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Antoine Mandel

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Changes in the firms behavior after the opening of an allowance market [1]

Antoine Mandel [2] [3]

Centre d'Economie de la Sorbonne, UMR 8174, CNRS-Université Paris 1.

Abstract

This paper focuses on the influence of the opening of a market of allowances, such as the European Union Emission Trading Scheme, on the general equilibrium of an economy. Assuming there existed an equilibrium before the opening of this new market, we describe the changes in the firms behavior which guarantee that an equilibrium can be reached in the enlarged economy. The existence of an equilibrium in this framework can then be interpreted as ensuring the economy has the capacity to undergo the opening of a market of allowances without too important modifications in its organisation.

Key Words: General Equilibrium Theory, Existence of Equilibrium, Externalities, Increasing Returns, Markets of allowances.

[1] This paper is a substantial improvement of previous work in collaboration with Alexandrine Jamin (see (15)).

[2] The author is grateful to Professor Jean-Marc Bonnisseau for his guidance and many useful comments. All remaining errors are mine.

[3] antoine.mandel@malix.univ-paris1.fr

1 Introduction

This paper proposes a general equilibrium analysis of an economy undergoing the opening of a market of allowances. The motivation for such a study comes from the promotion of greenhouse gases emissions trading as a key instrument to reach the objectives of the Kyoto Protocol. A general equilibrium approach on the issue seems necessary because the amounts of trades on emission allowance markets may be large enough to influence the whole economy and because emission trading can difficultly be considered separately from the energy markets. Also, markets of allowances maintain close relationships with economic theory as their origin can be found in the Coase theorem.

The previous general equilibrium literature (see Laffont (18), Boyd and al. (5), Conley and al. (9)) has focused on the existence of equilibrium with markets of allowances, taking the presence of such markets as a fact. We put the emphasis on the effects of the *creation* of an allowance market. The opening of new markets is a topic at the frontier of general equilibrium theory. Apart some recent contributions in the theory of incomplete markets (see Cass and al. (6) and Elul (12)), general equilibrium models usually consider the set of markets is fixed. This is emphasized by the assumption of market completeness or in the Schumpeterian analysis of economic evolution,(20), in which the opening of new markets is one of the dynamic phenomenon occurring in between, almost in opposition with, a sequence of general equilibria.

However, it seems to us that the actual *creation* of markets of greenhouse emissions allowances, such as the European Union Emission Trading Scheme (EUETS), raises inevitably the question of the consequences of the opening of a new market on the existence of a general equilibrium. Taking into consideration the dynamical perspective imposed by the notion of *creation* of a market, we formulate our main interrogation as: *"Which additional conditions ensure the existence of an equilibrium in an economy with a market of allowances knowing that there existed an equilibrium in the economy without such a market?"*

Of course, such a question is relevant only when one can not apply the standard existence results (in our framework Bonnisseau-Cornet (3) and Jouini (16)) to the economy with an allowance market. We argue this is the case. First it is unlikely that a global free-disposal assumption holds, because when it wastes part of its inputs a firm may incidentally pollute. Also, firms may suffer unbounded losses because of the cost of the allowances. Finally and most importantly, as its market is newly opened and as its *"legal essence"* makes it different from the other commodities, it seems disputable to posit directly assumptions on the agents characteristics in the enlarged economy which would neglect those differences.

Our analysis is conducted in a framework where the producers behavior is represented by general pricing rules. This allows us to encompass increasing returns to scale as well as competitive behavior. It seems important to encompass both cases as many of the firms subject to the greenhouse gases emissions reduction schemes are in the energy sector where the presence of increasing returns is commonly recognized and also because marginal pollution may well be decreasing. On another hand, pricing rules provide a convenient tool to represent changes in the firms behavior, after a slight change of perspective on their interpretation. They are not seen as the local couterpart of a general principle such as profit maximization or marginal pricing but rather as a set of constraints on the acceptable prices determining locally the firms behavior. Concerning the consumption side of the economy, the main particularity of our model is that agents may face a negative external effect because of the firms pollution. They can purchase allowances as a public good in order to prevent it.

Our approach to prove the existence of an equilibrium is to posit separately assumptions on the initial functioning of the economy and on the changes in the firms behavior following the opening of the allowance market. First, we use standard sufficient assumptions (see (3) and (16)) to ensure the existence of an equilibrium in the initial economy. Second we give conditions on the changes in the firms behavior which ensure that a gradual increase in the allowance price leads to a general equilibrium for arbitrary initial endowments in allowances. Accordingly, our results link the range of initial endowments in allowances for which there exists an equilibrium with the flexibility and the sensitivity of the pricing rules with regards to the price of the allowance. Meanwhile we provide a contribution to the theory of general equilibrium with increasing returns as we indeed prove existence of equilibrium without some of the standard assumptions such as free-disposability, bounded losses or positive values of the pricing rules (see Jouini (17) and Giraud (14)).

2 The Model

2.1 Initial economy

We consider an initial economy[4] with a finite number L of commodities labeled by $\ell = 1 \dots L$, n firms indexed by $j = 1 \dots n$ and m consumers indexed by $i = 1 \dots m$. This economy is lying within an environment whose state is denoted by a real parameter $\tau \in \mathbb{R}_-$. The state of the environment (for example the atmospheric concentration of greenhouse gases) is altered by the

[4] Notations: in the latter, R^L_{++} denotes the positive orthant of R^L, R^L_+ its closure, S the simplex of $\mathbb{R}^L$, S_{++} its relative interior and $\mathcal{H}$ the affine space it spans. Also e denotes the vector $(\frac{1}{L}, \dots, \frac{1}{L})$ of $\mathbb{R}^L$

production process and influences the consumers welfare. We focus on a situation where a market of allowances for environmental damages emerges whereas firms were used to pollute freely. Our aim is to study how the firms should then actualize their behavior in order to let a new general equilibrium come out. We formalize the situation as follows:

The production possibilities of firms in terms of 1 to L commodities are described by sets Y_j such that:

Assumption (Initial Production (IP)) *For all j,*

(1) Y_j *is closed;*
(2) $0 \in Y_j$;
(3) $Y_j - \mathbb{R}^L_+ \subset Y_j$;
(4) If , $(y_j) \in \prod_{j=1}^n \mathcal{A}Y_j$ *and* $\sum_{j=1}^n y_j \geq 0$ *then for all* j, $y_j = 0$.

Those assumptions are standard and ensure that, inaction is possible for every firm, firms can freely-dispose of commodities[5] , free-production is impossible asymptotically.

As they produce, firms influence the environment. We measure according to the function $f_j : \mathbb{R}^L \to \mathbb{R}_-$ the minimal damage caused to the environment by firm j (we speak of minimal damage because firms may be inefficient and pollute more than what they actually need to). The actual state of the environment when the firms choose a production scheme $(y_j) \in \prod_{j=1}^n Y_j$ is at least as bad as $\sum_{j=1}^n f_j(y_j)$ (the state of the environment is getting worse as this parameter decreases). We assume that the pollution function satisfies the following requirements:

Assumption (Pollution Function (PF)) *For all j, $f_j : \mathbb{R}^L \to \mathbb{R}_-$ is differentiable, has values in $\mathbb{R}_-$ and satisfies $f_j(0) = 0$*

In the initial economy, the environment has no economic value so that the commodities prices are the only relevant variable for the firms. We let each firm determine its choices of production according to a pricing rule $\phi_j : \partial Y_j \to R^L_+$. That is the price $p \in \mathbb{R}^L_+$ of the commodities 1 to L, is acceptable for firm j given a production plan $y_j \in Y_j$ if $p \in \phi_j(y_j)$. Such a behavior coincide with profit maximization when the Y_j are convex and ϕ_j is the normal cone to Y_j. We assume

Assumption (Initial Pricing Rules (IPR)) *For all j,*

(1) ϕ_j *has a closed graph.*

[5] Under this assumption, according to lemma 5 in Bonnisseau-Cornet (3), ∂Y_j can be endowed with a manifold structure by homeomorphism with $e^\perp$. In the latter we will consider that this identification holds.

(2) For all $y_j \in \partial Y_j$, $\phi_j(y_j)$ is a non-empty closed convex cone[6] *of $\mathbb{R}^L_+$ different of $\{0\}$.*

Concerning the consumers, they gain utility from the consumption of non-negative quantities of commodities 1 to L and also are sensitive to the state of the environment. Their preferences are represented by an utility function u_i defined on $\mathbb{R}^L_+ \times \mathbb{R}$ which associates to a bundle, $x \in \mathbb{R}^L_+$, of commodities and to an environmental parameter $\tau \in \mathbb{R}$, an utility level $u_i(x, \tau)$. Their wealth comes from an initial endowment in commodities, $\omega_i \in \mathbb{R}^L_{++}$ and from an amount $r_i(\pi_1, \ldots, \pi_n)$ of the firms profits and losses $(\pi_1, \ldots, \pi_n)$. The private property case where each agent i holds a share $\theta_{i,j}$ in firm j profits is encompassed in this setting and will serve as a benchmark. Those characteristics are assumed to satisfy the following assumptions:

Assumption (C) *For all i,*

(1) u_i is quasi-concave and C^1 on $\mathbb{R}^L_{++} \times \mathbb{R}$;
(2) u_i is monotonic;
(3) $\forall \tau \in \mathbb{R}_- \ \forall x \in \mathbb{R}^L_+ \ \forall v \in \mathbb{R}^L_+ - \{0\} \ \exists k \geq 0$ such that $u_i(x+kv, \tau) > u_i(x, 0)$;
(4) $\omega_i \in \mathbb{R}^L_{++}$;
(5) $r_i : \mathbb{R}^L \to \mathbb{R}$ is continuous and $\sum_{i=1}^m r_i(\pi_1, \ldots, \pi_n) = \sum_{j=1}^n \pi_j$.

All those assumptions are standard but $C(3)$ which guarantees that a large enough increase in the consumption of any commodity can always compensate the deterioration of the environment. The consumers behavior is then determined by the prices $p \in \mathbb{R}^L_+$ of the commodities 1 to L as they maximize the utility they gain from consumption of those commodities, under their budget constraint and taking the state of the environment as given.

We can then define an equilibrium of the initial economy as:

Definition 1 *An equilibrium of the initial economy is a collection $(\overline{p}, (\overline{x}_i), (\overline{y}_j, \overline{t}_j))$ in $S_{++} \times (\mathbb{R}^L_+)^m \times \prod_{j=1}^n (Y_j \times \mathbb{R}_-)$ satisfying*

(1) for every i, $\overline{x}_i$ maximizes $u_i(\cdot, \sum_{j=1}^n \overline{t}_j)$ in the budget set $B_i(\overline{p}, (\overline{y}_j)) := \{x_i \in \mathbb{R}^\ell_+ \mid \overline{p} \cdot x_i \leq \overline{p} \cdot w_i + r_i(\overline{p} \cdot \overline{y}_j)\}$;
(2) for every j, $\overline{y}_j \in \partial Y_j$, $\overline{t}_j \leq f_j(\overline{y}_j)$ and $\overline{p} \in \phi_j(\overline{y}_j)$.
(3) $\sum_{i=1}^m \overline{x}_i = \sum_{j=1}^n \overline{y}_j + \sum_{i=1}^m w_i$.

In order to ensure that there exists such an equilibrium we posit standard sufficient assumptions for existence of equilibrium with general pricing rules (see (4), (16)). On the one hand,we shall assume that the producers follow the marginal pricing rule or some pricing rule with bounded losses.

Assumption (Initial Standard Pricing Rules (ISPR)) *One of the fol-*

[6] By cone we mean a set C such that for all $c \in C$ and all $\lambda \geq 0$ one has $\lambda c \in C$.

lowing holds:

(1) For all j, ϕ_j has bounded losses: there exist $m_j \in \mathbb{R}$ such that if $(p, y_j) \in S \times \partial Y_j$ and $p \in \phi_j(y_j)$, one has $p \cdot y_j \geq m_j$.
(2) For all j, ϕ_j is the marginal pricing rule given by Clarke's Normal cone to Y_j, that is $\phi_j(y_j) = N_{Y_j}(y_j)$ (see (8)).

On the other hand, a survival assumption must ensure that the economy produces enough wealth in a sufficiently large range of situations.

Assumption (Initial Survival (IS)) *For all $\omega' \geq \omega$, for all $(p, (y_j)) \in S \times \prod_{j=1}^n Y_j$ such that $p \in \cap_j \phi_j(y_j)$ and $\sum_{j=1}^n y_j + \omega' \geq 0$ one has $p \cdot (\sum_{j=1}^n y_j + \omega') > 0$.*

Finally, in order to ensure each consumer receives a positive wealth, we posit:

Assumption (Initial Revenue (IR)) *For all $(p, (y_j)) \in S \times \prod_{j=1}^n Y_j$ such that $\sum_{j=1}^n y_j + \sum_{i=1}^m \omega_i \geq 0$ and $p \cdot (\sum_{j=1}^n y_j + \sum_{i=1}^m \omega_i) > 0$, one has for all i, $p \cdot \omega_i + r_i(p \cdot y_j) > 0$.*

Those assumptions guarantee the existence of an equilibrium in the initial economy in the sense of:

Theorem 1 *Under assumption (IP), (PF), (C), (IPR), (ISPR), (IS) and (IR), there exist an equilibrium in the initial economy.*

Proof: *Cf appendix.*

One can note that if the agents wealths are set according to a private property revenue scheme, the preceding assumptions clearly hold when the producers are competitive (i.e profit maximizers with convex production sets). More generally, they hold when the pricing rules are loss-free, i.e for all $(p, y_j) \in S \times \partial Y_j$ such that $p \in \phi_j(y_j)$, one has $p \cdot y_j \geq 0$. This encompasses the case of marginal pricing rule when the production sets are star shaped with respect to 0. Those particular cases are further discussed in the example section.

3 Economies with an allowance market

Let us now consider that in order to limit the environmental damages due to production, the government forces by legal means the firms to use as input in their production process a quantity of allowances corresponding to their actual influence on the environment. Namely, when firm j deteriorates the environment of t_j, it must use as input a quantity t_j of allowances. Meanwhile the government supplies allowances to the economy by initially allocating a quantity A to consumers and producers according to the vector

$a = (a_1, \cdots, a_m, a_{m+1}, \cdots, a_{m+n}) \in \mathbb{R}^{m+n}$ with $\sum_{i=1}^m a_i + \sum_{j=1}^n a_j = A$. The government hence limits the deterioration of the state of the environment to the level $-A$. Now, this initial allocation may not be efficient and agents may gain to trade allowances. Hence an allowance market emerges and the agents should consequently modify their behavior.

3.1 Technical changes in the production sector

First, the relevant production set for firm j now is:

$$Z_j := \{(y_j, t_j) \in Y_j \times \mathbb{R}_- \mid t_j \leq f_j(y_j)\}$$

Note that under assumptions (IP) and (PF), Z_j is closed, contains 0 and satisfies asymptotically a no free-production condition. However, given our assumption on the pollution function, Z_j does not necessarily satisfy a general free-disposability assumption of the type $Z_j - \mathbb{R}_+^{L+1} \subset Z_j$. Indeed firms may have to increase their use of allowance in order to dispose of their other inputs: for example when a firm burns its waste inputs it produces CO_2 emissions as a by-product.

On another hand firms face an additional cost whose magnitude depend on the allowance price q. Given a price $(p, q) \in \mathbb{R}^{L+1}$ and a production plan $(y_j, t_j) \in Z_j$ the profit of firm j is $p \cdot y_j + q(a_j + t_j)$. They should consequently modify their pricing behavior. We shall denote by $\psi_j : \partial Z_j \to \mathbb{R}^{L+1}$ the pricing rule adopted by firm j in the enlarged economy. Hence, the price vector $(p, q) \in R^{L+1}$ is acceptable for firm j given the production plan $(y_j, t_j) \in \partial Z_j$ if and only if $(p, q) \in \psi_j(y_j, t_j)$.

3.2 Changes in consumers behavior

The changes which affect consumers characteristics are the modification of their consumption set which now is $\mathbb{R}_+^{L+1}$ and the modification of their revenue induced by the initial allocation of allowances and the changes in the firms profits. Given a production scheme $(y_j, t_j) \in \prod_{j=1}^n Z_j$ and a price $(p, q) \in \mathbb{R}^{L+1}$, the wealth distributed to consumer i now is $(p, q) \cdot (\omega_i, a_i) + r_i((p, q) \cdot (y_j, t_j + a_j))$.

3.2.1 Private use of the allowance

Now the changes concerning properly the consumers' behavior depend on their access to the allowance market. If they do not have access to the market as buyers, they behave as in the initial economy: given an environment τ, they maximize the utility $u_i(x_i, \tau)$ they gain from consumption of bundles $x_i \in \mathbb{R}_+^L$ of commodities, under the budget constraint $p \cdot x_i \leq (p, q) \cdot (\omega_i, a_i) +$

$r_i((p,q)\cdot(y_j,t_j+a_j))$. In this case, the allowance is only used by firms and as a private good. Hence we can define an equilibrium with private use of allowance (denoted for short private equilibrium) as:

Definition 2 *A private equilibrium of the enlarged economy is a collection* $((\overline{p},\overline{q}),(\overline{x}_i),(\overline{y}_j,\overline{t}_j))$ *in* $(S^L\times\mathbb{R}_+)\times(\mathbb{R}^L)^m\times\prod_{j=1}^n\partial Z_j$ *satisfying:*

(1) for every i, $\overline{x}_i$ *maximizes* $u_i(\cdot,\sum_{j=1}^n\overline{t}_j)$ *in the budget set* $B_i(\overline{p},(\overline{y}_j)):=\{x_i\in\mathbb{R}_+^\ell\mid\overline{p}\cdot x_i\le(\overline{p},\overline{q})\cdot(\omega_i,a_i)+r_i((\overline{p},\overline{q})\cdot(\overline{y}_j,\overline{t}_j+a_j))\}$;
(2) for every j, $(\overline{p},\overline{q})\in\psi_j(\overline{y}_j,\overline{t}_j)$;
(3) $\sum_{i=1}^m\overline{x}_i=\sum_{j=1}^n\overline{y}_j+\sum_{i=1}^m w_i$;
(4) $\sum_{i=1}^m a_i+\sum_{j=1}^n a_j+\sum_{j=1}^n\overline{t}_j=0$.

One can remark that in this framework the equilibrium state of the environment is exogenously fixed by the government through the initial allocation of allowances at $\sum_{i=1}^m a_i+\sum_{j=1}^n a_j$. This is the situation that prevails in some markets of allowances such as the European Union Emission Trading Scheme.

3.3 Public use of the allowance

When the consumers access to the allowance market is unrestricted, they may purchase it in order to prevent its use by the producers and hence improve the state of the environment. Their purchases benefit the other consumers so that the allowance turns out to be a public good. Namely, the utility of a consumption bundle $(x_i,s_i)\in\mathbb{R}_+^{L+1}$ for agent i given the quantity of allowances $\sum_{i=1}^m a_i+\sum_{j=1}^n a_j$ initially endowed to the economy and the quantities $(s_k)_{k\ne i}$ purchased by the other consumers is $u_i(x_i,-(\sum_{j=1}^n a_j+\sum_{i=1}^m a_i)+(\sum_{k\ne i}s_k+s_i))$. Given an environment $-(\sum_{j=1}^n a_j+\sum_{i=1}^m a_i)+\sum_{k\ne i}s_k$, consumer i is set to maximize the utility of its consumption bundle $(x_i,s_i)\in\mathbb{R}_+^{L+1}$, under the budget constraint $p\cdot x_i+q\cdot s_i\le(p,q)\cdot(\omega_i,a_i)+r_i((p,q)\cdot(y_j,t_j+a_j))$. We then define an equilibrium with public use of the allowance (denoted for short public equilibrium) as:

Definition 3 *A public equilibrium of the enlarged economy is a collection* $((\overline{p},\overline{q}),(\overline{x}_i,\overline{s}_i),(\overline{y}_j,\overline{t}_j))$ *in* $(S^L\times\mathbb{R}_+)\times(\mathbb{R}_+^{L+1})^m\times\prod_{j=1}^n\partial Z_j$ *satisfying:*

(1) for every i, $(\overline{x}_i,\overline{s}_i)$ *maximizes* $u_i(x_i,-(\sum_{j=1}^n a_j+\sum_{i=1}^m a_i)+(\sum_{k\ne i}s_k+s_i))$ *in the budget set* $B_i(\overline{p},(\overline{y}_j)):=\{(x_i,s_i)\in\mathbb{R}_+^{L+1}\mid(\overline{p},\overline{q})\cdot(x_i,s_i)\le(\overline{p},\overline{q})\cdot(\omega_i,a_i)+r_i((\overline{p},\overline{q})\cdot(\overline{y}_j,\overline{t}_j+a_j))\}$;
(2) for every j, $(\overline{p},\overline{q})\in\psi_j(\overline{y}_j,\overline{t}_j)$;
(3) $\sum_{i=1}^m\overline{x}_i=\sum_{j=1}^n\overline{y}_j+\sum_{i=1}^m w_i$;
(4) $\sum_{i=1}^m\overline{s}_i=\sum_{j=1}^n\overline{t}_j+\sum_{i=1}^m a_i+\sum_{j=1}^n a_j$.

4 Changes in the firms behavior and existence of equilibrium.

The existence of an equilibrium in the enlarged economy relies heavily on the modification of the firms behavior following the opening of the allowance market. Indeed, the producers may consider they can only handle small variation of the quantity of pollution they cause so that an equilibrium will fail to exist if the initial allocation of allowances is too low. Also, firms may undergo important losses because of the cost of the allowance input. This may lead the revenue of certain consumers below 0 and hence prevent the existence of an equilibrium.

Our aim in the following is to give conditions on the firms behavior (i.e on the pricing rules) which are sufficient to ensure existence of equilibrium in the enlarged economies, knowing that sufficient conditions for the existence of an equilibrium were satisfied in the initial economy.

4.1 Stability of the initial equilibrium

First, in order to remain in a workable framework we shall assume that the newly set pricing rule satisfy the regularity and homogeneity properties commonly used in the literature:

Assumption (PR)

For all j, ψ_j has a closed graph and convex values values in $\mathbb{R}^{L+1}$.

Note that we do not assume the enlarged pricing rules have positive values. Indeed, the lack of free-disposability makes it doubtful that such a condition always holds. In particular, it is not necessarily satisfied in the case of marginal pricing (see the examples section).

A second natural requirement concerns the compatibility of the firms behavior with this it had in the initial economy. Indeed when the allowance price is null it is from the firms point of view as if it was available in arbitrary high quantity, so that they can behave as in the initial economy. Hence we state:

Assumption (Compatibility)

$\forall y_j \in \partial Y_j$, *one has* $\{p \in \mathbb{R}^L \mid (p, 0) \in \psi_j(y_j, f_j(y_j))\} = \phi_j(y_j)$

This implies the equilibria of the initial economy coincide with the private equilibria of the enlarged economy with zero allowance price:

Lemma 1 *Assume that for all j, ψ_j satisfies (Compatibility). Then $(\overline{p}, (\overline{x}_i), (\overline{y}_j, \overline{t}_j))$ is an equilibrium of the initial economy if and only if there exist an allowance* *allocation* $((a_i),(a_j)) \in (\mathbb{R}^L)^{m+n}_+$ *such that* $\sum_{j=1}^n a_j + \sum_{i=1}^m a_i + \sum_{j=1}^n \bar{t}_j \geq 0$ *and* $(\bar{p}, 0, (\bar{x}_i), (\bar{y}_j, \bar{t}_j))$ *is a private equilibrium of the enlarged economy.*

As a corollary, under (Compatibility) there can exist equilibria with improved state of the environment (compared to the initial situation) only if firms are ready to accept positive prices for the allowance and to modify consequently their behavior. In this respect let us define:

Definition 4 *An allowance price* q *is called acceptable for firm* j *at* y_j *if there exist* $p \in S_{++}$ *such that* $(p,q) \in \psi_j(y_j, f_j(y_j))$*. We shall denote by* $Q_j(y_j) = \{q \in \mathbb{R}_+ \mid \exists p \in S_{++} \ s.t \ (p,q) \in \psi_j(y_j, f_j(y_j))\}$ *the set of allowance prices acceptable for firm* j *at* y_j*.*

In order to introduce some flexibility in the firms reaction to a change in the allowance price, we assume:

Assumption (Flexibility) *For all* j*, for all* $y_j \in \partial Y_j$*, the set* $Q_j(y_j)$ *is open in* $\mathbb{R}_+$*.*

Although it may not seem very demanding this assumption implies (cf. appendix) that the firms are ready to readjust in function of the allowance price, the prices they accept for the 1 to L commodities until one of those is zero. It holds in particular in the case of marginal pricing (cf the examples section) or whenever the behavior of the firm is determined by some function depending of the profit (e.g zero profit pricing rule).

This flexibility requirement ensures existence of equilibrium is locally stable to the perturbation induced by the opening of the allowance market in the sense of:

Theorem 2 *Under assumptions (IP), (PF), (C), (IPR), (IS), (ISPR), (IR), (PR), (Compatibility) and (Flexibility), there exists a neighborhood of zero in* $\mathbb{R}_+$*,* $\mathcal{O}$*, such that for every allowance price* $q \in \mathcal{O}$*, there exist an initial endowment in allowance* $((a_i),(a_j)) \in \mathbb{R}^{m+n}_+$ *such that the enlarged economy has a private equilibrium with allowance price equal to* q*.*

Proof: *Cf appendix.* [7]

The fact that the allowance price turns positive does not necessarily imply that the state of the environment is improved. Indeed the initial allocation $((a_i),(a_j))$ given by the preceding theorem may be constant for every $q \in \mathcal{O}$. In order to ensure the economy may undergo positive reductions of its use of allowances, one must impose further conditions on the influence of the allowance price on the firms behavior.

[7] In fact, the flexibility assumption may here be weaken to: if $0 \in Q_j(y_j)$ then $Q_j(y_j)$ is a neighborhood of 0.

A prerequisite therefore is to ensure that the economic activity remains viable even though the allowance price increases significatively. The new costs induced by the use of allowance as input may lead the firms to use less productive technology for the production of commodities. In turn, this may modify the value of the outcome of the economic process. The economic activity as a whole remains viable only if this value remains above zero. Mathematically, this comes to:

Assumption (SA) *For all* $((p,q),(y_j)) \in (S \times \mathbb{R}_+) \times \prod_{j=1}^{n} \partial Y_j$ *such that* $\sum_{j=1}^{n} y_j + \omega \geq 0$ *and* $(p,q) \in \cap_j \psi_j(y_j, f_j(y_j))$ *one has* $p \cdot (\sum_{j=1}^{n} y_j + \omega) > 0$.

This is a weak form of survival assumption as, contrary to assumption (IS) and to the usual survival assumptions of the literature (see (4), see (3)), it bares only on the set of attainable allocations. Hence it states that firms do not actually choose production plans such that the aggregate wealth is zero, whereas the usual survival assumptions (which bare on a larger set than this of attainable allocations) posit that the firms do not choose production plans which would, for even greater resources, lead to a null aggregate wealth. Also note that (SA) concerns only the value of the production in terms of 1 to L commodities. The allowance does not enter into consideration here, as at equilibrium no wealth is created or lost because of the operation of the allowance market. The working of this market only causes lump-sum wealth transfers.

Assumption SA suffices to guarantee that whatever the allowance price may be, the economic process is beneficial and hence a private equilibrium may be reached:

Theorem 3 *Under assumptions (IP), (C),(PF), (IPR), (IS), (ISPR), (IR), (PR), (Compatibility), (Flexibility) and (SA), for every non-negative allowance price q, there exist an intial endowment in allowance* $((a_i),(a_j)) \in \mathbb{R}^{m+n}$ *such that the enlarged economy admits a private equilibrium with allowance price equal to* q.

Proof: *Cf appendix.*

Remark 1 *Theorem* 3 *can be seen as a result of existence of equilibrium with fixed price of the allowance. Existence of fixed price equilibria are usually obtained (see Drèze (11)) by fixing constraints on supply or demand in the economy. Here the constraints bare on the initial endowments in allowance.*

Let us underline a few cases where the assumption (SA) is satisfied.

First, because of the interiority of the initial endowments, it holds whenever the firms maintain a positive level of profit on the commodities markets:

Assumption (Enlarged Loss Free (ELF)) *For all* $((y_j), p, q)$ *such that* $(p, q) \in \cap_j \psi_j(y_j, f_j(y_j))$ *and* $\sum_{j=1}^{n} y_j + \sum_{i=1}^{m} w_i \geq 0$ *one has* $p \cdot y_j \geq 0$.

If the initial pricing rules were loss free, (ELF) holds provided the firms, which face a new cost on the allowance market, do not simultaneously accept a diminution of their profits on the commodities markets:

Assumption (Increasing Tarification) *For all* $((y_j), p, q)$ *such that* $(p, q) \in \cap_j \psi_j(y_j, f_j(y_j))$ *and* $\sum_{j=1}^{n} y_j + \sum_{i=1}^{m} w_i \geq 0$ *, for all* j *there exist* $p_0^j \in \phi_j(y_j)$ *such that* $\frac{p}{\|p\|_1} \cdot y_j \geq \frac{p_0^j}{\|p_0^j\|_1} \cdot y_j$.

When there are losses on the commodities markets, in order to ensure (SA) holds, one must exclude explicitly the possibility of a totally inefficient aggregate production:

Assumption (Minimal efficiency) *For all* $((y_j), p, q)$ *such that* $(p, q) \in \cap_j \psi_j(y_j, f_j(y_j))$ *and* $\sum_{j=1}^{n} y_j + \sum_{i=1}^{m} w_i \geq 0$ *, one has* $\sum_{j=1}^{n} y_j \notin \mathbb{R}_-^L$;

and also assume the outputs are valuated at a positive price. In our framework, this second assumption may be justified if the allowance is a necessary input for the operation of each production technique. Indeed one can then assume a general raise of the output prices to compensate the cost of allowance.

Assumption (Output prices Raise) *For all* $((y_j), p, q)$ *such that* $(p, q) \in \cap_j \psi_j(y_j, f_j(y_j))$*,* $q > 0$ *and* $\sum_{j=1}^{n} y_j + \sum_{i=1}^{m} w_i \geq 0$*,one has:*
if $(\sum_{j=1}^{n} y_j)_h > 0$ *there exist* j *and* $p_0 \in \phi_j(y_j)$ *such that* $\frac{p_h}{\|p\|_1} > \frac{(p_0)_h}{\|p_0\|_1}$.

(Output Price Raise), and (Minimal Efficiency) guarantee assumption SA holds, because there always is at least an output evaluated at a positive price. More generally if pricing rules have positive values, it suffices to assume that the economy never wastes its entire resources (i.e for all $((y_j), p, q)$ such that $(p, q) \in \cap_j \psi_j(y_j, f_j(y_j))$ and $\sum_{j=1}^{n} y_j + \sum_{i=1}^{m} w_i \geq 0$ one has $\sum_{j=1}^{n} y_j + \omega \neq 0$) in order to ensure SA holds.

4.3 On the revenue assumption in the enlarged economy

Even-though they do not influence the aggregate wealth, transfers occurring on the allowance market matter because of their influence on the consumers revenue. Indeed, in order to ensure the existence of an equilibrium, one must guarantee that each consumer receive a positive part of the aggregate wealth. This condition may fail to hold when the losses on the allowance market are not well distributed. In order to prevent this failure, one can extend the initial revenue assumption to:

Assumption (Revenue (R)) *For all* $((p,q),(y_j)) \in (S \times \mathbb{R}_+) \times \prod_{j=1}^n Y_j$ *such that* $(p,q) \in \cap_j \psi_j(y_j, f_j(y_j))$, *and* $(p,q) \cdot (\sum_{j=1}^n y_j + \sum_{i=1}^m \omega_i, \sum_{j=1}^n a_j + \sum_{i=1}^m a_i + \sum_{j=1}^n f_j(y_j)) > 0$, *one has for all* i $(p,q) \cdot (\omega_i, a_i) + r_i((p,q) \cdot (y_j, a_j + f_j(y_j)) > 0$.

This comes to consider that there exist an efficient mechanism of wealth transfers which allocates the firms' losses among consumers.

Note that the initial revenue assumption guaranteed the existence of such a mechanism for the standard commodities markets only, what is not sufficient to ensure each agent receives a positive wealth for arbitrary allocation of allowances. Indeed consider a firm which makes a zero profit on the 1 to L commodities market and uses large quantities of allowances, it is going to support heavy losses when the allowance's price raises. An agent who owns a large share of this firm may see its revenue turn negative.

Nevertheless if the government targets precisely the needs of each firm in allowance so that there is no trade of allowances at equilibrium (that is one has for all j, $a_j = -f_j(y_j)$), then there are no losses on the allowance market and the initial revenue assumption is sufficient to ensure each consumer receives a positive wealth. Hence if one wants to dispense with the enlarged revenue assumption, one can consider in the following that the government targets precisely the needs of each firm in allowance. Our existence results (theorems 4 and 5) then remain valid if one reads "for every aggregate level of allowance" (allocated so that there are no losses on the allowance market) instead of "for every initial allocation of allowance."

4.4 Existence of Private Equilibrium for arbitrary allowance allocation

Finally, in order to obtain equilibria for arbitrary allowance allocations, the firms behavior must be amenable enough to the allowance price. Hence we state,

Assumption (Amenabilty) *For all* $\epsilon > 0$ *there exist* $K \geq 0$ *such that for all* $(p,q,(y_j)) \in (S \times \mathbb{R}_+) \times \prod_{j=1}^n Y_j$ *satisfying* $\sum_{j=1}^n y_j + \omega \geq 0$, $(p,q) \in \cap_j \psi_j(y_j, f_j(y_j))$, $p \in S_{++}$, $q \geq K$,

$$\textit{one has for all } j,\ f_j(y_j) \geq -\epsilon.$$

This says that when the allowance price is large enough compared to the commodities price, the only production plans acceptable for the firms are those which generate arbitrary low pollution and hence necessitate arbitrary low use of allowances as input. It entitles us to state our main results concerning the existence of equilibrium for arbitrary initial allocations in allowances.

Theorem 4 *Under assumptions (IP), (PF), (C) , (IPR), (IS), (ISPR), (IR), (PR), (Compatibility), (Flexibility), (SA),(R) and (Amenability), for every initial allocation of allowance* $((a_i),(a_j)) \in \mathbb{R}_+^{m+n}$*, the enlarged economy has an equilibrium with private use of allowance.*

Proof: *cf. Appendix*

4.5 Existence of Public equilibrium for arbitrary allowance allocation

We now turn to the existence of equilibrium with public use of the allowance. In this framework the demand in allowance of the consumers tends to push up the price as soon as the market opens. Hence the analogous of theorem 2 and 3 do not hold. However, one has:

Theorem 5 *Under assumptions (IP), (PF), (C) (IPR), (IS), (ISPR), (IR), (Compatibility), (Flexibility),(SA),(R) and (Amenability), for every initial allocation of allowance* $((a_i),(a_j)) \in \mathbb{R}_+^{m+n}$*, the enlarged economy has an equilibrium with public use of allowance.*

Proof: *Cf appendix.*

5 Examples

We shall now discuss to which extent the results stated in the preceding sections apply to commonly used pricing rules.

5.1 Business as usual

In order to set a benchmark, let us first consider the *Business as usual* situation where firms do not modify their behavior following the opening of the allowance market and where consumers do not have access to the market. That is firms keep maximizing the profit they make on the 1 to L commodities market and then purchase the quantity of allowance they need whatever its price may be, while consumers are only affected by wealth transfers. In this framework all the previous assumptions but (Amenability) hold so that there exist equilibria for every allowance price. However these equilibria in fact coincide with those of the initial economy and require a corresponding supply of allowances. In particular the state of the environment is not improved.

5.2 Global Loss Free

Let us now focus on the case where pricing rules are globally loss-free in the sense of:

Assumption (Global Loss Free) *For all j, for all $y_j \in \partial Y_j$, for all $(p,q) \in \psi_j(y_j, f_j(y_j))$, $p \cdot y_j + q f_j(y_j) \geq 0$,*

then assumption (SA) holds. Moreover (Amenability) clearly holds because the use of a fixed positive quantity of allowance for arbitrary high allowance price would entail losses. Hence one obtains using theorems 4 and 5 :

Corollary 1 *Under assumptions (IP), (PF),(C),(IPR), (IS), (ISPR), (IR), (PR), (Compatibility), (Flexibility), (Global Loss Free) and (R), for every initial allocation of allowance $((a_i),(a_j)) \in \mathbb{R}_+^{m+n}$, the enlarged economy has an equilibrium with public (resp. private use) of allowance.*

Note that this encompasses in particular the case of competitive behavior when the Y_j are convex sets containing zero and the pollution functions are concave. That is to say when the marginal returns are decreasing and the marginal pollution is increasing.

5.3 Marginal Pricing and Competitive Behavior

Let us now deal with the case of marginal pricing behavior. That is we consider the firms follow the marginal pricing rule given by Clarke Normal cone (see (8)) in the initial and in the enlarged economy. This also encompasses the case of competitive behavior when the production sets are convex.

We restrict attention to the case where the marginal pricing rule is loss-free in the initial economy, that is we shall posit

Assumption (Star-Shaped) *For all j, Y_j is 0-star-shaped.*

We shall also assume that the pollution increases with the scale of production:

Assumption (Increasing Pollution) *For all $(y_j) \in \prod_{j=1}^n Y_j$ such that $\sum_{j=1}^n y_j + \omega \geq 0$ (and $f_j(y_j) < 0$) the application $\mu \to f_j(\mu y_j)$ is (strictly) decreasing.*

and that there exist an input whose use does not decrease the marginal pollution (what is fairly natural as the use of additional inputs is likely to increase pollution). In differentiable terms, the assumption is:

Assumption (Input Increase) *For all j, for all $y_j \in Y_j$, one has $\nabla f_j(y_j) \notin \mathbb{R}_{--}^L$*

This suffices to guarantee the existence of a marginal pricing equilibrium.

Corollary 2 *Assume assumptions (IP),(PF), (C), (Interiority), (Star-Shaped), (Increasing Pollution), (Input Increase) and (R) hold. If each firm follows the marginal pricing rule then for every initial allocation of allowance* $((a_i),(a_j)) \in \mathbb{R}_+^{m+n}$*, the enlarged economy has a public (resp a private) equilibrium.*

Proof: *The marginal pricing rule in the initial economy is given by*

$$\phi_j(y_j) = N_{Y_j}(y_j)$$

and satisfies assumption (IPR) and (ISPR).

As mentioned above (Star-Shaped) implies the marginal pricing rule is loss-free in the initial economy. Together with the interiority of the initial endowments this ensures the satisfaction of assumptions (IS) and (IR) and the existence of a marginal pricing equilibrium in the initial economy according to theorem 1.

Now, in the enlarged economy, the marginal pricing rule is given by (see Clarke (8)):

$$\psi_j(y_j, f_j(y_j)) = (N_{Y_j}(y_j), 0) - \{\lambda(\nabla f_j(y_j), 1)\}_{\lambda \geq 0}$$

and satisfies assumption (PR) as well as (Compatibility).

Due to (Increasing Pollution), one has for all $(y_j) \in \prod_{j=1}^n Y_j$ $\nabla f_j(y_j) \cdot y_j \leq 0$. *This implies the (Enlarged loss Free) condition and therefore SA holds.*

On another hand (Input Increase) implies that whenever $p \in S_{++}$ *and* $(p,\lambda) \in \psi(y)$*, there exist* $p_0 \neq 0$ *in* $N_{Y_j}(y_j) \cap \mathbb{R}_+^L$ *such that* $p = p_0 - \lambda \nabla f_j(y_j)$*. Hence for* $\epsilon > 0$ *small enough there exist* $\mu \geq 0$ *such that* $\mu p_0 - (\lambda + \epsilon)\nabla f_j(y_j) \in S_{++}$*. Therefore, (Flexibility) holds.*

Finally, let us focus on the (Amenabiltiy) requirement. Let us consider $\epsilon > 0$ *and* $(y_j) \in \prod_{j=1}^n Y_j$ *such that* $\sum_{j=1}^n y_j + \omega \geq 0$ *and* $f_j(y_j) \leq -\epsilon$*. Due to the compacity of the set of attainable production allocation*[8]*, AT, one clearly has*

- $m = \sup\{\nabla f_j(y_j) \cdot y_j \mid (y_j) \in AT, \inf_j f_j(y_j) \leq -\epsilon\} < 0$, *thanks to (Increasing Pollution)*
- $M = \sup\{\sum_{j=1}^n \|y_j\| \mid (y_j) \in AT\}$ *is bounded.*

Let $\lambda \geq -\frac{2M}{m}$*. Now, assume there exist* $p \in S_{++}$ *such that* $(p,\lambda) \in \psi_j(y_j, f_j(y_j))$*. One has* $p + \lambda \nabla f_j(y_j) \in N_{Y_j}(y_j)$*, but* $(p + \lambda \nabla f_j(y_j)) \cdot y_j \leq M + \lambda m < 0$ *which contradicts the fact that the marginal pricing rule on* Y_j *is loss-free. Hence the (Amenability) assumption holds.*

[8] See the appendix, section "Equilibrium Correspondence" for a proper definition.

All the necessary assumptions for theorems 4 and 5 hold. It suffices to apply those results to end the proof.

Similar results holds for arbitrary pricing rules whenever the $Star - Shaped$ assumption is replaced by the assumption that the initial pricing rules ϕ_j are loss free and when the pricing rules of the enlarged economy are obtained by adding the marginal cost of the allowance used as input in the production process to the initial pricing rules. Namely, one has:

Corollary 3 *Assume assumptions (IP),(C), (PF), (IPR), (Increasing Pollution) and (Input Increase) hold. If the initial pricing rules ϕ_j are loss-free and the pricing rules in the enlarged economy are of the form*

$$\psi_j(y_j, f_j(y_j)) = (\phi_j(y_j), 0) - \{\lambda(\nabla f_j(y_j), 1)\}_{\lambda \geq 0},$$

then for every initial allocation of allowance $((a_i), (a_j)) \in \mathbb{R}_+^{m+n}$, the enlarged economy has a public (resp a private) equilibrium.

6 Appendix, proofs

6.1 Foreword

In order to prove existence of an equilibrium in the enlarged economy we can not use the seminal literature on increasing returns (among others (3) and (16)) because of the presence of externalities, the lack of free-disposability in the production process, the value of the enlarged pricing rules outside the positive orthant (e.g in the case of marginal pricing), and also because losses on the allowance market may be unbounded. Nevertheless it is easy to obtain an existence result in the initial economy. Our approach then is to perturb the equilibrium correspondence of the initial economy in a way such that new zeroes correspond to equilibria of the enlarged economies. We then use invariance properties of the degree (see Cellina (7)) in order to show that there actually exist such equilibria.

6.2 Characterization of consumers behavior

Let us first define the consumers demands. We consider the demand of agent i in the enlarged economy when the allowance consumption is restricted at a certain level $H \geq 0$:

Definition 5 *The demand of agent i, $\Delta_i^H : \mathbb{R}_- \times (S_{++} \times]-1, +\infty[) \times \mathbb{R}_+ \rightarrow \mathbb{R}^{L+1}$, is the correspondence which associates to a collection $(\tau, (p, q), w)$ of* *environment, prices and wealth the set of elements:*

$$\Delta_i^H(\tau,(p,q),w) = \{(\overline{x}_i,\overline{s}_i) \in \mathbb{R}_+^L \times [0,H] \mid u_i(\overline{x}_i,\tau+\overline{s}_i) = \max_{B_i((p,q),w)} u_i(x_i,\tau+s_i)\}$$

where $B_i((p,q),w) = \{(x_i,s_i) \in \mathbb{R}_+^L \times [0,H] \mid p \cdot x_i + q \cdot s_i \leq w\}$.

The restriction of allowance consumption below H is a technical trick to be able to deal simultaneously with public and private use of allowance. In particular when $H = 0$, Δ_i^0 is the consumer demand in the initial economy (and at a private equilibrium). This restriction also makes it licit to define the demand for negative allowance prices (the use of negative allowance price also is a technical trick which ensure that the equilibria with zero allowance price do not lie on the boundary of the domain of the equilibrium correspondence). Under assumption C, Berge's maximum theorem ensures that Δ_i^H is well defined and upper-semi-continuous. Moreover thanks to assumption C(3) it satisfies the following boundary condition:

For all τ*, for all* $((p^n,q^n),w^n)$ *converging to* (p,q,w) *such that* $w > 0$ *and* $p \in \partial S$ *one has for all i,* $lim_n \ \|proj_{\mathbb{R}^L}((\Delta_i^H(\tau,(p^n,q^n),w_n))\| = +\infty$.

The wealth of agent i, given prices $(p,q) \in (S\times]-1,+\infty[)$, production choices $(y_j) \in \prod_{j=1}^n Y_j$ and an initial allocation $((a_i),(a_j)) \in \mathbb{R}_+^{n+m}$ of allowances is

$$w_i((p,q),(y_j),(a_i),(a_j)) = (p,q)\cdot(\omega_i,a_i) + r_i((p,q)\cdot(y_j,f_j(y_j)+a_j)).$$

As this wealth may fail to be positive at some point we introduce following lemma 2 in Jouini (16) auxiliary income functions, in order to be able to define the equilibrium correspondence on a sufficiently large set.

Lemma 2 *Let* $V = \{((p,q),(y_j),(a_i),(a_j)) \in S_{++}\times]-1,+\infty[) \times \prod_{j=1}^n \partial Y_j \times \mathbb{R}_+^m \times \mathbb{R}_+^n \mid (p,q)\cdot(\sum_{j=1}^n y_j + \omega, \sum_{i=1}^m a_i + \sum_{j=1}^n a_j + \sum_{j=1}^n f_j(y_j)) > 0\}$ *there exist functions* $\tilde{r}_i : V \to \mathbb{R}$ *such that for all* $((p,q),(y_j),(a_i),(a_j)) \in V$,

(1) $\sum_{i=1}^m \tilde{r}_i((p,q),(y_j),(a_i),(a_j)) = (p,q)\cdot(\sum_{j=1}^n (y_j, a_j+f_j(y_j)) + (\omega, \sum_{i=1}^m a_i))$;
(2) for all i, $\tilde{r}_i((p,q),(y_j),(a_i),(a_j)) > 0$;
(3) if for all i, $w_i((p,q),(y_j),(a_i),(a_j)) > 0$ *then for all i,* $w_i((p,q),(y_j),(a_i),(a_j)) = \tilde{r}_i((p,q),(y_j),(a_i),(a_j))$.

Proof: *It suffices to set following (16), for all* $((p,q),(y_j),(a_i),(a_j)) \in V$*:*

$$\tilde{r}_i((p,q),(y_j),(a_i),(a_j)) = (1-\theta(w))\frac{\sum_{i=1}^m w_i}{m} + \theta(w)w_i$$

where $w = (w_i) = w_i((p,q),(y_j),(a_i),(a_j))$

$$and\ \theta(w) = \begin{cases} 1, & if\ for\ all\ i \quad w_i > 0 \\ \frac{\sum_{i=1}^m w_i}{\sum_{i=1}^m w_i - m\ \inf_i w_i}, & otherwise \end{cases}$$

6.3 Proof of theorem 1

We can then characterize the equilibria of the initial economy through the correspondence E_0 defined on $\{(p,(x_i),(y_j)) \in S_{++} \times (\mathbb{R}^L)^m \times \prod_{j=1}^n \partial Y_j \mid p \cdot (\sum_{j=1}^n y_j + \sum_{i=1}^m \omega_i) > 0\}$ by $E_0(p,(y_j)) =$

$$(proj_{e^\perp}(\sum_{i=1}^m x_i - \sum_{j=1}^n y_j - \sum_{i=1}^m \omega_i), (x_i, 0) - \Delta_i^0(\sum_{j=1}^n f_j(y_j), (p,0), \tilde{r}_i((p,0),(y_j),(0),(0)), (\overline{\phi}_j(y_j) - p)).$$

where for all j and all $y_j \in \partial Y_j$, $\overline{\phi}_j(y_j) := \phi_j(y_j) \cap S$.

It is a direct consequence of 4.3 in (19) that under assumptions (IP), (PF), (C), (IPR), (IS) and (IR) the zeroes of E_0 coincide with the set of equilibria of the initial economy and that the degree of this correspondence is non-zero. Hence, there exist equilibria in the initial economy. This proves theorem 1.

6.4 Parametrization by the allowance market

The opening of the allowance market influences the commodities markets in two principal ways. First, the firms modify their pricing behavior in function of the allowance price, second the consumers wealth is modified by the transfers taking place on the allowance market. Those influences might be represented as parameters influencing the equilibrium on the commodities markets. We study in the following an equilibrium correspondence hence parametrized . The initial allocation of allowances for which there exist an equilibrium are then determined endogenously as the allocations which clear the allowance market for some values of the parameters.

The parameter influencing the firms pricing rules is the allowance price. However, we would like to define parametrized pricing rules for every non-negative real number (even if this number is not an admissible allowance price for the firm). Therefore we have to use the following trick. We set for $\lambda \geq 0$ and $y_j \in \partial Y_j$:

- $\gamma_j(\lambda, y_j) = sup\{q \leq \lambda \mid \exists p \in S\ s.t\ (p,q) \in \psi_j(y_j, f_j(y_j))\}$
- $\overline{\phi}_j(\lambda, y_j) = \{p \in S \mid (p, \gamma_j(\lambda, y_j)) \in \psi_j(y_j, f_j(y_j))\}$
- $\overline{\psi}_j(\lambda, y_j) = (\overline{\phi}_j(\lambda, y_j), \gamma_j(\lambda, y_j))$.

The value of $\gamma_j(\lambda, y_j)$ coincide with the allowance price whenever the pricing rule indeed admits λ as a possible value for the allowance price in y_j. Otherwise it is equal to the largest admissible allowance price below λ. Such an element exists thanks to assumption (Compatibility) and because ψ_j has a closed graph. The assumption (PR) also implies that $\overline{\phi}_j$ and $\overline{\psi}_j$ are s.c.s with non-empty convex compact values.

Concerning the influence of the allowance market on the consumers wealth, one can not represent it using the initial allocation of allowances as a parameter because this allocation must be endogenously determined. However at equilibrium the quantity of allowances used in the economy must be equal to the initial allocation. Hence in order to endogenize the wealth transfers taking place on the allowance market, we implement fictive initial allocations in allowances as functions of the quantities of allowances used by the agents. Namely, we consider continuous mappings $\alpha : \mathbb{R}^{m+n} \to \mathbb{R}_+^{m+n}$ such that $\sum_{i=1}^m \alpha_i((s_i),(t_j)) + \sum_{j=1}^n \alpha_j((s_i),(t_j)) \equiv \sum_{i=1}^m s_i + \sum_{j=1}^n t_j$, and we interpret $(\alpha_i(s_i,t_j), \alpha_j(s_i,t_j))$ as the quantity of allowances allocated to consumers and producers when $((s_i),(t_j))$ are the quantity of allowances used by producers and consumers respectively. Using such a representation, the demands (x_i, s_i) of consumers correspond to as situation where the transfers on the allowance market are balanced if and only if, $(x_i, s_i) \in \Delta_i^H(\sum_{j=1}^n f_j(y_j) - s_i, p, q, \tilde{r}_i((p,q),(y_j),(\alpha_i(f_j(y_j),(s_i))),(\alpha_j(f_j(y_j),(s_i)))))$. In the following, we shall abusively let $\Delta_i^{\alpha,H}((p,q),(y_j),s_i)$ stand for $\Delta_i^H(\sum_{j=1}^n f_j(y_j) - s_i, p, q, \tilde{r}_i((p,q),(y_j),(\alpha_i(f_j(y_j),s_i)),(\alpha_j(f_j(y_j),s_i))))$.

6.5 Equilibrium Correspondence

Under assumptions (IP) and (C), the set of attainable commodities allocation, $\{((x_i),(y_j)) \in (\mathbb{R}_+^L)^m \times \prod_{j=1}^n Y_j \mid \sum_{i=1}^m y_j + \sum_{i=1}^m \omega_i = \sum_{i=1}^m x_i\}$ is compact. Hence there exist a compact ball K of R^L such that K^{m+n} contains it in its interior. Let us set $U = \{((p,q),(x_i,s_i),(y_j)) \in (S_{++} \times]-1,+\infty[) \times (int(K) \times]-1, H+1[)^m \times \prod_{j=1}^n \partial Y_j \mid p \cdot (y_j + \omega) + q \sum_{i=1}^m s_i > 0\}$,

We can now define an equilibrium correspondence parametrized by (α, λ, H) by setting: $F_1^{(\alpha,\lambda,H)} : U \to e^\perp \times \mathbb{R} \times (\mathbb{R}^{L+1})^m \times (e^\perp)^n$ equal to

$$(proj_{e^\perp}(\sum_{i=1}^m x_i - \sum_{j=1}^n y_j - \omega), q - \lambda, (\Delta_i^{\alpha,H}((p,q),(y_j),s_i) - (x_i,s_i)), \overline{\phi}_j(\lambda, y) - p)$$

F_1 is an equilibrium correspondence in the sense of the following lemma:

Lemma 3 *Assume (IP), (PF), (C), (IPR), (IS), (IR), (PR) (Compatibility) and (Flexibility) holds. Let* $((p,q),(y_j),(x_i),(s_i)) \in (F_1^{(\alpha,\lambda,H)})^{-1}(0,0,0,0)$, *such that* $w_i((p,q),(y_j),\alpha_i(f_j(y_j),s_i),\alpha_j(f_j(y_j),s_i)) > 0$. *One has:*

(1) if $H = 0$, $((p,q),(x_i),(y_j,f_j(y_j)))$ *is a private equilibrium for the initial allocation of allowances* $(\alpha_i(f_j(y_j),0),\alpha_j(f_j(y_j),0))$, *and* $q = \lambda$.

(2) if $s_i < H$, $((p,q),(x_i,s_i),(y_j,f_j(y_j)))$ *is a public equilibrium for the initial allocation of allowances* $(\alpha_i(f_j(y_j),s_i),\alpha_j(f_j(y_j),s_i))$, *and* $q = \lambda$.

Proof: *Indeed let us consider* $((p,q),(x_i,s_i),(y_j)) \in (F_1^{(\alpha,\lambda,H)})^{-1}(0,0,0,0)$.

Let us first show that for all j, $(p,q) \in \psi_j(y_j,f_j(y_j))$. *First one clearly has* $q = \lambda \geq 0$ *and hence* $p \in \overline{\phi}_j(q,y_j)$. *Assume* $(p,q) \notin \psi_j(y_j,f_j(y_j))$. *Under Compatibility and (PR), the only possibility is that* $q > \gamma_j(q,y_j)$ *and* $(p,\gamma_j(q,y_j)) \in \psi_j(y_j,f_j(y_j))$. *As* $p \in S_{++}$, *assumption (Flexibility) then implies there exist* q_1 *such that* $q > q_1 > \gamma_j(q,y_j)$ *and* $(p,q_1) \in \psi_j(y_j,f_j(y_j))$. *This contradicts the definition of* $\gamma_j(q,y_j)$. *Hence one has* $(p,q) \in \psi_j(y_j,f_j(y_j))$.

As consumer i demand of allowances is equal to s_i *and one always has* $\sum_{i=1}^m \alpha_i(f_j(y_j),s_i) + \sum_{j=1}^n \alpha_j(f_j(y_j),s_i) = \sum_{j=1}^n f_j(y_j) + \sum_{i=1}^m s_i$, *the allowance market is clear provided the initial allocation is equal to* $(\alpha_i(f_j(y_j),s_i),\alpha_j(f_j(y_j),s_i))$.

Now, one has $proj_{e^\perp}(\sum_{i=1}^m x_i - \sum_{j=1}^n y_j - \sum_{i=1}^m \omega_i) = 0$. *Walras law and clearance of the allowance market then imply clearance of the* 1 *to* L *commodities markets.*

Now, as $w_i((p,q),(y_j),\alpha_i(f_j(y_j),s_i),\alpha_j(f_j(y_j),s_i)) > 0$, *the auxiliary incomes coincide with the original ones and hence the auxiliary demand coincide with the original demand of consumer* i *when its consumption of allowance is restricted below* H.

Finally, if $H = 0$ *the demand in allowance coincides with this at a private equilibrium of the economy.*

If $s_i < H$, *it coincides with this at a public equilibrium of the economy.*

6.6 Main Lemma

The proofs of theorems 2 to 5 are based on the following lemma which shows that the degree of F_1 can be related to the degree of the initial equilibrium correspondence. Indeed, given (α,λ,H) let us consider the family of correspondences $F_t^{(\alpha,\lambda,H)} : U \to e^\perp \times \mathbb{R} \times (\mathbb{R}^{L+1})^m \times (e^\perp)^n$ defined by

$$(proj_{e^\perp}(\sum_{i=1}^m x_i - \sum_{j=1}^n y_j - \omega), q - t\lambda, (\Delta_i^{\alpha,tH}((p,q),(y_j),s_i) - (x_i,s_i)), \overline{\phi}_j(t\lambda,y) - p)$$

Now, it is clear that under (Compatibility), $((p,q),(x_i,s_i),(y_j)) \in (F_0^{(\alpha,\lambda,H)})^{-1}(0,0,0,0)$ if and only if $q = 0$, $s_i = 0$ for all i and $(p,(x_i),(y_j))$ is a zero of E_0. Moreover it is clear that whatever may (α, λ, H) be the degree of $F_0^{(\alpha,\lambda,H)}$ is equal to this of E_0 and hence is non-zero according to the proof of theorem 1.

Finally we show the degree of F_0 is equal to this of F_1.

Assume

Assumption (SA$^\lambda$) *For all $\mu \in [0, \lambda]$, for all $(p, (y_j)) \in S \times \prod_{j=1}^n \partial Y_j$ such that $\sum_{j=1}^n y_j + \omega \geq 0$ and $p \in \cap_j \overline{\phi}_j(\mu, y)$ one has $p \cdot (\sum_{j=1}^n y_j + \omega) > 0$,*

then

Lemma 4 *Under assumption (IP), (PF),(C),(IPR), (IS), (ISPR), (IR), (PR), (Compatibility), (Flexibility) and (SA$^\lambda$), $deg(F_0^{(\alpha,\lambda,H)}, (0,0,0,0)) = deg(F_1^{(\alpha,\lambda,H)}, (0,0,0,0)$.*

Proof: *Let λ such that SA^λ holds. For sake of clarity let us denote F_t instead of $F_t^{(\alpha,\lambda,H)}$. It is clear that F_t defines an homotopy between F_0 and F_1. Let us show that the set $\cup_{t\in[0,1]} F_t^{-1}(0)$ is compact. The homotopy invariance property of the degree then implies the result (see (7)).*

Indeed consider a sequence $(p^n, q^n, (x_i^n, s_i^n), (y_j^n)) \in \cup_{t\in[0,1]} F_t^{-1}(0,0,0,0)$. For all n, there exist t^n such that $F_{(t^n)}(p^n, q^n, (x_i^n, s_i^n), (y_j^n)) = 0$.

By construction the transfers on the allowance market are balanced. Hence, using Walras one obtains that $\sum_{i=1}^m x_i^n - \sum_{j=1}^n y_j^n - \omega = 0$. Therefore for all n ,$((x_i^n), (y_j^n))$ lies in the set of attainable allocations which is compact. Moreover one has $t^n \in [0,1]$, $p^n \in S$, $q^n \in [-1, \lambda]$, $s_i^n \in [-\epsilon, H+\epsilon]$. Hence $(t^n, (x_i^n, s_i^n), (y_j^n), p^n, q^n,)$ lie in a compact set and hence has a subsequence converging to $(t, (x_i, s_i), (y_j), p, q)$ where $t \in [0,1]$, $x_i \in K$ and $s_i \in [0, H]$, $\sum_{j=1}^n y_j + \omega = \sum_{i=1}^m x_i \geq 0$, $(p, q) \in S \times [-1, +\infty[$.

It remains to show that $(p, q, (x_i, s_i), (y_j)) \in U$ and that $F_t(p, q, (x_i, s_i), (y_j)) = (0,0,0,0)$.

First as $((x_i), (y_j))$ is an attainable allocation, one has $x_i \in int(K)$.

Second as Δ_i^H has values in $\mathbb{R}^L \times [0, H]$ it is clear that $s_i \in [0, H] \subset]-\epsilon, H+\epsilon[$.

Third as $q = t\lambda \geq 0$ one clearly has $q > -1$.

Fourth as $\overline{\phi}_j$ is s.c.s, one has for all j $p \in \overline{\phi}_j(t\lambda, y_j)$ and as $\sum_{j=1}^n y_j + \omega \geq 0$, assumption SA^λ implies that $p \cdot (\sum_{j=1}^n y_j + \omega) > 0$. Hence $(p, q, (y_j), \alpha_i((f_j(y_j)), (s_i)), \alpha_j((f_j(y_j)), (s_i))) \in V$ and the auxiliary individual income, $\tilde{r}_i$, all are strictly positive. Given the fact that x_i^n is bounded, the boundary condition on the demand then implies that $p \in S_{++}$.This proves that $(p, q, (x_i, s_i), (y_j)) \in U$.

Given the continuity properties of F_t and Δ_i, one then has $(x_i, s_i) \in \Delta_i(p, q, (y_j), s_i)$ for all i and $F_t((y_j), p, q, (s_i)) = 0$. This ends the proof.

6.7 Proof of theorem 2

Given the compactness of the attainable allocations and the s.c.s of the pricing rules, it is clear that assumption (SA^λ) holds for all λ in a neighborhood of zero. Hence one has according to lemma 4 that for all (α, H) and for λ in a neighborhood of zero, $deg(F_1^{(\alpha,\lambda,H)})$ is non-zero. Let us then set $\alpha_j(f_j(y_j), s_i) = f_j(y_j)$ and $\alpha_i \equiv 0$. For such an α assumptions (SA^λ) and (IR) imply that for all $((p,q),(x_i,s_i),(y_j)) \in (F_1^{(\alpha,\lambda,H)})^{-1}(0,0,0,0)$, one has $w_i((p,q),(y_j),\alpha_i(f_j(y_j),s_i),\alpha_j(f_j(y_j),s_i)) > 0$. It then suffices to apply lemma 3 to end the proof.

6.8 Proof of theorem 3

Assumption SA implies SA^λ holds for all $\lambda \geq 0$. Now if one chooses α as in the proof of theorem 2, it is clear that for all λ, for all $((p,q),(x_i,s_i),(y_j)) \in (F_1^{(\alpha,\lambda,H)})^{-1}(0,0,0,0)$, one has $w_i((p,q),(y_j),\alpha_i(f_j(y_j),s_i),\alpha_j(f_j(y_j),s_i)) > 0$. It then suffices to apply lemma 3 to end the proof.

6.9 Connectedness Lemma

In order to prove theorems 4 and 5 we shall use the following lemma. Under assumption SA, lemma 4 implies that for all (λ,α,H) the degree of $F_1^{(\alpha,\lambda,H)}$ is non-zero. For a given $\overline{\lambda}$, let us consider the family of correspondences $G_t = F_1^{(\alpha,t\overline{\lambda},H)}$ and let f be a continuous function on U.

Lemma 5 *Assume there exist $b > a$ reals such that $\inf_{\eta\in G_0^{-1}(0)} f(\eta) = b$ and $\sup_{\eta\in G_1^{-1}(0)} f(\eta) = a$. Then for all $c \in [a,b]$ there exists $t \in [0,1]$ and $\eta \in G_t^{-1}(0)$ such that $f(\eta) = c$.*

Proof: *Assume this does not hold, that is there exists $c \in [a,b]$ such that $\forall t \in [0,1]$, $\forall \eta \in G_t^{-1}(0)$ $f(\eta) \neq c$. Let then $a_1 = \sup\{z \leq b \mid z \notin f(\cup_{t\in[0,1]} G_t^{-1}(0)\}$. As $\cup_{t\in[0,1]} G_t^{-1}(0)$ is compact in U and f is continuous one has $a_1 \in f(\cup_{t\in[0,1]} G_t^{-1}(0))$ Hence there must exist $a_2 < a_1$ such that $[a_2, a_1[\in (f(\cup_{t\in[0,1]} G_t^{-1}(0))^c$.*

Let $V = U \cap f^{-1}]a_2, +\infty[$. V is an open set such that $G_0^{-1}(0) \subset V$ and $\cup_{t\in[0,1]} G_t^{-1}(0) \cap \partial V = \emptyset$. This implies first that $deg((G_0)_{|V}, 0) = deg((G_0), 0) \neq 0$. Second it implies that $\cup_{t\in[0,1]}((G_t)_{|V})^{-1}(0)$ is compact in V. Using conservation of the degree by homotopy, one gets $deg((G_1)_{|V}, 0) = deg((G_0)_{|V}, 0) \neq 0$

This contradicts the fact that every zero η of G_1 satisfies $f(\eta) \leq a$ and hence belongs to V^c.

6.10 *Proof of theorem* 4

Let us show that there exist a private equilibrium for every initial endowment in allowance $a \in \mathbb{R}_+^{n+m}$. Therefore, let us set $\alpha_j(f_j(y_j), s_i) = \frac{a_j}{\sum_{i=1}^m a_i + \sum_{j=1}^n a_j}(\sum_{j=1}^n f_j(y_j) + \sum_{i=1}^m s_i)$ and $\alpha_i(f_j(y_j), s_i) = \frac{a_i}{\sum_{i=1}^m a_i + \sum_{j=1}^n a_j}(\sum_{j=1}^n f_j(y_j) + \sum_{i=1}^m s_i)$. Under assumption (R) and (SA) it is clear that for such an α, for all λ, for all $((p,q),(x_i,s_i),(y_j)) \in (F_1^{(\alpha,\lambda,H)})^{-1}(0,0,0,0)$, one has $w_i((p,q),(y_j),\alpha_i(f_j(y_j),s_i),\alpha_j(f_j(y_j),s_i)) > 0$. So as in the proof of theorem 3 there exist a private equilibrium for all non-negative allowance price λ with an initial allocation of allowance made according to α, that is proportional to $((a_i),(a_j))$. It then remains to show that there exist an equilibrium with aggregate allowance supply exactly equal to $\sum_{i=1}^m a_i + \sum_{j=1}^n a_j$.

Now wether $\sum_{i=1}^m a_i + \sum_{j=1}^n a_j \geq \overline{a} := inf\{\sum_{j=1}^n f_j(y_j) \mid \ (p,(x_i),(y_j))$ equilibrium of the initial economy $\}$ and the proof is straightforward using lemma 1 and free-disposability of the allowance. More generally one can state there exist equilibria for every allowance allocation proportional to $((a_i),(a_j))$ and whose sum is in $[\overline{a}, +\infty[$.

Now wether $\overline{a} = 0$ and the proof is complete wether $\overline{a} > 0$. In this case, let us consider ϵ such that $\overline{a} > \epsilon > 0$ and $\overline{\lambda}$ the corresponding bound on allowance price given by assumption (Amenabilty). Considering the family of applications $G_t = F_1^{(\alpha,t\overline{\lambda},0)}$, one has $sup_{x \in G_1^{-1}(0)} \sum_{j=1}^n f_j(y_j) \leq \epsilon$. Hence one can apply the preceding lemma to the function $\sum_{j=1}^n f_j$ in order to show that for every $c \in [\epsilon, \overline{a}]$ there exist $t \in [0,1]$ and $x \in G_t^{-1}(0)$ such that $\sum_{j=1}^n f_j(y_j) = c$. Hence there exist equilibria in the enlarged economy for every allowance allocation proportional to $((a_i),(a_j))$ and whose sum is in $[\epsilon, \overline{a}]$. As this holds for all $\epsilon > 0$ the proof is complete.

6.11 *Proof of theorem* 5

Let us show that there exist a public equilibrium for every initial endowment in allowance $a \in \mathbb{R}_+^{n+m}$. Let us consider α as in the proof of theorem 4, $H > 0$,$\overline{\lambda} > 0$, and the corresponding family of applications $G_t = F_1^{(\alpha,t\overline{\lambda},H)}$. Given the continuity of the marginal utility of the environment , for λ sufficiently high none of the consumers actually purchase allowance at equilibrium. Together with assumption (Amenability,) this implies that for λ high enough the total demand in allowance is below ϵ. Similar arguments to those in the proof of theorem 4 then imply that for every allowance allocation of the form $k((a_i),(a_j))$, there exist a zero of some $F_1^{(\alpha,t\overline{\lambda},H)}$ such that $\sum_{j=1}^n f_j(y_j) + \sum_{i=1}^m s_i = k(\sum_{i=1}^m a_i + \sum_{j=1}^n a_j)$. Now, according to lemma 7 such a zero is a public equilibrium if for all i, $s_i < H$. A sufficient condition therefore is that $\sum_{j=1}^{n} f_j(y_j) + \sum_{i=1}^{m} s_i < H$. As the preceding holds for arbitrary H, the proof is complete.

References

[1] IPCC (2001) IPCC Third Assessment Report – Climate Change 2001

[2] Arrow, K.J (1969) "The organization of economic activity : Issues pertinent to the Choice of Market versus Non-Market allocation", in Joint economic commitee, The analysis and evaluation of Public Expenditures ; The PPB System, Washington DC : Government Printing Office, pp 47-64.

[3] Bonnisseau, J-M. and Cornet, B. (1988) "Existence of equilibria when firms follow bounded losses pricing rules" Journal of Mathematical Economics, 1988, vol. 17, issue 2-3, pages 119-147

[4] Bonnisseau, J-M. (1992)"Existence of equilibria in the presence of increasing returns : A synthesis" J. Math. Econom. vol. 21(5), pp 441-452.

[5] Boyd, J. and Conley, John P. (1997) "Fundamental Nonconvexities in Arrovian Markets and a Coasian Solution to the Problem of Externalities" Journal of Economic Theory, Vol. 72, 1997, pp. 388-407.

[6] Cass, D. and Citanna, A. (1998) "Pareto Improving Financial Innovation in Incomplete Markets," Economic Theory 11: 467-494.

[7] Cellina, A. and Lasota, A. (1969). – "A New Approach to the Definition of Topological Degree for Multivalued Mappings", Atti della Academia Nazionale dei Lincei, Rendiconti. Classe de Scienze Fisiche, Mathematiche e Naturali, 47, pp. 434-440.

[8] Clarke, F., 1983, "Optimization and nonsmooth analysis" (Wiley, New York).

[9] Conley, J and Smith, S. (2005) , "Coasian equilibrium," Journal of Mathematical Economics, vol. 41(6), pp 687-704

[10] Dehez,P. and Drèze, J. (1988). "Competitive Equilibria with Quantity-taking Producers and Increasing Returns". JMathE; V.17, pp. 209-23

[11] Drèze,J. (1975) "Existence of an Exchange Equilibrium under Price Rigidities", IER.

[12] Elul, R., (1995) "Welfare Effects of Financial Innovation in Incomplete Markets Economies with Several Consumption Goods," Journal of Economic Theory 65: 43-7

[13] Guesnerie, R. (2003) (sous la direction de) "Kyoto et

l'économie de l'effet de serre ", Conseil d'analyse économique n° 39, Paris, La Documentation française.

[14] Giraud, G. (2001) "An algebraic index theorem for non-smooth economies" Journal of Mathematical Economics, V. 36(4), pp. 255-269.

[15] Jamin,A. and Mandel,A. (2006) "A general equilibrium analysis of emission allowances." Cahiers de la Maison des Sciences Economiques, série bleue 2006.03.

[16] Jouini, E (1992) "An Index Theorem for Nonconvex Production Economies", Journal of Economic Theory, 57 (1), 176-196.

[17] Jouini, E (1992) "Existence of the equilibria without free disposal assumption", Economics Letters, 38, 37-42.

[18] Laffont, J, J. (1978) Effets externes et théorie économique, Monographiee du Séminaire d'économétrie. Centre National de la Recherche Scientifique, Paris.

[19] Mandel A. (2006) "An index theorem for production economies with externalities",mimeo.

[20] Schumpeter, J. "Théorie de l'évolution économique Recherches sur le profit, le crédit, l'intérêt et le cycle de la conjoncture" Dalloz, Paris, 1999.