TECHNICAL CHANGE AND INVESTMENT LEVEL IN OPTIMAL NON-LINEAR PRICING.

Patrick GUY*

In this paper, we develop an adverse selection model where a monopoly choices a non-linear pricing associated with an investment level which defines the technology of production used. We show that, in general, to implement a non-linear pricing the monopoly choice a level of investment, which depends of the type of consumer and, also, that the level of investment for each type is correlated with the recovery degree of the investment.

Keys words : Adverse selection, Investment, Non linear pricing, Technology of production.

Prix non linéaire optimal et possibilité technologique en fonction du niveau d'investissement.

Dans cet article, nous développons un modèle de sélection adverse dans lequel un monopole choisit un prix non linéaire associé à un niveau d'investissement qui définit la technologie de production utilisée. Nous montrons qu'en général pour définir le prix non linéaire, le monopole choisit un niveau d'investissement qui dépend du type de consommateurs. De plus, le niveau d'investissement réalisé, pour les différents types, est en corrélation avec le degré de recouvrement des investissements engagés.

Mots Clefs : Sélection adverse, Investissement, Prix non linéaire, Technologie de production.

JEL Classification : D24, D42, D82

*LAMETA, Université de Montpellier I, Faculté des sciences économiques, avzenue de la mer, BP 9606, 34054 Montpellier Cedex 1.   Tel : 04 67 15 84 95   Fax : 04 67 15 84 67   E-mail : pguy@sceco.univ-montp1.fr

I Introduction :

In the most papers about the choice of a non-linear pricing set by a monopoly, which don't know the true characteristic of the consumers, the technology is a invariant parameter (see Maskin and Riley [1984]). In his article, Thomas [1999] shows that, when the monopoly has the choice between two technologies, he doesn't choice systematically the most efficient technology, because for the low type of the consumers the less efficient technology permits to reduce the information rent. This result is very important and significant but, in the most cases, the choice of a technology is not free of charge and the investment level depends of this choice. The target of this paper is to analyze the link between these two parameters and if this link has an influence on the choice of the non-linear pricing set.

II The model :

The monopoly sells a good in quantity q to a continuum of better informed consumers Each consumer knows is type, which is defined by the variable θ, and has an utility which is linear in his type. The transfer associated with the quantity q is a monetary transfer t, so the utility of the consumer can be written as :

$$u(\theta , q , t) = \theta q + v(q) - t \qquad (1)$$

The function $v(q)$ has the natural properties : $v(q) > 0$, $v'(q) > 0$ and $v''(q) < 0$, it is a concave function.

The monopoly don't know the true type of the consumers but it know the distribution function of the characteristics of the consumers : $F(\theta)$ with $\theta \in [\theta_l , \theta_h]$. It choices a non-linear pricing set $(q(\theta_a) , t(\theta_a))$ associated with a level investment set $I(\theta_a)$ which defines the technology used. The two sets depend of the type θ_a announced by the consumer. To extract the maximum gain from the consumer set, the monopoly must implement a mechanism which incites the consumer to participate and to reveal his true type θ. So, the program [P1] of the monopoly is :

$$\text{Max} \{ E_\theta[t(\theta) - C(q(\theta) , I(\theta))] \} \qquad (2)$$

With the two fundamental constraints :

$$u(\theta) \geq 0 , \forall \theta \in [\theta_l , \theta_h].$$

$$u(\theta) = u(\theta , \theta) \geq u(\theta , \theta_a) = \theta q(\theta_a) + v(q(\theta_a)) - t(\theta_a) , \forall (\theta , \theta_a) \in [\theta_l , \theta_h]^2.$$

A natural condition about the control variables (q , t , I) is to be non-negative. The cost function $C(q , I)$ defines the technology used. It is very important to define carefully this function. It depends of three terms. The two first terms are classically used but the last term is never used in the standard industrial economic, so we write :

$$C(q , I) = C_v(q , I) + C_f(I) + \alpha I , \alpha \in [0 , 1] \qquad (3)$$

The value of the coefficient α in the last term depends if the investment is a sunk cost

or not. Without depreciation of the investment, we get : $\alpha = 0$, but in general : $\alpha > 0$, and for complete sunk cost : $\alpha = 1$ [1] (see appendix). The two first terms are classical and have the regular characteristics. To simplify the study, we write :

$$C_v(q , I) = c_m(I) \, q \qquad (4)$$

The functions $c_m(I)$ and $C_f(I)$ are supposed to be non-negative, non-increasing and convex.

III The general solution :

We know from Guesnerie and Laffont [1984] that we can reformulate the two fundamental constraints by (see appendix) :

$$q'(\theta) \geq 0 , \, u'(\theta) = q(\theta) , \, u(\theta_l) = 0 \qquad (5)$$

And we have also (see appendix) :

$$\text{sign}[t'(\theta)] = \text{sign}[q'(\theta)] , \, t(\theta) = \theta \, q(\theta) + v(q(\theta)) - \int_{\theta l}^{\theta} q(s) \, ds \qquad (6)$$

If we use these equations, we can reformulate the program of the monopoly and we get (see appendix) :

$$\text{Max} \{ E_\theta[\theta \, q(\theta) + v(q(\theta)) - C(q(\theta) , I(\theta)) - q(\theta) \, (1 - F(\theta)) / f(\theta)] \} \qquad (7)$$

This program, combined with the two constraints : $u'(\theta) = q(\theta)$ and $u(\theta_l) = 0$, is a typical optimal control program where $u(\theta)$ is the state variable and the couple $(q(\theta) , I(\theta))$ is a control vector. The three constraints : $q(\theta) \geq 0$, $I(\theta) \geq 0$ and $q'(\theta) \geq 0$, characterize the control region. The Pontryagin Maximum Principle gives us a necessary condition set for the solution (see Seierstad and Sysdsaeter [1987]).

Theorem : For all intervals where a solution exists and the control functions are continuous and derivable, the following equations are verified :

$$v'(q(\theta)) - c_m(I(\theta)) = - \theta + (1 - F(\theta)) / f(\theta) \qquad (8)$$

$$c_m'(I(\theta)) \, q(\theta) + C_f'(I(\theta)) = - \alpha \qquad (9)$$

$$- v''(q(\theta)) \, (c_m''(I(\theta)) \, q(\theta) + C_f''(I(\theta)) \geq (c_m'(I(\theta))^2 \qquad (10)$$

Corollary : When the solution exists in agreement with the theorem, it verifies automatically the non-decreasing constraint and we have : $\text{sign}[I'(\theta)] = \text{sign}[q'(\theta)]$. For a given type of consumer θ , the offered quantity and the level of investment increase when the recovery degree of investment increases (the value of α decreases).

At each point of discontinuity θ_d , we have :

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[1] Sometime $\alpha < 0$ is possible (see appendix).

$$\nu'(q_-(\theta_d)) - c_m(I_-(\theta_d)) = \nu'(q_+(\theta_d)) - c_m(I_+(\theta_d)) \qquad (11)$$

Where $(q_-(\theta))$, $I_-(\theta))$ is the solution at left of the θ_d and $(q_+(\theta))$, $I_+(\theta))$ is the solution at right of the θ_d .

Proof : see appendix.

Remark : At θ_d , the continuity of the Hamiltonian implies the following equality :

$$(\theta_d \, q_+(\theta_d) + \nu(q_+(\theta_d)) - C(q_+(\theta_d) \, , \, I_+(\theta_d)) - (\theta_d \, q_-(\theta_d) + \nu(q_-(\theta_d)) - C(q_-(\theta_d) \, , \, I_-(\theta_d)) =$$

$$(q_+(\theta_d) - q_-(\theta_d))(1 - F(\theta_d)) / f(\theta_d) \qquad (12)$$

IV Discussion of the solution:

The different results generalize the results got by Thomas [1999]. In general case, the monopoly uses the more efficient technology for the high type, the investment is maximum and so the cost functions are minimum. When the type is decreasing, the investment is also decreasing and the used technology is less efficient. If discontinuities exist in the choice of the technology (and for the investment), it exists for each discontinuity a breakdown type defined by the discontinuity equations, and the decreasing relation between the type and the technology must be kept.

Thomas [1999] gives the reason for the choice of the more efficient technology only for the high type. With asymmetric information, consumers get an informational rent, which is increasing at rate of the quantity sold (see appendix) and, like we have seen, the quantity increases with the type of the consumer. In order to compensate that, the monopoly increases the investment to get more efficient technology and to increase its direct utility.

All of that is true when the solution can meet the second equation of the theorem, but sometime we can have : $\alpha < 0$ (see appendix). In that case, only a corner solution can exist for the investment. We don't mathematically demonstrate that, but from the corollary we know that the value of the investment increases when the value of the coefficient α decreases and so for : $\alpha < 0$, the only possibility is a maximum investment whatever the type of the consumers. So, the monopoly uses the more efficient technology whatever the type[2].

V Conclusion :

We have showed that, in general, to implement a non-linear pricing the monopoly choices a level of investment which depends of the type of consumer. But for special case, the monopoly uses the maximum investment whatever the type. In fact, the level of investment for each type is correlated with the recovery degree of the investment.

Appendix :

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[2] The value : $\alpha = 0$, is a turn out point for the solution. This is the special case that Thomas [1999] treats.

Cost function :

At the initial time the monopoly invests I, for example it buys a production machine. After the production, it sells the quantity q and receives the transfer t for the payment. If the depreciation allowance is : β I , $\beta \in [0 , 1]$, the true benefice is : $t - C_v(q , I) - C_f(I) - \beta I$. At the end of the operation, the monopoly sells the machine at the price : γI , $\gamma \in [0 , +\infty[$. So, in this operation, the wealth variation of the monopoly is :

$$(t - C_v(q , I) - C_f(I) - \beta I) + (\gamma I - I + \eta I) \qquad (13)$$

Where : $\eta I = \beta I$, because βI in the first parenthesis is a ghost cost[3]. We can write :

$$C(q , I) = C_v(q , I) + C_f(I) + \beta I + (I - \gamma I) - \eta I \qquad (14)$$

When the depreciation allowance is equal at the true economic depreciation of the investment, we have : $\beta I = I - \gamma I$, and so : $\alpha = \beta$. But in the other cases, following the value of γ , we can have : $\alpha \in]-\infty , 1]$, and if : $\gamma = 0$, we have : $\alpha = 1$.

Guesnerie / Laffont's formulation constraints and derived relations :

If we define : $v(\theta , \theta_a) = \theta q(\theta_a) + v(q(\theta_a))$, the revelation constraint gives the two conditions :

$$[\partial u(\theta , \theta_a)/\partial \theta_a]_{\theta a = \theta} = \theta q'(\theta) + v'(q(\theta)) q'(\theta) - t'(\theta) =$$

$$(\partial v(\theta , q(\theta))/\partial q(\theta)) q'(\theta) - t'(\theta) = 0 \qquad (15)$$

$$[\partial\partial u(\theta , \theta_a)/\partial \theta_a{}^2]_{\theta a = \theta} =$$

$$(\partial\partial v(\theta , q(\theta))/\partial q(\theta)^2) q'(\theta)^2 + (\partial v(\theta , q(\theta))/\partial q(\theta)) q''(\theta) - t''(\theta) \leq 0 \qquad (16)$$

If we differentiate around θ the first condition, it is straightforward to see that the second condition can be replaced by the condition :

$$(\partial\partial v(\theta , q(\theta))/\partial\theta\partial q(\theta)) q'(\theta) \geq 0 \qquad (17)$$

And with the definition of the function $v(\theta , q(\theta))$, we get the first relation : $q'(\theta) \geq 0$. Now, if we differentiate the function $u(\theta , \theta)$ around θ , we get :

$$du(\theta , \theta)/d\theta = u'(\theta) =$$

$$[\partial u(\theta , \theta_a)/\partial \theta]_{\theta a = \theta} + [\partial u(\theta , \theta_a)/\partial \theta_a]_{\theta a = \theta} [d\theta_a/d\theta]_{\theta a = \theta} \qquad (18)$$

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[3] That is clearer for an economic world with taxes. The first term in parenthesis is a benefice and the second term is a most-value. In general, the taxes are different for the two terms.

The second term is equal at zero by the first condition, so we get the second relation : $u'(\theta) = q(\theta)$.

Like the function $q(\theta)$ is non-negative and the participation constraint must be binding when it is possible, we get the third relation : $u(\theta_l) = 0$.

We can also rewrite the first condition like thus :

$$t'(\theta) = (\partial v(\theta , q(\theta))/\partial q(\theta)) \, q'(\theta) = dv(\theta , q(\theta))/d\theta - \partial v(\theta , q(\theta))/\partial\theta \qquad (19)$$

And after integration, if we use the third relation, we get the relation :

$$t(\theta) = v(\theta , q(\theta)) - \int_{\theta l}^{\theta} \partial v(s , q(s))/\partial s \, ds \qquad (20)$$

Now, if we use the definition of the function $v(\theta , q(\theta))$, we get the good relations for $t(\theta)$ and $t'(\theta)$ because we have :

$$\partial v(s , q(s))/\partial s = q(s) \qquad (21)$$

$$(\partial v(\theta , q(\theta))/\partial q(\theta)) = \theta + v'(q(\theta)) \geq 0 \qquad (22)$$

Program reformulation :

If we use the last relation in the main program, we get :

$$E_\theta[t(\theta) - C(q(\theta) , I(\theta))] = E_\theta[v(\theta , q(\theta)) - \int_{\theta l}^{\theta} \partial v(s , q(s))/\partial s \, ds - C(q(\theta) , I(\theta))] \qquad (23)$$

Now, we can integrate by part the third term, we get :

$$E_\theta[\int_{\theta l}^{\theta} \partial v(s , q(s))/\partial s \, ds] = \int_{\theta l}^{\theta h} (\int_{\theta l}^{\theta} \partial v(s , q(s))/\partial s \, ds) \, f(\theta) \, d\theta =$$

$$[(F(\theta) - 1) (\int_{\theta l}^{\theta} \partial v(s , q(s))/\partial s \, ds)]_{\theta l}^{\theta h} - \int_{\theta l}^{\theta h} (\partial v(\theta , q(\theta))/\partial\theta) (F(\theta) - 1) \, d\theta =$$

$$E_\theta[(\partial v(\theta , q(\theta))/\partial\theta) (1 - F(\theta)) / f(\theta)] \qquad (24)$$

With the definition of the function $v((\theta , q(\theta))$, we get the result searched.

Theorem and corollary demonstration :

The Hamiltonian of the program is :

$$H = (\theta q(\theta) + v(q(\theta)) - C(q(\theta) , I(\theta)) - q(\theta) (1 - F(\theta)) / f(\theta)) f(\theta) + \psi(\theta) q(\theta) \qquad (25)$$

Where the function $\psi(\theta)$ is the costate variable which is defined by the two equations : $\psi(\theta_h) = 0$ and $\psi'(\theta) = \partial H/\partial u$. So, because the costate variable must be a continuous function, these two equations implicate that the function $\psi(\theta)$ is equal at zero on the total interval.

Because the control function couple maximizes the Hamiltonian, on all intervals

where the control functions are continuous and derivable, we must have : $\partial H/\partial q = 0$, $\partial H/\partial I = 0$ and $((\partial\partial H/\partial q^2)\ \delta q^2 + (\partial\partial H/\partial I^2)\ \delta I^2 + 2\ (\partial\partial H/\partial q\partial I)\ \delta q\ \delta I) \leq 0$. It is highlight that the two first equations give the first equations of the theorem. The last equation is a quadratic form, its first term is non-positive $(\nu''(q(\theta))\ f(\theta)\ \delta q^2 \leq 0)$. So, we know from the quadratic form theory that the relation is true if we have : $(\partial\partial H/\partial q^2)\ (\partial\partial H/\partial I^2) - (\partial\partial H/\partial q\partial I)^2 \geq 0$. This last relation gives the last equation of the theorem.

Now, if we differentiate the two first equations around θ, we get :

$$\nu''(q(\theta))\ q'(\theta) - c_m'(I(\theta))\ I'(\theta) = - 1 + d((1 - F(\theta)) / f(\theta))/d\theta \qquad (26)$$

$$c_m''(I(\theta))\ I'(\theta)\ q(\theta) + c_m'(I(\theta))\ q'(\theta) + C_f''(I(\theta))\ I'(\theta) = 0 \qquad (27)$$

From the second equation, we get :

$$I'(\theta) = (- c_m'(I(\theta)) / (c_m''(I(\theta))\ q(\theta) + C_f''(I(\theta))))\ q'(\theta) \qquad (28)$$

And with the supposed properties for the cost functions, we have the first relation of the corollary. Now, if we use the relation between $I'(\theta)$ and $q'(\theta)$ in the first differentiated equation, we get :

$$(\nu''(q(\theta)) + c_m'(I(\theta))^2 / (c_m''(I(\theta))\ q(\theta) + C_f''(I(\theta))))\ q'(\theta) =$$

$$- 1 + d((1 - F(\theta)) / f(\theta))/d\theta \qquad (29)$$

In left term, the term in parenthesis is non-positive because the solution verifies the third equation of the theorem. The right term is non-positive for all the standard distribution functions. So, $q'(\theta)$ is a non-negative function and the necessary conditions give an admissible solution of the program.

We can also differentiate the two first equations around α with a given type of consumer, we get :

$$\nu''(q(\theta))\ dq(\theta) - c_m'(I(\theta))\ dI(\theta) = 0 \qquad (30)$$

$$c_m''(I(\theta))\ dI(\theta)\ q(\theta) + c_m'(I(\theta))\ dq(\theta) + C_f''(I(\theta))\ dI(\theta) = - d\alpha \qquad (31)$$

From the first equation, we get :

$$dq(\theta) = (c_m'(I(\theta)) / \nu''(I(\theta)))\ dI(\theta) \qquad (32)$$

And with the supposed properties for the marginal cost function and the function $\nu(q)$, we get : $\text{sign}[dq(\theta)] = \text{sign}[dI(\theta)]$. If we use the relation between $dq(\theta)$ and $dI(\theta)$ in the second differentiated equation, we have :

$$(c_m''(I(\theta))\ q(\theta) + (c_m'(I(\theta))^2 / \nu''(I(\theta))) + C_f''(I(\theta)))\ dI(\theta) = - d\alpha \qquad (33)$$

We can write this equation like that :

$- d\alpha = (1 / v''(I(\theta))) (v''(I(\theta)) (c_m''(I(\theta)) q(\theta) + C_f''(I(\theta))) + c_m'(I(\theta))^2) dI(\theta)$ (34)

The term before $dI(\theta)$ is non-negative, so we have : sign[$- d\alpha$] = sign[$dI(\theta)$] , which proves the second sentence of the corollary.

The last proposition of the corollary is easy to proof. If the program has a solution, the control functions have only discontinuity in some points and not on one interval. The first equation of the theorem is verified at left and also at right of one discontinuity point by the two different control function couples. The right function of the equation : $- \theta + (1 - F(\theta)) / f(\theta)$, is a continuous function, so the proposition is verified by the left term of the equation.

Informational rent :

If we use the different getting relations, we can easily write the reformulation program like that :

$$\text{Max } \{ E_\theta[\theta q(\theta) + v(q(\theta)) - C(q(\theta) , I(\theta)) - q(\theta) (1 - F(\theta)) / f(\theta)] \} =$$

$$\text{Max } \{ \int_{\theta l}^{\theta h} (u(\theta) + t(\theta) - C(q(\theta) , I(\theta))) f(\theta) d\theta - \int_{\theta l}^{\theta h} (1 - F(\theta)) du(\theta) \}$$ (35)

The first integral is the social surplus and the second is the total informational rent, so the informational rent for the type θ of consumer is equal at :

$$(1 - F(\theta)) du(\theta) = (1 - F(\theta)) q(\theta) d\theta$$ (36)

This relation proves our assertion.

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