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Monopolization through acquisitions in a differentiated product industry

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Abstract. This article analyzes the incentive to merge in a context of price competition with horizontal product differentiation. In contrast to the results obtained by Kamien and Zang (1990), we show that merged equilibria can appear in this game. Moreover monopolization of the industry occurs with a high number of firms.

Keywords: Mergers; Oligopoly; Cooperative game

JEL classification: L10; L11; L20

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1 Introduction

In a famous paper, Salant, Switzer and Reynolds (1983) have reported that in a Cournot oligopoly with homogeneous products, linear demand and cost functions, a merger is beneficial for participating firms if more than 80 per cent of all firms merge. It is because outsiders are more beneficial than the firms participating to the merger, the "insiders". Since production costs are linear, any coalition of firms is indifferent with respect to the way of splitting its total production among the members of the coalition, so, every coalition of firms behaves as if it were a single firm.

Perry and Porter (1985) but also Farrell and Shapiro (1990) have challenged the view that a merged firm is no larger than any of the constituent firms. These studies introduce the existence of some crucial assets that are in limited supply in order to capture the notion that some firms are larger than others in a homogeneous product industry. This assumption implies rising marginal cost of output production and, consequently, internal cost savings from mergers could make a merger profitable.

Deneckere and Davidson (1985) have found an opposite result in the case of price competition with differentiated products where a merger is always beneficial for the insiders.

Kamien and Zang (1990) have explored the possibility of endogenous monopolization of a homogeneous good Cournot oligopoly through one firm's acquisition of the others. They adopt two different approaches: firstly, an analysis of a centralized game: an owner who acquired several firms behaves as one entity (as in SSR, 1983). Secondly, they explore the possibility for an owner, possessing several firms, to choose the optimal number of active firms, each of them competing with each other: it is the decentralized game. More precisely, in this kind of game, they emphasize that an owner, possessing several firms, chooses to operate more than one firm. They disclose that, for the two kinds of game, monopolization can only occur in industries composed

ex-ante of a small number of firms. Moreover, in the centralized game (SSR context (1983)) with a large number of firms, merged equilibria (that means the number of active firms is fewer than the initial number of firms) are non-existent which reinforce the SSR's results (1983).

In Kamien and Zang (1993), a sequential noncooperative game in which the centralized game is played over and over is considered. They show that monopolization of the industry is impossible if the initial number of firms is relatively high.

A more recent literature takes into account strategic delegation (Gonzalez-Maestre and Lopez-Cunat (2001) or Ziss (2001)) to study merger profitability. What differs to the decentralized game in Kamien and Zang (1990) is the two types of competition: in the production and in the remuneration of managers. The delegation increases competition between entities inside the firm. Consequently, the incentives to merge and the profitability of merger, under delegation, are considerably increased with respect to the setting without delegation. Ziss (2001) argues that a merger will result in the merged entity operating only one firm.

Commitment through delegation may be limited by the possibility to renegotiate delegation contract in the absence of a strong enforcing institutional setting avoiding false disclosure and private renegotiation. Precommitment effects seem to rest on the crucial assumption that contracts, once publicly disclosed, cannot be secretly renegotiated. But this is at odds with reality: whether legally enforceable or of a more implicit nature, actual contracts can almost always be renegotiated if both parties agree (Caillaud, Jullien and Picard, 1995). In the same way, in the decentralized game of Kamien and Zang (1990), internal competition is not credible because if contracts were renegotiable ex-post, firms may act cooperatively. In our model, we consider that firms belonging to the same owner play in a cooperative way.

Our purpose is to analyse the incentives to merge in a context of price competition with horizontal product differentiation. As in Kamien and Zang (1990), a three-stage game is considered.

We study merger profitability by assuming that an owner of several firms chooses the number of firms he lets active. This optimal number is endogenously determined to maximize merger profit. The number of active firms plays a major role: since products are horizontally differentiated, demand increases with the number of active firms so a merger can gain market shares, but equilibrium price is lower. Active firms create internal competition but reinforces competition with the other firms in a same time.

We consider market structure with a high number of firms. We show that merged equilibria can appear in this game. Moreover monopolization of the industry occurs with this high number of firms. This is in contrast to the results obtained by Kamien and Zang (1990) who demonstrated that no merged equilibrium exists in a centralized game and that monopolization never occurs for a sufficiently large number of firms in their games (neither centralized nor decentralized).

The article is organized as follows. Section 2 presents the model. Characterizations of equilibrium are provided in section 3. Concluding remarks follow. Proofs of results appear in the appendix.

2 The model

We consider the following utility function derived from Häckner (2000):

$$U(\mathbf{q}, I) = \sum_{i=1}^{n} q_i - \frac{1}{2} \left[\sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{i \neq j} (q_i q_j) \right] + I$$
 (1)

The parameter $\gamma \in [0,1]$ measures the substitutability between the products. Utility is quadratic in the consumption of the n horizontally differentiated products and linear in the consumption of others goods: I, which

price is normalized to one.

Demand function is:

$$q_i(p_i, p_j, n) = \frac{1}{1 + \gamma(n-1)} \left[1 - \frac{1 + \gamma(n-2)}{1 - \gamma} p_i + \frac{\gamma}{1 - \gamma} \sum_{j \neq i} p_j \right]$$
(2)

We assume that entry into the industry is difficult and that each producer operates at a constant and identical marginal and average cost of c. Without loss of generality, we assume that c = 0. All the relevant variables and strategies available to the firms are common knowledge.

We posit an initial industry consisting of n=16 identical and independent firms.

Let us now turn to the formal description of our three-stage game.

• Stage 1: bidding stage.

One or two firms make offers simultaneously to other firms and each firm sets a ceasing price for its own firm. Each shareholder computes his willingness to pay for one, two, three or more firms.

Let K_j be the number of firms owned by a merger M_j and Z the number of outsiders which have not been bought.

A firm 'j' is sold to 'i' if the bid of 'i' is not smaller than the asking price and if it is the highest bid for this firm, it is sold to the willingness to pay of the buyer.

The allocation of firm 'i' is independent of the asking prices and bids received by every other firm. In the presence of tie, firm 'i' is not sold. A market structure is a Nash equilibrium in this subgame if no firm is able to purchase one or several firms and the others accept.

We authorize successive bidding by one or two firms until the equilibrium is reached.

• Stage 2: merger stage.

Each owner decides the number of his active firms.

Let k_j $(0 \le k_j \le K_j)$ be the number of active firms owned by M_j . A SPNE (Subgame Perfect Nash Equilibrium) in an acquisition game is said to be merged if the number of firms operated by all owners is fewer than the initial number of firms.

• Stage 3: competition stage.

Firms belonging to the same owner act cooperatively amongst one another but face competition with each other. The active firms in mergers and the outside firms compete in price.

We characterize pure strategy subgame perfect Nash equilibria of this game.

3 Analysis of equilibria

This section characterizes the set of equilibria.

proposition 1. At equilibrium, monopolization of the industry occurs.

We prove this result and exhibit two trajectories which conduct to the monopolization equilibrium. The first allows two firms to make bids on the others. The second allows only one firm to bid on the others.

3.1 Indirect monopolization

In this section, we consider the case where two mergers denoted M_1 and M_2 can buy firms. Each of them operates respectively k_1 and k_2 units. There are Z outside firms.

3.1.1 Equilibrium prices

 M_1 , M_2 and the outside firms simultaneously choose the price of each of their firm seeking to maximize their profit.

lemma 1. Equilibrium prices of the two mergers $(p_1^* \text{ and } p_2^*)$ and the outsiders (p^*) are given by:

$$\begin{cases} p_1^* = \frac{(1-\gamma)(2+2(Z-1)\gamma+2\gamma k_1+\gamma k_2)(2+(2Z-3)\gamma+2\gamma(k_1+k_2))}{A} \\ p_2^* = \frac{(1-\gamma)(2+(2Z-3)\gamma+2\gamma(k_1+k_2))(2+2(Z-1)\gamma+\gamma(k_1+2k_2))}{A} \\ p^* = \frac{(1-\gamma)(2+2(Z-1)\gamma+2\gamma k_1+\gamma k_2)(2+2(Z-1)\gamma+\gamma(k_1+2k_2))}{A} \end{cases}$$

with

$$A = 2\gamma^{2}k_{1}^{2}(4 - 4\gamma + 3Z\gamma + 3\gamma k_{2}) + 2(1 + (Z - 1)\gamma + \gamma k_{2})(2(2 + (Z - 3)\gamma)(1 + (Z - 1)\gamma) + \gamma(4 - 4\gamma + 3Z\gamma)k_{2}) + \gamma k_{1}(2(8 + 5(Z - 2)\gamma)(1 + (Z - 1)\gamma) + \gamma k_{2}(22 - 25\gamma + 17Z\gamma + 6\gamma k_{2})).$$

We check that A > 0.

Equilibrium profit of the merger M_1 is given by:

$$\pi^{M_1} = \frac{1}{(1 + \gamma(k_1 + k_2 + Z - 1))A^2} (1 - \gamma)k_1(1 + (Z - 1)\gamma + \gamma k_2)$$
$$(2 + 2(Z - 1)\gamma + 2\gamma k_1 + \gamma k_2)^2 (2 + (2Z - 3)\gamma + 2\gamma(k_1 + k_2))^2 \quad (3)$$

The expression for merger M_2 is symmetric.

3.1.2 Merger phase

We now determine if an owner of several firms will choose to close some of them or to keep all of them active.

proposition 2.

- If $Z \ge 2$ then merged equilibria can not occur in this game.
- If Z<2 the game can result in merged equilibria depending on the products substitutability. Merged equilibria can appear when the products are not too much differentiated.

Proof. See Appendix B.

The existence of merged equilibria depends on the number of outsiders. Precisely, the presence of outsiders increases the competitive pressure, so when the number of outsiders is high enough $(Z \ge 2)$, an owner of several firms will not close some of them to maintain its market power.

In the following of the paper, we assume $\gamma = 0.9$ in order to study all the different cases (merged or unmerged equilibria).

The objective now is to analyse if the number of firms owned by the mergers influences the number of firms they will let active.

Let us define the reaction function of the merger M_1 : $k_1^*(k_2)$.

lemma 2. The reaction function of the merger M_1 is:

$$\mathbf{k_1^*}(\mathbf{k_2}, \mathbf{Z}) = \begin{cases} K_1 & if \ Z > 1, \ \forall k_2 \\ f(k_2) < K_1 & if \ Z = 1 \ and \ k_2 \ge 8 \\ K_1 & if \ Z = 1 \ and \ k_2 < 8 \\ g(k_2) & if \ Z = 0 \end{cases}$$

The proof is obtained by numerical simulations. Appendix C gives the exact values of functions f and g as well as values of profit functions.

The reaction function of the merger K_2 is a symmetrical function of $k_1^*(k_2)$.

We observe that $k_1^*(k_2)$ is a decreasing function, so k_1 and k_2 are strategic substitutes.

3.1.3 Bidding stage

We suppose initially that each of the two shareholders M_1 and M_2 owns one firm and can not sell it; only the outside firms can be sold but only in their entirety. Each of the two shareholders simultaneously sets a vector of bids facing the number of firms owned by the other, that means, facing K_2 firms, the first shareholder has to decide how many firms to purchase.

After a deal, each shareholder owns one or more firms $(K_j \in (1,15), Z \in$

(0,14)).

Transactions occur in this acquisition stage until no additional purchase by one of the first two firms can be realized. The equilibrium condition is then defined by:

$$WTP_{M_i}(K_1, K_2, Z) < WTS_{out}^{M_i}(K_1, K_2, Z), \forall i = 1, 2.$$

 $WTP_{M_i}(K_1, K_2, Z)$ is the maximum price (Willingness To Pay) M_i is willing to pay for buying $K_i - 1$ firms, facing the other merger owning K_j firms. It is defined by:

$$WTP_{M_i}(K_i, K_i, Z) = \pi_{M_i}(K_i, K_i, Z) - \pi_{M_i}(1, K_i, Z + K_i - 1)$$
(4)

 $WTS_{out}^{M_i}(K_1,K_2,Z) \ \forall i=1,2$ is the total minimum selling price (Willingness To Sell) of outsiders towards the merger M_i and is expressed as .

$$WTS_{out}^{M_i}(K_i, K_j, Z) = \pi_{out}(K_i - 1, K_j, Z + 1) * nbr of firms bought (5)$$

The selling price of an outsider firm depends on the number of firms the two owners want to buy. More precisely, when the outsider sets its selling price, he forestalls his profit in the last stage if it declines the offer considering all the other firms the owner wants to buy have accepted and considering that the owner which buy others can close some of them after.

lemma 3.

- Mergers M_1 and M_2 buy all the outside firms so as to get: $K_1+K_2=16$ (Z=0).
- Since $K_1+K_2=16$ only market structures wherein one owner lets all his firms active and the other closes some of his firms can occur.

However, these structures are not Nash equilibria because the owners of M_1 and M_2 can benefit from purchasing themselves.

Facing K_2 , we compute the willingness to pay of M_1 to purchase M_2 in its entirety.

$$WTP_{M_1}^{2^{nd}}(K_1, K_2, 0) = \pi_{M_1}(16, 0, 0) - \pi_{M_1}(K_1, K_2, 0)$$
(6)

where $WTP_{M_1}^{2^{nd}}(K_1, K_2, 0)$ is the willingness to pay of M_1 and $\pi_{M_1}(K_1, K_2, 0)$ designs the duopoly profit of M_1 .

 M_2 is willing to sell its merger at any price above its profit : $\pi_{M_2}(K_1, K_2, 0)$. M_1 will purchase M_2 if :

$$WTP_{M_1}^{2^{nd}}(K_1, K_2, 0) > \pi_{M_2}(K_1, K_2, 0)$$
 (7)

We compute this for all the different values of K_2 (with $K_1 + K_2 = 16$) and we obtain the following proposition:

proposition 3. Monopolization of the industry:

- At equilibrium, only monopolization of the industry can occur.
- The only possible equilibria in this game consist of unmerged equilibria wherein one owner possessing all firms will let all of them active.

Proof. The first part is obtained by numerical simulations. The second is straightforward by considering: $\frac{\partial \pi^{M_1}}{\partial k_1}(16,0,0) > 0 \Rightarrow k_1^* = K_1$.

3.2 Direct monopolization

In this section, we exhibit a second trajectory in which only firm M_1 makes bids in order to purchase other firms. The game is solved as previously.

proposition 4.

• The merger M_1 will buy all the outside firms so as to monopolize the market.

• If the market results in a monopolization, then the owner will let active all his firms.

The first part of the proposition is verified comparing the willingness to pay of M_1 with the willingness to sell of all the outside firms. Proof of the second part is the same than for the previous proposition.

4 Concluding Remarks

We have considered the possibility of monopolization through acquisition in an industry composed initially of a high number of firms. We have modeled this in a three-stage game. In this model, firms compete in price in a differentiated product industry. Moreover, we assume that firms belonging to the same owner act cooperatively.

Our main conclusion is that, in contrast to the results obtained by Kamien and Zang (1990), merged equilibria can appear in this game. Moreover monopolization of the industry occurs with a high number of firms.

Our model can be extended to the case of coalitions structures in which a coalition, maximizing its joint payoff given the outside choice, decides the number of active firms which compete, but the other firms are not closed, they still exist but not compete (for example, they receive an allowance from the active firms).

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I would like to thank Laurent Flochel for encouragement and helpful discussions.

Appendix A. Proof of lemma 1

The maximisation program of a merger M_1 with k_1 active firms is :

$$\max_{p_1^{M_1}, p_2^{M_1}, \dots, p_{k_1}^{M_1}} (\pi^{M_1}) \tag{8}$$

where

$$\pi^{M_1} = \sum_{i=1}^{k_1} (p_i^{M_1} - c) \frac{1}{1 + \gamma(k_1 + k_2 + Z - 1)} *$$

$$\left[1 - \frac{1 + \gamma(k_1 + k_2 + Z - 2)}{1 - \gamma} p_i^{M_1} + \frac{\gamma}{1 - \gamma} (\sum_{i \neq j} p_j^{M_1} + \sum_{j \in M_2} p_j + \sum_{j \in out} p_j \right]$$
(9)

We design by $j \in out$ the firms which are outsiders.

We obtain k_1 First Order Conditions (FOC) which are symmetric, so $p_i^{M_1} = p^{M_1}, \forall i \in M_1$. After simplifications, we obtain the best response functions:

$$p^{M_1}(p^{-M_1}) = \frac{1 - \gamma + \gamma \sum_{i \notin M_1} (p_i)}{2[1 + \gamma(k_2 + Z - 1)]} + \frac{c}{2}$$
(10)

The best-reply function is symmetric for the merger M_2 .

The maximisation program of an outside firm is:

$$\max_{p_{i}}(p_{i}-c)\frac{1}{1+\gamma(\sum_{i=1}^{2}k_{i}+Z-1)}*
\left[1-\frac{1+\gamma(\sum_{i=1}^{2}k_{i}+Z-2)}{1-\gamma}p_{i}+\frac{\gamma}{1-\gamma}(\sum_{j\in M_{1}}p_{j}+\sum_{j\in M_{2}}p_{j}+\sum_{j\neq iand j\in out}p_{j})\right]\right]$$

As before, prices of outsiders are equal, we then replace, p_i^* by p^{out} for all $i \notin (M_1, M_2)$. We obtain:

$$p^{out}(p^{-out}) = \frac{1 - \gamma + \gamma(\sum_{j \in M_1} p_j + \sum_{j \in M_2} p_j + c[1 + \gamma(\sum_{i=1}^2 k_i + Z - 2)]}{2[1 + \gamma(\sum_{i=1}^2 k_i + Z - 2)] - \gamma(Z - 1)}$$
(12)

In order to simplify, we replace

$$(p^{M_1}(p^{-M_1}), p^{M_2}(p^{-M_2}), p^{out}(p^{-out})) (13)$$

by

$$(p_1, p_2, p). (14)$$

The intersection of best response functions yields to :

$$\begin{cases} p_1^* = & \frac{(1-\gamma)+\gamma(k_2p_2+Zp)}{2[1+\gamma(k_2+Z-1)]} \\ p_2^* = & \frac{(1-\gamma)+\gamma(k_1p_1+Zp)}{2[1+\gamma(k_1+Z-1)]} \\ p^* = & \frac{(1-\gamma)+\gamma(k_1p_1+k_2p_2)}{2[1+\gamma(k_1+k_2+Z-2)]-\gamma(Z-1)} \end{cases}$$

Appendix B. Proof of proposition 2

Numerical simulations give the number of active firms in the merger M_2 in order having $k_1^* < K_1$.

	$\gamma = 0.5$	$\gamma = 0.9$
Z=	$k2 \ge$	$k2 \ge$
0	11	1
1	17	8
2	23	14
3	30	20
4	36	26
5	42	33

We read this table like this for example:"for Z=3 and $\gamma=0.9,\ k_2$ must be higher than 20 to π^{M_1} have an interior maximum $(k_1^* < K_1)$ ".

Note that for $\gamma = 0.1, k_2$ must be very high to merger M_1 have a maximum.

Appendix C. Tables

Table 1

Reaction functions of the two mergers for Z = 1:

K_1	K_2	k_1	k_2	π^{M_1}	π^{M_2}
1	14	1	14	0.004226	0.024267
2	13	2	13	0.004656	0.016495
3	12	2.19	12	0.004818	0.015507
4	11	2.36	11	0.005013	0.014701
5	10	2.61	10	0.005255	0.013668
6	9	3.02	9	0.005562	0.012277
7	8	3.8	8	0.005965	0.010335
8	7	8	3.8	0.010335	0.005965
9	6	9	3.02	0.012277	0.005562
10	5	10	2.61	0.013668	0.005255
11	4	11	2.36	0.014701	0.005013
12	3	12	2.19	0.015507	0.004818
13	2	13	2	0.016495	0.004656
14	1	14	1	0.024267	0.004226

Table 2 Reaction functions of the two mergers for Z=0:

K_1	K_2	k_1	k_2	k_1	k_2	
1	15	Ø	0	0.156969	15	
2	14	2	0.247657	0.157632	14	
3	13	3	0.203337	0.158403	13	
4	12	4	0.186461	0.159311	12	
5	11	5	0.177512	0.160394	11	
6	10	6	0.171958	0.161711	10	
7	9	7	0.168174	0.163346	9	
8	8	8	0.165428	0.165428	8	
9	7	9	0.163346	0.168174	7	
10	6	10	0.161711	0.171958	6	
11	5	11	0.160394	0.177512	5	
12	4	12	0.159311	0.186461	4	
13	3	13	0.158403	0.203337	3	
14	2	14	0.157632	0.247657	2	
15	1	15	0.156969	\oslash	0	

For Z=0 and $K_i>1, \forall i=1,2$, two cases are possible for each structure (K_1,K_2) .

Table 3 Best-response functions $(k_1^*(K_1, \mathbb{Z}))$:

SI	-response functions $(\kappa_1(\mathbf{\Lambda}_1,\mathbf{Z}))$.													
		Z=0		Z=1	Z=2	Z=3	Z:	=4	Z=5	Z=6	Z=7	Z=8		
	K_1	$k_1 =$		$k_1 =$	$k_1 =$	$k_1 =$	k_1	_=	$k_1 =$	$k_1 =$	$k_1 =$	$k_1 =$		
	1	0.1569	969	1	1	1	1		1	1	1	1		
	2	(0.157)	(632;2)	2	2	2	2		2	2	2	2		
	3	(0.158)	3403;3)	2.19	3	3	3		3	3	3	3		
	4	(0.159)	311;4)	2.36	4	4	4		4	4	4	4		
	5	(0.160	394;5)	2.61	5	5	5		5	5	5	5		
	6	(0.161	711;6)	3.02	6	6	6		6	6	6	6		
	7	(0.163	346;7)	3.8	7	7	7		7	7	7	7		
	8	(0.165428;8) 8 8 8 8		8	8	8	0							
	9	(0.168	3174;9)	9	9	9	9		9	9	0	0		
	10	(0.171	958;10)	10	10	10	10)	10	0	0	0		
	11	(0.177)	7512;11)	11	11	11	11	L	\oslash	0	0	0		
	12	(0.186	5461;12)	12	12	12	Ø		\oslash	0	0	0		
	13	(0.203	337;13)	13	13	\oslash	Ø		\oslash	0	0	0		
	14	(0.247)	(657;14)	14	0	0	0		Ø		\oslash	0	0	0
	15	15		0	0	0	Ø		\oslash	0	0	0		
		Z=9	Z=10	Z=11	Z=12	2 Z=1	.3	Z=	:14					
	K_1	$k_1 =$	$k_1 =$	$k_1 =$	$k_1 =$	$k_1 =$		k_1	=					
	1	1	1	1	1	1		1						

	Z=9	Z=10	Z=11	Z=12	Z=13	Z=14
K_1	$k_1 =$					
1	1	1	1	1	1	1
2	2	2	2	2	2	0
3	3	3	3	3	0	0
4	4	4	4	0	0	0
5	5	5	0	0	0	0
6	6	0	0	0	0	0

Table 4Profit of the merger M_1 function of the number of outsider firms (Z) and the number of firms owned by the merger (K_1) : $K_1 = \begin{array}{|c|c|c|c|c|}\hline Z=0 & Z=1 \\\hline \end{array}$

$K_1 =$	Z=0		Z=1	Z=1 Z=2		Z=3				
1	0.024136			0.00229277		29277	0.0015	3169		
2	(0.0242078,0.11683)			0.004656 0.00310011		310011	0.0023	2071		
3	(0.0242909,0	.132968)	0.004818		0.00355813		0.00286077			
4	(0.0243881,0	.140544)	0.00393371		0.00332096		0.00291471			
5	(0.0245034,0	.144974)	0.00	5255	0.004	31892	0.00377761			
6	(0.0246423,0	.147886)	0.00	5562	0.004	76051	0.0042	7738		
7	(0.024813,0.1	49947)	0.00	5965	0.005	29569	0.0048	6037		
8	(0.0250277,0	.151484)	0.01	.0335	0.005	9674	0.0055	7287		
9	(0.025306,0.1	.52673)	0.01	2277	0.006	83702	0.0064	7999		
10	(0.025681,0.1	53622)	0.01	.3668	0.008	800361	0.0076	859		
11	(0.0262137,0	.154396)	0.01	4701	0.009	64303	0.0093	7582		
12	(0.0270303,0	.155039)	0.01	.5507	0.012	21038	0.0119	203		
13	(0.0284404,0	.155582)	0.01	6495	0.016	1899	\oslash			
14	(0.0314653,0	.156048)	0.02	24267	0		\oslash			
15	0.15645		Ø		0		\oslash			
$K_1=$	Z=4	Z=5		Z=6		Z=7		Z=8		
1	0.00114897	0.000927	7698	0.00074	43159 0.0006		394726 0.000		0630024	
2	0.00187127	0.001581	1	0.00139	0.0013		26817 0.003		17559	
3	0.00242304	0.002131	59	0.00193	0.0017		78781 0.003		68679	
4	0.00291471	0.002634	168	0.00243	3739 0.00229		9754 0.002		219966	
5	0.00340615	0.003145	529	0.00296083 0.002		0.0028	3193	0.002	274541	
6	0.00393964	0.003701	62	0.00353569		0.0034	2451	0.003	335683	
7	0.00455497	0.004342	28	0.00420094		0.0041	1473	0.004	107466	
8	0.0053	0.005118	314	0.00500784		0.00495664		0		
9	0.00624307	0.006099	978	0.0060336		0	\oslash		Ø	
10	0.00749451	0.007405	599	0	0		0			
11	0.00925262	\oslash		0		\oslash		\oslash		
$K_1=$	Z=9	Z=10		Z=11		Z=12		Z=13		
1	0.000584244	0.00055	51693	0.000528856		0.000513482		0.000504097		
2	0.0011098	0.00106	6365	0.0010326		0.00101364		0.00	0100481	
3	0.00161599	0.00156	8837	0.0015	3931	0.001	0.00152578			
4	0.00213386	0.00209	374	0.0020	7506	0		\oslash		
5	0.00269268	0.00266	814	0		0		0		
6	0.00332535	0		0		0		\oslash		
$K_1=$	Z=14	İ						,		
1	0.000499723									
where	for Z = 0, t	he first	num	ber des	igns t	he pro	fit of t	he m	erger M	

 $\overline{0}$, the first number designs the profit of the merger M_1 when

 $k_1^* < K_1$ and $k_2^* = K_2$ and for the second number it is the inverse.

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