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Educational quality, Stratification, and the Formation of communities: a Theoretical analysis.

Saïd HANCHANE^a; Tarek MOSTAFA ^{a*}

Abstract: In this paper, we develop a multicomunity model where public mixed finance and private schools coexist. Students are differentiated by income, ability and social capital. Schools maximize their profits under a quality constraint; the pricing function is dependent on the cost of producing education and on the position of an individual relatively to mean ability and mean social capital. Income plays an indirect role since it determines the type of schools and communities that can be afforded by a student given his ability and social capital.

Three dimensional stratification results from schools' profit maximization and individuals' utility maximization. This stratification is the corner stone of the distribution of students across communities and schools. Finally, we study majority voting over tax rates; property tax is used to finance educational quality not only in pure public schools but also in mixed finance schools. We provide the necessary conditions for the existence of a majority voting equilibrium determined by the median voter.

JEL Classification: I20, I21, H52, R31.

Keywords: Education market, majority voting equilibrium, peer group effects, formation of communities, school choice.

^a Laboratory of Sociology and Labor Economics, CNRS. Université de la méditerranée (Aix Marseille II).
E-mail addresses : said.hanchane@univmed.fr ; tarek.mostafa@univmed.fr

* Corresponding author: land phone. 0033-4-42378502. Fax: 0033-4-42267937. 35 avenue Jules Ferry, 13626 Aix en Provence, France.

I. INTRODUCTION.

Achieving social mixity, reducing inequalities in the access to education and in the outcomes of education has become a major preoccupation of educational reforms in most developed countries. Recent studies proved the existence of large disparities in the outcomes of education in most western countries. For instance in PISA 2000; Germany, the UK, the USA and France turned out to have a large students' dispersion of achievements over reading, numeracy, and basic science tests combined with low performances.¹ The recent French presidential elections triggered a vigorous debate on the quality of education in France. For instance, the debate on the abolishment of school districts and the introduction of public school choice made the headlines in most French newspapers. For example, *Le Monde* titled (16 September 2006) "School districts have become the instrument of social segregation", *Libération* titled (06 September 2006) "General offensive against school districts", and *Le Figaro* titled (03 January 2007) "The incoming reform of school districts".

As of September 2007 France will be experimenting new procedures in some school districts. Increased autonomy will be granted to educational institutions in order to increase educational quality, and School districts will be progressively abolished in order to grant parents more authority over the choice of the public school for their children. More precisely, the government announced that more students will be able to choose their public schools, and priority will be given to highly performing students.

The objective behind the abolishment of school districts is the reduction of social inequalities in the access to education. Not only highly advantaged households will be able to bypass school districts through the choice of private schools, but also ordinary students with high abilities will be able to choose their public school. It shall be said that in France students have the right to choose private schools outside their school district. Thus, sufficiently rich households have been able to bypass school districts; as a consequence segregation problems arose in poorer communities.

¹ Chapter 5 in "Education, Equality and Social Cohesion" by Green, Preston and Janmatt provides interesting results on outcomes' inequalities in five groups of countries: East Asia, the Nordic countries, the Germanic region, the Mediterranean countries and the predominantly English speaking countries. Also an article of Fuchs and Wobmann (March 2004) accounted for the low performances made by these countries.

In this theoretical paper, we are constructing the basis for the study of public school choice in a multicomunity environment, when private, mixed finance and public schools coexist. We analyze the formation of communities when education is a locally provided good. Public, mixed finance and private schools coexist. The interaction between students' utility maximization and schools' profit maximization leads to equilibrium on the market for schooling and that for housing and then to stratification across schools and communities. We also study voting over tax rates, and we determine the necessary conditions for the existence of a majority voting equilibrium determined by the median voter.

Yoram Barzel (1973), in a critical study of Robin Barlow (1970) introduced what is known as the "Ends against the middle phenomenon" in the provision of public goods when private alternatives coexist. The study of Barzel constituted the starting point for a more formalized approach to school choice, we may find the studies made by Joseph Stiglitz (1974), Clotfelter (1976), and Sonstelie (1979).

Five major articles are relevant to our analysis. Epple and Platt (1996) in which the authors construct an equilibrium in a multicomunity model where individuals have the choice between communities differentiated by housing prices and grant levels. Fernandez and Rogerson (1996), in which education is a locally, publicly provided good and where we have a strict hierarchy of communities following educational qualities, and a complete stratification by income. Glomm and Ravikumar (1998), in which the authors construct a model of the provision of education when both public and private alternatives coexist while analyzing majority voting over tax rates. Epple and Romano (1998 and 2006) where they study stratification across the market for education with individuals differentiated by income and ability.

Relatively to Epple and Platt (1996), we construct a multicomunity model of the provision of education where students are differentiated by income, ability, and social capital. Students have to choose a school to be enrolled into and a community to live in, the chosen school must not necessary belong to the chosen community. Housing prices, tuition, and transportation costs play a major role in the stratification of individuals across communities and schools. Students will be stratified according to their endowments in income, ability and social capital.

Similarly to Epple and Romano (1998 and 2006), the private sector is active, schools maximize profit under a quality constraint, and a discriminating pricing strategy is applied and is dependent on individuals' ability and social capital. However, we allow the existence of public, mixed finance and private schools in a multicomunity environment. Preferences are not single peaked, educational quality is three dimensional, students are stratified into communities and schools, and tax rates are chosen through majority voting.

Relatively to Fernandez and Rogerson (1996), education can be supplied privately; as a consequence preferences are no longer single peaked, this has major effects on majority voting. In our model there are no strict hierarchy of communities determined by school qualities, in fact each community may contain a different number of public, mixed finance and private schools with different qualities, and there may be some intersections of quality levels between communities. Our stratification is three dimensional allowing mixity according to income, ability and social capital, while their model has a complete stratification by income.

In relation to the article of Glomm and Ravikumar (1998), we are studying majority voting in a multicomunity model, education is a locally provided good. The level of rental in a community determines the quality of the school that can be attended. Students choosing a private school or a school outside their community prefer zero tax rates, while at very high tax rates some students will be willing to migrate. Majority equilibrium determined by the median voter would not occur unless some conditions are verified.

Section II presents the model, section III presents schools characteristics and profit maximization, section IV presents the characteristics of students and examines stratification across schools and communities, Section V analyses majority voting over tax rates, and finally, section VI concludes. Mathematical details are found in the appendix.

II. THE MODEL.

In our model, we consider an economy populated with a continuum of households differentiated by income, ability and social capital; each household has one student. The number of students is largely superior to that of schools. Social capital is defined to include the channels through which student acquisitions are affected by the social mix in their schools, these channels may include: social origins, the belonging to a professional class, the set of attitudes and mental dispositions governing different subjects like the superiority of the educational good. All these forms of social capital are represented through a single parameter, k . Individuals have identical preferences over private consumption. An individual i (with $i = 1, 2, 3, 4, \dots, P$) has an income y_i , an ability b_i and a social capital k_i . Note that i may designate a particular student or a type of students with the same combination of y , b , and k . These individuals live in communities, a community is designated by an index C with ($C = 1, 2, 3, \dots, C$).

Income, ability, and social capital are distributed in the population according to $f(b, y, k)$ which is positive and continuous on its support $S = (0, b_{\max}) \times (0, y_{\max}) \times (0, k_{\max})$.

Individual utility is assumed to be a function of private consumption and school quality. It is noted as $U(c, q)$; U is continuous, twice differentiable, and increasing in both arguments.

School quality is determined by expenditure per pupil, peer group effect, and neighborhood effect. Quality is increasing in peer effect, and the expenditure per pupil; while it is decreasing in neighborhood effect². Peer group effect is defined to be mean ability in a school; neighborhood effect is defined to be mean social capital in a school. A school is designated by an index j (with $j=1, 2, 3, \dots, j$).

Increasing quality in expenditure per pupil and in peer effect is consistent with the theoretical literature on this subject. Summers and Wolfe (1977), Henderson (1978), Sorensen and Hallinan (1986) and Kulik (1992) provided evidence that peer effect exists.

² Schools value diversity in term of social capital of their students.

Neighborhood effect is taken into account to shed the light on how school quality is affected by the social mix of enrolled students. Evidence from France, the USA, and other countries suggests that in order to make access to education more democratic, schools prefer a low level of mean social capital.³

The cost of housing in a community is given by the price of housing r^C multiplied by the number of unites of housing demanded H . r^C is the gross of tax price of housing; $r^C = (1+t)r_{net}$. Land is homogenous, and the amount of land may vary across communities. In this model we do not make a difference between renters and owners, we neglect the fact that owners may achieve a capital gain or loss from selling their house. We also neglect the existence of owners who receive the rent. All individuals pay their taxes even if they only use private schools.

We consider that individuals face a transportation cost equal to the price of transportation multiplied by the number of miles separating an individual's community from his school. It is given by T_j^C . The price of transportation is identical for all individuals, and is equal to zero for a school j geographically belonging to community C .

It should be noted that the number of individuals is larger than the number of schools and the latter is larger than the number of communities. $P > j > C$. Individuals can only be enrolled in one school, and they can not supplement this education elsewhere.

The proportion of students of type (b, y, k) in a particular school j is given by $\alpha_j(b, y, k)$; the number of students in school j is given by l_j . With:

$$l_j = \iiint_s \alpha_j(b, y, k) f(b, y, k) dbdydk \quad (1)$$

And $\alpha_j(b, y, k) \in [0,1]$

³ In France two types of scholarships are attributed to students, one is meritorious and is administered by the French ministry of education and the second is need based and is administered by the CROUS, in the latter case the scholarship is calculated using household socioeconomic data.

The proportion of students of type (b, y, k) in a particular community is given by $\alpha_c(b, y, k)$; the number of students is given by:

$$l_c = \iiint_s \alpha_c(b, y, k) f(b, y, k) dbdydk \quad (2)$$

And $\alpha_c(b, y, k) \in [0, 1]$

The production cost of education is dependent on the number of students enrolled in the school, it is given by $Co(l_j) = V(l_j) + F = n_1 l_j + n_2 l_j^2 + F$. $V' > 0$, $V'' > 0$, F is a fixed cost, n_1 and n_2 are positive constants⁴. The absence of economies of scale in the production of education means that a large number of schools exist and will be catering to each type of students.

In this model, we suppose that information is complete; individuals and schools make their decisions while having all information about educational quality, pricing, rental, tax rates and the distribution of income, ability and social capital.

III. SCHOOLS.

In our model, schools are assumed to maximize profit under a quality constraint. Funding is provided by three major sources; government subsidies, tuition paid by students and other earnings.

$$R_j = \iiint E_j \alpha_j(b, y, k) f(b, y, k) dbdydk + \iiint p_{ij} \alpha_j(b, y, k) f(b, y, k) dbdydk + G_j$$

E_j : Local government per student subsidies in school j . E_j is identical for all students belonging to the same school.

p_{ij} : Tuition paid by student i in school j .

G_j : Other earnings.

⁴ Contradicting evidence was provided by Lawrence Kenny, Ferris and West on the existence of economies of scale in the production of education. Kenny (1982) provided evidence that such economies of scale do exist. However, Ferris and West (2004) argued that large schools suffer from social problems; in other terms, external and invisible costs prevent the existence of economies of scale.

The government local subsidies in community C for school j are given by $E_j^C = \sum E_j$. The sum of government subsidies in a community C is equal to the sum of: the property tax rate in community C multiplied by the net of tax price of housing and by the number of units of housing $\sum E_j^C = \sum t^C r_{net} H_i$.

School quality is then given by:

$$q_j = q_j \left[\frac{R_j}{l_j}, \theta_j, O_j \right] \quad (3)$$

q_j is increasing in $\frac{R_j}{l_j}, \theta_j$, while it is decreasing in O_j .

With:

Expenditure per pupil given by $\frac{R_j}{l_j}$.

Peer group effect given by mean ability in a school: $\theta_j = \frac{1}{l_j} \iiint_s b_i \alpha_j(b, y, k) f(b, y, k) db dy dk$

(4)

Neighborhood effect given by mean social capital in a school:

$$O_j = \frac{1}{l_j} \iiint_s k_i \alpha_j(b, y, k) f(b, y, k) db dy dk \quad (5)$$

Optimization:

Schools maximize profit under a quality constraint; the level of quality chosen by a school determines the price for each type of students. Schools do not select directly their students; they admit any student who is able to pay the price corresponding to his type; displaying a prohibitive price is equivalent to refusing to admit a student. The constraints include: mean ability, mean social capital, and the number of students l_j enrolled in a school since it influences expenditure per pupil. Mathematical details are found in the appendix.

The optimal level of resources is the following⁵:

⁵ Results (6) and (7) are obtained by forming a Lagrangian function under the constraints (1), (4), and (5), and optimizing over α_j .

$$R^* = n_1 + 2n_2l_j + \mu'(\theta_j - b_i) + \mu''(O_j - k_i) \quad (6)$$

μ' , and μ'' are the Lagrangian multipliers. (7)

$$\mu' = \frac{1}{l_j} \iiint_s \frac{\partial R^*}{\partial \theta_j} \alpha_j(b, y, k) f(b, y, k) db dy dk$$

$$\mu'' = \frac{1}{l_j} \iiint_s \frac{\partial R^*}{\partial O_j} \alpha_j(b, y, k) f(b, y, k) db dy dk$$

μ' is positive and μ'' is negative.⁶ μ' represents the per student resources change deriving from a change in θ_j ; μ'' represents the per student resources change deriving from a change in O_j . For analytical convenience, they are considered to be the same for all schools.

When R^* is replaced by its value we obtain the following equation:

For pure private schools, we have:

$$p_{ij}^* = n_1 + 2n_2l_j + \mu'(\theta_j - b_i) + \mu''(O_j - k_i) \quad \text{With } E_j = 0.^7$$

For mixed finance schools, we have:

$$p_{ij}^* = -E_j + n_1 + 2n_2l_j + \mu'(\theta_j - b_i) + \mu''(O_j - k_i)$$

The level of per student subsidies E_j for mixed finance schools is determined by authorities and not by optimization, schools can only choose the level of pricing to apply conditioned by its level of quality and the type of students it will enroll.

For pure public schools, we have:

$$E_j^* = n_1 + 2n_2l_j + \mu'(\theta_j - b_i) + \mu''(O_j - k_i) \quad \text{With } p_{ij} = 0 \text{ for all } i.$$

Since pure public schools are free entry free of tuition schools, the type of student enrolled in pure public schools determines his marginal cost and thus the amount of subsidies needed. Thus E is noted with an asterisk, since it is determined by the optimization.

After optimization, profit is equal to zero, $\pi_j = 0$. New entries on the market of education are possible as long as $\pi_j > 0$. When $\pi_j = 0$, no new entries are possible.

⁶ See the definition of Lagrangian multipliers in Epple and Romano (1998) pp. 40.

⁷ Pricing is made according to the marginal cost of admitting a student of a particular type.

In this equation, R^{**} represents the resources needed to cover the marginal cost of admitting a student. The first term $n_1 + 2n_2l_j$ is the part of the marginal cost resulting from the cost of producing education. It is identical for all students in the same school.

The second term, $\mu'(\theta_j - b_i)$ represents the cost of one's ability on mean ability. Students with $\theta_j < b_i$ have a negative externality cost on the school. The reverse is true for $\theta_j > b_i$.

The third term, $\mu''(O_j - k_i)$ represents the cost of one's social capital on mean social capital. Students with $k_i < O_j$ have a negative externality cost on the school. The reverse is true for $k_i > O_j$.

Equation (6) is linear in ability and in social capital. For private and mixed finance schools, the price is decreasing in ability and increasing in social capital. For public schools, since the price is equal to zero, the ability and social capital of students would not have any effect on their disposable income but it would have an effect on the resources needed to cover the marginal cost of admitting them; E_j is decreasing in ability and increasing in social capital.

This equation allows us to overcome the strict distinction between public and private schools. Different types of education finance can be considered: we can start at the level of free of tuition public schools with $p_{ij} = 0$ for all i , and go through mixed finance schools where both p and E are positive until reaching purely private schools where $E_j = 0$.

It should also be noted that given the level of government subsidies; pricing will be both meritorious and need based. The second term of the equation represents the meritorious part of the pricing function; the third term of the equation represents the need based part of the pricing function. Another implication of equation (6) is the possibility that pricing can be negative depending on the level of government subsidies and the positioning of individual ability and social capital relatively to the means; in this case the negative price represents a scholarship.

The sequencing of decision making for individuals and schools.

The sequencing of decisions can be described through a multistage game. First, individuals choose the community in which to reside and the school to be enrolled into⁸. At this level the housing units demanded per household are determined. Second, individuals choose the property tax rate through majority voting. At this level the sum of subsidies in the community $\sum E_j^C = \sum t^C r_{net} H_i$ is determined. Thirdly, local authorities allocate their budget. At this level, it should be noted that we have three types of schools, pure publicly financed schools with $p_{ij} = 0$, mixed finance and pure private schools with $p_{ij} \neq 0$. Local authorities attribute subsidies to pure publicly financed schools first and according to their needs, and then the rest of funding is arbitrarily attributed to mixed finance schools. In other words, the type of students enrolled in pure public schools determines the funding needed, in this way these schools are free entry free of tuition schools. Causality in equation (6) goes from the right hand side to the left hand side; the marginal cost resulting from the type of students enrolled determines the funding allocated by local authorities. Conversely, for mixed finance schools, causality in equation (6) goes from the left hand side to the right hand side; after receiving local subsidies and choosing the level of educational quality, schools determine the type of students to be admitted and the pricing that goes with it.

Lemma 1:

(i) Educational quality q_j is strictly increasing in price p_{ij} for all i .

In this lemma, we show that higher prices are associated with higher qualities. In other words, in order to justify the existence of a higher price a higher educational quality should be associated with it⁹.

(ii) In each community, the schools with $p_j = 0$ are the lowest quality schools.

In the second part we show that pure public schools have the lowest quality in each community.

Proposition 1: The hierarchy of educational qualities in a community follows that of the prices. The proof for this proposition follows directly from lemma 1.

⁸ Complete and perfect information plays an important role in the decision making of individuals and schools.

⁹ It should be noted that the reverse is true; a higher quality is associated with a higher price.

IV. STUDENTS.

Individuals have two connected optimization problems; they have to choose a community and a school. They choose communities and schools simultaneously through utility maximization. It should be noted that since public, mixed finance and private schools exist in our model, individuals have a large set of choices, even outside their community. We have two cases; firstly, when public school choice is not allowed, individuals have a choice between the schools of their community and the mixed finance and private schools outside their community. Secondly, when public school choice is allowed, individuals have a choice between all schools in the economy. The cost of transportation combined with educational pricing and rental, play an important role in determining the affordable schools.

The budget constraint is given by: $y_i = c + T_j^C + p_{ij} + r^C H_i$ (8)¹⁰

Maximization of U subject to constraint (8) yields the indirect utility function:

$$U^*(c, q) = \text{Max} \left[y_i - T_j^C - p_{ij} - r^C H_i, q_j \left(\frac{R_j}{l_j}, \theta_j, O_j \right) \right] \quad (9)$$

$$= W_{ij}^C \left[(y_i, b_i, k_i), T_j^C, t^C, p_{ij}, r^C, H_i, q_j \right]$$

Assumption 1: pinning down preferences over education and housing.

a- We assume a positive (b, y, k) elasticity of educational quality. As (b, y, k) increases, the educational quality q_j demanded increases. Higher (b, y, k) prefer higher quality schools.

b- We assume a positive ability, income, and social capital elasticity of housing demand such

as: $\frac{\partial H}{\partial y} > 0$, $\frac{\partial H}{\partial b} > 0$, and $\frac{\partial H}{\partial k} > 0$. Housing demand is also assumed to be increasing

in (b, y, k) such as: $\frac{\partial H}{\partial (b, y, k)} > 0$. Higher (b, y, k) prefer larger houses. The case for

¹⁰ Notation remark: small letter c is used to denote consumption, capital letter C is used to denote a community.

higher (b, y, k) shall be considered because it accounts for situations where one or two of the variables b , y , or k is lower while others are higher.¹¹

Remark: The superiority of a combination (b, y, k) relatively to another one depends on how b , y , and k enter the utility function. It should be said that any (b, y, k) combination mentioned in this paper is not unique since b , y and k are substitutable in the pricing function (low ability or high social capital can be substituted by high income). Different types of students with different combinations of (b, y, k) may have the same level of utility. When we say higher (b, y, k) students, this means students with higher ability, higher income, or lower social capital, everything held equal.

Property 1: By virtue of the implicit function theorem, we obtain:

$$\begin{aligned} \text{a-} \quad \frac{\partial(r+p+T)}{\partial q} \Big|_{w=\bar{w}} &= \frac{\partial r + \partial p + \partial T}{\partial q} = \frac{\partial r}{\partial q} + \frac{\partial p}{\partial q} + \frac{\partial T}{\partial q} = \\ &-\frac{\partial W/\partial q}{\partial W/\partial r} - \frac{\partial W/\partial q}{\partial W/\partial p} - \frac{\partial W/\partial q}{\partial W/\partial T} = \frac{1}{H} + 1 + 1 = \frac{1}{H} + 2 > 0 \end{aligned}$$

Part a implies that indifference curves in the $[q, (r+p+T)]$ plane have positive slopes¹².

We are using the $[q, (r+p+T)]$ plane since community choice is not independent from school choice; the type of community chosen determines the type of schools that can be afforded, and the reverse is true.

$$\text{b-} \quad \partial \left(\frac{\partial(r+p+T)}{\partial q} \right) / \partial y = -\frac{H_y}{H^2} < 0$$

For students of the same ability and social capital, any indifference curve in the $[q, (r+p+T)]$ plane of a higher income student cuts any indifference curve of a lower income student from above. Thus for a given b and k , indifference curves exhibit single crossing in income. In other words, when income increases, the slope of the indifference curve decreases.

$$\text{c-} \quad \partial \left(\frac{\partial(r+p+T)}{\partial q} \right) / \partial b = -\frac{H_b}{H^2} < 0$$

For students of the same income and social capital, any indifference curve in the $[q, (r+p+T)]$ plane of a higher ability student cuts any indifference curve of a lower ability

¹¹ $\frac{\partial H}{\partial(b, y, k)} = \frac{\partial H}{\partial b} \partial b + \frac{\partial H}{\partial y} \partial y + \frac{\partial H}{\partial k} \partial k > 0$

¹² This part of the assumption means that for a higher educational quality, individuals are willing to spend more on rental, education, and transportation combined.

student from above. Thus for a given y and k , indifference curves exhibit single crossing in ability. In other words, when ability increases, the slope of the indifference curve decreases.

$$d- \quad \partial \left(\frac{\partial(r+p+T)}{\partial q} \right) / \partial k = -\frac{H_k}{H^2} < 0$$

For students of the same income and ability, any indifference curve in the $[q, (r+p+T)]$ plane of a higher social capital student cuts any indifference curve of a lower social capital student from above. Thus for a given b and k , indifference curves exhibit single crossing in social capital. In other words, when social capital increases, the slope of the indifference curve decreases.

$$e- \quad \partial \left(\frac{\partial(r+p+T)}{\partial q} \right) / \partial(b, y, k) = -\frac{H_{(b, y, k)}}{H^2} < 0$$

Any indifference curve in the $[q, (r+p+T)]$ plane of an individual with a higher combination of (b, y, k) cuts any indifference curve of a student with a lower combination of (b, y, k) from above. Thus, indifference curves exhibit single crossing in (b, y, k) . In other words, when (b, y, k) increases, the slope of the indifference curve decreases.

Lemma 2: Community and school choice: boundary indifference.

For $r^C < r^{C+1}$, and $q_j > q_{j'}$ there exists a combination $(\hat{b}, \hat{y}, \hat{k})$ such that:

$$W_{ij}^C \left\{ \hat{y}_i, \hat{b}_i, \hat{k}_i, T_j^C, t^C, p_{ij}, r^C, H_i, q_j \right\} = W_{ij'}^{C+1} \left\{ \hat{y}_i, \hat{b}_i, \hat{k}_i, T_{j'}^{C+1}, t^{C+1}, p_{ij'}, r^{C+1}, H_i, q_{j'} \right\} \quad (11)^{13}$$

With $C+1 \neq C$ and $j \neq j'$. Note that $(\hat{b}, \hat{y}, \hat{k})$ is not unique.¹⁴

Students choose a community and a school, as long as their indirect utility is higher than that obtained in any other combination of communities and schools.

In this Lemma we show that by choosing lower rental communities, higher (b, y, k) students would have higher numeraire consumption, higher school quality while having the same housing consumption as lower (b, y, k) students. Continuity of utility and of the distribution

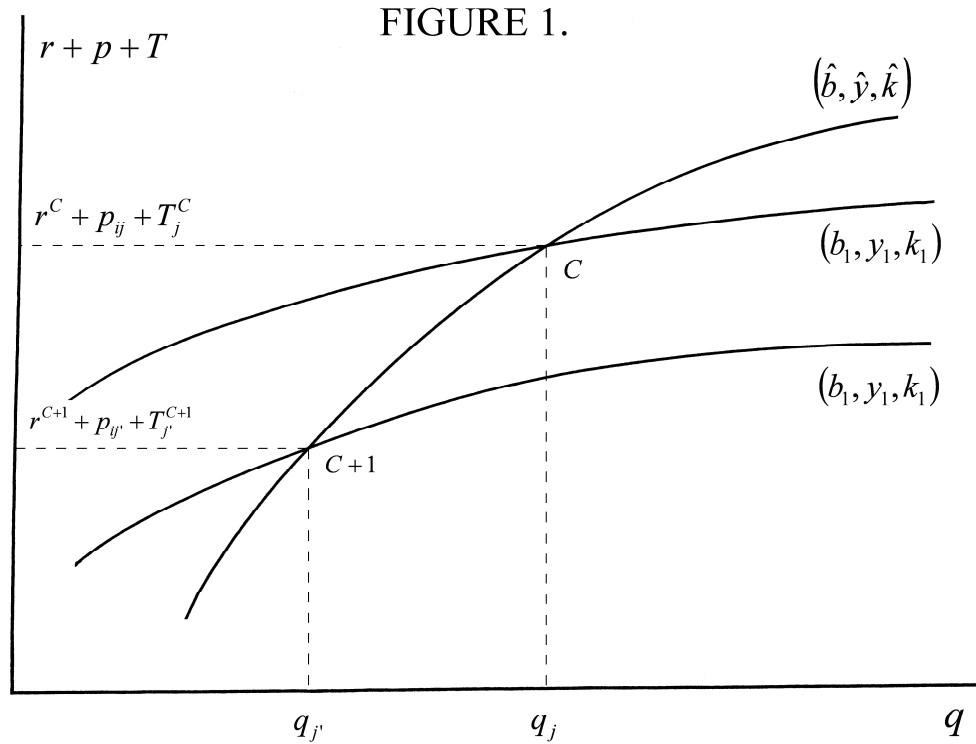
¹³ $(\hat{b}, \hat{y}, \hat{k})$ is not unique, any other equivalent combination of b , y and k can determine the boundary indifference.

¹⁴ See the remark on page 13.

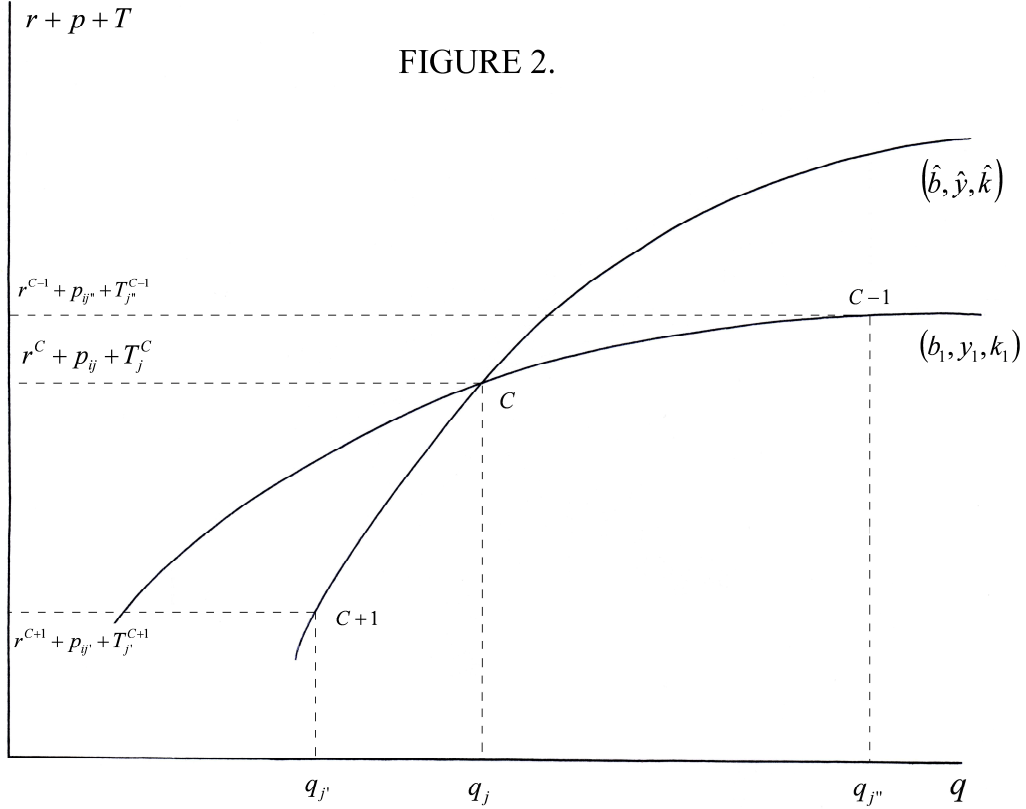
function of (b, y, k) ensures that there is a type of individuals that is indifferent between each two combinations of communities and schools.

Corollary 1: $(\hat{b}, \hat{y}, \hat{k})$ is strictly monotonically decreasing in b, y, k , and $(b, y, k) \forall C$.

In this corollary, we show that any student with a better combination than $(\hat{b}, \hat{y}, \hat{k})$ can not be indifferent between the two communities C and $C+1$.



Lemma 3: (i) Students with higher combinations of (b, y, k) spend more on rental, schooling, and transportation combined ($r + p + T$) and choose higher educational qualities. (ii) Higher (b, y, k) spend more on education, and less on rental, and they have a higher housing consumption. (iii) In the third part of this Lemma we examine individuals' spending on transportation.



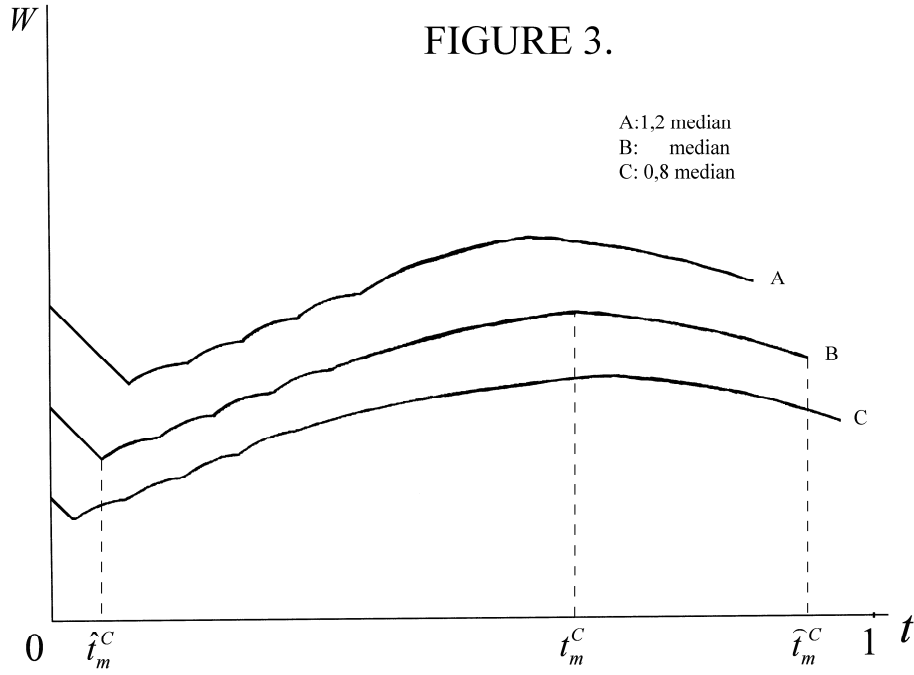
Proposition 2: Three dimensional stratification. Consider the same two individuals from Lemma 3. Individual $(\hat{b}, \hat{y}, \hat{k})$ is indifferent between these two combinations $(C+1, j')$ and (C, j) . Individual (b_1, y_1, k_1) is indifferent between these two combinations (C, j) and $(C-1, j'')$. Equilibrium exists if $(b, y, k) \in C$ when $(\hat{b}, \hat{y}, \hat{k}) < (b, y, k) < (b_1, y_1, k_1)$.

By virtue of proposition 2, and corollary 1 (Boundary Loci are strictly monotonically decreasing in b, y, k , and (b, y, k)), individuals are stratified into communities and schools according to their ability, income, and social capital; in other words, complete stratification by ability, income, or social capital would not occur. Thus, we will have a mixture of individuals with different combinations of (b, y, k) in each community.

V. MAJORITY VOTING.

In this part of our model we focus on individuals' choice of tax rates, we consider that all equilibrium conditions are satisfied. The tax rate has only an influence over expenditure per pupil as part of educational quality. Individuals have perfect knowledge of tax rates and school qualities in their communities; they anticipate any changes in educational quality and housing prices as a result of a change in the tax rate in their community; they assume that tax rates and educational qualities in other communities are fixed.

In our model, we considered three types of schools, pure public schools, mixed finance schools, and private schools. In each community, for very low tax rates the quality of public and mixed finance schools heavily relying on subsidies is low, students prefer private schools or even schools outside their community. A marginal increase in tax rates will not increase public school quality that much, but it will reduce numeraire consumption and utility; private are still preferred; at this stage individuals will not migrate from their community because they anticipate an increase in utility when the tax rate becomes sufficiently high. An important increase of the tax rate induces an important increase of quality in public and mixed finance schools; at some level of the tax rate, students are willing to move from their private school to the highest price (highest quality) mixed finance school, their utility increases until reaching its maximum for this school; at this point, the individual will move again to the second highest price mixed finance school, and so on. This process continues until the tax rate is high enough to encourage students to get enrolled in a public school, utility increases until reaching its highest level, from this point on, any increase in tax rates induces a decrease in utility. Once the tax rate becomes excessive such as the gross of tax price of housing is very high and income per capita is reduced, individuals anticipate that any further increase in the tax rate will just decrease their utility more and more; high (b, y, k) individuals migrate from this community. It should be noted that preferences are not single peaked, and that the "Ends against the Middle" phenomenon is present in our model. This is illustrated in Figure 3.



We define \hat{t}_i^C to be the tax rate at which individual i is indifferent between a private school and the highest price (highest quality) mixed finance school. We also define t_m^C to be the tax rate at which the utility of individual i is maximized in the public school, and finally \hat{t}_i^C the tax rate at which individual i is willing to migrate from community C. $i=m$ is used to designate the individual with median combination of (b, y, k) . It should be noted that individuals are not willing to migrate at \hat{t}_i^C because they anticipate an increase in utility when the tax rate becomes sufficiently high, conversely they are willing to migrate after \hat{t}_i^C because any further increase of the tax rate will reduce their disposable income and their utility. \hat{t}_i^C must satisfy the following equation: $c_{ij}^C = y_i - T_j^C - p_{ij} - (1 + \hat{t}_i^C)r_{net}H_i = 0$.

Conditions for the existence of a majority voting equilibrium:

- 1- Students with the median combination of ability, income, and social capital $(b, y, k)_m$ prefers a positive tax rate in a public school (c) over a zero tax rate in a private one (a) or a school outside community C. Note that $(b, y, k)_m$ is not unique.¹⁵

¹⁵ See the remark on page 13.

$W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$ is true.
 $T_c^C = 0$, while T_a^C may be different than zero.

- 2- As seen in figure 3, all individuals with $(b_i, y_i, k_i) > (b, y, k)$ have a migration tax rate \hat{t}_i^C higher than t_m^C (the tax rate that maximizes the utility of the individual with $(b, y, k)_m$ in the public school).
- 3- As seen in figure 3, t_i^C is decreasing in the combination (b, y, k) .
- 4- If two individuals 1 and 2 with $(b_1, y_1, k_1) > (b_2, y_2, k_2)$ are enrolled in the same school and live in the same community; individual 1 has a lower utility.

$$W_{2a}^C[(b_2, y_2, k_2), T_a^C, t^C, p_{2a}, r^C, H_2, q_a] > W_{1a}^C[(b_1, y_1, k_1), T_a^C, t^C, p_{1a}, r^C, H_1, q_a]$$

If they are enrolled in two different schools a and b respectively with $q_a > q_b$, then individual 1 has a higher utility.¹⁶

$$W_{2b}^C[(b_2, y_2, k_2), T_b^C, t^C, p_{2b}, r^C, H_2, q_b] < W_{1a}^C[(b_1, y_1, k_1), T_a^C, t^C, p_{1a}, r^C, H_1, q_a]$$

If 1 is not satisfied, then $t = 0$ is a majority voting equilibrium. If 2, 3, and 4 are not satisfied, then majority voting equilibrium may not exist. Restrictions on preferences alone do not allow the existence of a majority voting equilibrium; the following conditions represent restrictions on the distribution function of b , y , and k .

Definition: if t^C is to be a majority voting equilibrium, then there is no other tax rate that is preferred by more than 50% of the population of the community. In other terms, at least 50% and one individual choose this tax rate.

Lemma 4:

- (i) \hat{t}_i^C is increasing in (b, y, k) .

This part can be applied to any indifference tax rate between a private school and a mixed finance school, between two mixed finance schools or between a mixed finance and a public school. It can be explained intuitively. Higher (b, y, k) individuals need higher tax rates and thus higher school qualities to be encouraged to get enrolled in a mixed finance or a public school.

¹⁶ Epple and Romano (1996) used an assumption about education being a superior or a normal good, they also used a diminishing marginal utility of numeraire consumption, see page 300. Glomm and Ravikumar (1998) used a similar assumption where $\lim_{c \rightarrow +\infty} U_c(c, q) = 0$ and a homothetic utility function.

Remark: individuals choosing a school outside their community or a public school inside their community prefer a zero tax rate since they do not use public or mixed finance schools.

(ii) We consider three schools a, b and c, a and b are private schools or schools outside community C, c is a public school in community C.

1- If $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$ is true.¹⁷

Then $W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c] > W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b]$ for all $(b_1, y_1, k_1) < (b, y, k)_m$.

2- If $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] < W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$ is true.

Then $W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c] < W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b]$ for all $(b_1, y_1, k_1) > (b, y, k)_m$.

In this part of lemma 4 we prove that if the type of individuals with the median combination of (b, y, k) prefers (does not prefer) a positive tax rate in a public school over a zero tax rate in a private one or a school outside community C, all individuals with lower (higher) combinations of (b, y, k) have the same behavior.

It should be noted that according to condition 1, the type of individuals with the median combination of (b, y, k) prefer the tax rate t_m^C . It will be sufficient to prove that 50% of the population prefers t_m^C to prove the existence of a majority voting equilibrium. Note that the type of individuals with the median combination of (b, y, k) splits the population into two identical fractions of 50%.

Lemma 5:

(i) Over the interval $[t_m^C, 1]$, there does not exist a tax rate different than t_m^C that is preferred by more than 50% of the population.

(ii) Over the interval $[\hat{t}_m^C, t_m^C]$, there does not exist a tax rate different than t_m^C that is preferred by more than 50% of the population.

¹⁷ It should be noted that since c is a public school $p_c = 0$, and for schools belonging to community C, $T = 0$.

An important property used in the proof of Lemma 5 is based on the fact that indirect utility is increasing in t over the interval $[\hat{t}_m^C, t_m^C]$ and decreasing over the interval $[t_m^C, 1]$.

Proposition 3:

if $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$ is true, then t_m^C is a majority voting equilibrium.

If $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] < W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$ is true, then all individuals with $(b_i, y_i, k_i) > (b, y, k)_m$ prefer a zero tax rate in a private school over a positive tax rate in a public one; then $t = 0$ is a majority voting equilibrium.

Equilibrium:

1- Individuals.

- Utility maximization:

$$U^*(c, q) = \text{Max} \left[y_i - T_j^C - p_{ij} - r^C H_i, q_j \left(\frac{R_j}{l_j}, \theta_j, O_j \right) \right]$$

$$= W_{ij}^C[(y_i, b_i, k_i), T_j^C, t^C, p_{ij}, r^C, H_i, q_j]$$

2- Schools.

- Profit maximization and optimal level of resources: $\max \pi_j = R_j - V(l_j) - F$.

$$R^* = n_1 + 2n_2 l_j + \mu'_j(\theta_j - b_i) + \mu''_j(O_j - k_i)$$

With μ' is positive and μ'' is negative.

- At equilibrium $\pi_j = 0$; no new entries on the market are possible.
- The number of students in a school: $l_j = \iiint_s \alpha_j(b, y, k) f(b, y, k) db dy dk$.
- Mean ability in a schools: $\theta_j = \frac{1}{l_j} \iiint_s b_i \alpha_j(b, y, k) f(b, y, k) db dy dk$
- Mean social capital in a school: $O_j = \frac{1}{l_j} \iiint_s k_i \alpha_j(b, y, k) f(b, y, k) db dy dk$.

3- Communities.

- The community budget is balanced: $\sum E_j^C = \sum t^C r_{net} H_i$.
- The number of students in a community: $l_C = \iiint_s \alpha_C(b, y, k) f(b, y, k) dbdydk$.
- Boundaries indifference conditions are verified.
- t_m^C is a majority voting equilibrium.

4- Market clearance.

- All students go to school: $\sum_{j=1}^j l_j = \sum_{c=1}^C l_C = P$
- The housing demand in a community is equal to the housing supply.

VI. CONCLUSION.

In this paper, we have constructed an equilibrium on the market of education in a multicomunity environment where students are differentiated by their income, ability, and social capital. We have analyzed stratification across schools and communities to find that students with higher combinations of ability, income, and social capital spend more on rental, transportation and education combined, while choosing higher educational qualities; they spend more on education and less on rental taken separately and have higher housing consumption.

We have studied majority voting over tax rates; the presence of private, mixed finance, and public schools led to the existence of non single peaked preferences over tax rates. We have provided the necessary conditions for a majority voting equilibrium to be determined by the median voter. In future studies, this paper will be furthered by analyzing the consequences of introducing school choice between public schools belonging to different communities.

Appendix

School optimization:

Schools maximize profit under constraint:

$$\max \pi_j = R_j - V(l_j) - F$$

Under these constraints:

$$l_j = \iiint_s \alpha_j(b, y, k) f(b, y, k) dbdydk \quad (1)$$

$$\theta_j = \frac{1}{l_j} \iiint_s b_i \alpha_j(b, y, k) f(b, y, k) dbdydk \quad (4)$$

$$O_j = \frac{1}{l_j} \iiint_s k_i \alpha_j(b, y, k) f(b, y, k) dbdydk \quad (5)$$

With (i = 1, 2, 3, ..., P) and (j = 1, 2, 3, ..., j)

These constraints can be transformed in the following way:

$$1 - \theta_j \iiint_s \alpha_j(b, y, k) f(b, y, k) dbdydk - \iiint_s b_i \alpha_j(b, y, k) f(b, y, k) dbdydk = 0$$

$$2 - O_j \iiint_s \alpha_j(b, y, k) f(b, y, k) dbdydk - \iiint_s k_i \alpha_j(b, y, k) f(b, y, k) dbdydk = 0$$

The Lagrangian function is of the following form:

$$\begin{aligned} \Phi = & R_j - n_1 l_j - n_2 l_j^2 - F - \mu' [\theta_j \iiint_s \alpha_j(b, y, k) f(b, y, k) dbdydk - \iiint_s b_i \alpha_j(b, y, k) f(b, y, k) dbdydk] \\ & - \mu'' [O_j \iiint_s \alpha_j(b, y, k) f(b, y, k) dbdydk - \iiint_s k_i \alpha_j(b, y, k) f(b, y, k) dbdydk] \end{aligned}$$

With μ' , and μ'' the Lagrangian multipliers.

μ' is positive and μ'' is negative.

Optimization is made through the partial derivation of the Lagrangian function over $\alpha_j(b, y, k)$, the following results are obtained:

$$\frac{d\Phi}{d\alpha_j(b, y, k)} = R^* - n_1 - 2n_2 l_j - \mu' (\theta_j - b_i) - \mu'' (o_j - k_i) = 0$$

$$R^* = n_1 + 2n_2 l_j + \mu' (\theta_j - b_i) + \mu'' (o_j - k_i)$$

Proof of Lemma 1:

(i) The proof is by contradiction. Suppose that we have two schools 1 and 2 with $p_{i1} < p_{i2}$ and $q_1 \geq q_2$. In this case; rational individuals will choose school 1 since it has higher or equal quality at a lower price. No student will choose school 2; this school will be obliged to leave the market. In order for school 2 to exist it should have a quality $q_{i1} < q_{i2}$.

More precisely using equation (6):

$$p_{i1}^* = -E_1 + n_1 + 2n_2l_1 + \mu'(\theta_1 - b_i) + \mu''(O_1 - k_i)$$

$$p_{i2}^* = -E_2 + n_1 + 2n_2l_2 + \mu'(\theta_2 - b_i) + \mu''(O_2 - k_i)$$

With μ' is positive and μ'' is negative.

We can write, (for the same student):

$$p_{i1}^* - p_{i2}^* = E_2 - E_1 + 2n_2(l_1 - l_2) + \mu'(\theta_1 - \theta_2) + \mu''(O_1 - O_2) \text{ For all } i.$$

The right hand side is independent of (b_i, y_i, k_i) .

Higher mean ability and lower mean social capital schools (higher quality schools with $\frac{R_j}{l_j}$ held fixed) have higher prices. However for expenditure per pupil $\frac{R_j}{l_j}$, the explanation

is a little bit different. Consider schools 1 and 2 when they have the same mean ability and the same mean social capital and $l_1 < l_2$ or $E_1 > E_2$; school 1 has a higher quality but a lower price for all students. Even if this situation is possible; at equilibrium school 2 will be excluded of the market since it offers lower qualities at higher prices and no one would be willing to get enrolled in it. If two schools are of the same quality they should have the same pricing for the same student.

(ii) The second part of lemma 1 follows directly from the first part. Since educational quality is increasing in price, the lowest price $p_j = 0$ of pure public schools is associated with the lowest quality. An important assumption needed to the proof of this lemma is that made in the sequencing of decisions; since pure public schools are free entry free of tuition schools, any individual can be admitted in a public school, and thus no individual is restrained to choose a lower quality higher price school than a pure public school. There is no school that has a lower quality than pure public schools and a price $p_j > 0$.

Proof of Lemma 2:

We consider an individual i with (b_i, y_i, k_i) , he consumes the same amount of housing in both communities C and $C+1$.

His budget constraint in school j and community C is given by: $y_i = c_{ij}^C + T_j^C + p_{ij} + r^C H_i$

It can be written in the following form: $c_{ij}^C = y_i - T_j^C - p_{ij} - r^C H_i$

His budget constraint in school j' and community $C+1$ is given by:

$y_i = c_{ij'}^{C+1} + T_{j'}^{C+1} + p_{ij'} + r^{C+1} H_i$. It can be written in the following form:

$$c_{ij'}^{C+1} = y_i - T_{j'}^{C+1} - p_{ij'} - r^{C+1} H_i$$

The difference in consumption between community $C+1$ and C is given by:

$$c_{ij'}^{C+1} - c_{ij}^C = (T_j^C - T_{j'}^{C+1}) + (p_{ij} - p_{ij'}) + (r^C - r^{C+1}) H_i$$

By virtue of proposition 1, $q_j > q_{j'}$ implies that $p_{ij} > p_{ij'}$, thus $p_{ij} - p_{ij'} > 0$.

Since $r^C < r^{C+1}$, we can write $r^C - r^{C+1} < 0$. The difference between $(T_j^C - T_{j'}^{C+1})$ can be positive, negative, or equal to zero.

Even if the distribution of (b, y, k) has an upper bound determined by $(b_{\max}, y_{\max}, k_{\max})$ and a lower bound determined by $(0, 0, 0)$; it is analytically convenient for the proof of this lemma to consider $(b, y, k) \rightarrow +\infty$ and $(b, y, k) \rightarrow -\infty$.

We have two cases:

1- According to part b of assumption 1, for individuals with very high combinations of (b, y, k) ; such as $(b, y, k) \rightarrow +\infty$. We have $H_i \rightarrow +\infty$, and then $\lim_{(b, y, k) \rightarrow +\infty} (c_{ij'}^{C+1} - c_{ij}^C) = -\infty$.

In other terms, we can always find a (b, y, k) combination that is sufficiently high such as $(c_{ij'}^{C+1} - c_{ij}^C) < 0$.

According to part a of assumption 1; higher (b, y, k) individuals prefer higher educational qualities. As a consequence, this individual chooses school j .

If this individual is to locate to $C+1$; he will consume the same amount of housing, while having a lower consumption. So individual (b, y, k) strictly prefer community C .

2- According to part b of assumption 1, for individuals with very low combinations of (b, y, k) ; such as $(b, y, k) \rightarrow -\infty$. We have $H_i \rightarrow -\infty$, and then $\lim_{(b, y, k) \rightarrow +\infty} (c_{ij}^{C+1} - c_{ij}^C) = +\infty$.

In other terms, we can always find a (b, y, k) combination that is sufficiently low such as $(c_{ij}^{C+1} - c_{ij}^C) > 0$.

According to part a of assumption 1, lower (b, y, k) individuals prefer lower educational qualities. As a consequence, this individual chooses school j' .

If this individual is to locate to C+1; he will consume the same amount of housing, while having a higher consumption. So individual (b, y, k) strictly prefer community C+1.

We have seen, that high (b, y, k) students prefer communities with lower rental and schools with higher quality, and that students with low (b, y, k) prefer communities with high rental and schools with lower quality. The continuity of the utility function and of the distribution function of (b, y, k) implies that while going from an extreme to the other, there exist an individual with $(\hat{b}, \hat{y}, \hat{k})$ that is indifferent between the two communities and the two schools. The proof that is applied to two communities and schools can be extended to multiple communities and schools; between each two consecutive communities and schools there will be an individual indifferent between them.

Proof of corollary 1: The proof is by contradiction.

Suppose that $(\hat{b}, \hat{y}, \hat{k})$ is somewhere nondecreasing in b, y, k and (b, y, k) . Then we can find an individual with (b_1, y_1, k_1) indifferent between the two combinations of communities and schools, (C, j) and $(C+1, j')$ such as:

$$1- b_1 > \hat{b}, y_1 \geq \hat{y}, \text{ and } k_1 \geq \hat{k} . \qquad 4- b_1 > \hat{b}, y_1 > \hat{y}, \text{ and } k_1 \geq \hat{k}$$

$$2- b_1 \geq \hat{b}, y_1 > \hat{y}, \text{ and } k_1 \geq \hat{k} . \qquad 5- b_1 > \hat{b}, y_1 \geq \hat{y}, \text{ and } k_1 > \hat{k}$$

$$3- b_1 \geq \hat{b}, y_1 \geq \hat{y} \text{ and } k_1 > \hat{k} \qquad 6- b_1 \geq \hat{b}, y_1 > \hat{y}, \text{ and } k_1 > \hat{k}$$

$$7- b_1 > \hat{b}, y_1 > \hat{y}, k_1 > \hat{k} \text{ or } (b_1, y_1, k_1) > (\hat{b}, \hat{y}, \hat{k})$$

The case where $(b_1, y_1, k_1) > (\hat{b}, \hat{y}, \hat{k})$, shall be considered because it accounts for situations where one or two of the variables $b, y,$ and k is lower while the others are higher.

For case 1,

According to property 1-c, the slope of the indifference curve of (b_1, \hat{y}, \hat{k}) is lower than that of $(\hat{b}, \hat{y}, \hat{k})$. According to properties 1-b and 1-d, the slope of (b_1, y_1, k_1) is equal or lower than that of (b_1, \hat{y}, \hat{k}) .

$$\text{Slope}(b_1, y_1, k_1) \leq \text{Slope}(b_1, \hat{y}, \hat{k}) < \text{Slope}(\hat{b}, \hat{y}, \hat{k})$$

For case 2:

According to property 1-b, the slope of the indifference curve of (\hat{b}, y_1, \hat{k}) is lower than that of $(\hat{b}, \hat{y}, \hat{k})$. According to properties 1-c and 1-d, the slope of the indifference curve of (b_1, y_1, k_1) is lower or equal to that of (\hat{b}, y_1, \hat{k}) .

$$\text{Slope}(b_1, y_1, k_1) \leq \text{Slope}(\hat{b}, y_1, \hat{k}) < \text{Slope}(\hat{b}, \hat{y}, \hat{k})$$

For case 3:

According to property 1-d, the slope of the indifference curve of (\hat{b}, \hat{y}, k_1) is lower than that of $(\hat{b}, \hat{y}, \hat{k})$. According to properties 1-b and 1-c, the slope of the indifference curve of (b_1, y_1, k_1) is lower or equal to that of (\hat{b}, \hat{y}, k_1) .

$$\text{Slope}(b_1, y_1, k_1) \leq \text{Slope}(\hat{b}, \hat{y}, k_1) < \text{Slope}(\hat{b}, \hat{y}, \hat{k})$$

For Case 4:

According to properties 1-b and 1-c, the slope of the indifference curve of (b_1, y_1, \hat{k}) is lower than that of $(\hat{b}, \hat{y}, \hat{k})$. According to property 1-d, the slope of the indifference curve of (b_1, y_1, k_1) is equal or lower than that of (b_1, y_1, \hat{k}) .

$$\text{Slope}(b_1, y_1, k_1) \leq \text{Slope}(b_1, y_1, \hat{k}) < \text{Slope}(\hat{b}, \hat{y}, \hat{k})$$

For case 5:

According to properties 1-c and 1-d, the slope of the indifference curve of (b_1, \hat{y}, k_1) is lower than that of $(\hat{b}, \hat{y}, \hat{k})$. According to property 1-b, the slope of the indifference curve of (b_1, y_1, k_1) is equal or lower than that of (b_1, \hat{y}, k_1) .

$$\text{Slope}(b_1, y_1, k_1) \leq \text{Slope}(b_1, \hat{y}, k_1) < \text{Slope}(\hat{b}, \hat{y}, \hat{k})$$

For case 6:

According to properties 1-b and 1-d, the slope of the indifference curve of (\hat{b}, y_1, k_1) is lower than that of $(\hat{b}, \hat{y}, \hat{k})$. According to property 1-c, the slope of (b_1, y_1, k_1) is equal or lower than that of (\hat{b}, y_1, k_1) .

$$\text{Slope } (b_1, y_1, k_1) \leq \text{Slope } (\hat{b}, y_1, k_1) < \text{Slope } (\hat{b}, \hat{y}, \hat{k})$$

For case 7:

According to properties 1-b, 1-c, 1-d and 1-e. the slope of the indifference curve of (b_1, y_1, k_1) is lower than that of $(\hat{b}, \hat{y}, \hat{k})$.

$$\text{Slope } (b_1, y_1, k_1) < \text{Slope } (\hat{b}, \hat{y}, \hat{k})$$

For a student with (b_1, y_1, k_1) to be indifferent between the two combinations of communities and schools (C, j) and $(C+1, j')$; his indifference curve shall pass through the two points $[q_j, (r^C + p_{ij} + T_j^C)]$ and $[q_{j'}, (r^{C+1} + p_{ij'} + T_{j'}^{C+1})]$. Thus, the slope of the indifferent curve of (b_1, y_1, k_1) must be at some point greater than that of $(\hat{b}, \hat{y}, \hat{k})$ in order to cross it at these two points. Following the 7 cases previously developed and figure 1, this is a contradiction; the slope of the indifference curve of (b_1, y_1, k_1) is always smaller than that of $(\hat{b}, \hat{y}, \hat{k})$, and there is only one intersection between the two indifference curves. The student with (b_1, y_1, k_1) can not be indifferent between these two combinations, and the boundary loci between communities are strictly monotonically decreasing in b, y, k , and (b, y, k) .

Proof of Lemma 3:

Consider two students with $(b_1, y_1, k_1) > (\hat{b}, \hat{y}, \hat{k})$; according to property (1-e), the indifference curve of student (b_1, y_1, k_1) has a lower slope than that of student $(\hat{b}, \hat{y}, \hat{k})$. There is only one intersection between their indifference curves, such as they can not be indifferent between the same two combinations of communities and schools. See figure 2.

(i) Note that $C + 1 \neq C \neq C - 1$ and $j'' \neq j \neq j'$.

We consider that student $(\hat{b}, \hat{y}, \hat{k})$ is indifferent between these two combinations $(C + 1, j')$ and (C, j) ; his indifference curve passes through these two points $[q_{j'}, (r^{C+1} + p_{ij'} + T_{j'}^{C+1})]$ and $[q_j, (r^C + p_{ij} + T_j^C)]$. According to Lemma 2 we have:

$$q_j > q_{j'} \text{ and } r^C < r^{C+1}.$$

We consider that student (b_1, y_1, k_1) is indifferent between these two combinations (C, j) and $(C - 1, j'')$, his indifference curve passes through these two points $[q_j, (r^C + p_{ij} + T_j^C)]$ and $[q_{j'}, (r^{C-1} + p_{ij''} + T_{j''}^{C-1})]$. According to Lemma 2 we have:

$$q_{j''} > q_j \text{ and } r^{C-1} < r^C.$$

We can see that there is only one intersection of indifference curves at point $[q_j, (r^C + p_{ij} + T_j^C)]$.

From the explanation mentioned above, we can write $q_{j''} > q_j > q_{j'}$. From Property (1-a), we know that indifference curves have positive slopes in the $[q, (r + p + T)]$ plane, thus $q_{j''} > q_j > q_{j'}$ implies $(r^{C-1} + p_{ij''} + T_{j''}^{C-1}) > (r^C + p_{ij} + T_j^C) > (r^{C+1} + p_{ij'} + T_{j'}^{C+1})$.

Individuals with a higher combination of (b, y, k) spend more on rental, schooling, and transportation combined, while choosing higher quality schools.

(ii) From lemma 1 we know that higher qualities are associated with higher prices; thus

$$q_{j''} > q_j > q_{j'} \Rightarrow p_{ij''} > p_{ij} > p_{ij'}.$$

Higher (b, y, k) students spend more on education.

From part (i) of this Lemma we can write: $r^{C-1} < r^C < r^{C+1}$.

Higher (b, y, k) students spend less on rental.

From Assumption (1-b), we know that: $\frac{\partial H}{\partial(b, y, k)} > 0$

Thus higher (b, y, k) students have higher housing consumption.

(iii) From part (i) and (ii) of this lemma we can analyze the different possible cases of spending on transportation.

We have $r^C + p_{ij} + T_j^C > r^{C+1} + p_{ij'} + T_j^{C+1}$, $p_{ij} > p_{ij'}$, and $r^C < r^{C+1}$.

We can write $(T_j^C - T_j^{C+1}) > (p_{ij'} - p_{ij}) + (r^{C+1} - r^C)$ with $p_{ij'} - p_{ij} < 0$, and $r^{C+1} - r^C > 0$.

Then $(T_j^C - T_j^{C+1})$ can be either positive, negative, or equal to zero.

a- $(T_j^C - T_j^{C+1}) > 0$; high (b, y, k) students choose far from school communities in comparison with lower (b, y, k) students.

b- $(T_j^C - T_j^{C+1}) < 0$; high (b, y, k) students choose close to school communities in comparison with lower (b, y, k) students.

c- $(T_j^C - T_j^{C+1}) = 0$; high (b, y, k) students live at the same distance from school in comparison with lower (b, y, k) students.

Proof of Proposition 2:

From property 1, we know that the indifference curve of individual (b, y, k) has a steeper slope than (b_1, y_1, k_1) , since $(b, y, k) < (b_1, y_1, k_1)$. From lemma 3, we know that he spends less on rental, schooling and transportation combined and he chooses lower quality schools.

Similarly, the indifference curve of individual $(\hat{b}, \hat{y}, \hat{k})$ has a flatterer slope than $(\hat{b}, \hat{y}, \hat{k})$, since $(\hat{b}, \hat{y}, \hat{k}) < (b, y, k)$. From lemma 3, we know that he spends more on rental, schooling and transportation combined and he chooses higher quality schools.

By virtue of Lemma 3 and figure 2, we can write.

$$r^{C-1} + p_{ij''} + T_j^{C-1} > r^C + p_{ij} + T_j^C > r^{C+1} + p_{ij'} + T_j^{C+1} \text{ and,}$$

$$q_{j''} > q_j > q_{j'}$$

Thus individual (b, y, k) satisfying $(\hat{b}, \hat{y}, \hat{k}) < (b, y, k) < (b_1, y_1, k_1)$ must strictly prefer the combination (C, j) , with $[q_j, (r^C + p_{ij} + T_j^C)]$.

Proof of lemma 4:

(i) The proof is by contradiction.

Consider two individuals 1 and 2 with $(b_1, y_1, k_1) > (b_2, y_2, k_2)$; and three schools a, b, and c; a and b are two private schools with $q_a > q_b$ (they may be inside or outside community C), c is the highest price (highest quality) mixed finance school in community C.¹⁸

We suppose $\hat{t}_1^C < \hat{t}_2^C$ to be true.

For individual 1; \hat{t}_1^C is his indifference tax rate between a and c. we can write:

$$W_{1a}^C[(b_1, y_1, k_1), T_a^C, \hat{t}_1^C, p_{1a}, r^C, H_1, q_a] = W_{1c}^C[(b_1, y_1, k_1), T_c^C, \hat{t}_1^C, p_{1c}, r^C, H_1, q_c] \quad (a)$$

For individual 2; \hat{t}_2^C is the indifference tax rate between b and c. we can write:

$$W_{2b}^C[(b_2, y_2, k_2), T_b^C, \hat{t}_2^C, p_{2b}, r^C, H_2, q_b] = W_{2c}^C[(b_2, y_2, k_2), T_c^C, \hat{t}_2^C, p_{2c}, r^C, H_2, q_c]$$

By virtue of figure 3 we know that for any $t < \hat{t}_2^C$. Individual 2 prefers school b.

$$W_{2b}^C[(b_2, y_2, k_2), T_b^C, t, p_{2b}, r^C, H_2, q_b] > W_{2c}^C[(b_2, y_2, k_2), T_c^C, t, p_{2c}, r^C, H_2, q_c]$$

Since we supposed that $\hat{t}_1^C < \hat{t}_2^C$, we can write:

$$W_{2b}^C[(b_2, y_2, k_2), T_b^C, \hat{t}_1^C, p_{2b}, r^C, H_2, q_b] > W_{2c}^C[(b_2, y_2, k_2), T_c^C, \hat{t}_1^C, p_{2c}, r^C, H_2, q_c] \quad (b)$$

Since $(b_1, y_1, k_1) > (b_2, y_2, k_2)$, and by virtue condition 4 we have:

$$W_{1a}^C[(b_1, y_1, k_1), T_a^C, \hat{t}_1^C, p_{1a}, r^C, H_1, q_a] > W_{2b}^C[(b_2, y_2, k_2), T_b^C, \hat{t}_1^C, p_{2b}, r^C, H_2, q_b].^{19} \quad (c)$$

From (a), (b) and (c), we can write:

$$W_{1c}^C[(b_1, y_1, k_1), T_c^C, \hat{t}_1^C, p_{1c}, r^C, H_1, q_c] > W_{2c}^C[(b_2, y_2, k_2), T_c^C, \hat{t}_1^C, p_{2c}, r^C, H_2, q_c]. \quad (d)$$

We know from condition 4 that if $(b_1, y_1, k_1) > (b_2, y_2, k_2)$ is true then:

$$W_{1c}^C[(b_1, y_1, k_1), T_c^C, \hat{t}_1^C, p_{1c}, r^C, H_1, q_c] < W_{2c}^C[(b_2, y_2, k_2), T_c^C, \hat{t}_1^C, p_{2c}, r^C, H_2, q_c].^2$$

This is a contradiction with (d); our assumption that $\hat{t}_1^C < \hat{t}_2^C$ is false. This implies that the reverse is true: $\hat{t}_1^C > \hat{t}_2^C$. Thus the indifference tax rate is increasing in the combination (b, y, k) .

¹⁸ Since c is in community C, then $T_c^C = 0$, while T_a^C and T_b^C may be different than zero.

¹⁹ Higher (b, y, k) achieve higher utilities when they have access to higher educational qualities and lower utilities when they are enrolled in the same school and live in the same community as lower (b, y, k) individuals; this is assumed to be true in condition 4.

(ii) Consider two individuals m and 1 with $(b, y, k)_m$ and (b_1, y_1, k_1) respectively; and three schools $a, b,$ and c ; a and b are two private schools or schools outside community C , c is a public school.

Part 1:

For $(b, y, k)_m > (b_1, y_1, k_1)$ and $q_a > q_b$.

By virtue, we can write:

$$W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] < W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c].^{20}$$

$$W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a] > W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b].^{21}$$

And if $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$ is true

We can write:

$$W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b] < W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a] < W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] < W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c]$$

Thus:

$$W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b] < W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c]$$

Part 2:

For $(b, y, k)_m < (b_1, y_1, k_1)$ and $q_a < q_b$.

From part a of assumption 2, we can write:

$$W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c].$$

$$W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a] < W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b].$$

And if $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] < W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$ is true

We can write:

$$W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b] > W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a] > W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c]$$

Thus:

$$W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b] > W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c]$$

²⁰ m and 1 are in the same public school in the same community.

²¹ m and 1 are in two different private schools.

Proof of Lemma 5:

(i) For $t \in [t_m^C, 1] \Rightarrow t > t_m^C$.

Consider an individual denoted 1 with, $(b_1, y_1, k_1) > (b, y, k)_m$.

From the third equilibrium condition, we know that for $(b_1, y_1, k_1) > (b, y, k)_m$ we have $t_1^C < t_m^C$.

Two situation shall be analyzed:

1- For $\hat{t}_1^C < t_1^C$.

From figure 3, we know that this individual is enrolled in a public school for t_1^C . We can write: $\hat{t}_1^C < t_1^C < t_m^C < t$. Since indirect utility is decreasing in the tax rate for all individuals with $(b_i, y_i, k_i) > (b, y, k)_m$ over the interval $[t_m^C, 1]$; individual 1 prefers the lowest tax rate over the interval which is t_m^C .

2- For $\hat{t}_1^C > t_1^C$.

From figure 3, we know that this individual is enrolled in a private school for t_1^C . Since indirect utility (in a private school) is decreasing in the tax rate for all individuals in the economy; individual 1 prefers the lowest tax rate over the interval which is t_m^C .²²

Over the interval $[t_m^C, 1]$, there does not exist a tax rate different than t_m^C that is preferred by more than 50% of the population.²³

(ii) For $t \in [\hat{t}_m^C, t_m^C] \Rightarrow t < t_m^C$, consider an individual with $(b_1, y_1, k_1) < (b, y, k)_m$.

As indirect utility is increasing over this interval, individual 1 is going to choose the highest attainable tax rate over the interval, thus he chooses t_m^C .

Over the interval $[\hat{t}_m^C, t_m^C]$, there does not exist a tax rate different than t_m^C that is preferred by more than 50% of the population.²⁴

²² Individuals enrolled in a private school prefer a zero tax rate since they do not use public or mixed finance schools.

²³ The second and third equilibrium conditions are essential to the proof.

²⁴ The third equilibrium condition is essential to the proof.

Proof of proposition 3:

We have to demonstrate that over the interval $[0, \hat{t}_m^C]$, there does not exist a tax rate different than t_m^C that is preferred by more than 50% of the population. For an individual 1 with $(b_1, y_1, k_1) < (b, y, k)_m$ we have two situations to analyze corresponding to the intervals $[0, \hat{t}_1^C]$ and $[\hat{t}_1^C, \hat{t}_m^C]$.

1- Over the interval $[0, \hat{t}_1^C]$.

As indirect utility is decreasing over the interval $[0, \hat{t}_1^C]$, individual 1 prefers a zero tax rate over any other tax rate t on the interval.

$$W[(b_1, y_1, k_1), T_b^C, 0, p_{mb}, r^C, H_m, q_b] > W[(b_1, y_1, k_1), T_b^C, t, p_{mb}, r^C, H_m, q_b]$$

In Lemma 4 we have provided the proof that if $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$ is true, all individuals with $(b_i, y_i, k_i) < (b, y, k)_m$ have the same behavior, such as:

$$W_{ic}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{ic}, r^C, H_1, q_c] > W_{ib}^C[(b_1, y_1, k_1), T_b^C, 0, p_{ib}, r^C, H_1, q_b]$$

Thus, individual 1 prefers the tax rate t_m^C over the interval $[0, \hat{t}_1^C]$.

2- Over the interval $[\hat{t}_1^C, \hat{t}_m^C]$.

Indirect utility is increasing in the tax rate over this interval; t_m^C is higher than any other tax rate $t \in [\hat{t}_1^C, \hat{t}_m^C]$, thus individual 1 prefers t_m^C over any other tax rate.

By Lemma 4 and proposition 2, we find that there does not exist a tax rate different than t_m^C that is preferred by more than 50% of the population. t_m^C is a majority voting equilibrium.

References

Calabrese S, Epple D, Romer T, and Sieg H. 2006. Local public good provision: Voting, peer effects, and mobility. *Journal of Public Economics* 90 (959-981).

Clotfelter C. 1976. School desegregation, Tipping and private school enrollment. *The Journal of human resources* Vol 11 No 1 (28-50).

Benabou R 1996. Heterogeneity, Stratification, and Growth: Macroeconomic implications of community structure and school finance. *The American economic Review*, Vol 86, N3 (584-609).

Berg Dale S, and Krueger A, 1999. Estimating the Payoff to Attending a more Selective College: an Application of Selection on Observable and Unobservable Variables. NBER, Working Paper 7322.

Dertouzos M, Lester R, and Solow R 1989. *Made in America: regaining the productive edge*, report of the MIT commission on productivity. Cambridge, MA: MIT press.

Epple D, Filimon R, and Romer T. 1984. Equilibrium among local jurisdictions: toward an integrated treatment of voting and residential choice. *Journal of Public Economics* 24 (281-304).

Epple D, Filimon R, and Romer T. 1993. Existence of voting and housing equilibrium in a system of communities with property taxes. *Regional Science and Urban Economics* 23 (585-610).

Epple D, and Platt G. 1998. Equilibrium and local redistribution in an urban economy when households differ in both preferences and income. *Journal of Urban Economics* 43 (23-51).

Epple D, and Romano R, 1996. Ends against the middle: Determining Public service provision when there are private alternatives. *Journal of Public Economics*, Vol 62 (297-325).

Epple D, and Romano R, 1998. Competition between public and private schools, Vouchers, and peer group effects. *The American Economic Review* Vol 88 No 1 (33-62).

Epple D, and Romano R, 2002. Educational Vouchers and Cream Skimming. NBER, Working Paper 9354.

Epple D, and Romano R, 2006. Admission, Tuition, and financial aid policies in the market for higher education. *Econometrica*, Vol 74, No4 (885-928).

Fernandez R, and Rogerson R, 1996. Income distribution, communities, and the quality of public education. *The Quarterly Journal of Economics*, Vol 111, No1 (135-164).

Ferris J. S, West E. G, 2004. Economies of scale, school violence and the optimal size of schools. *Applied Economics*, Vol 36 (1677-1684).

- Glomm G, and Ravikumar B, 1998. Opting out of publicly provided services: A majority voting Equilibrium. *Social Choice and Welfare* Vol 15 (187-199).
- Green A, Preston J, and Janmatt J. Comprehensive schooling and educational inequality in "Education, Equality and Social Cohesion". Palgrave Macmillan.
- Henderson V, Mieszkowski P, and Sauvageau Y, 1978. Peer group effects in educational production functions. *Journal of Public Economics* Vol 10 No 1 (97-106).
- Hoxby C, Weingarth G 2005. Taking race out of the equation: School reassignment and the structure of peer effect. Working paper, Harvard University.
- Kenny L. 1982. Economies of scale in schooling. *Economics of Education Review* Vol 2 No1 (1-24).
- Kulik J, and Kulik C, 1992. Effects of ability groupings on secondary school students: A Meta analysis of evaluation findings. *American Education Research Journal* Vol 19 No 3 (415-428).
- Scotchmer S, and Wooders M, 1987. Competitive equilibrium and the core in club economics with non anonymous crowding. *Journal of Public Economics*, Vol34 No 2. (159-173).
- Sonstelie J, 1979. Public school quality and private school enrollments. *National Tax Journal* (343-353).
- Sorensen A, and Hallinan M, 1986. Effects of ability grouping on growth in academic achievement. *American Educational Research Journal* Vol 24 No 4 (519-542).
- Stiglitz J. 1974. The demand for education in public and private school systems. *Journal of public economics* Vol 3 No 4 (349-385).
- Summers A, and Wolfe B, 1977. Do schools make a difference? *American Economic Review* Vol 67 No 4 (639-652).