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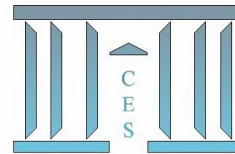
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Public debt and aggregate risk[★]

Audrey Desbonnet^a and Sumudu Kankanamge^b

^a*University of Paris I Panthéon-Sorbonne, CNRS, France*

^b*University of Paris I Panthéon-Sorbonne, CNRS, France*

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Résumé :

Cet article reconsidère l'impact de l'introduction de la dette publique dans le cadre de marchés incomplets et de fluctuations macroéconomiques. Aiyagari et Mc Grattan (1998) montrent que l'incomplétude des marchés et l'existence d'une contrainte d'endettement créent un motif d'épargne de précaution qui réaffirme le rôle de la dette publique. Parce qu'elle offre un moyen additionnel aux ménages de lisser leur consommation, l'introduction de la dette publique est bénéfique. Cependant, cette analyse ignore une source non négligeable d'exacerbation du risque idiosyncrasique de revenu non assurable qu'est le risque agrégé et risque donc d'avoir sous estimé le rôle de la dette. L'accroissement du taux et de la durée du chômage lors des phases de récession entame la capacité assurantielle de l'épargne de précaution. En outre, l'occurrence de faibles taux d'intérêt durant les périodes de récession rend plus coûteuse la constitution d'une épargne de précaution. L'introduction de la dette publique accroît le taux d'intérêt. L'épargne de précaution devient moins coûteuse. C'est la raison pour laquelle le niveau optimal de dette publique est égal à 180% du produit trimestriel et s'établit à un niveau plus élevé que dans un environnement dépourvu de fluctuations macroéconomiques.

Abstract :

This paper assesses the optimal level of public debt in a new framework where aggregate fluctuations are taken into account. Agents are subject to both aggregate and idiosyncratic shocks and the market structure prevents them from perfectly insuring against the risk. We find that the optimal level of debt is very different when aggregate risk is taken into account : a simple idiosyncratic model generates a quarterly optimal level of debt of 60% of GDP, our benchmark model embedding aggregate risk finds the quarterly optimal level of debt to be 180% of GDP. Thus aggregate fluctuations have a strong positive impact on the level of public debt in the economy. Aggregate fluctuations exacerbate the overall risk level in the economy and households are forced to increase their precautionary saving in response. Public debt and the implied higher interest rate generate a strong effect that helps precautionary saving behavior.

Mots-Clés : dette, risque agrégé, épargne de précaution

Key words : debt, aggregate risk, precautionary saving

JEL classification : E32, E63, H31

1 Introduction

The introduction of uninsurable risk was a major step in the recent literature on public debt. The first author to depart from the complete markets framework in this way is Woodford (1990). He describes a simple economy embedding credit constraints and finds that public debt can be efficient because it keeps interest rates higher and closer to time preference rates. Aiyagari and McGrattan (1998) quantitatively address the question of the optimal level of public debt in a heterogeneous agent, incomplete market model. Calibrating on the U.S. economy they find the annual positive debt over GDP ratio of 2/3 to be optimal. Finally Floden (2001) also uses the former framework to look at public debt/transfers optimal combinations. This author underlines the strong uncertainty and inequality implications behind public debt policies. Very much in the same way, the introduction of uninsurable risk in the literature on the cost of business cycles has generated rich implications. In an effort to reconsider the small welfare effect of business cycles found by Lucas (1987), both Krusell and Smith (2002) and Storesletten, Telmer and Yaron (2001) draw a link between the aggregate risk and the cross sectional distribution. In other words, the aggregate productivity shock is correlated to the individual specific shocks to produce strong distributional effects of aggregate fluctuations. In such a framework, these authors find a greater cost of business cycles than Lucas (1987). Even more interestingly, Imrohorglu (1989) briefly suggest that economic policies could be used to help individuals bear the cost of business cycles. In this paper, we explore this latter suggestion and find novel implications for public debt.

This paper's main objective is to introduce a framework exhibiting aggregate fluctuations to quantify the optimal level of public debt. As aggregate fluctuations impact agents' saving behavior, it is an important feature to take into account for public debt policies that has been ignored so far. Our framework can also be used to decompose the effects of public debt along the cycle or across the population. Following Bewley-Huggett-Aiyagari type models, we build a production economy with capital market imperfections where a large number of ex-ante identical infinitely-lived agents face idiosyncratic income shocks and aggregate productivity shocks. Households' saving behavior is influenced by precautionary saving motives and borrowing constraints. Private capital and government bonds, both yielding the same interest, can be claimed to insure against future risk. Government levies proportional taxes on households and issues debt in order to finance its consumption.

In this setup, our benchmark calibration yields an optimal level of public debt of 180% of GDP on a quarterly basis (roughly 50% in annual equivalent).

This is to be compared to the simple idiosyncratic risk model¹ that yield the lower optimal level of public debt of 60% on a quarterly basis (roughly 15% in annual equivalent). We also find that the average consumption gain of being at the optimal level instead of the benchmark level is about 60 times higher in the aggregate risk model than in the idiosyncratic risk model. This gain can be dramatically higher in recessions or for some percentiles of the population. Thus, we find aggregate uncertainty to have strong implications on the optimal level of public debt. The intuition behind our results is the following. Credit constraints and uncertainty lead agents to engage in precautionary saving. The result of precautionary saving is a higher level of capital what in turn mechanically lowers the interest rate away from the time preference rate. Aggregate fluctuations exacerbate overall risk level in the economy. An *employment fluctuations effect* makes it more difficult for agents to smooth their consumption because there is a risk that they spend more time without a job in bad periods. A *price fluctuation effect* changes the level of prices between recessions and booms making it more difficult to accumulate precautionary saving in bad periods. As a result agents engage in even more precautionary saving. The capital stock rises and the interest rates move further away from the time preference rate. A positive level of public debt has a strong effect here : a higher level of debt increases the interest rate and reduces the cost of precautionary saving what helps consumption smoothing. Thus households are better off with a positive level of debt when the economy is subject to aggregate fluctuations. We show that the interest rate is the key variable to understand why agents prefer higher levels of public debt. In a price fixed small open economy, the optimal level of debt is much lower than the levels mentioned above and is no more positive.

We also find that the welfare implications of public debt vary between groups of individuals. As discussed by Ball and Mankiw (1995) and Floden (2001), our model accounts that rich agents prefer higher levels of debt whereas poor agents prefer lower levels of debt. Higher taxes and crowding out of capital harm all agents but only rich people, whose main income is the return on the assets they hold, benefit from high levels of interest rates. Finally, we extend the model to replicate European labor market features and find that the optimal level of public debt is higher than in the benchmark economy.

The rest of the paper is organized as follows. Next section describes the benchmark economy. Section 3 details the results. In section 4 we characterize the optimal level of public debt for the European labor market. The last section concludes.

¹ We switch from the aggregate risk model to the simpler idiosyncratic risk model following Imrohoroğlu (1989) : we average the employment process over bad and good periods and keep the same calibration as in the general model otherwise.

2 The Benchmark Model

Our benchmark economy is a Bewley-Huggett-Aiyagari type dynamic stochastic general equilibrium model augmented to allow aggregate fluctuations à la Krusell and Smith (1998) and public debt. Insurance markets are incomplete. Agents face idiosyncratic and aggregate risks and are borrowing constrained. These three assumptions lead agents into precautionary saving (Aiyagari (1994)).

2.1 Firms

We assume that there is a continuum of firms which have a neoclassical production technology² and behave competitively in product and factor markets. The output is given by :

$$Y_t = z_t F(K_t, N_t)$$

where K is aggregate capital and N aggregate labor used in production. Capital depreciates at a constant rate δ . The economy is subject to an exogenous aggregate shock noted z . There are two possible aggregate states : a good state where $z = z_g$ and a bad state where $z = z_b$. The aggregate shock follows a first-order Markov process with transition probability $\eta_{z|z'} = \Pr(z_{t+1} = z' / z_t = z)$. Thus $\eta_{z|z'}$ is the probability that the aggregate state tomorrow is z' given that it is z today. We note η the matrix that describes the transition from one aggregate state to another such that :

$$\eta = \begin{pmatrix} \eta_{gg} & \eta_{gb} \\ \eta_{bg} & \eta_{bb} \end{pmatrix}$$

Finally, our setting assumes that inputs market are competitive. The wage w and the interest rate r verify :

$$\begin{aligned} r_t + \delta &= z_t F_K(K_t, N_t) \\ w_t &= z_t F_N(K_t, N_t) \end{aligned}$$

² The function F exhibits constant returns to scale with respect to K and N , has positive and strictly diminishing marginal products, and satisfies the Inada conditions.

2.2 The government

The government issues public debt and levies taxes to finance public expenses. Both the revenue of capital and labor are taxed proportionally at an identical rate τ . The government's budget constraint verifies :

$$G_t + r_t B_t + TR_t = B_{t+1} - B_t + T_t$$

with

$$T_t = \tau_t(w_t N_t + r_t A_t)$$

G_t is the level of public expenses, B_t the level of public debt, T_t tax revenues and TR_t a lump sum transfer to households that amounts to zero in equilibrium. We assume that public expenses are constant in every period. A accounts for total wealth in the economy and thus includes physical capital and public debt such that :

$$A_t = K_t + B_t$$

2.3 Households

The economy is populated by a continuum of *ex ante* identical infinitely lived households of unit mass. Their preferences are summarized by the function V :

$$V = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\prod_{j=0}^t \beta_j u(c_t) \right) \right\}$$

where β is the discount factor. We assume that this discount factor is random. Thus the discount factor β can differ across agents and varies over time. We specify that the latter follows a three-states first-order Markov process. This assumption on the discount factor helps reproduce the wealth distribution as shown in Krusell and Smith (1998). The discount factor verifies :

$$\begin{cases} \beta_0 = 1 \\ \beta_{j \geq 1} \in]0; 1[\end{cases}$$

c_t is the household level consumption. The utility function we use has a stan-

standard CRRA specification and writes :

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} & \text{if } \sigma \neq 1 \\ \log(c) & \text{if } \sigma = 1 \end{cases}$$

Agents are subject to idiosyncratic unemployment shocks. Let s be the household's labor market status. A household can either be unemployed ($s = u$) or employed ($s = e$). Households are also subject to shocks at the aggregate level. Aggregate shocks exacerbate idiosyncratic unemployment risks. The unemployment rate and the unemployment spells are higher in recessions than what they are in booms. Therefore, transitions on the labor market are correlated to the aggregate state. We note $\Pi_{zz'|ss'}$ the joint transition probability to a state (s', z') conditional on a state (s, z) . The matrix that jointly describes the transition from a state (s, z) to a state (s', z') is the following :

$$\Pi = \begin{pmatrix} \Pi_{bbuu} & \Pi_{bbue} & \Pi_{bguu} & \Pi_{bgue} \\ \Pi_{bbeu} & \Pi_{bbee} & \Pi_{bgeu} & \Pi_{bgee} \\ \Pi_{gbuu} & \Pi_{gbue} & \Pi_{gguu} & \Pi_{ggue} \\ \Pi_{gbeu} & \Pi_{gbee} & \Pi_{ggeu} & \Pi_{ggee} \end{pmatrix}$$

where $\Pi_{ggee} = \Pr(z_{t+1} = z_g, s_{t+1} = e | z_t = z_g, s_t = e)$.

When agents are in an employed state, they receive the wage w . However when agents are unemployed their income corresponds to their home production that we assume equal to a fraction θ of their employed wage³. Insurance markets are incomplete so that agents can only partially self-insure against idiosyncratic risk. Following Aiyagari and McGrattan (1998) no borrowing is allowed. The only way for households to self-insure against idiosyncratic risk is to accumulate physical capital and government bonds both yielding the same return r . Their overall holding in the later assets is noted a . Therefore a typical household solves the following problem :

$$\max_{c_t, a_{t+1}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\prod_{j=0}^t \beta_j \right) u(c_t) \right\},$$

subject to :

$$a_{t+1} + c_t = (1 + (1 - \tau_t)r_t)a_t + \chi(s_t)w_t + TR_t$$

$$c_t \geq 0$$

$$a_{t+1} \geq 0$$

³ The same assumption is made in Krusell and Smith (1998).

with

$$\chi(s_t) = \begin{cases} \theta & \text{if } s_t = u, \\ (1 - \tau_t) & \text{if } s_t = e \end{cases}$$

The existence of aggregate risk leads us to distinguish between individual state variables and aggregate state variables. The individual state variables are given by the vector (a, s, β) . The aggregate state variables are summarized by the vector (z, Γ) where $\Gamma(a, s, \beta)$ is a distribution of agents over asset holdings, employment status and preferences. To determine the wage and the interest rate, households need to forecast the aggregate stock of physical capital. Therefore, they need to know the wealth distribution. That is why wage and interest rate depend on that wealth distribution.

We detail the computational strategy we used to solve the model in appendix C.

2.4 Equilibrium

The recursive equilibrium consists of a set of decision rules for consumption and asset holding $\{c(a, s, \beta; z, \Gamma), a'(a, s, \beta; z, \Gamma)\}$, aggregate capital $K(z, \Gamma)$, factor prices $\{r(z, \Gamma), w(z, \Gamma)\}$, tax rate τ and a law of motion for the distribution $\Gamma' = H(\Gamma, z, z')$ which satisfies these conditions :

(i) Given the aggregate states, $\{z, \Gamma\}$, prices $\{r(z, \Gamma), w(z, \Gamma)\}$ and the law of motion for the distribution $\Gamma' = H(\Gamma, z, z')$, the decision rules $\{c(a, s, \beta; z, \Gamma), a'(a, \epsilon, \beta; z, \Gamma)\}$ solve the following dynamic programming problem :

$$v(a, s, \beta; z, \Gamma) = \max_{c, a'} \{u(c) + \beta E [v(a', s', \beta'; z', \Gamma') | (s, \beta; z, \Gamma)]\}$$

subject to :

$$c + a' = (1 + r(z, \Gamma)(1 - \tau))a + w(z, \Gamma)\chi(s) + TR$$

$$c \geq 0$$

$$a' \geq 0$$

and

$$\Gamma' = H(\Gamma, z, z')$$

(ii) Market price arrangements are :

$$r(z, \Gamma) = zF_K(K, N) - \delta$$

$$w(z, \Gamma) = zF_N(K, N)$$

(iii) Government budget constraint holds.

(iv) Capital market verify :

$$K + B = \int a'(a, \epsilon, \beta; \Gamma, z) d\Gamma$$

(v) Consistency : agents' optimization problem is satisfied given the law of motion H and the law of motion is consistent with individual behavior.

2.5 Calibration

For the sake of comparison, the model economy is calibrated to match certain observations in the U.S. data. We let one period in the model be one quarter in the data. To remain simple and allow comparisons, we closely follow Krusell and Smith (1998) when calibrating the characteristics of the labor market and the aggregate risk.

2.5.1 Technology

We choose the production function to be Cobb-Douglas :

$$Y_t = z_t F(K_t, N_t) = z_t K_t^\alpha N_t^{1-\alpha} \quad 0 < \alpha < 1$$

Technology parameters are standard. The capital share of output α is set to 0.36 and the capital depreciation rate δ is 0.025. As Krusell and Smith (1998) we assume that the value of the aggregate shock z is equal to 0.99 in recessions (z_b) and 1.01 in booms (z_g).

2.5.2 Preferences and discount factor

In the benchmark economy we assume a logarithmic utility function. We now detail the calibration steps to generate a realistic wealth distribution, the observed U.S. wealth Gini index and the capital-output ratio. Here our calibration differ from Krusell and Smith (1998). In their economy agents can borrow whereas here for the sake of comparison and simplicity we follow Aiyagari and McGrattan (1998) and no borrowing is allowed. To reproduce the shape of the U.S. wealth distribution we first assume that unemployed agents

receive income too and fix the home production ratio θ to be 0.10. This assumption produces a large group of poor agents. Next we use the preference heterogeneity setting discussed in Krusell and Smith (1998) to generate a long thick right tail. We impose that the discount factor β takes on three values $\{\beta_l, \beta_m, \beta_h\}$ where $\beta_l < \beta_m < \beta_h$:

$$\begin{pmatrix} \beta_l \\ \beta_m \\ \beta_h \end{pmatrix} = \begin{pmatrix} 0.9750 \\ 0.9850 \\ 0.9981 \end{pmatrix}$$

Thus an agent with a discount factor β_m is more patient than an agent with a discount factor β_l . To calibrate the transition matrix, we impose that the invariant distribution for β 's has 10% of the population at the lowest discount rate β_l , 70% at the medium discount factor β_m and 20% at the highest discount factor β_h . As Krusell and Smith (1998) we assume that there is no immediate transition between extreme values of the discount factors. Finally, we set the average duration of the lowest discount factor and the highest discount factor to be 50 years (200 quarters). These assumptions yield the following transition matrix⁴ :

$$\Upsilon = \begin{pmatrix} 0.9950 & 0.005 & 0.0000 \\ 0.0007 & 0.9979 & 0.0014 \\ 0.0000 & 0.0050 & 0.9950 \end{pmatrix}$$

The conditions that we have described so far specify 10 parameters so 10 targets are needed. When we account for the Gini index and the capital-output ratio, we need 8 additional targets. Given our calibration strategy, those 8 targets would be 8 points from the U.S. wealth distribution. In practice, instead of targeting 8 specific points, we searched for a set of parameters such that the wealth distribution of the model economy is as similar as possible to its U.S. counterpart.

As shown in Table 1 this calibration does a fairly good job at approximating the shape and the skewness in the U.S. wealth distribution⁵ and yields a Gini index of 0.78 and a capital-output ratio of 10.4⁶.

⁴ For further details on the calibration of this matrix, see appendix A.

⁵ The data we report on the U.S. distribution comes from Krusell and Smith (1998) and Budria-Rodriguez, Diaz-Gimenez, Quadrini and Rios-Rull (2002).

TAB. 1
Distribution of wealth : Model and Data

Model	Held by Top				Gini
	5%	10%	20%	30%	
Benchmark Model	41	63	85	93	.78
Data	51	64	79	88	.79
Model	Held by Bottom				
	20%	40%	60%	80%	
Benchmark Model	1	2	5	14	
Data	0	1	6	18	

2.5.3 Labor market processes

For the sake of simplicity our calibration of the aggregate shock and the labor market process follows Krusell and Smith (1998). The process for z is set so that the average duration of good and bad times is 8 quarters. Therefore, the transition matrix η for aggregate state switches is defined by :

$$\eta = \begin{pmatrix} 0.8750 & 0.1250 \\ 0.1250 & 0.8750 \end{pmatrix}$$

The average duration of an unemployment spell is 1.5 quarters in good times and 2.5 quarters in bad times. We also set the unemployment rate accordingly : in good periods it is 4% and in bad periods it is 10%. These assumptions enable us to define the transition matrixes for labor market status for each aggregate state switch : Π^{gg} for a transition from a good period to a good period, Π^{bb} for a transition from a bad period to a bad period, Π^{gb} for a transition from a good period to a bad period and Π^{bg} for a transition from a bad period to

⁶ The value of the capital-output ratio can change with the definition of capital. Here we adopt the definition in Quadrini (2000). Thus aggregate capital results from the aggregation of plant and equipment, inventories, land at market value, and residential structures. This definition is close to the findings of Prescott (1986) and is also used for instance in Floden and Linde (2001). This yield a capital-output ratio of 2.6 on an annual basis that we convert to its quarterly equivalent of 10.4.

a good period⁷ :

$$\Pi^{bb} = \begin{pmatrix} 0.6000 & 0.4000 \\ 0.0445 & 0.9555 \end{pmatrix} \quad \Pi^{bg} = \begin{pmatrix} 0.2500 & 0.7500 \\ 0.0167 & 0.9833 \end{pmatrix}$$

$$\Pi^{gb} = \begin{pmatrix} 0.7500 & 0.2500 \\ 0.0729 & 0.9271 \end{pmatrix} \quad \Pi^{gg} = \begin{pmatrix} 0.3333 & 0.6667 \\ 0.0278 & 0.9722 \end{pmatrix}$$

Finally the joint transition matrix Π for labor market statuses and aggregate state can be defined as :

$$\Pi = \begin{pmatrix} \eta_{bb}\Pi^{bb} & \eta_{bg}\Pi^{bg} \\ \eta_{gb}\Pi^{gb} & \eta_{gg}\Pi^{gg} \end{pmatrix} = \begin{pmatrix} 0.5250 & 0.3500 & 0.0313 & 0.0938 \\ 0.0388 & 0.8361 & 0.0021 & 0.1229 \\ 0.0938 & 0.0313 & 0.2916 & 0.5884 \\ 0.0911 & 0.1158 & 0.0243 & 0.8507 \end{pmatrix}$$

2.5.4 Government

We fix the ratio of government purchases to GDP to 0.217 and the debt over GDP ratio, noted b , to $\frac{2}{3}$. Those values are the observed ratios in the U.S. as reported by Aiyagari and McGrattan (1998). Table 2 report the benchmark calibration.

TAB. 2
Benchmark calibration

Parameters	β_l	β_m	β_h	σ	α	δ	θ
Values	0.9750	0.9851	0.9981	1.00	0.36	0.025	0.10

3 Results

We now present the results obtained with our benchmark economy. A first section reports the aggregate behavior of the model. A second examines the long-run welfare effects of public debt in an aggregate fluctuations setting. In a third we move on to the business cycle and distributional effects of public

⁷ Further details on this calibration can be found in Appendix A.

debt. Finally we compare the benchmark economy to a simpler idiosyncratic risk only model.

3.1 Public debt in an aggregate fluctuations setting

TAB. 3
Steady state value of macro variables

Statistics	Level of debt (% of output)								
	2/3	70	90	110	130	150	170	180	190
Agg. Capital	11.80	11.79	11.74	11.70	11.65	11.60	11.55	11.53	11.51
Agg. Wealth (capital+debt)	12.55	12.58	12.76	12.94	13.11	13.29	13.47	13.56	13.64
Output	1.1354	1.1351	1.1335	1.1319	1.1302	1.1286	1.1270	1.1262	1.1253
Capital-Output Ratio	10.40	10.38	10.36	10.33	10.30	10.28	10.25	10.24	10.23
Before Tax Interest Rate (%)	0.966	0.967	0.976	0.985	0.994	1.003	1.012	1.017	1.021
After Tax Interest Rate (%)	0.677	0.678	0.682	0.687	0.692	0.696	0.701	0.703	0.705
Tax Rate (%)	29.92	29.94	30.11	30.27	30.44	0.3061	30.78	30.86	30.95
Wage	2.3883	2.3877	2.3843	2.3808	2.3774	2.3740	2.3705	2.3688	2.3671

We start by discussing the aggregate behavior of our benchmark model. Our computations are reported in Table 3. Increasing the level of public debt increases the supply of safe assets in the economy. Consequently, the after tax and before tax interest rates increase. In turn, public debt has a crowding-out effect on private capital : higher levels of debt decrease the aggregate amount of private capital in the economy. The crowding-out of capital induces the observed decline in output⁸. Nevertheless, the decline in physical capital is smaller than the increase in public debt. The increase in the after-tax interest rate reduces the gap between the after-tax interest rate and the rate of time preference. The cost of postponing consumption to build up a buffer stock of saving is then reduced. Households choose to hold more assets at the steady state equilibrium. That is why the overall wealth level A , which is the combination of private capital and public debt, is higher. Also, because the repayment of debt interests increases, the income tax rate increases.

3.2 Welfare analysis and optimal level of debt

We define the optimal level of public debt as the debt over GDP ratio that maximizes the traditional utilitarian welfare criterion μ . This criterion measures the amount of consumption that one would have to remove or add in

⁸ In the absence of a crowding out effect of physical capital, the increase in wealth would have been the same as the increase in public debt. The steady state consumption would have been higher.

order to make the agent indifferent between the benchmark debt over GDP ratio and some other level of public debt⁹. It verifies :

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\prod_{j=0}^t \beta_j \right) \log((1 + \mu)c_t^{bench.}) \right] = E_0 \left[\sum_{t=0}^{\infty} \left(\prod_{j=0}^t \beta_j \right) \log(c_t) \right]$$

with $\{c_t^{bench.}\}_{t=0}^{\infty}$ the consumption stream in the benchmark model when the debt over GDP ratio is equal to $\frac{2}{3}$. $\{c_t\}_{t=0}^{\infty}$ is the consumption stream when the debt over GDP ratio is some other level than the benchmark one. We can write :

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\prod_{j=0}^t \beta_j \right) \right] \log(1 + \mu) + V^{bench.} = V,$$

and finally that :

$$\mu = \exp \left(\left[V - V^{bench.} \right] / S \right) - 1$$

with

$$S = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\prod_{j=0}^t \beta_j \right) \right]_{bench.}$$

The result of the introduction of public debt on welfare is *a priori* undetermined because of two opposing effects. The first effect is a *crowding-out effect*. The crowding out of physical capital clearly reduces consumption and then welfare. Moreover the increase in the income tax rate tends to amplify the negative impact of public debt on welfare. The second effect is a *liquidity effect* : the increase in the after-tax interest rate makes it less costly to accumulate precautionary saving in order to smooth consumption as the interest rate gets closer to the time preference rate. This second effect is welfare enhancing¹⁰. It is difficult to predict which effect overcomes the other analytically.

Figure 1 depicts the optimal level of debt in the benchmark economy. In a setting embedding aggregate risk and calibrated on the U.S. economy, the optimal public debt level is 180% of output on a quarterly basis¹¹. For a debt over

⁹ This is the methodology introduced by Lucas (1987).

¹⁰ Woodford (1990) argues that welfare can be enhanced if the interest rates are kept high enough, that is, closer to time preference rates in a liquidity-constrained economy. We find the same effect here.

¹¹ As the model period is the quarter, all our results are quarterly values. To make comparison easier, a quarterly debt over GDP ratio of 180% is equivalent to roughly a ratio of 45% on an annual basis.

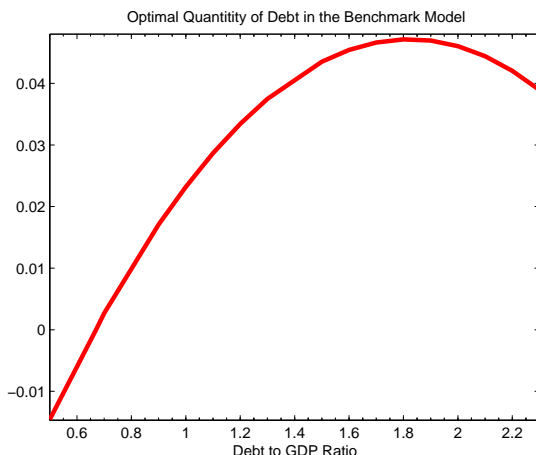


FIG. 1. Welfare gain versus debt/GDP ratio in the benchmark model

GDP ratio smaller than 180%, the *liquidity effect* dominates the *crowding-out effect*. Aggregate fluctuations, idiosyncratic risk and credit constraints lead agents to engage in precautionary saving in order to smooth consumption. Without public debt the cost of precautionary saving is high because when households accumulate, the interest rate mechanically lowers. A positive level of public debt, raises the interest rate and reduces the cost of precautionary saving. As a result welfare is enhanced. At the optimal level of debt the *liquidity effect* balances the *crowding-out effect*. Beyond that point, the loss of consumption due to the crowding-out of capital is greater than the benefit of a reduction in the cost of precautionary saving. Thus the *crowding-out effect* dominates the *liquidity effect*.

TAB. 4

Consumption gain (%) of switching to the optimal level of debt

Population	Business Cycle		
	Average	Recessions	Booms
All	0.0470	0.1210	-0.0080
Bottom 10%	-0.4070	-0.3890	-0.4430
Top 10%	5.0100	4.9600	5.0200

As illustrated in Table 4, switching from the benchmark level of debt to the optimal level of debt is welfare enhancing. The consumption gain of being at the optimal level of debt instead of the benchmark level is 0.047%. On the contrary, it is welfare decreasing to switch to a smaller level of public debt. In an economy where there is no public debt, the consumption loss not to be at the benchmark level (resp. optimal level) would be 0.07% (resp. 0.117%). As long as the *liquidity effect* dominates the *crowding-out effect* it can be costly to switch to a lower level of debt : although the output is higher when there is no debt in the economy, households have to bear the same level of risk but without the help of high interest rates to reduce the cost of precautionary

saving.

These results suggest that the role that the interest rate plays in reducing the cost of asset holding is central to understand why agents prefer higher levels of public debt. As illustrated in Table 5, in a price fixed small open economy¹², it appears that increasing public debt is no longer welfare enhancing. The welfare cost of being at the optimal level of debt instead of the benchmark level is now 1.35%. It is straightforward to see why agents loose that much in a price fixed small open economy. We impose that agents face the same series of prices as in the benchmark economy and at the same time we raise the debt over GDP ratio. The prices are lower and agents accumulate less than in the general equilibrium corresponding to the level of debt we impose. Thus the *crowding-out effect* is stronger because there is less private saving although the ratio of debt is higher. On the other hand as interest rates remain constant, there is no *liquidity effect* to balance the *crowding-out effect*. The overall effect on welfare can only be negative.

TAB. 5

Consumption gain (%) of switching to the optimal level of debt in a small open economy

Population	Benchmark to optimal level of debt in	
	General equilibrium	Small open economy
All	0.0470	-1.3500
Bottom 10%	-0.4070	-1.8600
Top 10%	5.0100	-0.8000

3.3 Business cycles and distributional effects

In this section we move on to the effects of public debt along the cycle and across the distribution of the population. The last two columns of Table 4 illustrate that switching from the benchmark to the optimal level of debt is welfare increasing when we account for recessions and welfare decreasing when we account for booms. This shows that when we account only for recessions, even at the optimal level of debt the *liquidity effect* dominates the *crowding-out effect*. This is due to a *price fluctuation effect* : in recessions, the cost of postponing consumption to build up a buffer stock of savings is higher because as shown on Table 6 the interest rate is lower. Thus the consumption gain of increasing public debt is higher in recessions. On the contrary, prices are higher during booms. When we only account for booms, switching from

¹² In a small open economy we prevent interest rates (resp. the wages) from increasing (resp. decreasing) when we switch from the benchmark level of public debt to the optimal level

the benchmark to the optimal level of debt is welfare decreasing because at the optimal level the *crowding-out effect* dominates the *liquidity effect*. But this is not true for any level of debt. Our computations reveal that a higher level of debt is welfare increasing even in booms as long as the debt over GDP ratio is below 110%. Beyond that level, increasing public debt is not welfare enhancing in booms.

The fluctuation of prices is not the only relevant effect we need to account for the optimal level of public debt. When we take aggregate risk into consideration, an *employment fluctuation effect* modifies the precautionary saving motive. Along the cycle, the unemployment rate and duration increase in recessions and reduce in booms. In recessions, the precautionary motive becomes stronger : employed agents face a higher risk of loosing their job and unemployed agents find it more difficult to find a job. In recessions agents want to save more for precautionary motives because of the *employment fluctuation effect* but at the same time the *price fluctuation effect* raise the cost of saving. Thus public debt plays a major role in reducing the cost of precautionary saving in recessions. In booms, agents want to save less because of the *employment fluctuation effect*. At the same time the *price fluctuation effect* makes it less costly to save. Thus public debt is less useful in booms.

TAB. 6
Macro variables along the cycle

Statistics	Level of debt (% of output)					
	2/3			180		
	Booms	Average	Recessions	Booms	Average	Recessions
Agg. Capital	11.86	11.80	11.73	11.59	11.53	11.46
Output	1.17	1.13	1.10	1.16	1.12	1.09
Before Tax Interest Rate (%)	1.06	0.97	0.87	1.11	1.017	0.92
Wage	2.3890	2.3883	2.3873	2.3677	2.3688	2.3697

The overall consumption gain of a higher level of public debt is shared very differently in the population. To show that we decompose the welfare gains of a higher level of public debt across the population. The last two rows of Table 4 shows this decomposition. For instance the row *Bottom 10%* shows the welfare gap between the 10% least fortunate people living in an economy with the benchmark level of public debt and those living in an economy with the optimal level of public debt. This decomposition closely matches a decomposition by wealth levels and we use alternately the terms *least (resp. most) fortunate* or *poorest (resp. richest)*¹³. The poorest agents would be better off with a lower level of public debt. Switching from the benchmark level of debt to the optimal

¹³ The lowest (highest) expected utilities refer to people who have experienced the highest unemployment (employment) spells and who end up with the lowest (highest) level of assets.

debt level leads to a decrease in consumption of 0.407% for the poorest 10% of the population. In the meantime, the richest 10% of the population would gain 5.01% of consumption. This is explained by the fact that rich people's income is mainly capital income whereas poor people's income is mostly labor income. Thus when the level of public debt is higher, poor people suffers from the reduction in output caused by higher tax rates, lower wages and crowding out of capital. On the contrary, as interest rates raise with higher level of public debt, rich people are better off¹⁴.

3.4 Optimal level of debt without aggregate fluctuations

TAB. 7
Macro variables in the idiosyncratic risk model

Statistics	Level of debt (%)					
	0	20	40	60	2/3	80
Agg. Capital	11.61	11.56	11.52	11.47	11.46	11.43
Agg. Wealth (capital+debt)	11.61	11.79	11.97	12.15	12.21	12.33
Output	1.1285	1.1270	1.1254	1.1238	1.1233	1.1222
Capital-output ratio	10.28	10.26	10.23	10.21	10.20	10.18
Before Tax Interest rate (%)	1.000	1.010	1.020	1.026	1.029	1.035
After Tax Interest rate (%)	0.708	0.713	0.717	0.721	0.723	0.726
Tax Rate (%)	29.21	29.37	29.56	29.72	29.72	29.89
Wage	2.3743	2.3710	2.3677	2.364	2.365	2.361

In this section we look at a setup without aggregate fluctuations and compare it with our benchmark economy. There are several ways of eliminating business cycles¹⁵. For the sake of simplicity we take a basic approach : we first average the aggregate shock to its mean and then we follow Imrohorglu (1989) to derive the labor market process. The transition probabilities of this economy are set so that the average rate of unemployment and the average duration of unemployment are the same between this economy and the benchmark economy. All other calibrated parameters are kept to their benchmark values, especially time preference rates and the risk aversion parameter¹⁶. Table 7 summarizes the aggregate behavior of this model. This model is similar to the benchmark model in many ways : higher levels of debt raise interest rates and crowds-out private capital, overall wealth increase with debt and taxes are higher. But because the risk faced by agents is significantly lower, agents save less and interest rates are higher in this model.

¹⁴ This effect is also discussed in Ball and Mankiw (1995) and Floden (2001).

¹⁵ For a survey, see Barlevy (2004).

¹⁶ For more details, see appendix B. The model without aggregate shocks is presented in detail.

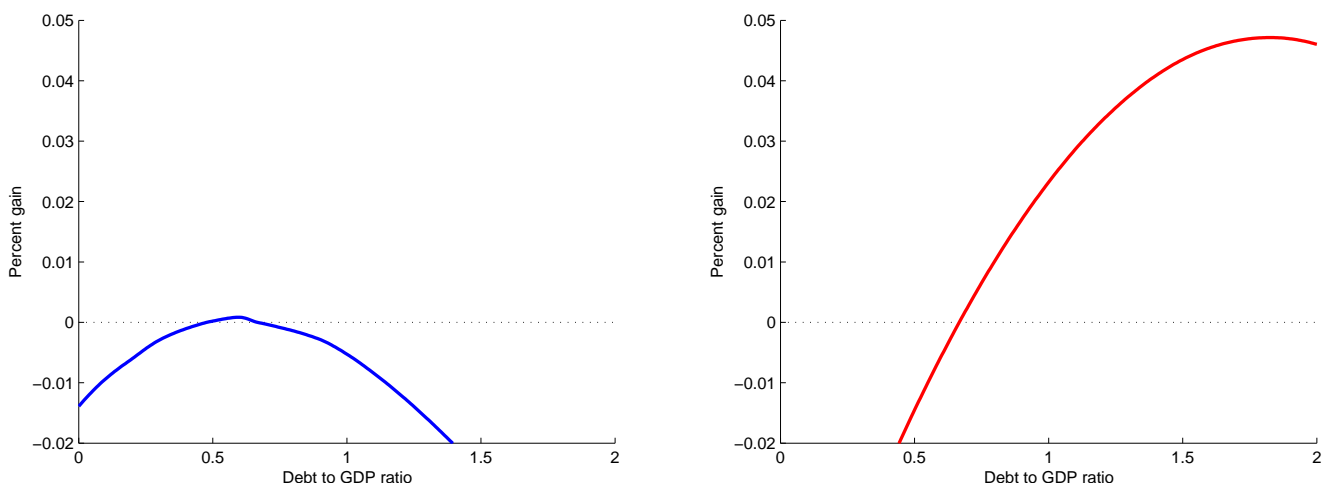


FIG. 2. Welfare gain versus debt/GDP ratio in the model without aggregate fluctuations (left) and the benchmark model (right)

Figure 2 depicts the welfare profile we find in this idiosyncratic risk model. The optimal level of public debt is 60% of output on a quarterly basis¹⁷. This level is significantly lower than the level we found in the benchmark economy. This is because there is no *price fluctuation effect* and *employment fluctuation effect* in this economy. In the benchmark economy, as the unemployment rate and the unemployment duration increase during recessions, the precautionary motive is stronger. Moreover, in the benchmark economy the interest rate and the physical capital are smaller in recessions. Therefore, it is more costly to save for precautionary motive. That is why the need for public debt is more important in the benchmark economy. In the idiosyncratic model the risk is lower because agents are subject to a less risky labor market and at the same time they no longer face price fluctuations. The precautionary motive is weaker here than in the benchmark model and agents save less. As a consequence interest rates are at a higher level in this economy. For instance, on average the before tax interest rate is 3.5% lower in the benchmark economy than in this economy for a debt over GDP ratio of 2/3. In recessions, the interest rate is 15% lower in the benchmark economy than in this economy. As interest rates are higher, the *liquidity effect* is weaker in this economy and is balanced by the *crowding-out effect* for a lower level of debt than in the benchmark economy. This explains the significant difference in the optimal levels of debt between the two economies and shows the importance of the *price fluctuation effect* and the *employment fluctuation effect*.

Table 8 underlines that consumption costs are very different when considering rich and poor people. The poorest agents benefit from having lower levels

¹⁷ Roughly 15% on an annual basis.

TAB. 8

Consumption gain (%) of switching to the optimal level of debt

Benchmark level of debt to :	Optimal level
Benchmark economy (all)	0.0471
Idiosyncratic economy (all)	0.0008
Idiosyncratic economy (bottom 10%)	0.0300
Idiosyncratic economy (top 10%)	-0.3100

of debt because the tax rate decreases, the wage increases and the physical capital increases. Conversely, the richest agents suffer from lower levels of debt because the interest rate decreases. Moreover, it appears that decreasing or increasing public debt has smaller effects on welfare here because the need for public debt is weaker in an economy without aggregate debt. The role of interest rates remains central here although it is weaker than in the benchmark economy. In a price fixed small open economy everybody benefits from a lower public debt level. This result is reported in Table 9. In a small open economy a policy that consists in switching from a debt over GDP ratio of 2/3 to a ratio of 60% is welfare enhancing. As prices are constant, the richest agents (and to a lesser extent the poorest agents) do not suffer from the decrease in interest rates as they would in a general equilibrium context.

TAB. 9

Consumption gain (%) of switching to the optimal level of debt

Benchmark level of debt to optimal level in :	General equilibrium	Small open economy
All	0.0008	0.1450
Bottom 10%	0.0300	0.1350
Top 10%	-0.3100	0.2660

4 Extension : European labor market

Our benchmark calibration reflects the behavior of the U.S. labor market.

In this section we consider an alternate calibration to reproduce European labor market characteristics. We follow the methodology used in Algan and Allais (2004) and the data set of Blanchard and Wolfers (2000). We only modify the labor market features and leave the rest of the calibration unchanged. We fix the unemployment rate to be 13% in recessions and 7% in booms. We also set the duration of an unemployment spell to be 6 quarters in recessions and 4 quarters in booms. We now find the optimal level of debt to be 320% on a quarterly basis¹⁸ as depicted in Figure 3. Longer unemployment spells

¹⁸ Roughly 80% of GDP on an annual basis.

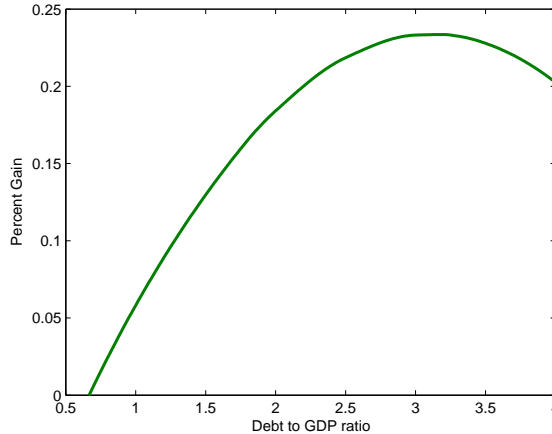


FIG. 3. Welfare gain versus debt/GDP ratio for the European economy

and higher unemployment rates tend to raise the optimal level of debt. The consumption gains are also higher when compared to the benchmark model as illustrated in Table 10. Here switching from the benchmark level of debt of 2/3 of GDP to the optimal level induce a consumption gain of 0.23%.

This calibration purposely strengthens the *employment fluctuation effect* : unemployed agents have a harder time finding a job in this economy when compared to the benchmark economy and employed agents face a higher risk of loosing their jobs. In recessions, this is amplified. The precautionary motive is stronger here than in the benchmark economy. The harder it is for households to smooth consumption and the higher is the need for public debt. Here a higher level of public debt is needed to help households effectively smooth consumption.

TAB. 10
Consumption gain (%) of switching to the optimal level of debt

Benchmark level of debt to :	Optimal level	Optimal level in the benchmark economy
Benchmark economy	0.0471	0.0471
European economy	0.2335	0.1651

The same type of results as above can be obtained by simply lengthening the duration of the cycle. In our computations we also modified the benchmark model to allow longer cycle duration. We doubled the benchmark cycle duration of 8 quarters to make it 16 quarters and adjusted the transition matrixes accordingly. The optimal level of debt we found was 350% of GDP on a quarterly basis. This is a way of strengthening the *price fluctuation effect* : in recessions households have to bear for a longer time the cost of postponing consumption. Thus a higher level of public debt is needed.

5 Conclusion

This paper reconsidered the optimal level of public debt in an environment of aggregate fluctuations. Our benchmark model calibrated on the U.S. economy finds that a positive public debt level of 180% of output on a quarterly basis is optimal. This level is higher than in an economy without aggregate fluctuations where we find the optimal level to be only 60% of output on a quarterly basis. Our benchmark economy shows that in an aggregate fluctuations setting, households are subject to an *employment fluctuation effect* and to a *price fluctuation effect*. These effects make the precautionary saving motive stronger and at the same time the cost of saving higher. Our results suggest that the higher supply of safe assets induced by higher levels of public debt tends to raise the interest rate. This helps households reduce the cost of precautionary saving and make smoothing of consumption easier. As a result there are welfare gains in the economy. However for the poorest agents, higher levels of debt are not optimal. Poor agents rely mainly on labor income and are dependent on higher wages and lower taxes. Public debt decreases wages and increases tax level thus poor agents prefer lower levels of public debt. Also, we find that longer cycles or longer unemployment spells tend to raise the optimal level of public debt.

Appendix

In this appendix, we first go into details about several points of our benchmark calibration. Next, we describe the economy without aggregate risk. Finally, the computational strategy adopted to solve the benchmark model is detailed.

A Calibration

We now show how we derived the transition matrices for aggregate state switches (η), for joint transition between aggregate states and labor market statuses (Π) and for discount factor switches (Υ).

A.1 Aggregate state switch transition matrix

To deduce the aggregate state switch transition matrix η , we solve the following system :

$$\left. \begin{array}{l} \eta_{gg} = \eta_{bb} \\ \eta_{bg} = \eta_{gb} \\ \eta_{gg} + \eta_{gb} = 1 \\ \eta_{bb} + \eta_{bg} = 1 \\ \eta_{bg} = \frac{1}{8} \end{array} \right\} \implies \begin{pmatrix} 0.875 & 0.125 \\ 0.125 & 0.875 \end{pmatrix}$$

As we assumed that the duration of a boom or a recession is the same, we deduce the two first equations. Moreover, the duration of a cycle is set to 8 quarters, it follows that $\eta_{bg} = Pr(z_{t+1} = g/z_t = b) = \frac{1}{8}$ and $\eta_{gb} = Pr(z_{t+1} = b/z_t = g) = \frac{1}{8}$.

A.2 Matrix for joint transition between aggregate states and labor market statuses

The determination of the matrix Π that describes the transition between unemployment and employment requires the identification of the aggregate shock (whether we are in a recession or in a boom). The transition matrix Π is built thanks to the matrix η and to the transition matrixes Π^{gg} , Π^{bb} , Π^{gb} and Π^{bg} . Π verifies :

$$\Pi = \begin{pmatrix} \eta_{bb}\Pi^{bb} & \eta_{bg}\Pi^{bg} \\ \eta_{gb}\Pi^{gb} & \eta_{gg}\Pi^{gg} \end{pmatrix}$$

We assumed that in recessions the duration of the unemployment, that we note $durub$, amounts to 2.5 quarters and the unemployment rate u_b is set to 10%. In booms, the duration of unemployment, $durug$, is equal to 1.5 quarters and the unemployment rate, u_g , is set to 4%. From these informations, we can deduce the matrices Π^{gg} and Π^{bb} .

The transition matrix Π^{gg} corresponds to the case $(z, z') = (g, g)$. It verifies :

$$\Pi^{gg} = \begin{pmatrix} \Pi_{uu}^{gg} & \Pi_{ue}^{gg} \\ \Pi_{eu}^{gg} & \Pi_{ee}^{gg} \end{pmatrix}$$

Solving the system below gives the values of Π_{ee}^{gg} , Π_{eu}^{gg} , Π_{ue}^{gg} and Π_{uu}^{gg} :

$$\left\{ \begin{array}{l} \Pi_{ee}^{gg} + \Pi_{eu}^{gg} = 1 \\ \Pi_{ue}^{gg} + \Pi_{uu}^{gg} = 1 \\ \Pi_{ue}^{gg} = \frac{1}{durug} \\ \Pi_{ee}^{gg} = 1 - \frac{u_g \Psi_{ue}^{gg}}{1 - u_g} \end{array} \right. \implies \Pi^{gg} = \begin{pmatrix} 0.3333 & 0.6667 \\ 0.0278 & 0.9722 \end{pmatrix}$$

The transition matrix Π^{bb} corresponds to the case $(z, z') = (b, b)$. It verifies :

$$\Pi^{bb} = \begin{pmatrix} \Pi_{uu}^{bb} & \Pi_{ue}^{bb} \\ \Pi_{eu}^{bb} & \Pi_{ee}^{bb} \end{pmatrix}$$

Solving the system below gives the values of Π_{ee}^{bb} , Π_{eu}^{bb} , Π_{ue}^{bb} and Π_{uu}^{bb} :

$$\left\{ \begin{array}{l} \Pi_{ee}^{bb} + \Pi_{eu}^{bb} = 1 \\ \Pi_{ue}^{bb} + \Pi_{uu}^{bb} = 1 \\ \Pi_{ue}^{bb} = \frac{1}{durub} \\ \Pi_{ee}^{bb} = 1 - \frac{u_b \Pi_{ue}^{bb}}{1 - u_b} \end{array} \right. \implies \Pi^{bb} = \begin{pmatrix} 0,6 & 0.4 \\ 0.0445 & 0.9555 \end{pmatrix}$$

When the cycle changes the unemployment rate changes. The transitions between unemployment and employment get modified. We make the same assumptions as Krusell and Smith (1998) :

$$\begin{cases} \Pi_{uu}^{bg} = \Pr(\epsilon_{t+1} = u^g / \epsilon_t = u^b) = 0.75\Pi_{uu}^{gg} \\ \Pi_{uu}^{gb} = \Pr(\epsilon_{t+1} = u^b / \epsilon_t = u^g) = 1.25\Pi_{uu}^{bb} \end{cases}$$

The probability to remain unemployed when the next period is a recession (resp. boom), increases (resp. decreases) since by assumption the unemployment rate is higher in recession than in boom.

The transition matrix Π^{bg} corresponds to the case $(z, z') = (b, g)$. It verifies :

$$\Pi^{bg} = \begin{pmatrix} \Pi_{uu}^{bg} & \Pi_{ue}^{bg} \\ \Pi_{eu}^{bg} & \Pi_{ee}^{bg} \end{pmatrix}$$

The system below gives us Π_{ee}^{bg} , Π_{eu}^{bg} , Π_{ue}^{bg} and Π_{uu}^{bg} :

$$\begin{cases} \Pi_{ee}^{bg} + \Pi_{eu}^{bg} = 1 \\ \Pi_{ue}^{bg} + \Pi_{uu}^{bg} = 1 \\ \Pi_{uu}^{bg} = 0.75\Pi_{uu}^{gg} \\ \Pi_{ee}^{bg} = \frac{((1 - u_g) - u_b\Pi_{ue}^{bg})}{1 - u_b} \end{cases} \implies \Pi^{bg} = \begin{pmatrix} 0.25 & 0.75 \\ 0.0167 & 0.9833 \end{pmatrix}$$

The transition matrix Π^{gb} corresponds to the case $(z, z') = (g, b)$.

$$\Pi^{gb} = \begin{pmatrix} \Pi_{ee}^{gb} & \Pi_{eu}^{gb} \\ \Pi_{ue}^{gb} & \Pi_{uu}^{gb} \end{pmatrix}$$

The system below gives us Π_{ee}^{gb} , Π_{eu}^{gb} , Π_{ue}^{gb} and Π_{uu}^{gb} :

$$\begin{cases} \Pi_{ee}^{gb} + \Pi_{eu}^{gb} = 1 \\ \Pi_{ue}^{gb} + \Pi_{uu}^{gb} = 1 \\ \Pi_{uu}^{gb} = 1.25\Pi_{uu}^{bb} \\ \Pi_{ee}^{gb} = \frac{((1 - u_b) - u_g\Pi_{ue}^{gb})}{1 - u_g} \end{cases} \implies \Pi^{gb} = \begin{pmatrix} 0.75 & 0.25 \\ 0.0729 & 0.9271 \end{pmatrix}$$

A.3 Matrix for discount factor switches

We assumed that the discount factors follow a three-states first-order Markov process. Therefore, the matrix describing the transition from the discount factor β_i to the discount factor β_j is the following :

$$\Upsilon = \begin{pmatrix} \Upsilon_{ll} & \Upsilon_{lm} & \Upsilon_{lh} \\ \Upsilon_{ml} & \Upsilon_{mm} & \Upsilon_{mh} \\ \Upsilon_{hl} & \Upsilon_{hm} & \Upsilon_{hh} \end{pmatrix}$$

As we assumed that there is no immediate transition between β_l and β_h as in Krusell and Smith (1998), it involves that $\Upsilon_{lh} = \Upsilon_{hl} = 0$. Moreover, as we set the duration of the extreme states (β_l and β_h) to 50 years namely 200 quarters, we have $\Upsilon_{lm} = \frac{1}{200} = \Upsilon_{hm}$. Solving the following system gives us the transition matrix Υ :

$$\left. \begin{array}{l} \Upsilon_{ll} + \Upsilon_{lm} + \Upsilon_{lh} = 1 \\ \Upsilon_{ml} + \Upsilon_{mm} + \Upsilon_{mh} = 1 \\ \Upsilon_{hl} + \Upsilon_{hm} + \Upsilon_{hh} = 1 \\ \Upsilon_{lh} = \Upsilon_{hl} = 0 \\ \Upsilon_{lm} = \frac{1}{200} = \Upsilon_{hm} \\ \Upsilon_{ml} = \frac{\Pr(\beta_t=\beta_l)\Upsilon_{lm}}{\Pr(\beta_t=\beta_m)} \\ \Upsilon_{mh} = \frac{\Pr(\beta_t=\beta_h)\Upsilon_{hm}}{\Pr(\beta_t=\beta_m)} \end{array} \right\} \implies \Upsilon = \begin{pmatrix} 0.995 & 0.005 & 0 \\ 0.0007 & 0.9979 & 0.0014 \\ 0 & 0.005 & 0.995 \end{pmatrix}$$

B Model without aggregate risk

We now briefly detail the model without aggregate risk and its calibration. Most of this model is similar to the benchmark model, thus we underline only differences.

Model

As this model serves comparison purposes most of the benchmark assumptions remain unchanged. The assumptions about the firm are similar with the exception of the technical progress z . In the absence of aggregate risk, z is fixed

to its average value. In the absence of aggregate risk, the budget constraint of the government is :

$$G_t + r_t B_t = B_{t+1} - B_t + T_t$$

Household preferences are unchanged. In the absence of aggregate risk, the matrix that describes the transition on the labor market becomes :

$$\pi = \begin{pmatrix} \pi_{uu} & \pi_{ue} \\ \pi_{eu} & \pi_{ee} \end{pmatrix}$$

with $\pi_{uu} = \Pr(s_{t+1} = u | s_t = u)$. When there is no aggregate risk, it is no longer necessary to distinguish the nature of the cycle. In the absence of aggregate risk, the state variables are summarized by the vector (a, s, β) . The program the household solves is :

$$v(a, s, \beta) = \max_{c, a'} \{u(c) + \beta E[v(a', s', \beta') | (s, \beta)]\} \quad (\text{B.1})$$

subject to :

$$c + a' = (1 + r(1 - \tau))a + w\chi(s) \quad (\text{B.2})$$

$$c \geq 0 \quad (\text{B.3})$$

$$a' \geq 0 \quad (\text{B.4})$$

with

$$\chi(s) = \begin{cases} \theta & \text{if } s = u, \\ (1 - \tau) & \text{if } s = e \end{cases}$$

Equilibrium

The recursive equilibrium consists of a set of decision rules for consumption and asset holding $\{c(a, s, \beta), a'(a, s, \beta; z, \Gamma)\}$, aggregate capital K , factor prices $\{r, w\}$, tax rate τ satisfying these conditions :

- (1) Given the prices $\{r, w\}$, the decision rules $\{c(a, s, \beta), a'(a, \epsilon, \beta)\}$ solve the dynamic programming problem (B.1) subject to the constraints (B.2), (B.3) and (B.4)

(2) Market price arrangements are :

$$\begin{aligned} r &= \alpha z K^{\alpha-1} N^{1-\alpha} - \delta \\ w &= (1 - \alpha) z K^{\alpha} N^{-\alpha} \end{aligned}$$

(3) Government budget constraint is balanced.

(4) Capital Market clears when :

$$K + B = \int a'(a, \epsilon, \beta) d\Gamma(a, s, \beta)$$

with $\Gamma(a, s, \beta)$ the distribution of agents over asset holdings, employment status and preference.

Calibration

We present here the calibration strategy in an economy without aggregate risk. The calibration of the preferences, the discount factors and the behavior of the government are unmodified. The calibration of z and the characteristics of the labor market differ in the absence of aggregate risk. In an economy without aggregate risk, z is constant and set to the average value of the aggregate shock, namely the unit value. The unemployment rate u and the unemployment spells $duru$ are respectively set to 7% and 2 quarters (we average over good and bad periods in the benchmark model). These two assumptions define the transition matrix π :

$$\begin{cases} \pi_{uu} + \pi_{ue} = 1 \\ \pi_{eu} + \pi_{ee} = 1 \\ \pi_{ue} = \frac{1}{duru} \\ \pi_{ee} = 1 - \frac{u\pi_{ue}}{1-u} \end{cases} \implies \begin{pmatrix} 0.5 & 0.5 \\ 0.0376 & 0.9624 \end{pmatrix}$$

C Computational strategy

We solve the model using the methodology developed by Krusell and Smith (1998). They show that agents need only a restrictive set of statistics about the wealth distribution to determine prices. This set includes the mean of the wealth distribution and the aggregate productivity shock. A linear prediction rule based only on the average level of capital provides an accurate prediction. This result comes from the near linearity of the decision rule $a'(a, s, \beta; z, \Gamma)$.

As the aggregate capital stock is mainly held by rich people who have approximately the same propensity to save, next period's aggregate capital is accurately predicted by current period's aggregate capital. In our model, we assume the following law of motion :

$$\log(\bar{K}') = a_0 + a_1 z + a_2 \log(\bar{K})$$

where \bar{K}' and \bar{K} denote respectively the average stock of capital of the next period and of the current period. Thus the strategy is the following :

(Step 1) : Given a set of parameter values (a_0, a_1, a_2) for the law of motion, we solve the individual problem. Unlike Krusell and Smith (1998) whom solve the Bellman equation by value function iteration on a discrete grid, we use the Parameterized Expectations Approach (PEA)¹⁹. This method consists in the approximation of the conditional expectation in the Euler equation,

$$U'(c) = E [\beta' U'(c') (1 + r'(z', \Gamma') (1 - \tau)) / s, \beta; z, \Gamma],$$

by a flexible polynomial $h(a, s, \beta, z, \bar{K}, \psi)$ that depend on the state variables and some coefficients ψ that we must estimate. As the aggregate shock is discrete, $z \in \{z_b, z_g\}$, in lieu of approximating one Euler equation, we approximate 12 rules. We define as many rules as there are statuses on the labor market, impatience levels and aggregate shock levels. To identify those coefficients, we use projection methods on the state space grid. Projection methods turn out to be efficient when using orthogonal polynomials such as the Chebyshev polynomial family. We only approximate the conditional expectation when the borrowing constraint is not binding. Otherwise consumption can be deduced from the budget constraint.

(Step 2) : Given the parameter values for individual decision rules, we solve the aggregate problem *i.e.* the coefficients of the law of motion.

(Step 3) : If the parameters (a_0, a_1, a_2) found are close to the parameter values used to solve (Step 1), the algorithm has converged. Otherwise (Step 1), (Step 2) and (Step 3) are repeated until convergence.

¹⁹ The Parameterized Expectations Approach is discussed in Marcet (1988) and Marcet and Lorenzini (1999).

Références

- [1] Aiyagari, R., 1994, Uninsured Idiosyncratic Risk and Aggregate Saving, *The Quarterly Journal of Economics*, 109, 659-84.
- [2] Aiyagari, R., McGrattan, R.E., 1998, The Optimum Quantity of Debt, *Journal of Monetary Economics*, 42, 447-469.
- [3] Algan, Y., Allais, O., 2004, Incomplete Unemployment Insurance under Aggregate Fluctuations, *Economics Bulletin*, 5(12), 1-7.
- [4] Ball, L., Mankiw, G., 1995, What Do Budget Deficits Do?, in *Budget Deficits and Debt : Issues and Options. Symposium Proceedings, Federal Reserve Bank of Kansas City*, 95-119.
- [5] Barlevy, G., 2004, The Cost of Business Cycles and the Benefits of Stabilization : A Survey, *NBER Working Paper Series*, 10926.
- [6] Blanchard, O., Wolfer, J., 2000, The Role of Shocks and Institutions in the rise of European Unemployment : The Aggregate Evidence, in *Economic Journal*, 110, 1-33.
- [7] Budria-Rodriguez, S., Diaz-Gimenez, J., Quadrini, V. Rios-Rull, J.-V., 2002, Updated Facts on the U.S. Distributions of Earnings, Income, and Wealth, *Quarterly Review, Federal Reserve Bank of Minneapolis*, 26-3, 2-35.
- [8] Diamond, P., 1965, National Debt in a Neoclassical Growth Model, *American Economic Review*, 55, 1126-1150.
- [9] Floden, M., 2001, The Effectiveness of Government Debt and Transfers as Insurance, *Journal of Monetary Economics*, 48, 81-108.
- [10] Floden, M., Linde, J., 2001, Idiosyncratic Risk in the United States and Sweden : Is There a Role for Government Insurance, *Review of Economic Dynamics*, 4, 406-437.
- [11] Imrohoroğlu, A., 1989, Cost of business Cycles with Indivisibilities and Liquidity Constraints, *Journal of Political Economy*, 97, 1364-1383.
- [12] Krusell, P., Smith, A., 1998, Income and Wealth Heterogeneity in the Macroeconomy, *Journal of Political Economy*, 106, 867-896.
- [13] Krusell, P., Smith, A., 2002, Revisiting the Welfare Effects of Eliminating Business Cycles, *Working Paper*.
- [14] Lucas, R., 1987, Models of Business Cycles, *Basil Blackwell, New York*.
- [15] Marcet, A., 1988, Solving nonlinear stochastic models by parameterizing expectations, *Working paper. Carnegie Mellon University*.
- [16] Marcet, A., Lorenzoni, G., 1999, The parameterized expectation approach : some practical issues, in *Marimon, R., Scott, A. (Eds.), Computational methods for study of dynamic economies. Oxford University Press, New York*, 143-171.
- [17] Prescott, E., 1986, Theory Ahead of Business Cycle Measurement, *Carnegie-Rochester Conference Series on Public Policy*, 25, 11-44.
- [18] Quadrini, V., 2000, Entrepreneurship, Saving and Social Mobility, *Review of Economic Dynamics*, 3, 1-40.
- [19] Storesletten, K., Telmer, C., Yaron, A., 2001, The welfare cost of business

- cycles revisited : finite lives and cyclical variation in idiosyncratic risk,
European Economic Review, 45, 1311-1339
- [20] Woodford, M., 1990, Public Debt as Private Liquidity, *American Economic Review*, 80, 382-388.