

# Overlooking mathematical justifications in the Sanskrit tradition: the nuanced case of G. F. Thibaut

Agathe Keller

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# Overlooking mathematical justifications in the Sanskrit tradition: the nuanced case of G. F. Thibaut

This is a Draft

#### Agathe Keller

#### Abstract

How did the narratives on the history of mathematics in India omit the tradition of mathematical justifications that existed in medieval Sanskrit commentaries? This article will look at a key moment of the history of mathematics in India, when in 1874 and 1875, the German philologer G. F. Thibaut published a translation of a set of Vedic geometrical texts, thus establishing that India had known other mathematical activities than "practical calculations". We will observe how his philological work and historiographical values influences his approach of these texts, providing a bias for his approach of the reasonings and efforts that may have existed to assert the validity of algorithms existing in this set of texts.

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### Introduction

Dhruv Raina has shown that the overlooking of the tradition of mathematical justifications existing in medieval Sanskrit mathematical commentaries springs partly from the fact that during most of the XIXth century, the English philologists and French mathematicians that looked at Indian astronomical and mathematical texts considered geometry to be the measure of mathematical activity. Computational reasonings and algorithmic verifications were considered as "practical" and devoid of the rigor and prestige of a real logical and geometrical demonstration. This article then looks at the moment when a German philologist, Georg Friedrich Wilhem Thibaut (1848-1914), published the oldest known mathematical texts in Sanskrit, which happens to be solely concerned with geometry. After having situated this author, we will look at the method he openly used to approach this corpus of texts. We will focus on his historiography of mathematics, especially on his perception of mathematical justificationss in this text.

## 1 Thibaut's intellectual background

G. F. Thibaut's approach of the *śulbasūtras* can be seen as a blend of what was 50 years before him two conflicting traditions, as described by D. Raina (and F. Charette?): that of an acute philologist and a somewhat dogmatic historian of science.

#### 1.1 A philologist

Thibaut's training was indeed that of a German Sanskritist<sup>1</sup>. Born in 1848 in Heidelberg, he studied Indology in Germany, his European career culminating when he left for England in 1870 to work as an assistant for Max Müeller's edition of the Vedas. In 1875 he is appointed Sanskrit professor at Benares Sanskrit College. This is the time when his edition and studies of the śulbasūtras (SS), which will be the focus of the present article, where published<sup>2</sup>. Consequently, Thibaut spent the following 20 years in India, teaching Sanskrit, publishing translations and editing numerous texts. He was responsable with P. Griffith of the Benares Sanskrit Series, from 1880 onwards. Specialized in the study of mimāmsa philosophy and sanskrit scholarly grammar, he made regular incursions in the history of mathematics and astronomy. His interest in this field can be at first understood by his interest in  $mim\bar{a}msa$ , since the authors commented upon the ancilliary parts of the Vedas  $(ved\bar{a}nqa)$  devoted to ritual. It is in this litterature, that the first texts of astronomy and mathematics in Sanskrit that have been handed down to us, have been found. Thibaut published several articles on vedic math-

<sup>&</sup>lt;sup>1</sup>The following paragraph rests mainly on [Stachen-Rose 1990].

<sup>&</sup>lt;sup>2</sup>See [Thibaut 1874], [Thibaut 1875], [Thibaut 1877].

ematics and astronomy, between 1875 and 1878<sup>3</sup>. The first volume of the Benares Sanskrit Series, of which Thibaut was the general scientific editor, was none but the Siddhāntatattvaviveka of Bhatta Kamalākara, an astronomical treatise written in the Wilkinson milieu, a Sanskrit apology of Western Astronomy and its relation to Puranic cosmology<sup>4</sup>. His next direct incursion in the history of mathematics and astronomy in India, will be on the medieval astronomical treatise of Varahāmihira, the Pañcasiddhānta (PS), which was edited and translated with S. Dvivedi in 1888. He will then heatedly debate with H. Jacobi on the latters attempt to date the Veda on the basis of descriptions of heavenly bodies in ancient texts. He will, at the end of his life, publish several synthetical texts on the subject of ancient Indian mathematics and astronomy<sup>5</sup>. His main *oeuvre*, will however not be in the field of history of science, but the three volume translation of one of the main Mimāmsa textes: Śaikarācārya's commentary on the Vedāntasūtras, published in the series initiated by his teacher Max Müeller, the Sacred Books of the East <sup>6</sup>. Thibaut died in Berlin at the beginning of the first world war, in october  $1914^7$ .

The *sulbasūtras* are a set of treatises treating of the geometry of the Vedic altar, composed by four different authors, the two most famous being Baudhāyaṇa and Āpastamba, which Thibaut focussed on, occasionally examining also Katyanyana's *śulbapariśiṣṭa*. He notes the existence of the

<sup>&</sup>lt;sup>3</sup>The last being a study of the *jyotisavedānga*, in [Thibaut 1878].

<sup>&</sup>lt;sup>4</sup>Studied in [Minkowski 2001].

<sup>&</sup>lt;sup>5</sup>[Thibaut 1899], [Thibaut 1907].

<sup>&</sup>lt;sup>6</sup>[Thibaut 1904].

<sup>&</sup>lt;sup>7</sup>See [Stachen-Rose 1990].

Manavasulbasūtra, but doesn't seem to have had any access to it. Written in the typical style of aphoristic sūtras and dated from 600 to 200 B. C. They are accompanied by a certain number of late commentaries. Thibaut used for his discussion Dvārakānatha Yajvan's commentary<sup>8</sup> on and Rāma's commentary to Katyayana's text. He also quotes occasionally Kapardisvāmin's commentary of Āpastamba<sup>9</sup>.

Thibaut's introductory study of these texts shows that he was familiar with the philological and historical literature that had been produced before him on the subject of Indian mathematics and astronomy. However, he doesn't make any direct reference to other scholars. The only author he acknowledges directly is A. C. Burnell's catalogue of manuscripts<sup>10</sup>. We can therefore find him quoting Colebrooke's translation of  $L\bar{\imath}l\bar{a}vat\bar{\imath}^{-1}$ , and silently refer to other publications as well. He thus seems to have read on the history of mathematics in Greece, or at least a general textbook on the subject, as his references that we will se below to Plato's meno indiquate. We do not know what were his sources. His approach of the texts shows the importance he gave to acute philological studies. For instance, he writes<sup>12</sup>:

"the want of complete manuscripts of this latter work (the  $Baudh\bar{a}yana$   $Kalpas\bar{u}tra$ ) prevents me from being positive on this point (that the  $Baudh\bar{a}yana\acute{s}ulvas\bar{u}tra$  belongs to the Kalpas $\bar{u}tra$  of the same author)

<sup>&</sup>lt;sup>8</sup>[Thibaut 1875, p. 3]

<sup>&</sup>lt;sup>9</sup>[Thibaut 1877, p. 75]

<sup>&</sup>lt;sup>10</sup>[Thibaut 1875 p. 3].

<sup>&</sup>lt;sup>11</sup>[Thibaut 1875 p. 61].

<sup>&</sup>lt;sup>12</sup>[Thibaut 1875 p. 4].

Most subsequent authors will not be as cautious as him, on this point.

Thibaut often comments on how important commentaries are to read the treatises<sup>13</sup>:

"the sūtra-s themselves are of an enigmatical shortness (...) but the commentaries leave no doubt about the real meaning"

The importance of the commentary is also underlined in his introduction of the  $Pa\tilde{n}casiddh\bar{a}nta^{14}$ :

"Commentaries can be hardly done without in the case of any Sanskrit astronomical work..."

But he also remarks that being composed much later than the treatises they should be taken with critical distance:<sup>15</sup>:

"Trustworthy guides as they are in the greater number of cases, their tendency of sacrificing geometrical constructions to numerical calculation, their excessive fondness, as it might be styled, of doing sums renders them sometimes entirely misleading."

Indeed, Thibaut will illustrate, by devoting to them an entire labelled paragraph, in his 1875 article some of what he considers the commentaries' misreadings. However, the same article will sometimes provide commentator's

 $<sup>^{13}{\</sup>rm op.}$  cit. p. 18

<sup>&</sup>lt;sup>14</sup>[Thibaut 1888, p. v].

<sup>&</sup>lt;sup>15</sup>[Thibaut 1875, p. 61-62].

procedures, precisely because the method is "purely geometrical and perfectly satisfactory" <sup>16</sup>. So that, and Thibaut does this continuously, there is a discrepancy in between Thibaut's essentializing statements, and what as a philologer he does. We also see him pulling the mathematics he is looking for towards "pure geometry", we will come back to this later. The disregard of commentaries is important since the tradition of "discussions on the validity of procedures", which sometimes have technical names <sup>17</sup> known so far in Sanskrit, are to be found in medieval commentaries. It is thus possible that the omission of mathematical justifications from the narrative on the history of mathematics in India, has not only to do with a conception of proofs, but also with that of what a mathematical text was, and consequently on how commentaries where read and considered. The comment above also shows implicitly how Thibaut values geometrical reasoning over arithmetics, a fact we will come back to later.

Thibaut's conception of the Sanskrit scholarly tradition and texts are those that were common at the time. He thus hesitates between a vision of Indian society and culture as homogenous and a-historical and the subtleties that a philological study of Sanskrit texts compels him to introduce. His value of history would naturally lead him to consider these texts within an evolution, without essentializing them. In 1884 as Principal of Benares Sanskrit College (he was appointed at this post in 1879) he entered a heated debat with Bapu Pramadadas Mitra, one of the Sanskrit tutors of the col-

<sup>&</sup>lt;sup>16</sup>This concludes a description of how to transform a square into a rectange as described by Dvarakantha in [Thibaut 1875, p. 27-28].

<sup>&</sup>lt;sup>17</sup>We will set aside, for now, the question of wether such reasonings can be given the name "proof" in a mathematical sense or not.

lege, on the question of the methodology of scholarly Sanskrit pandits. A respectful philologist to the pandits who helped him in his work- they were always mentioned in his publications- he nonetheless openly advocated a "Europeanization" of Sanskrit Studies in Benares, sparkling a controversy on the need for Pandits to learn English and history of linguistics and litterature. His main argument was the absence of historical perspective in Pandits thought, which led them often to be too reverent towards the past<sup>18</sup>. And indeed he will often criticize commentaries for reading into the text, their own ways of doing, irrespective of what was the treatises original intention. However a promoter of history, this did not prevent him from essentializing at the same time Indian astronomical litterature. He explains in the introduction of the PS that knowledge presented in Sanskrit works, especially astronomy, is always within a tradition, be it a revealed knowledge or based on previous works<sup>19</sup>:

"(...) these works claim for themselves direct or derived infallibility, propound their doctrines in a calmly dogmatic tone, and either pay no attention whatever to views diverging from their own or else refer to such only occasionally, and mostly in the tone of contemptuous depreciation."

This belief in the contemptuous arrogance of treatises implicitly denies it any claim for reasonable mathematical justifications, as we will see later. However, part of their criticized clumsiness is atributed to their old age<sup>20</sup>:

<sup>&</sup>lt;sup>18</sup>See [Dalmia 1996, p. 328 sqq].

<sup>&</sup>lt;sup>19</sup>[Thibaut 1888, p. vii].

<sup>&</sup>lt;sup>20</sup>[Thibaut 1875, p. 60]

"Besides the quaint and clumsy terminology often employed for the expression of very simple operations (...) is another proof for the high antiquity of these rules of the cord, and seperates them by a wide gulf from the products of later Indian science with their abstract and refined terms."

We can note that Thibaut does not seem to struggle over what defines science, mathematics or astronomy. He does not discuss either his legitimacy as a philologer to discuss such matters. This is a sign that the debates that had raged in the first half of the XIXth century seemed to have died out by the time the century came to a close. In fact, Thibaut clearly states that subtle philology is not required of mathematical texts. He thus writes at the beginning of the PS:

"texts of purely mathematical or astronomical contents may, without great disadvantages, be submitted to a much rougher and bolder treatment than texts of other kinds. What interests us in these works, is almost exclusively their matter, not either their general style or the particular words employed, and the peculiar nature of the subject often enables us to restore with nearly absolute certainty the general meaning of passages the single words of which are past trustworthy emendation." (PS, p.v)

This can be often seen for instance in his philological but somewhat callous translation of technical vocabulary. He thus translates variably  $d\bar{\imath}$  phacaturaśra, literally "oblong quadrilateral". Thus it is at times a "rectangular oblong". The expression "rectangular oblong" is quite strange. Indeed, if the idea is

Thibaut's translations aims at expressing the fact that a  $d\bar{\imath}$ rghacaturaśra has right-angles, but the idea of orthogonality is never explicit in the Sanskrit works used here, or even in the later litterature. At other times it is only an "oblong" <sup>21</sup>. Thibaut's translation then, is not litteral, but transmits partly his own idea of what a  $d\bar{\imath}$ rghacaturaśra is. Similarly, he calls the rules and verses of the treatises, the Sanskrit  $s\bar{\imath}$ tras, "proposition(s)", which gives a clue to what he expects of a scientific text, and thus towards what kind of scientific text he would like to pull the  $sulbas\bar{\imath}$ tras.

#### 1.2 Thibaut's Historiography of Science

For Thibaut, "true science" does not have a practical bent. In this sense, the science embodied in the *śulbas*, which he considers to be motivated by a practical religious purpose, is "primitive" <sup>22</sup>:

"The way in which the sūtraka-s found the cases enumerated above, must of course be imagined as a very primitive one. Nothing in the sūtra-s would justify the assumption that they were expert in long calculations."

However, he considers it worth while knowledge especially because it is geometrical<sup>23</sup>:

"It certainly is a matter of some interest to see the old ācārya-s attempting this problem (squaring of the circle), which has since

<sup>&</sup>lt;sup>21</sup>See for instance, [Thibaut 1875, p. 6].

<sup>&</sup>lt;sup>22</sup>[Thibaut 1875, p. 3].

<sup>&</sup>lt;sup>23</sup>[Thibaut 1875, p. 33].

haunted so m[an]y unquiet minds. It is true the motives leading them to the investigation were vastly different from those of their followers in this arduous task. Theirs was not the disinterested love of research which distinguishes true science<sup>24</sup>, nor the inordinate craving of undisciplined minds for the solution of riddles which reason tells us cannot be solved; theirs was simply the earnest desire to render their sacrifice in all its particulars acceptable to the gods, and to deserve the boons which the gods confer in return upon the faithful and conscientious worshipper."

The whole paragraph is typical of those parts of the text, that give us the impression that Thibaut had done some general reading on the history of mathematics. Or again<sup>25</sup>:

"...we must remember that they were interested in geometrical truths only as far as they were of practical use, and that they accordingly gave to them the most practical expression"

Reversely, as well, it was the practical aspect of these primitive mathematics that explains why the methods they used were geometrical<sup>26</sup>:

"It is true that the exclusively practical purpose of the śulvasūtras necessitated in some way the employment of practical, that means in this case, geometrical terms, ..."

<sup>&</sup>lt;sup>24</sup>Emphasis is mine.

<sup>&</sup>lt;sup>25</sup>[Thibaut 1875, p. 9].

<sup>&</sup>lt;sup>26</sup>[Thibaut 1875, p. 61].

This distinguishes the SS from medieval or classical Indian mathematical treatises, which is an occasion for Thibaut to once again show how he privileges geometry over arithmetic<sup>27</sup>:

"Clumsy and ungainly as these old sūtra-s undoubtedly are, they have at least the advantage of dealing with geometrical operations in really geometrical terms, and are in this point superior to the treatment of geometrical questions which we find in the Līlāvatī and similar works".

As made clear from the above quotation, Thibaut is a presentist historian of science who possesses a set of criteria enabling him to judge the contents and the form of ancient texts. Here is another striking instance, which gives us a clue that Euclide is one of his references. Commenting on rules to make a new square the area of which is the sum or the difference of two known square he states, in the middle of his own translation of Baudhāyana's SS<sup>28</sup>:

Concerning the methods, which the  $\acute{S}ulvas\bar{u}tras$  teach for caturas- $rasam\bar{a}sa$  (sum of squares) and  $caturas ranirh\bar{a}ra$  (subtraction of squares). I will only remark that they are perfectly legitimate; they are at the bottom the same which Euclid employs.

He also possesses an idea of what should be the form of a mathematical text. We can note that contemptuous as he may be, he does not believe that the SS were influenced by Greek geometry<sup>29</sup>.

<sup>&</sup>lt;sup>27</sup>[Thibaut 1875, p. 60].

<sup>&</sup>lt;sup>28</sup>[Thibaut 1877, p. 76].

<sup>&</sup>lt;sup>29</sup>[Thibaut 1875, p. 4]. This however will still be discussed as late as [] with the subsequent reactions the article provoked.

For Thibaut history of mathematics should reconstruct then the whole process from the origin of an idea to the way it was justified. This leads him to complain about Indian astronomical and mathematical texts, which do not give such information.

He clearly considers the texts as arranged haphazardly because the rules do not obey this generative logic. He thus defines his task<sup>30</sup>:

"I shall extract and fully explain the most important sūtra-s (...) and so try to exhibit in some systematic order the knowledge embodied in these ancient sacrificial tracts"

The assumption is that these works, not treatises but "tracts" (there must be a deregoratory idea implied), are not clear and systematic and that Thibaut needs to be here to disentangle ("extract") the knowledge they contain.

This knowledge may be quite remarkable but it is ill presented. Thus commenting a couple years later on the  $Ved\bar{a}igajyotisa$ , he remarks<sup>31</sup>:

The first obstacle in our way is of course the style of the treatise itself with its enigmatical shortness of expression, its strange archaic forms and its utter want of connexion between the single verses.

He thus sometimes remarks where the rules should according to his logic be placed. All the different texts of the SS start by describing how to construct

<sup>&</sup>lt;sup>30</sup>[Thibaut 1875, p. 5].

<sup>&</sup>lt;sup>31</sup>[Thibaut 1877, p.411], emphasis is mine.

a square, and particularly how to make a square from a rectangle. However, for Thibaut, this isn't right<sup>32</sup>:

"their (the rules for making a square from a rectangle) right place is here, after the general propositions about the diagonal of squares and oblongs, upon which they are founded"

Consequently, Thibaut considers the śulbas a general body of text, and uses all the authors as ways of completing the parts of a process he wants to reconstruct. But at the same time he distinguishes the different author's of the śulba's and repeatedly insists that Āpastamba is more "practical" than Baudhāyana, which he prefers. For instance, an example of his method<sup>33</sup>:

"Baudhāyana does not give the numbers expressing the length of the diagonals of his oblongs or the hypotenuses of the rectangular triangles, and I subjoin therefore some rules from Āpastamba, which supply this want, while they show at the same time the practical use, to which the knowledge embodied in Baudhāyana's sūtra could be turned"

When using several authors is not sufficient he will supply his own presuppositions.

The whole of Thibaut's text is indeed full of reconstructions:<sup>34</sup>:

"The author of the sūtra -s do not give us any hint as to the way in which they found their proposition regarding the diagonal of a

<sup>&</sup>lt;sup>32</sup>[Thibaut 1875, p. 28].

<sup>&</sup>lt;sup>33</sup>[Thibaut 1875, p. 12].

<sup>&</sup>lt;sup>34</sup>[Thibaut 1875, p.11, 18, 49].

square; but we may suppose ..." (...) "The question arises: how did Baudhāyana or Āpastamba or whoever may have the merit of the first investigation, find this value? (...) I suppose that they arrived at their result by the following method which accounts for the exact degree of accuracy they reached." (...) "Baudhāyana does not state at the outset what the shape of his wheel will be, but from the result of his rules we may conclude his intention." etc.

Because he has an acute idea of what was logical to be done, he thus has an idea of what is adequate or not when reconstructing processes. As we have noted above, the arithmetics of Dvakaranatha, are not part of what is proper.

In fact, reading Thibaut, we cannot but notice a misunderstanding due to different conceptions of what an astronomical treatise should be. Because for Thibaut, such texts should be constructed logically and clearly, with propositioons regularly demonstrated, he thus seems o overlook what he surely must have known from his familiarity with Sanskrit scholarly texts, that is that a long sanskrit philological tradition explains that the elaborate character of a  $s\bar{u}tra$  is shown precisely by the diverse readings that one can extract from it. In other words, when a commentary extracts a new reading of the text, he is showing the fruitfullness of the rule; its aim is not to retrieve a univoqual singular meaning, but on the contrary underline the multiple reading it can be submitted to, implicitly considering that it belongs to the commentator. These expectations and misunderstandings may in fact have something to do with the fact that Thibaut finds no mathematical justificationss in theses

texts. Let us thus look more closely at the type of reconstruction, Thibaut can proceeds to. Focussing on the cases of proofs

# 2 Practices and Readings in the History of Science

Thibaut's use of the word "proof" is, relevantly, more concerned with philology than mathematics. Thus, as we have seen above, the word is used to say that the clumsiness of the vocabulary establishes the SS's antiquity.

#### 2.1 No mathematical justifications in the śulbasūtras

However, for Thibaut there is no doubt that Baudhyāna and probably other "abstractly bent" treatise writers wanted to justify their procedures. More often than not the modes of justification are not known, this is why he develops fictions historical procedures. For instance<sup>35</sup>:

"The author of the sūtra -s do not give us any hint as to the way in which they found their proposition regarding the diagonal of a square (e.g. the Pythagorean proposition in a square); but we may suppose that they, too, were observant of the fact that the square on the diagonal is divided by its own diagonals into four triangles, one of which is equal to half the first square. This is at the same time an immediately convincing mathematical justifications of the Pythagorean proposition as far as squares or equilat-

 $<sup>^{35}</sup>$ [Thibaut 1875, p. 11-12].

eral rectangular triangles are concerned." (...) But how did the sūtrakāra-s satisfy themselves of the general truth of their second proposition regarding the diagonal of rectangular oblongs? Here there was no such simple diagram as that which demonstrates the truth of the proposition regarding the diagonal of the square, and other means of mathematical justifications had to be devised."

Thibaut thus implicitely believes that diagrams were used to "show", litterally, the reasoning and thus "proove" it. This indeed seems to be the idea that authors of the medieval period of Sanskrit mathematics could have had on what a geometrical justification consisted of<sup>36</sup>. Thibaut then expresses, concerning Āpastamba's methods to construct fire altars<sup>37</sup>:

"In this manner Āpastamba turns the Pythagorean triangles known to him to practical use (...) but after all Baudhāyana's way of mentioning these triangles as proving his proposition about the diagonal of an oblong is more judicious. It was no practical want which could have given the impulse to such a research - for right angles could be drawn as soon as one of the *vijñeya* (e.g. determined) oblongs (for instance that of 3, 4, 5) was knownbut the want of some mathematical justifications which might establish a firm conviction of the truth of the proposition."

<sup>&</sup>lt;sup>36</sup>[Keller 2005]. Bhāskara's commentary on the Āryabhaṭīya was not yet published at the time, but I sometimes had the feeling that he had read it, or worked with a pandit who had read it. The discussion on viṣamacaturaśra and samacaturaśra, in [Thibaut 1875, p. 10], thus echoes Bhāskara I's discussion on verse 3 of chapter 2 of the Āryabhaṭīya.
<sup>37</sup>[Thibaut 1875, p. 17].

So in both cases the existence and knowledge of several Pythagorean triplets is seen as the want of mathematical justifications of the "Pythagorean proposition", and this gives him criteria to judge both Āpastamba's and Baudhāyana's use of them. These statements show the tension and contradictions of Thibaut's approach of the texts. His search of an appropriate geometrical mathematical justifications in the *śulbasūtra*s may have made him overlook a striking phenomenon.

#### 2.2 Two different rules for a same result

Indeed, Thibaut underlines how several algorithms are given to obtain a same result. This puzzles him at time. For instance<sup>38</sup>, Thibaut examines the many different methods given by different authors to construct a square, the *caturaśrakaraṇa*. Each author, Apastamba, Baudhāyana and Kātyāyana give two methods to accomplish this task. I will not expose these methods here, they have been explained amply nd clearly elsewhere<sup>39</sup>. But they may have been maybe not so much as "mathematical justificationss" as "modes to verify that know mistake has been made precedently". They have to do with asserting that the correct result has been obtained. It is striking that in one case the rule is given in general terms, and that in the other, it is usually given with specific values applied to it. each time, one method requires few steps, while the other seems more complicated. In the case of the complex construction described by Baudhāyana and reproduced in Figure, most probably the method, althought not simple, concinces one litterally that

<sup>&</sup>lt;sup>38</sup>[Thibaut 1875, p. 28-30].

 $<sup>^{39}[\</sup>mbox{Thibaut 1875, p. 28-30}],$  [Datta, p. ], [Bag, p.] and finally [Delire, p. ]

a square is being constructed. Indeed the succession of circles that are traced, all have the same diameter's length, that of the square constructed. It further rests upon the simple property of the orthogonality of triangles constructed on the diameter of a circle. In the second rule, who's mathematical grounding isn't as apparant as the first, and rests upon the pythagoran theorem, the construction is described with the help of a specific measure (a square of 12 aigulas long).

Apastamba states the rule given by Baudhayāna in a particular case, in general terms, and then illustrates it with a particular number. It is when illustrating it, that we understand how it works? He gives a second method in general terms.

Thibaut also remarks that in some cases, Baudhāyana gives a rule and its reverse, althought the reverse cannot be grounded in geometry. This is the case with the procedure to turn a circle into a square<sup>40</sup>:

Considering this rule closer, we find that it is nothing but the reverse of the rule for turning a square into a circle. It is clear, however, that the steps taken according to this latter rule could not be traced back by means of a geometrical construction, for if we have a circle given to us, nothing indicates what part of the diameter is to be taken as the *atiśayaṭṛṭīya* (e.g. the segment of the diameter which is outised of the square)

I am no specialist of Sulbageometry, and do not know if we should see these doubling of procedures and inverting of procedures, as sorts of "proofs", at least as efforts to convince one that the procedures were correct.

<sup>&</sup>lt;sup>40</sup>[Thibaut 1875, p. 35]

As argued in another article of this volume, this would precisely have been a way of verifying the correctness of an algorithm.

# Conclusion

The assumption of the practical mind of the "Hindu astronomers", the search for proper visual demonstrations in what was an algorithmic tradition, and finally a disregard of commentaries in favor of the treatises all have contributed to a narration of the tradition of mathematics in India devoid of modes of mathematical justifications.

# **Bibliography**

- Dalmia1996 . "Sanskrit Scholars and Pandits of the old School: The Benares Sanskrit College and the Constitution of authority in the late nineteenth century." in *Journal of Indian philosophy* 24(4): 321-337.
- Keller 2005 "Making Diagrams Speak, in Bhāskara I's commentary on the Āryabhaṭīya." Historia Mathematica 32: 275-302.
- Stache-Rosen 1990 German Indologists: Biographies of Scholars in Indian Studies Writing in German, with a summary of Indology in German, Max Mueller Bhavan, New-Delhi.
  - Thibaut 1874 . "Baudhayānaśulbasūtra." in *The Paṇḍit*. Reprinted and Re-edited by D. Chattopadhyaya in *Mathematics in the making in Ancient India*, K.P. Bagchi & Co, Delhi, 1984. Pagings refer to this edition.

- Thibaut 1875 "On the Sulvasutras." in *Journal of the Asiatic Society of Bengal*.

  Reprinted and Re-edited by D. Chattopadhyaya in *Mathematics in the making in Ancient India*, K.P. Bagchi & Co, Delhi, 1984. Pagings refer to this edition.
- Thibaut 1877. "Baudhayānaśulbasūtra." in The Paṇḍit. Reprinted and Re-edited by
  D. Chattopadhyaya. in Mathematics in the making in Ancient India,
  K.P. Bagchi & Co, Delhi, 1984. Pagings refer to this edition.
- Thibaut 1877 "Contributions to the explanation of the Jyotishavedānga". Journal of the Asiatic Society of Bengal 46: 411-437.
- Thibaut 1880 "On the Sūryaprajapti". Journal of the Asiatic Society of Bengal 49: 107-127; 181-206.
- Thibaut 1882 "Katyayana śulbapariśiṣṭa with the commentary of Ràma son of Sùryadasa".

  The Paṇḍit NS 4: 94-103; 328-339; 382-389; 487-491.
- Thibaut 1884 "Notes from Varāha Mihira's Pachasiddhāntikā". *Journal of the Asiatic Society of Bengal* 53: 259-293.
- Thibaut 1885 "The Number of Stars Constituting the Several Nakṣatras According to Brahmagupta and Vriddha-Garga." em Indian Antiquary 14: 43-45.
- Thibaut & Dvivedi 1888 *Pañcasiddhāntika*, In Chowkhamba Sanskrit Studies, Vol. LXVIII Chowkhamba Sanskrit Series Office, Varanasi. Reprinted in 1968.
  - Thibaut 1894 On the Hypthesis of the Babylonian Origin of the so-called Lunar Zodiac. Journal of the Asiatic Society of Bengal 63: 144-163.

- Thibaut 1895 On some recent attempts to Determine the Antiquity of Vedic Civilization. Indian Antiquary 24: 85-100.
- Thibaut 1899 Astronomie, Astrologie une Mathematik. Strasburg, Trbner, K. J.
- Thibaut 1904 Vedānta sūtras with the commentary by Shankarācarya, Tranlated by Goerge Thibaut. Delhi; Patna; Varanasi.
- Thibaut 1907 Indian Astronomy. Indian Thought 1: 81-96; 313-334; 422-433.