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Indeterminacy under Input-specific Externalities and Implications for Fiscal Policy

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Abstract

In this paper, we introduce input-specific externalities in a dynamic general equilibrium model with heterogeneous households and a finance constraint (Woodford (1986)). In contrast to existing papers, average labor and capital have not a positive impact on the total productivity of factors, but respectively on labor and capital efficiencies. Focusing on not too low degrees of capital-labor substitution, we show that indeterminacy requires not only a lower bound for the elasticity of capital-labor substitution, but also an upper bound, although the returns are increasing. As a direct implication, the well known wrong slopes condition (labor demand steeper than labor supply) is neither a necessary nor a sufficient condition for indeterminacy and larger increasing returns promote sadlle-path stability when inputs are high substitutes. Using this framework, we are also able to analyze the role of variable tax rates on capital and labor income on the dynamics. In contrast to existing results, we show that tax rates decreasing with their tax base do not promote instability due to self-fulfilling expectations when capital and labor are sufficiently high substitutes, but rather have a stabilizing effect.

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1 Introduction

Considering dynamic general equilibrium one-sector models, a lot of papers have introduced productive externalities to analyze the role of increasing returns on local indeterminacy and the occurrence of endogenous cycles. In most of them, production, characterized by constant returns at the private level, benefits from externalities because the total productivity of factors increases with respect to average capital and labor ($Y = A(\overline{K}, \overline{L})F(K, L)$).[1] The purpose of this paper is to go one step further by first analyzing the robustness of these results when productive externalities do no more affect the total productivity of factors, but rather have an influence on capital and labor efficiencies. In a second step, we will analyze the implications of such a specification of externalities on fiscal policy issues. More specifically, we will show that, in contrast to the existing literature, variable tax rates decreasing with their tax bases do not always promote fluctuations due to the volatility of expectations.

To become more precise, we introduce externalities as follows: we assume that average capital increases capital efficiency ($C(\overline{K})$) and average labor increases labor efficiency ($D(\overline{L})$). Therefore, average capital and labor do no more improve the aggregate production, but rather affect the capital-labor ratio, measured in efficient units ($Y = F(C(\overline{K})K, D(\overline{L})L)$).[2] Obviously, we note that the existence of these input-specific externalities ensures that returns to scale are increasing at the social level.

We introduce such a production sector in a finance constrained economy with heterogeneous households as initially developed by Woodford (1986).[3]

[1]Among others, one can refer to Barinci and Chéron (2001), Benhabib and Farmer (1994), Cazzavillan, Lloyd-Braga and Pintus (1998), Farmer and Guo (1994), Harrison and Weder (2002), Hintermaier (2003) or Pintus (2006). See Benhabib and Farmer (1999) for a survey.

[2]In a previous version of this paper, we have considered a more general specification of externalities where average capital and labor increase both capital and labor efficiencies ($C(\overline{K}, \overline{L})$, $D(\overline{K}, \overline{L})$). It is shown that new results on indeterminacy emerge if the effect of average capital on labor efficiency and the effect of average labor on capital efficiency are not too large, providing a justification of the simpler specification we use in this work. For more details, see Seegmuller (2006).

[3]In the last decade, this model has been extensively used in the literature. See, for instance, Barinci and Chéron (2001), Cazzavillan, Lloyd-Braga and Pintus (1998), Gokan

Using this framework, we can easily analyze the stability properties of the steady state, i.e. the local indeterminacy and the occurrence of endogenous cycles, and compare our results with the case where productive externalities are introduced through the total productivity of factors, as analyzed by Cazzavillan, Lloyd-Braga, and Pintus (1998).

For sake of conciseness, we focus on cases where capital and labor are not weak substitutes.[4] We show that indeterminacy and endogenous cycles require a not too weak effect of labor on labor efficiency ($D(L)$). If this necessary condition is satisfied, we prove that indeterminacy occurs if the capital-labor substitution is greater than a lower bound (σ_H). Note that this result is quite similar to the one obtained by Cazzavillan, Lloyd-Braga and Pintus (1998), even if the economic mechanism is different. However, in our framework, indeterminacy also requires a finite upper bound (σ_T) for the elasticity of capital-labor substitution: for all values of this elasticity larger than this upper bound, the steady state becomes a saddle.

As a direct implication, we show that the steady state can be locally determinate when the wrong slopes condition is met on the labor market, i.e. the labor demand is steeper than the labor supply. Furthermore, since the upper bound value σ_T is decreasing with the degree of increasing returns, we obtain another new conclusion. In contrast to the literature, larger increasing returns promote saddle-path stability for sufficiently high capital-labor substitutions.

To provide an economic intuition of our main result, i.e. indeterminacy does no more occur for a large enough elasticity of capital-labor substitution, we first underline that expectations can be self-fulfilling only if the real wage and real interest rate are sufficiently sensitive to capital and labor, respectively. When externalities affect the total productivity of factors (Cazzavillan, Lloyd-Braga and Pintus (1998)), this is possible as soon as externalities are not too weak. Indeed, in this case, externalities increase production and hence both the marginal productivities of capital and labor. On the contrary, in our framework, externalities mainly modify the capital-labor ratio. Since the elasticities of the marginal productivities of capital and labor with respect to the capital-labor ratio are inversely related to the degree of capital-labor substitution, the externalities do no more play a great role on dynamics if capital and labor are sufficiently high substitutes, explaining that the steady state becomes a saddle in such a case.

(2006), Grandmont (2006), Grandmont, Pintus and de Vilder (1998), Lloyd-Braga and Modesto (2007), Lloyd-Braga, Modesto and Seegmuller (2006) and Pintus (2003, 2004).

[4]Indeed, it is well known that in the Woodford (1986) model, endogenous fluctuations occur without externality for a weak elasticity of capital-labor substitution. See Grandmont, Pintus and de Vilder (1998).

Since the seminal contributions of Guo and Lansing (1998) and Schmitt-Grohé and Uribe (1997), several contributions have analyzed the (de-)stabilizing role of variable tax rates under a balanced budget. In one-sector models, one of the main results states that tax rates (at least on labor income) decreasing with their tax base have a destabilizing effect by promoting fluctuations due to the volatility of expectations.[5] We reexamine this issue in our framework and obtain a less clear-cut conclusion. Indeed, variable tax rates decreasing with their tax base can be a source of determinacy if the capital-labor substitution is sufficiently large. The explanation of this result is quite simple: the introduction of tax rates decreasing with their tax base reduces the upper bound σ_T, promoting saddle-path stability.

This paper is organized as follows. In the next section, we present the model and define the intertemporal equilibrium. In Section 3, we establish the existence of a steady state. In Section 4, we analyze the occurrence of local indeterminacy and endogenous cycles, and provide an economic interpretation. In Section 5, we introduce variable tax rates through a balanced-budget and discuss dynamic implications of fiscal policy. Our main findings are summarized in Section 6.

2 The Model

We consider a competitive monetary economy with discrete time, $t = 1, 2, ..., +\infty$, and two types of infinitely lived households, workers and capitalists.[6] Only workers supply labor, whereas both workers and capitalists consume the final good. Moreover, workers are more impatient than capitalists, i.e. they discount the future more than the latter. Following Woodford (1986), we assume that there is a financial market imperfection that prevent workers from borrowing against their wage earnings. Therefore, in a neighborhood of a monetary steady state, capitalists hold the whole capital stock and no money, and workers save their wage income in the form of money balances. In the production sector, firms produce the final good. The production benefits from productive externalities specific to each input.

[5]For instance, the reader can refer to Guo and Lansing (1998), Gokan (2006), Lloyd-Braga, Modesto and Seegmuller (2006), Pintus (2003) and Schmitt-Grohé and Uribe (1997).

[6]This model with heterogeneous households has been introduced in the seminal contribution of Woodford (1986) and has been extensively used in the literature. See among others Barinci and Chéron (2001), Cazzavillan, Lloyd-Braga and Pintus (1998), Gokan (2006), Grandmont (2006), Grandmont, Pintus and de Vilder (1998), Lloyd-Braga and Modesto (2007), Lloyd-Braga, Modesto and Seegmuller (2006) and Pintus (2003, 2004).

2.1 Workers

The population size of these households is normalized to one. Each worker chooses his labor supply and his consumption maximizing the utility function:

$$\sum_{t=1}^{\infty}\left[\lambda^{t-1}U(C_t^w/B)-\lambda^t V(L_t)\right] \tag{1}$$

where C_t^w is the consumption in period t, L_t the labor supply, $B>0$ a scaling parameter and $\lambda \in (0,1)$ the discount factor. The utility functions U and V satisfy the following assumption:[7]

Assumption 1 *The functions $U(x)$ and $V(L)$ are continuous for all $x \geq 0$ and $0 \leq L \leq \overline{L}$, where the labor endowment $\overline{L} > 1$ may be finite or infinite.*[8] *They are $\mathcal{C}^n$ for $x > 0$, $0 < L < \overline{L}$ and n large enough, with $U'(x) > 0$, $U''(x) < 0$, $V'(L) > 0$ and $V''(L) > 0$. Moreover, $lim_{L\to\overline{L}}V'(L) = +\infty$ and consumption and leisure are gross substitutes, i.e. $-xU''(x)/U'(x) < 1$.*

In what follows, we respectively note M_t^w and K_t^w the money balances and the capital stock held by workers, $\delta \in (0,1)$ the depreciation rate of capital, r_t the nominal interest rate, w_t the nominal wage and P_t the price of the final good. Each worker maximizes his utility function (1) under the constraints:

$$P_t\left(C_t^w + K_t^w - (1-\delta)K_{t-1}^w\right) + M_t^w = M_{t-1}^w + r_t K_{t-1}^w + w_t L_t \tag{2}$$

$$P_t\left(C_t^w + K_t^w - (1-\delta)K_{t-1}^w\right) \leq M_{t-1}^w + r_t K_{t-1}^w \tag{3}$$

Equation (2) represents the usual budget constraint, while (3) defines the liquidity constraint. The equilibria considered here are defined by:

$$U'(C_t^w/B) > \lambda U'(C_{t+1}^w/B)\left[(1-\delta) + r_{t+1}/P_{t+1}\right] \tag{4}$$

$$(1-\delta)P_{t+1} + r_{t+1} > P_t \tag{5}$$

Workers always choose $K_t^w = 0$ and the financial constraint is binding, i.e. $P_t C_t^w = M_{t-1}^w$. Therefore, we deduce:

$$u\left(C_{t+1}^w/B\right) = v(L_t) \tag{6}$$

$$P_{t+1}C_{t+1}^w = w_t L_t \tag{7}$$

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[7]For simplification, we note $x \equiv C^w/B$.

[8]We assume that the labor endowment is strictly greater than 1, because as we will see in the next section, the labor supply will be normalized to 1 at the steady state.

where $u(x) = xU'(x)$ and $v(L) = LV'(L)$. Under Assumption 1, there exists a function $\gamma \equiv u^{-1} \circ v$, such that $C^w_{t+1}/B = \gamma(L_t)$. Since consumption and leisure are gross substitutes, $\varepsilon_\gamma(L) \equiv \gamma'(L)L/\gamma(L) = [1+LV''(L)/V'(L)]/[1+xU''(x)/U'(x)] > 1$. Hence, the labor supply is increasing in the real wage, with an elasticity $1/(\varepsilon_\gamma(L) - 1) > 0$.

2.2 Capitalists

Capitalists behavior is represented by a representative agent who maximizes his lifetime utility function:

$$\sum_{t=1}^{\infty} \beta^t \ln C^c_t \tag{8}$$

where $\beta \in (\lambda, 1)$ is his discount factor and C^c_t his consumption. At period t, the representative agent faces the following budget constraint:

$$P_t \left(C^c_t + K^c_t - (1-\delta) K^c_{t-1} \right) + M^c_t = M^c_{t-1} + r_t K^c_{t-1} \tag{9}$$

where M^c_t is the money balances at period t and K^c_t the capital stock. Since we focus on equilibria satisfying $(1-\delta)P_{t+1} + r_{t+1} > P_t$, capitalists do not hold money ($M^c_t = 0$) because it has a lower return than capital. We obtain the optimal solution:

$$C^c_t = (1-\beta) R_t K_{t-1} \tag{10}$$

$$K_t = \beta R_t K_{t-1} \tag{11}$$

where $R_t \equiv 1 - \delta + r_t/P_t$ is the real gross return on capital.[9]

2.3 Production Sector

A continuum of firms, of unit size, produce the final good using labor L_t and capital K_{t-1} with an internal constant returns to scale technology. However, production benefiting from externalities, returns to scale are increasing at the social level. In existing one-sector models, externalities are usually introduced through the total productivity of factors that increases with the average capital and labor.[10] In this paper, we rather consider externalities specific to each input, assuming that efficiency of capital (labor) is increasing in the average capital (average labor).

[9]The superscript on K^c_t is dropped because workers hold no capital.

[10]See among others Barinci and Chéron (2001), Benhabib and Farmer (1994), Cazzavillan, Lloyd-Braga, and Pintus (1998), Farmer and Guo (1994), Harrison and Weder (2002), Hintermaier (2003), Pintus (2006). For a survey, one can refer to Benhabib and Farmer (1999).

More specifically, $F(K_{t-1}, L_t)$ is a well defined strictly concave production function, homogeneous of degree one, increasing with each argument, and $f(a_t)$ the intensive production function satisfying the following assumption:

Assumption 2 *The intensive production function $f(a)$ is continuous for $a \geq 0$, positively valued and differentiable as many times as needed for $a > 0$, with $f'(a) > 0$ and $f''(a) < 0$.*

At each period, the quantity of final good produced is given by:

$$\begin{aligned} Y_t &= AF(C(\overline{K}_{t-1})K_{t-1}, D(\overline{L}_t)L_t) \\ &= Af(a_t)D(\overline{L}_t)L_t, \end{aligned} \tag{12}$$

where

$$a_t \equiv \frac{C(\overline{K}_{t-1})K_{t-1}}{D(\overline{L}_t)L_t} \tag{13}$$

is the capital-labor ratio measured in efficient units, $A > 0$ a scaling parameter, $\overline{K}_{t-1}$ average capital and $\overline{L}_t$ average labor. Furthermore, $C(\overline{K})$ represents externalities specific to capital and $D(\overline{L})$ externalities specific to labor. Note also that $C(\overline{K})K$ ($D(\overline{L})L$) can be interpreted as capital (labor) measured in efficient units. We further assume:

Assumption 3 *The functions $C(K)$ and $D(L)$ are continuous for all $K \geq 0$ and $L \geq 0$, positively valued and differentiable as many times as needed for $K > 0$ and $L > 0$. Moreover, we assume that $\varepsilon_{C,K}(K) \equiv C'(K)K/C(K) \geq 0$ and $\varepsilon_{D,L}(L) \equiv D'(L)L/D(L) \geq 0$.*

Especially, this assumption states that the efficiency of capital $C(\overline{K})$ (labor $D(\overline{L})$) increases with average capital $\overline{K}$ (average labor $\overline{L}$).

Maximizing their profits, the producers take as given the level of externalities. We deduce the real interest rate ϱ_t and the real wage Ω_t:

$$\varrho_t = C(\overline{K}_{t-1})A\rho\,(a_t) \tag{14}$$

$$\Omega_t = D(\overline{L}_t)A\omega\,(a_t) \tag{15}$$

with

$$\rho(a_t) \equiv f'(a_t) \ \text{ and } \ \omega(a_t) \equiv f(a_t) - a_t f'(a_t)$$

We notice that $\widetilde{\varrho}_t \equiv \varrho_t/C(\overline{K}_{t-1})$ and $\widetilde{\Omega}_t \equiv \Omega_t/D(\overline{L}_t)$ represent the marginal productivities of capital and labor measured in efficient units. Before determining the equilibrium, it is useful to define the following relationships. First, we note $s(a) \equiv \rho(a)a/f(a) \in (0,1)$ the capital share in income. Moreover, the elasticity of capital-labor substitution in efficient units is defined by $\sigma(a) = dlna/dln(\widetilde{\Omega}/\widetilde{\varrho}) \geq 0$.[11] This implies that $1/\sigma(a) = d\ln\omega(a)/d\ln a - d\ln\rho(a)/d\ln a$. Since $\omega'(a) = -a\rho'(a)$, we deduce that:

$$\varepsilon_\omega(a) \equiv \frac{\omega'(a)a}{\omega(a)} = \frac{s(a)}{\sigma(a)} \quad \text{and} \quad \varepsilon_\rho(a) \equiv \frac{\rho'(a)a}{\rho(a)} = -\frac{1-s(a)}{\sigma(a)} \tag{16}$$

Finally, note that the degree of returns to scale is determined by $1 + s(a)\varepsilon_{C,K}(\overline{K}) + (1-s(a))\varepsilon_{D,L}(\overline{L})$.[12]

2.4 Intertemporal Equilibrium

Equilibrium on labor market requires that $\overline{L}_t = L_t$ and:

$$w_t/P_t = \Omega_t = D(L_t)A\omega\,(a_t) \equiv A\Omega(K_{t-1}, L_t) \tag{17}$$

with $a_t = C(K_{t-1})K_{t-1}/[D(L_t)L_t]$. Equilibrium on capital market is ensured by $\overline{K}_{t-1} = K_{t-1}$ and:

$$r_t/P_t = \varrho_t = C(K_{t-1})A\rho\,(a_t) \equiv A\varrho(K_{t-1}, L_t) \tag{18}$$

Let $M > 0$ be the constant money supply. Since workers save their wage income in the form of money and capitalists do not hold money, the equilibrium condition on money market can be written:

$$C_t^w = M/P_t = \Omega_t L_t \tag{19}$$

Finally, good market equilibrium is ensured by Walras law. Then, substituting (17) and (18) in (6) and (11), we obtain the two dynamic equations:

$$A\Omega(K_t, L_{t+1})L_{t+1}/B = \gamma\,(L_t) \tag{20}$$

$$K_t = \beta[1 - \delta + A\varrho(K_{t-1}, L_t)]K_{t-1} \tag{21}$$

and an intertemporal equilibrium is defined by:[13]

Definition 1 *Given $K_0 > 0$, an intertemporal equilibrium with perfect foresight is a sequence $(K_{t-1}, L_t) \in \mathbb{R}^2_{++}$, $t = 1, 2, ..., \infty$, such that (20) and (21) are satisfied.*

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[11] A quite similar definition of the elasticity of capital-labor substitution is used by Pintus (2004).

[12] Obviously, returns to scale are increasing when $s(a)\varepsilon_{C,K}(\overline{K}) + (1-s(a))\varepsilon_{D,L}(\overline{L}) > 0$.

[13] Note that capital K_{t-1} is the only one predetermined variable.

3 Existence of a Steady State

A steady state of the dynamic system (20)-(21) is a solution $(K, L) = (K_{t-1}, L_t)$ for all t, such that:

$$\frac{\theta\Omega(K,L)}{B\beta\varrho(K,L)} = \frac{\gamma(L)}{L} \tag{22}$$

$$A\varrho(K,L) = \theta/\beta \tag{23}$$

where $\theta \equiv 1 - \beta(1-\delta) \in (0,1)$.

Following Cazzavillan, Lloyd-Braga, and Pintus (1998), the existence of a steady state is established by choosing appropriately the two scaling parameters $A > 0$ and $B > 0$ so as to ensure that one stationary solution coincides with $(K, L) = (1, 1)$.[14] From equation (23), we obtain a unique solution $A = A^*$, with:

$$A^* = \frac{\theta}{\beta\varrho(1,1)} > 0 \tag{24}$$

Using (22), we get:

$$u\left(\frac{\theta\Omega(1,1)}{B\beta\varrho(1,1)}\right) = v(1) \tag{25}$$

From Assumption 1, u is decreasing in B. Therefore, there exists a unique $B = B^* > 0$ satisfying (25) if and only if the following assumption is satisfied:

Assumption 4 *$lim_{x\to 0}u(x) < v(1) < lim_{x\to+\infty}u(x)$.*

This result is summarized as follows:

Proposition 1 *Under Assumptions 1-4, $(K, L) = (1, 1)$ is a stationary solution of the dynamic system (20)-(21) if and only if A^* and B^* are the unique solutions of (24) and (25).*

4 Local Dynamics

In this section, we analyze the stability properties of the steady state. More specifically, we focus on local indeterminacy and bifurcations to analyze the occurrence of fluctuations due to the volatility of expectations and endogenous cycles.

We first differentiate the dynamic system (20)-(21). If we note ε_γ the elasticity of $\gamma(L)$, and $\varepsilon_{\varrho,K}$, $\varepsilon_{\varrho,L}$, $\varepsilon_{\Omega,K}$ and $\varepsilon_{\Omega,L}$ the elasticities of $\varrho(K,L)$ and

[14] For sake of conciseness, we do not analyze uniqueness or multiplicity of steady states.

$\Omega(K, L)$ with respect to K and L, evaluated at the steady state defined in Proposition 1, we get:

$$\frac{dK_t}{K} = (\theta\varepsilon_{\varrho,K} + 1)\frac{dK_{t-1}}{K} + \theta\varepsilon_{\varrho,L}\frac{dL_t}{L} \tag{26}$$

$$\frac{dL_{t+1}}{L} = -\frac{\varepsilon_{\Omega,K}(1+\theta\varepsilon_{\varrho,K})}{1+\varepsilon_{\Omega,L}}\frac{dK_{t-1}}{K} + \frac{\varepsilon_\gamma - \theta\varepsilon_{\Omega,K}\varepsilon_{\varrho,L}}{1+\varepsilon_{\Omega,L}}\frac{dL_t}{L} \tag{27}$$

We also note $\varepsilon_{C,K} \equiv \varepsilon_{C,K}(1)$, $\varepsilon_{D,L} \equiv \varepsilon_{D,L}(1)$, $s \equiv s(C(1)/D(1))$ and $\sigma \equiv \sigma(C(1)/D(1))$. Using (16), (17) and (18), we obtain the following relationships:

$$\begin{aligned}
\varepsilon_{\varrho,K} &= \varepsilon_{C,K} - \frac{(1-s)(1+\varepsilon_{C,K})}{\sigma} \\
\varepsilon_{\varrho,L} &= \frac{(1-s)(1+\varepsilon_{D,L})}{\sigma} \\
\varepsilon_{\Omega,K} &= \frac{s(1+\varepsilon_{C,K})}{\sigma} \\
\varepsilon_{\Omega,L} &= \varepsilon_{D,L} - \frac{s(1+\varepsilon_{D,L})}{\sigma}
\end{aligned} \tag{28}$$

Before determining the trace T and the determinant D of the associated Jacobian matrix, we clarify that, for simplification and briefness, we restrict our analysis to the case of an infinitely elastic labor supply,[15] i.e.

Assumption 5 $\varepsilon_\gamma = 1$.

Using this assumption and equations (26), (27), (28), we have:

$$T = 1 + \frac{\sigma[1+\theta\varepsilon_{C,K}(1+\varepsilon_{D,L})] - \theta(1+\varepsilon_{D,L})(1-s+\varepsilon_{C,K})}{(\sigma-s)(1+\varepsilon_{D,L})} \tag{29}$$

$$D = \frac{\sigma(1+\theta\varepsilon_{C,K}) - \theta(1-s)(1+\varepsilon_{C,K})}{(\sigma-s)(1+\varepsilon_{D,L})} \tag{30}$$

Recall that when $1-T+D>0$, $1+T+D>0$ and $D<1$, the steady state is a sink, i.e. locally indeterminate since one variable (the capital) is predetermined. When $1-T+D>(<)0$ and $1+T+D<(>)0$, the steady state is a saddle. Otherwise, it is a source. In these last two cases, the steady state is locally determinate. Moreover, when a parameter varies

[15]Note that this assumption is often used in the literature. The interested reader can however refer to a previous version of this paper (Seegmuller (2006)) where the analysis is led without assuming an infinitely elastic labor supply.

continuously, a flip bifurcation generically occurs when $1+T+D=0$, a transcritical bifurcation generically occurs when $1-T+D=0$ and a Hopf bifurcation generically occurs when $1-T+D>0$, $1+T+D>0$ and $D=1$.[16]

Before beginning the analysis, we further assume:

Assumption 6

(i) $s > \theta(1-s)(1+\varepsilon_{C,K})(2+\varepsilon_{D,L})/(2+\theta\varepsilon_{C,K})$*;*

(ii) $\sigma > s$*.*

Note that, in this model, the length of period is small, implying values of the depreciation rate of capital δ close to 0 and of the discount factor β close to 1. Therefore, θ is small and close to 0, providing a justification of inequality (i).[17] Assumption 6 (ii) imposes that we do not focus on sufficiently weak degrees of capital-labor substitution. In addition, it is already well known that, in the Woodford (1986) framework, local indeterminacy and endogenous cycles can occur without externalities only if $\sigma < s$ (see Grandmont, Pintus and de Vilder (1998)).

To determine the local stability properties of the steady state, we first compute $1+T+D$. Using (29) and (30), we obtain:

$$1+T+D = \frac{2(2+\varepsilon_{D,L})(2+\theta\varepsilon_{C,K})(\sigma-\sigma_F)}{(\sigma-s)(1+\varepsilon_{D,L})} \tag{31}$$

with

$$\sigma_F \equiv \frac{(1+\varepsilon_{D,L})[2s+\theta(1-s+\varepsilon_{C,K})]+\theta(1-s)(1+\varepsilon_{C,K})}{2(2+\varepsilon_{D,L})(2+\theta\varepsilon_{C,K})} \tag{32}$$

Using Assumption 6 (i), we can prove that $\sigma_F < s$. Therefore, $1+T+D > 0$ for all $\sigma > s$. We compute now $1-T+D$:

$$1-T+D = \frac{\theta\varepsilon_{C,K}\varepsilon_{D,L}(\sigma_T-\sigma)}{(\sigma-s)(1+\varepsilon_{D,L})} \tag{33}$$

with

$$\sigma_T \equiv 1 + \frac{1-s}{\varepsilon_{C,K}} + \frac{s}{\varepsilon_{D,L}} \tag{34}$$

[16]See Azariadis (1993) and Grandmont, Pintus and de Vilder (1998) for more details.

[17]We remark that without externalities, inequality (i) becomes $s > \theta(1-s)$, which is an assumption introduced in the perfectly competitive economy analyzed by Grandmont, Pintus and de Vilder (1998).

We deduce that $1 - T + D > 0$ for $s < \sigma < \sigma_T$, $1 - T + D = 0$ for $\sigma = \sigma_T$ and $1 - T + D < 0$ for $\sigma > \sigma_T$. Note that in the particular case where $\varepsilon_{C,K} = 0$ and/or $\varepsilon_{D,L} = 0$, σ_T is not defined and $1 - T + D > 0$ for all $\sigma > s$.

Using Assumption 6 (i), we remark that D is a decreasing function of σ (see equation (30)), from $+\infty$ when $\sigma \to s$ to $(1 + \theta\varepsilon_{C,K})/(1 + \varepsilon_{D,L})$ when $\sigma \to +\infty$. Therefore, the inequality

$$\varepsilon_{D,L} > \theta\varepsilon_{C,K} \tag{35}$$

is a necessary condition to have $D < 1$. Indeed, when (35) is satisfied, $D > 1$ for $s < \sigma < \sigma_H$, $D = 1$ for $\sigma = \sigma_H$ and $D < 1$ for $\sigma > \sigma_H$, with:

$$\sigma_H \equiv \frac{s(1 + \varepsilon_{D,L}) - \theta(1 - s)(1 + \varepsilon_{C,K})}{\varepsilon_{D,L} - \theta\varepsilon_{C,K}} \tag{36}$$

Otherwise, $D > 1$ for all $\sigma > s$. We finally remark that $\sigma_T > \sigma_H$ if and only if:

$$\frac{\varepsilon_{D,L}^2}{1 + \varepsilon_{D,L}} > \theta\frac{s}{1 - s}\frac{\varepsilon_{C,K}^2}{1 + \varepsilon_{C,K}} \tag{37}$$

Using all these results, we deduce the stability properties of the steady state:

Proposition 2 *Under Assumptions 1-6, the following generically holds.*

(i) *If $\varepsilon_{D,L} \leq \theta\varepsilon_{C,K}$ or $\varepsilon_{D,L} > \theta\varepsilon_{C,K}$ and $\frac{\varepsilon_{D,L}^2}{1+\varepsilon_{D,L}} < \theta\frac{s}{1-s}\frac{\varepsilon_{C,K}^2}{1+\varepsilon_{C,K}}$, the steady state is a source for $s < \sigma < \sigma_T$ and is a saddle for $\sigma > \sigma_T$.*

(ii) *If $\varepsilon_{D,L} > \theta\varepsilon_{C,K}$ and $\frac{\varepsilon_{D,L}^2}{1+\varepsilon_{D,L}} > \theta\frac{s}{1-s}\frac{\varepsilon_{C,K}^2}{1+\varepsilon_{C,K}}$, the steady state is a source for $s < \sigma < \sigma_H$, is a sink for $\sigma_H < \sigma < \sigma_T$ and is a saddle for $\sigma > \sigma_T$.*

A transcritical bifurcation generically occurs when σ crosses σ_T and a Hopf bifurcation generically occurs when σ crosses σ_H.

This proposition states first that endogenous fluctuations can occur only if inequalities (35) and (37) are satisfied. This means that the response of labor efficiency ($D(L)$) to an increase of labor has to be large enough with respect to the response of capital efficiency ($C(K)$) to an increase of capital. In such a case, indeterminacy requires elasticities of capital-labor substitution higher than a lower bound (σ_H). This result can be related to Cazzavillan, Lloyd-Braga and Pintus (1998) who analyze, in a Woodford (1986) model, the more usual case where externalities affect the total productivity of factors.[18] They show that, when capital and labor are not too weak substitutes, indeterminacy and endogenous cycles require a contribution of labor to externalities sufficiently large with respect to the contribution of capital to these externalities. Even if these two results seem to be connected, they have different economic meanings. Indeed, in Cazzavillan, Lloyd-Braga and Pintus (1998), an increase of average labor (capital) increases production through the total productivity, whereas in our paper, this modifies labor (capital) efficiency and therefore capital-labor ratio.

Another difference between the two papers concerns the range of elasticities of capital-labor substitution such that indeterminacy emerges. In contrast to Cazzavillan, Lloyd-Braga and Pintus (1998), local indeterminacy requires a capital-labor substitution less than an upper bound (σ_T). Hence, in our paper, a high enough capital-labor substitution ($\sigma > \sigma_T$) does not promote indeterminacy, but is rather a source of saddle-path stability. Since the upper bound σ_T decreases with $\varepsilon_{C,K}$ and $\varepsilon_{D,L}$ (see equation (34)), that is, with the degree of increasing returns, determinacy emerges more easily for larger degree of increasing returns, if capital and labor are sufficiently high substitutes. This result is also new since most of the existing papers rather show that higher increasing returns promote fluctuations due to self-fulfilling expectations.

It is also interesting to connect our analysis to Benhabib and Farmer (1994). As it is well known, these authors have shown, considering an infinitely lived agent model, that local indeterminacy requires wrong slopes on the labor market, i.e. a slope of labor demand greater than the slope of labor supply. Since this seminal contribution, several authors have enriched the model to show that fluctuations can occur without the wrong slopes.[19] In our model, the slope of labor supply $\varepsilon_\gamma - 1$ is equal to 0 (Assumption 5), whereas the slope of labor demand is given by $\varepsilon_{\Omega,L} = \varepsilon_{D,L} - s(1+\varepsilon_{D,L})/\sigma$ (see (28)). Therefore, we get the wrong slopes if:

$$\sigma > \frac{s(1+\varepsilon_{D,L})}{\varepsilon_{D,L}} \equiv \widetilde{\sigma} \tag{38}$$

where $\sigma_H < \widetilde{\sigma} < \sigma_T$. Since $\varepsilon_{\Omega,L}$ is an increasing function of σ, when $\sigma_H < \sigma < \widetilde{\sigma}$, indeterminacy occurs with a negatively slopped labor demand, i.e. without the wrong slopes.[20] On the contrary, when $\widetilde{\sigma} < \sigma < \sigma_T$, there

[18] In their model, the production is given by $Y_t = A(\overline{K}_{t-1}, \overline{L}_t)F(K_{t-1}, L_t)$, where the total productivity of factors $A(\overline{K}_{t-1}, \overline{L}_t)$ is increasing in the average capital $\overline{K}_{t-1}$ and labor $\overline{L}_t$.

[19] See Benhabib and Farmer (1999) for a survey.

[20] See Barinci and Chéron (2001) for a related result under a Cobb-Douglas technology.

is indeterminacy under the wrong slopes condition. More interestingly, the steady state is a saddle for all $\sigma > \sigma_T$, i.e. when the condition on the wrong slopes is ensured. Hence, the wrong slopes condition is neither a necessary nor a sufficient condition for indeterminacy in our framework.

To give now an economic interpretation of our results, we recall that the dynamics are defined by:

$$\begin{aligned} C^w_{t+1}/B &= \gamma(L_t) &(39)\\ K_t &= \beta(1-\delta+\varrho_t)K_{t-1} &(40)\end{aligned}$$

where $C^w_{t+1} = (P_t/P_{t+1})\Omega_t L_t = \Omega_{t+1}L_{t+1}$. To provide an intuition on the occurrence of fluctuations due to self-fulfilling expectations, assume that workers expect a decrease of the inflation factor (P_{t+1}/P_t), i.e. an increase of future consumption. Since consumption and leisure are gross substitutes, they increase their labor supply L_t (see (39)), which has a positive effect on the real interest rate ϱ_t. Therefore, capital accumulation K_t increases (see (40)), which implies a larger real wage Ω_{t+1}.

To summarize, following a decrease of inflation, future consumption will effectively increase if two effects are large enough:

(i) the positive impact of labor on the real interest rate;

(ii) the positive impact of capital on the real wage.

Under perfect competition (no externality), these two effects require a sufficiently weak capital-labor substitution (see (28)).[21] When production benefits from externalities through the total productivity of factors (Cazzavillan, Lloyd-Braga and Pintus (1998)), expectations can be self-fulfilling under a high degree of capital-labor substitution, because a larger level of labor (capital) increases the productivity, i.e. the aggregate production, and therefore the real wage and the real interest rate.

In our framework, the mechanism is quite different because now externalities affect the capital-labor ratio $a = [C(K)K]/[D(L)L]$ measured in efficient units. Hence, a higher level of labor implies a sufficiently large increase of the real interest rate $\varrho = C(K)A\rho(a)$, because labor efficiency $D(L)$, increasing in labor, reinforces the decrease of the capital-labor ratio a. In a quite similar way, a higher level of capital has a sufficiently positive impact on the real wage $\Omega = D(L)A\omega(a)$, because it raises capital efficiency $C(K)$, which reinforces the increase of the capital-labor ratio a. However, in contrast to Cazzavillan, Lloyd-Braga and Pintus (1998), indeterminacy does no more occur if the elasticity of capital-labor substitution is sufficiently large because

[21] See Grandmont, Pintus and de Vilder (1998) and Woodford (1986) for more details.

the two effects just described are dampened. Indeed, when σ is high enough, a decrease (increase) of the capital-labor ratio a measured in efficient units only induces a small variation of the real interest rate (wage) (see (28)).

5 Implications for Fiscal Policy

Since the seminal contributions of Guo and Lansing (1998) and Schmitt-Grohé and Uribe (1997), several papers have analyzed the (de-)stabilizing role of fiscal policy rules under a balanced budget. In this section, we contribute to this debate by introducing variable tax rates on labor and capital incomes in the previous model. While existing results show that, in one-sector models, tax rates decreasing with their tax base always have a destabilizing effect,[22] we will show that less clear-cut conclusions are obtained in our framework.

To be as short as possible we do not present the model with taxation in details but only focus on the main differences regarding the model developed in Section 2. We assume that there exists now a government that levies taxes on labor income $\Omega_t L_t$ and capital income $\varrho_t K_{t-1}$, at the rate τ_t^L and τ_t^K respectively. These two tax rates that depend on the tax base are defined by the government as follows:[23]

$$\tau_t^L = 1 - z_L \left(\frac{\Omega(K_{t-1}, L_t) L_t}{\Omega(1,1)} \right)^{\xi_L} \equiv \tau^L(K_{t-1}, L_t) \tag{41}$$

$$\tau_t^K = 1 - z_K \left(\frac{\varrho(K_{t-1}, L_t) K_{t-1}}{\varrho(1,1)} \right)^{\xi_K} \equiv \tau^K(K_{t-1}, L_t) \tag{42}$$

with $z_i \in (0,1)$ for $i = K, L$. Note that the term into the brackets corresponds to the tax base (labor and capital income respectively), divided by its value at the steady state. Therefore, $1 - z_i \in (0,1)$ determines the level of the tax rate at the steady state and when the tax rate does not depend on the tax base ($\xi_i = 0$). Finally, the tax rate τ_t^i is decreasing (increasing) in the tax base if $\xi_i > 0$ ($\xi_i < 0$).

The government determines the level of public spending G_t according to the balanced-budget rule:[24]

$$G_t = \tau_t^L \Omega_t L_t + \tau_t^K \varrho_t K_{t-1} \tag{43}$$

[22]See among others Guo and Lansing (1998), Gokan (2006), Lloyd-Braga, Modesto and Seegmuller (2006), Pintus (2003) and Schmitt-Grohé and Uribe (1997).

[23]In their paper, Guo and Lansing (1998) use quite similar specifications of the tax rates.

[24]Note that G_t neither enters the production function nor the households preferences.

Following Gokan (2006), Lloyd-Braga, Modesto and Seegmuller (2006), Pintus (2003) and Schmitt-Grohé and Uribe (1997), we assume that households know the tax rates but take them as given when they determine their optimal behaviors. We deduce that all the analysis of Section 2 applies, except that the real wage $\Omega_t = A\Omega(K_{t-1}, L_t)$ and the real interest rate $\varrho_t = A\varrho(K_{t-1}, L_t)$ have to be replaced by the after tax real wage $\overline{\Omega}_t$ and real interest rate $\overline{\varrho}_t$. Using equations (41) and (42), we get:

$$\begin{aligned}\overline{\Omega}_t &= (1-\tau_t^L)\Omega_t = z_L \left(\frac{\Omega(K_{t-1}, L_t)L_t}{\Omega(1,1)}\right)^{\xi_L} A\Omega(K_{t-1}, L_t) \\ &\equiv A\overline{\Omega}(K_{t-1}, L_t)\end{aligned} \tag{44}$$

$$\begin{aligned}\overline{\varrho}_t &= (1-\tau_t^K)\varrho_t = z_K \left(\frac{\varrho(K_{t-1}, L_t)K_{t-1}}{\varrho(1,1)}\right)^{\xi_K} A\varrho(K_{t-1}, L_t) \\ &\equiv A\overline{\varrho}(K_{t-1}, L_t)\end{aligned} \tag{45}$$

Substituting these two expressions into (20) and (21), the definition of the intertemporal equilibrium and the existence of a steady state can be conducted in a similar way. Focusing now on the dynamic implications of the fiscal policy, we observe that the introduction of the two variable tax rates only modify the elasticities of the (after tax) real wage and real interest rate. Defining $\varepsilon_{\overline{\varrho},K}$, $\varepsilon_{\overline{\varrho},L}$, $\varepsilon_{\overline{\Omega},K}$ and $\varepsilon_{\overline{\Omega},L}$ the elasticities of $\overline{\varrho}(K, L)$ and $\overline{\Omega}(K, L)$ with respect to K and L, evaluated at the steady state $(K, L) = (1, 1)$, and using (28), (44) and (45), we get:

$$\begin{aligned}\varepsilon_{\overline{\varrho},K} &= \overline{\varepsilon}_{C,K} - \frac{(1-s)(1+\overline{\varepsilon}_{C,K})}{\sigma} \\ \varepsilon_{\overline{\varrho},L} &= \frac{(1-s)(1+\overline{\varepsilon}_{D,L})}{\sigma} \\ \varepsilon_{\overline{\Omega},K} &= \frac{s(1+\overline{\varepsilon}_{C,K})}{\sigma} \\ \varepsilon_{\overline{\Omega},L} &= \overline{\varepsilon}_{D,L} - \frac{s(1+\overline{\varepsilon}_{D,L})}{\sigma}\end{aligned} \tag{46}$$

where $\overline{\varepsilon}_{C,K}$ and $\overline{\varepsilon}_{D,L}$ are defined by:

$$\begin{aligned}\overline{\varepsilon}_{C,K} &\equiv \xi_K(1+\varepsilon_{C,K}) + \varepsilon_{C,K} \\ \overline{\varepsilon}_{D,L} &\equiv \xi_L(1+\varepsilon_{D,L}) + \varepsilon_{D,L}\end{aligned} \tag{47}$$

We notice that substituting $\overline{\varepsilon}_{C,K}$ and $\overline{\varepsilon}_{D,L}$ by $\varepsilon_{C,K}$ and $\varepsilon_{D,L}$ respectively, the elasticities $\varepsilon_{\overline{\varrho},K}$, $\varepsilon_{\overline{\varrho},L}$, $\varepsilon_{\overline{\Omega},K}$ and $\varepsilon_{\overline{\Omega},L}$ are identical to $\varepsilon_{\varrho,K}$, $\varepsilon_{\varrho,L}$, $\varepsilon_{\Omega,K}$ and

$\varepsilon_{\Omega,L}$. Therefore, the analysis of Section 4 applies and the dynamic stability properties of the steady state are still given by Proposition 2, substituting $\varepsilon_{C,K}$ and $\varepsilon_{D,L}$ by $\overline{\varepsilon}_{C,K}$ and $\overline{\varepsilon}_{D,L}$. However, we need to impose $\overline{\varepsilon}_{C,K} \geq 0$ and $\overline{\varepsilon}_{D,L} \geq 0$, i.e.

Assumption 7 *$\xi_K \geq -\varepsilon_{C,K}/(1+\varepsilon_{C,K})$ and $\xi_L \geq -\varepsilon_{D,L}/(1+\varepsilon_{D,L})$.*

On the one hand, note that when the tax rates are decreasing in their tax base, i.e. $\xi_K > 0$ and $\xi_L > 0$, this assumption is satisfied. In this case, we further note that $\overline{\varepsilon}_{C,K} > \varepsilon_{C,K}$ and $\overline{\varepsilon}_{D,L} > \varepsilon_{D,L}$. On the other hand, when $\xi_K = \xi_L = 0$, $\varepsilon_{\overline{\varrho},K}$, $\varepsilon_{\overline{\varrho},L}$, $\varepsilon_{\overline{\Omega},K}$ and $\varepsilon_{\overline{\Omega},L}$ are equal to $\varepsilon_{\varrho,K}$, $\varepsilon_{\varrho,L}$, $\varepsilon_{\Omega,K}$ and $\varepsilon_{\Omega,L}$, respectively. Hence, the fiscal policy has no influence on local dynamics when the tax rates are constant.

In what follows, focusing on tax rates decreasing with their tax base ($\xi_K > 0$ and $\xi_L > 0$), we analyze the implications of fiscal policy on indeterminacy. Applying the results of Proposition 2, we notice that indeterminacy requires a sufficiently large $\overline{\varepsilon}_{D,L}$ regarding the value of $\overline{\varepsilon}_{C,K}$. This is ensured by a not too flat tax rate on labor income and a not too decreasing tax rate on capital income. Otherwise, indeterminacy is ruled out. Considering now that this condition is satisfied, Proposition 2 states that indeterminacy occurs for $\sigma_H < \sigma < \sigma_T$, with:

$$\sigma_H \equiv \frac{s(1+\overline{\varepsilon}_{D,L}) - \theta(1-s)(1+\overline{\varepsilon}_{C,K})}{\overline{\varepsilon}_{D,L} - \theta\overline{\varepsilon}_{C,K}} \tag{48}$$

$$\sigma_T \equiv 1 + \frac{1-s}{\overline{\varepsilon}_{C,K}} + \frac{s}{\overline{\varepsilon}_{D,L}} \tag{49}$$

Obviously, we remark that the higher bound σ_T is decreasing in $\overline{\varepsilon}_{C,K}$ and $\overline{\varepsilon}_{D,L}$, i.e. in ξ_K and ξ_L. Moreover, we can prove that under not restrictive conditions, σ_H is decreasing in $\overline{\varepsilon}_{D,L}$ (or ξ_L) and increasing in $\overline{\varepsilon}_{C,K}$ (or ξ_K).[25]

We deduce that a more variable (decreasing) tax rate on capital income (ξ_K higher) promotes stability by increasing σ_H and reducing σ_T. A more variable (decreasing) tax rate on labor income (ξ_L higher) implies a decrease of both σ_H and σ_T. Therefore, under more decreasing tax rates, indeterminacy occurs less easily for high elasticities of capital-labor substitution (σ close to σ_T). Tax rates characterized by larger values of ξ_K and ξ_L stabilize the economy by promoting saddle-path stability. As we have already underlined, this result is in contrast with the existing literature introducing distortionary taxes and balanced-budget rules in one-sector models.

[25]Using equation (48), we obtain $\partial\sigma_H/\partial\overline{\varepsilon}_{D,L} < 0$ if $\theta(1-2s)\overline{\varepsilon}_{C,K} < s - \theta(1-s)$ and $\partial\sigma_H/\partial\overline{\varepsilon}_{C,K} > 0$ if $\theta(1-2s)\overline{\varepsilon}_{D,L} < s - \theta(1-s)$. These two inequalities are satisfied if $\overline{\varepsilon}_{C,K}$ and $\overline{\varepsilon}_{D,L}$ are not excessively large and θ is small.

6 Conclusion

In this paper, we introduce input-specific externalities in a model with infinitely lived agents and a finance constraint as developed in the seminal contribution of Woodford (1986). Instead of assuming, as usually, that average labor and capital increase the total productivity of factors, we consider that average labor (capital) raises labor (capital) efficiency. Focusing on not too weak elasticities of capital-labor substitution, we show that indeterminacy does not only require a lower bound for the capital-labor substitution, but also a finite higher bound. As a direct implication of this result, the wrong slopes condition (slope of labor demand greater than slope of labor supply) does not ensure an indeterminate steady state. Moreover, larger increasing returns improve the scope of capital-labor substitution for saddle-path stability. Introducing variable tax rates on capital and labor income in our framework, we reexamine the role of a balanced-budget fiscal policy rule on endogenous fluctuations. In contrast to existing results in one-sector models, tax rates decreasing with their tax base do not always promote instability coming from the volatility of expectations.

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