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## **From theory to modelling : urban systems as complex systems<sup>1</sup>**

### **La complexité dans les systèmes urbains : de la théorie au modèle**

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#### **Abstract :**

The question of complexity and its increasing application to social sciences is challenging the modelling of spatial systems. New concepts and new methods have been proposed and invite to reformulate classical modelling frames. This approach is demanding to open a better informed dialogue between the disciplines which supply models and tools and those where the existing knowledge is reformulated inside this new frame. Actually, creating an « artificial geography » is not straightforward. It is rather easy to translate urban theories within the paradigm of complex systems, but their modelling, for instance by using multi-agents systems, still raises many conceptual and practical difficulties. We underline here some problems in defining significant urban entities and exploring the evolution of their spatial relationships over time. We briefly present which options have been selected for developing the SIMPOP2 model which is conceived for simulating the evolution of systems of cities over long periods of time.

**Key-words :** complexity, emerging structures, self-organisation, urban systems, space-time convergence, multi-agents systems

#### **Résumé:**

La modélisation des systèmes spatiaux se confronte aux théories de la complexité et à leurs applications en sciences sociales. De nouveaux concepts et de nouvelles méthodes sont proposés, qui invitent à reformuler les modèles classiques. Cette démarche suppose d'ouvrir un dialogue mieux informé entre les disciplines qui offrent les modèles et les outils et celles qui tentent de reformuler leurs connaissances dans ce nouveau cadre. En effet, la création d'une « géographie artificielle » ne va pas de soi. S'il est relativement aisé de traduire les théories urbaines en termes de systèmes complexes, leur modélisation, par exemple au moyen de systèmes multi-agents, soulève encore de nombreuses difficultés conceptuelles et pratiques. Nous insistons notamment sur la définition des entités urbaines et sur les variations de leurs relations dans l'espace au cours du temps. Nous présentons enfin rapidement les principales options retenues pour le modèle SIMPOP2 conçu pour simuler le devenir des systèmes de villes dans la longue durée.

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**Mots-clés:** complexité, structures émergentes, auto-organisation, systèmes urbains, convergence espace-temps, systèmes multi-agents

## **Introduction :**

During the last thirty years, new developments in the theory of self-organisation in physics (Prigogine, 1996, Haken, 1977), evolution of living species in biology and adaptive cognitive systems in economy or social networks (Arthur, 1994, Lesourne, Orléan, 1998, Anderson, Arrow and Pines, 1988, Arthur, Durlauf and Lane, 1997, Weidlich, 2000) have changed our representations of system dynamics, especially by emphasizing the conditions of emergence of new structures from local interactions between adapting individuals. The main epistemological questions have therefore shifted from the autonomy of systems relative to their environment, towards the identification of attractors governing their dynamics, and lastly to their capacity of innovation within a context of uncertain and changing rules of social interactions (Pumain, 2003). A large variety of complex systems ranges from “simple” non linear dynamics of physical or chemical systems, where particles are in very large numbers and have passive (reactive) behaviour, to the more sophisticated representation of social systems involving intelligent agents capable of innovation and anticipation. After Casti (1997), the new computational techniques from now on allow social scientists to conduct “controlled, repeatable experiments” with “silicon surrogates”, and such models could contribute to create the first real theory of complex systems.

The main distinctive feature of complex adaptive systems is their ability to exhibit emerging properties, or, as quoted by Batty and Torrens (2001), to give rise to a “surprise” for the observer. But of course there are many possible definitions of complexity, from the realistic (“la complexité est un ordre dont on ne connaît pas le code”, Atlan, 1979) to the constructivist (“la complexité est le nombre d’interprétations non équivalentes qu’un observateur peut se faire d’un système”, Livet, 1983). The challenge for geographers is now to establish fully explicit connections between agent’s attributes as their supposed representations and behaviours (like learning, invention or adaptation), and the “surprising” (actually very well-known for long, but quite unpredictable in their detailed further features) collective properties that emerge from their interaction. The attempt is in designing a model and tuning parameters in a way both theoretically parsimonious and factually consistent with the state of knowledge, from surveys at the individual level on one hand and statistical observation of aggregated urban systems on the other, in order to come as close as possible of what could be a “validation” by a simulation model. We try to do this about a few selected properties of the evolution of cities and urban systems, especially the relations between their mass, the space, and time, as expressed by the transformations of the hierarchical structure within the system. The problem is how interactions are regulating the relative size of the urban elements, according to the speed and intensity of spatial interaction: at the intra-urban scale, through the spatial dilatation of the built-up area, and at the scale of a system of cities where the reinforcement of the urban hierarchy is produced by the short-circuiting of smaller intermediary centres and the various processes of diffusion of the innovations (Bretagnolle, 1999).

Multi-agents systems are fully adapted instruments for answering such a challenge. They are much more flexible than differential equations for simulating spatial and evolving interactions, including quantitative and qualitative effects. Through the definition of rules at

individual level, they can reproduce the circulation of information between cognitive and decision making agents. They simulate at the upper level the emergence of collective or aggregated structures which can be tested statistically. The rules can be adapted for varying space and time scales of interaction under the course of history. A series of such models could help to a new formalisation of geographical theories by developing an “*artificial geography*”. Of course multi-agents systems do not solve the problem of choosing a “good” theoretical representation. We suggest here a few improvements which are brought to a former version of the SIMPOP model (Bura et al. 1996, Sanders et al., 1997). This version simulated the emergence of a functionally differentiated system of towns and cities from an initial more or less homogeneous rural settlement system. A better representation of the competitive interaction between cities within the model is introduced by two new agents which explicit the role of the functions of innovation and governance within the dynamics of the urban system.

## 1 Towards an « artificial geography »?

Complex systems theory has been recently developed and is sometimes conceived as a discipline per se, for instance in institutions like the Santa Fe Institute (created in 1984). The main incentive has come from mathematicians and physicists who started applying their models of emerging properties to biological then to social sciences. We are not far from the kind of attempt by Bertalanffy (1971) to build a “general system’s theory” which would give a global framework for explaining systems dynamics in a wide range of fields of knowledge. Some social scientists are sceptical, for instance as Durlauf (2003) considering that “econophysics” is a new discipline insufficiently rooted in economic theory and empirical observation. Another peculiarity of new complex systems theory is to focus on the emergence of properties at a macrolevel as resulting from the interactions between individual behaviour at a micro level. Most of applications, for instance in cognitive economy or in social networks, refer to a “social ontology” which does not include aggregated entities as having an important role in the structuring of social systems. The generation of institutions and their intervention in economic modelling remains as a black hole in economic theory (Walliser, 2000). Even if we think it interesting to complete the theory of evolving urban systems, by relating their observed or simulated general properties to the social processes which are shaping them (which social practices and, if possible, which intentions are behind the observed statistical aggregate properties?), we are not sure that this can in a near future produce a spectacular advance in research. On one hand this view reflects a new fashion in science with a taste for considering bottom-up constructions and sometimes neglecting too many other determinants from aggregate levels (Pumain, 2005). On the other, our empirical knowledge about detailed intercity flows (of persons, goods, information) and their corresponding motives is still very limited. So the empirical testing of the hypothesis which will be put in such models has to be very cautious. But in any case, it is important to define this as a promising research program.

When simulation tools are required, the sciences of complexity widely use multi-agents systems for formalising and testing hypothesis about different aspects of dynamics. Former computational software as those provided for distributed artificial intelligence (DAI) or artificial life, have open the way for the development of multi-agents systems. There is now a large variety of models that apply such methodologies to our discipline.

## 1.1 Artificial life and distributed artificial intelligence

Imagining artificial life by exploring different possibilities is an interesting challenge. For instance, the observable species puzzle biologists who wonder why imaginable creatures do not exist (Lewontin, 2003). From the simplest “life game” conceived by G. Conway in 1982 (Berlekamp, Conway, Guy, 1982) towards the more sophisticated models replicating elements of evolution theory (Wolfram, 1994, Langton, 1988 and 1991), a variety of simulations have explored different ways of representing existing or imaginary life processes and their corresponding outcomes. In a parallel way, the computational models of DAI have elaborated task solving methodologies involving different degrees of co-operation between individual agents. From the eighties on, they have contributed to develop and enrich concepts as agents, their environment and interactions, receiving new definitions from a variety of applications, in disciplinary fields as economy, ecology, ethology or geography (Testafion, 2002).

The multi-agents systems rely upon specific software<sup>2</sup> inspired by the object-oriented approach which includes passive attributes and active rules for designated classes of objects. Objects belonging to the same class share common processes but may have different data; and classes of objects are organised in a hierarchic way according to an heritage principle of their properties. Of course other computational devices as cellular automata or simulation modelling techniques (as for instance large scale free networks, see Anderson et alii, 2003) may be of special interest for geographers. But multi-agents systems are especially useful as simulation tools for modelling a dynamics, when it is essentially explained by the heterogeneity of individual features and their interaction. They allow the modeller to associate qualitative and quantitative rules, and to integrate several levels of organisation as well as diverse time scales and dynamics relationships. They are used in natural, social or cognitive sciences, as artificial laboratories (*in silico*) for observing the behaviour of agents at an individual or collective level, and analysing the evolution of the structures which emerge at a macro-level (Daudé, 2005).

## 1.2 Properties of multi-agents systems

Multi-agents systems are characterized by their ability to consider the environment of a system, their acceptance of a wide conceptual diversity of agents, a diversity of scales enabling multi-level analysis, and their flexibility regarding interaction rules, especially in spatial relationships.

Defining the environment of the agents is not an absolute necessity but may be of interest for many geographical applications. It is represented most of the time by a frame of cells associated in a network (Figure 1). In that case, an agent’s state may depend on its network of connected cells and sometimes on the actions of the agents which are located there. Conversely, the agents are constrained in their actions by the connections between cells, for their moves, actions and representations. According to different applications, the environment in a multi-agents system can be a space supporting the movement of agents (Page, 1998); a complex whole including agents or objects which are parts of it (Bura et al., 1996); a resource for agents which use its attributes for their actions (Epstein and Axtell, 1996); a

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<sup>2</sup> Examples of languages used for programming are Smalltalk, C++, Objective C, Lisp, Java and recently more integrated dedicated platforms as Starlogo or Swarm.

communication field for the agents (Drogoul and Ferber, 1994); an entity with its own dynamics (Bousquet, Gautier, 1999).

There is a large variety of definitions for agents according to the applications. A common characteristic is that agents represent autonomous entities which are distinct from their environment and can act independently from other agents, even if in most multi-agents systems agents are interacting through co-operation, competition, or simply because of their co-existence. Agents can be mobile (as in many ethologic models) or immobile (when they represent fixed geographical entities as in SIMPOP). They have capabilities of acting on themselves or on their environment, of communicating with other agents (receiving and sending information) and of behaving according to their resources, observations, knowledge and interactions with other agents (Ferber, 1995). They can be both reactive (as the simple entities governed by external stimuli) and cognitive (as agents taking decisions after their information and representation about themselves, other agents and their environment), also as learning and adaptive agents they may change their behaviour over time as a result of their past experience or according to their chances of success in the future. Reactive agents are often considered in ethology (Deneubourg and Pasteels, 1987) or robotics (Brooks, 1986) while cognitive agents are conceived in psychological, sociological or economic applications. In geography, this question is linked to the scale of applications.

There are no interactions for systems in which all agents have the same aim, the same level of resources and can reach separately their objective (as for instance in economic models where a market is defined exogenously). However, most of the time, agents are defined as interacting entities, between them and/or with their environment. Interactions are stemming from the exchanges of messages between the agents. They are responsible for the possible emergence of structures and properties at different levels of aggregation within the system. In geographical models, interactions may be spatially defined according to various distance or territorial effects (multi-agents systems are then more flexible than cellular automata for simulating such rules) or can be non spatial as well (Figure 2). In the SIMPOP2 model, a special attention is devoted to the diversity and evolution of spatial interaction (see below section 4).

### 1.3 Applications in geography

In geography, there is no general agreement about a concern for “methodological individualism”. Neither are there formal and well accepted theories about what a “homo geographicus” should have as fundamental attributes and rules of behaviour (Pumain, Racine, 1999). However, most applications of multi-agents systems deal with a representation of agents as individuals (Daudé 2002, Daudé 2004), immobile (as farmers exploiting simultaneously limited resources) or mobile (in the case of urban agents relocating according to their social or housing preferences, Bonnefoy, 2003, Portugali, 2000). Intermediate entities as institutions or social groups, or physical and social neighbourhoods, are sometimes included in models; for instance, F. Dureau and alii (2001) introduce families as individual decision making entities for a migration model.

As much regularity has been observed by geographers at aggregate levels leading to the identification of geographical objects like regions of cities, a few multi-agents systems do represent collective territorial entities as the individual agents (cities and towns in the Simpop model, for example). Interactions refer then to spatially and temporally aggregated flows (as

migrants, or information or investment flows) which structure the system at a macro-scale by influencing the qualitative and quantitative evolution of the territorial entities. The “behaviour” of such aggregated objects is not reducible to the behaviour of individual persons, as it would be for instance in models of cognitive economy. Even the concepts of urban governance, or innovation function, which are introduced in a second version of the model (SIMPOP2) actually refer to collective entities. They are conceived as a way of introducing the effects of some “cognitive” behaviour within the model.

When using such collective territorial entities as agents in the simulating model, we have to define these objects in a non ambiguous way, which raises in the case of cities and towns different ontological questions.

## **2 Defining the levels and the agents: an ontological problem**

Urban systems can be conceptualised as systems where emerging properties are produced mainly at two levels of observation through the interactions between “agents” occurring at a lower level: the morphological and social structures of a city are emerging from the multiple interactive decisions of the residents or groups of citizens, while the spatial organisation, hierarchical and socio-economic differentiation at the scale of a system of cities are created by the adaptive strategies and mainly competitive relations between the cities considered as “agents” at this level of analysis. This would signify no more than a linguistic updating of the famous formula “cities as systems within systems of cities” already coined by B. Berry as soon as 1964, and it would merely be playing with fashionable words to speak of urban systems as complex systems if we were not able to represent the agents and their interactions in a way proper to reproduce and predict some of the emerging collective properties in simulation models.

When experimenting with urban systems, a first difficulty is to define precisely which are the objects under study, or in other words, to identify clearly the content and limits of each level of organisation. Indeed, urban objects are essentially relational (for instance when compared to rural settlements which mainly exploit ecological local resources, towns and cities are developing by capturing and maintaining positions within networks). At every level of organisation, there are so many relations and interactions, within and between the different levels, that it is a complicate task to isolate them, theoretically and practically, for measurement purposes.

### **2.1 The city, an adaptive object with fuzzy limits**

In recent urbanisation, the urban sprawl associated to the diffusion of the automobile has blurred the spatial marks of the pre-industrial urban patterns. The usual morpho-statistical criteria are not sufficient to delimitate the outskirts of large metropolises. But the functional criteria are not entirely satisfying either: when applied to the French urban system, there are 350 urban units (aires urbaines) instead of the 2 200 that are produced when applying the definition of urban agglomeration. A quantity of information about small towns is lost when functional criteria are applied (that is why the US Census decided in 2000 to add information about “micropolitan areas” to the classical one of “metropolitan areas”, by reducing the size of eligible core from 50 000 to 10 000 jobs).

Technical solutions have then become insufficient and invite us to engage a common reflection about the comparative approach of delimitation of cities, especially at the European level. There is an urgent need for that, from institutions as well as from research. The recent experience of the second version of the “urban audit”, however piloted by Eurostat with the concourse of many national statistical institutes, did not succeed in solving this problem. The adoption of common morphological and statistical criteria would mark a progress, but more precise instructions are needed as shown by the results of the comparison made in co-operation with Marianne Guérois (2003) about a few Italian cities. Three data bases, using the same morphological and statistical criteria than the French Statistics Board (INSEE), have been compared with a GIS for the 21 cities larger than 200 000 inhabitants (which are the most difficult to delimitate, because of the intensity of the urban sprawl) : the NUREC database (Network on Urban Research in the European Community), sponsored by Eurostat (NUREC, 1994), the Geopolis database, elaborated by a French geographer, François Moriconi-Ebrard (1994), and the delimitation of Italian urban systems (including morphological agglomerations), proposed by two Italian geographers, Remo Madella and Giovanni A. Rabino (2003). The results of the comparison (Table 1 and Figure 3) give an idea of the fuzziness of the limits of such objects: about three quarters of the 21 agglomerations show significant differences (more than 20%) between the populations defined according to the three data bases. Figure 4 focuses on Firenze, which shows dramatic variations of populations between the three data bases (from about 450 000 to about 950 000 inhabitants). We can follow, with the CORINE land cover image, the extension of the urban sprawl, including an initial nucleus in the valley of Arno (NUREC perimeter), a latter junction with secondary valleys of the North-West, filled by Prato (Madella-Rabino delimitation) and Pistoia (Geopolis perimeter).

## 2.2 Urban systems as open systems

Urban systems are themselves very difficult to delimitate. The trading networks are evolving through time. Furthermore, urban systems are open (the exchanges with their environment enable, for instance, the introduction of technical or social innovations) and are overlapping, as an articulation of interlocked networks.

If the general structures of urban systems are complex, the behaviours that can be observed are far from being entirely stochastic. Despite the inequalities in the weight and influence of urban nodes within the system, their evolutionary trajectories reveal some collective common behaviour. Among these common features are: adaptation to change, selection, cooperation or imitation. The various networks which have flourished among cities for so many different purposes (cities located at less than one hour from Paris, Atlantic coast line, Baltic sea...) as well as the corporate networks revealed by linkages between firms headquarters and their branches in foreign cities (Rozenblat, Pumain, 2004) illustrate this kind of collective behaviour, which is not initiated or controlled by a unique institution but self-organised. These collective behaviours contribute to ensure a social regulation of the system. A demonstration is provided by the work of F. Paulus and D. Pumain (2003) about the functional evolution of the French cities during the last fifty years. The trajectories of the cities are extremely parallel, despite the differences in the specialisation profiles or in the sizes, which show that these cities have adapted themselves very quickly to the different waves of technical and social innovations, through imitation behaviours. This kind of regulation is not a global one which would fit every city of the system as in a strict managerial hierarchical organisation, but it looks more like a hierarchy of overlapping fragments, which



are impossible to enumerate totally or even to classify in a hierarchical way. Emerging properties, characterising relationships between the micro level and the macro level of the system, contribute to ensure this global regulation.

## **2. 3 Dynamic interactions**

Urban systems are characterised by very complex interactions. A full variety of networks connecting cities interfere in different time and spatial scales. Moreover, geographic space is in constant evolution, especially since the industrial revolution. The increasing speed of communication has tremendously changed the relative positions of cities and towns. The “time-space contraction” or “time-space convergence”, which describes the progressive reduction of travel time between two locations, introduces different effects according to the scale of analysis: at the local scale (one city), there is an apparent dilatation of the “centre”, enlarging the “urban space” or “urban field” through the gain in accessibility to the centre, whereas at the inter-urban scale (between cities) there is an apparent contraction (cities become closer to each other because of the increasing communication speed) and smaller towns are short-circuited.

At the intra-urban scale, the metaphor illustrating the consequences of the increasing speed of communication is of waves of growth, inducing a progressive dilatation of the urban space. This is mentioned for instance by Korcelli or Klaassen (Bretagnolle and alii, 2003). Quoting Balzac, “everything seems like if Rouen was at the gates of Paris”. The boundaries of the familiar territory, frequently visited by the residents of the centre, are moving forward according to the innovations in transport technology. The destiny of the places which are integrated in these waves of dilatation can then be completely reversed in a few decades. The example of Saint-Denis (Figure 5), located in the north of Paris, illustrates these transformations. During Middle Age, Saint-Denis was a real city, separated from Paris by a distance of two hours walking (about 7 km). It was a stop on the Roman road which led to Rouen, then receiving important royal privileges (abbey and fair). With the transport revolution, at the beginning of the nineteenth century, Saint-Denis felt under the isochronic line of one hour distance from Paris, and was propelled to the role of an industrial suburb (metallurgy, chemical industry and stocking activities). The population was multiplied by a factor 15 within less than a century. With the revolution in communication and the fast railway lines of the second half of the twentieth century, the status of Saint-Denis is changing again, because it now belongs to the Parisian hyper-centre, which is delimited roughly by the isochronic line of half an hour (an argument which is often presented by urban developers or actors who want to attract new firms and residents). Registered offices and high technology activities (multimedia, information processing) settle in this locality and the former industrial activities and workers are moving away to farther suburbs.

At the inter-urban scale, there is a slow but systematic short-circuiting of the small and medium size cities which are in an intermediary position between larger cities. The examples of Reims, Troyes and Châlons-sur-Marne (Figure 5), located at about 200 km from Paris to the east, illustrate these transformations. During the Middle Age, these three cities were important nodes, at the national scale and even at the international level. From the eighteenth and the beginning of the nineteenth century, various technical improvements concerning roads and coaches have almost divided travel times by two. The three cities remained however important nodes at the national and regional levels. With the railway transports, they felt under the isochronic line of one day from Paris. While entering the influence field of the

Parisian region, they underwent a relative decline at the national and international scale. During the second half of twentieth century, they were not served by an airport (except a regional one), fast train (TGV) or highway crossing (Bretagnolle, 2003). From their situation, now at less than two hours from Paris, it can be expected that they will become soon integrated in the dilatation wave of the Parisian urban field (the process will be accelerated through the building of the East TGV line, including a stop at Reims: the share of commuters toward Paris has already begun to increase).

### **3 Urban hierarchy: an emerging property involving local fluctuations and dynamic stability**

#### **3.1 From self-organisation to complex systems theory**

The conception of self-organised systems where a structure observed at a macro-level is supposed to be produced by the interactions between elements at a micro-level has stimulated urban dynamic modelling since the eighties (Allen, Sanglier, 1979, Wilson, 1981, Lombardo, Rabino, 1984, Pumain et al., 1989). The first models were mainly conceived as systems of non linear differential equations describing the evolution of state variables at a macro-level, the lower level interactions being summarised in relations or in parameters. As interactions are non linear, the systems are not attracted towards a pre-determined equilibrium (in physics they would be said non ergodic), a shock linked with the amplification of some internal fluctuation, or with an external perturbation, that is, a small change in the parameters of the model, can modify the dynamic trajectory of the systems and persist as a determinant of their further qualitative structure, according to a bifurcation. For instance, a small change in preference of consumers for large size and diversity of shops, as well as a variation in the price of oil, can produce a spatial concentration of trade in a major centre or its dispersion in a multitude of small centres (Wilson, 1981, Allen et al., 1981).

Even if some models made analytically more explicit connections between the individual behaviour and the resulting aggregated interactions (as for instance the synergetic model of interregional or interurban migrations first developed by W. Weidlich and G. Haag (1988) using as a starting point a description by a master equation), in practice there was very limited correspondence established with observations at a micro-level, since an “average” behaviour was supposed to be representative of the individuals and the applications were conducted with statistics on aggregated flows (for instance, at city level by Sanders, 1992). On the contrary, models of micro-simulation integrated a lot of details about the behaviour and familial or professional career of individuals, but did not pay so much attention to the evolution of the resulting structures at the macro level (Holm, Sanders, 2001). Compared to these earlier representations of self-organisation in models, the actual notion of emergent properties refers to a more explicit modelling of individual behaviour and interactions, usually in agent based models or in multi-agent systems. Emergent properties appear at a higher level of aggregation than the original description of the system.

When applied to urban systems, these ideas can lead to a diversity of types of models. We analyse here one example connecting the meso and macro levels of observations.

### 3.2 Emergence of a hierarchical organisation

The hierarchical organisation and functional differentiation are emergent properties which characterize the level of observation of systems of towns and cities. They are produced by the multiple interactions which occur between individual towns and cities. The fact that a town or a city maintains its size within a given proportion of the other cities' size, and that over time there is a rather consistent persistence of the hierarchical order and the social specialisation (both have slow dynamics, even if the former takes much longer to change, centuries sometimes, than the latter which normally may change after a few decades) cannot be inferred from the nature and function of one single city. Indeed, these interactions between individual towns and cities, when observed at very short time intervals, seem like local fluctuations, which bring about stochastic variations of city sizes. After the French statistician Robert Gibrat (1931), if denoting  $dx$  the variable which describes the variations of city sizes between two very close dates, the relative variations  $dx/x$  (in other words the growth rates of the cities) are observed to be distributed as a Gaussian law. These variations are independent from the initial sizes of the cities<sup>3</sup>. This observation, qualified as "surprising" by the statistician, may be explained, according to him, by the multitude of factors (economical, political, social...) which interact on urban populations, in these short time intervals. Following the same argument about the firm sizes, he mentions: "We don't need to know them individually to think that they are numerous and that each of them little modify the number of workers", and the author quotes the expression of the Italian statistician and economist Pareto, "a set of unknown factors, acting either in a sense, either in another".

These local stochastic fluctuations, described in a theoretical way by Gibrat for infinitely short  $dx$  variations, can be roughly estimated for larger time intervals. If we use the census data for the French cities, the length of the intervals is between five and ten years, from 1800 to 2000. We actually observe growth rates which are fluctuating around the average, even in the case of largest cities, which are nevertheless continuously growing during these two centuries when characterised by a cluster analysis (Pumain, Saint-Julien, 1995). But the evolution of their relative weight within the urban system is quite different, with oscillations around zero which mean short term fluctuations of the relative attractiveness of each metropolis within the urban system. We have plotted for the largest French cities (Figure 6) its relative weight at one census date (on X-axis) and at the following census date (on Y-axis). A trajectory of constant relative growth (or growing relative attractiveness) would be represented by a diagonal on this graph of phase space, crossing the X-axis at a 45° angle. On the contrary, we observe spiral patterns, which characterise cyclic variations of growth, sometimes positive, sometimes negative. The cycles are not synchronous, but exhibit more or less regular patterns. Paris, Lyon and Marseilles, for instance, seem to follow a very regular cycle (increasing then decreasing their relative weight), whereas Lille, Toulouse or Nantes have more complicated stories.

This fluctuating behaviour of individual cities, at short time intervals, contrasts in a surprising way with the stability of the structure and evolution of the whole system. Gibrat uses the expression "dynamic stability" to qualify the fact that the system remains bounded through time, without brutal jumps (a city which would grow up "infinitely") or crashing down (case of "ghost cities", characterised by a strong demographic decay). The urban system is self-regulating, by the way of the multiple interactions which allow successive adjustments and progressive adaptations to the external perturbations or to the endogenous innovations. This

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<sup>3</sup> One necessary condition is, of course, that the variations  $dx$  have to be measured on a very short time interval.

dynamic stability can be perceived, for instance, when plotting rank-size distribution over several centuries (Figure 7). The different curves are very similar, rendering invisible the multiple local variations in the individual rankings between census dates, which were yet well established by F. Guérin-Pace and D. Pumain (1993). The similarity of the curves can also be characterised by measuring the slopes of the adjusted Zipf rank-size distributions, which are interpreted as indices of concentration. The values are growing up very slightly and very continuously, from the eighteenth century to nowadays<sup>4</sup> (Bretagnolle and alii, 2003).

Another significant example is given by the long run dynamics of European towns and cities, from the Middle Age to the nineteenth century. The discovery of the New World in 1492 and the invention of the steam engine, in 1769, gave twice a decisive advantage to the Atlantic cities, compared to Mediterranean ones, but these tremendous changes did not generate a sudden bifurcation of the whole system. On the contrary, we observe a very slow modification of the spatial pattern of the most prominent cities (as defined by the population potential model) from the south to the north, with a bi-polar structure which maintains the equilibrium of the system from the 14th to the 17th century, transferring the centre of the world economy of the time from northern Italy to the Belgian, Dutch and then English metropolises (Braudel, 1949, de Vries, 1984, Bretagnolle and alii, 1998).

## 4 The SIMPOP2 model

SIMPOP is designed for simulating the emergence, structuring and evolution of a system of cities, starting from an initial spatial distribution of settlements in a large region, state or set of states, a repartition of resources which can be assessed randomly or exogenously, and rules defining how cities interact, grow and specialise. In this model, the environment is represented by cells having different kinds of resources, for agricultural production or industrial purposes (exploited only from year 1800 on), and various facilities or obstacles for circulation. They can be allocated randomly or according to a specific pattern. Towns are emerging as centres of accumulation of population and wealth, through first the trading of agricultural surplus in their surrounding cells, then from their competition for the acquisition of other urban functions, as other types of trades, or administrative roles. Interurban competition is simulated by relating profits (from trade or taxes) and growth rates, with a random factor. Meanwhile, the spatial range of interactions is increased when cities acquire new functions and with technological progress going. As a result, different patterns of towns and cities in terms of spatial and hierarchical distribution are emerging.

The second version of the SIMPOP model, named SIMPOP2 is now experimented. It is conceived for a larger number of agents (several thousands). It also includes a better representation of the competitive interaction between cities, through the introduction of two new agents which make explicit the role of the functions of innovation and governance within the dynamics of the urban system. This is a way to complete the theory of urban systems within the framework of complex systems theory, by substantiating the growth process in social terms. What make cities growth? Since Schumpeter, innovations, especially entrepreneurial ones, are theorised as making the economic basis for further urban growth. But if the process of diffusing innovation is well documented, since mainly Hägerstrand's work (1952), the question of its appearance remains much more difficult. A recent review of

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<sup>4</sup> In order to take into account the urban sprawl, in 1999, we have used functional urban areas when they were defined, and we have completed this information with smaller urban units (urban agglomerations).

the literature on industrial districts, innovation and learning processes in regional and urban systems (McKinnon et al. 2003) underlines many uncertainties in our actual knowledge about such processes. The SIMPOP2 model may help in establishing a few dynamic conditions of this emergence of “second nature” advantages.

## **4.1 Interurban competition**

The SIMPOP model is evolutionary because a major underlying hypothesis is that the pervasive structural features of urban systems which we can observe are produced by an historical evolution. This evolution involves systematic, time-oriented changes in major circumstances of the system over time, including the demographic and urban transitions, the increase in gross and per capita economic wealth, the trendy increase in the speed of transportation means, as well as the recurrent appearance of technical, economic and cultural innovation. Thus, it is this social, historical evolution which supports the dynamics of urban systems, even if in a concrete way it is made through the mechanism of interurban interactions. Those are the “bottom-up” processes leading to the emergence of the structure of the system, whereas the evolutionary trends can be thought of as emerging trends, which are produced as feed-back effects by the system of cities itself, and become new constraints on the dynamics of individual cities. Actually, we do not know yet how to make these large evolutionary trends to emerge, and they are represented in an exogenous way within the model, whereas it is possible to represent the endogenous process of building an urban hierarchy from the interactions between cities.

Interactions between cities keep over time some permanent features, among them the most important is their competition for adopting social change and capturing the benefits from innovation. A city participates to this interurban competition through the functions (or economic specialisation) that it successively acquires over time. A function enable a city to supply a type of product or service to other cities, which provide more or less returns in terms of economic growth and attractivity on population, according to the level of productivity of that function. The criteria for establishing a list of relevant specialisation for the definition of urban functions are related to an evolutionary perspective, under the main hypothesis that a narrow connexion exist between the relative dynamics of an urban entity in the system of cities and the innovation cycles that the city has adopted (or to which it has better adapted). The question is to identify, for the entire system of cities, which innovation cycles have produced noticeable urban specialisation, affecting in a durable way the relative evolution of the specialised cities, and for each city, which are the specialisation that correspond at best to its actual and potential trajectory. A limited number of urban functions were selected as representative of the major economic cycles which gave rise to differential urban growth and cities specialisation over the past four centuries (Figure 8). Cities as agents have a total or partial (as constrained by the network of their partner cities) information about the emergence of new functions (which remains exogenous to the model). Cities also have a power of decision to invest in that innovation, according to the wealth they have previously accumulated and to their line of urban strategy, that can be more or less risk-oriented. This decision process is represented by a “cognitive” attribute named “urban governance” The urban governance also may represent in the model the possible intervention of the individual actors, which represent a third level in the modelling of urban systems. This level can be lower than the city level (for instance, an investor choosing a specific location for a firm, or a mayor defining a type of urban policy) or “above” the system of cities (for instance a political system imposing a centralised administration can lead to the emergence of a prominent capital

in an urban system (example of France), whereas a more decentralised government may lead to a more regular urban hierarchy (example of Germany)).

## 4.2 Simulation of stylised interactions

What are interurban interactions in the SIMPOP2 model? As our intention is to simulate the development of urban systems which include a large number of towns and cities, it would be unrealistic to think of interactions as “real” flows of exchanged goods, people or information. The interactions which are simulated in the model are not these “first order” interurban exchanges, but more abstract, “second order” interactions, which represent an interpretation of the effect of concrete flows on the relative dynamics of cities. For example, the urban functions are essential attributes of the cities. They do not give an exhaustive representation of the economic profiles of the cities, since they are attributed only to the cities having developed a major specialisation in a particular sector of activity during the corresponding innovation cycle. In a similar way, the exchange of products and resources among cities on the “market place” (cities selling and buying according to their level of supply and demand) does not reflect the totality of the urban economy but only the specialised part of the interurban market, the one that is likely to give rise to urban growth differentials.

The rules which define the ability of a city to adopt innovations are partly deterministic, in order to reproduce the powerful trend to hierarchical diffusion of urban innovation (this is the case for most of central functions, a given level cannot be acquired if the other are not yet there), and partly random: when new urban specialisations appear, they can select locations (or become acquired according to some decision of urban governance) which do not necessarily correspond to the largest cities. There are sometimes necessary “seeds” for such location of specialised activities, as mineral resources for manufacturing industries of the 19<sup>th</sup> century, or knowledge (human capital) in the case of technopolises of the 20<sup>th</sup>.

## 4.3 Three types of spatial interaction

Our model is a geographic model, in the sense that spatial interaction are supposed to reflect the power of cities in terms of range of influence of their activities and support for new developments from their access to more or less extended markets. Three types of spatial interactions are distinguished for reflecting the most frequent types of interurban exchanges, linked to different constraints: 1) proximity constrained interactions are representative of many activities for which the distance between supply and demand is an essential constraint, they are the rule for all central place functions, whatever their level and range, and even if that spatial range is increasing over time; under that rule, the probability of exchanges are distributed according to a model of gravity type ; 2) territorially constrained interactions are limiting a city’s influence within boundaries, regional or national, they correspond to all types of administrative or political activities; the interaction rule is modulated according to the nature of the activity, for instance, a capital can levy taxes in an exhaustive way on all cities belonging to its region or state, whereas in the case of other activities this rule can attribute only a preference for a territorial market ; 3) interactions within specialised networks are free from distance constraints, even if exploring them for developing new markets along this line may have a differential costs according to the distance. Long distance trade, maritime transport, part of tourism activities, or manufacturing industry, are following this type of spatial interaction rule (Figure 8).

## Conclusion

Very stimulating ideas come from the complex systems theory and simulation tools including reflexive and cognitive aspects in social systems, as exemplified by « small worlds » modelling. While such ideas appear challenging for geography, we want to draw attention on what could be a more specific contribution of our discipline to this developing field of research. Because interested in the world's diversity and its various scale effects, geography could foster types of model which take into account intermediary groupings between micro behaviour and macro scale, and document interactions between aggregate entities as well as their evolution. Until now, geography has provided most of its scientific results at such diverse intermediate scales, instead of producing a standard theory of any "homo geographicus".

Of course simulation models can be conceived as entertainment tools, and designed for building games, or imagining fictive worlds, as utopias always did. But if we want to learn something about the real world from such exercises, by confronting the results of simulation with observation, there are two essential points: first is that these models should refer to the existing knowledge when defining the agents relevant attributes and behaviour, for instance from surveys in demography, social or economic, instead of inventing intuitive and untested rules; and simulated results should be not just presented but evaluated by using statistics or spatial analysis as benchmarks for testing the plausibility of the simulated emergent structures. One could avoid rejections of these new instruments because of mutual ignorance between modellers and the non modelling community of geographers, and contempt judgements as the one formulated by Durlauf about "econophysics": "there is also a strong tendency in the econophysics community to denigrate the body of existing economic theory, leading both to a misunderstanding of that theory as well as a failure to integrate complex systems perspectives into the theory. Instead, one sees theoretical models proposed that all too often make little sense to a social scientist" (2003). We should then keep in mind that models are useful if they can reasonably be integrated in the existing knowledge, formalising it and possibly producing new results.

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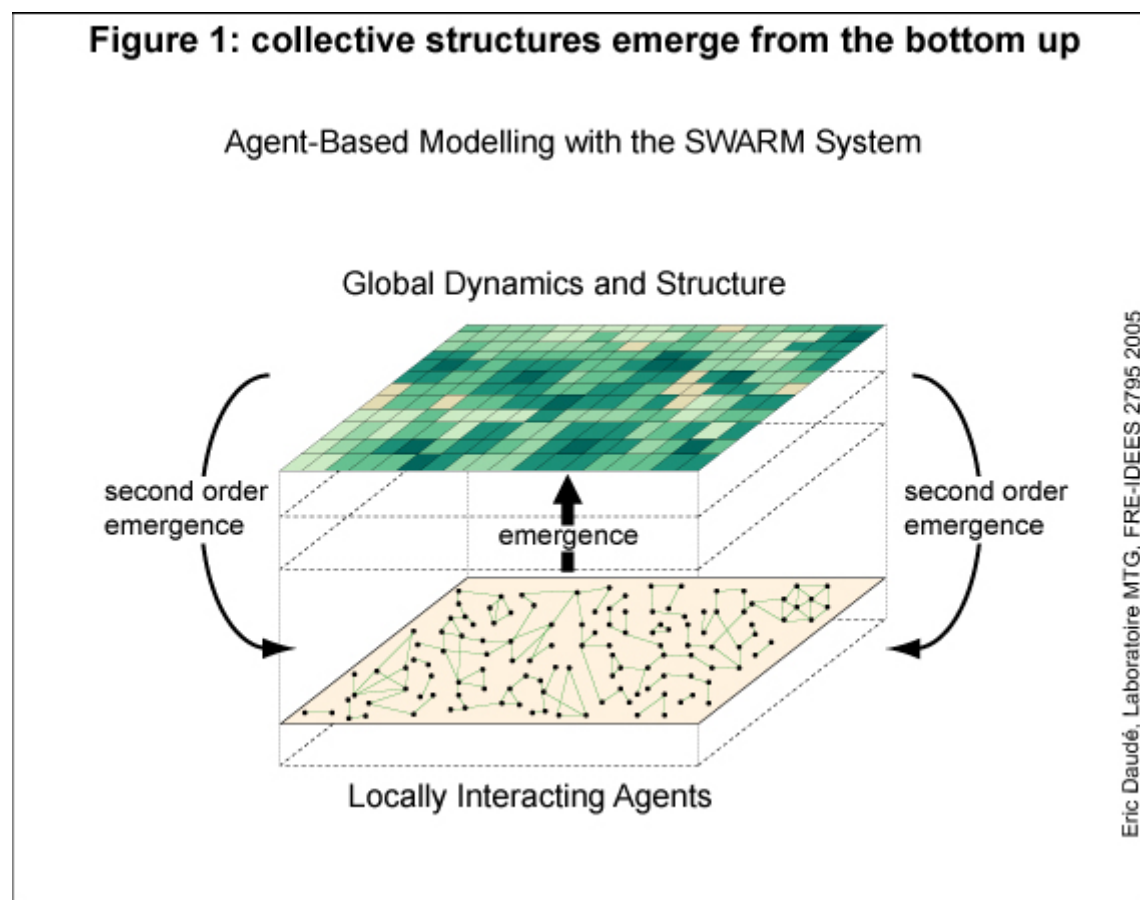
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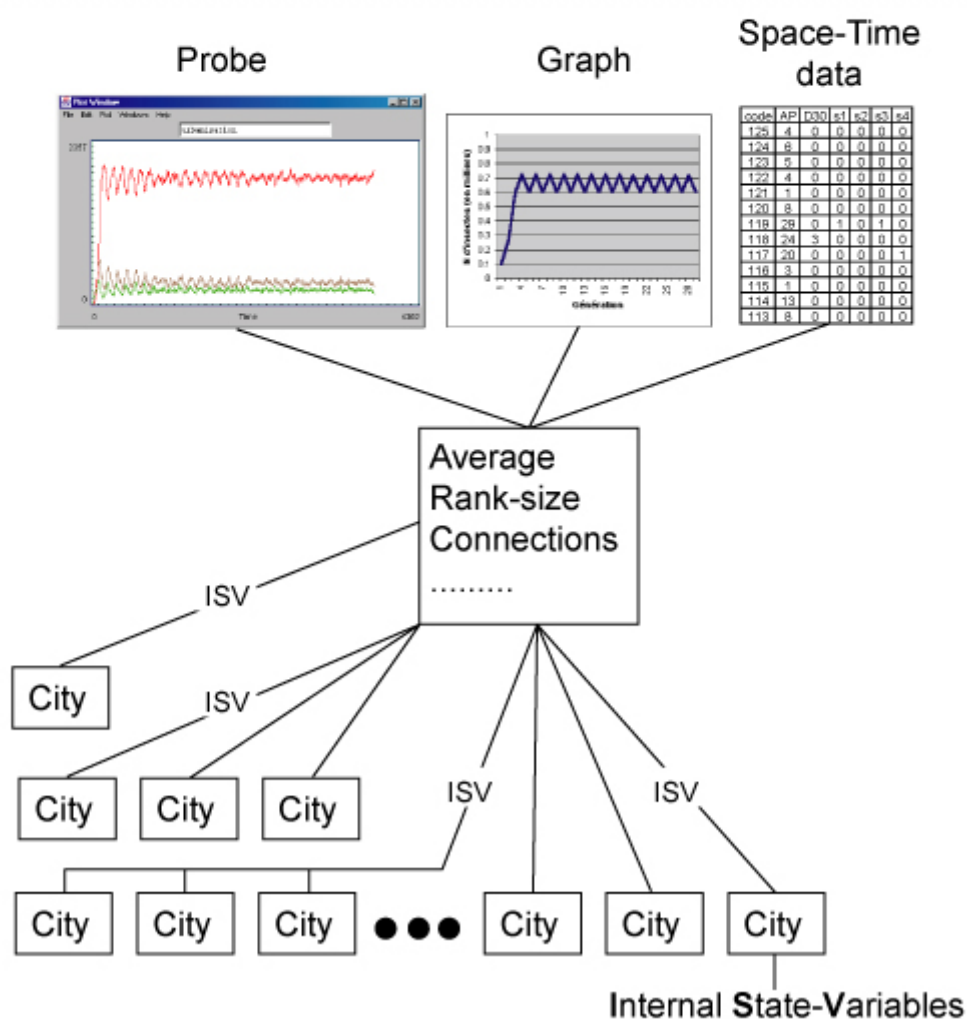
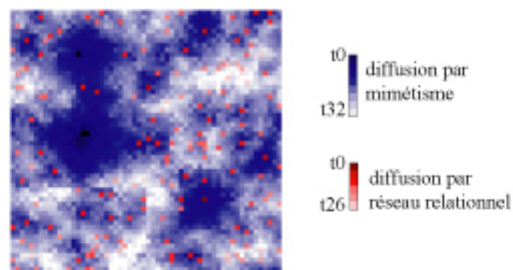
	Geopolis (1)	NUREC (2)	Madella/Rabino (3)	Variation (%) between 1 and 2	Variation (%) between 2 and 3	Variation (%) between 1 and 3
Milano	3895	1912	3752	51	49	4
Roma	2962	2775	2844	6	2	4
Napoli	2888	1260	2184	56	42	24
Torino	1460	1066	1302	27	18	11
Firenze	948	466	791	51	41	17
Genova	881	692	704	21	2	20
Palermo	776	699	819	10	15	-6
Bari	653	342	483	48	29	26
Catania	612	362	533	41	32	13
Bologna	576	404	523	30	23	9
Salerno	524	149	167	72	11	68
Venezia	462	334	378	28	12	18
Padova	375	245	345	35	29	8
Bergamo	348	123	397	65	69	-14
Massa/Carrara	262	183	189	30	3	28
Brescia	314	194	319	38	39	-2
Verona	290	256	276	12	7	5
Cagliari	304	204	301	33	32	1
Taranto	270	232	232	14	0	14
Caserta	261	102	279	61	63	-7

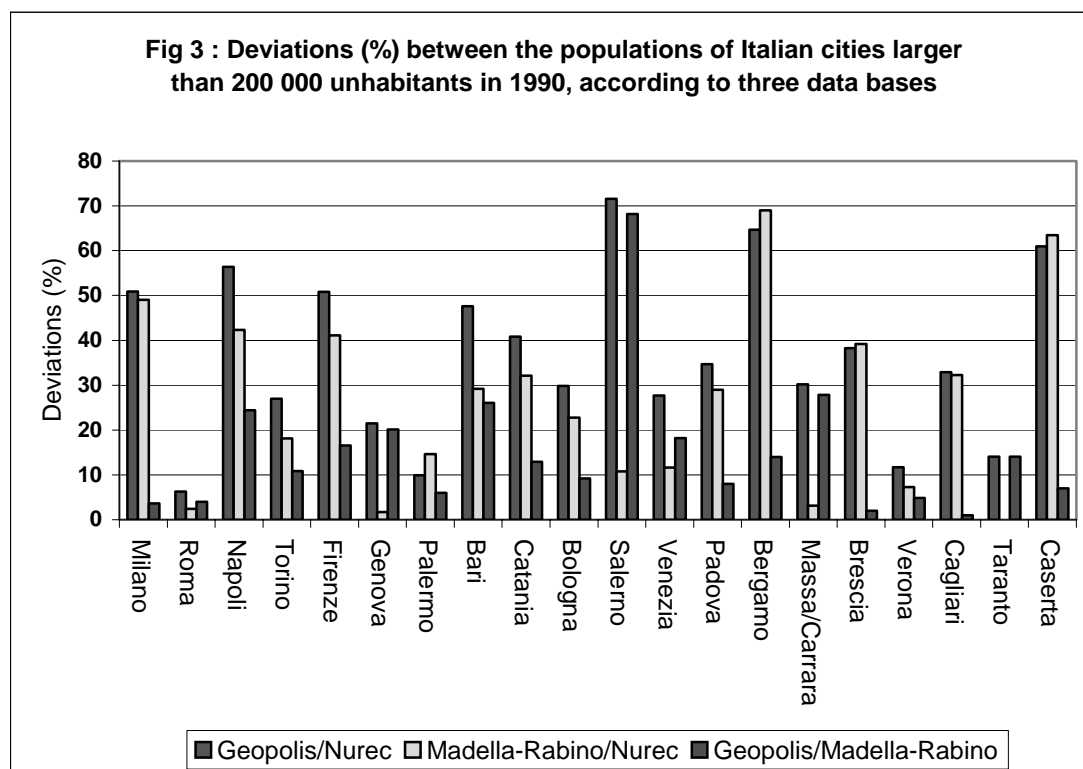
**Table 1: Population of Italian agglomerations larger than 200 000 inhabitants in 1990, according to three data bases (populations in thousands)**

Sources : Moriconi-Ebrard (2003, web site), NUREC (1994), Madella and Rabino (2003)

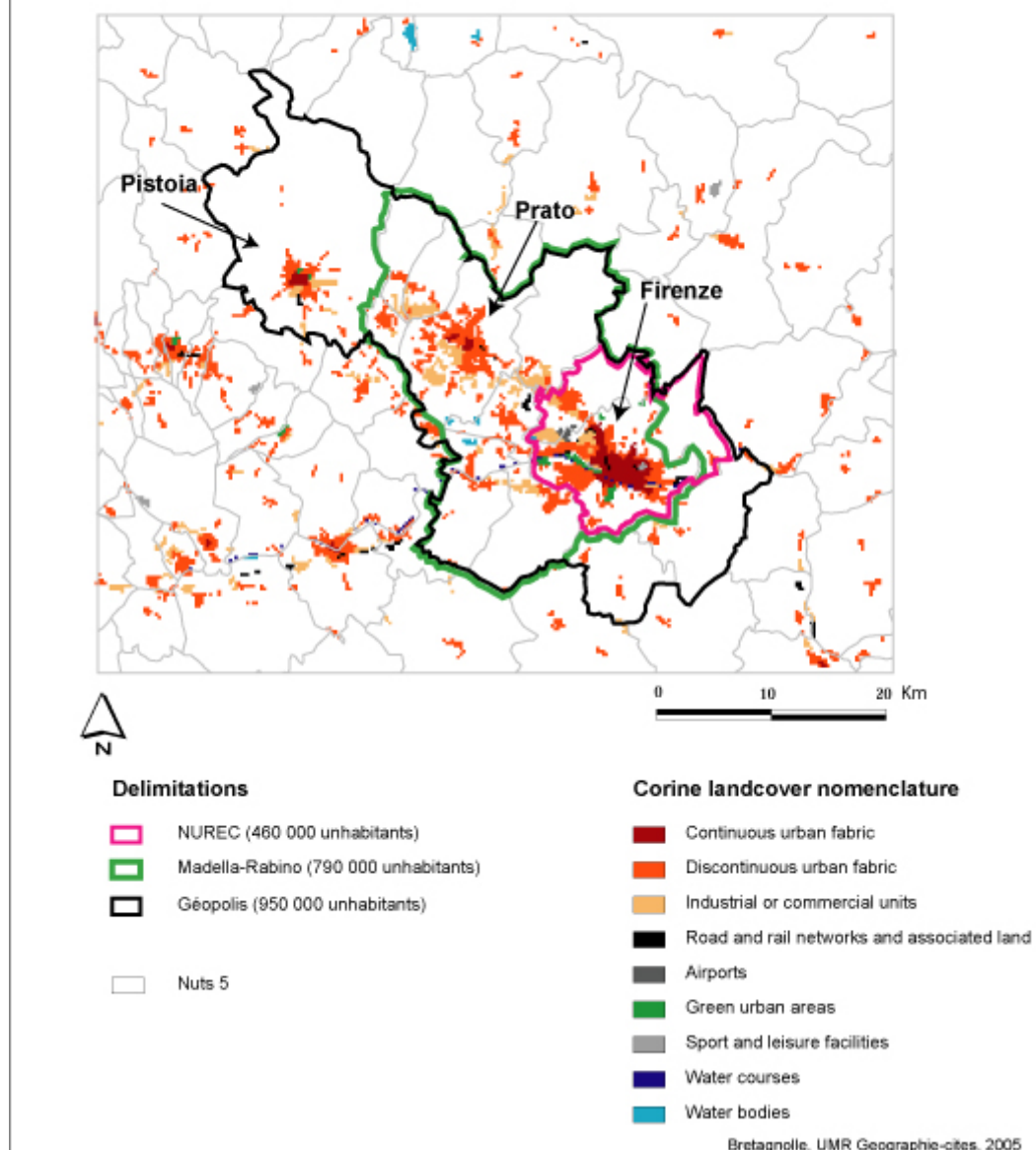


A 2-dimensional image of the screen





**Figure 4: Three delimitations of the agglomeration of Firenze (1990), using the same morphological and statistical criteria**



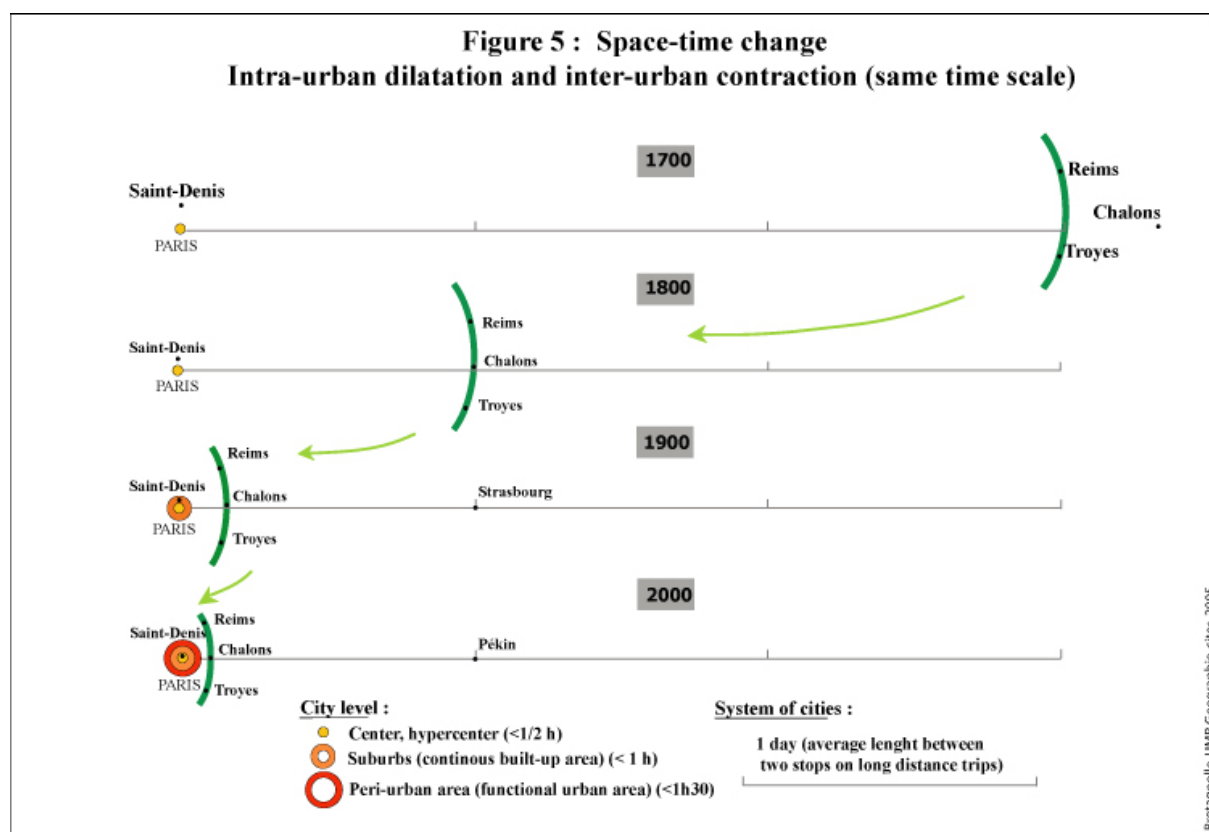
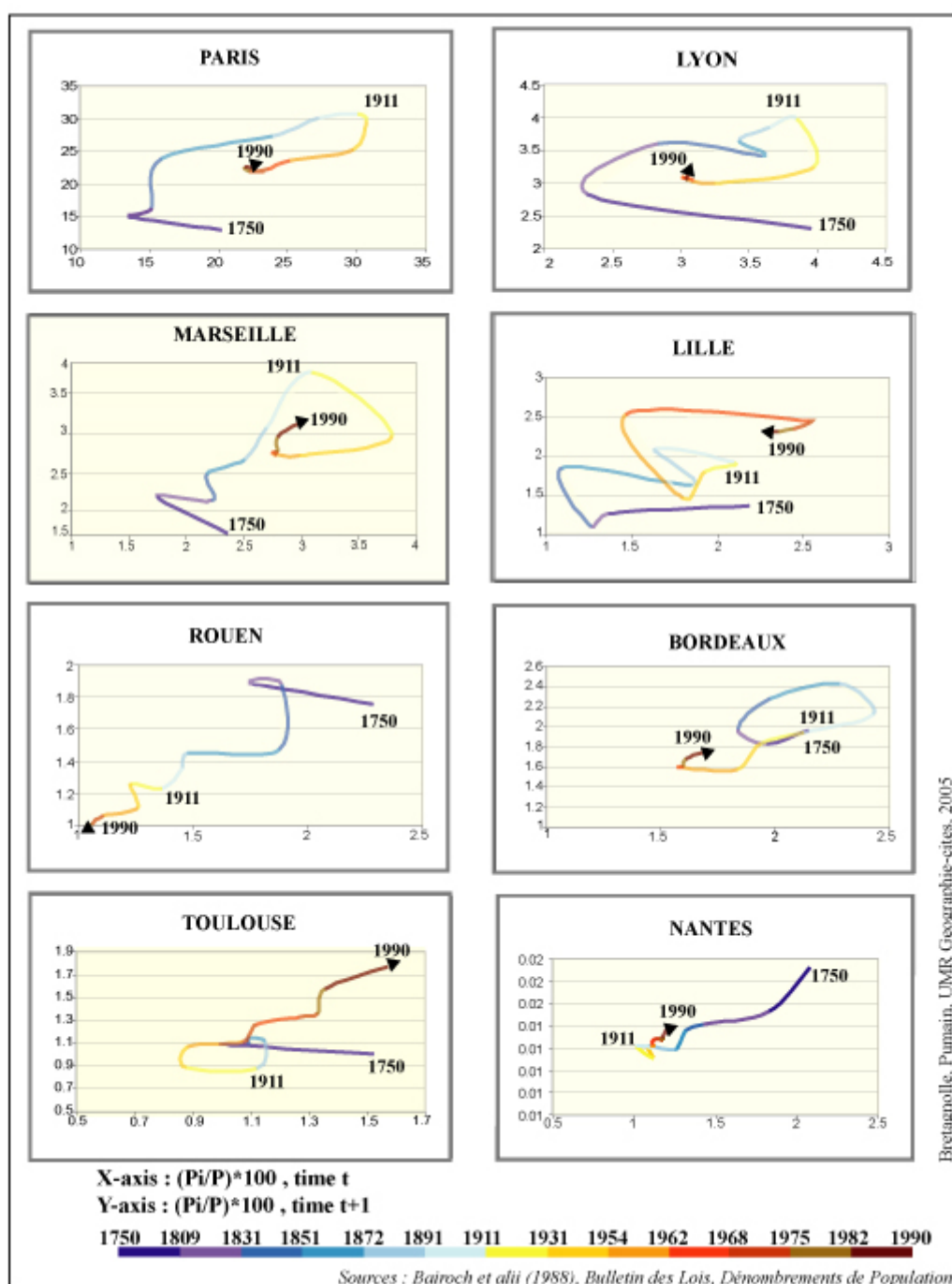
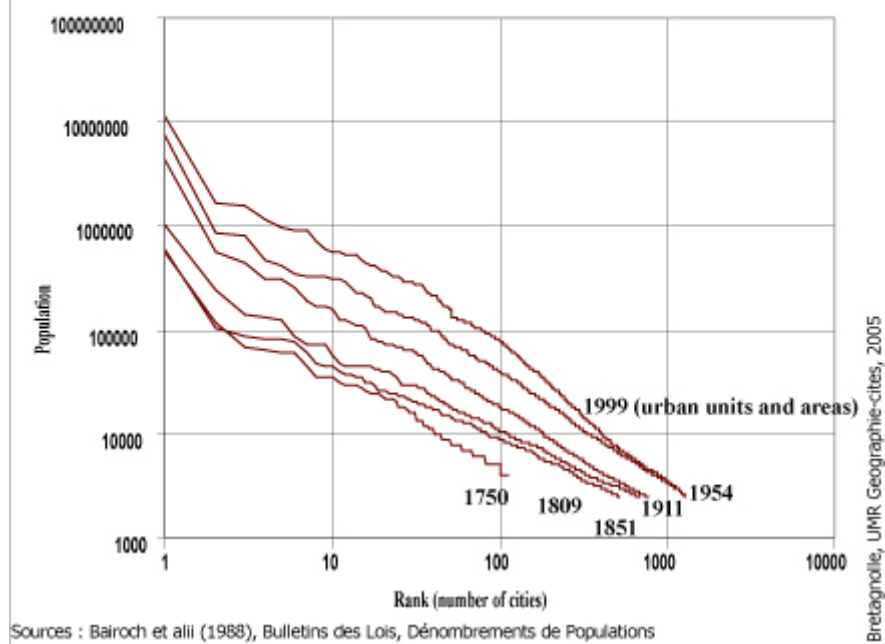




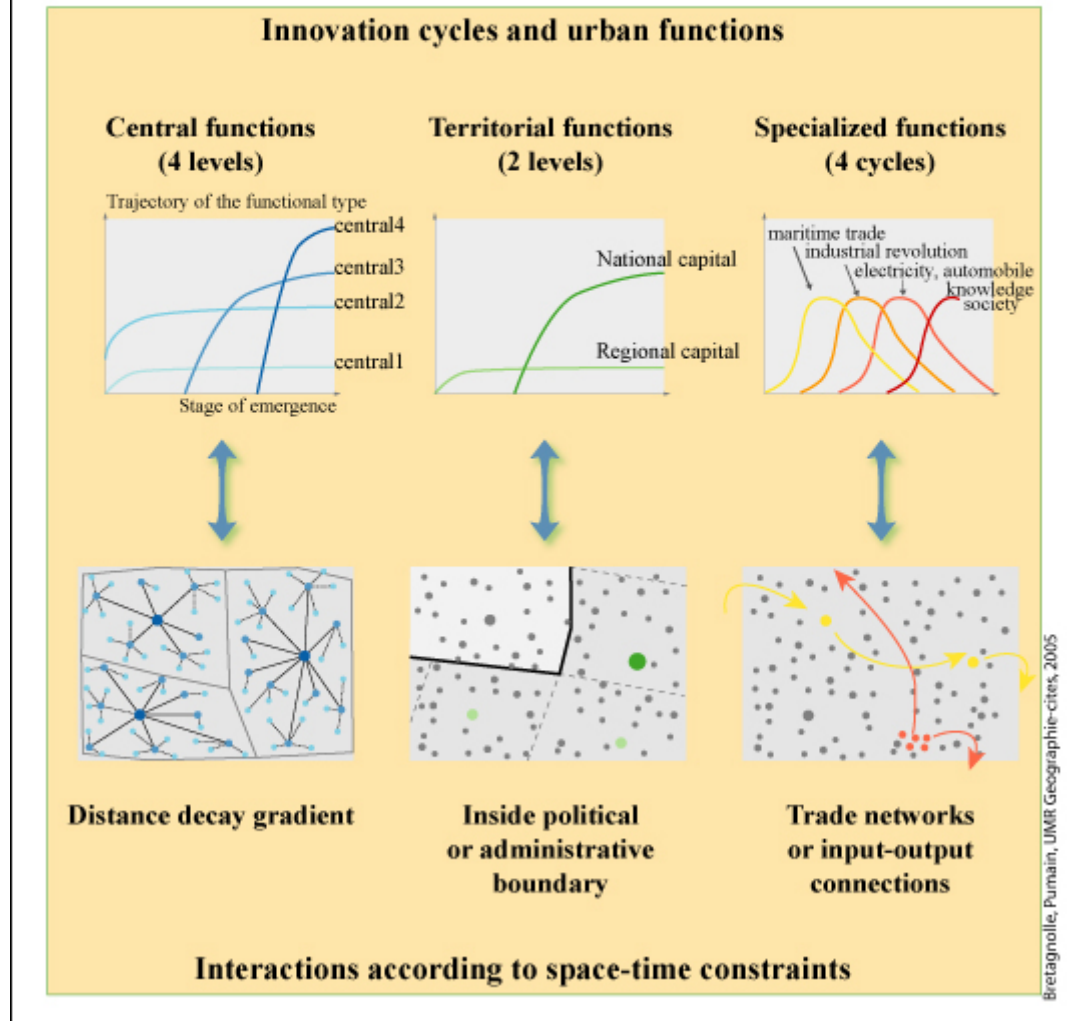
Figure 6 : Trajectories of relative sizes of the largest French cities



**Figure 7 : Rank-size distributions of the French towns and cities (urban units), 1750-2000**



**Figure 8 : Urban functions and spatial interactions modelling rules (Simpop2)**



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