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HUFF'S MODEL AND THEORY OF GRAPHS APPLIED TO RENEWABLE ENERGY MUTUALISATION

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ABSTRACT

The Huff model is generally used to determine trading areas around particular facilities. It may however be applied to other issues such as the supply of solar energy in cities. When combined with the theory of graphs, having taken into account the technical constraints relative to the productivity of solar panels, the Huff model can be used to demonstrate that mutualisation of energy production and supply via mini networks is the most efficient solution for urban areas.

EXPLOITING URBAN RENEWABLE ENERGY (RE): ACCOMMODATING TWO OPPOSED UTILITIES

Considering the city as a physical place endowed with resources is tantamount to stating that a significant part of the demand that is “naturally” made on the urban environment (aeraulic demand, solar radiance –see for instance [Compagnon, 2004] and [Santamouris, 2001], geothermal demand [Maïzia, 2007]) forms a mobilisable capital justifying the set up of technical infrastructure provided this demand as converted into calorific or electrical energy (aeraulics thanks to Aeolian technology, solar energy with thermal or photovoltaic panels, geothermal energy using a heat pump) and considered exploitable to meet the needs of city dwellers.

Thus for energy applications on an urban scale, one can design a distribution network of minimal length (to avoid energy losses as much as possible) that links one or more power plants to the greatest feasible number of – and ideally all - inhabitants. To accomplish this task, one must at the onset contend with the double constraint of accommodating the two opposed utility functions that express the logic the inhabitants will use to choose their RE equipment. Let us take the example of thermal solar energy, which usually ensures the supply of domestic hot water by means of solar panels placed on the roof of buildings. How do the two opposed utility functions manifest themselves? On the one hand, if we assume that all inhabitants behave with the same degree of rationality, they will all want to maximize their solar energy production utility and require to be connected to plants with the solar panels that give the highest yield per square metre. This is of course the case because the higher the yield, the smaller the solar panel surface, hence the lower the cost of initial investment and the quicker the equipment pays for itself. On the other hand, they will be eager to minimize the effects of negative utility due to distribution: the shorter the distance between consumption points (their home) and solar panels, the greater the overall yield of the equipment and the lower the economic cost. In other words, in the case of production by thermal solar panels, it will always be in the interest of households to be connected by the shortest route possible to the panels with the highest RE yield.

In addition, one is not only faced with the double constraint due to the inhabitants' reasoning about connection but also with limitations resulting from the physical shape of the city itself. Distribution network layout is obviously governed by urban morphology, which is by and large determined by the shape of buildings (the supposed physical support for energy production) and the road framework (the supposed distribution support). Energy resource can therefore be regarded as a consequence of urban morphology: with respect to solar RE, the ideal urban fabric is one that maximizes optimal solar panel surfaces whilst offering multiple distribution channels.

RE MUTUALISATION LEVEL: A TOPOLOGICAL DECIDING FACTOR

The simplest form of solar fitting is one where the energy production of each building is dedicated to the exclusive consumption of its inhabitants (this is so in the majority of cases: the solar panel equipment on the roof meets the sole needs of the building's occupants). This particular case of "every man for himself" can in some ways be regarded as an extreme and therefore as a case too unique to be of interest to us. Indeed, it indiscriminately excludes from exploiting RE resources all buildings (and hence all their inhabitants) that cannot meet their own needs due to unfortunate orientation with respect to the sun whilst significantly under exploiting buildings that are very favourably oriented and could produce large quantities of solar energy (relative to their own requirements).

The most satisfactory idea that springs to mind in order to go beyond this simplistic one-to-one correspondence and remedy such under exploitation is to redistribute the surplus of unexploited resources. This means the inhabitants of buildings more favourably endowed would offer part of their production to the less favoured. This is what we call RE mutualisation. More specifically, mutualisation would involve ensuring the same total energy production by increasing the solar panel surface on the roofs of the most favourably oriented buildings whilst reducing the panel surface on the roofs of the less well-exposed buildings. Mutualisation defined in these terms would allow meeting the overall requirements of a city in the most effective way possible by reducing the total panel surface and hence the economic cost of installation to a minimum. By the same logic, full mutualisation would consist in concentrating all the production in a single place or building with ideal solar exposure. However, experience has shown this solution is not very satisfactory either (energy losses, network vulnerability, etc.).

What suitable solutions have we got left? There is of course the large array of "moderate" types of mutualisation ranging between the two extremes we have just described. This type of mutualisation, which is neither non-existent nor total, consists in building mini distribution networks over areas that span from a single islet to entire neighbourhoods.

This having been said, how can we most accurately define the adequate "level" of mutualisation? The level of optimal mutualisation does not exist if we analyse the issue from a strictly theoretical point of view (the equations involved cannot be used to calculate this). It is however possible to tend towards optimisation empirically. Without going into highly technical details, it is possible to characterise urban morphology with respect to roof orientation by expressing the level of mutualisation as a function of the productivity of potential solar panels placed on them (*Figure 1*). Upon analysing the points of inflexion of the curve of this function, it is not economically viable to increase mutualisation beyond a certain level since this will not necessarily lead to a significant gain in productivity.

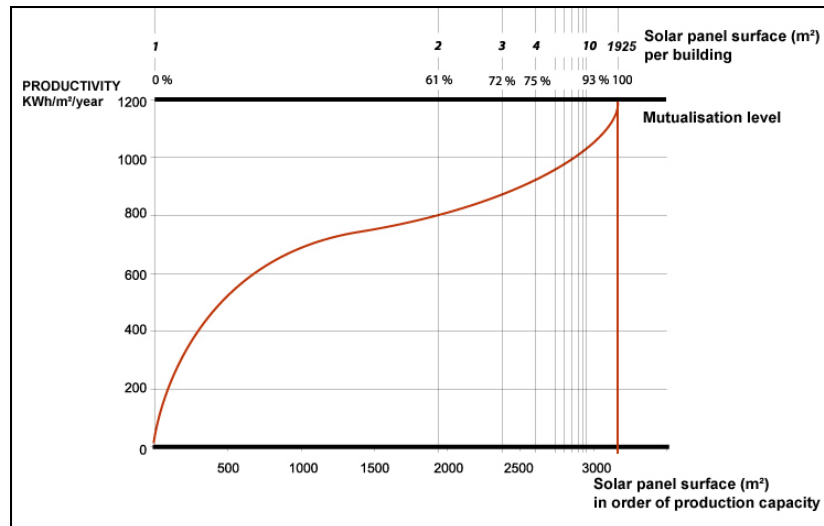


Figure 1: Mutualisation and solar panel productivity (applied to the town of Compiègne)

A DOUBLE GRAPH TO REPRESENT OFFER AND DEMAND FOR URBAN RE

Resorting to the theory of graphs to represent these opposing forces can be extremely useful. Regarding energy, the city can thus be represented in terms of nodes that express energy requirements (buildings housing people) as well as the production of energy that should meet this demand. The nodes are linked by a potential network that is totally connected topologically and superimposes itself geometrically on the road system (or any land that can harbour a grid of cables and pipes). In theory, such a network initially contains all the connection solutions offered to households. However, only one of these solutions can maximize their utility in terms of solar energy requirements.

From a more general perspective, modelisation of a city can be achieved with a “double” graph comprising two structurally analogous layers (i.e. layers of identical geometry and topology) that are perfectly “superimposed” in a theoretical (or vectorial) urban space. In the double graph, the host of nodes in the top layer symbolise the locations where energy provided by hypothetical solar panels is offered. The nodes in the bottom layer represent the locations where the potential energy needs of local inhabitants create a demand. This very convenient theoretical construct makes it possible to differentiate buildings that might constitute energy production centres from those that are evaluated as being consumption areas.

In the case study presented in this paper, the solar productivity of every potential panel –the top layer- was calculated by using SOLO methodology [CSTB, 1995]. The functions between the potential productivity, the slope and azimuth of roofs were implemented as algorithms into a GIS data base –*Erreur ! Source du renvoi introuvable.*

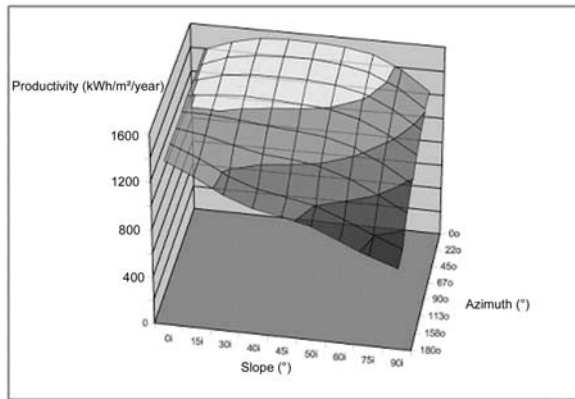


Figure 2: Potential panel productivity



Figure 3: Solar panel surface (m²) necessary to meet 50% of individual needs for domestic hot water (applied to the town of Compiègne)

Slopes were deduced from the morphological analysis and the azimuth from the streets orientations. According to the relative homogeneity of the buildings height, the effects of mask were therefore not considered in the calculation. The productivity was calculated by using buildings as unit of analysis. Each building was described as a vector every component of which was associated streets network and architectural typology data bases. Combination between data bases and SOLO algorithms gave the final output database

HUFF'S MODEL: A GRAVITARY PROBABILISTIC REPRESENTATION DEDICATED TO THE CONSTRUCTION OF MINI RE DISTRIBUTION NETWORKS

Seen from another angle, the production or consumption of the remaining nodes each represents a kind of “mass”. For example, in the case of the top layer, the greater the production of a node, the higher its mass. The same logic applies to the bottom layer and consumption. As most economists and geographers know, it allows one to approach the problem in gravitary terms.

The top layer represents the localized masses of energy offer. In principle, such masses are in strong competition with one another and repel each other as if they had the same electrical charge (to pursue the analogy with particle physics). This competition allows each mass to build a surrounding field of attraction that acts upon the nodes in the bottom layer. Economics and marketing call this field a “trading area”.

Meanwhile, the lower layer is subject to the influence of attraction induced by the upper layer. The demand of a building is very likely to be met by the offer from another building (or by itself) particularly if it is located inside the area of influence of the latter (inside its trading area). The gravitary or Reilly relationships that exist between buildings as a result of mass, whether they are hives of energy production or consumption, have been remarkably described using the marketing model developed by Huff [Huff, 1964].

The Huff model hypotheses can be simplified by using the theory of graphs [Berge, 1970] as a framework. These hypotheses consider that for each user located at node i , the opportunity to use – or in this case be connected to – a piece of facility situated at node j depends, as in Reilly's model, on the service (quantified by the facility mass M_j) that the facility can offer and the difficulty involved in reaching it or being connected to it (the resistance $r(D_{ij})$, which is simplified in Huff's model to the function $D_{ij}^{-\beta}$). Such an opportunity, denoted O_{ij} , represents the product of the facility mass by the corresponding resistance to reach it:

$$O_{ij} = M_j \cdot r(D_{ij}) = \frac{M_j}{D_{ij}^\beta} \quad (1)$$

The total number of opportunities available to a user located at i is seen as the potential access to this type of service given the facility available in the city.

$$U_i = \sum_k O_{ik} \quad (2)$$

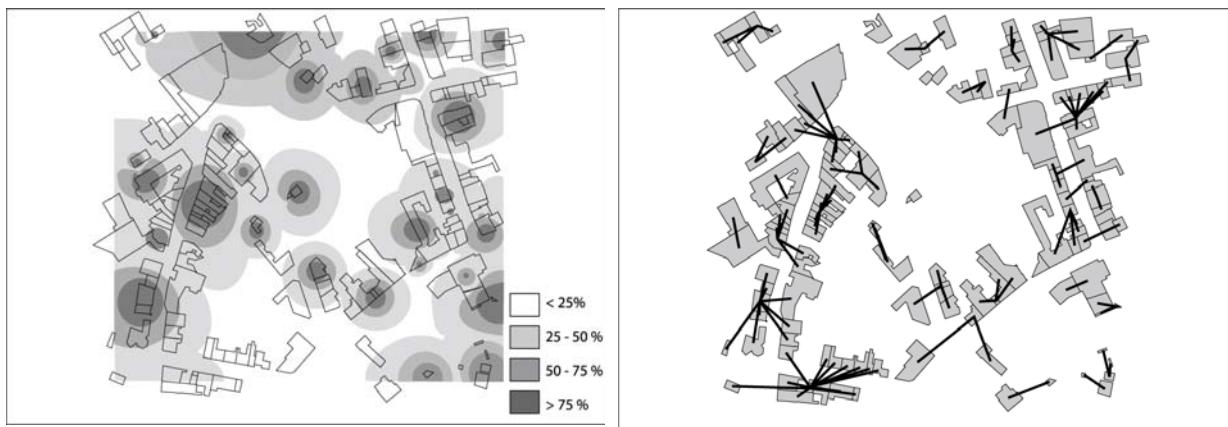
The probability that a user located at i will connect to the facility situated in j is therefore the ratio of the opportunity to be connected at j over all the opportunities available in the city or in other words, the ratio of the opportunity to go to j over the potential available to the user:

$$p_{ij} = \frac{O_{ij}}{U_i} = \frac{O_{ij}}{\sum_k O_{ik}} \quad (3)$$

How can these hypotheses be applied to RE mutualisation?

Masses M_j in Huff's model replace the nodes in the top layer of the double graph. They express the solar energy production of buildings selected for mutualisation in terms of $kWh/m^2/year$ (where m^2 applies to the panel surface). The greater the mass, the more users will be likely to make their utility positive. Of course, the chances of this happening will more often than not be counterbalanced by the distance effect, i.e. the distance that separates them from production masses. Such distances, represented as D_{ij} in Huff's model, account for network energy losses, which cannot be described using an affine function and are represented by the exponent β . Conversely, the greater the distance, the greater the negative impact on user utility.

Thus a user's opportunity to be connected to a building that mutualises solar energy production will be expressed as $kWh/m^2/year/lm^\beta$ (lm : linear metre of network). If we work out the ratio of this opportunity relative to the offer potential of all the production masses, we obtain the percentage probability that a user will be connected to a given production point. When applied to the city as a whole, this method makes it possible to determine trading areas around production points – *Figure 4*– and one has to do is lay out the various micro networks!



Proximity zones around sectors according to their potential to produce solar domestic hot water *Automatic translation into direct-link micro-networks*

Figure 4: Application of Huff's model to solar energy production by hypothetical micro power plants (applied to the town of Compiègne)

In order to map this network, one can use various algorithms depending on the technological option chosen for the system: a direct link between users and solar energy power plants corresponds to a parallel set-up while linkage using the shortest path algorithm (Floyd, Shimmel, etc.) corresponds to a set-up in series.

CONCLUSION

In spite of the linear form of this text, the Huff model transposed to the theory of graphs and applied to the exploitation of solar urban resources should not be interpreted solely as a method for determining the size of an energy distribution system or as a simple engineering “recipe”. The purpose of discussing this application is rather to demonstrate that probabilistic gravitary theories can contribute to determining the best way to distribute a given product as long as they are coupled with theories applicable to networks. The advantage of this type of representation is that it does not limit one’s analysis of urban spaces to their geometrical aspect, which is often the case in spatial research [Traisnel, Maïzia, 2005]. Integrating topological aspects makes the demonstration more subtle and better adapted to the technical realities of urban situations in addition to being rather satisfying from a theoretical standpoint.

Mass in a gravitary model should not be solely viewed as a point of Euclidian space earmarked with a quantitative attribute. It also represents a topological location that depends on connections with an environment composed of other topological locations. These connections define distribution modalities and explicitly indicate the effect of distance on the quality of the product reaching the final consumer (or to be more general, the final node). Distance in a gravitary model of this type is also therefore a topological distance that is equally if not more important than traditional Euclidian distance. The topological characterization of the network that links “mass” to the final node offers a crucial indication of the quality of exchanges between origins and destination by including - amongst other things - notions relative to the fragility of the system under study (for instance, the fragility of networks as a function of mutualisation). The chances that an origin be connected to a particular destination can thus be integrated into the probabilistic approach of Huff’s model by taking into account the connectivity of the network that links them up. However, integrating topological vulnerability within a gravitary model is quite another story...

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