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► **To cite this version:**

Minh Ha-Duong. Fusion of experts' opinions on climate sensitivity with the Transferable Belief Model. 2006. halshs-00112129v1

**HAL Id: halshs-00112129**

**<https://shs.hal.science/halshs-00112129v1>**

Preprint submitted on 7 Nov 2006 (v1), last revised 19 May 2008 (v3)

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# Fusion of experts' opinions on climate sensitivity with the Transferable Belief Model

Minh Ha-Duong\*

November 6, 2006

## Abstract

This paper aggregates experts' opinion on the climate sensitivity parameter from Morgan and Keith [1995] using the Transferable Belief Model. The non-independence of experts issue is dealt with by organizing experts into schools of thought, requiring non-interactivity only across but not within groups. The issue of dissonance, that is wide qualitative differences in beliefs, is dealt with by using a disjunctive rather than conjunctive rule to combine the groups. We find the fusion do not support the idea that climate sensitivity must necessarily be within the IPCC range at any level, that the plausibility of the [4.5, 12] range is about 0.62, and for [6.0, 12] it is 0.31.

## 1 Introduction

This paper aggregates experts' opinions on the climate sensitivity parameter from Morgan and Keith [1995] using the Transferable Belief Model.

Data come from structured interviews using "expert elicitation" methods drawn from decision analysis with 16 leading U.S. climate scientists. The study obtained quantitative, probabilistic judgments about a number of key climate variables. One of them is the climate sensitivity parameter, which can be defined as how much global warming there would be in the long term, if atmospheric CO<sub>2</sub> concentration was doubled relative to its pre-industrial levels.

Reichert and Keith [2001] noted that two aspects of this dataset are critical if one tries to combine them into a single judgment on climate sensitivity. First, the 16 experts are not independent, they are part of a research community regularly sharing data, models and ideas. Second, opinions on climate sensitivity are widely different in qualitative terms: in terms of Evidence Theory, there is a high degree of dissonance in beliefs.

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\*Chargé de recherche au CIRED-CNRS. Mail to haduong@centre-cired.fr. This research was supported by the Centre National de la Recherche Scientifique, France and by the Center for Integrated Assessment of Human Dimensions of Global Change, Pittsburgh PA (created through a cooperative agreement between the National Science Foundation (SBR-9521914) and Carnegie Mellon University

To deal with this dissonance, this paper groups experts' probability distributions in four classes, with a low dissonance within each group. Classes are based on the elicited range of possible values for the climate sensitivity parameter (see Figure 2). Distributions from experts 2, 3 and 6 are the widest, they allow both a positive probability to cooling and to climate sensitivity well above 6°C. Distributions from experts 4, 7, 8, 9 do not give weight to cooling, but have an upper bound above 8°C. Probability distributions of experts numbered 1 and 10 to 16 are formulated on a range with width between 4.2 and 5.5°C. Expert's 5 probability distribution lie in the narrowest range [0°C, 1°C].

Section 2 immediately below reviews prior works on combining experts opinion on climate sensitivity. In section 3, some acquaintance with the Dempster-Shafer theory of evidence Shafer [1976] is assumed, as the text briefly reminds the mathematical notations for the elements of the Transferable Belief Model used in this paper. In that paper, only three different combination rules will be used. They differ as follows:

- The fusion of independent experts is computed using operators appropriate for “non-interactive” information sources. When there is no independence, the fusion of interactive sources is computed using different operators, called “cautious” combination rules below.
- In the same way that logical propositions can be connected by conjunction (AND) or disjunction (OR), fusion operators can be conjunctive or disjunctive. The former are used to combine information sources when assuming that they are all correct. The latter are used when assuming that at least one source is right, but one does not know which one.

The three combination rules used below are a noninteractive conjunction, a noninteractive disjunction and a cautious combination operator, the latter introduced by Denoeux [2006].

Section 4 aggregates the experts' opinion using these rules. It uses a two-stages procedure, based on idea that each of the four experts groups distinguished above is a “school of thought”. The first stage is to combine within groups beliefs using a cautious combination operator. The second stage is to combine the four groups together using a non-interactive disjunction operator. Section 5 discusses, 6 concludes.

## 2 Literature

IPCC [2001b, Technical Summary F.3] states that “climate sensitivity is likely to be in the range of 1.5 to 4.5°C”.

This parameter is critical for climate policy. According to current trends, humankind is well on track to double the CO<sub>2</sub> concentration in the Earth's atmosphere. According to IPCC [2001a] assessment, 2°C of global warming already raises several serious reasons for concern such as the risks to many unique and threatened ecosystems, or a large increase in the risk from extreme climate

|           | $T_{2x} \in [0^\circ\text{C}, 1.5^\circ\text{C}]$ | $[1.5^\circ\text{C}, 4.5^\circ\text{C}]$ | $[4.5^\circ\text{C}, 10^\circ\text{C}]$ |
|-----------|---|--|---|
| Prior     | [0, 0.07]   | [0.31, 0.98]                             | [0.02, 0.62]                            |
| Posterior | [0, 0.00]   | [0.53, 0.99]                             | [0.01, 0.47]                            |

Table 1: Imprecise probability results from Kriegler (2005) on the climate sensitivity parameter  $T_{2x}$ . The prior summarizes PDFs from the literature, the posterior is updated with Dempster’s rule.

events. Thus, if climate sensitivity was at the lower end of the uncertainty range, 1.5K, one could argue that doubling the CO<sub>2</sub> concentration would be somewhat consistent with UNFCCC ultimate objective. But if climate sensitivity was at the upper end, 4.5K, then doubling the CO<sub>2</sub> concentration would almost certainly be a dangerous interference with the climate system. This is why over the last two decades, climate sensitivity has become one of the main communication anchors between the scientists and policymakers to quantify the seriousness of the climate change issue, as discussed by van der Sluijs [1997].

The mainstream way to represent the uncertainty about climate sensitivity is to give a probability density function (PDF). Such PDFs are discussed for example in Hegerl et al. [2006, Figure 3], where the authors estimate that the 5-95 per cent confidence range of climate sensitivity is 1.5 - 6.2°C, and argue that the previous literature overestimated that range’s upper bound.

A more sophisticated way represent to uncertainty about climate sensitivity is to use imprecise probabilities. This approach implies that probability ranges rather than point values are reported. In Kriegler [2005] the author first estimates a prior imprecise distribution based on the literature, and then updates it using a climate model and observational data for 1870–2002. Updating was done by both Dempster’s rule and the Generalized Bayes Rule, but only Dempster’s rule produced meaningful results. Table 4.3 in the reference summarizes those results as follows:

For example, the posterior results suggest that the probability of climate sensitivity being less than 1.5°C is very small (0.00 meaning less than 1 per thousand). In the posterior, the probability that climate sensitivity falls IPCC range is between 0.53 and 0.99. This interval is consistent, if somewhat wider, with IPCC’s qualification that the range is “likely”, meaning with a probability between 66 and 90 percent.

[Reichert and Keith, 2001] previously looked at the question of how to combine the experts judgments on climate sensitivity from Morgan and Keith [1995]. The 5-95 percent fractile shall not be reported here, since the authors make it clear that they do not draw conclusion about future temperature change. Their point is rather that it is necessary to use imprecise probabilities. Based on this, they offer a bayesian aggregation technique where the combined probability distribution do not necessarily narrows as the number of experts is increased, and that is more robust with respect to extreme experts judgments than previously published techniques.

This paper deals very much with the same subject matter as [Reichert and

Keith, 2001], and Kriegler [2005], but using a different flavor of imprecise probabilities: the Transferable Belief Model.

### 3 Elements of the Transferable Belief Model

#### 3.1 Cautious and noninteractive combination rules

The Transferable Belief Model represents uncertainty by allocating the unit mass of belief among subsets of a given frame of reference  $\Omega$ . It allows a non-zero mass to the empty subset  $\emptyset$ . Formally, a basic belief assignment (BBA) is a function  $m : 2^\Omega \rightarrow [0, 1]$  such that:

$$\sum_{A \subset \Omega} m(A) = 1 \quad (1)$$

For any two real-valued subset functions  $\mu_1$  and  $\mu_2$ , we denote  $\mu_1 \cap \mu_2$  the product of the  $\cap$  combination rule defined by:

$$\mu^{1 \cap 2}(A) = \sum_{B \cap C = A} \mu_1(B) \mu_2(C) \quad (2)$$

When  $\mu_1$  and  $\mu_2$  are BBAs, this is the TBM noninteractive conjunction. For any subset  $A \subset \Omega$  and real number  $w$ , we denote  $A^w$  the function  $\mu : 2^\Omega \rightarrow \mathfrak{R}$  such that:

$$\begin{aligned} \mu(A) &= 1 - e^{-w} \\ \mu(\Omega) &= e^{-w} \\ \mu(B) &= 0 \quad \text{otherwise} \end{aligned} \quad (3)$$

When  $w \geq 0$ , the function  $A^w$  is a BBA. A BBA  $m$  is said to be:

**simple** if there is a subset  $A \subset \Omega$  and a weight  $w \geq 0$  such that  $m = A^w$ . It represents reason to believe that the state of the world is in  $A$  to an extent  $w$ .

**non-dogmatic** if  $m(\Omega) > 0$ . Here all simple BBAs are non-dogmatic.

**vacuous** if  $m(\Omega) = 1$ , and  $m(A) = 0$  if  $A \neq \Omega$ . This is also  $A^0$  for any  $A$ .

**separable** if there is a weight function  $w(\cdot) : 2^\Omega \rightarrow [0, +\infty)$  such that:

$$m(A) = \cap_{A \subset \Omega, A \neq \Omega} A^{w(A)} \quad (4)$$

Note that the value of  $w(\Omega)$  is irrelevant, since the vacuous BBA is the identity element for  $\cap$ .

The negation  $\bar{m}$  of a BBA  $m$  is defined by  $\bar{m}(A) = m(\bar{A})$ , where  $\bar{A}$  denotes  $A$ 's complement in  $\Omega$ . For any two subset functions  $m_1$  and  $m_2$ , the TBM noninteractive disjunction  $m_1 \cup m_2$  is defined by:

$$m^{1 \cup 2}(A) = \overline{\bar{m}_1 \cap \bar{m}_2} = \sum_{B \cup C = A} m_1(B) m_2(C) \quad (5)$$

For any non-dogmatic BBA  $m$ , there is a weight function  $w(\cdot)$  in  $\mathfrak{R}$  (it may take negative values) such that equation 4 holds. The weights can be computed using a function  $q$  called the commonality function associated with BBA  $m$  (in what follows  $|A|$  denotes the number of elements of subset  $A$ ):

$$q(A) = \sum_{B \supseteq A} m(B) \quad (6)$$

$$w(A) = \sum_{B \supseteq A} (-1)^{|B|-|A|} \ln(q(B)) \quad (7)$$

For any two non-dogmatic BBA  $m_1$  and  $m_2$ , with corresponding weight functions  $w_1$  and  $w_2$ , the non-interactive conjunction can be computed by adding the weight functions:

$$m_{1 \cap 2}(A) = \cap_{A \subset \Omega, A \neq \Omega} A^{w_1(A)+w_2(A)} \quad (8)$$

Denoeux [2006] defines the cautious combination  $m_1 \wedge m_2$  denoted  $m_{1 \wedge 2}$  by taking the maximum of the weight functions<sup>1</sup>:

$$m_{1 \wedge 2}(A) = \cap_{A \subset \Omega, A \neq \Omega} A^{\max(w_1(A), w_2(A))} \quad (9)$$

All three combination rules are commutative and associative, ensuring that experts are treated symmetrically when combined with them. The cautious rule combination is idempotent, i.e.  $m \wedge m = m$  for all  $m$ . This ensures that an expert's opinion cannot be double-counted. Moreover, the cautious rule distributes over the noninteractive rule:

$$(m_1 \cap m_2) \wedge (m_1 \cap m_3) = m_1 \cap (m_2 \wedge m_3) \quad (10)$$

This implies that when expert A's belief result from the noninteractive conjunction of two pieces of evidence represented by  $m_1 \cap m_2$ , and expert B's belief result from  $m_B = m_1 \cap m_3$ , in the fusion of A's and B's belief the shared evidence  $m_1$  is not counted twice.

### 3.2 Relations between BBAs, probabilities and possibilities

Any probability function  $p : \Omega \rightarrow [0, 1]$  immediately defines a BBA  $m$  by:

$$\begin{aligned} m(\{\omega\}) &= p(\omega) && \text{for any element } \omega \in \Omega \\ m(A) &= 0 && \text{if } |A| > 2 \text{ or } |A| = 0 \end{aligned} \quad (11)$$

Any BBA  $m$  such  $m(\emptyset) \neq 1$  that defines a probability function  $p$ , called the pignistic probability function of  $m$ , by:

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<sup>1</sup>In this paper,  $w$  is the opposite of the log of what Denoeux [2006] denotes with  $w$

$$p(\omega) = \frac{1}{1 - m(\emptyset)} \sum_{\omega \in A} \frac{m(A)}{|A|} \quad (12)$$

Any possibility function  $\pi : \Omega \rightarrow [0, 1]$  immediately defines a BBA  $m$  which can be computed via its commonality function as follows<sup>2</sup>:

$$q(A) = \min_{\omega \in A} \pi(\omega) \quad (13)$$

$$m(A) = \sum_{B \supseteq A} (-1)^{|B|-|A|} q(B) \quad (14)$$

For any BBA  $m$ , one can consider the commonality of singletons :

$$\pi(\omega) = q(\{\omega\}) = \sum_{\omega \in A} m(A) \quad (15)$$

This function  $\pi$  in  $[0, 1]$  is not always a possibility distribution, its maximum may be less than 1. For example, if we start from a probability function  $p$ , using equation 11 then equation 15 gives back  $p$  itself.

Smets [2000] defines the following transformation between a probability  $p$  and a possibility  $\pi$ . Order the probability levels such that  $p_{n_1} > \dots > p_{n_{|\Omega|}}$ , consider the sets  $A_k = \{\omega_{n_1}, \dots, \omega_{n_k}\}$  and assign to  $A_k$  the belief mass:

$$m(A_k) = |A| \times (p_{n_k} - p_{n_{k+1}}) \quad (16)$$

with the convention that  $p_{|\Omega|+1} = 0$ . The procedure is illustrated Figure 1, which makes it is apparent that equation 1 is verified and  $m$  is indeed a belief mass assignment: instead of cutting the unit belief mass distribution in vertical slices, one is cutting it in horizontal slices. It is obvious that  $q(\omega_{n_1}) = 1$ , thus equation 15 defines a possibility distribution.

The pignistic probability of the  $m$  obtained with this procedure is  $p$  itself. Smets [2000] justifies the probability-possibility transformation used here by a principle similar to maximum entropy: among all belief assignments admitting pignistic probability distribution  $p$  (this set is never empty as it contains at least the BBA immediately defined by  $p$  itself), choose the one that is the most uncertain (i.e. that maximizes  $q$  pointwise).

## 4 The aggregation

### 4.1 Implementation

The equations above were implemented in *Mathematica*, using matrix calculus for belief functions described in Smets [2001]. For the sake of numerical tractability, the climate sensitivity range  $[-6^\circ\text{C}, 12^\circ\text{C}]$  was subdivided in seven of mutually exclusive and collectively exhaustive ranges:

<sup>2</sup>Equation 14 is the converse of equation 6 and parallels equation 7.

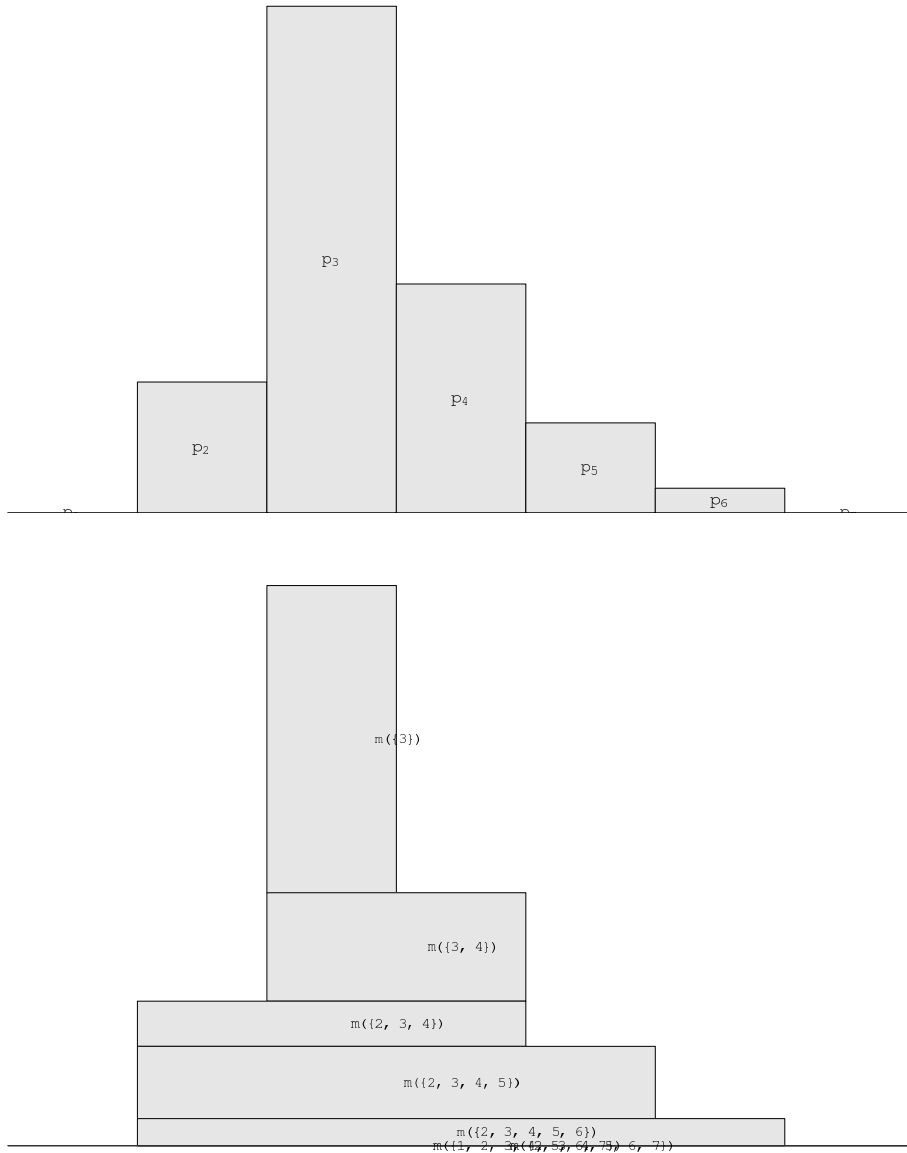


Figure 1: Top, probability distribution from expert 1. The width of each rectangle is 1, and its height is proportional to the probability. Bottom, the corresponding belief function. The outline remains the same, but the slices are cut horizontally.



$$\begin{aligned}\Omega &= \{\omega_1, \dots, \omega_7\} \\ &= \{-6, 0\}, [0, 1.5], [1.5, 2.5], [2.5, 3.5], [3.5, 4.5], [4.5, 6], [6, 12]\end{aligned}$$

Each expert's probability distribution on  $\Omega$  were computed from the elicited PDFs  $P_i$ :

$$\begin{aligned}p_i\{\omega_1\} &= P_i\left(-6 \leq \Delta T_{2 \times \text{CO}_2} < 0\right) \\ p_i\{\omega_2\} &= P_i\left(0 \leq \Delta T_{2 \times \text{CO}_2} < 1.5\right) \\ &\dots\end{aligned}$$

Then the procedure described above equation 16 was used to compute the implicit BBA  $m_i$  and the corresponding possibility distribution  $\pi_i$  associated with each expert's probability distribution  $p_i$ . Figure 2 represent  $p_i$  and  $\pi_i$  for the 16 experts. Four qualitatively different groups of distributions can be seen: Experts  $\{2,3,6\}$  allow cooling. Experts  $\{4,7,8,9\}$  allow high outcomes but no cooling. Experts  $\{1,10-16\}$  disallow extreme cases, and expert  $\{5\}$  distribution is concentrated on  $[0^\circ\text{C}, 1^\circ\text{C}]$ .

The implicit BBAs  $m_i$  are dogmatic except for experts  $\{2,3,6\}$ . This is technically a problem, as weights are defined for non dogmatic BBAs. It was fixed by discounting experts beliefs by 0.01. Denoting  $m_0$  the vacuous BBA, we replaced  $m_i$  by  $m'_i$  defined as:

$$m'_i = 0.99m_i + 0.01m_0 \quad (17)$$

This can be justified by three reasons. One, no expert is 100% reliable. Two, the elicitation of expert's probability distributions was necessarily coarse, so experts who allocated no significant probability weight to extreme outcomes might have agreed that there was a very small possibility. Third, the theoretical literature suggests that the fusion operators can be extended by continuity to dogmatic BBAs.

## 4.2 Result

The fusion uses a two-stages procedure, based on idea that each of the four experts groups distinguished above is a "school of thought". The first stage assumes that within a school of thought, all experts are right, but they are not independent. Thus, their beliefs are combined using a cautious combination operator:

$$m_A = m'_2 \wedge m'_3 \wedge m'_6 \quad (18)$$

$$m_B = m'_4 \wedge m'_7 \wedge m'_8 \wedge m'_9 \quad (19)$$

$$m_C = m'_1 \wedge m'_{10} \wedge \dots \wedge m'_{16} \quad (20)$$

$$m_D = m'_5 \quad (21)$$

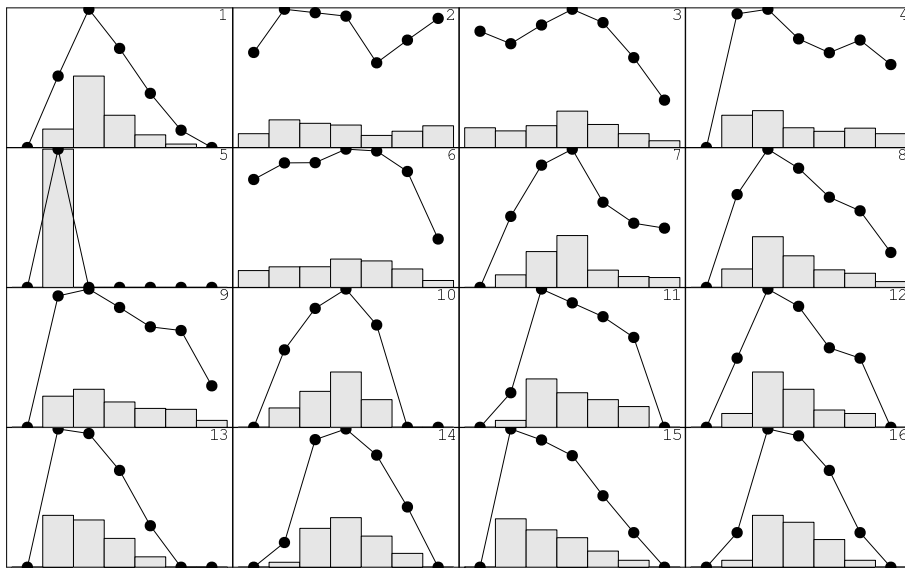


Figure 2: The probability (grey histograms) and implicit possibility (dotted lines) for the 16 experts in [Morgan and Keith, 1995]. The vertical axis goes from 0 to 1. The horizontal axis discretizes the  $[-6^{\circ}\text{C}, 12^{\circ}\text{C}]$  climate sensitivity range into seven intervals using a non-uniform subdivision at  $-6, 0, 1.5, 2.5, 3.5, 4.5, 6$  and  $12^{\circ}\text{C}$ . Four qualitatively different groups of distributions can be seen: Experts 2,3,6 allow cooling, 4,7,8,9 allow high outcomes but no cooling, 1,10-16 disallow extreme cases, and 5 is concentrated on  $[0^{\circ}\text{C}, 1^{\circ}\text{C}]$ .

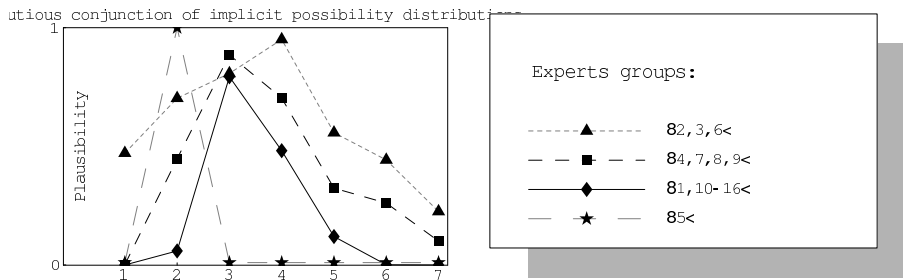


Figure 3: Result of the fusion’s first stage, within-group.

To give an idea of the resulting BBAs, figure 3 represents the commonality of singletons. Note that these BBA do not derive from a possibility function in the meaning of equation 14, but it is not convenient to display the full set functions  $m_A$ ,  $m_B$ ,  $m_C$ , and  $m_D$ .

Within each group, the qualitative divergence is mostly eliminated, so that the fusion do not lead to a collapse of beliefs. This can be quantified as the Transferable Belief Model takes the belief mass associated with the empty set as a measure of dissonance. The worst group is  $C$ , with  $m_C(\emptyset) = 0.13$ . This compares very well to the outcome of the cautious combination of all 16 experts together, where the disagreement index is 0.94.

The second stage is to combine the four groups together using a non-interactive disjunction operator. When several scientific theories compete to explain the same observations, it should not be assumed that both are true at the same time (conjunction), but that at least one will remain (disjunction).

$$m^* = m_A \cup m_B \cup m_C \cup m_D \quad (22)$$

Figure 4 and the first two lines of table 2 display the pignistic probability and the commonality on singletons associated with the fusion  $m^*$ . The complete  $m^*$  is tabulated in Annex A.

In order to compare with table 1, we computed the imprecise probability ranges defined by  $m^*$  for the outcomes “climate sensitivity is below, within and above the IPCC range”. The lower bounds are zero. In particular, the fusion do not support the idea that climate sensitivity must necessarily lie within the IPCC range at any level. This differs from Kriegler’s results, which found the degree of belief in the IPCC range to be .31 in the prior and .53 in the posterior. This difference can be attributed in part to the effect of expert 5, as removing  $m_D$  from the fusion yields a belief level for the IPCC range of 0.13.

Regarding the most worrisome outcomes, that climate sensitivity is above  $4.5^\circ\text{C}$ , the fusion’s result is  $[0, 0.62]$ . This matches table 1 perfectly, although it is based on different data.

| Outcome $\omega$      | [-6,0] | [0,1.5] | [1.5,2.5] | [2.5,3.5] | [3.5,4.5] | [4.5,6.0] | [6.0,12] |
|-----------------------|--------|---------|-----------|-----------|-----------|-----------|----------|
| $q(\{\omega\})$       | 0.48   | 1.      | 1.        | 0.99      | 0.74      | 0.59      | 0.31     |
| Pignistic probability | 0.08   | 0.21    | 0.21      | 0.21      | 0.14      | 0.10      | 0.05     |

| Sensitivity analysis, $q(\{\omega\})$ for different fusion rules |      |      |      |      |      |      |      |
|--|------|------|------|------|------|------|------|
| A: Discount 0.001  | 0.47 | 1.   | 1.   | 0.99 | 0.73 | 0.58 | 0.30 |
| B: $\cap(0.5m_i + 0.5m_0)$                                       | 0    | 0.05 | 0.35 | 0.22 | 0.03 | 0    | 0    |
| C: $\wedge(0.5m_i + 0.5m_0)$                                     | 0.03 | 0.34 | 0.45 | 0.37 | 0.18 | 0.08 | 0.03 |
| D: $\wedge(m'_i)$  | 0    | 0.05 | 0.01 | 0.01 | 0    | 0    | 0    |
| E: $\cup(p_i)$   | 0.32 | 1    | 1    | 0.99 | 0.91 | 0.74 | 0.42 |

Table 2: The fusion of experts opinion on climate sensitivity. Top two lines, commonality of singletons  $q(\{\omega\})$  and pignistic probability associated with  $m^*$ , the result of the staged fusion procedure. Case A to E below, commonality of singletons for various alternative fusion rules considered less justified for this dataset. The commonality of singletons is identical to the level of plausibility of singletons, it can be seen as an upper bound of admissible probability.

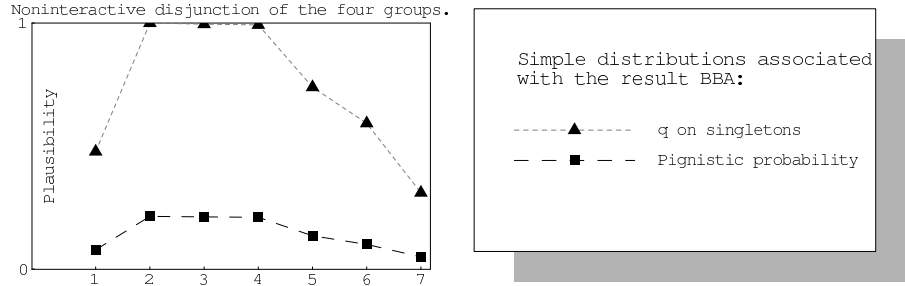


Figure 4: Result of the fusion, using the non interactive disjunction across groups.

## 5 Discussion

### 5.1 Alternative combination rules

Table 2 bottom half presents the results of alternative fusion rules, numbered from A to E.

Case A is obtained by replacing the discount weight .01 by .001 in the staged fusion procedure. This is to check that results are not significantly affected by the technical discounting coefficient introduced equation 17. Indeed, this did not change the results' belief masses by more than 0.01.

Case B display the result of the procedure that discount experts heavily, then fusion all experts using the non interactive conjunction:  $\cap(0.5m_i + 0.5m_0)$ . This procedure was not chosen for  $m^*$  since the choice of the reliability factor 0.5 is hard to justify and experts do interact. Because there is a lot of dissonance, the empty subset receives a large belief mass. Dempster rule of combination would multiply the resulting BBA by  $\frac{1}{1-m(\emptyset)}$  to renormalize it, but this add another step that we find hard to justify.

Case C improves the previous procedure by using a cautious rather than noninteractive combination rule:  $\wedge(0.5m_i + 0.5m_0)$ . The arbitrariness of the 0.5 factor remains. We did not study procedures that gave different discounting weights to experts, as there is no clear guidelines on how to do that. It may well be that in practice, people tend to give more attention to experts that have a small range of uncertainty. That would be problematic from a mathematical point of view for the fusion, as this would increase rather than decrease the issue of dissonance.

Case D illustrates several issues with conjunctive rules. It displays  $\wedge m'_i$ , differing from C by changing the discount factor back to 0.01. Outcomes 1, 6 and 7 have zero possibility, and outcomes 3 and 4 have possibility 0.01. This illustrates the “veto” problem: as soon as one expert gives probability zero to an outcome, it can't appear in the conjunction at a level higher than the discount rate. Since that one is not elicited from the experts, it is a serious issue. The remaining belief mass concentrates on outcome 2. Renormalizing the distribution to cancel the contradiction would lead to a misleading result suggesting that outcome 2 is a concensus view, while in fact it is only the outcome present with a nonnegative but small probability for all experts. This is a well known paradox that justifies why in the Transferable Belief Model  $m(\emptyset) > 0$  is allowed.

Finally, case E shows  $\cup(p_i)$  the fusion is done using the BBAs immediately defined by expert's probabilities. This is a noninteractive disjunction, since the noninteractive conjunction has the same issues with dissonance as the previous case D. This fusion operator is interesting, as it does not require to apply the probability-possibility transformation first, without discounting or renormalization. However, experts in this dataset do interact. For the record, the pignistic probability function in this case E is (0.05, 0.19, 0.19, 0.19, 0.17, 0.17, 0.07).

Cautious disjunction was not used in this paper mainly because we did not find it properly analyzed in the theoretical literature. The literature offers are

many more rules to combine beliefs, there is no space to discuss them all, see Smets [2005] for a survey and Dezert [2002] for interesting developments.

## 5.2 Alternatives to combination rules

This paper uses a mathematical procedure to aggregate experts opinion. In this kind of procedure, assumptions about the independence and the reliability of the information sources have to be made. These assumptions are personal to the entity performing the fusion. As we have seen, they largely determine the ultimate result, especially when information sources are in conflict. This needs discussion, since ideally the results should depend mostly on what the experts said, rather than on the author's choices. The combination of experts' opinion do not has to be done by the middlemen, it can also be left to the decision-maker or to the experts themselves.

One way to avoid the use of mathematical procedures altogether is to rely on behavioral approaches where experts directly interact with each other. To facilitate the interactions and ensure the rationality of the outcome a variety of tools can be used such as the well-known DELPHI method; the IPCC writing process; or having the experts trade contingent assets on prediction markets Wolfers and Zitzewitz [2004]. Another method based on imprecise probabilities was proposed by Nau [2001]. Advantages of behavioral approaches is that collective deliberation is a natural social process, and that the group judgment is more legitimate since it comes from the experts themselves. Drawbacks is that experts interaction are time consuming, and these approaches cannot be used in automated information processing to model learning.

Expert aggregation can only lead to a loss of information, so it may be sometimes better to leave with the decision-maker the task of the combining the judgment of all experts. This is the position explicit in Morgan and Keith [1995]. Keith [1996] argue in more details why that combining experts is rarely appropriate, and suggests instead to use alternative analysis framework such as seeking robust adaptative strategies or using scenarios analysis to bound the problem.

I concur that if the goal is to obtain a single precise probability distribution from which expected utility maximization can provide an optimal answer to all policy issues, then combining experts is a gross oversimplification of reality. But when decisions involve different parties, each will tend to break the symmetry of the elicitation process by myopically focusing on the results best supporting their interest. Another risk is that organizations seeking a balanced point of view would overemphasize the most extreme positions in the group, even when they are actually a minority not representative of the expert's general opinion.

Moreover, if the goal is to obtain an imprecise probability distributions, then it do not necessarily follows that one is laying ground for an exercise in expected utility-maximization. Less single-minded rules for decision making under uncertainty are discussed in [Bewley, 2002, Walley, 1991]. Combining experts judgement can be used to quantify the parameters in the alternative robust analysis frameworks. For example, this paper's possibility distribution

was actually computed as part of a larger exercise to build global warming scenarios.

## 6 Conclusion

An original information fusion method has been applied to combine expert's opinion on climate sensitivity. It is based on a simple model of experts' social relations: they are divided in schools of thought. This method requires to formalize in logical terms the meta-information about the experts, but avoids the problem with discounting and/or renormalization of beliefs.

Within each school experts opinions are aggregated using the cautious combination operator of the Transferable Belief Model. This operator do not assume independence, which is a preferable assumption since in this dataset experts were obviously highly interactive sources of information.

Then the groups beliefs were aggregated using the noninteractive disjunction. This do not assume that all competing theories are right, but that at least one is. When several scientific theories compete to explain the same observations, it should not be assumed that both are true at the same time (conjunction), but that at least one will remain (disjunction).

As in the previous literature, the fusion allows a high (0.62) plausibility that climate sensitivity is actually above  $4.5^{\circ}\text{C}$ . The policy-relevance of this result is mitigated by the fact that the data is outdated compared to the future IPCC Assessment Report 4. However, there is no evidence that experts beliefs on climate sensitivity have evolved downwards since 1995. Evidence theory could also be used to represent learning in a dynamic context, when information on a changing world is collected sequentially, but this requires more research.

In the context of this special issue, this paper shows how learning can proceed from conflicting sources of information. It reminds that learning is not simply an accumulation of observational data over time towards smaller uncertainty ranges. While evidence from repeated controlled experiments should indeed be fused with conjunctive rules, when a new scientific theory or explanation emerges it should be treated as disjunctive learning, increasing uncertainty ranges.

## Acknowledgements

Thanks to David Keith for providing the data of the experts survey. This research was supported by the Centre National de la Recherche Scientifique, France and by the Center for Integrated Assessment of Human Dimensions of Global Change, Pittsburgh PA (created through a cooperative agreement between the National Science Foundation (SBR-9521914) and Carnegie Mellon University).

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## Annex A: The full $m^*$ result of the fusion

| subset $A$            | $m^*(A)$ |
|-----------------------|----------|
| {2}                   | 0.0001   |
| {3, 2}                | 0.0074   |
| {4, 2}                | 0.0033   |
| {4, 3, 2}             | 0.1587   |
| {4, 3, 2, 1}          | 0.0064   |
| {5, 4, 2}             | 0.0011   |
| {5, 4, 3, 2}          | 0.1321   |
| {5, 4, 3, 2, 1}       | 0.0709   |
| {6, 4, 3, 2}          | 0.0267   |
| {6, 4, 3, 2, 1}       | 0.0129   |
| {6, 5, 4, 3, 2}       | 0.0888   |
| {6, 5, 4, 3, 2, 1}    | 0.1811   |
| {7, 4, 3, 2}          | 0.0211   |
| {7, 5, 4, 3, 2}       | 0.0063   |
| {7, 6, 4, 3, 2}       | 0.0135   |
| {7, 6, 4, 3, 2, 1}    | 0.0105   |
| {7, 6, 5, 4, 3, 2}    | 0.0632   |
| {7, 6, 5, 4, 3, 2, 1} | 0.1956   |