

## Overlooking mathematical justifications in the Sanskrit tradition: the nuanced case of G. F. Thibaut

Agathe Keller

#### ▶ To cite this version:

Agathe Keller. Overlooking mathematical justifications in the Sanskrit tradition: the nuanced case of G. F. Thibaut. History of Mathematical Proof in Ancient Traditions, Cambridge University Press, pp.260-273, 2012. halshs-00150736v2

## HAL Id: halshs-00150736 https://shs.hal.science/halshs-00150736v2

Submitted on 21 Jan 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



# Overlooking mathematical justifications in the Sanskrit tradition: the nuanced case of G. F. Thibaut

#### Agathe Keller

#### Abstract

How did the narratives of the history of Indian mathematics explain the tradition of mathematical justifications that existed in medieval Sanskrit commentaries? When the German philologer G. F.Thibaut published a translation of a set of Vedic geometrical texts in 1874 and 1875, he established that India had known other mathematical activities than 'practical calculations'. Thibaut's philological work and historiographical values determined his approach to these texts and provided a bias for understanding the reasonings and efforts which establish the validity of algorithms in this set of texts.

### Introduction

<sup>1</sup> Until the 1990s, the historiography of Indian mathematics largely held that Indians did not use "proofs" in their mathematical texts. Dhruv Raina has shown that this interpretation arose partly from the fact that during the second half of the nineteenth century, the French mathematicians who analyzed Indian astronomical and mathematical texts considered geometry to be the

<sup>&</sup>lt;sup>1</sup>I would like to thank K. Chemla and M. Ross for their close reading of this article. They have considerably helped in improving it.

<sup>&</sup>lt;sup>2</sup>Srinivas 1990, Hayashi 1995.

measure of mathematical activity<sup>3</sup>. The French mathematicians relied on the work of the English philologers of the previous generation, who considered the computational reasonings and algorithmic verifications merely 'practical' and devoid of the rigor and prestige of a real logical and geometrical demonstration. Against this historiographical backdrop, the German philologer Georg Friedrich Wilhem Thibaut (1848-1914) published the oldest known mathematical texts in Sanskrit, which are devoted only to geometry.

These texts,  $\acute{sulbasutras}^4$  (sometimes called the sulvasutras) contain treatises by different authors (Baudhāyana, Āpastamba, Kātyāyana and Mānava) and consider the geometry of the Vedic altar. These texts were written in the style typical of aphoristic sutras between 600 and 200 BCE. They were sometimes accompanied by later commentaries, the earliest of which may be assigned to roughly the thirteenth century. In order to understand the methods which he openly employed for this corpus of texts, Thibaut must be situated as a scholar. This analysis will focus on Thibaut's historiography of mathematics, especially on his perception of mathematical justifications.

## 1 Thibaut's intellectual background

G. F. Thibaut's approach to the *śulbasūtras* combines what half a century before him had been two conflicting traditions. As described by D. Raina and F. Charette, Thibaut was equal parts acute philologer and scientist investigating the history of mathematics.

<sup>&</sup>lt;sup>3</sup>See Raina 1999: chapter VI.

<sup>&</sup>lt;sup>4</sup>We will adopt the usual transliteration of Sanskrit words which will be marked in italics, except for the word Veda, which belongs also to English dictionaries.

#### 1.1 A philologer

Thibaut trained according to the German model of a Sanskritist<sup>5</sup>. Born in 1848 in Heidelberg, he studied Indology in Germany. His European career culminated when he left for England in 1870 to work as an assistant for Max Müller's edition of the Vedas. In 1875, he became Sanskrit professor at Benares Sanskrit College. At this time, he produced his edition and studies of the śulbasūtras, the focus of the present article<sup>6</sup>. Afterwards, Thibaut spent the following 20 years in India, teaching Sanskrit, publishing translations and editing numerous texts. With P. Griffith, he was responsible for the Benares Sanskrit Series, from 1880 onwards. As a specialist in the study of the ritualistic mimāṃsa school of philosophy and Sanskrit scholarly grammar, Thibaut made regular incursions in the history of mathematics and astronomy.

Thibaut's interest in mathematics and astronomy in part derives from his interest in  $mim\bar{a}msa$ . The authors of this school commented upon the ancillary parts of the Vedas ( $ved\bar{a}niga$ ) devoted to ritual. The  $\acute{s}ulbas\bar{u}tras$  can be found in this auxiliary literature on the Vedas. As a result of having studied these texts, between 1875 and 1878<sup>7</sup>, Thibaut published several articles on vedic mathematics and astronomy. These studies sparked his curiosity about the later traditions of astronomy and mathematics in the Indian subcontinent and the first volume of the Benares Sanskrit Series, of which Thibaut was the general scientific editor, was the Siddhāntatattvaviveka of Bhatta Kamalākara. This astronomical treatise written in the seventeenth

<sup>&</sup>lt;sup>5</sup>The following paragraph rests mainly on Stachen-Rose 1990.

<sup>&</sup>lt;sup>6</sup>See Thibaut 1874, Thibaut 1875, Thibaut 1877.

<sup>&</sup>lt;sup>7</sup>The last being a study of the *jyotisavedānga*, in Thibaut 1878.

century in Benares attempts to synthesize the re-workings of theoretical astronomy made by the astronomers under the patronage of Ulug Begh with the traditional Hindu  $siddh\bar{a}ntas^8$ .

Thibaut's next direct contribution to the history of mathematics and astronomy in India was a study on the medieval astronomical treatise, the  $Pa\tilde{n}casiddh\bar{a}nta$  of Varāhamihira. In 1888, he also edited and translated this treatise with S. Dvivedi and consequently entered in a heated debate with H. Jacobi on the latter's attempt to date the Veda on the basis of descriptions of heavenly bodies in ancient texts. At the end of his life, Thibaut published several syntheses of ancient Indian mathematics and astronomy<sup>9</sup>. His main oeuvre, was not in the field of history of science but a three volume translation of one of the main  $mim\bar{a}msa$  texts:  $\acute{S}aikar\bar{a}c\bar{a}rya$ 's commentary on the  $Ved\bar{a}ntas\bar{u}tras$ , published in the  $Sacred\ Books\ of\ the\ East$ , the series initiated by his teacher Max Müller <sup>10</sup>. Thibaut died in Berlin at the beginning of the first world war, in October 1914.

Among the  $\acute{sulbasutras}$ , Thibaut focussed on Baudhāyana (ca. 600 BCE)<sup>11</sup> and Āpastamba's texts, occasionally examining Kātyāyana's  $\acute{sulbapari\acute{siṣta}}$ . Thibaut noted the existence of the  $M\bar{a}navasulbas\bar{u}tra$  but seems not to have had access to it<sup>12</sup>. For his discussion of the text, Thibaut used Dvārakānātha

<sup>&</sup>lt;sup>8</sup>See Minkowski 2001 and CEES, Vol 2: 21.

<sup>&</sup>lt;sup>9</sup>Thibaut 1899, Thibaut 1907.

<sup>&</sup>lt;sup>10</sup>Thibaut 1904.

<sup>&</sup>lt;sup>11</sup>Unless stated otherwise, all dates refer to the CESS. When no date is given, the CESS likewise gives no date.

<sup>&</sup>lt;sup>12</sup>For general comments on these texts, see Bag & Sen 1983, CESS, Vol 1: 50; Vol 2: 30; Vol 4: 252. For the portions of Dvārakānātha's and Venkateśvara's commentaries on Baudhāyana's treatise, see Delire 2002 (in French).

Yajvan's commentary<sup>13</sup> on the Baudhāyana  $sulbas\bar{u}tra$  and Rāma's (fl. 1447/1449) commentary on Kātyāyana's text. Thibaut also occasionally quotes Kapardisvāmin's (fl. before 1250) commentary of Āpastamba<sup>14</sup>. Thibaut's introductory study of these texts shows that he was familiar with the extant philological and historical literature on the subject of Indian mathematics and astronomy. However, Thibaut does not refer directly to any other scholars. The only work he acknowledges directly is A. C. Burnell's catalogue of manuscripts<sup>15</sup>. For instance, Thibaut quotes Colebrooke's translation of  $L\bar{u}l\bar{u}vat\bar{\iota}^{16}$  but does not refer to the work explicitly. Thibaut also reveals some general reading on the the history of mathematics. For example, he implicitly refers to a large history of attempts to square the circle, but Thibaut's sources are unknown.

His approach to the texts shows the importance he ascribed to acute philological studies<sup>17</sup>. Thibaut often emphasizes how important commentaries are for reading the treatises<sup>18</sup>:

the  $s\bar{u}tra$ -s themselves are of an enigmatical shortness (...) but the commentaries leave no doubt about the real meaning

The importance of the commentary is also underlined in his introduction of the  $Pa\tilde{n}casiddh\bar{a}nta^{19}$ :

 $<sup>^{13}</sup>$ Thibaut 1875: 3.

<sup>&</sup>lt;sup>14</sup>Thibaut 1877: 75.

<sup>&</sup>lt;sup>15</sup>Thibaut 1875: 3.

<sup>&</sup>lt;sup>16</sup>Thibaut 1875: 61.

 $<sup>^{17}</sup>$ See for instance Thibaut 1874: 75-76 and his long discussions on the translations of vrddha.

 $<sup>^{18}</sup>op.\ cit.: 18.$ 

<sup>&</sup>lt;sup>19</sup>Thibaut 1888: v.

Commentaries can be hardly done without in the case of any Sanskrit astronomical work...

However, Thibaut also remarks that because they were composed much later than the treatises, such commentaries should be taken with critical distance<sup>20</sup>:

Trustworthy guides as they are in the greater number of cases, their tendency of sacrificing geometrical constructions to numerical calculation, their excessive fondness, as it might be styled, of doing sums renders them sometimes entirely misleading.

Indeed, Thibaut illustrated some of the commentaries' 'mis-readings' and devoted an entire paragraph of his 1875 article to this topic. Thibaut explained that he had focussed on commentaries to read the treatises but disregarded what was evidently their own input into the texts. Thibaut's method of openly discarding the specific mathematical contents of commentaries is crucial here. Indeed, according to the best evidence, the tradition of 'discussions on the validity of procedures' appear in only the medieval and modern commentaries. True, the commentaries described mathematics of a period different than the texts upon which they commented. However, Thibaut valued his own reconstructions of the *śulbasūtra*s proofs more than the ones given by commentaries.

The quote given above shows how Thibaut implicitly values geometrical reasoning over arithmetical arguments, a fact to which we will return later.

<sup>&</sup>lt;sup>20</sup>Thibaut 1875: 61-62.

 $<sup>^{21}</sup>$ These are discussed, in a specific case, in the other article in this volume I have written, Keller same volume.

It is also possible that the omission of mathematical justifications from the narrative of the history of mathematics in India concerns not only the conception of what counts as proof but also concerns the conception of what counts as a mathematical text. For Thibaut, the only real mathematical text was the treatise, and consequently commentaries were read for clarification but not considered for the mathematics they put forward.

In contradiction to what has been underlined here, the same 1875 article sometimes included commentator's procedures, precisely because the method they give is 'purely geometrical and perfectly satisfactory'<sup>22</sup>. Thus there was a discrepancy in between Thibaut's statements concerning his methodology and his philological practice.

Thibaut's conception of the Sanskrit scholarly tradition and texts is also contradictory. He alternates between a vision of a homogenous and a-historical Indian society and culture and the subtleties demanded by the philological study of Sanskrit texts.

In 1884, as Principal of Benares Sanskrit College (a position to which he had been appointed in 1879), Thibaut entered a heated debate with Bapu Pramadadas Mitra, one of the Sanskrit tutors of the college, on the question of the methodology of scholarly Sanskrit pandits. Always respectful to the pandits who helped him in his work, Thibaut always mentioned their contributions in his publications. Nonetheless, Thibaut openly advocated a 'Europeanization' of Sanskrit Studies in Benares and sparked a controversy about the need for Pandits to learn English and history of linguistics and

<sup>&</sup>lt;sup>22</sup>This concludes a description of how to transform a square into a rectangle as described by Dvārakantha in Thibaut 1875: 27-28.

literature. Thibaut despaired of an absence of historical perspective in Pandits reasonings—an absence which led them often to be too reverent towards the past<sup>23</sup>. Indeed, he often criticized commentators for reading their own methods and practices into the text, regardless of the treatises' original intentions. His concern for history then ought to have led him lead to consider the different mathematical and astronomical texts as evidence of an evolution.

However, although he was a promoter of history, this did not prevent him from making his own sweeping generalizations on all the texts of the Hindu tradition in astronomy and mathematics. He writes in the introduction of the  $Pa\tilde{n}casiddh\bar{a}nta^{24}$ :

(...) these works [astronomical treatises by Brahmagupta and Bhāskarācarya]<sup>25</sup> claim for themselves direct or derived infallibility, propound their doctrines in a calmly dogmatic tone, and either pay no attention whatever to views diverging from their own or else refer to such only occasionally, and mostly in the tone of contemptuous depreciation.

Through his belief in a contemptuous arrogance on the part of the writers, Thibaut implicitly denies the treatises any claim for reasonable mathematical justifications, as we will see later. Thibaut attributed part of the clumsiness which he criticized to their old age<sup>26</sup>:

<sup>&</sup>lt;sup>23</sup>See Dalmia 1996: 328 sqq.

<sup>&</sup>lt;sup>24</sup>Thibaut 1888: vii. I am setting aside here the fact that he argues in this introduction for a Greek origin of Indian astronomy.

<sup>&</sup>lt;sup>25</sup>[] indicate the author's addenda for the sake of clarity.

<sup>&</sup>lt;sup>26</sup>Thibaut 1875: 60.

Besides the quaint and clumsy terminology often employed for the expression of very simple operations (...) is another proof for the high antiquity of these rules of the cord, and separates them by a wide gulf from the products of later Indian science with their abstract and refined terms.

After claiming that the treatises had a dogmatic nature, Thibaut extends this to the whole of "Hindu literature" <sup>27</sup>:

The astronomical writers (...) therein only exemplify a general mental tendency which displays itself in almost every department of Hindu Literature; but mere dogmatic assertion appears more than ordinarily misplaced in an exact science like astronomy...

Thibaut does not seem to struggle with definitions of science, mathematics or astronomy, nor does he does discuss his competency as a philologer in undertaking such a study. In fact, Thibaut clearly states that subtle philology is not required for mathematical texts. He thus writes at the beginning of the  $Pa\tilde{n}casiddh\bar{a}nta^{28}$ :

...texts of purely mathematical or astronomical contents may, without great disadvantages, be submitted to a much rougher and bolder treatment than texts of other kinds. What interests us in these works, is almost exclusively their matter, not either their general style or the particular words employed, and the peculiar

 $<sup>^{27}</sup>$ Thibaut 1888: vii.

<sup>&</sup>lt;sup>28</sup>Thibaut 1888: v.

nature of the subject often enables us to restore with nearly absolute certainty the general meaning of passages the single words of which are past trustworthy emendation.

This "rougher and bolder treatment" is evidence, for instance, in his philologically accurate but somewhat clumsy translation of technical vocabulary. He thus translates  $d\bar{\imath} r g hacatura s' ra$  (literally 'oblong quadrilateral') variously; it is at some times a 'rectangular oblong', and at others an 'oblong'. The expression 'rectangular oblong' is quite strange. Indeed, if the purpose is to underline the fact that it is elongated, then why repeat the idea? The first of Thibaut's translations seems to aim at expressing the fact that a  $d\bar{\imath} r g hacatura s' ra$  has right-angles, but the idea of orthogonality is never explicit in the Sanskrit works used here, or even in later literature. Thibaut's translation, then, is not literal but colored by his own idea of what a  $d\bar{\imath} r g hacatura s' ra$  is. Similarly, he calls the rules and verses of the treatises, the Sanskrit  $s\bar{\imath} t r as$ , 'proposition(s)', which gives a clue to what he expects of a scientific text, and thus also an inkling about what kind of scientific text he suspected spawned the  $sulbas\bar{\imath} t r as$ .

#### 1.2 Thibaut's Historiography of Science

For Thibaut, 'true science' did not have a practical bent. In this sense, the science embodied in the *śulbas*, which he considered motivated by a practical religious purpose, is 'primitive'<sup>30</sup>:

The way in which the  $s\bar{u}trak\bar{a}ra$ -s [those who compose treatises]

<sup>&</sup>lt;sup>29</sup>See for instance, Thibaut 1875: 6.

<sup>&</sup>lt;sup>30</sup>Thibaut 1875: 17.

found the cases enumerated above, must of course be imagined as a very primitive one. Nothing in the  $s\bar{u}tra$ -s [the aphorisms with which treatises are composed] would justify the assumption that they were expert in long calculations.'

However, he considered the knowledge worthwhile especially because it was geometrical<sup>31</sup>:

It certainly is a matter of some interest to see the old  $\bar{a}c\bar{a}rya$ -s [masters] attempting to solve this problem [squaring of the circle], which has since haunted so m[an]y unquiet minds. It is true the motives leading them to the investigation were vastly different from those of their followers in this arduous task. Theirs was not the disinterested love of research which distinguishes true science<sup>32</sup>, nor the inordinate craving of undisciplined minds for the solution of riddles which reason tells us cannot be solved; theirs was simply the earnest desire to render their sacrifice in all its particulars acceptable to the gods, and to deserve the boons which the gods confer in return upon the faithful and conscientious worshipper.'

#### Or again<sup>33</sup>:

...we must remember that they were interested in geometrical truths only as far as they were of practical use, and that they accordingly gave to them the most practical expression'

<sup>&</sup>lt;sup>31</sup>Thibaut 1875: 33.

<sup>&</sup>lt;sup>32</sup>Emphasis is mine.

<sup>&</sup>lt;sup>33</sup>Thibaut 1875: 9.

Conversely, the practical aspect of these primitive mathematics explains why the methods they used were geometrical<sup>34</sup>:

It is true that the exclusively practical purpose of the  $\acute{S}ulvas\bar{u}tra$ -s necessitated in some way the employment of practical, that means in this case, geometrical terms,...

This geometrical basis distinguished the  $\acute{sulbas\bar{u}tras}$  from medieval or classical Indian mathematical treatises. Once again, Thibaut took this occasion to show how his preference for geometry over arithmetic<sup>35</sup>:

Clumsy and ungainly as these old  $s\bar{u}tra$ -s undoubtedly are, they have at least the advantage of dealing with geometrical operations in really geometrical terms, and are in this point superior to the treatment of geometrical questions which we find in the  $L\bar{u}l\bar{u}vat\bar{u}$  and similar works.

As made clear from the above quotation, Thibaut was a presentist historian of science who possessed a set of criteria which enabled him to judge the contents and the form of ancient texts. In another striking instance, Thibaut gives us a clue that Euclid is one of his references. Commenting on rules to make a new square of which the

area is the sum or the difference of two known squares, Thibaut states in the middle of his own translation of Baudhāyana's  $\acute{sulbas\bar{u}tras}^{36}$ :

Concerning the methods, which the  $\acute{S}ulvas\bar{u}tras$  teach for caturas- $rasam\bar{a}sa$  (sum of squares) and  $caturas ranirh\bar{a}ra$  (subtraction of

<sup>&</sup>lt;sup>34</sup>Thibaut 1875: 61.

<sup>&</sup>lt;sup>35</sup>Thibaut 1875: 60.

<sup>&</sup>lt;sup>36</sup>Thibaut 1877: 76.

squares). I will only remark that they are perfectly legitimate; they are at the bottom the same which Euclid employs.

Contemptuous as he may be of the state of Indian mathematics, Thibaut did not believe that the  $\acute{sulbas\bar{u}tras}$  were influenced by Greek geometry<sup>37</sup>.

For Thibaut, history of mathematics ought to reconstruct the entire deductive process from the origin of an idea to the way it was justified. Although later commentaries may include some useful information, they do not give us the key to understanding how these ideas were developed at the time when the treatises were composed. This lack of information provoked Thibaut to complain about Indian astronomical and mathematical texts.

Thibaut clearly considered the texts to have been arranged haphazardly because the order of the rules do not obey generative logic. He thus defined his task<sup>38</sup>:

...I shall extract and fully explain the most important  $s\bar{u}tra$ -s (...) and so try to exhibit in some systematic order the knowledge embodied in these ancient sacrificial tracts

Here, Thibaut assumed that these works—not treatises but 'tracts' (presumably with derogatory connotations)—are not clear and systematic. Further, Thibaut felt the need to disentangle ('extract') the knowledge they contain.

In his view, this knowledge may be quite remarkable but it was ill presented. Thus commenting a couple years later on the  $Ved\bar{a}igajyotiṣa$ , he remarked<sup>39</sup>:

<sup>&</sup>lt;sup>37</sup>Thibaut 1875: 4. This however will still be discussed as late as Staal 1999.

<sup>&</sup>lt;sup>38</sup>Thibaut 1875: 5.

<sup>&</sup>lt;sup>39</sup>Thibaut 1877:411, emphasis is mine.

The first obstacle in our way is of course the style of the treatise itself with its enigmatical shortness of expression, its strange archaic forms and its utter want of connection between the single verses.

He thus sometimes remarked where the rules should have been placed according to his logic. All the various texts of the  $\acute{sulbasutra}$ s start by describing how to construct a square, particularly how to make a square from a rectangle.

However, Thibaut objected<sup>40</sup>:

their [the rules for making a square from a rectangle] right place is here, after the general propositions about the diagonal of squares and oblongs, upon which they are founded

Consequently, Thibaut considered the  $\acute{sulbas\bar{u}tras}$  as a single general body of text and selected the scattered pieces of the process he hoped to reconstruct from among all the  $s\bar{u}tras$  composed by various authors. At the same time, he distinguished the different authors of the  $\acute{sulbas\bar{u}tras}$  and repeatedly insisted that  $\bar{A}$ pastamba is more 'practical' than Baudhāyana, whom he preferred. For instance, an example of his method<sup>41</sup>:

Baudhāyana does not give the numbers expressing the length of the diagonals of his oblongs or the hypotenuses of the rectangular triangles, and I subjoin therefore some rules from Āpastamba, which supply this want, while they show at the same time the practical use, to which the knowledge embodied in Baudhāyana's  $s\bar{u}tra$  could be turned

<sup>&</sup>lt;sup>40</sup>Thibaut 1875: 28.

<sup>&</sup>lt;sup>41</sup>Thibaut 1875: 12.

When alternating among several authors was insufficient for his purposes, Thibaut supplied his own presuppositions.

Indeed, Thibaut peppered his text with such reconstructions:<sup>42</sup>:

The authors of the  $s\bar{u}tra$ -s do not give us any hint as to the way in which they found their proposition regarding the diagonal of a square; but we may suppose ...(...) The question arises: how did Baudhāyana or Āpastamba or whoever may have the merit of the first investigation, find this value? (...) I suppose that they arrived at their result by the following method which accounts for the exact degree of accuracy they reached.' (...) Baudhāyana does not state at the outset what the shape of his wheel will be, but from the result of his rules we may conclude his intention. etc.

Because he had an acute idea of what was logically necessary, Thibaut thus had a clear idea of what was sufficient and insufficient for reconstructing the processes. As a result, Thibaut did not deem the arithmetical reasoning of Dvārakānātha adequate evidence of mathematical reasoning.

The misunderstandings on which Thibaut's judgments rest are evident. For him, astronomical and mathematical texts should be constructed logically and clearly, with all propositions regularly demonstrated. This presumption compelled him to overlook what he surely must have known from his familiarity with Sanskrit scholarly texts: the elaborate character of a  $s\bar{u}tra$ -marked by the diverse readings that one can extract from it-enjoyed

<sup>&</sup>lt;sup>42</sup>Thibaut 1875: 11, 18, 49.

a long Sanskrit philological tradition. In other words, when a commentator extracts a new reading from one or several  $s\bar{u}tras$ , he demonstrates the fruit-fulness of the  $s\bar{u}tras$ . The commentator does not aim to retrieve a univocal singular meaning but on the contrary underline the multiple readings the  $s\bar{u}tra$  can generate. Additionally, as Thibaut rightly underlined, geometrical reasoning represented no special landmark of correctness in reasoning to medieval Indian authors.

Because of these expectations and misunderstandings Thibaut was enable to find the mathematical justifications that maybe were in these texts. Let us thus look more closely at the type of reconstruction which Thibaut employed, particularly in the case of proofs

## 2 Practices and Readings in the History of Science

It is telling that the word 'proof' is used more often by Thibaut in relation to philological reasonings than in relation to mathematics. Thus, as we have seen above, the word is used to indicate that the clumsiness of the vocabulary establishes the  $\acute{s}ulbas\bar{u}tras$ 's antiquity.

#### 2.1 No mathematical justifications in the $\acute{s}ulbas\bar{u}tras$

However, for Thibaut, Baudhyāna and probably other 'abstractly bent' treatise writers doubtlessly wanted to justify their procedures. More often than not, these authors did not disclose their modes of justification. Thus, when the authors are silent, Thibaut developed fictional historical procedures. For instance  $^{43}$ :

The authors of the  $s\bar{u}tra$  -s do not give us any hint as to the way in which they found their proposition regarding the diagonal of a square [e.g. the Pythagorean proposition in a square]; but we may suppose that they, too, were observant of the fact that the square on the diagonal is divided by its own diagonals into four triangles, one of which is equal to half the first square. This is at the same time an immediately convincing proof of the Pythagorean proposition as far as squares or equilateral rectangular triangles are concerned.' (...) But how did the  $s\bar{u}trak\bar{u}ra$ -s [composers of treatises] satisfy themselves of the general truth of their second proposition regarding the diagonal of rectangular oblongs? Here there was no such simple diagram as that which demonstrates the truth of the proposition regarding the diagonal of the square, and other means of proof had to be devised.

Thibaut thus implied that diagrams were used to 'show' the reasoning literally and thus 'prove' it. This method seems to hint that authors of the medieval period of Sanskrit mathematics could have had some sort of geometrical justification<sup>44</sup>. Concerning Āpastamba's methods of constructing

<sup>&</sup>lt;sup>43</sup>Thibaut 1875: 11-12.

 $<sup>^{44}</sup>$ See Keller 2005. Bhāskara's commentary on the  $\bar{A}ryabhat\bar{t}ya$  was not published during Thibaut's lifetime, but I sometimes suspect that either he or a pandit with whom he worked had read it. The discussion on visamacaturasra and samacaturasra, in Thibaut 1875: 10, thus echoes Bhāskara I's discussion on verse 3 of chapter 2 of the  $\bar{A}ryabhat\bar{t}ya$ . Thibaut's conception of geometrical proof is similar to Bhāskara's as well.

fire altars, which was based on known Pythagorean triplets, Thibaut stated<sup>45</sup>:

In this manner Āpastamba turns the Pythagorean triangles known to him to practical use (...) but after all Baudhāyana's way of mentioning these triangles as proving his proposition about the diagonal of an oblong is more judicious. It was no practical want which could have given the impulse to such a research [on how to measure and construct the sides and diagonals of rectangles]- for right angles could be drawn as soon as one of the *vijñeya* [determined] oblongs (for instance that of 3, 4, 5) was known- but the want of some mathematical justifications which might establish a firm conviction of the truth of the proposition.

So, in both cases, Thibaut represented the existence and knowledge of several Pythagorean triplets as the result of not having any mathematical justification for the Pythagorean Theorem. Thibaut proceeded to use this fact as a criterion by which to judge both Āpastamba's and Baudhāyana's use of Pythagorean triplets. Thibaut's search for an appropriate geometrical mathematical justification in the *śulbasūtras* may have made him overlook a striking phenomenon.

#### 2.2 Two different rules for a same result

Indeed, Thibaut underlined that several algorithms are occasionally given in order to obtain the same result. This redundancy puzzled him at times. For instance<sup>46</sup>, Thibaut examined the many various *caturaśrakarana*—methods to

<sup>&</sup>lt;sup>45</sup>Thibaut 1875: 17.

<sup>&</sup>lt;sup>46</sup>Thibaut 1875: 28-30.

construct a square—given by different authors. Āpastamba, Baudhāyana and Kātyāyana each gave two methods to accomplish this task. I will not expose these methods here; they have been explained amply and clearly elsewhere<sup>47</sup>. Thibaut also remarked that in some cases, Baudhāyana gives a rule and its reverse, although the reverse cannot be grounded in geometry. Such is the case with the procedure to turn a circle into a square<sup>48</sup>:

Considering this rule closer, we find that it is nothing but the reverse of the rule for turning a square into a circle. It is clear, however, that the steps taken according to this latter rule could not be traced back by means of a geometrical construction, for if we have a circle given to us, nothing indicates what part of the diameter is to be taken as the *atiśayaṭṛṭīya* (e.g. the segment of the diameter which is outside of the square)

I am no specialist of  $\acute{sulba}$  geometry and do not know if we should see the doubling of procedures and inverting of procedures as some sort of 'proofs', but at the very least they can be considered efforts to convince the reader that the procedures were correct. The necessity within the  $\acute{sulbas\bar{u}tras}$  to convinced and to verify has often been noted in the secondary literature, but has never fully or precisely studied<sup>49</sup>. Thibaut, although puzzled by the fact, never addressed this topic. Similarly, later historians of mathematics have noted that commentators of the  $\acute{sulbas\bar{u}tras}$  sought to verify the procedures while setting aside the idea of a regular demonstration in these texts. Thus

<sup>&</sup>lt;sup>47</sup>Thibaut 1875: 28-30, Datta 1993: 55-62, Bag & Sen 1983 and finally Delire 2002:75

<sup>&</sup>lt;sup>48</sup>Thibaut 1875: 35.

<sup>&</sup>lt;sup>49</sup>See for instance Datta 1993: 50-51.

Delire notes that Dvārakānātha used arithmetical computations as an easy method of verification (in this case of the Pythagorean Theorem)<sup>50</sup>. The use of two separate procedures to arrive at the same result, as argued in another article of this volume<sup>51</sup>, could have been a way of mathematically verifying the correctness of an algorithm—an interpretation that did not occur to Thibaut.

#### Conclusion

Thibaut, as we have thus seen, embodied contradictions. On the one hand, he swept aside the Sanskrit literary tradition and criticized its concise  $s\bar{u}tras$  as obscure, dogmatic and following no logic whatsoever. On the other hand, as an acute philologer, he produced nuanced studies on the differences among the approaches of different authors. Through his naive assumption of a practical mind of the 'Hindu astronomers', his fruitless search for proper visual demonstrations in an algorithmic tradition, and a disregard of commentaries in favor of the treatises, Thibaut envisioned a tradition of mathematics in India blind to the logic that could have been used to justify the algorithms which he studied. Such arguments could have been perceived through the case of the 'doubled' procedures in the  $\acute{sulbas\bar{u}tras}$ , and maybe even through the arithmetical readings of these geometrical texts found in later commentaries.

 $<sup>^{50}</sup>$ Delire op. cit. : 129.

<sup>&</sup>lt;sup>51</sup>See Keller same volume.

## **Bibliography**

CESS Pingree, D. (1970-1994) Census of the Exact Sciences in Sanskrit. Series A, Five volumes. Philadelphia.

Dalmia (1996) 'Sanskrit Scholars and Pandits of the old School: The Benares Sanskrit College and the Constitution of authority in the late nineteenth century', *Journal of Indian philosophy* 24(4): 321-337.

Datta, B. (1993) Ancient Hindu Geometry: The science of the sulba. New-Delhi.

Delire, J. M (2002) "Vers une édition critique des sulbadīpikā et sulbamīmāmsā commentaires du Baudhāyana sulbasūtra: Contribution à l'histoire des mathématiques Sanskrites", Ph.D. Diss., Université Libre de Bruxelles.

Hayashi, Takao (1995) The Bakhshālī Manuscript. An ancient Indian mathematical treatise. Groningen.

Keller, A. (2005) 'Making Diagrams Speak, in Bhāskara I's commentary on the  $\bar{A}ryabhat\bar{\imath}ya$ ',  $Historia\ Mathematica\ 32:\ 275-302.$ 

Keller, A. (same volume) 'Dispelling mathematical doubts, Assessing the mathematical corectness of algorithms in Bhāskara's commentary on the mathematical chapter of the  $\bar{A}ryabhat\bar{\imath}ya$ '.

Minkowski, C. (2001) 'The Pandit as public intellectual: the controversy over virodha or inconsistency in the astronomical sciences?, in *The Pandit*. *Traditional scholarship in India*, ed. A. Michaels. New Delhi: 79-96.

Raina, D. (1999) "Nationalism, Institutional Science and the Politics of Knowledge; Ancient Indian Astronomy and Mathematics in the Landscape of French Enlightenment Historiography", Ph.D. Diss., Goteborgs Universitet.

Srinivas, M. D. (1990) 'The Methodology of Indian Mathematics and its contemporary relevance', in *History of Science and Technology in India*, ed. S. Prakashan. Vol. II, Chap. 2. New Delhi.

Staal, F. (1999) 'Greek and Vedic geometry', Journal of Indian philosophy 27: 105-127.

Stache-Rosen (1990) German Indologists: Biographies of Scholars in Indian Studies Writing in German, with a summary of Indology in German. New-Delhi.

Thibaut, G. F. (1874). 'Baudhayānaśulbasūtra', *The Paṇḍit*. Reprinted and Re-edited in Chattopadhyaya, D. (1984) *Mathematics in the making in Ancient India*. Delhi. Pagings refer to this edition.

Thibaut, G. F. (1875) 'On the Sulvasutras', in Journal of the

Asiatic Society of Bengal. Reprinted and Re-edited in Chattopadhyaya, D. (1984) Mathematics in the making in Ancient India, Delhi. Pagings refer to this edition.

Thibaut, G. F. (1877a). 'Baudhayānaśulbasūtra', in *The Paṇḍit*. Reprinted and Re-edited in Chattopadhyaya, D. (1984) *Mathematics in the making in Ancient India*. Delhi. Pagings refer to this edition.

Thibaut, G. F. (1877b) 'Contributions to the explanation of the Jyotishavedānga', *Journal of the Asiatic Society of Bengal* 46: 411-437.

Thibaut, G. F. (1880) 'On the Sūryaprajapti', Journal of the Asiatic Society of Bengal 49: 107-127; 181-206.

Thibaut, G. F. (1882) 'Katyayana śulbapariśiṣṭa with the commentary of Ràma son of Sùryadasa', *The Paṇḍit* NS 4: 94-103; 328-339; 382-389; 487-491.

Thibaut, G. F. (1884) 'Notes from Varāha Mihira's Pachasiddhāntikā', Journal of the Asiatic Society of Bengal 53: 259-293.

Thibaut, G. F. (1885) 'The Number of Stars Constituting the Several Nakṣatras According to Brahmagupta and Vriddha-Garga', *Indian Antiquary* 14: 43-45.

Thibaut, G. F. & Dvivedi (1888) *Pañcasiddhāntika*, Varanasi: Chowkhamba Sanskrit Studies, Vol. LXVIII, Chowkhamba Sanskrit Series. Reprinted in 1968.

Thibaut, G. F. (1894) 'On the Hypothesis of the Babylonian Origin of the so-called Lunar Zodiac', *Journal of the Asiatic Society of Bengal* 63: 144-163.

Thibaut, G. F. (1895) 'On some recent attempts to Determine the Antiquity of Vedic Civilization', *Indian Antiquary* 24: 85-100.

Thibaut, G. F. (1899) Astronomie, Astrologie und Mathematik. Strasburg.

Thibaut, G. F. (1904) Vedānta sūtras with the commentary by Shankarācarya, Translated by George Thibaut. Delhi; Patna; Varanasi.

Thibaut, G. F. (1907) 'Indian Astronomy', *Indian Thought* 1: 81-96; 313-334; 422-433.