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# Monetary Policy and Inflation Divergences in a Heterogeneous Monetary Union

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## **Abstract**

*It is widely recognized that the Euro area is an asymmetric monetary union which assembles countries with heterogeneous structures on financial, goods and labour markets stricken by asymmetric shocks. However, the main objective of the European Central Bank (ECB) is to preserve price stability for the euro area as a whole, and the ECB pays most of its attention to union-wide output and (principally) inflation, neglecting, at least on the level of principles, inflation and output divergences in union. In this paper, we wonder, at a theoretical level, about the social loss associated with such an objective based on aggregate magnitudes, and we search for solutions, namely an “optimal” contract for a common central bank. We show in particular that it is not necessarily a good thing that a common central bank worries about inflation divergences without being concerned about output divergences in union.*

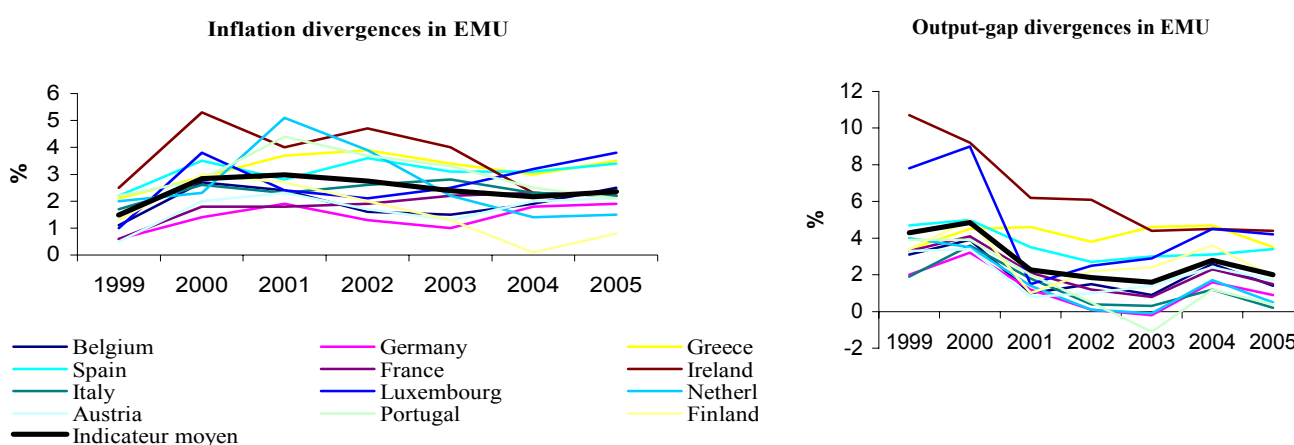
Should a common central bank in a heterogeneous monetary union consider national divergences, and how should she do it? This question is at the heart of the monetary policy matter in the Euro area. It is widely recognized that the Euro area is an asymmetric monetary Union which assembles countries with heterogeneous structures on financial, goods and labour markets stricken by asymmetric shocks. However, the main objective of the European Central Bank (ECB) is to preserve price stability for the euro area as a whole, and the ECB pays most of its attention to Union-wide output and (principally) inflation, neglecting, at least on the level of principles, inflation and output divergences in Union. In this paper, we wonder, at a theoretical level, about the social loss associated with such an objective based on aggregate magnitudes, and we search for solutions, namely an “optimal” contract for a common central bank.

Inflation and output-growth divergences in the Euro area are well documented<sup>1</sup>. In 2005, for instance, the degree of inflation and of output dispersion, measured as the un-weighted standard deviation among the 12 EMU countries was respectively about 0.86 and 1.5 percentage points. These values reflect very different inflation and output-gap positions in the area (see *Table 1*)

*Table 1: Inflation and output-gap in EMU*

<i>Year</i>	<i>1999</i>	<i>2000</i>	<i>2001</i>	<i>2002</i>	<i>2003</i>	<i>2004</i>	<i>2005</i>
<i>INFLATION</i>	<i>0.72</i>	<i>1.03</i>	<i>1.06</i>	<i>1.14</i>	<i>0.97</i>	<i>0.85</i>	<i>0.86</i>
<i>OUTPUT-GAP</i>	<i>2.52</i>	<i>2.06</i>	<i>1.67</i>	<i>1.77</i>	<i>1.84</i>	<i>1.29</i>	<i>1.5</i>

*Un-weighted standard deviation of inflation and output-gap in EMU countries*



Perhaps more worrying is the persistent character of these divergences. Since the launch of the EMU, the inflation rate was more than one percentage point higher than average in Spain, Greece, Portugal and Ireland, while was continuously below average in Germany (OECD, 2005). Furthermore, the cross-countries dispersion did not decrease since the launch of the EMU, and most countries have systematically maintained either a positive or a negative inflation gap against the Euro area average (Buseti *et al.*, 2006, ECB 2005), even if some arguments plead for a decrease of asymmetries in the long-run (Park, 2002). Moreover, the enlargement of EMU is likely to emphasize the relative size of output-gap and inflation divergences, since the new participants’ relatively low level of economic development will

<sup>1</sup> See, for example, the recent papers of Andres, Ortega & Vallès (2003), Altissimo, Angeloni & Ehrmann (2004), Musso & Westermann (2005) or Benigno & Palenzuela (2005). The ECB (2005) provides an analysis of the causes and implications of those divergences for the conduct of the monetary policy.

structurally produce higher than average output growth and inflation rates (through a Balassa-Samuelson effect), during the catching-up path. Enlargement of the EMU will also probably increase the heterogeneity in the transmission channel of the common monetary policy, since the new participants experience different processes of financial liberalization.

A number of recent theoretical papers have shown that the existence of asymmetries in the transmission of monetary policy calls for a design of monetary policies that takes into account national data and not only average or aggregated (EMU) data. The superiority of strategies based upon national data has notably been established by De Grauwe (2000), Gros & Hefeker (2002) and De Grauwe & Senegas (2003)<sup>2</sup>. Despite these results, it seems that the monetary policy design in the Euro area is mostly (and perhaps exclusively) based upon aggregated magnitudes. For example, in the Governing Council congregation of September, 09, 1999, President Duisenberg asserted: “...our decisions today, again and as always, were based on a euro area-wide analysis (...) –and nothing else”. This point is strengthened by the declaration following the Governing Council meeting dated from March, 30 (2000): “as laid down in the Treaty, each member of the Governing Council is therefore well aware that he or she is not a representative of a country (...) but acts (...) in deciding the appropriate conduct of monetary policy for the euro area as a whole”<sup>3</sup>. However, while the common monetary policy does not address regional economic dispersion in its policymaking, a great part of the recent debate in monetary policy has tended to focus upon the implications of inflation differentials for the formulation of monetary policy (with few or no reference to output differentials).

In this paper, we construct a theoretical model to assess the benefits of a common central bank that worries about inflation differentials in a monetary union. We model a simple two-country monetary union in which monetary policy is designed by a common central bank that is only concerned about average magnitudes (inflation and output-gap). The common central bank possesses its own loss function (that we call “centralized” loss function), which differs from the union loss function, which is the average of national loss functions (that we call “coordinated” loss function). What is the cost of this “wrong” objective, in terms of social welfare?

In a homogenous union, without “structural” heterogeneity neither in the transmission process of monetary policy nor in the objective defended by the member states of union, the difference between the two loss function is irrelevant: using a centralized loss function or a coordinated loss function gives rise to the same solution, even if each country is stricken by idiosyncratic shocks. In such a homogenous union, neglecting regional inflation or output differentials does not involve any cost.

Thus, in order to deal with costs associated with a centralized policymaking, compared with a coordinated one, one has to consider some degree of “structural” heterogeneity in the union. If the transmission channel of monetary policy is asymmetric, for example, a centralized policymaking is less efficient, from the union-wide welfare point of view, than a “coordinated” one, a result established by Gros & Hefeker (2002) and De Grauwe & Senegas (2003). In this model, we introduce heterogeneity in the simplest way, namely in the

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<sup>2</sup> In addition, several results in literature emphasize the benefits of using national information (De Grauwe, Dewachter & Aksoy, 1999, De Grauwe, 2000). Monteforte & Siviero (2003) find that the cost of neglecting national information may be very high (a welfare loss near than 50%), in contrast with De Grauwe & Piskorski (2001) who find a welfare loss about 5 to 10%. De Grauwe & Sénégas (2003) find that uncertainty reinforces the case for using a national perspective.

<sup>3</sup> But the Gonzales-Paramo (2005) indicates that “this does not mean that the ECB looks exclusively at the euro area-wide information”, but “analyzes all relevant information relating to the various sectors and countries of the euro area”.

transmission channel of the common interest rate to aggregate demand. Our model neglects other sources of heterogeneity in Union, such as labour market heterogeneity (in link with divergences in wage setting) or goods market heterogeneity (in link with differences in cyclical positions, levels of economic development or economic openness degrees, for example); not because these sources of heterogeneity are less significant, but because they exceed the pure monetary dimension. Furthermore, the heterogeneity of the transmission channel of monetary policy is a direct clause of concern for the ECB<sup>4</sup>. In addition to this “structural” asymmetry, we introduce idiosyncratic supply and demand shocks. So, in some sense, our paper can be viewed as an extension of Gros & Hefeker (2002) and De Grauwe & Senegas (2003), with a precise modelling of asymmetry in the monetary transmission process and in the shocks that affect the different countries, in which we explicitly study the optimal contract for the common central bank<sup>5</sup>.

Our model shows that, under general assumptions, the inefficiency associated with a “centralized” monetary policy design (relative to a “cooperative” one) can be removed by setting an “optimal” contract for the central bank. This optimal contract penalizes the common central bank from inflation and output divergences in the union. We show that the form of this optimal contract is very simple: the penalties imposed on inflation (respectively on output) divergences correspond to the relative weight of inflation (respectively output) in the social welfare function.

The interpretation of the “optimal contract” is straightforward: for monetary policy to take account of union heterogeneity, one has to force the common central bank to feel some aversion towards inflation and output divergences. If this degree of aversion towards divergences is well defined, as it is the case under the optimal contract, the common monetary policy produces the first best.

This result must receive some qualifications. It only holds if the different member states and the common central bank share the same relative preferences for output and inflation stabilization. In others cases, the optimal contract for the central bank is more complicated, becomes model-dependent and does not necessary produce the first best solution. Thus, introducing inflation differential in the loss function of the common central bank is welfare improving if she possess the “right” relative preference for output stabilization and if she also takes account of output differential.

However these two conditions are unlikely to be fulfilled in EMU. Then, we turn our attention to situations in which the simple optimal contract cannot be implemented. If the common central bank possesses peculiar relative preferences for the stabilization of output and inflation, for example, our model shows that one can still find an optimal contract for monetary policy; but in the optimal contract, the penalties imposed on inflation (respectively on output) divergences are higher than the relative weights of inflation (respectively output) in the social welfare function and are inversely linked with the degree of heterogeneity in the Union. If it is difficult to design a monetary policy that responds to output divergences in the Union, on the other hand, no optimal contract can be implemented. In such a situation, we search for “second” best contracts for the central bank. Our model shows notably that, if output divergences are not a clause of concern for the common central bank, the second best coefficient for inflation divergence is not necessarily positive. Thus, attempting to reduce

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<sup>4</sup> The relative size of “credit channel” or “interest channel” of monetary policy in EMU countries, for example, may produces divergent effects of monetary policy impulses. Such divergences are studied in particular by Coudert & Mojon (1995), Cecchetti (1999), Issing & al. (2001), Peersman & Smets (2001), Mojon & Peersman (2001) and Angeloni & al. (2002), Clausen, Hayo (2002), Penot, Pollin (2003), Ruth (2004), Berben, Locarno., Morgan, Vallés (2005), for example. Moreover, the enlargement of EMU will probably heighten uncertainty about the transmission channels (Hefeker, 2004).

<sup>5</sup> Gros & Hefeker (2002) and De Grauwe & Senegas (2005) only consider symmetric shocks, but, as we shall see, the mix between symmetric and asymmetric shocks strongly affects the form of the central bank contract.

inflation divergences in a heterogeneous monetary Union is not necessarily a good prescription, if this prescription is not supported by an output divergences-oriented device.

The paper is structured as follows. Section 1 introduces the main characteristics of the model. Besides demand and supply shocks, we introduce structural asymmetries on the interest rate transmission channel. Section 2 investigates the cost of a centralized monetary policy design, relative to the optimal “cooperative” solution. In section 3 we assess the optimal contract for the common central bank, and in section 4 and section 5 we study respectively how the optimal contract changes when the central bank does not share social preferences for the stabilization of output relative to inflation, and the form of “second best” contracts when she does not take output divergences into consideration.

## 1/ The model

Our model depicts a two-country closed monetary union. The two countries have the same size and are indexed by  $h$  and  $f$  respectively. Supply functions are defined by:

$$y_i^s = \alpha\pi_i + \mu_i, \quad i = h, f \quad (1)$$

where  $\pi_i$  is the inflation rate and  $\mu_i$  is a white noise supply shock with variance  $\sigma_{\mu_i}^2$ . All variables are specified in log-deviations (in particular, the natural level of output is zero, and all expected quantities are set to zero). Thus, relation (1) depicts a “Lucas supply function”, in which equilibrium output can exceed natural product only when some “surprises” are present, either because of an exogenous supply shock or because of an inflation surprise which produces an ex post under-indexation of wages.

Notice that, compared to Gros & Hefeker (2002) and De Grauwe & Senegas (2003); we suppose here that inflation rates may be different in the two countries. It is an important characteristic of our model, since we want to study the optimal way for the common central bank to take account of inflation divergences in the union. Thus, we cannot suppose, as these authors, that the central bank directly controls the inflation rate. In contrast, we must specify demand functions and study the monetary transmission process. In order to focus on heterogeneity in the union, we specify very simple demand functions. In country  $i$ , demand depends on the union-wide interest rate ( $r$ ), which is the monetary policy instrument set by the common central bank, on the inflation differential ( $\pi - \pi_i$ ), which depicts country  $i$  competitiveness ( $\pi = \frac{1}{2}(\pi_h + \pi_f)$  is the average inflation in the union) and is affected by a white noise demand shock ( $\delta_i$ ) with variance  $\sigma_{\delta_i}^2$ :

$$y_i^d = a(\pi - \pi_i) - b_i r + \delta_i, \quad i = h, f \quad (2)$$

Through the paper, we consider that in each country, demand shocks are made up of fiscal policy shocks ( $g_i$ ) and autonomous demand shocks ( $v_i$ ):  $\delta_i = g_i + v_i$ .

In addition to supply and demand idiosyncratic shocks, we introduce some « structural » heterogeneity in the union, and more precisely in the monetary policy transmission channel ( $b_i$ ). In order to deal with « pure » heterogeneity effects, independently

of average effects, we define coefficient  $b_i$  in deviation from its mean. Let  $b \equiv \frac{1}{2}(b_h + b_f)$  be the average interest-elasticity of demand; we define  $\varepsilon^2 < 1$  as the degree of heterogeneity in the monetary policy transmission channel, with:  $b_h = (1 + \varepsilon)b$  and  $b_f = (1 - \varepsilon)b$ , with  $\varepsilon >> 0$ . In the *Appendix*, we show that equilibrium solutions are independent of coefficient  $b$ , so we can normalize this coefficient to:  $b = 1$ . Thus, average supply and demand functions in the union become:

$$y^s = \alpha\pi + \mu \quad (3a)$$

$$y^d = \delta - r \quad (4a)$$

and, in deviation:

$$\bar{y}^s = \alpha\bar{\pi} + \bar{\mu} \quad (3b)$$

$$\bar{y}^d = \bar{\delta} - \alpha\bar{\pi} - \varepsilon r \quad (4b)$$

For all variable  $x_i$ , we define a symmetric component:  $x = \frac{1}{2}(x_h + x_f)$  and an asymmetric (or rather an anti-symmetric) component:  $\bar{x} = \frac{1}{2}(x_h - x_f)$ . In addition, we have:  $x_h = x + \bar{x}$  and  $x_f = x - \bar{x}$ . We consider below that these two components are independently distributed, thus:  $\sigma_{x\bar{x}} = 0$ .

To solve the model, we write equilibrium in average variables ( $y^s = y^d$ ) and in deviation ( $\bar{y}^s = \bar{y}^d$ ). We obtain the following relations for inflation, on average and in deviation:

$$\pi = \frac{\delta - r - \mu}{\alpha} \quad (5a)$$

$$\bar{\pi} = \frac{\bar{\delta} - \varepsilon r - \bar{\mu}}{\alpha + a} \quad (5b)$$

And we can easily compute union product, on average and in deviation:

$$y = \alpha\pi + \mu = \delta - r \quad (6a)$$

$$\bar{y} = \alpha\bar{\pi} + \bar{\mu} = \frac{\alpha\bar{\delta} - \alpha\varepsilon r + a\bar{\mu}}{\alpha + a} \quad (6b)$$

In equation (6a), we can notice that union average income does not depend on the heterogeneity coefficient ( $\varepsilon$ ). This is also the case for all average variables in union<sup>6</sup>.

We suppose that each country of the union is endowed with a social loss function that depends on deviations of income and inflation:

$$L_i = \frac{1}{2}[\lambda y_i^2 + \pi_i^2], \quad i = h, f \quad (7)$$

where  $\lambda$  depicts social preferences for income stabilization relative to inflation stabilization. We also suppose that  $\lambda$  is the same in both countries, in order to focus on “structural” heterogeneity ( $\varepsilon^2$ ) in the union. The problem of heterogeneity of preferences in a monetary union is an important, but distinct, question. Moreover, one can wonder why some countries decide to form a monetary union if they do not share the same preferences. Our model describes a union where there are no preference conflicts, but simply differences in the functioning of economies.

Since both countries have the same size, the union-wide social loss is:

$$L^U = \frac{1}{2}L_h + \frac{1}{2}L_f \quad (8)$$

In contrast with this social loss function, based on the average of national loss functions in the union, we suppose that the common central bank chooses the union-wide interest rate<sup>7</sup>  $r$ , in order to minimize a loss function that depends on deviations of income and inflation, *based on the average variables of the Union*:

$$L^C = \frac{1}{2}[\lambda y^2 + \pi^2] \quad (9)$$

The common central bank chooses its interest rate by minimizing (9), knowing the values of demand and supply shocks. From the union-wide welfare point of view, on the contrary, what matters is the *ex ante* value of the social loss function  $EL^U$  where  $E$  denotes the rational expectations operator. It is widely accepted that the decisions of the ECB are designed for minimizing an objective based on average euro variables rather than an objective made up of national loss functions. In our model, we depict this situation by the fact that the common central bank minimizes a “centralized” loss function ( $L^C$ ) and not the union-wide social loss function, which is a “cooperative” loss function ( $L^U$ ).

We first suppose that the common central bank shares the social preference parameter for the stabilization of output relative to inflation ( $\lambda$ ), in order to focus on the impact of “centralized” versus “coordinated” monetary policies. In section 4, we consider the alternative case in which the central bank possesses peculiar preferences ( $\lambda^c \neq \lambda$ )

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<sup>6</sup> This would no longer be the case if coefficients such as  $a$  or  $\alpha$  were affected by heterogeneity.

<sup>7</sup> Since expected inflation is zero,  $r$  denotes either nominal or real interest rate.



To keep the model simple, we also choose to focus exclusively on a stabilization problem for monetary policy, and we ignore eventual average bias in monetary policy, resulting from credibility problems that could arise when the central bank has an output target higher than the natural product (here zero). Such an inconsequential objective would lead to a well-known inflation bias, which can easily be solved by the adoption of an optimal contract that penalizes the common central bank from inflation deviations. Walsh (1995) shows, in particular, that the optimal contract does not depend on the variance of inflation, but only on its average level, and that this contract results in a linear penalty on average inflation<sup>8</sup>.

In our model, on the contrary, the minimization of (9) relative to (8) will raise a stabilization problem for the monetary policy. In consequence, it will become necessary to modify the common central bank preferences for the stabilization of output and inflation – since only quadratic contracts may affect the stabilization properties of monetary policies (see in particular Rogoff, 1985 and Villieu, 2003).

## 2/ The cost of a centralized monetary policy

Let us now characterize the inefficiencies in monetary policy associated to the minimization of (9) rather than (8), considering first that the different member states and the common central bank share the same relative preferences for output and inflation stabilization ( $\lambda^c = \lambda$ ). By the minimization of (9), the common central bank sets its interest rate at (see the *Appendix*):

$$r = r^c = \psi_1^c \mu + \psi_2^c \delta \quad (10)$$

where:  $\psi_1^c = -\left(\frac{1}{1 + \lambda\alpha^2}\right)$  and:  $\psi_2^c = 1$ .

The optimal interest rate, obtained by minimizing (8) with respect to  $r$  is (see the *Appendix*):

$$r = r^u = \psi_1^u \mu + \psi_2^u \delta + \psi_3^u \bar{\mu} + \psi_4^u \bar{\delta} \quad (11)$$

where:

$$\psi_1^u = -\frac{\omega_1}{\omega}, \quad \psi_2^u = \frac{(1 + \lambda\alpha^2)\omega_1}{\omega}, \quad \psi_3^u = \frac{(\lambda\alpha a - 1)\omega_2}{\omega\varepsilon} \quad \text{and:} \quad \psi_4^u = \frac{(1 + \lambda\alpha^2)\omega_2}{\omega\varepsilon}.$$

All along the paper, we use the notations:  $\omega_1 = (\alpha + a)^2$ ,  $\omega_2 = \alpha^2 \varepsilon^2$ , and:  $\omega = (1 + \lambda\alpha^2)(\omega_1 + \omega_2)$ .

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<sup>8</sup> In our model, if  $k$  is the output target of the common central bank, the optimal penalty for inflation deviations is:  $c = \lambda k \alpha$ , so that the common central bank minimizes:  $L^C = \frac{1}{2} \left[ \lambda (y - k)^2 + \pi^2 + 2c\pi \right]$ .

A direct comparison between (10) and (11) allows identifying the inefficiencies in monetary policy. Results are summarized by the following *Proposition*:

**Proposition 1**

*In a heterogeneous monetary union, symmetric shocks have to be less stabilized and anti-symmetric shocks have to be more stabilized than in a homogenous monetary union. If the common central bank minimizes a “centralized” loss function, which take account only on average quantities in union, its interest rate policy will involve an over-reaction to symmetric shocks and an insufficient reaction to asymmetric shocks.*

**Proof:**

Concerning symmetric shocks, since:  $\frac{\omega_1}{\omega} < \frac{1}{1 + \lambda\alpha^2}$ , we have:  $|\psi_1^u| < |\psi_1^c|$  and:  $|\psi_2^u| < |\psi_2^c|$  if  $\varepsilon^2 > 0$ .

The reaction of interest rate to symmetric supply shocks is too large with a centralized monetary policy relative to a cooperative one. As a result, the union-wide average product will be insufficiently stabilized in (6a), while average inflation will be too much stabilized in (5a). On the other hand, in the cooperative regime, demand shocks are perfectly stabilized if the monetary union is homogenous ( $\varepsilon^2 = 0$ ), but only partially stabilized in a heterogeneous union (since  $|\psi_2^u| < 1$ ). Yet, with a centralized loss function, the common central bank continues to completely stabilize symmetric demand shocks, in spite of heterogeneity. As a result, average inflation and output in union are too much stabilized, to the detriment of the stabilization of deviations ( $\bar{y}$  and  $\bar{\pi}$ ).

These findings show that, concerning symmetric shocks, in a heterogeneous monetary union, one needs a less reactive monetary policy than in a homogenous union<sup>9</sup>.

Concerning anti-symmetric shocks, by focusing on average variables of the union, we can directly see from (10) and (11) that the common central bank does not take these shocks into account, while she should do under the optimal interest policy ( $\psi_3^u \neq 0$  and  $\psi_4^u \neq 0$ ). Thus, asymmetric shocks are not sufficiently stabilized in the union. Average output and inflation in Union are not affected, but the use of a centralized loss function increases divergences in the area: national quantities are not adequately stabilized.

Notice that in a homogenous monetary union ( $\varepsilon = 0$ ), our model gives rise to the well-known equivalence between minimizing  $L^U$  or  $L^C$ , since  $\omega_2 = 0$ . Thus, if there is no cross-countries divergence, monetary authorities can rely on a loss function based on area wide variables only, without implying any welfare loss in union.

In a heterogeneous Union, on the other hand, the social loss will be higher with the interest rate rule (10) than with (11). To show this, let us suppose from now that fiscal policy is able to perfectly stabilize demand shocks ( $g_i = -v_i$ , thus:  $\delta = \bar{\delta} = 0$ , we examine the alternative situation below). Thus, the union-wide ex ante social loss is, under the optimal interest rate policy (11):

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<sup>9</sup> A finding that can be compared to the « *precaution principle* » in models of monetary policy with uncertainty.

$$EL^U(r^u) = \frac{(1-X)}{2\alpha^2} \sigma_\mu^2 + \frac{(\lambda a^2 + 1 - \bar{X})}{2\omega_1} \sigma_{\bar{\mu}}^2 \quad (12a)$$

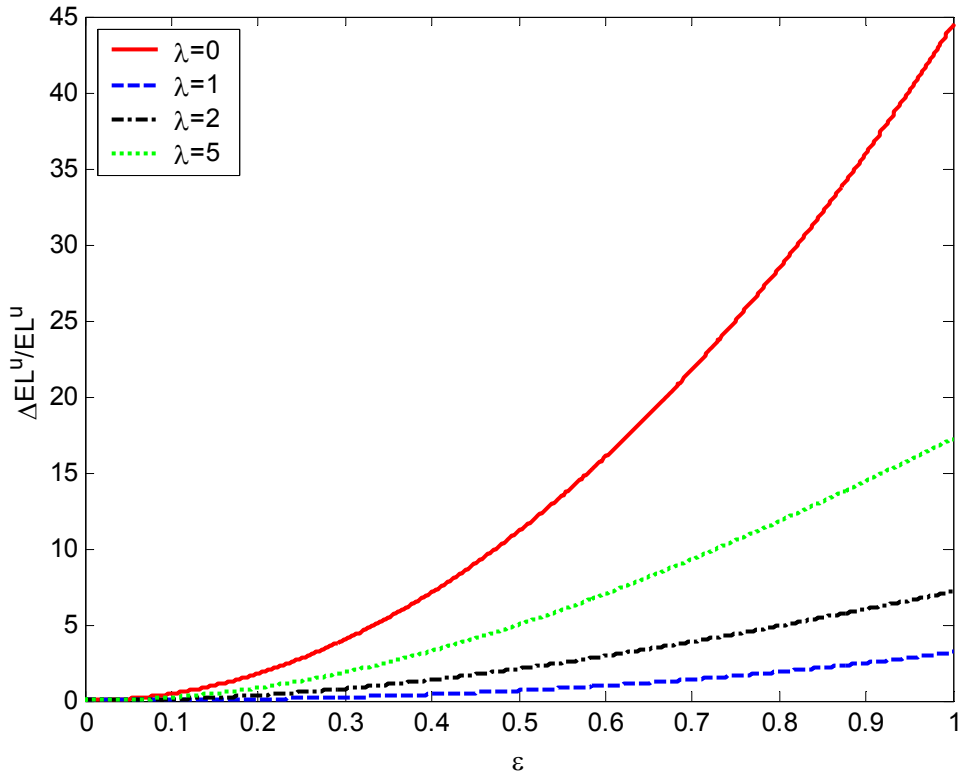
where:  $X = \omega_1 / \omega$  and:  $\bar{X} = (\alpha\lambda a - 1)^2 (\omega_2 / \omega)$ ; and, under the centralized policy (10):

$$EL^U(r^c) = \frac{(1-Y)}{2\alpha^2} \sigma_\mu^2 + \frac{(\lambda a^2 + 1)}{2\omega_1} \sigma_{\bar{\mu}}^2 \quad (12b)$$

where:  $Y = \frac{1}{1 + \lambda\alpha^2} \left( \frac{\omega_1 - \omega_2}{\omega_1} \right)$

Since:  $Y < X < 1$  and:  $0 \leq \bar{X} < \lambda a^2 + 1$ , we can easily verify that:  $EL^U(r^u) < EL^U(r^c)$ .

Figure 1 – Welfare differential as a function of heterogeneity<sup>10</sup> (in %)



Let us now study how the welfare differential ( $\Delta EL \equiv EL^U(r^c) - EL^U(r^u)$ ) changes in response to variations in parameters. From (12a-b), we obtain the value of the welfare differential:

<sup>10</sup> Unless other information, we choose:  $\alpha = 2$ ,  $a = 1$ ,  $\lambda = 1$ ,  $\sigma_\mu^2 = \sigma_{\bar{\mu}}^2 = 1$  and  $\varepsilon^2 = 0.25$  in all simulations.

$$\Delta EL^U = \frac{(X-Y)}{2\alpha^2} \sigma_\mu^2 + \frac{\bar{X}}{2\omega_1} \sigma_{\bar{\mu}}^2 = \frac{\omega_2}{2\omega\omega_1} \left[ \varepsilon^2 \sigma_\mu^2 + (\alpha a \lambda - 1)^2 \sigma_{\bar{\mu}}^2 \right] \quad (13)$$

We can easily notice that:  $\frac{d\Delta EL^U}{d\varepsilon^2} > 0$ . Thus, the more heterogeneous the union is, the highest the relative cost of a centralized policymaking is. In *Figure 1*, we can see that this cost is significant for admissible parameter values, notably for “high” or “low” values of  $\lambda$ .

### 3/ Introducing aversion to divergences in the central bank loss function

Let us suppose now that the common central bank has some degree of aversion to inflation and income divergences in the monetary union. We depict this fact by modifying its loss function: beyond stabilizing average variables in the Union, the central bank attempts to stabilize inflation and income differentials, measured as the cross section standard error of these variables:

$$L^C = \frac{1}{2} \left[ \lambda y^2 + \pi^2 + \theta_y \bar{y}^2 + \theta_\pi \bar{\pi}^2 \right] \quad (14a)$$

where  $\theta_y$  and  $\theta_\pi$  are the coefficients of aversion to income and inflation divergences, respectively. In this section, we search for optimal values for  $\theta_y$  and  $\theta_\pi$ . The following proposition shows that we can find a simple optimal contract for the common central bank, such that minimizing  $L^C$  in (14a) amounts to minimizing  $L^U$  in (8).

#### Proposition 2:

*If the different member states of the monetary union and the common central bank share the same preferences for the stabilization of output and inflation (say,  $\lambda$  and 1 respectively), the first best solution for monetary policy can be obtained by an optimal contract that penalizes the common central bank from inflation and output divergences in the union. In the optimal contract, the penalties imposed on inflation (respectively on output) divergences correspond to the relative weight of inflation (respectively output) in the social welfare function. Thus, the optimal contract for the common central bank is such as:  $\theta_y^* = \lambda$  and  $\theta_\pi^* = 1$ .*

**Proof:** We prove *Proposition 2* for a general case with  $n$  countries and without specifying any supply or demand relation.

Suppose that the Union is composed of  $n$  countries. The social loss function is described by:

$$L^U(r) = \frac{1}{n} \sum_{i=1}^n L_i = \frac{1}{n} \sum_{i=1}^n \frac{1}{2} (\lambda y_i^2 + \pi_i^2) \quad (15a)$$

By minimizing this function with respect to  $r$ , we obtain:

$$\frac{\partial L^U(r)}{\partial r} = \frac{\lambda}{n} \sum_{i=1}^n y_i \frac{\partial y_i}{\partial r} + \frac{1}{n} \sum_{i=1}^n \pi_i \frac{\partial \pi_i}{\partial r} \quad (16a)$$

If the common central bank minimizes a social loss functions based on average quantities and with aversion to divergences<sup>11</sup>:

$$L^C(r) = \frac{1}{2} [\lambda y^2 + \pi^2 + \theta_y \bar{y}^2 + \theta_\pi \bar{\pi}^2] \quad (15b)$$

$$\text{where: } y = \frac{1}{n} \sum_{i=1}^n y_i, \pi = \frac{1}{n} \sum_{i=1}^n \pi_i, \bar{y} = \left[ \frac{1}{n} \sum_{i=1}^n (y_i - y)^2 \right]^{1/2} \text{ and } \bar{\pi} = \left[ \frac{1}{n} \sum_{i=1}^n (\pi_i - \pi)^2 \right]^{1/2}$$

By minimizing this function with respect to  $r$  and rearranging, we obtain:

$$\frac{\partial L^C(r)}{\partial r} = y(\lambda - \theta_y) \frac{\partial y}{\partial r} + \pi(1 - \theta_\pi) \frac{\partial \pi}{\partial r} + \frac{\theta_y}{n} \sum_{i=1}^n y_i \frac{\partial y_i}{\partial r} + \frac{\theta_\pi}{n} \sum_{i=1}^n \pi_i \frac{\partial \pi_i}{\partial r} \quad (16b)$$

We can easily observe that expressions (16a) and (16b) are identical if  $\theta_y = \lambda$  and  $\theta_\pi = 1$ , so are (15a) and (15b) identical in this case. Thus, under the optimal contract, the centralized monetary regime with aversion to divergences is efficient and conducts to the optimal regime.

*Proposition 2* shows that a simple “optimal contract” for the central bank can enforce the “coordinate” optimal solution. This result is similar to Walsh (1995), except that Walsh deals with inflation bias of monetary policy, while we exclusively deal with a stabilization problem of monetary policy. One important limit about *Proposition 2* is that the different member states of the union and the common central bank must share the same preferences. If it is not the case (with different relative preferences for output stabilization  $\lambda_i$  for example), it is in general impossible to completely remove the inefficiency associated with a centralized monetary policy, and only “second best” contracts can be enforced, but such contracts are model-dependent.

*Proposition 2* is established for a general case. In our two-country model, minimizing (14a) provides the following relation:

$$r = r^c = \psi_1^c \mu + \psi_2^c \delta + \psi_3^c \bar{\mu} + \psi_4^c \bar{\delta} \quad (17a)$$

with:

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<sup>11</sup> Divergences in the Union are synthesized by the standard error operator, for variable  $x$ :  $\left[ \frac{1}{n} \sum_{i=1}^n (x_i - x)^2 \right]^{1/2}$ . Applied to the case of a monetary union formed by two countries only, this variance

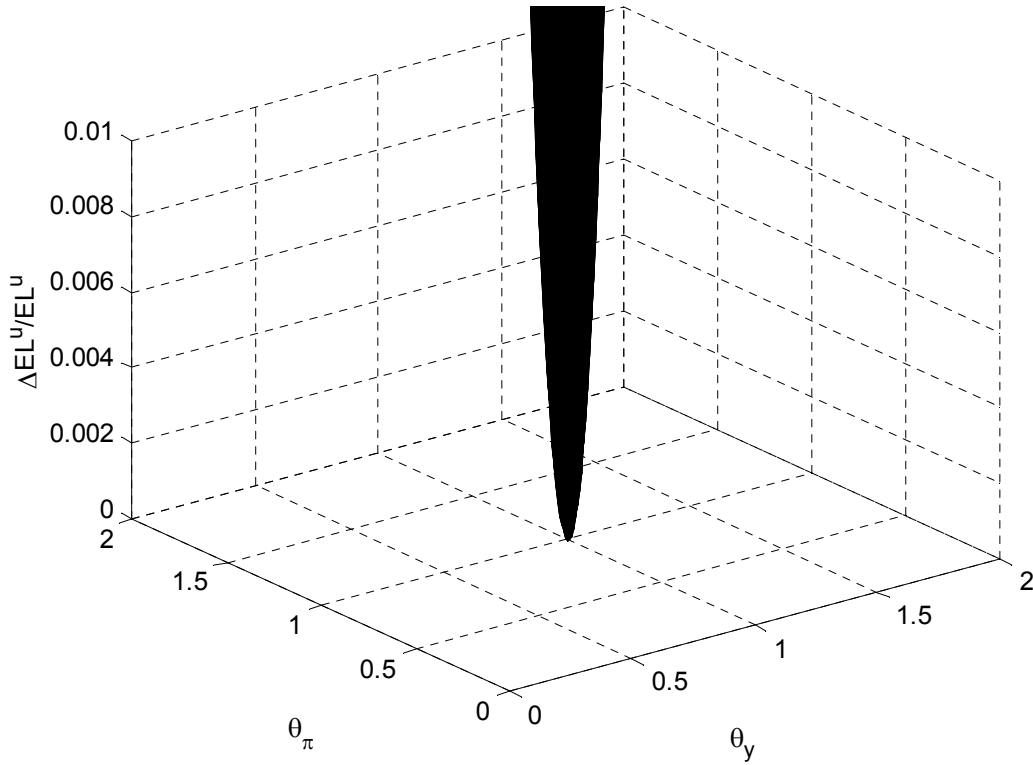
operator gives rise to the term:  $\bar{x} = \frac{x_h - x_f}{2}$  used in the paper.

$$\psi_1^c = \frac{-\omega_1}{\omega + \omega_2 \Phi_1}, \psi_2^c = \frac{\omega_1(1 + \lambda \alpha^2)}{\omega + \omega_2 \Phi_1}, \psi_3^c = \frac{\omega_2(\alpha a \lambda - 1 + \Phi_2)}{(\omega + \omega_2 \Phi_1)\varepsilon} \text{ and: } \psi_4^c = \frac{\omega_2(1 + \lambda \alpha^2 + \Phi_1)}{(\omega + \omega_2 \Phi_1)\varepsilon}$$

and we use the notation:  $\Phi_1 = \alpha^2(\theta_y - \lambda) + \theta_\pi - 1$  and:  $\Phi_2 = \alpha a(\theta_y - \lambda) + 1 - \theta_\pi$ .

One can easily verify that (17a) corresponds to (11) if:  $\theta_y^* = \lambda$  and  $\theta_\pi^* = 1$ , and to (10) if:  $\theta_y = \theta_\pi = 0$ . *Figure 3* depicts the social welfare differential ( $\Delta EL \equiv EL^U(r^c) - EL^U(r^u)$ ), computed with the interest rule (17a), as a function of  $\theta_y$  and  $\theta_\pi$ , verifying the optimality of the  $\theta_y^* = \lambda = 1$  and  $\theta_\pi^* = 1$  point.

*Figure 3 – Welfare differential as a function of  $\theta_y$  and  $\theta_\pi$  (in %)*



Let us compute the expected social loss  $EL^U$  under the interest rule (17a) for any values of  $\theta_y$  and  $\theta_\pi$ . Assuming that fiscal policy perfectly stabilizes demand shocks ( $\delta = \bar{\delta} = 0$ ), we have:

$$EL^U(r^c) = \frac{(1-Z)}{2\alpha^2} \sigma_\mu^2 + \frac{(\lambda\alpha^2 + 1 - \bar{Z})}{2\omega_1} \sigma_\pi^2 \quad (18a)$$

where:

$$Z = \frac{\omega_1(\omega + 2\omega_2\Phi_1)}{(\omega_2\Phi_1 + \omega)^2}$$

$$\bar{Z} = \frac{\omega_2(\Phi_2 + \alpha a \lambda - 1)[(2\omega_2\Phi_1 + \omega)(\lambda \alpha a - 1) - \omega\Phi_2]}{(\omega_2\Phi_1 + \omega)^2}$$

We can verify that  $Z = X$  and  $\bar{Z} = \bar{X}$  for  $\theta_y = \lambda$  and  $\theta_\pi = 1$ , and that  $Z = Y$  and  $\bar{Z} = 0$  for  $\theta_y = 0$  and  $\theta_\pi = 0$ , so that expression (18a) corresponds respectively to expressions (12a) and (12b) in these cases.

Let us now compute the differential of welfare associated with a centralized policymaking compared to a cooperative one, namely:

$$\Delta EL = EL^U(r^c) - EL^U(r^u) = \frac{(X - Z)}{2\alpha^2} \sigma_\mu^2 + \frac{(\bar{X} - \bar{Z})}{2\omega_1} \sigma_{\bar{\mu}}^2 \quad (19a)$$

where:

$$X - Z = \frac{\omega_1\omega_2^2(\Phi_1)^2}{\omega(\omega + \omega_2\Phi_1)^2}$$

$$\bar{X} - \bar{Z} = \frac{\omega_2[\omega_1(1 + \lambda\alpha^2)\Phi_2 + \alpha\omega_2(a + \alpha)\Phi_3]^2}{\omega(\omega + \omega_2\Phi_1)^2}$$

and we use the notation:  $\Phi_3 = (\theta_y - \lambda) - \lambda(\theta_\pi - 1)$ .

Concerning the symmetric component of supply shocks, a centralized policymaking reaches the same social loss than the optimal one if:  $\Phi_1 = 0$ , namely if:

$$\theta_\pi = \theta_\pi^s = 1 - \alpha^2(\theta_y - \lambda) \quad (20a)$$

this value is also the one that minimizes the social loss function  $\left( \frac{dEL^U(r^c)}{d\theta_\pi} = 0 \right)$  if there is no

anti-symmetric shock. Concerning the anti-symmetric component of supply shocks, a centralized policymaking reaches the same social loss than the optimal one if:  $\bar{X} = \bar{Z}$ , namely if:

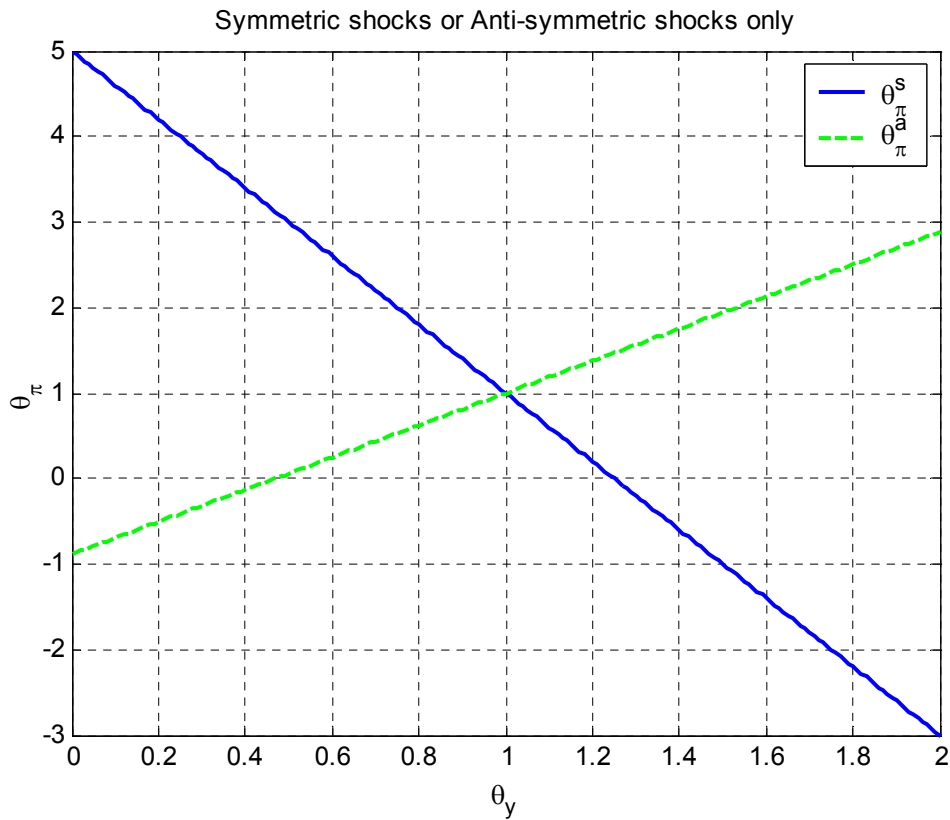
$$\theta_\pi = \theta_\pi^a = 1 + \alpha^2\Omega(\theta_y - \lambda) \quad (21a)$$

where: 
$$\Omega = \frac{\omega_2 + a(a + \alpha)(1 + \alpha^2 \lambda)}{\alpha(a + \alpha)(1 + \lambda \alpha^2) + \lambda \alpha^2 \omega_2}$$

this value is also the one that minimizes the social loss function  $\left( \frac{dEL^U(r^c)}{d\theta_\pi} = 0 \right)$  if there is no symmetric shock.

We can notice that:  $\theta_\pi^s = \theta_\pi^a = 1$  if  $\theta_y = \lambda$ , finding the optimal contract of *Proposition 2*. But for non-optimal values of the central bank aversion for output divergences (that is  $\theta_y \neq \lambda$ ), there is a conflict between stabilizing symmetric and anti-symmetric components of supply shocks. Effectively,  $\theta_\pi^s$  negatively depends on  $\theta_y$ , while  $\theta_\pi^a$  positively depends on it. Thus, if  $\theta_y < \lambda$ , a situation that we favour in section 5, stabilizing symmetric supply shocks would call for a higher than one coefficient of aversion to inflation divergences ( $\theta_\pi^s > 1$ ), but stabilizing anti-symmetric supply shocks would require a lower than one coefficient of aversion to inflation divergences ( $\theta_\pi^a < 1$ ). The reverse is true if  $\theta_y > \lambda$ . These results are summarized in *Figure 4a*:

*Figure 4a – Best value for  $\theta_\pi$  in function of  $\theta_y$*





Before examining (in section 5) the potential conflict between stabilizing symmetric and anti-symmetric shocks when the optimal contract cannot be implemented, we study in section 4 how the optimal contract have to be changed when the central bank does not share the social relative preferences for output and inflation stabilization.

#### 4/ The optimal contract with independent central bank preferences for output and inflation stabilization

Let us suppose now that the common central bank possess its own preferences for the stabilization of output relative to inflation, namely:  $\lambda^c \neq \lambda$ . The objective of the central bank becomes:

$$L^c = \frac{1}{2} [\lambda^c y^2 + \pi^2 + \theta_y \bar{y}^2 + \theta_\pi \bar{\pi}^2] \quad (14b)$$

We suppose, as usual, that the central bank is more concerned with inflation stabilization than society ( $\lambda^c \leq \lambda$ ). The interest rule that comes from the minimization of (14b) is analogous to equation (17a) above:

$$r^c = \Psi_1^c \mu + \Psi_2^c \delta + \Psi_3^c \bar{\mu} + \Psi_4^c \bar{\delta} \quad (17b)$$

where:

$$\Psi_1^c = \frac{-\omega_1}{\omega^c + \omega_2 \Phi_1^c}, \quad \Psi_2^c = \frac{\omega_1 (1 + \lambda^c \alpha^2)}{\omega^c + \omega_2 \Phi_1^c}, \quad \Psi_3^c = \frac{\omega_2 (\alpha \lambda^c - 1 + \Phi_2^c)}{(\omega^c + \omega_2 \Phi_1^c) \varepsilon} \quad \text{and} \quad \Psi_4^c = \frac{\omega_1 (1 + \lambda^c \alpha^2 + \Phi_1^c)}{(\omega^c + \omega_2 \Phi_1^c)},$$

with:  $\Phi_1^c = \alpha^2 (\theta_y - \lambda^c) + \theta_\pi - 1$ ,  $\Phi_2^c = \alpha a (\theta_y - \lambda^c) + 1 - \theta_\pi$  and  $\omega^c = (1 + \alpha^2 \lambda^c) (\omega_1 + \omega_2)$ .

#### Proposition 3:

*If the common central bank is more concerned with the stabilization of inflation relative to output, compared with social preferences in the Union, namely if  $\lambda^c \leq \lambda$ , the first best solution for monetary policy can be obtained by an optimal contract that penalizes the common central bank from inflation and output divergences in the union. In the optimal contract, the penalties imposed on inflation (respectively on output) divergences are higher than the relative weighs of inflation (respectively output) in the social welfare function. Thus, the optimal contract for the common central bank is such as:  $\theta_y^* \geq \lambda$  and  $\theta_\pi^* \geq 1$ .*

**Proof:** Suppose, as in the previous section, that fiscal policy perfectly stabilizes demand shocks. The social loss function associated to the interest rule (17b) is now:

$$EL^U(r^c) = \frac{(1 - Z^c)}{2\alpha^2} \sigma_\mu^2 + \frac{(\lambda a^2 + 1 - \bar{Z}^c)}{2\omega_1} \sigma_\mu^2 \quad (18b)$$

where<sup>12</sup>:

<sup>12</sup> We can easily verify that:  $Z^c = Z$ ,  $\bar{Z}^c = \bar{Z}$  if  $\lambda = \lambda^c$ .

$$Z^c = \frac{\omega_1 \left[ \omega + 2(\omega_2 \Phi_1 - \omega_1 \alpha^2 (\lambda - \lambda^c)) \right]}{(\omega^c + \omega_2 \Phi_1^c)^2}$$

$$\bar{Z}^c = \frac{\omega_2 (\Phi_2 + \alpha \alpha \lambda - 1) \left[ (\alpha \alpha \lambda - 1) \left[ 2\omega_2 \Phi_1^c + \omega^c - \alpha^2 (\omega_1 + \omega_2) (\lambda - \lambda^c) \right] - \omega \Phi_2 \right]}{(\omega^c + \omega_2 \Phi_1^c)^2}$$

And the differential of welfare associated with a centralized policymaking compared to a cooperative one is:

$$\Delta EL = EL^U(r^c) - EL^U(r^u) = \frac{X - Z^c}{2\alpha^2} \sigma_\mu^2 + \frac{\bar{X} - \bar{Z}^c}{2\omega_1} \sigma_{\bar{\mu}}^2 \quad (19b)$$

where:

$$X - Z^c = \frac{\omega_1 \left[ \omega_2 \Phi_1 - \alpha^2 \omega_1 (\lambda - \lambda^c) \right]^2}{\omega (\omega^c + \omega_2 \Phi_1^c)^2} \geq 0$$

$$\bar{X} - \bar{Z}^c = \frac{\omega_2 \left[ \omega_1 \left[ (1 + \alpha^2 \lambda) \Phi_2 + \alpha^2 (\alpha \alpha \lambda - 1) (\lambda - \lambda^c) \right] + \alpha (a + \alpha) \omega_2 \Phi_3 \right]^2}{\omega (\omega^c + \omega_2 \Phi_1^c)^2} \geq 0$$

If  $\lambda^c = \lambda = \theta_y$  and  $\theta_\pi = 1$ , the differential of welfare is zero. But if  $\lambda^c \leq \lambda$ , the differential of welfare is positive even if  $\theta_y = \lambda^c$  and  $\theta_\pi = 1$ <sup>13</sup>. Consequently, the optimal contract for monetary policy is not  $\theta_y = \lambda^c$  and  $\theta_\pi = 1$ .

Concerning the symmetric component of supply shocks, a centralized regime with aversion toward divergences produces the same social loss as the optimal “cooperative” regime if:

$$\theta_\pi = \theta_\pi^s = 1 - \alpha^2 (\theta_y - \lambda) + \alpha^2 \frac{\omega_1}{\omega_2} (\lambda - \lambda^c) \quad (20b)$$

Concerning the asymmetric component of supply shocks, we obtain the same social loss under both regimes if:

$$\theta_\pi = \theta_\pi^a = 1 + \alpha^2 \left[ \Omega (\theta_y - \lambda) + \frac{\alpha (a + \alpha) (\alpha \alpha \lambda - 1) (\lambda - \lambda^c)}{\alpha (a + \alpha) (1 + \alpha^2 \lambda) + \alpha^2 \omega_2 \lambda} \right] \quad (21b)$$

---

<sup>13</sup> Effectively, for  $\theta_y = \lambda^c$  and  $\theta_\pi = 1$ , we have:  $X - Z^c = \frac{\omega_1 \left[ \alpha^2 \omega_1 (\lambda - \lambda^c) \right]^2}{\omega (\omega^c + \omega_2 \Phi_1^c)^2} \geq 0$  and

$$\bar{X} - \bar{Z}^c = \frac{\omega_2 \left[ \omega_1 \alpha^2 (\alpha \alpha \lambda - 1) (\lambda - \lambda^c) \right]^2}{\omega (\omega^c + \omega_2 \Phi_1^c)^2} \geq 0.$$

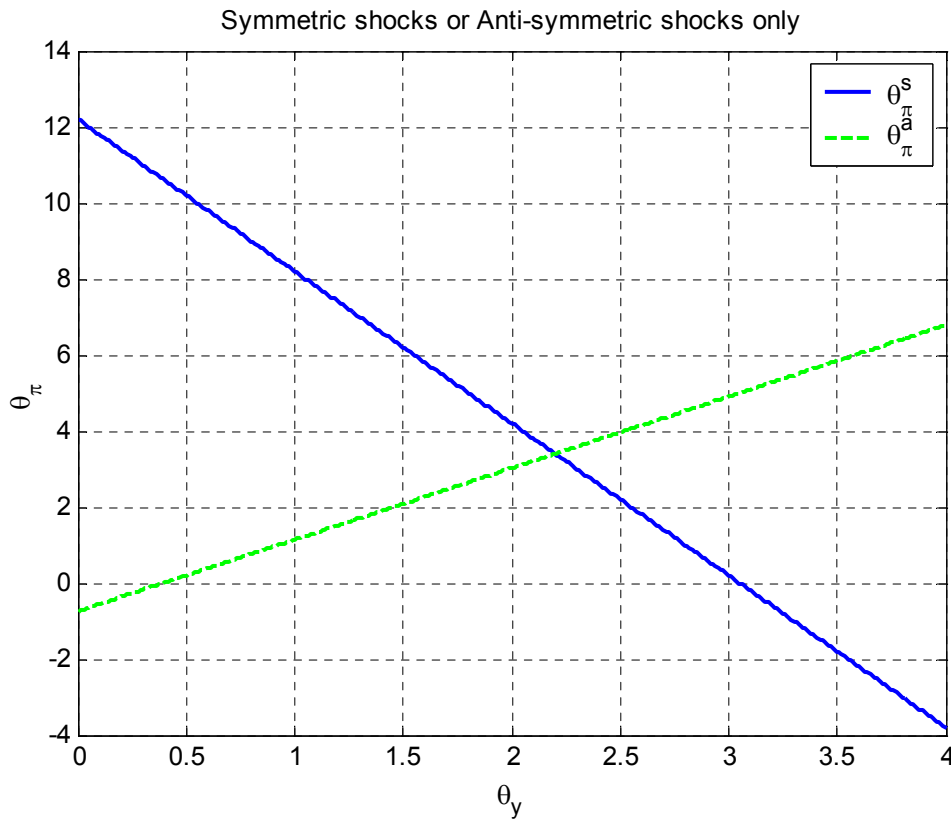
From (20b)-(21b), we easily obtain the values of the coefficients of aversion toward output and inflation divergences under the optimal contract, namely:

$$\begin{aligned}\theta_y^* &= \lambda + \frac{(a+\alpha)}{\alpha\varepsilon^2}(\lambda - \lambda^c) \geq \lambda \\ \theta_\pi^* &= 1 + \left[ \frac{a(a+\alpha)}{\varepsilon^2} \right] (\lambda - \lambda^c) \geq 1\end{aligned}\tag{22}$$

This proves Proposition 3.

These results are depicted in *Figure 4b* for  $\lambda^c = 0.8 < \lambda = 1$ .  $\theta_\pi^s$  and  $\theta_\pi^a$  are still respectively decreasing and increasing function in  $\theta_y$ , but, when  $\lambda^c < \lambda$ , the two curves moves upwards compared with *Figure 4a*. As a result, the optimal values of  $\theta_\pi^s$  and  $\theta_\pi^a$  are higher than they was when  $\lambda^c = \lambda$ . In *Figure 4b* special case:  $\theta_y^* = 2.2$  and  $\theta_\pi^* = 3.4$ , instead of  $\theta_\pi^s = \theta_\pi^a = 1$ .

*Figure 4b – Best value for  $\theta_\pi$  in function of  $\theta_y$  ( $\lambda^c = 0.8$ )*



In expressions (22), we can see that the optimal values of aversion towards inflation and output divergences are decreasing function of the degree of heterogeneity in the union ( $\varepsilon^2$ ). The more heterogeneous the union is, the lesser the common central bank should worry about inflation and output divergences under the optimal contract.

Let us examine more closely this apparent paradox. If the union was homogenous ( $\varepsilon^2 = 0 \Rightarrow \bar{y} = \bar{\pi} = 0$ ), minimizing  $L^U$  in (8) would correspond to minimizing  $L = \frac{1}{2}[\lambda y^2 + \pi^2]$ , and no contract would allow equalizing this quantity to  $L^c = \frac{1}{2}[\lambda^c y^2 + \pi^2]$ . Thus, no contract would totally remove the inefficiencies associated with the “wrong” central bank loss function, as show the values of penalties in equation (22) which tend to infinity.

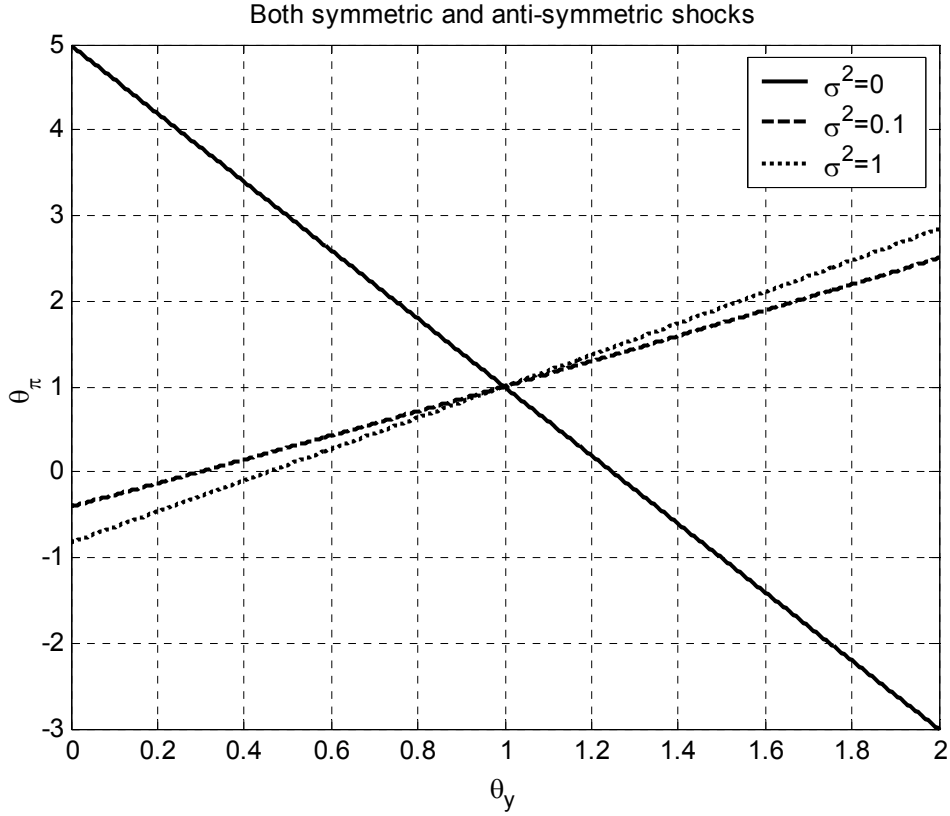
In a heterogeneous union, on the contrary, imposing penalties on inflation and output divergences allows totally removing the inefficiencies associated with the “wrong” central bank loss function. Sufficiently “high” values of the penalties in equation (22) correct both the bias associated with a centralized policymaking *and* the bias associated with the particular preference for output/inflation stabilization. In the optimal contract, penalties have to be higher than if  $\lambda^c = \lambda$  (namely, 1), but the more heterogeneous the union is, the lesser these penalties have to be. Thus, the gap between optimal penalties when  $\lambda^c = \lambda$  and their values when  $\lambda^c < \lambda$  negatively depends on the degree of union heterogeneity, because heterogeneity gives an instrument for correcting the bias associated with the “wrong” preference parameter  $\lambda^c$ . Taking the logic to extreme, in a strongly heterogeneous union ( $\varepsilon^2 \rightarrow +\infty$ ), the relative preference for output/inflation stabilization is irrelevant, because the optimal policy is:  $r^c = r^U = 0$  in (11) and (17b), and the optimal contract is the same as in section 3.

## 5/ Second-best contracts for monetary policy

However, the optimal contract does not seem to characterize the behaviour of the ECB. On the contrary, the recent monetary policy debate in EMU has tended to focus on the cost of a monetary policy uniquely designed on the base of union-wide quantities and on the difficulties for defining an adequate common monetary policy in the presence of large inflation differentials, with few references to income divergences. Effectively, it seems difficult to design monetary policy in function of growth differentials in the Euro area, since these differentials reflect structural adjustment and catching up of less developed member states, and arte outside the province of current interest rate policy. Even if inflation divergences also possess a structural component, they directly affect the central bank ability of defining a “good” inflation rate for the area, and the ECB does probably keep more watch on inflation differentials than on output differentials movements.

In what follows, we wonder about the interest if “second best” contracts, in which the common central bank shares the social relative preferences for output and inflation stabilization ( $\lambda^c = \lambda$ ), as in sections 2 and 3, and worries about inflation differentials but without being endowed with the optimal degree of aversion towards output divergences ( $\theta_y \neq \theta_y^* = \lambda$ ). In other words, we search for the optimal degree of aversion towards inflation divergences in function of different (possibly null) degrees of aversion towards output divergences.

Figure 4c – Best value for  $\theta_\pi$  in function of  $\theta_y$



With both symmetric and anti-symmetric shocks, the degree of aversion towards inflation divergences ( $\theta_\pi^*$ ) that minimizes the welfare differential in function of the coefficient of aversion for output divergences ( $\theta_y$ ) can be expressed as<sup>14</sup>:

$$\theta_\pi^* - 1 = \Theta(\theta_y - \lambda) \quad (23)$$

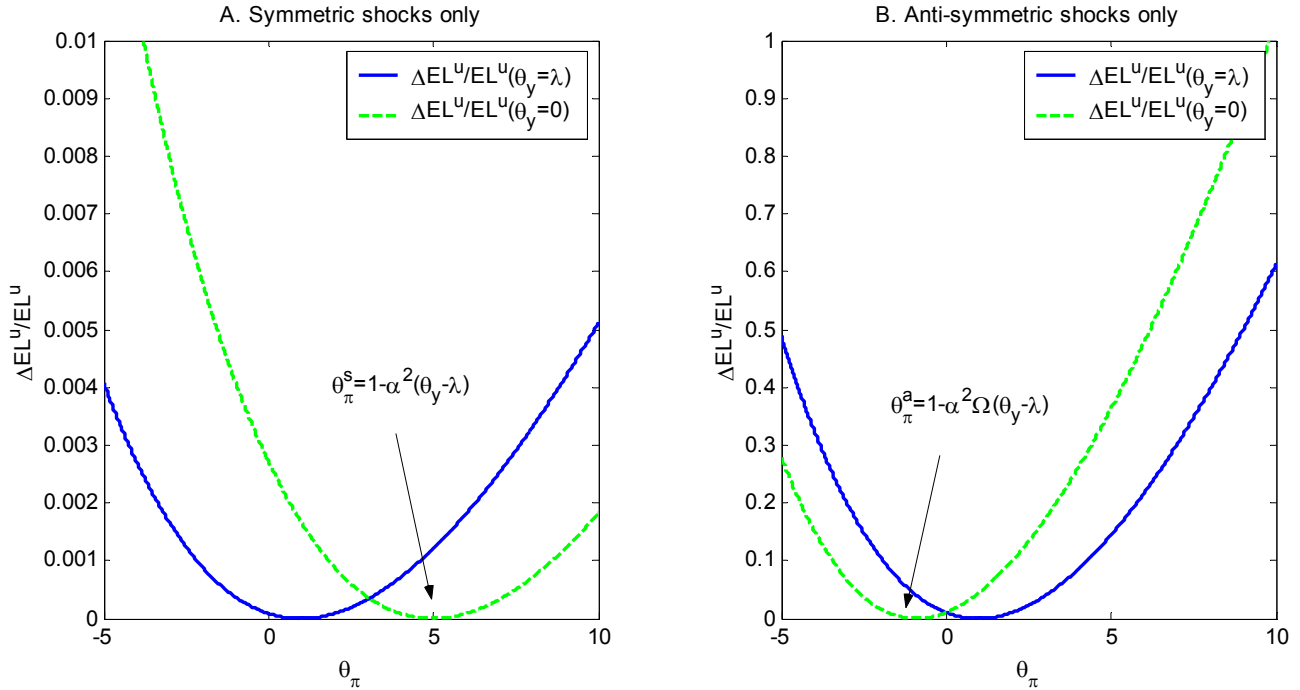
Coefficient  $\Theta$  depends on the variance of symmetric and anti-symmetric shocks, but for admissible parameter values, anti-symmetric shocks dominate even if their variance is very small compared to the variance of symmetric shocks. In *Figure 4c*, we represent  $\theta_\pi^*$  as a function of  $\theta_y$ , for different values of the ratio of variances  $\sigma = \sigma_\mu^2 / \sigma_\pi^2$ , showing that anti-symmetric shocks dominate even if  $\sigma = 0.1$ . Thus, the relation between  $\theta_y$  and  $\theta_\pi^*$  is most probably positive, pointing out the fact that, from the union-wide welfare perspective, a

<sup>14</sup> In this expression:  $\Theta = \frac{-\omega_1 \omega_2 \sigma_\mu^2 + \alpha(\omega_2 + a\tau)[\tau + \alpha\omega_2 \theta_y] \sigma_\mu^2}{\omega_1(\sigma_\mu^2 \varepsilon^2 + \sigma_\mu^2(1 + \alpha^2 \lambda)^2) + \omega_2 \alpha \sigma_\mu^2 [\tau \lambda + \theta_y(\tau + \lambda \alpha \omega_2)]}$ , with:

$$\tau = (a + \alpha)(1 + \alpha^2 \lambda).$$

common central bank that worries about inflation divergences without taking care of output differentials would not be a good idea.

Figure 5 – Welfare differential as a function of  $\theta_\pi$  (in %)



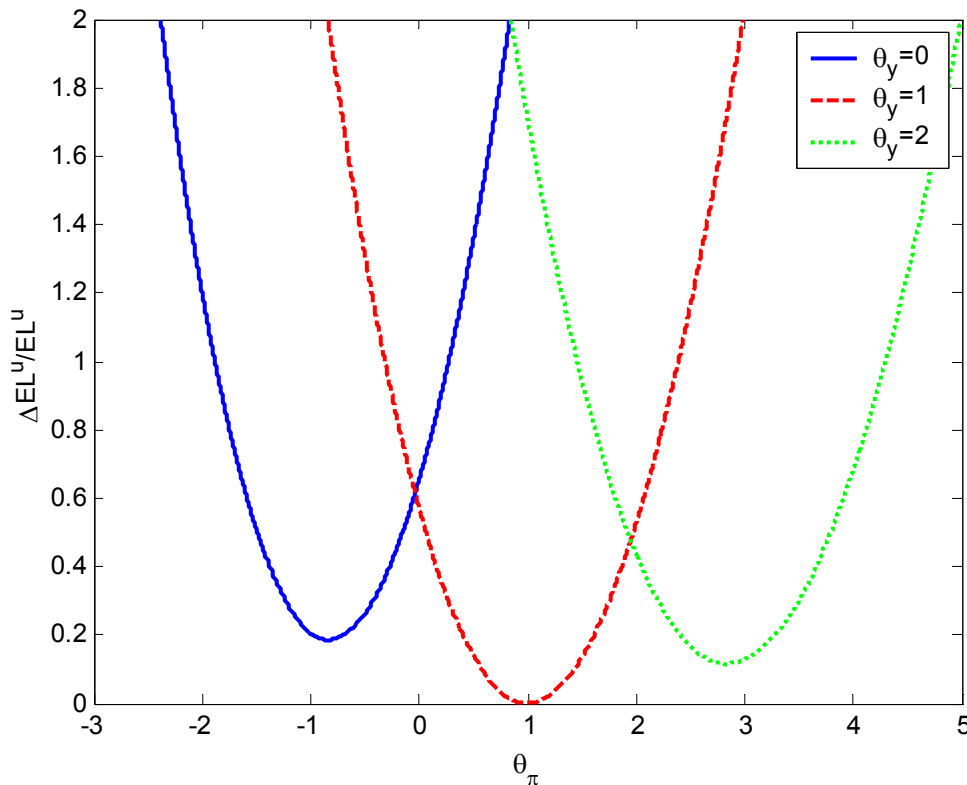
In Figures 5a and 5b, we represent the social welfare differential ( $\Delta EL = EL^U(r^c) - EL^U(r^u)$ ) for two cases: a case with symmetric shocks only (Figure 5a) and a case with asymmetric shocks only (Figure 5b). We can notice that the bias associated to centralized policymaking can be totally removed ( $\Delta EL = 0$ ) for  $\theta_\pi = \theta_\pi^s$  if there are only symmetric shocks and for  $\theta_\pi = \theta_\pi^a$  if there are only anti-symmetric shocks. If both symmetric and anti-symmetric shocks are present, however, this bias cannot be completely removed unless  $\theta_y = \lambda$  (in which case  $\theta_\pi^s = \theta_\pi^a = 1$ ).

We can notice that when the common central bank is not concerned by output divergences ( $\theta_y = 0$ ), solving the problem of anti-symmetric shocks requires a *negative* optimal degree of aversion towards inflation divergences (for our simulation parameters). This is also the case when both symmetric and anti-symmetric shocks are present, as show Figures 4c and 6.1 below. Independently of parameter values, removing the inefficiencies associated to the sub-optimal stabilization of symmetric shocks when  $\theta_y = 0$  requires a higher than one value for  $\theta_\pi$  ( $\theta_\pi^s = 5$  in Figure 5a), but removing the inefficiencies associated to the sub-optimal stabilization of anti-symmetric shocks requires a lower than one value for  $\theta_\pi$  ( $\theta_\pi^a = -0.8$  in Figure 5b).

In Figures 6, we depict the differential of welfare when there are both symmetric and anti-symmetric shocks. As we have seen, the differential is fully removed only on the optimal

contract (here:  $\theta_y = \theta_\pi = 1$ ). If the degree of aversion towards output divergences is set sub-optimally ( $\theta_y \neq 1$ ), on the contrary, the second best value for the coefficient for inflation differentials conducts to a net loss of welfare compared to the “cooperative” monetary policy. Notice in particular that, if the central bank is not concerned about output differentials ( $\theta_y = 0$ ), the best coefficient for inflation divergences is not necessarily positive, notably if  $\lambda$  is small.

Figure 6 – Welfare differential as a function of  $\theta_\pi$  (in %)



#### National losses under the “optimal” contract

A central question about the feasibility of the optimal contract for the common central bank concerns its effects on national welfare in each country of the union. On this point, our simulations show that, in most cases, there is a conflict between the two member states. Effectively, for a given degree of aversion towards output divergences ( $\theta_y$ ), welfare is an increasing function of the degree of aversion towards inflation divergences ( $\theta_\pi$ ) in one country, and a decreasing function in the other. Thus, one country only takes benefits from the fact that the common central bank considers inflation differentials in the union, while this is detrimental to the other.

Moreover, if the common central bank takes care of inflation divergences ( $\theta_\pi > 0$ ), welfare is higher than if she does not ( $\theta_\pi = 0$ ) in one country, while it is lower in the other country. This property arises whatever the degree of aversion towards output divergences is. In Figure 8 we plot national welfare loss differentials between a centralized monetary policy

with  $\theta_\pi = \theta_y = 0$  and a policy in which the common central bank has the “right” coefficient of aversion towards output divergences ( $\theta_y = \lambda$ ), for different values of the coefficient of aversion towards inflation differentials. We can notice that only one country takes advantages of  $\theta_\pi > 0$  (country  $h$  if  $\theta_\pi$  is small or country  $f$  if  $\theta_\pi$  is large).

*Figure 9 depicts* national welfare loss differentials between a centralized monetary policy with  $\theta_\pi = \theta_y = 0$  and the optimal (from the union-wide welfare point of view) policy with  $\theta_\pi = 1$  and  $\theta_y = \lambda$ , for different values of the degree of heterogeneity in union ( $\varepsilon$ ). This *Figure* confirms the fact that only one country takes benefits from the implementation of the optimal contract. Furthermore, we can notice that national welfare differentials are large, compared to the union-wide welfare differential.

Therefore, modifying common central bank preferences, even to implement the optimal contract, would be a source of potential conflicts between member states of the union, especially in a heterogeneous union.

## 6/ Conclusion

Should a common central bank in a heterogeneous monetary union worry about inflation differentials? In this paper, we have shown at a theoretical level that an optimal contract for the common central bank can be found, from the point of view of union-wide welfare. This contract penalizes the central bank from inflation and output divergences in union. On the other hand, penalizing the central bank from inflation divergences only is not necessarily a better solution than a “centralized” policymaking that is only concerned with union-wide magnitudes. Furthermore, even the optimal contract is difficult to implement, because only some member states takes advantage of this contract, while it is detrimental to the welfare of others. Modifying common central bank preferences is therefore a source of potential conflicts between member states of the union.



Figure 8 – Aggregate and national differentials of welfare

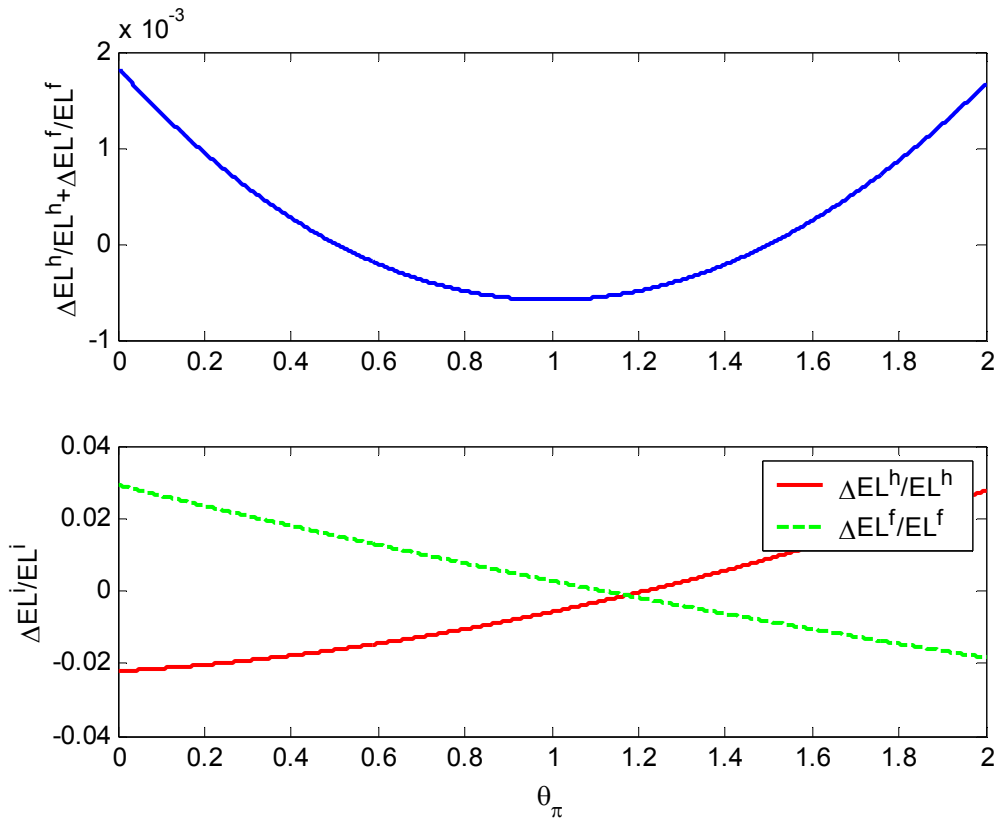
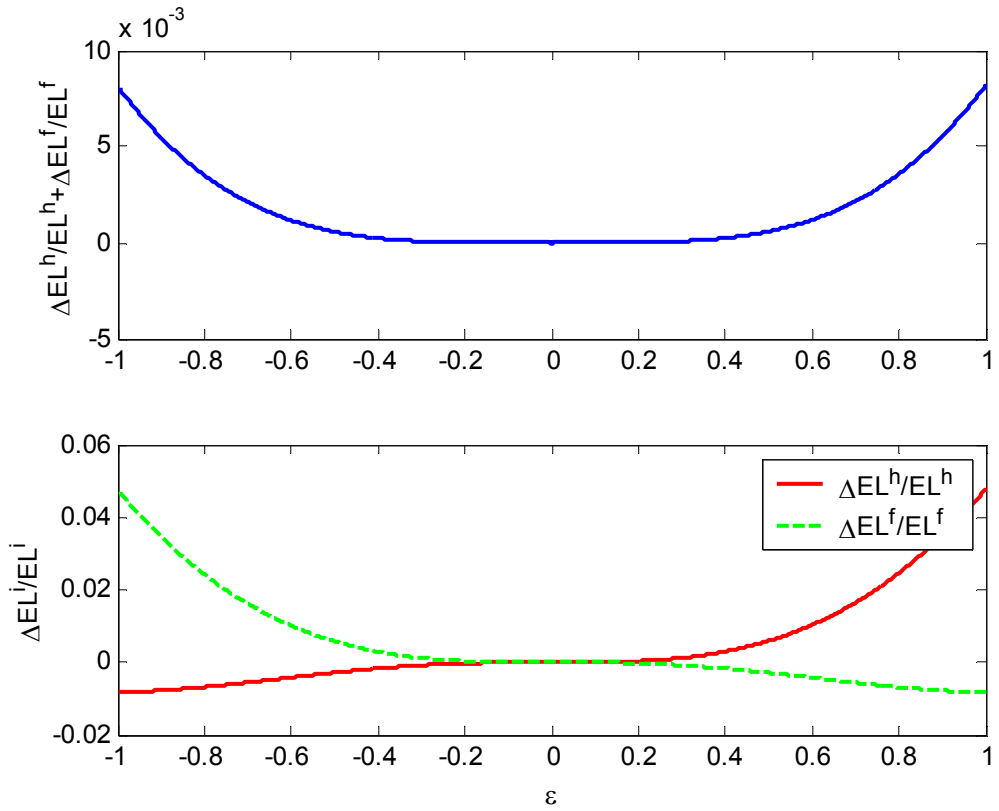


Figure 9 – National welfare differential in function of the degree of heterogeneity



## Appendix: Resolution of the model

To minimize the Union-wide loss function (8), we first write national magnitudes:

$$y_h = y + \bar{y} = \delta - br + \frac{\alpha\bar{\delta} - \alpha b\epsilon r + a\bar{\mu}}{\alpha + a} \quad (\text{A1a})$$

$$y_f = y - \bar{y} = \delta - br - \frac{\alpha\bar{\delta} - \alpha b\epsilon r + a\bar{\mu}}{\alpha + a} \quad (\text{A1b})$$

$$\pi_h = \pi + \bar{\pi} = \frac{\delta - br - \mu}{\alpha} + \frac{\bar{\delta} - b\epsilon r - \bar{\mu}}{\alpha + a} \quad (\text{A1c})$$

$$\pi_f = \pi - \bar{\pi} = \frac{\delta - br - \mu}{\alpha} - \frac{\bar{\delta} - b\epsilon r - \bar{\mu}}{\alpha + a} \quad (\text{A1d})$$

The first order condition for the minimization of (8) is:

$$\frac{dL^U}{dr} = \lambda(\delta - br) + \frac{\lambda\alpha\epsilon(\alpha\bar{\delta} - \alpha b\epsilon r + a\bar{\mu})}{(\alpha + a)^2} + \frac{(\delta - br - \mu)}{\alpha^2} + \frac{\epsilon(\bar{\delta} - b\epsilon r - \bar{\mu})}{(\alpha + a)^2} = 0 \quad (\text{A2})$$

and the optimal interest rate is:

$$br = \frac{(\alpha + a)^2 \left[ (1 + \alpha^2 \lambda) \delta - \mu \right] + \alpha^2 (1 + \alpha^2 \lambda) \epsilon \bar{\delta} + \alpha^2 (\lambda \alpha a - 1) \bar{\mu} \epsilon}{(1 + \alpha^2 \lambda) \left[ (\alpha + a)^2 + \alpha^2 \epsilon^2 \right]}$$

If:  $\omega_1 = (\alpha + a)^2$ ,  $\omega_2 = \alpha^2 \epsilon^2$ , and:  $\omega = (1 + \lambda \alpha^2)(\omega_1 + \omega_2)$ :

$$br = -\frac{\omega_1}{\omega} \mu + \frac{(1 + \lambda \alpha^2) \omega_1}{\omega} \delta + \frac{\omega_2 (\alpha \lambda a - 1)}{\epsilon \omega} \bar{\mu} + \frac{(1 + \lambda \alpha^2) \omega_2}{\epsilon \omega} \bar{\delta} \quad (\text{A3})$$

and we find equation (11) in text, which corresponds to the minimization of (8). We can notice that all equilibrium solutions will be independent of  $b$ , justifying the choice  $b=1$  in the main text.

The first order condition for the maximization of the common central bank loss function with aversion towards divergences (eq. 14a) is:

$$\frac{dL^C}{dr} = \lambda(\delta - br) + \frac{(\delta - br - \mu)}{\alpha^2} + \frac{\alpha\epsilon\theta_y(\alpha\bar{\delta} - \alpha b\epsilon r + a\bar{\mu})}{(\alpha + a)^2} + \frac{\epsilon\theta_\pi(\bar{\delta} - b\epsilon r - \bar{\mu})}{(\alpha + a)^2} = 0 \quad (\text{A4})$$

If  $\theta_y = \theta_\pi = 0$  in (A3), the interest rate becomes:

$$br = \frac{(1 + \lambda\alpha^2)\delta - \mu}{1 + \lambda\alpha^2} \quad (\text{A5})$$

and we find equation (10) in text, which corresponds to the minimization of (19).

In the general case  $\theta_y \neq 0$  and  $\theta_\pi \neq 0$ , defining  $\omega_1 = (\alpha + a)^2$  and  $\omega_2 = \alpha^2 \varepsilon^2$ , we obtain:

$$\frac{dL^C}{dr} = \alpha^2 \lambda \omega_1 (\delta - br) + \omega_1 (\delta - br - \mu) + \alpha \frac{\omega_2}{\varepsilon} \theta_y (\alpha \bar{\delta} - ab\varepsilon r + a\bar{\mu}) + \frac{\omega_2}{\varepsilon} \theta_\pi (\bar{\delta} - b\varepsilon r - \bar{\mu}) = 0$$

from which we obtain the interest rate:

$$br^c = \frac{(1 + \lambda\alpha^2)\omega_1 \delta - \omega_1 \mu + \frac{\omega_2}{\varepsilon} (\theta_\pi + \theta_y \alpha^2) \bar{\delta} + \frac{\omega_2}{\varepsilon} (a\alpha\theta_y - \theta_\pi) \bar{\mu}}{\left[ (1 + \lambda\alpha^2)\omega_1 + (\theta_\pi + \theta_y \alpha^2)\omega_2 \right]} \quad (\text{A6})$$

Thus, if  $\omega = (1 + \lambda\alpha^2)(\omega_1 + \omega_2)$ ,  $\Phi_1 = \alpha^2(\theta_y - \lambda) + \theta_\pi - 1$  and  $\Phi_2 = a\alpha(\theta_y - \lambda) - (\theta_\pi - 1)$ , we find equation (17a) in text, which corresponds to the minimization of (14a). Reintroducing these values in (A1a-d), we find:

- in the optimal regime

$$\begin{aligned} y_h &= \frac{(\alpha + a)A_h}{\omega} \mu + \frac{a\omega - \alpha^2 A_h [\lambda a\alpha - 1] \varepsilon}{\omega(\alpha + a)} \bar{\mu} \\ y_f &= \frac{(\alpha + a)A_f}{\omega} \mu - \frac{a\omega + \alpha^2 A_f [\lambda a\alpha - 1] \varepsilon}{\omega(\alpha + a)} \bar{\mu} \\ \pi_h &= \frac{(\alpha + a)A_h - \omega}{\alpha\omega} \mu - \frac{\alpha^2 A_h (\lambda a\alpha - 1) \varepsilon + \omega\alpha}{\alpha\omega(\alpha + a)} \bar{\mu} \\ \pi_f &= \frac{(\alpha + a)A_f - \omega}{\alpha\omega} \mu - \frac{\alpha^2 A_f (\lambda a\alpha - 1) \varepsilon - \omega\alpha}{\alpha\omega(\alpha + a)} \bar{\mu} \end{aligned}$$

where:

$$\begin{aligned} A_h &= \alpha(1 + \varepsilon) + a \\ A_f &= \alpha(1 - \varepsilon) + a \\ \omega &= (1 + \lambda\alpha^2) [\alpha^2 \varepsilon^2 + (\alpha + a)^2] \end{aligned}$$

- In the centralized regime with aversion towards divergences:

$$y_h = \frac{(\alpha + a)A_h}{Z} \mu + \frac{aZ - \alpha^2 A_h [\theta_y a\alpha - \theta_\pi] \varepsilon}{Z(\alpha + a)} \bar{\mu}$$

$$y_f = \frac{(\alpha + a)A_f}{Z} \mu - \frac{aZ + \alpha^2 A_f [\theta_y a \alpha - \theta_\pi] \varepsilon}{Z(\alpha + a)} \bar{\mu}$$

$$\pi_h = \frac{(\alpha + a)A_h - Z}{\alpha Z} \mu - \frac{\alpha^2 A_h [\theta_y a \alpha - \theta_\pi] \varepsilon + Z\alpha}{\alpha Z(\alpha + a)} \bar{\mu}$$

$$\pi_f = \frac{(\alpha + a)A_h - Z}{\alpha Z} \mu - \frac{\alpha^2 A_f [\theta_y a \alpha - \theta_\pi] \varepsilon - Z\alpha}{\alpha Z(\alpha + a)} \bar{\mu}$$

where:  $Z = (1 + \lambda \alpha^2)(\alpha + a)^2 + \alpha^2 \varepsilon^2 (\alpha^2 \theta_y + \theta_\pi)$ .

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