



**HAL**  
open science

# Imperfect competition, technical progress and capital accumulation

Biancamaria d'Onofrio, Bertrand Wigniolle

► **To cite this version:**

Biancamaria d'Onofrio, Bertrand Wigniolle. Imperfect competition, technical progress and capital accumulation. 2006. halshs-00118568v1

**HAL Id: halshs-00118568**

**<https://shs.hal.science/halshs-00118568v1>**

Submitted on 5 Dec 2006 (v1), last revised 3 Oct 2007 (v2)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



# Centre d'Economie de la Sorbonne

UMR 8174

C  
a  
h  
i  
e  
r  
s  
de  
la  
M  
S  
E

## Imperfect competition, technical progress and capital accumulation

Biancamaria d'ONOFRIO

Bertrand WIGNIOLLE

2006.46



CENTRE NATIONAL  
DE LA RECHERCHE  
SCIENTIFIQUE

Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13  
<http://mse.univ-paris1.fr/Publicat.htm>

ISSN : 1624-0340

# Imperfect competition, technical progress and capital accumulation\*

Biancamaria d'Onofrio<sup>†</sup>      Bertrand Wigniolle<sup>‡</sup>

May 9, 2006

---

\*This research was started during a stay by Bertrand Wigniolle in Roma. The invitation of the Universita di Roma "La Sapienza" and of the Banca d'Italia is gratefully acknowledged. The authors thank participants of the GED seminar (Centre d'Economie de la Sorbonne) for their helpful comments, and Nicholas Sowels for a careful reading.

<sup>†</sup>Dipartimento di Matematica, Universita di Roma, "La Sapienza", Piazzale Aldo Moro 2, 00185 Rome, Italy. Tel: +39 06 49913253.

<sup>‡</sup>C.E.S., Université de Paris I. Address: EUREQua, Maison des Sciences Economiques, 106-112, boulevard de l'hôpital, 75647 Paris Cedex 13, France. Email : wigniolle@univ-paris1.fr.

## Résumé

Ce travail étudie l'impact de la concurrence imparfaite sur l'accumulation du capital. Le cadre théorique est le modèle à générations imbriquées avec agents altruistes. Deux types d'équilibres de long terme existent alors, qualifiés d'altruistes (avec transferts) et égoïstes (sans transferts). Nous supposons que deux types de firmes existent : les firmes concurrentielles et les firmes non-concurrentielles qui disposent d'une technologie plus productive. Les firmes non-concurrentielles ont un comportement stratégique sur le marché du travail, demandant moins de travail afin de faire baisser le salaire d'équilibre. Un progrès technique bénéficiant à une firme non-concurrentielle aura un impact différent selon la productivité initiale de cette firme, et selon le type d'état stationnaire de l'économie. Un accroissement de la productivité de la firme la plus productive aura toujours un impact négatif sur l'accumulation pour un équilibre égoïste, et un impact positif pour un équilibre altruiste. Si le progrès technique affecte le secteur concurrentiel, il pourra également avoir différents effets sur l'accumulation. Si les niveaux de productivité des firmes non-concurrentielles sont proches, l'accumulation croîtra dans un équilibre égoïste et diminuera dans un équilibre altruiste. Mais, le résultat pourra être inversé dans le cas de niveaux de productivité éloignés.

JEL classification: D43, D9, O3

Mots-clés : concurrence imparfaite, accumulation du capital, progrès technique.

## Abstract

This paper explores the consequences of imperfect competition on capital accumulation. The framework is an OLG growth model with altruistic agents. Two types of long run equilibria exist: egoistic or altruistic. We assume both competitive and non-competitive firms exist, the latter being endowed with more productive technology. They behave strategically on the labor market: they take into account the impact of their demand for labor on the equilibrium wage and on their profit. The effect of technical progress for a non-competitive firm depends on the initial productivity of the firm and on the type of steady state (egoistic or altruistic). An increase in the productivity of the most productive firm has a negative impact on capital accumulation in an egoistic steady state, and a positive one in an altruistic steady state. An increase in the productivity of the competitive sector can have various effects on capital accumulation. If the productivity levels of the non-competitive firms are close enough, capital accumulation increases in an egoistic steady state and decreases in an altruistic one. But, the impact of increasing productivity in the competitive sector can be reversed if the productivity of the less productive non-competitive firm is low enough.

JEL classification: D43, D9, O3

Keywords: imperfect competition, capital accumulation, technical progress.

# 1 Introduction

This paper explores the consequences of imperfect competition on capital accumulation. We consider an economy populated by firms producing the same good and endowed with technologies that differ by their productivity. In a perfectly competitive economy, only the firm endowed with the most productive technology would be active. But the assumption of perfect competition becomes meaningless if only one firm (or a small number of firms) remains on the market. Therefore, an economy populated by heterogeneous firms is the natural framework for considering imperfect competition. We assume that the most productive firms realize that they hold some market power on the labor market, and take into account the impact of their demand for labor on wages. We thus obtain an equilibrium with strategic behavior in the labor market.

More precisely, we assume that the single good of the economy is produced using capital and labor by different types of firms. First, competitive firms exist which all use the same basic Cobb-Douglas technology with constant returns. For a fixed (pre-determined) level of capital, their competitive labor demand is a decreasing function of the real wage. Second, there exist  $m \geq 1$  non-competitive firms using their own technology that differ from the basic technology by having higher total factor productivity. This higher productivity can result from a past innovation process, and is exogenous. Moreover, we generally assume in this paper, for the sake of simplicity, that this production structure is identical in all periods. Non-competitive firms behave as an oligopsony on the labor market: they take into account the impact of their labor demand on the equilibrium wage and on their profits. They play a Cournot-Nash game among themselves.

This production sector is part of a growth model with overlapping generations. The capital stock of each firm results from agents' investments in the preceding period. As agents can arbitrate between returns provided by different firms, all capital returns must be equal at equilibrium. This last condition added to equilibrium conditions of the Cournot-Walras game allow an equilibrium to be defined in which the shares of capital and labor used by each firm are endogenous.

We assume that agents are altruistic (as in Barro (1974) and Weil (1987)) and we focus on the long run steady state of the economy. It is well known that two types of steady state may exist, depending on whether bequest is operative or not. Moreover, we assume a logarithmic instantaneous utility function. These assumptions have particular consequences on capital accumulation. In an egoistic steady state (without bequest), capital accumulation only depends on the equilibrium wage of the economy, as only labor earnings are saved. In contrast, in an altruistic steady state (with positive bequest), capital accumulation only depends on the rate of return of capital. The non-competitive behavior of some firms tends to decrease the equilibrium wage of the economy and to increase the rate of return of capital. Therefore, such behavior is detrimental to capital accumulation in an egoistic steady state and beneficial in an altruistic one.

In this framework, we study the impact on long run capital accumulation of an

exogenous technical progress that affects one non-competitive firm, or the competitive sector. As a benchmark, it is worth noting that in a perfectly competitive economy, an increase in productivity always gives rise to an increase in capital accumulation.

Considering a non-competitive firm, it is proved that the impact of technical progress depends on the initial productivity of the firm and on the type of steady state (egoistic or altruistic). First, it should be noted that the increase in productivity of a non-competitive firm has opposing effects on factors remuneration at equilibrium: if the equilibrium wage increases, the rate of return decreases and vice versa. Therefore, if capital accumulation increases for an egoistic steady state, it will decrease for an altruistic one and vice versa. Second, an increase of one firm's productivity can have various consequences on factor remunerations, since it induces two effects. The direct effect is that the firm reduces the quantity of labor it uses per unit of capital. This effect tends to decrease the equilibrium value of the wage and to increase the capital return. The indirect effect is that the firm at equilibrium holds a higher share of the total capital of the economy. It acts in the same way as the first one, if the firm that benefits from the technical progress is initially the most productive one. But it acts in the opposing direction if the firm that benefits from this productivity increase is not initially the most productive one. The second effect may be dominant, particularly, for the less-productive non-competitive firm: an increase of its productivity can rise the equilibrium wage and decrease the capital return.

We consider now the impact on accumulation of an increase of productivity in the competitive sector. If the productivity levels of the non-competitive firms are close enough, we show that capital accumulation increases for an egoistic steady state and decreases for an altruistic one. This effect results from the increase of the wage and the decrease in the capital return. But, we show that the impact of increasing productivity in the competitive sector can be reversed if the productivity of the less-productive non-competitive firm is low enough.

These results can be understood through comparison with existing literature. Indeed, our contribution appears as a theoretical model that can be viewed as the symmetrical to Sorger's (2002) and Becker's (2003) contributions. In these articles, firms are perfectly competitive and consumers behave non-competitively in the capital market. In our model, consumers are perfectly competitive and some firms behave non-competitively in the labor market. In Sorger and Becker, a long run equilibrium exists in which consumers endowed with different rates of time preference hold positive amounts of capital. This result is in contrast with the competitive economy, in which only the most patient agent holds capital in the long run. In our contribution, a long run equilibrium exists in which firms endowed with different productivities are productive, while only the most productive ones would be active in a competitive setting. In both cases, the interpretation is the same: through his/her non-competitive behavior, an agent increases his/her gain, but it exerts a positive influence on the gain of other players. And agents who have the lowest market power are those who benefit the most from the non-competitive behavior of

other agents. Therefore, all agents may remain in the market.

In Sorger and Becker, consumers endowed with the lowest rate of time preference have the greatest market power. They underinvest in capital in order to increase the rate of interest. But this behavior increases the gain of all other agents, and particularly the gain of agents who exercise "more competitive" behavior. In our framework, the most productive firms have stronger market power. They employ less labor for a given quantity of capital in order to diminish the equilibrium wage. But this behavior is beneficial to less-productive firms who employ more labor. Their lower technical progress is balanced by their higher demand of labor which increases their capital productivity.

The imperfect competition mechanism that we introduce in this paper can also be viewed in the line of Cournot-Walras equilibrium (cf. Gabszewicz and Vial (1972), Codognato and Gabszewicz (1993) and Gabszewicz and Michel (1997)). Following this concept, some agents, having a significant size compared to the whole economy, take into account the influence of their action on the equilibrium. A Walrasian equilibrium is formed which depends on the quantities chosen by the strategic agents who play a game of the Cournot-Nash type between themselves. Recent papers have used this concept in various frameworks. Belan, Michel and Wigniolle (2002) show that it can be fruitful to interpret pension funds behavior. Belan, Michel and Wigniolle (2005) wonder if imperfect competition can foster capital accumulation in a developing economy.

In the last part of the paper, a simple extension of the model is considered, which allows an evolving production structure to be studied. In each period, one firm may receive an innovation that increases its productivity by some given factor with respect to the common technology. These events occur at random and can be interpreted as the appearance of an innovation. The innovating firm has an exclusive use of its new technology during one period. In the next period, all firms benefit from the preceding innovation that becomes freely available. Therefore, there is one non-competitive firm in each period in which an innovation occurs, and there are no non-competitive firm in periods without innovation. In this framework, we study the dynamics of an egoistic equilibrium. We show that the occurrence of an innovation has a negative short run impact on the growth rate, because imperfect competition decreases the equilibrium wage. But, after one period, the effect is positive as the technology is available for the competitive sector.

The paper is organized as follows. Section 2 presents consumers' behavior. Section 3 studies the game played by non-competitive firms. Section 4 presents the intertemporal long run equilibrium. Section 5 analyses the impact of an exogenous technical change on capital accumulation. Section 6 concludes. The most complex proofs are given in the appendix.

## 2 Consumers' behavior

The model is based on Diamond (1965) and Barro (1974)-Weil (1987). We consider an overlapping generations model of agents living for two periods. The size of generation  $t$  is  $N_t$  and each agent has  $(1+n)$  children. We assume that parents care about their children's welfare by weighting their children's utility in their own utility function. The utility of a generation born at time  $t$ ,  $V_t$ , is given by

$$V_t = U(c_t, d_{t+1}) + \gamma V_{t+1}, \quad 0 < \gamma < 1$$

with  $U(c_t, d_{t+1}) = (1-a)\ln(c_t) + a\ln(d_{t+1})$

In their first period of life, individuals born in  $t$  work and receive a wage  $w_t$ . In addition to their wage income, they receive a bequest  $x_t$  from their parents. They consume  $c_t$  and save an amount  $s_t$ . Gross returns on savings are equal to  $R_{t+1}$ : at equilibrium, all firms must provide the same return on capital. In their second period of life, people receive returns on savings and allocate net resources between consumption  $d_{t+1}$  and bequests  $x_{t+1}$  to their  $(1+n)$  children. Thus

$$x_t + w_t = c_t + s_t \tag{1}$$

$$R_{t+1}s_t = d_{t+1} + (1+n)x_{t+1} \tag{2}$$

Bequests must be non-negative :

$$x_{t+1} \geq 0 \tag{3}$$

The maximum of total utility is given by the following recursive relation:

$$V_t^*(x_t) = \max_{c_t, s_t, d_{t+1}, x_{t+1}} \{U(c_t, d_{t+1}) + \gamma V_{t+1}^*(x_{t+1})\}$$

subject to (1), (2) and (3).

We will mainly focus on the long run steady state of the economy. It is well-known that two types of long run equilibria may exist, as constraint (3) may or may not be binding.

For an altruistic long run equilibrium ( $x > 0$ ), the maximization problem of the consumer leads to the modified golden rule equation:

$$R = \frac{1+n}{\gamma} \tag{4}$$

This equation will determine the long run value of capital per young agent.

For an egoistic long run equilibrium ( $x = 0$ ), the maximization problem of the consumer leads to his/her optimal savings behavior:

$$s = aw \tag{5}$$

The equilibrium values of prices  $w$  and  $R$  will depend on the equilibrium between non-competitive firms.



### 3 Equilibrium between non-competitive firms

#### 3.1 Definition of the equilibrium concept

Firms employ capital and labor. Capital depreciates fully in one period. There exist two types of firms: competitive and non-competitive. Competitive firms have the same Cobb-Douglas technology given by:

$$F(K, L) = A_0 K^\alpha L^{1-\alpha}$$

Without loss of generality, it is possible to consider only one competitive firm, and we denote by  $K_{0,t}$  and  $L_{0,t}$  its amounts of capital and labor.

There exist  $m$  non-competitive firms respectively endowed with Cobb-Douglas production technologies given by:

$$\text{For } i = 1, \dots, m, \quad A_i K_{i,t}^\alpha L_{i,t}^{1-\alpha}$$

We assume the following inequalities:

$$A_0 < A_1 < \dots < A_m$$

Non-competitive firms have a higher total factor productivity than competitive firms. This higher productivity may result for instance from past innovations. We will assume that this production structure is identical in all periods.

In each period  $t$ , we consider the total capital stock  $K_t$  as given, and we study the general Nash-equilibrium resulting from the following assumptions:

1. The competitive firm maximizes its profit in choosing  $L_{0,t} = \arg \max_L F(K_{0,t}, L) - w_t L$ , or:

$$w_t = A_0(1 - \alpha) K_{0,t}^\alpha L_{0,t}^{-\alpha} \quad (6)$$

As at equilibrium,  $L_{0,t} = N_t - \sum_{i=1}^m L_{i,t}$ , this equation gives the competitive wage  $w_t$  as a function of the demand of labor by non-competitive firms  $L_{i,t}$ ,  $i = 1, \dots, m$ ,

$$w_t = A_0(1 - \alpha) K_{0,t}^\alpha \left( N_t - \sum_{i=1}^m L_{i,t} \right)^{-\alpha} \equiv \omega_t(L_{1,t}, \dots, L_{m,t}) \quad (7)$$

2. Each non-competitive firm  $i$  maximizes its profit, taking into account the impact of its labor demand on the equilibrium wage:

$$\max_{L_{i,t}} A_i K_{i,t}^\alpha L_{i,t}^{1-\alpha} - \omega_t(L_{1,t}, \dots, L_{m,t}) L_{i,t} \quad (8)$$

3. All capital returns are equal:

$$\forall i = 1, \dots, m, \quad \frac{A_i K_{i,t}^\alpha L_{i,t}^{1-\alpha} - w_t L_{i,t}}{K_{i,t}} = \alpha A_0 K_{0,t}^{\alpha-1} L_{0,t}^{1-\alpha} \quad (9)$$

4. the capital market is balanced:

$$K_{0,t} + \sum_{i=1}^m K_{i,t} = K_t \quad (10)$$

The first item corresponds to the optimal demand for labor by the competitive firm. It allows the equilibrium wage to be defined as a function of all labor demands by the non-competitive firms. The second item describes the non-competitive behavior of other firms. They behave as oligopsonies on the labor market by taking into account the impact of their demand for labor on the equilibrium wage. The resulting optimal labor demand defines the best response function of each firm. The third item requires all capital returns to be equal at equilibrium, considering that consumers can freely arbitrate between all firms when they invest their savings. Finally, the fourth item is the equilibrium condition on the capital market. It is worth noting that by construction (from item 1), the labor market is balanced.

It is convenient to introduce the following notations:  $q_i = A_i/A_0$ ,  $l_{i,t} = L_{i,t}/K_{i,t}$ ,  $\lambda_{i,t} = l_{i,t}/l_{0,t}$ ,  $p_{i,t} = K_{i,t}/K_{0,t}$  and  $\tilde{p}_{i,t} = p_{i,t}/p_{0,t}$ .

Considering the optimal choice of the non-competitive firm  $i$  (8), the first order condition leads to:

$$A_i l_{i,t}^{-\alpha} - A_0 l_{0,t}^{-\alpha} - \alpha A_0 \frac{K_{i,t}}{K_{0,t}} l_{i,t} l_{0,t}^{-\alpha-1} = 0$$

The third term of this equation stems from the non-competitive behavior. Dividing by  $A_0 l_{0,t}^{-\alpha}$  we obtain:

$$q_i \lambda_{i,t}^{-\alpha} - 1 - \alpha \tilde{p}_{i,t} \lambda_{i,t} = 0 \quad (11)$$

The equality of capital returns for each firm (9) gives the condition:

$$R_t = \alpha A_0 l_{0,t}^{1-\alpha} = A_i l_{i,t}^{1-\alpha} - A_0 (1-\alpha) l_{0,t}^{-\alpha} l_{i,t}$$

Dividing by  $A_0 l_{0,t}^{-\alpha}$  we obtain:

$$q_i \lambda_{i,t}^{1-\alpha} - (1-\alpha) \lambda_{i,t} = \alpha \quad (12)$$

Finally, the equilibrium of the capital market (10) leads to:

$$p_{0,t} + \sum_{i=1}^m p_{i,t} = 1$$

or:

$$p_{0,t} \left( 1 + \sum_{i=1}^m \tilde{p}_{i,t} \right) = 1 \quad (13)$$

### 3.2 Existence of the equilibrium between non-competitive firms

In this section, we prove that our equilibrium concept leads to a unique equilibrium, and we describe its properties. We first remark that (11), (12) and (13) define a system of  $2m + 1$  equations for  $2m + 1$  variables, and that these equations do not depend on the period  $t$ . Consequently,  $\forall i = 1, \dots, m$ ,  $\lambda_{i,t}$ ,  $\tilde{p}_{i,t}$  and  $p_{0,t}$  are constant variables, that we will write further  $\lambda_i$ ,  $\tilde{p}_i$  and  $p_0$ .

From equation (11), we obtain that  $\lambda_i$  is such that:  $q_i \lambda_i^{-\alpha} > 1$ , or  $\lambda_i < (q_i)^{1/\alpha}$ . Equation (12) has a unique solution  $\lambda_i$  such that  $\lambda_i < (q_i)^{1/\alpha}$ , and this solution defines  $\lambda_i$  as a decreasing function of  $q_i$ . Moreover, as  $q_i \lambda_i^{-\alpha} > 1$ , (12) implies that:

$$\lambda_i - (1 - \alpha)\lambda_i - \alpha < \lambda_i q_i \lambda_i^{-\alpha} - (1 - \alpha)\lambda_i - \alpha = 0$$

Thus,

$$\lambda_i < 1$$

We have finally proved that all non-competitive firms have a smaller labor-capital ratio than the ratio in the competitive sector. The more productive a non-competitive firm is, the smaller its labor-capital ratio is, as  $\lambda_i$  is a decreasing function of  $q_i$ . This property results from the higher market power of the more productive firms: they reduce their labor demand more in order to decrease the equilibrium wage.

As for all  $i = 1, \dots, m$ ,  $\lambda_i$  is well-defined, we deduce from (11) the value of  $\tilde{p}_i$ ,  $\forall i = 1, \dots, m$ :

$$\tilde{p}_i = \frac{q_i \lambda_i^{-\alpha} - 1}{\alpha \lambda_i}$$

which is an increasing function of  $q_i$ .

Finally  $p_0$  is given by (13), and is a decreasing function of  $q_i$ . The share of capital held by firm  $i$   $p_i$  is given by:

$$p_i = p_0 \tilde{p}_i = \frac{\tilde{p}_i}{1 + \sum_{j=1}^m \tilde{p}_j}$$

which is an increasing function of  $\tilde{p}_i$ , and therefore an increasing function of  $q_i$ .

We have finally proved that, for each value of total capital, there exists a non-competitive equilibrium in which all firms are productive. The higher the productivity of a firm is, the higher the share of capital that it employs at equilibrium is and the lower its labor-capital ratio will be. It is worth noting that, in an equilibrium with perfect competition, only the most productive firm would be active. With imperfect competition, the more productive firms strategically diminishes their labor demand to decrease the equilibrium wage. This behavior exerts a positive externality on less productive firms, which employ a higher labor-capital ratio, and which can attain the same level of capital productivity.

### 3.3 Equilibrium prices

At some period  $t$ , if  $K_t$  is the total capital stock and  $N_t$  the number of young people, it is possible to determine the equilibrium level of the wage  $w_t$  and the gross interest rate  $R_t$ .

From the labor market equilibrium, we have:

$$\begin{aligned} N_t &= \sum_{i=0}^m L_{i,t} = \sum_{i=0}^m l_{i,t} p_i K_t \\ N_t &= l_{0,t} p_0 K_t \left[ 1 + \sum_{i=1}^m \lambda_i \tilde{p}_i \right] \end{aligned}$$

Finally, with  $k_t = K_t/N_t$  denoting the ratio of capital per young agent, we obtain from (13):

$$\begin{aligned} l_{0,t}^{-1} &= k_t X \\ \text{with } X &= \frac{1 + \sum_{i=1}^m \lambda_i \tilde{p}_i}{1 + \sum_{i=1}^m \tilde{p}_i} \end{aligned}$$

In an economy with perfect competition, we would obtain  $X = 1$ . In our economy with imperfect competition, we have  $X < 1$  as  $\lambda_i < 1$ .

We can then deduce the values of the equilibrium wage and gross interest rate:

$$\begin{aligned} w_t &= (1 - \alpha) A_0 l_{0,t}^{-\alpha} = (1 - \alpha) A_0 k_t^\alpha X^\alpha \\ R_t &= \alpha A_0 l_{0,t}^{1-\alpha} = \alpha A_0 k_t^{\alpha-1} X^{\alpha-1} \end{aligned}$$

In these two equations, the variable  $X$  results from imperfect competition. As  $X < 1$ , we see that imperfect competition tends to decrease the equilibrium wage, and to increase the gross interest rate, with respect to the case of perfect competition with the less productive technology.

## 4 The intertemporal long run equilibrium

Two types of long run intertemporal equilibria may exist: an altruistic equilibrium with operative bequest ( $x > 0$ ) and an egoistic equilibrium ( $x = 0$ ).

At an altruistic long run equilibrium, the capital per young agent ratio  $k_t$  converges towards a value  $\bar{k}$ , which is determined by the modified golden rule (4):

$$R = \alpha A_0 \bar{k}^{\alpha-1} X^{\alpha-1} = \frac{1+n}{\gamma}$$

or:

$$\bar{k} = \left( \frac{\alpha \gamma}{1+n} \right)^{\frac{1}{1-\alpha}} \frac{(A_0)^{\frac{1}{1-\alpha}}}{X} \quad (14)$$

Along an egoistic long run equilibrium, the capital per young agent ratio  $k_t$  converges toward a value  $k^*$ , which is determined by the savings behavior of the agents (5):

$$(1+n)k^* = aw$$

or:

$$k^* = \left( \frac{(1-\alpha)a}{1+n} \right)^{\frac{1}{1-\alpha}} (A_0)^{\frac{1}{1-\alpha}} (X)^{\frac{\alpha}{1-\alpha}} \quad (15)$$

In both equations (14) and (15) the impact of imperfect competition on capital accumulation results from the variable  $X < 1$ . Within an altruistic steady state, capital accumulation in the long run only depends on the return of capital. As imperfect competition tends to increase the gross interest rate, the capital per young agent ratio  $\bar{k}$  is higher than under perfect competition. In contrast, within an egoistic steady state, capital accumulation in the long run only depends on savings that only depend on the equilibrium wage for a log-linear utility function. Thus, as imperfect competition tends to decrease the equilibrium wage, the capital per young agent ratio  $k^*$  is smaller than under perfect competition.

It is straightforward to show that the condition ensuring positive bequests in Weil (1987) remains the same in our framework. The steady state will be altruistic if  $\bar{k} > k^*$ . This inequality gives the following condition:

$$(1-\alpha)aX < \alpha\gamma$$

As  $X$  is smaller than 1, this condition shows that the existence of an altruistic steady state is furthered by imperfect competition. This property was expected as imperfect competition tends to increase the return to capital .

## 5 Technical progress and capital accumulation

In this section, we study how technical progress (i.e. an increase of some  $A_i$ ,  $i = 0, \dots, m$ ) modifies capital accumulation in the long run.

### 5.1 Increasing productivity of a non-competitive firm

We first study the impact of an increase of some  $A_i$ ,  $i \geq 1$ . This is equivalent to consider that some  $q_i$  increases, for  $i \geq 1$ . Such technical progress will affect capital accumulation through the variable  $X$ . It is worth noting that  $X$  has an opposite effect on the two types of steady states, altruistic or egoistic. An increase of  $X$  diminishes  $\bar{k}$  and rises  $k^*$ .

As a benchmark, we know that a technical progress in a competitive economy always increases capital accumulation, in both types of long run equilibria.

**Proposition 1** *it is possible to define some increasing function  $\kappa(q)$  with  $\kappa(1) = 0$ , such that, for each  $q_i$ ,  $i = 1, \dots, m$ , the interval  $K_i = (1, q_i + \kappa(q_i))$  satisfies:*

1. *If for all  $j \neq i$ ,  $q_j \in K_i$ ,  $\frac{\partial X}{\partial q_i} < 0$ . An increase of  $q_i$  rises  $\bar{k}$  and diminishes  $k^*$ .*

2. If for all  $j \neq i$ ,  $q_j \notin K_i$ ,  $\frac{\partial X}{\partial q_i} > 0$ . An increase of  $q_i$  diminishes  $\bar{k}$  and raises  $k^*$ .

**Corollary 1** 1. For  $i = m$ , as all  $q_j \in K_m$ , we have  $\frac{\partial X}{\partial q_m} < 0$ . An increase of  $q_m$  raises  $\bar{k}$  and diminishes  $k^*$ .

2.  $q_1$  is the only variable that can satisfy point 2 of proposition (1). Particularly, if  $q_1$  is sufficiently small, ( $q_1$  tends toward 1),  $\frac{\partial X}{\partial q_1} > 0$ . An increase of  $q_1$  diminishes  $\bar{k}$  and raises  $k^*$ .

These results show that a technical progress can have various effects on capital accumulation. Its impact depends on the initial productivity of the firm experiencing an increase in its productivity, and on the type of long-run equilibrium - egoistic or altruistic.

This latter property can be understood, having in mind that the rise of productivity of a non-competitive firm has opposite effects on factors remuneration at equilibrium: if the equilibrium wage increases, the rate of return decreases and vice versa. Therefore, if capital accumulation increases for an egoistic steady state, it will decrease for an altruistic one and conversely.

The former property can be explained, as an increase of one firm's productivity leads to two opposite effects. First, the firm reduces the quantity of labor used per unit of capital. This effect tends to decrease the equilibrium value of the wage and to increase the capital return. Second, since all capital returns are equal at equilibrium, the firm holds a higher share of the total capital of the economy. If the firm that benefits from this productivity increase is the most productive ( $i = m$ ), this second effect acts in the same sense as the first one. Therefore, the equilibrium wage decreases and the capital return increases, which decreases accumulation for an egoistic steady state and increases accumulation for an altruistic one. But, if the firm that benefits from this productivity increase is not the most productive one ( $i < m$ ), the second effect acts in opposite direction to the first one, and the global effect is ambiguous. Particularly, it is proved that for the less productive non-competitive firm, it is possible that a rise in its productivity increases the equilibrium wage and decreases the capital return.

## 5.2 Increasing productivity in the competitive sector

We study the impact of an increase of  $A_0$  on both types of stationary equilibrium  $\bar{k}$  and  $k^*$ . From (14) and (15),  $A_0$  has a direct effect and an indirect effect via the variable  $X$ . Indeed, increasing  $A_0$  implies a decrease for all  $q_i$ ,  $i \geq 1$ .

The following proposition shows that an increase of  $A_0$  can have various effects on capital accumulation.

**Proposition 2** 1. When  $q_m \rightarrow +\infty$ ,  $\frac{d \ln k^*}{d A_0} \rightarrow \frac{1}{A_0(1-\alpha)}$  and  $\frac{d \ln \bar{k}}{d A_0} \rightarrow 0$ .

2. When  $q_1 \rightarrow 1$ ,  $\frac{d \ln k^*}{d A_0} \rightarrow -\infty$  and  $\frac{d \ln \bar{k}}{d A_0} \rightarrow +\infty$ .

3. When  $\forall i \geq 1$ ,  $q_i = q$ ,  $\frac{d \ln k^*}{d A_0} > 0$  and  $\frac{d \ln \bar{k}}{d A_0} < 0$ . By continuity, these properties hold when the  $q_i$ ,  $i \geq 1$ , are sufficiently close together.

These results show that when the total factor productivities of non-competitive firms are close, an increase of  $A_0$  tends to increase capital accumulation in an egoistic steady state, and to decrease capital accumulation in an altruistic steady state. This effect results from the increase of the wage and the decrease of the capital return. In contrast, the results may be reversed when the total factor productivities of non-competitive firms are distant.

These properties can be interpreted as follows. The direct effect of an increase of the productivity in the non-competitive sector is a rise of both wage and capital returns. But an indirect effect stems from the fall of the relative productivity of all non-competitive firms, which modify their market power. If non-competitive firms have close productivities (case 3), the productivity increase in the competitive sector implies a fall in the market power of all non-competitive firms, which causes an increase of wages and a decrease of capital returns. If non-competitive firms have distant productivities (cases 1 and 2), the productivity increase in the competitive sector redistributes market power in favor of the most productive firms, and to the detriment of the less productive ones. The resulting effect on capital accumulation may be reversed with respect to case 3.

### 5.3 Growth with random innovations

In this last section, we provide a simple extension of the model, introducing random innovations. We assume that at each period, with a probability  $\pi$ , one firm receives an innovation (and with probability  $1 - \pi$ , no innovation occurs in the whole economy). This innovation increases the productivity by a factor  $\delta > 1$ , with respect to the common technology. Finally, each innovator has an exclusive use of its new technology during only one period. After that period, there is free access to this technology.

From the preceding assumptions, at each period  $t$ , either the economy is purely competitive and the common productivity is  $A_{0,t}$ , or one non-competitive firm has a productivity level  $A_{1,t} = \delta A_{0,t}$  while the other firms are competitive with the common productivity  $A_{0,t}$ .

We only consider the egoistic equilibrium in this part, as we want to analyse the dynamics of capital accumulation. The dynamics of  $k_t$  with random innovations become:

$$(1 + n)k_{t+1} = a(1 - \alpha)A_{0,t}k_t^\alpha X_t^\alpha$$

With probability  $\pi$  (arrival of an innovation in  $t$ )

$$\begin{aligned} X_t &= \frac{x}{1 + x(x - 1)} \\ \text{with } x &> 1 \text{ the solution of } \delta = \alpha x^{1-\alpha} + (1 - \alpha)x^{-\alpha} \\ A_{0,t+1} &= \delta A_{0,t} \end{aligned}$$

With probability  $1 - \pi$  (no innovation in  $t$ )

$$\begin{aligned} X_t &= 1 \\ A_{0,t+1} &= A_{0,t} \end{aligned}$$

As  $x > 1$ , we have  $\frac{x}{1+x(x-1)} < 1$ . The arrival of an innovation has a negative short run effect on the growth rate, because imperfect competition decreases the equilibrium wage. But, after one period, the effect is positive as the technology is available for the competitive sector. The greater the size of the innovation  $\delta$  is, the larger both short run and long run effects will be.

## 6 Conclusion

This paper has studied how long run growth can be affected by strategic behavior of firms in the labor market. The main results show the paradoxical effects associated with imperfect competition: a technical progress may decrease capital accumulation if it leads to distortions due to imperfect competition.

Our work could lead to further developments, mostly in endogenizing the technical progress by an explicit innovative activity of the firms. Growth models in which innovation is the source of growth are natural frameworks to develop our analysis, since they make endogenous productivity differences of firms.

### Appendix 1: proof of proposition 1

We study the impact of an increase of  $q_i$  on  $X$ . It is possible to write

$$X = \frac{1 + \sum_{j=1}^m (x_j - 1)}{1 + \sum_{j=1}^m x_j (x_j - 1)}$$

with  $x_j = 1/\lambda_j$ . Consequently,  $1 < x_1 < \dots < x_m$ .

Computing the derivation  $\frac{\partial X}{\partial x_i}$ , its sign is the same as the sign of  $Z$  such that:

$$Z = 1 + \sum_{j=1}^m x_j (x_j - 1) - \left[ 1 + \sum_j (x_j - 1) \right] (2x_i - 1)$$

We take as given the value of  $x_i$ , and we study  $Z$  as a function of  $x_j$ ,  $j \neq i$ . It is possible to write:

$$Z = 1 - x_i^2 + \sum_{j \neq i} P(x_j, x_i)$$

with  $P(x_j, x_i)$  such that:

$$P(x_j, x_i) = x_j^2 - x_j - (2x_i - 1)(x_j - 1) = (x_j - 1)(x_j - 2x_i + 1)$$



For a given  $x_i$ , we study  $P(x_j, x_i)$  as a function of  $x_j$ . This function is quadratic and reaches its minimum in  $x_j$  solution of  $\partial P(x_j, x_i)/\partial x_j = 0$ , which gives

$$2x_j - 1 - (2x_i - 1) = 0$$

or:

$$x_j = x_i$$

Thus  $Z$  reaches a minimum for  $x_j = x_i \forall j \neq i$ , and the minimum value  $Z_{\min}$  is:

$$Z_{\min} = 1 - x_i^2 - (m-1)(x_i - 1)^2 = -(x_i - 1)[m(x_i - 1) + 2] < 0$$

As all functions  $x_j \mapsto P(x_j, x_i)$  are identical and symmetrical with respect to the minimum reached in  $x_j = x_i$ , there exists an interval  $I_i = (x_i - \rho(x_i), x_i + \rho(x_i))$ , such that, if all  $x_j$  are in  $I_i$ , then  $Z < 0$ , and if all  $x_j \notin I_i$ , then  $Z > 0$ .  $\rho(x_i)$  is defined by the property: if  $\forall j \neq i, x_j = x_i \pm \rho(x_i)$ ,  $X = 0$ .

But, it is easy to see that the lower bound of the interval  $x_i - \rho(x_i)$  is smaller than 1. Indeed, for  $x_j = 1 \forall j \neq i$ , we have  $X = 1 - x_i^2 < 0$ , which means that  $x_i - \rho(x_i) < 1$ . Thus, the relevant interval is in fact:  $J_i = (1, x_i + \rho(x_i))$ .

Finally, the value of  $\rho(x_i)$  is computed as the value of  $\rho$  solution of the equation:  $X = 0$  with  $\forall j \neq i, x_j = x_i + \rho$ . This leads to the equation:

$$\rho^2 = \frac{mx_i^2 - 2(m-1)x_i + m - 2}{m-1}$$

which defines  $\rho$  as a function of  $x_i$ :

$$\rho(x_i) = \sqrt{\frac{mx_i^2 - 2(m-1)x_i + m - 2}{m-1}}$$

The interval  $J_i = (1, x_i + \rho(x_i))$  implicitly defines an interval  $K_i$  on  $q_i$  under the form  $(1, q_i + \kappa(q_i))$ . Indeed, equation (12) implicitly defines  $x_i$  as a function of  $q_i$ :  $x_i$  is the solution  $> 1$  of the equation:

$$q_i = \alpha x_i^{1-\alpha} + (1-\alpha)x_i^{-\alpha}$$

We denote by  $\chi$  this function.  $x_i = \chi(q_i)$  is an increasing (bijective) function from  $[1, +\infty)$  on to  $[1, +\infty)$ .

We have:

$$\frac{\partial X}{\partial q_i} = \frac{\partial X}{\partial x_i} \frac{dx_i}{dq_i}$$

with

$$\frac{dx_i}{dq_i} = \chi'(x_i) = \frac{1}{\alpha(1-\alpha)x_i^{-1-\alpha}(x_i-1)} > 0$$

Therefore  $\frac{\partial X}{\partial q_i}$  and  $\frac{\partial X}{\partial x_i}$  have the same sign.

Finally, we obtain the interval  $K_i = (1, q_i + \kappa(q_i))$  such that, for all  $q_j$  in this interval,  $\frac{\partial X}{\partial q_i} < 0$ . The function  $\kappa(q_i)$  is defined as:

$$\begin{aligned} q_i + \kappa(q_i) &= \chi^{-1}(x_i) \\ \text{with } x_i &= \chi(q_i) \end{aligned}$$

or:

$$q_i + \kappa(q_i) = \alpha (\chi(q_i) + \rho(\chi(q_i)))^{1-\alpha} + (1 - \alpha) (\chi(q_i) + \rho(\chi(q_i)))^{-\alpha}$$

## Appendix 2: proof of proposition 2

We first compute the expression of  $\frac{dx_i}{dA_0}$  for all  $i$ .

As  $q_i = A_i/A_0$ , we have:  $\frac{dq_i}{dA_0} = -\frac{q_i}{A_0}$ . From equation (12),  $x_i$  is defined as a function of  $q_i$ , and we have:

$$\frac{dx_i}{dq_i} = \frac{1}{\alpha(1 - \alpha)x_i^{-1-\alpha}(x_i - 1)}$$

Therefore,

$$\begin{aligned} \frac{dx_i}{dA_0} &= \frac{dx_i}{dq_i} \frac{dq_i}{dA_0} \\ &= -\frac{q_i}{A_0} \frac{1}{\alpha(1 - \alpha)x_i^{-1-\alpha}(x_i - 1)} \end{aligned}$$

and using equation (12) to eliminate  $q_i$ , we obtain:

$$\frac{dx_i}{dA_0} = \frac{-1}{\alpha(1 - \alpha)A_0} \frac{x_i}{x_i - 1} (\alpha x_i + 1 - \alpha)$$

Second, we compute the expression of  $\frac{d \ln X}{dA_0}$ , with  $X$  defined as a function of  $(x_1, \dots, x_m)$ :

$$X = \frac{1 + \sum_{j=1}^m (x_j - 1)}{1 + \sum_{j=1}^m x_j (x_j - 1)}$$

We obtain:

$$\frac{d \ln X}{dA_0} = \frac{1}{\alpha(1 - \alpha)A_0} \sum_{i=1}^m \frac{x_i(\alpha x_i + 1 - \alpha)}{x_i - 1} \frac{\left[ (2x_i - 1) \left( 1 + \sum_{j=1}^m (x_j - 1) \right) - \left( 1 + \sum_{j=1}^m x_j (x_j - 1) \right) \right]}{\left( 1 + \sum_{j=1}^m x_j (x_j - 1) \right) \left( 1 + \sum_{j=1}^m (x_j - 1) \right)}$$

From this formula, we have:

$$\lim_{x_m \rightarrow +\infty} \frac{d \ln X}{dA_0} = \frac{1}{(1 - \alpha)A_0}$$

$$\lim_{x_1 \rightarrow 1} \frac{d \ln X}{dA_0} = -\infty$$

And, if we take  $x_i = x \forall i$ , we obtain:

$$\frac{d \ln X}{dA_0} = \frac{1}{\alpha(1-\alpha)A_0} \frac{mx(\alpha x + 1 - \alpha) [m(x-1) + 2]}{[1 + mx(x-1)] [1 + m(x-1)]} \quad (16)$$

Finally we have to consider the two stationary states.

For an altruistic steady state, the sign of  $\frac{d \ln \bar{k}}{dA_0}$  is given, from (14), by the sign of

$$\frac{1}{(1-\alpha)A_0} - \frac{d \ln X}{dA_0}$$

For an egoistic steady state, the sign of  $\frac{d \ln k^*}{dA_0}$  is given, from (15), by the sign of

$$\frac{1}{A_0} + \alpha \frac{d \ln X}{dA_0}$$

Items 1 and 2 of proposition 2 immediately follow from the preceding results.

The result  $\frac{d \ln k^*}{dA_0} > 0$  of point 3 is obtained as  $\frac{d \ln X}{dA_0} > 0$  from (16). Finally,  $\frac{d \ln \bar{k}}{dA_0} < 0$  is obtained in using (16) to calculate:

$$\begin{aligned} \frac{1}{(1-\alpha)A_0} - \frac{d \ln X}{dA_0} &= \frac{1}{\alpha(1-\alpha)A_0} \left\{ \alpha - \frac{mx(\alpha x + 1 - \alpha) [m(x-1) + 2]}{[1 + mx(x-1)] [1 + m(x-1)]} \right\} \\ &= \frac{\frac{m^2}{\alpha} x(1-x) - mx^2 - 2mx \frac{1-\alpha}{\alpha} - m + 1}{\alpha(1-\alpha)A_0 [1 + mx(x-1)] [1 + m(x-1)]} \end{aligned}$$

In the last expression, the denominator is positive and the numerator is always negative for  $x > 1$  and  $m \geq 1$ .

## References

- Barro, R.J.: Are government bonds net wealth ? *Journal of Political Economy* **82**, 379-402 (1974).
- Belan, P., Michel, P., Wigniolle, B.: Pension funds and capital accumulation. *Economics Bulletin* **4**, 1-8 (2002).
- Belan, P., Michel, P., Wigniolle, B.: Does imperfect competition foster capital accumulation in a developing economy?. *Research in Economics* **59**, 189-208 (2005).
- Becker, R. A.: Stationary strategic Ramsey equilibrium, mimeo, Department of Economics, Indiana University (2003).
- Codognato, G., Gabszewicz, J. J.: Cournot-Walras equilibria in markets with a continuum of traders. *Economic Theory* **3**, 453-464 (1993).
- Diamond P.: National debt in a neoclassical growth model. *American Economic Review* **55**, 1126-1150 (1965).
- Gabszewicz, J.J., Michel, P.: Oligopoly equilibrium in exchange economies. In B. Curtis Eaton and Richard G. Harris (ed.) *Trade, Technology and Economics*, Essays in honor of Richard Lipsey, pp. 217-240. Cheltenham, Edward Elgar 1997.
- Gabszewicz, J.J., Vial, J.P.: Oligopoly à la Cournot in general equilibrium analysis. *Journal of Economic Theory* **49**, 10-32 (1972).
- Sorger, G.: On the long-run distribution of capital in the Ramsey model. *Journal of Economic Theory* **105**, 226-243 (2002).
- Weil, P.: 'Love Thy Children': Reflections on the Barro debt neutrality theorem", *Journal of Monetary Economics* **19**, 379-391 (1987).