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► **To cite this version:**

Ralf Krömer. Compte-rendu de Klaus Volkert, Das Homöomorphieproblem, insbesondere der 3-Mannigfaltigkeiten, in der Topologie 1892-1935 (cahier spécial 4 de Philosophia scientiae, Paris: éditions Kimé 2002). *Historia Mathematica*, 2006, 33 (2), pp.259-260. <10.1016/j.hm.2005.08.003>. <halshs-00132726>

HAL Id: halshs-00132726

<https://shs.hal.science/halshs-00132726v1>

Submitted on 27 Oct 2025

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Das Homöomorphieproblem, insbesondere der 3-Mannigfaltigkeiten, in der Topologie 1892–1935

By Klaus Volkert. *Philosophia scientiæ, Cahier spécial 4*. Paris (Kimé). 2002. ISBN 2-84174-275-X.

343 pp. €29

This book is a history of three-dimensional topological manifolds (hereafter 3-manifolds), from their debut in the work of Poincaré until 1935. The main part of the book is preceded by a short introduction and a chapter on the history of the classification of surfaces, analyzing contributions by Riemann, Listing, Möbius, Klein, Jordan, von Dyck, Dehn and Heegard, and others. Although this chapter holds few surprises, it lays the ground for the following chapter on Poincaré's investigations in higher dimensions [Poincaré, 1895], which relied on the solution of the classification problem for surfaces. Here the author discusses the role of Poincaré's work in topology for his philosophical thought, and he presents the first detailed account of the "5th supplement" [Poincaré, 1904], stressing Poincaré's discussion of various examples of manifolds as test cases for tentative classifications. Volkert later shows that these and other examples were crucial to most of the progress made in this area. He points out connections to Poincaré's work on automorphic functions, and indicates how the French mathematician came to formulate a special case of the classification problem, otherwise known as the Poincaré Conjecture.

The fourth chapter presents work that appeared immediately after Poincaré's contributions, which continued the latter in some ways, in particular in the study of the fundamental group. Novel elements were also introduced at this time, especially on a methodological level—for example, the axiomatic approach of Steinitz [1908], and Dehn's applications of groups in knot theory [1910]. The period 1919–1935 is dealt with in Chapter 5, starting with a setback: Alexander's proof [1919] that the fundamental group is not a strong enough invariant for classification of the 3-manifolds. The development of additional invariants led to partial results by Threlfall and Seifert, in particular [1931; 1933], and to a solution of the problem for a certain class of spaces (lense spaces) by Reidemeister [1936]. The author argues that the problem was viewed as overly ambitious around 1935, when topology began a two-decade-long focus on an algebraically oriented revision of its methodology, which largely ignored the classification problem. This shift of attention is briefly analyzed in Chapter 6, and illustrated by a comparison of textbook tables of contents from Veblen [1922] to Eilenberg and Steenrod [1952]. In Chapter 7, the author takes up more recent developments, which renew approaches developed before the algebraic turn, including the work of Smale [1960] and Thurston [1978]. The author holds that the approach of the Seifert–Threlfall textbook [1934] provided greater inspiration for recent research than did its successors. The book's last chapter is a short but suggestive general essay on patterns in the genesis of a mathematical discipline, that profits from the foregoing view of the classification problem.

By contributing to the history of so-called “low-dimensional topology,” the author aims to fill a significant gap in the historiography of topology. This topic was outside the scope of earlier work in the field, including that of Jean Dieudonné [1989]. While Volkert’s account is as reliable as Dieudonné’s in mathematical details, its historical method is markedly superior. For example, Volkert conserves original notation, and in lieu of teleology, stresses the original motivations and contexts of the historical contributions. This, without neglecting their eventual relevance for current research. Volkert’s work is all the more valuable in that the problems of 3-manifolds are central to contemporary research, and have been, as he shows, important for topology’s development into an independent subdiscipline of mathematics. The book usefully extends the history of the manifold concept inaugurated by Scholz [1980].

This excellent book is not free of blemishes. Unpublished sources are rarely referred to, and a proper discussion of relevant sources is lacking. There is no index, rendering the book useless as a source of reference. A considerable number of misprints are to be regretted, although these do not tend to mar comprehension. The bibliography is extensive and precise in general, but the six-year delay in publication of the manuscript has led to the neglect of a few titles appearing after 1996, including recent advances on the Poincaré conjecture, and historical work such as Sarkaria [1999]. These details aside, the book includes a multitude of useful illustrations; it is a real bargain, and I recommend it strongly.

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Available online 19 October 2005