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A note on price-taking and price-making behaviours in general equilibrium oligopoly models^{*}

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Abstract. *This paper explores the rationale of price-taking and price-making behaviours in the context of Walrasian and Cournotian pure exchange economies. Beside the influence of the number of agents, we underline the role of the structure of preferences. Through three equilibrium variations of the same basic economy, we obtain several results about price manipulation, about asymptotic identifications for large economies and for degenerate preferences, and about welfare comparisons. Perfect competition does not only correspond to the case of large economies, but may also concern economies where market powers are more or less equivalent.*

JEL Classification: D43, D51

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1. Introduction

In Debreu's *Theory of Value* (1959), perfect competition is formally defined¹. There are a finite number of agents (consumers and firms) and a finite list of goods. Each good is associated a single price expressed in a *numéraire*. Perfectly competitive behaviour is then put forward. Each rational consumer selects the net transactions that maximise her/his utility under the assumptions that unlimited quantities can be bought or sold at the specified prices and that these plans do not influence the profits received. Equivalently, each rational firm selects the inputs and outputs that maximise its net receipts, given it can buy and sell any quantities without manipulating prices. A Walrasian equilibrium is characterised by a price vector and perfectly competitive individual choices such that all markets simultaneously clear.

Two essential specificities of perfectly competitive economies are their major results of Pareto optimality and their specific assumption of absence of strategic behaviour. If the welfare properties are endogenously explained, the competitive behaviour assumption may appear as an enigmatic principle. For what type of economies is the parametric behaviour founded? Under what conditions is perfect competition justified? More substantially, is a competitive economy an economy where agents have no market power, or an economy where they do not use the market power they may have? These questions might justify the attempts to find some strategic foundations for the competitive behaviour, possibly considered as a limiting case of a more general theory of markets (see Gale (2000)). In the

¹ This point is developed by Arrow and Hahn (1971) and discussed in Roberts (1989).

literature, the concept of perfect competition is often justified by economic negligibility of any agent and characterises economies with numerous agents (Mas-Colell (1982)).

Two major lines of research were developed throughout the literature in order to give a rational foundation to perfect competition. The relation between the core and the competitive allocations has been studied in the perspective of the asymptotic approach through replication procedure by Debreu and Scarf (1963) and also with the atomic approach where the set of agents is indexed by a continuum with an atomless measure space (Aumann (1964)). In these Edgeworthian perspectives, the price-taking and the coalitional strategic behaviours can be identified under individual negligibility and for a great number of traders. The second line of research introduces non cooperative strategic behaviour, turning the Walrasian equilibrium into a type of Cournotian general equilibrium². We here focus on the approach opened by Gabszewicz and Vial (1972) in an economy with production and pursued by Codognato and Gabszewicz (1991), (1993) in the context of pure exchange economies. Different concepts of Cournot-Walras equilibria can be developed, depending on the way strategic behaviour is introduced (see Gabszewicz and Michel (1997))³. In these general equilibrium Nashian perspectives, the Cournot-Walras equilibria can be identified to the competitive equilibrium in the case of large economies.

² This field of research mixing the general approach *à la* Walras and the little number competition *à la* Cournot is not unified. Opened by Shapley and Shubik (1977), the strategic market games treat all traders symmetrically and allow them to manipulate the price structure. It essentially aims at replacing the Walrasian auctioneer. Another view, particularly developed by Gabszewicz (2002), considers that the behaviour of agents is no longer symmetric, 'significant' agents trying to manipulate the price system.

³ These authors define a general notion of non cooperative equilibrium for a quantity setting oligopoly in pure exchange economies. They thus capture a large variety of market structures.

The objective of this paper is to question the rationale of price-taking or price-making behaviour in the framework of general equilibrium oligopoly, when all traders may or may not manipulate the price of the good they own. We thus consider a class of pure exchange economies with two goods similar to these analysed in Gabszewicz (2002). About endowments, the market sizes are the same in the economy, the market shares are the same in each sector, but the market concentration may be different or identical between the two sectors. About preferences, we assume an identical Cobb-Douglas specification for every individual and discuss the role of the α parameter as an index of the preference of good 1 relatively to good 2. Regarding three variations of this pure exchange economy, we obtain the following results. First, *compared to the level of the competitive price, the level of the oligopoly price is in favour of the oligopolists when they trade with price-takers.* Second, *the oligopoly behaviour tends to the competitive behaviour when the number of agents goes to infinity.* Third, *the oligopoly behaviour tends to the competitive one when the preferences of agents are strongly unbalanced.* Fourth, *there is no possible Pareto domination among the four equilibria, but between the competitive equilibrium and the symmetric oligopoly equilibrium.* We show that price-taking behaviour does not only concern large economies, but maybe also economies where market powers are more or less equivalent.

The paper is organised as follows. In section 2, we determine three distinct types of equilibrium for the considered pure exchange economy and give several results. In section 3, in reference with these three concepts of equilibrium, we finally discuss the rationale of perfect competition and price-taking behaviour.

2. Three variations on a pure exchange economy

Consider a pure exchange economy with two consumption goods (1 and 2) and $(m+n)$ consumers. We assume the following Cobb-Douglas specification for the utility function:

$$U_h = x_{h1}^\alpha x_{h2}^{1-\alpha}, \quad 0 < \alpha < 1, \quad \forall h. \quad (1)$$

The structure of the initial endowments is assumed to be the same as in the case of the homogeneous oligopoly developed by Gabszewicz-Michel (1997):

$$\begin{aligned} \omega_h &= \left(\frac{1}{m}, 0 \right), \quad h = 1, 2, \dots, m \\ \omega_h &= \left(0, \frac{1}{n} \right), \quad h = m+1, \dots, m+n. \end{aligned} \quad (2)$$

It is assumed that good 2 is taken as the *numéraire*, so p is the price of good 1 as expressed in units of good 2. We consider three versions of the economy, each corresponding to an equilibrium concept: the competitive or the Walrasian one, the asymmetric oligopoly or the Cournot-Walras ones and the symmetric oligopoly or the Nash-Walras one.

2.1. Walrasian equilibrium

In this context, the behaviour of each agent is competitive. Thus the individual plans come from a non-strategic maximization of the utility subject to the budget constraint, which can be written:

$$\text{Arg max}_{\{z_{h1}\}} \left(\frac{1}{m} - z_{h1} \right)^\alpha (pz_{h1})^{1-\alpha}, \quad h = 1, 2, \dots, m \quad (3)$$

$$\underset{\{z_{h2}\}}{\text{Arg max}} \left(\frac{1}{p} z_{h2} \right)^\alpha \left(\frac{1}{n} - z_{h2} \right)^{1-\alpha}, h = m+1, \dots, m+n \quad (4)$$

where z_{h1} and z_{h2} respectively represent the competitive supply of good 1 by agent h , $h = 1, 2, \dots, m$, and the competitive supply of good 2 by agent h , $h = m+1, \dots, m+n$. From (3) and (4), we deduce the competitive individual offer plans and the demand functions:

$$z_{h1} = \frac{1-\alpha}{m} \text{ and } (x_{h1}, x_{h2}) = \left(\frac{\alpha}{m}, (1-\alpha) \frac{p}{m} \right), h = 1, 2, \dots, m \quad (5)$$

$$z_{h2} = \frac{\alpha}{n} \text{ and } (x_{h1}, x_{h2}) = \left(\frac{\alpha}{np}, (1-\alpha) \frac{1}{n} \right), h = m+1, \dots, m+n \quad (6)$$

The competitive equilibrium price p^* is given by $\sum_{h=1}^{h=m} z_{h1} = \sum_{h=m+1}^{h=m+n} \alpha / (np^*)$,

which yields $1-\alpha = \alpha / p^*$, and finally $p^* = \frac{\alpha}{1-\alpha}$. We deduce the competitive

equilibrium allocations:

$$(x_{h1}^*, x_{h2}^*) = \left(\frac{\alpha}{m}, \frac{\alpha}{m} \right), h = 1, 2, \dots, m \quad (7)$$

$$(x_{h1}^*, x_{h2}^*) = \left(\frac{1-\alpha}{n}, \frac{1-\alpha}{n} \right), h = m+1, \dots, m+n \quad (8)$$

The utility levels reached by each type of agents are respectively

$$U_h^* = \alpha / m, h = 1, 2, \dots, m \text{ and } U_h^* = (1-\alpha) / n, h = m+1, \dots, m+n.$$

2.2. Cournot-Walras equilibria

The agents who have an endowment in good 1 adopt a strategic behaviour in manipulating the price by means of the quantity of good 1 they offer, whereas agents who have an endowment in good 2 behave competitively. We denote s_{h1}

the pure strategy of agents $h = 1, 2, \dots, m$, with $s_{h1} \in [0, 1/m]$. The equilibrium price

$$\text{verifies } \sum_{h=1}^{h=m} s_{h1} = \sum_{h=m+1}^{h=m+n} \frac{\alpha}{np(s_{11}, s_{21}, \dots, s_{m1})}, \text{ so } p = \frac{\alpha}{\sum_{h=1}^{h=m} s_{h1}}.$$

Under this assumption, a *Cournot-Walras equilibrium* is given by a m -uple of strategies $(\hat{s}_{11}, \hat{s}_{21}, \dots, \hat{s}_{m1})$, with $\hat{s}_{h1} \in [0, 1/m]$, $h = 1, 2, \dots, m$, and an allocation $(\hat{x}_1, \hat{x}_2, \dots, \hat{x}_m) \in \mathbb{R}_+^{2m}$ such that (i) $\hat{x}_h = x_h(\hat{s}_{h1}, \hat{s}_{-h1})$ and (ii) $U_h(x_h(\hat{s}_{h1}, \hat{s}_{-h1})) \geq U_h(x_h(s_{h1}, \hat{s}_{-h1}))$, $h = 1, 2, \dots, m$, where s_{-h1} denotes the strategy of every agent who owns a quantity of good 1 and who is different from agent h . The non-cooperative equilibrium is associated to the resolution of the simultaneous strategic programmes:

$$\text{Arg max}_{\{\hat{s}_{h1}\}} \left(\frac{1}{m} - s_{h1} \right)^\alpha \left\{ \frac{\alpha s_{h1}}{[s_{h1} + (m-1)s_{-h1}]} \right\}^{1-\alpha}, h = 1, 2, \dots, m \quad (9)$$

All the agents of the same sector being identical, we have $\hat{s}_{h1} = \hat{s}_{-h1}$, so the m equilibrium strategies are:

$$\hat{s}_{h1} = \frac{(1-\alpha)(m-1)}{m[m-(1-\alpha)]}, h = 1, 2, \dots, m \quad (10)$$

We thus have:

$$\hat{p} = \left(\frac{\alpha}{1-\alpha} \right) \frac{[m-(1-\alpha)]}{(m-1)} \quad (11)$$

The individual allocations are:

$$(\hat{x}_{h1}, \hat{x}_{h2}) = \left(\frac{\alpha}{m-(1-\alpha)}, \frac{\alpha}{m} \right), h = 1, 2, \dots, m \quad (12)$$

$$(\hat{x}_{h1}, \hat{x}_{h2}) = \left(\frac{(1-\alpha)(m-1)}{n[m-(1-\alpha)]}, \frac{1-\alpha}{n} \right), h = m+1, \dots, m+n \quad (13)$$

The associated utility levels can be written $\widehat{U}_h = \frac{\alpha}{m} \left(\frac{m}{m - (1 - \alpha)} \right)^\alpha$, $h = 1, 2, \dots, m$

and $\widehat{U}_h = \frac{(1 - \alpha)}{n} \left\{ \frac{m - 1}{[m - (1 - \alpha)]} \right\}^\alpha$, $h = m + 1, \dots, m + n$.

It is also possible to consider the other asymmetric case, where the strategists are the last n agents, the first m agents behaving as price-takers. We denote s_{h2} the pure strategy of agents $h = m + 1, \dots, m + n$, with $s_{h2} \in [0, 1/n]$. The optimal strategies and the price are then $\check{s}_{h2} = \alpha(n - 1)/n(n - \alpha)$, $h = m + 1, \dots, m + n$ and $\check{p} = [\alpha/(1 - \alpha)][(n - 1)/(n - \alpha)]$. The allocations and the utility levels are $(\check{x}_{h1}, \check{x}_{h2}) = (\alpha/m, \alpha(n - 1)/m(n - \alpha))$ and $\check{U}_h = (\alpha/m)[(n - 1)/(n - \alpha)]^{1 - \alpha}$ for $h = 1, 2, \dots, m$. They are $(\check{x}_{h1}, \check{x}_{h2}) = ((1 - \alpha)/n, (1 - \alpha)/(n - \alpha))$ and $\check{U}_h = [(1 - \alpha)/n][n/(n - \alpha)]^{1 - \alpha}$ for $h = m + 1, \dots, m + n$.

2.3. Nash-Walras equilibrium

We here consider that each agent is an oligopolist and behaves strategically. Each agent h tries to manipulate the price by contracting his/her supply. The

market clearing condition implies that the price must be $p = \frac{\sum_{h=m+1}^{h=m+n} s_{h2}}{\sum_{h=1}^{h=m} s_{h1}} = \frac{s_2}{s_1}$.

Under that assumption, a *symmetric oligopoly equilibrium* is a $(m + n)$ -uple of strategies $(\tilde{s}_{11}, \dots, \tilde{s}_{m1}, \tilde{s}_{m+12}, \dots, \tilde{s}_{m+n2})$, with $\tilde{s}_{h1} \in [0, 1/m]$ for $h = 1, 2, \dots, m$ and $\tilde{s}_{h2} \in [0, 1/n]$ for $h = m + 1, \dots, m + n$, and an allocation $(\tilde{x}_1, \dots, \tilde{x}_m, \tilde{x}_{m+1}, \dots, \tilde{x}_{m+n}) \in \mathbb{R}_+^{2(m+n)}$ such that (i) $\tilde{x}_h = x_h(\tilde{s}_{h1}, \tilde{s}_{-h1})$ and $U_h(x_h(\tilde{s}_{h1}, \tilde{s}_{-h1})) \geq U_h(x_h(s_{h1}, \tilde{s}_{-h1}))$ for $h = 1, 2, \dots, m$ and (ii) $\tilde{x}_h = x_h(\tilde{s}_{h2}, \tilde{s}_{-h2})$

and $U_h(x_h(\tilde{s}_{h2}, \tilde{s}_{-h2})) \geq U_h(x_h(s_{h2}, \tilde{s}_{-h2}))$ for $h = m+1, \dots, m+n$. The non-cooperative equilibrium is associated to the resolution of the simultaneous strategic programmes:

$$\text{Arg max}_{\{\tilde{s}_{h1}\}} \left(\frac{1}{m} - s_{h1} \right)^\alpha \left(\frac{s_2}{s_1} s_{h1} \right)^{1-\alpha}, h = 1, 2, \dots, m \quad (14)$$

$$\text{Arg max}_{\{\tilde{s}_{h2}\}} \left(\frac{s_1}{s_2} s_{h2} \right)^\alpha \left(\frac{1}{n} - s_{h2} \right)^{1-\alpha}, h = m+1, \dots, m+n, \quad (15)$$

which gives the following optimal strategies:

$$\tilde{s}_{h1} = \frac{(1-\alpha)(m-1)}{m[m-(1-\alpha)]}, h = 1, 2, \dots, m \quad (16)$$

$$\tilde{s}_{h2} = \frac{\alpha(n-1)}{n(n-\alpha)}, h = m+1, \dots, m+n. \quad (17)$$

We deduce the price $\tilde{p} = \frac{\sum_{h=m+1}^{h=m+n} \tilde{s}_{h2}}{\sum_{h=1}^{h=m} \tilde{s}_{h1}}$, which is:

$$\tilde{p} = \left(\frac{\alpha}{1-\alpha} \right) \frac{(n-1)[m-(1-\alpha)]}{(n-\alpha)(m-1)}. \quad (18)$$

The individual allocations are:

$$(\tilde{x}_{h1}, \tilde{x}_{h2}) = \left(\frac{\alpha}{m-(1-\alpha)}, \frac{\alpha(n-1)}{m(n-\alpha)} \right), h = 1, 2, \dots, m \quad (19)$$

$$(\tilde{x}_{h1}, \tilde{x}_{h2}) = \left(\frac{(1-\alpha)(m-1)}{n[m-(1-\alpha)]}, \frac{1-\alpha}{n-\alpha} \right), h = m+1, \dots, m+n \quad (20)$$

The utility levels reached are respectively $\tilde{U}_h = \frac{\alpha}{m} \left[\frac{m}{m-(1-\alpha)} \right]^\alpha \left(\frac{n-1}{n-\alpha} \right)^{1-\alpha}$ for

$h = 1, 2, \dots, m$ and $\tilde{U}_h = \left(\frac{1-\alpha}{n} \right)^\alpha \left[\frac{m-1}{m-(1-\alpha)} \right]^\alpha \left(\frac{n}{n-\alpha} \right)^{1-\alpha}$ for $h = m+1, \dots, m+n$.

The three variations on the considered pure exchange economy lead to the following four results.

Result 1. *Compared to the level of the competitive price, the level of the oligopoly price is in favour of the oligopolists when they trade with price-takers.*

Proof. The price level depends on either strategic or competitive aggregate supplies of goods 1 and 2: $p^* = z_2^* / z_1^*$, $\hat{p} = \hat{z}_2 / \hat{s}_1$, $\check{p} = \check{s}_2 / \check{z}_1$ and $\tilde{p} = \tilde{s}_2 / \tilde{s}_1$,

where $z_1^* = \sum_{h=1}^{h=m} z_{h1}^*$, $z_2^* = \sum_{h=m+1}^{h=m+n} z_{h2}^*$, $\hat{s}_1 = \sum_{h=1}^{h=m} \hat{s}_{h1}$, $\hat{z}_2 = \sum_{h=m+1}^{h=m+n} \hat{z}_{h2}$,

$\check{z}_1 = \sum_{h=1}^{h=m} \check{z}_{h1}$, $\check{s}_2 = \sum_{h=m+1}^{h=m+n} \check{s}_{h2}$, $\tilde{s}_1 = \sum_{h=1}^{h=m} \tilde{s}_{h1}$ and $\tilde{s}_2 = \sum_{h=m+1}^{h=m+n} \tilde{s}_{h2}$. First, $\hat{p} > p^*$

because $\hat{s}_1 < z_1^*$ and $\hat{z}_2 = z_2^*$. Second, $\check{p} < p^*$ because $\check{z}_1 = z_1^*$ and $\check{s}_2 < z_2^*$. Third,

\tilde{p} may be bigger or equal or smaller than p^* as $\tilde{s}_1 < z_1^*$ and $\tilde{s}_2 < z_2^*$.

The oligopolists “à la Cournot” try to influence the price level in the direction that favours them by a contraction of their supply (see Gabszewicz (2002)).

Result 2. *The oligopoly behaviour tends to the competitive behaviour when the number of agents goes to infinity.*

Proof. For every kind of equilibrium concept, let's denote by E the set of equilibrium outcomes, i.e. $E = \{p, x, U\}$, where p is the relative price, x is the allocation and U is the associated utility level in each case. It is easy to verify that

$$\lim_{m \rightarrow \infty} \hat{E} = E^*, \lim_{n \rightarrow \infty} \check{E} = E^*, \lim_{n \rightarrow \infty} \tilde{E} = \hat{E}, \lim_{m \rightarrow \infty} \tilde{E} = \check{E} \text{ and } \lim_{\substack{m \rightarrow \infty \\ n \rightarrow \infty}} \tilde{E} = E^*.$$

The price making behaviour and the price taking behaviour lead to supplied quantities that tend to be identical in large economies. This happens because when the number of agents on the considered side of the exchange is “very” big, adopting a strategic behaviour is no longer effective and is eventually equivalent to adopting a parametric behaviour.

Result 3. *The oligopoly behaviour tends to the competitive behaviour when the preferences of the agents are strongly unbalanced.*

Proof. When $\alpha \rightarrow 0$, $\widehat{s}_{h1} = \widetilde{s}_{h1}$ tends to $z_{h1}^* = \widetilde{z}_{h1}$, $\forall h = 1, 2, \dots, m$. So, \widehat{E} tends to E^* and \widetilde{E} tends to \widetilde{E} . When $\alpha \rightarrow 1$, $\widetilde{s}_{h2} = \widehat{s}_{h2}$ tends to $z_{h2}^* = \widehat{z}_{h2}$, $\forall h = m + 1, \dots, m + n$. So, \widetilde{E} tends to E^* and \widehat{E} tends to \widehat{E} . Moreover, we can easily verify that $\lim_{\alpha \rightarrow 0}(\widetilde{p} / p^*) = \lim_{\alpha \rightarrow 0}(\widetilde{p} / p^*) = (n - 1) / n$ and $\lim_{\alpha \rightarrow 1}(\widetilde{p} / p^*) = \lim_{\alpha \rightarrow 1}(\widehat{p} / p^*) = m / (m - 1)$.

The price making behaviour and the price taking behaviour lead to supplied quantities that tend to be identical for the sellers of a strongly undervalued good. This happens because when the good supplied by the considered side of the exchange is strongly depreciated, adopting a strategic behaviour is no longer effective and is eventually equivalent to adopting a parametric behaviour⁴.

⁴ About the price limits for limit values of α , one might notice the following three cases (i) $\lim_{\alpha \rightarrow 0}(p^*, \widehat{p}, \widetilde{p}, \widetilde{p}) = (0, 0, 0, 0)$, (ii) $\lim_{\alpha \rightarrow 1}(p^*, \widehat{p}, \widetilde{p}, \widetilde{p}) = (\infty, \infty, \infty, \infty)$ and (iii) if $\alpha = 1/2$, then $p^* = 1$.

Result 4. *There is no possible Pareto domination among the four equilibria, but between the competitive equilibrium and the symmetric oligopoly equilibrium.*

Proof. We state the proof in three steps. First, we show that there is no Pareto domination between the Walrasian equilibrium and the Cournot-Walras equilibria. Second, we show that there is no Pareto domination between the Nash-Walras equilibrium and the Cournot-Walras equilibria. Third, we show that the Nash-Walras equilibrium is Pareto dominated by the Walrasian equilibrium only in the case of a neutralisation of the relative advantages based on the agents' preferences and on the number of agents.

Step 1: $\widehat{U}_h = U_h^* \{m/[m - (1 - \alpha)]\}^\alpha$ implies $\widehat{U}_h > U_h^*$ for $h = 1, 2, \dots, m$ and $\widehat{U}_h = U_h^* \{(m - 1)/[m - (1 - \alpha)]\}^\alpha$ implies $\widehat{U}_h < U_h^*$ for $h = m + 1, \dots, m + n$. The result is reverse for \widetilde{U}_h .

Step 2: $\widetilde{U}_h = \widehat{U}_h [(n - 1)/(n - \alpha)]^{1 - \alpha}$ implies $\widetilde{U}_h < \widehat{U}_h$ for $h = 1, 2, \dots, m$ and $\widetilde{U}_h = \widehat{U}_h [n/(n - \alpha)]^{1 - \alpha}$ implies $\widetilde{U}_h > \widehat{U}_h$ for $h = m + 1, \dots, m + n$. The result is again reverse for \widetilde{U}_h .

Step 3 has two parts. We firstly show that the Nash-Walras equilibrium is Pareto dominated by the competitive equilibrium when there no net relative advantage on one side of the market. The neutralisation of the relative advantages between the two kinds of agents might be translated by $\tilde{p} = p^*$, which is equivalent to $1 = [(n - 1)/(n - \alpha)].[m - (1 - \alpha)]/(m - 1)$. Does this imply $\widetilde{U}_h < U_h^*$ for $h = 1, 2, \dots, m$ and $\widetilde{U}_h < U_h^*$ for $h = m + 1, \dots, m + n$? We see that $\tilde{p} = p^*$ is equivalent to $1 = [(n - 1)/(n - \alpha)].[m - (1 - \alpha)]/(m - 1)$. We have $\widetilde{U}_h < U_h^*$ for

$h = 1, 2, \dots, m$ if and only if $(m-1)^{1-\alpha} m^\alpha < m - (1-\alpha)$. This leads to prove that $[1 + 1/(m-1)]^\alpha < 1 + [\alpha/(m-1)]$. Define $\Gamma(x) \equiv \alpha \log(1+x) - \log(1+\alpha x)$, where $x = 1/(m-1)$, with $0 < x \leq 1$. We must verify that $\Gamma(x) < 0$ when $x \in]0, 1[$. As $\Gamma(0) = 0$ and $\Gamma'(x) = \alpha^2(x-1)/[(1+\alpha)(1+\alpha x)] < 0$, we have $\Gamma(x) < 0, \forall x \in]0, 1[$. The argument is similar for $h = m+1, \dots, m+n$. Secondly, the absence of Pareto domination between the two equilibria means that $\tilde{U}_h > U_h^*$ for $h = 1, 2, \dots, m$ and $\tilde{U}_h < U_h^*$ for $h = m+1, \dots, m+n$ (or conversely). Little algebra shows that these two inequalities require $(1-1/m)^{1-\alpha} < (1-1/m)^\alpha$. If $\alpha = 1/2$ (absence of relative advantage due to preferences), this condition stands if $m < n$ (relative advantage due to the endowments in favour of the m first agents). If $m = n$, this condition stands if $\alpha > 1/2$. The argument is similar for $\tilde{U}_h < U_h^*$ for $h = 1, 2, \dots, m$ and $\tilde{U}_h > U_h^*$ for $h = m+1, \dots, m+n$.

3. Competitive or strategic behaviour: *fundamental and behavioural market powers*

In the models developed, the strategic behaviour brings a higher satisfaction compared to the competitive behaviour under two kinds of condition. Firstly, the absence of evanescence of the effectiveness of the oligopoly behaviour is required. For the strategic behaviour to be *effective*, it must imply a significant contraction of the offer in order to really push for a more favourable price. This sensitive reduction of the supply occurs for the first type of agents when good 1 is not greatly unappreciated (α does not tend to 0) and when the supply of good 1 is not very scattered (m is finite). It occurs for the second type of agents when good 2 is

not greatly unappreciated (α does not tend to 1) and when the supply of the good 2 is not very scattered (n is finite). Secondly, the absence of neutralisation of the price manipulation is needed, in case this strategy would be engaged at the same time by the two types of agents. The reduction of the supply must indeed involve a more favourable price for the strategic behaviour to be *efficient*. The achievement of such an advantageous price occurs for one kind of agents when the other kind has a parametric behaviour.

But what is the relevance of each type of behaviour and of each concept of equilibrium? More specifically, under what conditions are the price taking behaviour and the Walrasian equilibrium justified? A first way to answer these problems is to emphasize the institutional framework of each equilibrium concept. In particular, one might say that the Walrasian equilibrium is grounded when all the logistics of the *tâtonnement* is truly at work, around the auctioneer (see Arrow (1959)). A second way is to determine the conditions under which the different types of equilibrium outcomes would be relevant. We follow throughout the paper this latter logic. Different non-cooperative approaches have been conceived in the literature⁵. We here propose to explore some strategic and non-strategic foundations of price-taking behaviour.

Three elements have been considered: the possible agents' relative preference for one good, the comparative numbers of suppliers in sectors 1 and 2 and the

⁵ Postlewaite and Roberts (1976) show that the utility gain that each agent can achieve in manipulating prices through the adoption of a non-competitive behaviour tends to zero as the number of consumers becomes large, so she/he acts as a price taker. Through a dynamic matching and bargaining framework, Gale (2000) shows how strategic interactions between rational agents can lead to price-taking behaviour. Another argument, based on the 'non surplus condition', is developed by Ostroy and Makowski (2001).

either competitive or strategic behaviour of each kind of agents. And we have seen that the four types of price are functions of these involved parameters, i.e. $p^* = p^*(\alpha)$, $\hat{p} = \hat{p}(\alpha, m)$, $\check{p} = \check{p}(\alpha, n)$ and $\tilde{p} = \tilde{p}(\alpha, m, n)$. In order to analyse the relations between all these elements, let's propose a basic distinction about market power for the pure exchange economy previously developed. One might call *fundamental market power* the relative advantages grounded on the fundamental elements identifying the agents (preferences and endowments), and *behavioural market power* the adoption of the strategic price-making behaviour, or the rejection of the parametric price-taking behaviour.

The value of α represents the unanimous preference for good 1 relatively to good 2⁶. When $\alpha \rightarrow 1$, good 1 is commonly more appreciated, so the initial owners of commodity 1 detain a relative fundamental market power. When $\alpha \rightarrow 0$, such a market power is granted to the initial owners of good 2. Finally, when α is around 1/2, the two goods are equally valued by all the consumers, so there is no relative fundamental market power due to preferences.

The specification of the endowments gives the structure of the private property: each agent owns only one good and a same part as every other agent of the same sector⁷. The comparative value m/n represents the relative degree of concentration of sector two compared to sector one. When $m < n$, sector one is more concentrated and that might be provide a fundamental market power to the owners

⁶ We could have introduced another utility function for agents who own good 2 under the form $U_h = x_{h1}^\beta x_{h2}^{1-\beta}$ with $\beta \neq \alpha$, in order to model heterogeneity in preferences or differentiation in the structure of preferences. The results are not affected by this modification.

⁷ Agents' negligibility then emerges when m and n go to infinity. The concept of negligibility has been largely developed in the literature devoted to the relation between the Edgeworthian core and the Walrasian allocations.

of good 1. Conversely, when $m > n$, sector one is less concentrated, and that might involve a fundamental advantage for the agents of sector 2. When $m = n$, the two sectors are equivalently concentrated, so there is no relative advantage due to the distribution of initial endowments.

Preferences and endowments may provide, or not (when $\alpha = 1/2$ and $m = n$), a relative fundamental market power. They may play in the same direction (for example when $\alpha \rightarrow 1$ and $m < n$), giving rise to a reinforced relative fundamental market power. They might also play in opposite directions (when $\alpha \rightarrow 1$ and $m > n$), giving rise to a net relative fundamental market power being zero, or positive for the agents owning good 1, or positive for the agents owning good 2.

The adoption of a strategic behaviour implies the endeavour either to create or to develop a relative exchange advantage on the others. Conversely, the adoption of a competitive behaviour either reveals a naïve belief (each agent takes the given price as granted or true), or means a pacific attitude (each agent will not try to push some objective relative exchange she/he might have).

Two different views can be developed about the possible link between these two types of market power. According to the first one, the fundamental market power and the behavioural one are independent, and the adoption of a strategic behaviour is just a question of pure individual choice. According to the opposite view, the behavioural market power must be based on a net relative fundamental market power, and the adoption of a strategic behaviour is a simple consequence of a net relative advantage. Let's now develop these alternative views.

If taking or making the price is a matter of choice, then we might consider some game-theoretic foundation of the types of behaviour, leading to a strategic justification of the types of equilibrium. Let i be a representative agent of the first m agents, and j a representative agent of the last n agents. Players i and j have the options to be either price-taker (PT) or price-maker (PM). The following matrix represents the simultaneous *meta-game*, where the associated payoffs are the utility levels reached by each type of agent under the four equilibria⁸:

	j	
		PT PM
i		
	PT	(U_i^*, U_j^*) $(\tilde{U}_i, \tilde{U}_j)$
	PM	(\hat{U}_i, \hat{U}_j) $(\tilde{U}_i, \tilde{U}_j)$

Both agents have PM as a dominant strategy. This gives a strategic foundation of the PM behaviour and a justification of the Nash-Walras equilibrium. Moreover, from result 4 (see section 2), we know that the strategies PM-PM may constitute or not an optimal situation. If there is a net fundamental relative advantage in favour of one sector, then the equilibrium is optimal and the game features an *invisible hand* structure. In the opposite case, the equilibrium is not optimal, as it is dominated by PT-PT, and the game features a *prisoner's dilemma* structure.

Alternatively, if taking or making the price is a given structural characterisation, then it is an exogenous element. There is nevertheless a way to justify it, as a consequence of an objective basis. In case of a net fundamental relative advantage

⁸ There are two versions of the Cournot-Walras equilibrium (see section 2.2).

for one kind of agents, the asymmetric Cournot-Walras equilibrium is justified: the advantaged type of agents becomes the price making side of the exchange, the disadvantaged agents taking the price made by the others as given. In case of a zero net relative fundamental advantage for any kind of agents, a symmetric equilibrium is justified: either the Nash-Walras equilibrium or the Walrasian equilibrium (especially when there is no fundamental market power of any kind). The idea that the behavioural market power needs to be backed up by the fundamental market power is here captured.

4. Conclusion

The common view in economics states that the price taking behaviour is relevant when the number of agents is ‘very’ big, because in these limit circumstances the strategic behaviour is no longer effective, and meets the competitive behaviour. Our analysis leads us to suggest a possible extension of the validity field of perfect competition.

Beyond the usual cases of large economies, the cases of homogeneous economies ($\alpha = 1/2$ and $m = n$), may also be situations for which the Walrasian conception is relevant, both for behavioural and fundamental reasons. Under a normative point of view, in these homogeneous economies, the Walrasian equilibrium dominates the Nash-Walras equilibrium: the universal price-taking behaviour is more efficient than the universal price-making behaviour. Under a positive point of view, if the asymmetric equilibrium concepts such as the

Cournot-Walras equilibrium match heterogeneous economies, the symmetric equilibrium concepts such as the competitive equilibrium or the Nash-Walras equilibrium capture homogeneous economies.

References

- Arrow, K.J. [1959], "Toward a theory of price adjustment", in M. Abramovitz (Eds), *The Allocation of Economic Resources*, Stanford University Press, Stanford, pp. 41-51.
- Arrow, K.J., Hahn, F.H. (1971), *General Competitive Analysis*, Holden Day, San Francisco.
- Aumann, R. (1964), "Markets with a continuum of traders", *Econometrica*, 32, pp. 39-50.
- Codognato, J., Gabszewicz, J.J. (1993), "Cournot-Walras equilibria with a continuum of traders", *Economic Theory*, 3, pp. 453-464.
- Codognato, J., Gabszewicz, J.J. (1991), "Equilibres de Cournot-Walras dans une économie d'échange pure", *Revue Economique*, 42, pp. 1013-1026.
- Debreu, G. (1959), *Theory of Value*, John Wiley and sons, New-York.
- Debreu, G., Scarf, H. (1963), A limit theorem of the core of an economy, *International Economic Review*, 4, pp. 235-246.
- Friedman, J. (1982), "Oligopoly theory", in K.J. Arrow and M. Intriligator (Eds), *Handbook of Mathematical Economics*, Elsevier, North-Holland, pp. 491-534.
- Gabszewicz, J.J. (2002), *Strategic multilateral exchange, general equilibrium with imperfect competition*, Edward-Elgar, Cheltenham.

- Gabszewicz, J.J., Michel, P. (1997), "Oligopoly equilibria in exchange economies", in B.C. Eaton and R.G. Harris (Eds), *Trade, Technology and Economics*. Essays in Honour of R.G. Lipsey, Edward-Elgar, Cheltenham, pp. 217-240.
- Gabszewicz, J.J., Vial, J.P. (1972), "Oligopoly 'à la Cournot' in a general equilibrium analysis", *Journal of Economic Theory*, 4, pp. 381-400.
- Gale, D. (2000), *Strategic Foundations of General Equilibrium: Dynamic Matching and Bargaining Games*, CUP.
- Mas-Colell, A. (1982), *Non Cooperative Approaches to the Theory of Perfect Competition*, Academic Press, New-York.
- Ostroy, J.M., Makowski, L. (2001), "Perfect competition and the creativity of the market", *Journal of Economic Literature*, 39, pp. 479-535.
- Postlewaite, A., Roberts, J. (1976), "The incentives for price-taking behaviour in large exchange economies", *Econometrica*, 44, pp. 115-127.
- Roberts, J. (1989), "Perfectly and imperfectly competitive markets", in J. Eatwell and R. Milgate (Eds), *The New Palgrave*, pp. 837-841.
- Shapley, L., Shubik, M. (1977), "Trade using one commodity as a means of payment", *Journal of Political Economy*, 85, pp. 937-967.