Journal* (343-353).
- Sorensen A. and M. Hallinan, 1986. Effects of ability grouping on growth in academic achievement. *American Educational Research Journal* Vol 24 No 4 (519-542).
- Stiglitz J. 1974. The demand for education in public and private school systems. *Journal of public economics* Vol 3 No 4 (349-385).
- Summers A. and B. Wolfe, 1977. Do schools make a difference? *American Economic Review* Vol 67 No 4 (639-652).