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The Exact Insensitivity of Market Budget Shares and the "Balancing Effect"*

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Abstract.— We reformulate Grandmont's and its successors' notion of behavioral heterogeneity such as to get the exact insensitivity of the aggregate budget share function with respect to changes in prices and income, instead of a mere approximate insensitivity. We propose a non parametric set-up such that, if the population is distributed according to some "uniform" probability measure, the aggregate budget share function is constant. The important contribution is that this exact insensitivity is not explained by any insensitivity at the microeconomic level but rather by an exact "balancing effect". We give illustrative examples of populations that fulfill our requirements.

KEYWORDS: aggregation of demand, behavioral heterogeneity, balancing effect, large economy, Law of Demand.

Résumé.— Ce papier propose une reformulation de la notion d'hétérogénéité de comportement forgée par Grandmont et ses successeurs, de manière à obtenir l'insensibilité exacte de la fonction de part de budget agrégée à l'égard des modifications de prix et de revenu, au lieu d'une simple insensibilité approchée. Pour cela, nous proposons un paradigme non-paramétrique tel que, si la population est distribuée selon une probabilité uniforme, la fonction de part de budget agrégée est constante. La contribution de cette approche est qu'elle n'explique pas l'insensibilité macro'-économique par une quelconque homogénéité micro-économique, mais plutôt par un effet de « balancement » exact. Des exemples illustrent notre reformulation.

Mots-clefs: Agrégation de la demande, hétérogénéité, effet de balancement, économie non-atomique, loi de la demande.

JEL CLASSIFICATION NUMBERS: D11, D12, D30, D41, D50, E1

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1. Introduction

These last two decades many authors have pointed out that the way to develop appropriate micro-foundations for macroeconomics is not to be found from the study of individuals in isolation but rests in an essential way on studying the aggregate activity resulting from the interaction between different individuals. Therefore, an alternative approach to the neoclassical paradigm based on individual rationality has been developed. The fundamentals of the models remain individual behaviors described by individual characteristics. However, the assumptions formalized at the individual level are very weak so that individual rationality might even be redundant. The major assumptions rely on the whole population and, more precisely, on the distribution of agents' characteristics. The purpose of this paper is to apply this approach that we call the "statistical approach".[1] to demand theory[2] For a complete survey on the statistical approach see [24].

Consider a market consisting of households with different demand functions. The typical question in aggregation theory is to look for the conditions which guarantee that the market demand that describes the behavior of this population takes on a particular property. There are two mutually non-exclusive approaches to problems of this sort.

The traditional microeconomic approach is simply to ask for the conditions on individual behavior which guarantee that the aggregate property holds. Nevertheless, in demand theory all the properties induced by individual rationality are not aggregable. In particular, the weak axiom of revealed preferences does not hold in the aggregate even when all households in the population satisfy this property. In order to explain the weak axiom in the aggregate, one should introduce a stronger property at the household level, such as the Law of Demand. To conclude, the pure microeconomic approach to aggregation problems is distinguished by three features: First, the property required at the individual level is at least as strong as the aggregate property required. Second, once it is satisfied and therefore the support of the distribution is restricted, the precise shape of the distribution of individual characteristics on this support does not matter. Finally, the aggregate property is sensitive to small perturbations of this support.

This motivates the statistical approach where these features are reversed. In this alternative approach, the assumptions made at the individual level are weaker than, or at least different from, the aggregate property required. The aggregate property arises because of the distributional assumptions imposed and it is, in addition, robust to small perturbations of this distribution.

Examples of the statistical approach abound. An application of Lyapunov's theorem guarantees that when the measure space of agents is atomless, the aggregate demand correspondence is convex valued, even when each agent may not have a convex valued demand correspondence (see [14]). Beyond this, there is a substantial literature where the continuity and smoothness of market demand is obtained when these properties, due to non-convexities, do not hold at the household level (see, for example, [8]). Another well known example of the statistical approach is the result due to [15]. It says that a market in which all households have the same demand function which obeys the weak axiom will satisfy a stronger property – the Law of Demand – provided the income distribution has a downward sloping density function[3].

[1] This terminology was introduced by [21].

[2] This approach has also been applied to other economic models, see, for example, [22] for an application to an intertemporal framework in the overlapping generations model of a pure exchange economy, or [5] for an application to financial asset economies with heterogeneous beliefs.

[3] Notice that [6] and more recently [25] and [29] obtained the Law of Demand for more general income distributions. However, this was done at the cost of additional requirements on individual behaviors (or on the aggregate substitution effect matrix). Hildenbrand subsequently showed in [16] that an assumption over the distribution of individual demand vectors ensures the positive semi-definiteness in the aggregate of the income effect matrix.

The purpose of this paper is to extend the family of models first studied by [10]. Grandmont considers a population consisting of agents with the same income and different demand functions generated by a given demand function through the class of affine transformations. Affine transformations are defined in section 3; what should be noted here is that an agent and its transformed react differently to price changes. Assuming that there are l commodities in the economy, demand functions can then be parametrized by elements in $\mathbb{R}^l$. When the density function on the parameter is sufficiently flat in some precise sense, market budget share of each commodity becomes increasingly insensitive to changes in prices. Note that, under the additional assumption that households are not victims of money illusion, this property implies, in addition, that the market budget share of each commodity is approximately insensitive to changes in income.

Kneip extended this formalism in [20] to a non parametric set-up where the class of affine transformations is slightly wider since it implies that an agent and its transformed react distinctly to changes in prices and/or income. He proves that when the population is described by a probability measure close to the invariant measure (with respect to the group of these affine transformations), the market budget share of each commodity is approximately insensitive to changes in prices and/or income. Note that the latter property does not require any longer the absence of money illusion at the individual level.

An issue which has attracted some debate recently is the precise nature of behavioral heterogeneity at work in this family of models. First, the approximate insensitivity of the market budget shares might follow from an almost insensitivity at the individual level. The intuition is the following. Consider a population represented by a probability distribution of parameters α over $\mathbb{R}_+^{L+1}$. Denote by $\bar{w}$ the budget share function generating the whole population through the class of affine transformations.[4]

Let us assume, as commonly done in demand theory, that the generator $\bar{w}$ is almost insensitive to changes in prices and/or income outside a compact set K of the price-income vector.[5] By construction, any budget share function, w^α, in the population is almost insensitive to changes in prices and/or income outside the compact set K^α, which is the image of K through the translation of α in $\mathbb{R}_+^{L+1}$. Hence, different agents are sensitive to price-income changes at different parts of the price-income space. In addition, as the probability distribution of the parameters α becomes more flat (in the sense that all compact subsets of equal size in $\mathbb{R}_+^{L+1}$ tend to have the same weight in the population), at any single price-income vector, the weight of the population sensitive to changes in prices and/or income tends towards zero. This implies that at each price-income level almost all households are insensitive to changes in prices and/or income. This type of behavioral heterogeneity has been formalized for a finite population by [17].[6] These authors show that Grandmont's model, with some additional assumptions, can effectively be understood as an example of such a behavioral heterogeneity.[7]

This is a legitimate view of the type of behavioral heterogeneity that might induces the insensitivity

The Law of Demand follows then from the Slutsky decomposition of the Jacobian matrix of market demand. In this approach, individual rationality was still required to give an account of the negative semi-definiteness in the aggregate of the substitution effect matrix.

[4]More precisely, the behavior of household α is described by

$w^\alpha(p,x) = \bar{w}(e^\alpha \otimes (p,x)), \forall (p,x)$.

[5]Note that this property holds, in particular, for the class of CES budget share functions. Indeed, it holds for any function $\bar{w}$ with the property that $\lim_{(p,x)\to(\bar{p},\bar{x})} w(p,x)$ exist for any $(\bar{p},\bar{x})$ such that ($\bar{x}=0$ or $\bar{x}=+\infty$) and/or ($\bar{p}_l = 0$ or $\bar{p}_l = +\infty$) for at least one commodity $l \in \{1,2,\ldots,L\}$.

[6]See also [2] and [13] for populations described by a continuum of households.

[7]The crucial assumption in Hildenbrand and Kneip's interpretation of Grandmont's result is that the expenditure share function generating the affine class has, in certain sense, finitely many turning points ([17, Assumption 3]). Note that this assumption is fulfilled when the generator is almost insensitive to changes in prices and/or income outside a compact subset of the price-income space.

property in the aggregate, but it is not the only possible view. A second possible view, where the statistical approach is more fully used, is that the insensitivity property in the aggregate is induced by a "balancing effect" (The term is borrowed from [23]). In other words, the mean budget share for each commodity is approximately constant because, while some agents may increase their budget share as prices and/or income change, this is balanced by other agent who choose to reduce their budget share. Note that a very similar type of behavioral heterogeneity was already present in [12, p.64] to give an account of the Law of Demand. Hicks underlines that the property emerges in aggregate for the excess demand function if income effects cancel out when aggregating over buyers and sellers.

The purpose of this paper is to offer the most general analytical framework that formalizes this second type of behavioral heterogeneity. For this purpose we shall introduce a broader class of transformations over the set of individual budget share functions than the class of affine transformations.[8] In this formalism the type of behavioral heterogeneity introduced depends both on the generator and on the class of transformations applied to this generator to construct the population of households. Note that assumptions introduced on the generator will specify the individual rationality introduced in the model while restrictions on the class of transformations specify in which way households react differently to changes in prices and income.

In section 2, we first recall the economic applications of two alternate aggregate properties ; the insensitivity of market budget shares to changes in prices and their insensitivity to changes in income. We then give a careful examination of the mathematical feature which ensures that there exists a probability distribution over the set of feasible budget share functions such that these properties hold in aggregate while they never hold at the microeconomic level. The contribution is twofold. First, the insensitivity property is an *exact* insensitivity rather than a mere approximate insensitivity. Second, in contrast to [10], this exact insensitivity is necessarily induced by an exact balancing effect. More precisely, we identify the assumptions which guarantee that the distribution of demand is such that the market budget share on each commodity is independent of prices. This result is therefore an exact version of Grandmont's theorem. We also identify distributional conditions under which market demand becomes exactly linear in income and the aggregate of income effects is exactly positive semi-definite. These two results give an exact version of Kneip's theorems. In addition, they emerge while they never hold at the household level. Section 2 also points out that an approximate insensitivity of market budget shares with respect to prices and/or income is still obtained for a probability distribution sufficiently close to the limit. The interesting feature of this approximate result is that it is established for a finite population rather than a theoretical atomless population. Note that this approximate insensitivity is sufficient to get relevant properties of maket demand such as the Law of Demand.

Section 3 gives a much simpler proof of our results in the case of affine transformations. With this type of heterogeneous reactions to changes in prices and/or income we prove the intuition given previously that to generate the balancing effect one has to restrict the behavior of individual budget shares on the hedge of the price-income space. More precisely, individual budget shares must have no limit when the price-income vector converges to the hedge of the price-income space. Nevertheless, this first restriction at the microeconomic level is not sufficient to ensure that the insensitivity in the aggregate is induced by the "balancing" effect. We offer such a sufficient condition that requires at the microeconomic level a periodicity property of the budget share function. Section 3 also includes a brief discussion of the relationship between this paper and the exact aggregation results in [9]. We point out that these authors prove the existence of a probability measure such that market budget shares of the corresponding population are insensitive to changes in prices and/or income. Nevertheless, since they

[8] A first step in this direction was made by Maret in [23].

do not introduce any periodicity requirement of the individual budget share function nothing ensures in their framework that this insensitivity property emerges in the aggregate from an exact balancing effect. In other words, the exact insensitivity in the aggregate might follow from the exact insensitivity of any household. In this sense, their model is not an illustration of the statistical approach adopted in our set-up.

In section 4, we give examples of populations fulfilling our assumptions and discuss their economic interpretation. In two first examples the population is built following [10] and its successors by using the class of affine transformations. In a third example the population is built by introducing a new class of transformations defined through rotations of the price-income vector (rather than the class of affine transformations). With this new type of heterogeneous reactions to changes in prices and/or income we prove that the balancing effect is generated with individual rationality essentially restricted to the absence of money illusion. In particular, the periodicity requirement that has to be introduced in the case of affine transformations is no longer needed.

As soon as one departs from the class of affine transformations, the problem that emerges is that the class of transformations used to generate the set of feasible budget share functions does not necessarily preserve the weak axiom of revealed preferences (WARP). However, when the household behavior is arbitrary, Becker has already offered in [1] a solution to give an account of the insensitivity property in the aggregate by the balancing effect. In addition, Becker's formalism does not require to introduce any heavy formalism such as a class of transformations over the set of feasible budget share functions. Nevertheless, Becker's solution cannot be applied if household behaviors are not arbitrary. The motivation of our paper is that non arbitrary household behaviors is a more reasonable assumption. To be more precise, the collective approach (see for example [7]) has pointed out that if the WARP is not well supported empirically at the household level (when the household includes more than one member), other properties of the budget share function will hold. This legitimizes to consider a compact set of budget share functions that do not satisfy the WARP but are not arbitrary in contrast to [1]. Given any such compact set, we are able to explain the insensitivity properties of the mean budget share function by the "balancing effect".

In [19], the celebrated example of [1] is formalized by a "uniform" distribution over the space of individual characteristics which induces the insensitivity of the aggregate budget share function. The mathematical structure of this example is essentially the following: Consider the set of budget share functions $\mathcal{W} := \Sigma^{\mathbb{R}_{++}^{(L+1)}}$ as an uncountable product over the unit-simplex $\Sigma := \{x \in \mathbb{R}_+^L \ : \ \sum_i x_i = 1\}$. Equip this space with the product topology. Thanks to Tychonov's theorem, $\mathcal{W}$ is compact. Kolmogorov's extension theorem insures furthermore that the infinite product of the Lebesgue measure $\lambda^{\mathbb{R}_{++}^{(L+1)}}$ is well-defined. John [19] proves that a population whose budget share functions are distributed according to the "uniform" probability $\lambda^{\mathbb{R}_{++}^{(L+1)}}$ has a market demand of the symmetric Cobb-Douglas type. The strength of this example is that, like in the general theory developed in this paper, no continuity assumption on the budget share functions is required. However, since $\mathbb{R}_{++}^{(L+1)}$ is non-countable, the product topology is not metrizable, so our result are not valid in this setting. In particular, nothing ensures in John's framework that the exact insensitivity of the market budget shares is induced by an exact balancing effect.

In a somewhat similar context, [8][9] prove that aggregation has a smoothing effect on the demand behavior in a fashion that looks very much like ours. Interpreting a price as a linear operator on the commodity space, they define an action of the group of normalized prices on individual preferences; the notion of "price-dispersed preferences" is then defined by requiring that the distribution on the functional space of smooth utilities be absolutely continuous with respect to the Haar measure on the group. By

[9]See also [30] and the references therein, especially [27].

comparison, the framework employed in this paper ensures that the aggregate budget share function is constant, or almost constant.[10] The price to pay, however, is that we cannot content ourselves with the absolute continuity with respect to some 'uniform' distribution: we need the distribution of households' characteristics itself to be 'uniform' in a certain sense, or approximately 'uniform'. However, in our set-up market demand is generally not smooth, allowing to consider 'uniform' distributions which are not restricted to the Haar measure.

Section 5 contains concluding remarks and section 6 contains the proofs.

2. Towards insensitive aggregate budget shares

2.1. The problem

Consider[11] an economy with $L \geq 1$ commodities. Each household is characterized by a demand function f:

$$f : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_{+}, \tag{2}$$

which associates to each pair (p, x) of prices and income, a point in the consumption set. As convincingly argued by [20], it is more convenient to work with the corresponding budget share function $w : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to [0, \gamma]^L$ where $\gamma > 0$, defined by:

$$\forall (p, x), \quad w(p, x) = \frac{p \otimes f(p, x)}{x}. \tag{3}$$

We consider a subpopulation of households with identical income. Households differ in their budget share functions, hence in their characteristics affecting demand independently of prices and income. Let denote by $\mathcal{W}^*$ the space of budget share functions of the economy at hand, endowed with the topology genrerated by the sup-norm $\|\cdot\|_\infty$. The joint distribution of households' characteristics induces a distribution ν of budget share functions on $\mathcal{W}^*$. The assumption that all households have the same income is common to all the previous literature, and could be relaxed. Indeed, one easily sees that the properties obtained below for the aggregate budget share of a given subpopulation are preserved through aggregation. Hence, subsequent analysis could apply to suitable sub-economies populated by individuals with identical incomes.

The statistical approach to the aggregation problem consists in asking whether, for a large set $\mathcal{W}$, there exists a Borel probability distribution ν such that certain properties (e.g., the Law of Demand) are

[10] On the other hand, our angle of attack is quite different: neither do we need to rely on the existence proof of a Haar measure on some locally compact topological group in order to exhibit a 'uniform' distribution on agents' characteristics, nor do we restrict ourselves to individual preferences that can be represented by smooth utility functions or to homogeneous budget constraints.

[11] **Notations:** For any pair of vectors $x, y \in \mathbb{R}^L$, $x \cdot y$ denotes the Euclidean scalar product, and $x \otimes y = (x_1 y_1, ..., x_L y_L)$ the tensor product. If $p \in \mathbb{R}^L_{++}$, p^{-1} denotes the vector $(\frac{1}{p_1}, ..., \frac{1}{p_L})$. For any bijective mapping $T : X \to X$ and any integer n, T^n stands for $T \circ ... \circ T$, the nth composition of T with itself. Any Euclidean space is equipped with its Euclidean norm. $B(x, \varepsilon)$ is the open ball of center x and of radius ε. δ_x is the Dirac measure with support $\{x\}$; $\#X$ is the cardinality of the set X. ??Consider the equivalence relation

$$\forall x \in X, \forall x' \in X, \quad x \sim x' \Leftrightarrow \exists n \in \mathbb{Z} \quad / \quad x' = T^n(x). \tag{1}$$

?? For any topological space $X, \mathcal{C}^0(X)$ [respectively $L_\infty(X)$] is the space of continuous functions [respectively equivalence classes of bounded functions] $f : X \to X$.

fulfilled by the aggregate budget share function

$$(p,x) \mapsto W(p,x) := \int_{\mathcal{W}} w(p,x)\nu(dw). \tag{4}$$

In other words, we want to take the space $\mathcal{W}$ itself as given, provided it belongs to a convenient class of functional spaces, and to prove that an adequate choice of the distribution of households' characteristics, which can be interpreted as representing a perfectly heterogeneous population, can induce per se economically sound properties on the macro-economic level. In this sense, we view the approach taken in this paper as quite distinct from the one adopted, e.g., by [3]. There, it is argued, loosely speaking, that, given a budget share function it is always possible to construct a complementary one such that their sum satisfies the Law of Demand.

What kind of behavior can be expected from the aggregate budget share function of a large, heterogeneous population? The most demanding property is certainly the insensitivity of the map W with respect to changes in prices and income. This property means that market demand behaves as if it was induced by the maximization of a Cobb-Douglas utility function. This property induces indeed most of the aggregate properties one could dream of.

First, the insensitivity of market budget shares with respect to prices induces the celebrated "Law of Demand" in aggregate. The latter can be expressed in terms of the aggregate budget share function: $\forall x \in \mathbb{R}_{++}$

$$\forall p,q \in \mathbb{R}^L_{++}, \quad (p-q)\cdot\Big(p^{-1}\otimes W(p,x) - q^{-1}\otimes W(q,x)\Big) \leq 0. \tag{5}$$

When market budget shares are insensitive to changes in prices, one has: $\forall x \in \mathbb{R}_{++}, \forall p,q \in \mathbb{R}^L_{++}$,

$$(p-q)\cdot\Big(p^{-1}\otimes W(x) - q^{-1}\otimes W(x)\Big) = -\sum_{l=1}^{L}\frac{(p_l-q_l)^2}{p_l q_l}W_l(x) \leq 0. \tag{6}$$

Hence, the Law of Demand follows directly from (4).

Second, the insensitivity property of market budget shares with respect to income implies that market demand is linear in income, in other words, market demand takes on homothetic-like (but not necessarily Cobb-Douglas like) properties. Effectively, market demand for commodity l can be written

$$F_l(p,x) = \frac{W_l(p)}{p_l}x = g_l(p)x$$

$\forall x \in \mathbb{R}_{++}, \forall p,q \in \mathbb{R}^L_{++}$, where $g_l : \mathbb{R}^L_{++} \to \mathbb{R}^L_{+}$. Under the additional assumption that, individual budget share functions are C^1, this property is well known to ensure that the aggregate of income effects is negative. This implies, in turn, when the aggregate of substitution effects is negative (obtained in particular when WARP holds at the microeconomic level), the Law of Demand in aggregate. Indeed, the negative aggregate of income effects is known as *increasing dispersion* and empirical tests of the Law of Demand typically test this property. See [16] for some empirical work and also [18] for theoretical discussions of this property and its variants.

Note that the two first properties might hold simultaneously or not. Nevertheless, whenever households are not victims of money illusion, one easily checks that if market budget shares are insensitive to prices they are also insensitive to income.

Finally, the insensitivity property of market budget shares with respect to prices and income yields a market excess demand which obeys gross substitutability and eventually the uniqueness and global

stability (for the Walrasian tâtonnement) of the equilibrium of a pure exchange economy. Suppose that income x at price p is defined by:

$$x := p \cdot \omega \tag{7}$$

where $\omega \in \mathbb{R}^L_+$ is the initial endowment in commodities of any household in our population. Indeed, consider two price systems p and q such that $q_l > p_l$ and $q_k = p_k$ for $k \neq l$. Denote by $Z_k(p) = F_k(p, p \cdot \omega) - \omega_k$ the market excess demand for commodity k at the price system p. The insensitivity property implies that

$$\frac{p_k F_k(p, p \cdot \omega)}{p \cdot \omega} = \frac{q_k F_k(q, p' \cdot \omega)}{q \cdot \omega}, \tag{8}$$

where by assumption $p \cdot \omega < q \cdot \omega$. Hence, for any pair $(p, q) \in (\mathbb{R}^L_{++})^2$ if $q_l > p_l$ and $q_k = p_k$ for $k \neq l$ one has

$$Z_k(q) > Z_k(p). \tag{9}$$

Thus, there is a unique equilibrium price, which is moreover globally stable in any standard tâtonnement process.

It is important to note that these aggregate properties are preserved as long as market budget shares are sufficiently insensitive to changes in prices and/or income. In particular, if we further introduce the desirability requirement that for any given compact price set K, $W(p) > 0$, $\forall p \in K$, then we can deduce from the almost insensitivity of market budget shares to changes in prices that the Law of Demand holds in K (see section 6.1). It is important to observe that, in contrast to [10] and [20], the latter assumption is not required for all prices but only for prices in K. In addition when the desirability requirement is assumed for all $p \in \mathbb{R}^L_{++}$, the Law of Demand will also be valid for all prices $p \in \mathbb{R}^L_{++}$.

Similarly, it is easy to prove that, if the aggregate budget share function is approximately insensitive to changes in income, then, market demand F is almost linear in income. Under the additional assumption that, individual budget share functions are C^1, this ensures that the aggregate of income effects is negative.

Finally, the insensitivity property of market budget shares with respect to prices and income yields a market excess demand which obeys gross substitutability on any given compact price set K (when the desirability property is required on K) for all prices $p \in \mathbb{R}^L_{++}$ (when the desirability property is extended to all prices $p \in \mathbb{R}^L_{++}$). As mentioned previously, under some standard additional assumption which guarantees that no equilibrium price exists outside the compact set of prices K, this again ensures uniqueness and global stability of the price equilibrium. Observe moreover, that nothing prevents from interpreting the collection $\{1, ..., L\}$ of "commodities" as composed of consumption goods and securities, possibly within an incomplete markets setting.

2.2. The main results

In order to formally define a population of households that react heterogeneously to a 'perturbation' of the price-income vector, the literature has focused so far on the class of affine transformations on the functional space $\mathcal{W}^*$ (see, for example, [10] and [20]). However, this is just one of many possible classes that can be used. We shall consider a broader class of transformations T.

We shall first make more precise the space $\mathcal{W}^*$ on which our results apply.

Assumption 1:

(i) The space $\mathcal{W}^*$ of admissible budget share functions is a subset of the set of all functions from $\mathbb{R}^L_{++} \times \mathbb{R}_{++}$ to $[0, \gamma]^L$ where $\gamma > 0$.

(ii) The space $\mathcal{W}^*$ is convex.

(iii) $\mathcal{W}^*$ is compact with respect to the topology induced by the sup-norm.
(iv) $\mathcal{W}^*$ is not restricted to a finite set of constant functions over $\mathbb{R}^L_{++} \times \mathbb{R}_{++}$.

Convexity guarantees that the market budget share function generated through (4) from a probability distribution ν on $\mathcal{W}^*$ still belong to it. Compactness is the topological analogue of finiteness, and was already assumed by [15, p. 17], $\mathcal{M}_{\text{com}}$. It can be thought of as arising from the continuity of some mapping that associates to each individual in, say, the real interval $[0, 1]$ her budget share function. In other words, in a parametric setting, all we need is that the parameter set describing the set of feasible budget share functions be compact (see examples below). Assumption 1(iv) excludes the specific case where any household in the feasible set possess a constant budget share function. Note that if this assumption is necessary to avoid that the insensitivity of the market budget share does not follow from the insensitivity property at the microeconomic level, it is not sufficient. Effectively, the distribution ν might still give all the weight to a subset of households with constant budget share functions.

For the transformations T, we consider a group, G, of transformations endowed with the composition law "$\circ$" operating on $\mathcal{W}^*$. This group of transformations fulfills the following assumption.

Assumption 2:

(i) For all $T \in G$, the map $w \mapsto T[w]$ is an isometry over $\mathcal{W}^*$.
(ii) The group G is stable through composition, $T_1 \circ T_2$ belongs to G whenever T_1 and T_2 both do.
(iii) Every function $w \in$W*, which is invariant with respect to every transformation $T \in G$, is constant over $\mathbb{R}^L_{++} \times \mathbb{R}_{++}$.

As an illustration, the class of affine transformations considered by [20], defined by

$$\forall t \in \mathbb{R}^{L+1}, \quad T_t[w](p, x) = w(e^t \otimes (p, x)), \forall w \in \mathcal{W}^*, \forall (p, x) \in \mathbb{R}^L_{++} \times \mathbb{R}_{++}, \tag{10}$$

fulfills Assumption 2.

When the behavioral heterogeneity is restricted to heterogeneous reactions of households to a perturbation of the price system Assumption 2(iii) is substituted by:

Assumption 2p(iii):

Every function $w \in$W*, which is invariant with respect to every transformation $T \in G$, is constant with respect to $p \in \mathbb{R}^L_{++}$.

As an illustration, the group of affine transformations considered by [10], defined by

$$\forall w \in \mathcal{W}^*, \forall \alpha \in \mathbb{R}^L, \forall (p, x) \in \mathbb{R}^L_{++} \times \mathbb{R}_{++}, \quad T_\alpha[w](p, x) = w(e^\alpha \otimes p, x),$$

fulfills Assumption 2p.

Alternately, when the behavioral heterogeneity is restricted to heterogeneous reactions of households to a perturbation of income, Assumption 2(iii) is substituted by:

Assumption 2x(iii):

Every function $W : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_+$, $(p, x) \mapsto w(p, x)$, which is invariant with respect to every transformation $T \in G$, is constant with respect to $x \in \mathbb{R}_{++}$.

As an illustration, the group of homothetic transformations considered by [28], defined by

$$\forall w \in \mathcal{W}^*, \forall \beta \in \mathbb{R}, \forall (p,x) \in \mathbb{R}^L_{++} \times \mathbb{R}_{++}, \quad T_\beta[w](p,x) = w(p, e^{-\beta}x),$$

fulfills Assumption 2x.[12]

Note that, by assumption, G operates isometrically on $\mathcal{W}^*$:

$$\forall T \in G, \forall w, v \in \mathcal{W}^*, d(T(w), T(v)) = d(w, v). \tag{11}$$

We shall formalize a perfectly heterogeneous population (in terms of households' reactions to changes in prices and/or income) by an invariant measure ν on $\mathcal{W}^*$ with respect to the operation of the group G. Assumptions 1 and 2 ensure that this invariant measure exists. In addition, one can also prove that any finite population not too far away from a perfectly heterogeneous population admits a market budget share function that is approximately constant with respect to prices and/or income. The proofs of these results are available by the authors upon request.

We shall now introduce an assumption ensuring that the insensitivity property in aggregate is the outcome of the balancing effect rather than the insensitivity at the microeconomic level. To make this point, it will suffice to show that any non-empty open subset of the set of feasible budget share functions is non-negligible. This can be done by restricting our attention to a narrower equivalence subclass $\mathcal{W}$ induced by a single generator $\bar{w}$, i.e. the orbit of $\bar{w}$ by all elements of the group G. More precisely, we introduce the following assumption.

Assumption 3

Consider $\bar{w}$ a non constant function from $\mathbb{R}^L_{++} \times \mathbb{R}_{++}$ to $[0, \gamma]^L$ where $\gamma > 0$.

(i) The group G is compact.

(ii) The set of feasibe budget share functions $\mathcal{W}$ is the orbit of $\bar{w}$ by all elements of the group G.

Assumption 3 means that we restrict ourselves to the type of heterogeneity generated by the group transformations T: it is possible to go from one's budget share function to another by composing transformations T. This requirement was made in [10] for the specific group of affine transformations. Here, we allow for a broader group of transformations. Nevertheless, we add the assumption that the group G is compact. In this formalism the type of behavioral heterogeneity introduced depends both on the generator and on the group of transformations applied to this generator to construct the population of households. Note that assumptions introduced on the generator will specify the individual rationality introduced in the model while restrictions on the compact group of transformations specify in which way households react differently to changes in prices and income. The crucial point is to interpret economically these two types of restrictions introduced by Assumption 3. This is the purpose of sections 3 and 4.

By forbidding the concentration of the measure ν over any closed proper subset of $\mathcal{W}$, Assumptions 1 to 3 truly impose the behavioral heterogeneity we are looking for in this paper.

Proposition 1 *Under Assumptions 1 to 3, the measure ν satisfies:*

$$\nu(O) > 0 \quad \forall O \text{ non-empty, open subset of } \mathcal{W}.$$

[12]Note that, under the assumption that each household is not victim of money illusion, the class of homothetic transformations is a subclass of Grandmont's class of affine transformations where $\alpha_l = \beta$ for all l.

The next result establishes that there is a unique way for the space of budget share functions $\mathcal{W}$ to be perfectly heterogeneously distributed.

Theorem 1 *Under Assumptions 1 to 3, there exists a unique probability distribution ν on $\mathcal{W}$ invariant with respect to the operation of the group G.*

The proof of this theorem clearly establishes the relationship between probability measures over the equivalence subclass $\mathcal{W}$ and probability measures λ on the group G of transformations. In particular, it is shown that for a probability distribution ν on $\mathcal{W}$ to generate constant market budget shares through (3), it is necessary and sufficient that it is the image on $\mathcal{W}$, in some well defined sense (?? to be specified??), of an invariant Haar probability measure on the topological group G. Existence an uniqueness follows from the existence of a unique invariant Haar probability measure under compactness of the group G.

?? Remark : In the proof, we have to clarify the relationship of the following assumption with compactness of G, hence of the equivalence subclass $\mathcal{W}$ of a single generator $\bar{w}$).

Assumption : For any sequence (T_n) in $\left(\mathcal{T}\right)^{\mathbb{N}}$, and any continuous map $f : \mathcal{W} \to \mathcal{W}$, if $T_n[w]$ converges to $f(w)$ uniformly on $\mathcal{W}$, then $\exists\, T \in \mathcal{T}$, such that $f(w) = T[w]$, $\forall w \in \mathcal{W}$.??

3. The special case of affine transformations

When the transformations T are affine, our results can be demonstrated in a much simpler and more direct way.

Consider a function $\bar{w} : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_+$. With any t in $\mathbb{R}^{L+1}$, we can define w^t, a function from $\mathbb{R}^L_{++} \times \mathbb{R}_{++}$ to $\mathbb{R}^L_+$, by $w^t(p,x) = T_t[\bar{w}](p,x) := \bar{w}(e^t \otimes (p,x))$. In this section, we consider a population of households with the same income and budget share functions drawn from the set

$$\mathcal{W} = \{w^t : t \in \mathbb{R}^{L+1}\}$$

Note that for all $t \in \mathbb{R}^{L+1}$ and $t' \in \mathbb{R}^{L+1}$, one has $T_t[T_{t'}[\bar{w}]] = T_{t+t'}[\bar{w}]] = w^{t+t'}$. Hence, the budget share functions in $\mathcal{W}$ are deduced from one to the other through translations in $\mathbb{R}^{L+1}$.

If w is measurable and bounded, then for any density function $h : \mathbb{R}^{L+1} \to \mathbb{R}_+$, the integral $\int w^t(p,x)h(t)dt$ exists. So, if the distribution of t is governed by the measure μ then the mean budget share on good l is $W_l(p,x,\mu) = \int_{\mathbb{R}^{L+1}} w^t(p,x)\mu(dt)$. The discussion below tells us that there exists a unique probability distribution μ such that the mean expenditure devoted to any commodity is independent of prices and income. In addition, this insensitivity property follows from the balancing effect rather than the insensitivity at the microeconomic level.

For every vector $\eta = (\eta_1, ..., \eta_{L+1}) \in \mathbb{R}^{L+1}$ and every index $i = 1, ..., L+1$, we write $e^{\eta^i} := (0, ..., 0, e^{\eta_i}, 0, ..., 0) \in \mathbb{R}^{L+1}$, where e^{η_i} stands in the ith position. (Obviously, $e^\eta = \sum_i e^{\eta^i}$.)

We make the following assumption on $\bar{w}$:

Assumption 5 The generator $\bar{w}$ is a non constant function of (p,x) that is bounded, Borel-measurable and periodic in the following sense: there exists $\eta \in \mathbb{R}^{L+1}$ such that

$$\bar{w}(e^{n\eta} \otimes (p,x)) = \bar{w}(p,x), \quad \forall p \in \mathbb{R}^L, \forall x \in \mathbb{R}, \forall n \in \mathbb{Z}. \tag{12}$$

This assumption implies that for any $t \in \mathbb{R}^{L+1}$ and any $n \in \mathbb{Z}$ the budget share functions w^t and $w^{t+n\eta}$ are identical. Therefore, we shall consider the Abelian, additive group, G, obtained by quotienting $\mathbb{R}^{L+1}$ with respect to the equivalence relation :[13]

[13] In mathematical notations, G is usually designed by $\mathbb{R}^{L+1}/\eta\mathbb{Z}^{L+1}$.

$$t \sim t' \iff \big(t = t' + n\eta, \quad n \in \mathbb{Z},\big).$$

We denote by $\dot{t} \in G$ the equivalence class associated to $t \in \mathbb{R}^{L+1}$. According to Assumption 5, $\mathcal{W}$ coincides with:

$$\mathcal{W} = \{w^{\dot{t}},\ \dot{t} \in G\}.$$

When equipped with the quotient topology (i.e., the coarsest topology that makes the projection map $t \mapsto \dot{t}$ continuous), the group G is a compact topological group (It is actually homeomorphic to the torus, a doughnut when $L+1 = 2$). Therefore, it admits a (unique up to a scalar multiple) finite (Borel) Haar measure, denoted by λ. When the population of households is distributed according to this Haar measure, the mean budget share on good l can be written

$$W_l^{\lambda}(p,x) = \int_G w^{\dot{t}}(p,x)\lambda(d\dot{t}).$$

By definition of a Haar measure, λ is invariant with respect to the group addition, i.e., for every Borel subset $A \subset G$, $\lambda(A + \dot{t}) = \lambda(A)$ for every $\dot{t} \in G$. Translating this property in terms of the mean budget share on good l, one gets:

$$W^T(\cdot,x,\lambda) := \int_G w^{\dot{t}+T}(\cdot,x)\lambda(d\dot{t}) = \int_G w^{\dot{t}}(\cdot,x)\lambda(d\dot{t}) = W(\cdot,x,\lambda), \quad T \in G, x \in \mathbb{R}$$

In other words, the aggregate budget share function W^{λ} is invariant with respect to any translation on G. This implies that W^{λ} is a constant function of prices and income. In addition, since by assumption $\bar{w}$ is a non constant function, all functions in $\mathcal{W}$ are non constant, and by definition of the Haar measure any subset of the population described by a non empty compact subset $A \in G$ has a strictly positive measure. In other words, households are not insensitive to changes in prices and income, but the insensitivity property emerges in aggregate from the balancing effect. One easily checks that the population described by $\mathcal{W} = \{w^{\dot{t}},\ \dot{t} \in G\}$, where the generator $\bar{w}$ fulfills Assumption 1-5 and the 'uniform' probability distribution alluded to in Theorem 1 and Theorem 4 is λ.

The exact aggregation result in [9] is obtained for this special case of affine transformations. Nevertheless, these authors do not introduce any periodicity requirement such as our Assumption 5. Hence, they can prove the existence of a probability measure such that market budget shares of the corresponding population are insensitive to changes in prices and/or income but nothing ensures in their framework that this insensitivity property emerges in the aggregate from an exact balancing effect. In other words, the exact insensitivity in the aggregate might follow from the exact insensitivity of any household. In this sense, their model is not an illustration of the statistical approach adopted in our set-up.

Note, that what makes affine transformations special is their preservation of the possible rationality properties. It is straightforward to check that if a function w defined on $\mathbb{R}^L_{++} \times \mathbb{R}_{++}$ satisfies the weak axiom, then so will its transformation $T_{\dot{t}}[w]$. Furthermore, if w is generated by the maximization of a utility function $u(\cdot)$ over the budget set $B(p,x) = \{z \in \mathbb{R}^L_+ : p \cdot z \leq x\}$, then its transformation is generated by the maximization of the utility function $u^t(c_1,\ldots,c_L) = u(\frac{e^{t_{L+1}}}{e^{t_1}}c_1,\ldots,\frac{e^{t_{L+1}}}{e^{t_L}}c_L)$ over $(c_1,\ldots,c_L) \in B(p,x)$. This shows incidentally that it is possible, in the case of affine transformations, to formulate all the assumptions put on the record in this paper on the (more fundamental ?) level of individual preferences, rather than on demand functions.

It is not difficult to see that one consequence of the periodicity requirement (12) is that any individual budget share function $w^{\dot{t}}$ must have some "erratic" behavior near the boundary of the domain of the

price-income vector: $w^{\dot{t}}$ wiggles as income or some price tends to zero or infinity. This is, in fact, a general property of the space $\mathcal{W}$ obtained through affine transformations:

Proposition 2 *Suppose that the space $\mathcal{W}$ satisfies Assumption 1, and that the class $\mathcal{T}$ consists of affine transformations. Then, for every $w \in \mathcal{W}, (\overline{p}, \overline{x}) \in \mathbb{R}^{L+1}_{++}$ and $i, j \in \{1, ..., L\}$,*

(i) either $w_j(\overline{p}, \cdot)$ is constant or the limits $\lim_{x\to 0} w_j(\overline{p}, x)$ and $\lim_{x\to\infty} w_j(\overline{p}, x)$ do not exist;

(ii) either $w_j(\overline{p}_1, ..., \overline{p}_{i-1}, \cdot, \overline{p}_{i+1}, ..., \overline{x})$ is constant or the limits $\lim_{p_i\to 0} w_j(\overline{p}_1, ..., \overline{p}_{i-1}, p_i, \overline{p}_{i+1}, ..., \overline{x})$ and $\lim_{p_i\to\infty} w_j(\overline{p}_1, ..., \overline{p}_{i-1}, p_i, \overline{p}_{i+1}, ..., \overline{x})$ do not exist.

The problem even occurs for more general classes of transformations. Suppose Assumption 2 is in force. In addition, assume that $\overline{w}$ is "nice", in the sense that the partial derivatives $\frac{\partial \overline{w}_j}{\partial p_j}$ or $\frac{\partial \overline{w}_j}{\partial x}$ exist and have constant sign. If the transformations in $\mathcal{T}$ have the property of preserving the sign of these derivatives (let us qualify such transformations sign-preserving transformations), the corresponding derivatives of the aggregate budget share function have the same sign. Consequently, they can never be equal to zero.

4. Examples and interpretation

In this section, we exhibit examples illustrating the theory outlined in the previous section, and discuss their economic interpretation.

4.1 Example 1.

We shall first use the setting of affine transformations and more precisely the class of homothetic transformations to build a population of households heterogeneous in terms of their reaction to changes in income.

Consider a function $\bar{w} : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_+$. With any β in $\mathbb{R}$, we can define w^β, a function from $\mathbb{R}^L_{++} \times \mathbb{R}_{++}$ to $\mathbb{R}^L_+$, by $w^\beta(p, x) := \bar{w}(p, e^{-\beta}x)$. We consider a population of households with the same income and budget share functions drawn from the set

$$\mathcal{W} = \{w^{\dot{\beta}} : \dot{\beta} \in G\},$$

where $G = \mathbb{R}/\eta\mathbb{Z}$ and the generator $\bar{w}$ fulfills the following assumption.

Assumption 5x For any $p \in \mathbb{R}^L_{++}$, the generator $\bar{w}(p, \cdot)$ is a non constant function of x that is bounded, Borel-measurable and periodic in the following sense: there exists $\eta \in \mathbb{R}$ such that

$$\bar{w}(p, e^{n\eta}x) = \bar{w}(p, x), \quad x \in \mathbb{R}, n \in \mathbb{Z}. \tag{13}$$

When equipped with the quotient topology, the group G is a compact topological group. Therefore, it admits a (unique up to a scalar multiple) finite (Borel) Haar measure, denoted by λ. We consider the population of households distributed according to this Haar measure.

To be more specific, one could consider for the generator the 2π-periodic budget share function defined by $\bar{w}(p, x) = 1 + \sin \ln x$. In this case $w^{\dot{\beta}}(p, x) = 1 + \sin(\dot{\beta} + \ln x)$ and $\mathcal{W} = \{w^{\dot{\beta}} : \dot{\beta} \in \mathbb{R}/e^{2\pi}\mathbb{Z}\}$

¿From Section 3, we know that this population fulfills Assumptions 1x, 2 to 4 and possesses mean budget shares that are constant with income. In addition, this insensitivity in the aggregate emerges from the balancing effect since all households are sensitive to changes in income. Hence, the mean

demand function of the population is linear in income. It behaves as if it was generated from homothetic preferences while the tastes of any household are not even assumed to be represented by a preference relationship.

Let us be more precise about the restrictions introduced at the household level. First, note that the household is not even assumed to choose a consumption bundle in her budget set $B(p,x)$: The budget shares are only required to be bounded. The main assumption at the individual level is the periodicity requirement. As underlined in section 3 this implies, in particular, that any individual budget share function $w^{\dot{\beta}}$ must have some "erratic" behavior near the boundary of the income domain: $w^{\dot{\beta}}$ wiggles as income tends to zero or infinity. This implies, in particular, that $w^{\dot{\beta}}(p,x)$ has no limit as income tends to zero or infinity.

Note also that for many purposes (in particular for uniqueness and global stability of the price equilibrium under the standard boundary condition of individual preferences. See, for example, [26].) we are only interested in the behavior of market demand on a proper compact subset, K, of prices and income. Hence, it is enough to require that the household described by $\dot{\beta}$ has a budget share function which coincides with $w_{\dot{\beta}}$ on K and is not restricted outside K. In this case, every function $w^{\dot{\beta}}$ in the population is of bounded variations over K and has no erratic behavior near the boundary of the income domain.

To complete this example one should point out that the restriction imposed by Assumption (5x) on the individual demand behavior is not atypical. In particular, one can find a budget share function induced by standard utility maximization that fulfills this assumption. Hence, we shall build the example of a generator $\bar{w}$ that fulfills Assumption (5x) and the following assumption:

Assumption 6 The generator $\bar{w}(p,x)$ is continuous in $x>0$ and is generated by the maximization of a utility function $u(\cdot)$ over the budget set
$B(p,x)=\left\{(y_1,y_2)\in\mathbb{R}^2_+ \mid p_1x_1+p_2x_2=x\right\}$.

This generator can be built by pieces. Let us explain graphically the construction. Start with the income $x=e^{-\frac{\eta}{2}}$, for a given $n\in\mathbb{N}$, the household demand $f\left(\bar{p}_1,\bar{p}_2,e^{-\frac{\eta}{2}}\right)$ is represented on figure 1 by the point A and the corresponding budget shares by the point W. Denote by $B(x)=\left\{(x_1,x_2)\in\mathbb{R}^2_+ \mid \bar{p}_1x_1+\bar{p}_2=x\right\}$. The "weak periodicity" requirement implies that for the income level $e^{\frac{\eta}{2}}$ the household demand $f\left(\bar{p}_1,\bar{p}_2,e^{\frac{\eta}{2}}\right)$ should correspond to the same budget shares W, it is, therefore, represented by point B, the intersection of the household budget line and the diagonal OA.

The next step is to build a continuous demand induced by utility maximization for $e^{-\frac{\eta}{2}}\leq x\leq e^{\frac{\eta}{2}}$. This is obtained, in particular, for a household that first increases its demand in the first commodity up to reach $f_1\left(\bar{p}_1,\bar{p}_2,e^{\frac{\eta}{2}}\right)$ then it increases its demand in the second commodity up to reach the level $f_2\left(\bar{p}_1,\bar{p}_2,e^{\frac{\eta}{2}}\right)$. Such a demand behavior is obtained for a rational household that maximizes a utility function that is quasi-linear with respect to the first commodity in the area (I) and quasi-linear with respect to the second commodity in the area (II).

The demand behavior should be "replicated" outside the income interval $\left[e^{-\frac{\eta}{2}},e^{\frac{\eta}{2}}\right]$ in order to ensure that

$$\bar{s}\left(\bar{p}_1,\bar{p}_2,xe^{n\eta}\right)=\bar{s}\left(\bar{p}_1,\bar{p}_2,x\right)$$

for all $x>0$ and all $n\in\mathbb{N}$. In particular, the demand behavior on $\left[e^{-\frac{\eta}{2}},e^{\frac{\eta}{2}}\right]$ generates the demand behavior on $\left[e^{\frac{\eta}{2}},e^{\frac{3\eta}{2}}\right]$ with the requirement that

$$\bar{s}\left(\bar{p}_1,\bar{p}_2,xe^{\eta}\right)=\bar{s}\left(\bar{p}_1,\bar{p}_2,x\right)$$

for all $x\in\left[e^{-\frac{\eta}{2}},e^{\frac{\eta}{2}}\right]$. The consumption bundle chosen by the household at income $e^{\frac{\eta}{2}}$ represented by point B yields to the same budget shares as the consumption bundle represented by point A chosen by

the household at income $e^{-\frac{\eta}{2}}$. Symmetrically B' matches A', B'' matches A'', B''' matches A''' and so on.

Formally, this demand behavior corresponds to the budget share function $\bar{w}(\bar{p}_1, \bar{p}_2, \cdot)$ defined by

$$\begin{pmatrix} \bar{w}_1(\bar{p}_1, \bar{p}_2, x) \\ \bar{w}_2(\bar{p}_1, \bar{p}_2, x) \end{pmatrix} = \begin{cases} \begin{pmatrix} 1 - a_n \frac{\bar{p}_1}{x} \\ a_n \frac{\bar{p}_1}{x} \end{pmatrix} & \text{for } e^{\eta n - \frac{\eta}{2}} \leq x \leq e^{\eta n} \text{ and } n \in \mathbb{Z} \\ \begin{pmatrix} b_n \frac{\bar{p}_2}{x} \\ 1 - b_n \frac{\bar{p}_2}{x} \end{pmatrix} & \text{for } e^{\eta n} \leq x \leq e^{\eta n + \frac{\eta}{2}} \text{ and } n \in \mathbb{Z} \end{cases},$$

where the sequence of parameters $(a_n, b_n)_{n \in \mathbb{Z}}$ is defined by

$$\begin{cases} a_n = \frac{e^{\eta n}\left(e^{\frac{\eta}{2}} - 1\right)}{\bar{p}_1 (e^{\eta} - 1)} \\ b_n = \frac{e^{\eta n} - a_n \bar{p}_1}{\bar{p}_2} = \frac{e^{\eta n}}{\bar{p}_2}\left(\frac{e^{\eta} - e^{\frac{\eta}{2}}}{e^{\eta} - 1}\right) \\ a_{n+1} = e^{\eta} a_n \\ b_{n+1} = e^{\eta} b_n \end{cases}.$$

Hence, by construction this budget share function fulfills assumptions 5x and 6.

4.2 Example 2.

We shall now use the setting of affine transformations to build, in the spirit of Grandmont's (1992) seminal construction, a population of households heterogeneous in terms of their reaction to changes in prices.

Consider a function $\bar{w} : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_+$. With any β in $\mathbb{R}^L$, we can define w^β, a function from $\mathbb{R}^L_{++} \times \mathbb{R}_{++}$ to $\mathbb{R}^L_+$, by $w^\beta(p, x) = \bar{w}(e^\beta \otimes p, x)$. We consider a population of households with the same income and budget share functions drawn from the set

$$\mathcal{W} = \{w^{\dot{\beta}} : \dot{\beta} \in G\},$$

where $G = \mathbb{R}^L / \eta \mathbb{Z}^L$ and the generator $\bar{w}$ fulfills the following assumption.

Assumption 5p For any $p \in \mathbb{R}^L_{++}$, the generator $\bar{w}(p, \cdot)$ is a non constant function of x that is bounded, Borel-measurable and periodic in the following sense: For $i = 1, \ldots, L$ there exists $\eta_i \in \mathbb{R}$ such that

$$\bar{w}(e^{n\eta_i} \otimes p, x) = \bar{w}(p, x), \quad x \in \mathbb{R}, n \in \mathbb{Z}. \tag{14}$$

When equipped with the quotient topology, the group G is a compact topological group. Therefore, it admits a (unique up to a scalar multiple) finite (Borel) Haar measure, denoted by λ. We consider the population of households distributed according to this Haar measure.

¿From Section 3, we know that this population fulfills Assumptions 1p, 2 to 4 and possesses mean budget shares that are constant with prices. In addition, this insensitivity in aggregate emerges from the balancing effect since all households are sensitive to changes in prices. Hence, the mean demand function behaves as if generated from Cobb-Douglas preferences while the tastes of any household are not even assumed to be represented by a preference relationship.

Let us be more precise about the restrictions introduced at the household level. First, note that the household is not even assumed to choose a consumption bundle in her budget set $B(p, x)$: The budget shares are only required to be bounded. The main assumption at the individual level is the periodicity requirement. As underlined in section 3 this implies, in particular, that any individual budget share function $w^{\dot{\beta}}$ must have some "erratic" behavior near the boundary of the price domain : $w^{\dot{\beta}}$ wiggles as the price of one commodity tends to zero or infinity. This implies, in particular, that $w^{\dot{\beta}}(p, x)$ has no limit as the price of one commodity tends to zero or infinity.

Again, for many purposes (in particular for uniqueness and global stability of the price equilibrium under the standard boundary condition of individual preferences) we are only interested in the behavior of market demand on a proper compact subset, K, of prices and income. Hence, it is enough to require that the household described by $\dot{\beta}$ has a budget share function which coincides with $w^{\dot{\beta}}$ on K and is not restricted outside K. In this case, every function $w^{\dot{\beta}}$ in the population is of bounded variations over K and has no erratic behavior near the boundary of the price domain.

To complete this example one should point out that the restraint imposed by Assumption (5p) on the individual demand behavior is not atypical. In particular, one can find a budget share function induced by standard utility maximization that fulfills this assumption. Consider[14] any indirect utility function $V : \mathbb{R}^L_+ \times \mathbb{R}_+ \to \mathbb{R}$. By Roy's identity, we know that the demand function $\bar{f}$ induced by this indirect utility function is defined by

$$\bar{f}_\ell(p,x) = -\frac{\frac{\partial V(p,x)}{\partial p_\ell}}{\frac{\partial V(p,x)}{\partial x}}.$$

Therefore, the corresponding budget share function verifies:

$$\overline{w}_l(p,x) = -\frac{p_\ell \frac{\partial V(p,x)}{\partial p_\ell}}{x \frac{\partial V(p,x)}{\partial x}}. \tag{15}$$

Since, V is homogenous of degree zero in (p,x), one gets from Euler's identity:

$$\overline{w}_l(p,x) = -\frac{p_\ell \frac{\partial V(p,x)}{\partial p_\ell}}{\sum_k p_k \frac{\partial V(p,x)}{\partial p_k}}. \tag{16}$$

Let us assume in addition that, for any fixed income x, $V(\cdot,x)$ is positively homogeneous of degree $\alpha > 0$. Hence, one gets from Euler's identity:

$$\overline{w}(p,x) = \frac{1}{\alpha}\frac{p_l}{V(p,x)}\frac{\partial V(p,x)}{\partial p_l}.$$

Thus, for $\overline{w}$ to be periodic wrt prices, it suffices that the elasticity $\frac{p_l}{V(p,x)}\frac{\partial V(p,x)}{\partial p_l}$ of $V(p,x)$ be itself periodic with respect to prices. This is obviously the case whenever V arises from a Cobb-Douglas utility function, but it is easy to find examples of non-Cobb-Douglas utilities fulfilling this requirement.

4.3. Example 3.

As shown in the section 3, "erratic" individual behavior are needed near the boundary of the price/income domain whenever heterogeneity is checked against sign-preserving transformations. We shall now build an example where behavioral heterogeneity of households with respect to a perturbation of the price-income vector is defined with the use of the (non-sign preserving) class of <u>rotations</u> of the price-income vector. This time, Assumption 1 can be fulfilled without imposing any erratic individual behavior. However, another stumbling block is to be encountered in terms of interpretation, as we now show.

To simplify the presentation we consider an economy with two commodities and we focus on heterogeneity of the households' share functions with respect to the price vector. All households possess the same income level. Denote again by $\overline{w}$ the generator of the population. Suppose that that we are concern with the behavior of the population on the compact set of the price-income vector, K. We assume that $\overline{w}$ is a non constant function continuous and homogeneous of degree zero in (p,x) over K. Hence, our

[14] This example was communicated to us by John Quah.

main assumption at the household level is the absence of money illusion. We furthermore extend $\overline{w}$ by continuity outside K in such a way that: $\forall x$

$$\overline{w}(0,1,x) = \overline{w}(1,0,x). \tag{17}$$

Note that this extension is always feasible and does not restrict the behavior of $\overline{w}$ over K. The set of prices (p_1, p_2) can be identified with the non negative orthant of the complex plan:

$$\mathbb{C}_+ := \{z_p = p_1 + ip_2 \in \mathbb{C} \ : \ (p_1, p_2) \in \mathbb{R}^2_+\}. \tag{18}$$

Moreover, thanks to the homogeneity assumption, prices can be normalized so that the price space can be identified to $\mathbb{U}_+ := \mathbb{U} \cap \mathbb{C}_+$, where $\mathbb{U} := \{z \in \mathbb{C} : |z| = 1\}$. To define our population, we introduce a second function $\tilde{w}$ defined over $\mathbb{U} \times \mathbb{R}_{++}$ by

$$\tilde{w}(z_p, x) = \overline{w}(p_1, p_2, x). \tag{19}$$

The population is a collection of functions $\{w_\theta\}$ where for $\theta \in \mathbb{U}$, w_θ is defined by:

$$w_\theta(p, x) = T_\theta[\overline{w}](p, x) := \tilde{w}\big(\theta z_p, x\big), \tag{20}$$

where $\theta \in \mathbb{U}$ is some unitary complex number. This means that w_θ is essentially deduced from the generator $\overline{w}$ by applying the rotation of angle θ to the price vector. We shall now specify the set of parameters θ. From (18) we deduce that for all $n \in \mathbb{Z}$, one has

$$\tilde{w}(e^{in\frac{\Pi}{2}} z_p, x) = \tilde{w}(z_p, x). \tag{21}$$

We can therefore consider the equivalence relation

$$z \sim z' \Leftrightarrow \exists n \in \mathbb{Z} \quad / \quad z' = i^n z. \tag{22}$$

The quotiented space $\mathbb{U}_+/\sim$ is denoted Π. Let us define the set of feasible budget share functions $\mathcal{W}$ by (the uniform closure of) the collection of functions $\{w_{\dot{\theta}}\}_{\dot{\theta} \in \Pi}$. Since the price space $\mathbb{U}_+$ is compact, one can apply Ascoli's Theorem and prove that the family of such transformed budget share functions is relatively compact for the uniform topology. Taking its uniform closure yields compactness. Thus, Assumption 1 and 2 follow and according to Theorem 1, we can conclude that $\mathcal{W}$ admits a probability measure with respect to which the market budget share function is constant. In addition, thanks to the compactness of the set of rotation parameters, Π, one can prove that Assumptions 3 and 4 hold.

As in the previous subsection, this second example leaves us with an interpretational issue. Condition (18) requires indeed a specific household behavior on the boundary of the price space, namely that budget shares be identical whether the price of the first commodity or the price of the second commodity is equal to zero. On one hand, this can be viewed as unrealistic. On the other hand, whatever being the household behavior over K the budget share function can always be extended by continuity outside K in order to fulfill (18). Hence, as long as one is only interested in the behavior of market demand on a compact set of the price-income vector K, assumptions made at the household level in this example are rather weak. The major requirement is that households are not victims of money illusion — an assumption that is commonly made in demand theory.

5. Concluding Remarks

According to the angle of attack adopted in this paper, three ingredients drive the exact insensitivity of aggregate budget share: (1) one needs a "large" population (this is Assumption 2(i),(iii) and (iv)),

where the type of behavioral heterogeneity introduced is formalized by a class of transformations of the budget share functions, (2) whose characteristics are in a compact set (Assumptions 2(iii)) and (3) are uniformly distributed (this is the G-invariance of λ and Assumption 1). The crucial assumption introduced in this paper is the "specific type of heterogeneity" requirement (Assumption 3 combined with the compactness assumption) that ensures that this uniformity requirement describes the exact balancing effect. It requires that the only behavioral heterogeneity in the population is the one introduced by the class of transformations $\mathcal{T}$. Finally, note that for a given type of behavioral heterogeneity (for a given class of transformations $\mathcal{T}$) the population yielding the exact balancing effect is unambiguously defined (i.e. unique) when a last technical (closedness) assumption is introduced (Assumption 4).

6. Proofs of the results

6.1. The approximate insensitivity property and the Law of Demand

Consider a finite population of households such that the mean budget share function is almost insensitive to changes in prices, i.e. $\forall p \in \mathbb{R}^L_{++}$, $\forall \eta > 0$ there exists $\varepsilon > 0$ such that $\forall q \in \mathbb{R}^L_{++}$ with $\|q - p\| \leq \eta$ one has

$$\|W^\nu(p,x) - W^\nu(q,x)\| \leq \varepsilon.$$

Hence,

$$(p-q)\cdot\Big(p^{-1}\otimes W^\nu(p,x) - q^{-1}\otimes W^\nu(q,x)\Big) \leq (p-q)(p^{-1}\otimes(W^\nu(q,x)+\varepsilon^{p,q}) - q^{-1}\otimes W^\nu(q,x))$$

where $\varepsilon^{p,q} \in \mathbb{R}^L$ with $\varepsilon^{p,q}_l = \varepsilon$ if $p_l - q_l \geq 0$ and $\varepsilon^{p,q}_l = -\varepsilon$ otherwise. Under the desirability requirement that for any given compact price set K, $W^\nu(p,x) \in \mathbb{R}^L_{++}$, $\forall p \in K$, we deduce that

$$(p-q)\cdot\Big(p^{-1}\otimes W^\nu(p,x) - q^{-1}\otimes W^\nu(q,x)\Big) \leq 0$$

In other words the Law of Demand holds in K.

6.1. Proof of Theorem 1

Since $\mathcal{W}$ is pre-compact with respect to d, for any $\varepsilon > 0$, there exists at least one finite subset $R(\varepsilon)$ of $\mathcal{W}$, such that, for any $w \in \mathcal{W}$, $\inf\{d(w,r) \ : \ r \in R(\varepsilon)\} \leq \varepsilon$. Let call $R(\varepsilon)$ a ε-network, and denote by $N(\varepsilon)$ the minimal cardinality of such ε-networks.

Claim. Let $\varepsilon > 0$, and R and R' two ε-networks of $\mathcal{W}$, of minimal cardinality $N(\varepsilon)$. There exists a bijection $\psi : R \to R'$, such that:

$$d(w,\psi(w)) \leq 2\varepsilon \quad \forall w \in \mathcal{W}. \tag{23}$$

To prove this claim, take $w \in R$, and consider the following set $A_w \subset R'$ of elements of R' which are "closely related" to w:

$$A_w = \big\{v \in R' \ : \ B(w,\varepsilon) \cap B(v,\varepsilon) \neq \emptyset\big\}. \tag{24}$$

Take, now, any subset $I \subset R$, and consider the set R'' obtained by replacing every element from I by the family of its "close" points:

$$R'' := \big(R \setminus I\big) \cup \big(\cup_{w\in I} A_w\big). \tag{25}$$

It is not difficult to see that R'' is still an ε-network of $\mathcal{W}$. Indeed, for any $x \in \mathcal{W}$, there exists some $w \in R$. If $w \notin I$, we are done. Otherwise, there must also exist some $v \in R'$ such that $d(v,x) \leq \varepsilon$. Hence $v \in A_w$, which implies that $v \in R''$.

Before going further, let us first recall the following well-known "wedding lemma" (see, for example, [11]):[15]

Lemma 1 *Let Y be a nonempty set, n some integer ≥ 1 and $A_1, ..., A_n$ be finite subsets of Y such that:*

$$\forall I \subset \{1, ..., n\}, \quad \#\big(\cup_{i \in I} A_i\big) \geq \#I. \tag{26}$$

Then, there exists a one-to-one mapping from I to $\prod_i A_i$.

In order to apply our wedding lemma, we need to verify:

$$\#R \leq \#R'' \leq \#(R \setminus I) + \#\big(\cup_{w \in I} A_w\big) \tag{27}$$

$$= \#R - \#I + \#\big(\cup_{w \in I} A_w\big). \tag{28}$$

This implies that $\#\big(\cup_{w \in I} A_w\big) \geq \#I$. Hence, there exists some one-to-one mapping $\psi : R \to \cup_{w \in R} A_w \subset R'$ such that $\psi(w) \in A_w$, $\forall w \in R$. Since $\#R = \#R'$, ψ is also onto. The inequality announced in the claim follows from the triangle inequality of the distance d.

In order to prove the theorem, take any sequence $(\varepsilon_n)_n$ of positive real numbers $\varepsilon_n \leq \varepsilon$ converging to 0 and, for every n, an ε_n-network R_n, of minimal cardinality $N(\varepsilon_n) = N_n$. Let us denote by:

$$\lambda_n := \frac{1}{n} \sum_{w \in R_n} \delta_w \tag{29}$$

the uniform probability measure over R_n. For any element $g \in G$, the finite network $R'_n := gR_n$ is still an ε-network of $\mathcal{W}$. Indeed, if $w \in \mathcal{W}$ and $x \in R_n$ such that $d(g^{-1}w, x) \leq \varepsilon_n$, one has:

$$d(w, gx) = d(g^{-1}w, x) \leq \varepsilon_n \leq \varepsilon. \tag{30}$$

Take any bijection ψ as in the preceding claim, any function $F \in \mathcal{C}^0(\mathcal{W})$, and denote:

$$\alpha_n := \sup\big\{|F(w) - F(v)|,\ w, v \in \mathcal{W} \ / \ d(w, v) \leq 2\varepsilon_n\big\}. \tag{31}$$

We have:

$$\int_{\mathcal{W}} F(gw)\lambda_n(dw) - \int_{\mathcal{W}} F(w)\lambda_n(dw) = \frac{1}{N_n}\Big[\sum_{w \in R_n} F(gw) - \sum_{w \in R_n} F(w)\Big] \tag{32}$$

$$= \frac{1}{N_n}\Big[\sum_{w \in R'_n} F(w) - \sum_{w \in R_n} F(w)\Big] = \frac{1}{N_n}\Big[\sum_{w \in R_n} (F(\psi(w) - F(w))\Big]. \tag{33}$$

It follows that:

$$|\int_{\mathcal{W}} F(gw)\lambda_n(dw) - \int_{\mathcal{W}} F(w)\lambda_n(dw)| = \frac{1}{N_n} \sum_{w \in R_n} |F(\psi(w) - F(w)| \leq \alpha_n. \tag{34}$$

Banach-Alaoglu's theorem implies that the sequence $(\lambda_n)_n$ of probability measures admits a subsequence that converges for the weak-$*$ topology to some probability measure, say, λ. On the other hand, since $\mathcal{W}$

[15] The interpretation should be clear: A_i is the set of boyfriends of Ms. i; if a certain collection of ladies I put their boyfriends in common, the number of men they get is at least as high as $\#I$. The conclusion of the lemma becomes then obvious: the n ladies will be able to marry without practicing polyandry.

is $\sigma(L_\infty, L_1)$-compact, F is uniformly continuous, so that $\alpha_n \to 0$ as n grows to infinity. Moreover, the mapping $w \mapsto F \circ g(w)$ is $\sigma(L_\infty, L_1)$-continuous. Hence, (34) yields, by passing to the limit:

$$\int_{\mathcal{W}} F(gw)\lambda(dw) = \int_{\mathcal{W}} F(w)\lambda(dw), \quad \forall g \in G \tag{35}$$

In order to conclude the proof of the Theorem, consider the application $F_{p,x} : w \mapsto w(p,x)$. $F_{p,x} : (\mathcal{W}, d) \to \mathbb{R}$ is continuous. Thus, the preceding equality yields:

$$\int_{\mathcal{W}} T[w](p,x)d\lambda = \int_{\mathcal{W}} wd\lambda, \quad \forall p, x, T \in \mathcal{T} \tag{36}$$

□

It should be noted that the measure λ is, in general, <u>not</u> the Haar measure of any (locally compact) group. What theorem 2 does is essentially to provide sufficient conditions ensuring that λ can be viewed as <u>the</u> Haar measure on the group G, and to take advantage from the uniqueness of this last measure.

6.2. Proof of Corollary 1

Since $\mathcal{W}$ is compact, the space of continuous functions on $\mathcal{W}$ is separable, i.e., admits a countable and dense subset $(f_n)_n$. Hence, the weak$-*$ topology on $\Delta(\mathcal{W})$ can be metrized by, e.g., the distance induced by the countable collection of semi-norms $p_f(\mu) := \int_{\mathcal{W}} |f(w)|\mu(dw)$:

$$d(\mu, \nu) = \sum_{i=1}^{\infty} \frac{1}{2^i} \frac{p_{f_i}(\nu - \mu)}{1 + p_{f_i}(\nu - \mu)}. \tag{37}$$

With the notations introduced in the proof of Theorem 1:

$$|\int_{\mathcal{W}} F(gw)\lambda_n(dw) - \int_{\mathcal{W}} F(w)\lambda_n(dw)| \leq |\int_{\mathcal{W}} F(gw)\lambda_n(dw) - \int_{\mathcal{W}} f_i(gw)\lambda_n(dw)| +$$

$$|\int_{\mathcal{W}} f_i(gw)\lambda_n(dw) - \int_{\mathcal{W}} f_i(w)\lambda_n(dw)| + |\int_{\mathcal{W}} F(w)\lambda_n(dw) - \int_{\mathcal{W}} f_i(w)\lambda_n(dw)|. \tag{38}$$

For f_i $\frac{\varepsilon}{3}$-close to F, if $\alpha_n = \frac{\varepsilon}{3}$, this yields:

$$|\int_{\mathcal{W}} F(gw)d\lambda_n - \int_{\mathcal{W}} F(w)d\lambda_n| \leq \varepsilon. \tag{39}$$

Hence, it suffices to take $\nu = \lambda_n$ for n large enough.

□

6.3. Proof of Proposition 1

It suffices to show that, for any $w \in \mathcal{W}$ and any $\varepsilon > 0$, there exists a collection $(g_1, ..., g_n) \in G^n$ such that

$$\mathcal{W} = \cup_{i=1}^{n} B(g_i w, \varepsilon). \tag{40}$$

This easily follows from the pre-compactness of $\mathcal{W}$ and assumption 2. In turn, (40) implies that each open ball $B(g_i w, \varepsilon)$ must be non-negligible with respect to λ. Indeed,

$$1 = \lambda(\mathcal{W}) \leq \sum_i \lambda\big(B(g_i w, \varepsilon)\big) = \sum_i \lambda\big(g_i B(w, \varepsilon)\big) \tag{41}$$

$$= \sum_i \lambda\big(B(w, \varepsilon)\big) = n\lambda\big(B(w, \varepsilon)\big). \tag{42}$$

The first equality comes from the fact that G operates isometrically; the second from the G-invariance of λ.

□

6.4. Proof of Theorem 2

Thanks to assumption 4 (i), we can identify each element $g \in G$ with its (continuous) canonically associated mapping on $\mathcal{W}$, $\varphi_g : \mathcal{W} \to \mathcal{W}$:

$\forall w \in \mathcal{W}, \quad \varphi_g(w) = gw.$

On the other hand, let endow G with the following metric:

$$\delta(g,h) := \sup_{w \in \mathcal{W}} d(g(w), h(w)) \quad g,h \in G. \tag{43}$$

It easily follows that the family of mappings $\varphi_g : w \mapsto gw,\ g \in G$ is equi-continuous. Indeed, for any $\varepsilon > 0$, one has:

$$\forall w, v \in \mathcal{W}, \forall g \in G,\ d(w,v) \leq \varepsilon \Rightarrow d(g(w), g(v)) \leq \varepsilon. \tag{44}$$

Thanks to Ascoli's theorem, (G, δ) is therefore relatively compact. But assumption 3 (ii) says precisely that (G, δ) is closed. Hence, G is now a compact topological group. Consider the right-hand translation:

$$R_g(h) = hg \quad g,h \in G. \tag{45}$$

One has:

$$\delta(R_g(h) - R_g(h')) = \sup_w d(hg(w), h'g(w)) = \sup_w d(h(w), h'(w)) \tag{46}$$

because of assumption 2 and of the distance-preserving property of any g in G. Thus, $R_g(\cdot)$ is an isometry. We therefore can apply Theorem 1 on the group G itself, viewed as operating on itself <u>via</u> $R_g(\cdot)$. Thus, that there exists a probability μ on (G, d) verifying, for any continuous mapping $F : (G, d) \to (G, d)$:

$$\int_G F(hg)\mu(dh) = \int_G F(h)\mu(dh) \quad g \in G. \tag{47}$$

Obviously, μ is the Haar measure associated with (G, d). Let fix λ, a 'uniform distribution', $w \in \mathcal{W}$, $g \in G$ and $F \in \mathcal{C}^0(\mathcal{W}, \mathcal{W})$. One has:

$$\int_{\mathcal{W}} F(w)\lambda(dw) = \int_{\mathcal{W}} F(gw)\lambda(dw). \tag{48}$$

Let us integrate both terms of the last equality with respect to $\mu(dg)$. Since F is continuous, hence bounded, Fubini's theorem yields:

$$\int_{\mathcal{W}} F(w)\lambda(dw) = \int_{\mathcal{W}} \lambda(dw)\Big[\int_G F(gw)\mu(dg)\Big]. \tag{49}$$

By assumption 3, for each $(v, w) \in \mathcal{W}^2$, there exists a $h \in G$ such that $w = hv$. Thus,

$$\int_G F(gw)\mu(dg) = \int_G F(ghv)\mu(dg) = \int_G F(gv)\mu(dg). \tag{50}$$

It follows that:

$$\int_{\mathcal{W}} F(w)\lambda(dw) = \int_{\mathcal{W}} \lambda(dw)\Big[\int_G F(gw)\mu(dg)\Big] = \int_G F(gw)\mu(dg). \tag{51}$$

Hence the result follows from the uniqueness of the Haar measure (see [4, chap. 7(1), Theorem 1, p.13]).

□

4.5. Proof of Proposition 2

Since the arguments are the same in both cases, we focus on case (i). Suppose that $w_j(\overline{p},\cdot)$ is not constant, and that $s := \lim_{x\to\infty} w_j(\overline{p},x)$ exists. By Assumption 2(iii), $T_\alpha[w] \in \mathcal{W}$, where $T_\alpha[w](p,x) := w(p,\alpha x)$ with $\alpha > 0$. Then, $(w^n)_{n\in\mathbb{N}}$ is a sequence in $\mathcal{W}$ such that, for any fixed $x > 0$, the sequence $\big(w_j^n(\overline{p},x)\big)_n = \big(w_j(\overline{p},nx)\big)_n$ converges to s, i.e., the sequence $\big(w_j^n(\overline{p},\cdot)\big)_n$ of functions converges pointwise to the constant function $w_j^\infty(\overline{p},\cdot) \equiv s$.

Since $w_j(\overline{p},\cdot)$ is not constant, there must exist some $x_0 > 0$ such that $w_j(\overline{p},x_0) \neq s$. This implies for $n \in \mathbb{N}$:

$$\begin{aligned}
\|w_j^n(\overline{p},\cdot) - w_j^\infty(\overline{p},\cdot)\|_\infty &= \sup_{x>0} |w_j^n(\overline{p},x) - s| \\
&\geq |w_j(\overline{p},n\frac{x_0}{n}) - s| \\
&= |w_j(\overline{p},x_0) - s| > 0.
\end{aligned}$$

Hence, any subsequence of $\big(w_j^n(\overline{p},\cdot)\big)_n$ does not converge uniformly to the pointwise limit $w^\infty(\overline{p},\cdot)$. As a consequence, $(w_j^n)_n$ and $(w^n)_n$ have no converging subsequence in $\big(\mathcal{W},\|\cdot\|_\infty\big)$. This implies that $\big(\mathcal{W},\|\cdot\|_\infty\big)$ is not compact, contradicting Assumption 2(ii).

□

7. References

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