Contagion Between the Financial Sphere and the Real Economy. Parametric and non Parametric Tools: A Comparison

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1 Introduction

In this chapter we try to participate to the debate concerning the detection of contagion between the business and financial spheres or between different national economies.

To understand how a national economy's evolution influences another country's economy, we are going to investigate the purpose of contagion or interdependence. We will extend it to international financial markets. Indeed, we observe on international markets an increase of volatility whose exact causes are not yet known. On financial markets, an important objective is to reduce volatility and thus contagion. This could be based on a thorough understanding of the causes and consequences of contagion. It seems important to reduce the high degree of volatility in international capital flows and also the high susceptibility of international capital markets to contagion. But, until now, there is not yet an uniform definition of what constitutes contagion. Exact causes of contagion are not known and robust methods to measure it are not yet totally investigated. Thus, the debate is opened.

In order to give some answers or to propose some thinking's tracks on this subject, we need to specify first the notion of contagion. Then, we tackle the different methods that can be used to measure these behaviors and we compare them. The methods proposed here belong to specific domains of statistics, called parametric modelling and non parametric tools. We illustrate our purpose using some real data sets. We discuss also the limitations and the interest of the different tools.

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The chapter is organized as follows: in Section two, we specify the notion of contagion. In Section three, we introduce some models which permit to investigate them. In Section four, non parametric tools are discussed. In Section five, we study the contagion's phenomena between different economies and between the real and financial spheres. We will focus on the copula approach and the switching models. We compare the both methods. Section 6 concludes.

2 Contagion's concept

Before 1997, the term "contagion" usually referred to the spread of a medical issue, and not to turmoil in international financial markets or international economies. In July 1997, a currency crisis in Thailand quickly spread throughout Asia and then to Russia and Brazil, even developed markets in North America and Europe were affected. To understand these phenomena, the first question is to define what is contagion. We propose different "ways of thinking".

1. Contagion is a "disease": recent financial crisis are certainly as devastating as many diseases.

2. Contagion refers to the "transmission" of a disease: as the Thai crisis spreads to other countries in Asia, the understanding why this crisis spreads so quick is as important as understanding the initial event.

3. Contagion can occur through "direct or indirect" contacts: the debate concerns the national and international context; local crisis spreads internationally.

4. The contagion's channels can be through the economies, bilateral trade flows, international investors.

Now the problem is really to understand how a crisis spreads and creates such contagion. We have to understand also why a national economy has such pervasive global ramifications to pollute the international economy, when a crisis appears. In a first approach, we can retain as main causes: common shocks, devaluation, financial links with privilege partner, irrational investor behavior. Indeed, contagion refers to the spread of market disturbances mostly on the downside from one country to another one. The importance of this phenomena is the inter-dependence of the market economies (the shocks are transmitted across countries). Another point comes from the behavior of the investors or financial agents (they do not response to fundamentals or global shocks). This corresponds to actions of specific financial agents in propagating shocks.

The term contagion refers more to the crises' spreads in financial markets than to the crises' spreads between the national economies. In that latter case, inter-dependence or interaction are more commonly used by the authors. They do not refer to something so bad as contagion.

The poor understanding of the transmission of financial crises in the past few years has prompted a range of interest in international financial contagion. The necessity to understand the relative importance of real linkages versus financial contagion is fundamental to reduce financial contagion in the future as importance of the volatility on the markets. Contagion can be defined as a significant increase in cross-market linkages after a shock to an individual country or a group of countries. It can also be defined as a shock on one market or country that is transmitted to another market or country, but generally it is not related to fundamentals. Now, these shocks, in financial markets, can affect real sector GDP for several potential reasons. We retain the following ones. First, the price of assets affects national wealth and hence aggregates demand. Second, the liquidity of financial markets and the price of the assets affect business desires and ability to raise money for investment. This has implications for aggregate demand today, and for aggregate in the future. Thus, the behavior of financial markets influence GDP through their effect on aggregate demand in a country. This is due to the market imperfections and the role of intermediaries.

Here, we argue that a shock can begin with an economy and spread to a set of economies linked to the first one. Looking at the inter-dependence between the real economies, we examine the influence of the "real" shocks to the real sectors of the economy. Looking at the dependence or contagion between the financial sphere and the real economy, we consider the influence of "intermediary-specific" shocks.

In the literature, a lot of papers investigate the different shocks and contagion on financial markets, see for instance Kodres and Pritser (2000), Allen and Cale (2000) and de Bandt and Hartman (2000). Very few concern the measure of inter-dependence between the national economies and the influence of shocks on financial markets and their impact on national aggregate economies.

Most of the works concerning national economies are related to the study of cycles. There are different reasons for taking an interest into the cycles. The evolution of the cycles carries with it an evolution in variables of considerable consequence for policy-makers. A closely related interest has been in the use of business cycle evidence in the context of optimal currency area theory. It represents a positive indicator for monetary union. It permits to explain expansion and recession events in a national economy. The first works come from a group of researchers belonging to the National Bureau of Economic Research (NBER) in New York in the 1940s and the 1950s. These researchers produce a lot of statistics to characterize the business cycle. One of their purpose is to explain how long are the expansion and recession episodes. Until the 1990s, most of the methods proposed to study these cycles and to provide a description of the recession and expansion episodes are based on "graphical" methods, see Burns and Mitchell (1946) and Bry and Boschan (1971). Some non parametric methods have also been proposed, we refer to Kaiser and Maravall (2001) and a spectral analysis is proposed by Croux, Forni and Reichlin (2001). Parametric models have also been used, see Hamilton (1988).

Here, we argue that, studying the degrees of co-movement in crisis periods relative to that in tranquil times, may illustrate the normal dependence of the economies. To describe these crises, we work with a multivariate setting using parametric and nonparametric tools. The non parametric methods include the linear correlation's coefficient, the conditional correlation's coefficient and the τ of Kendall. These measures concern the second order moments of the joint distribution of the data sets that we investigate. We consider also the tail dependence index computed via the copulas. It uses all the information given by the joint distribution of the data sets. The parametric models include models describing existence of several states inside the data. These tools permit to investigate the possible channels of transmission between national economies in period of turmoil as also the contamination between the financial sphere and the national economy. The methods developed in this chapter are complementary with those proposed in Avouyi-Dovi, Guégan and Ladoucette (2002). Here, we base mainly our approach on the use of switching models and on the behavior of the tail dependence of the different factors which characterize the data sets under investigation.

3 Parametric models

Non-linear models have the great advantage to be flexible enough to take into account certain stylized facts of the economic business cycle, such as asymmetries in the phases. They are also interesting to model contagion's phenomena which implies possibility of changes in the series. In this respect, much of attention has concentrated on the class of non-linear dynamic models that accommodate the possibility of regime changes. Two classes of models take into account this kind of behavior. The threshold autoregressive (TAR) model, proposed first by Tong and Lim (1980), which produces limit cycle, time irreversibility and asymmetry behavior. In this model the transition variable is observed: it may be either an exogenous variable, such as a leading index for example, or a linear combination of lagged values of the series. In this latter case, the model is referred to a self-exciting threshold autoregressive (SETAR) model. The Markov-Switching model introduced, first, by Quandt (1958), then reconsidered by Neftçi (1982, 1984) and popularized later in economy by Hamilton in 1988 permits to describe existence of different states inside data. The autoregressive data generating process varies according to the states of a latent Markov chain. These two classes of models are complementary because the notion captured under investigation is not exactly the same. We specify now these two classes of models.

3.1 The SETAR models

The mean stationary process $(Y_t)_t$ follows a SETAR process if it verifies the following equation:

$$
\begin{aligned}
Y_t &= (\phi_{0,1} + \phi_{1,1} Y_{t-1})(1 - I_{[Y_{t-d}>c]}) \qquad (1)\\
&+ (\phi_{0,2} + \phi_{1,2} Y_{t-1}) I_{[Y_{t-d}>c]} + \varepsilon_t. \qquad (2)
\end{aligned}
$$

For a given threshold c and the position of the random variable Y_{t-d} with respect to this threshold c, the process $(Y_t)_t$ follows here a particular AR(1) model: $\phi_{0,2} + \phi_{1,2}Y_{t-1} + \varepsilon_t$ or $\phi_{0,1} + \phi_{1,1}Y_{t-1} + \varepsilon_t$. The model's parameters are $\phi_{i,j}$, for $i = 0, 1$ and $j = 1, 2$, the threshold c and the delay d. Details on this class of models can be found in Tong (1990). On each state, it is possible to propose more complex stationary models like the ARMA(p,q) processes or non linear models (see Guégan, 1994, 2003a and references therein).

We can use a smooth transition variable to characterize the states of the model and then, we get the smooth transition autoregressive (STAR) process. In that case the process $(Y_t)_t$ follows the recursive scheme, $\forall t$:

$$
\begin{aligned}
Y_t &= (\phi_{0,1} + \phi_{1,1} Y_{t-1})(1 - G(Y_{t-d}, \gamma, c)) \qquad (3)\\
&+ (\phi_{0,2} + \phi_{1,2} Y_{t-1}) G(Y_{t-d}, \gamma, c) + \varepsilon_t, \qquad (4)
\end{aligned}
$$

where G is some continuous function, for instance the logistic one:

$$
G(Y_{t-d}, \gamma, c) = \frac{1}{1 + \exp(-\gamma(Y_{t-d} - c))}. \qquad (5)
$$

Note that the transition function G is bounded between 0 and 1. The parameter c can be interpreted as the threshold between the two regimes in the sense that the logistic function changes monotonically from 0 to 1 with respect to the value of the lagged endogenous variable Y_{t-d}. The parameter γ determines the smoothness of the change in the value of the logistic function, and thus, the smoothness of the transition of one regime to the other. As γ becomes very large, the logistic function (5) approaches the indicator function $I_{Y_{t-d>c}}$, defined as $I_A = 1$ if A is true and $I_A = 0$ otherwise. Consequently, the change of $G(Y_{t-d}, \gamma, c)$ from 0 to 1 becomes instantaneous at

$Y_{t-d} = c$. Then, we find the SETAR model as a particular case of this STAR model. When $\gamma \rightarrow 0$, the logistic function approaches a constant (equal to 0.5) and when $\gamma = 0$, the STAR model reduces to a linear autoregressive model. This model has been introduced by Terasvirta and Anderson (1992).

A stationary SETAR model with changes in the variance may also be considered. We refer to Pfann, Schotman and Tchernig (1996). If we want to make long memory dynamics, we can consider a SETAR process $(Y_t)_t$ defined as follows, $\forall t$:

$$\begin{cases} (1-B)^d Y_t = \varepsilon_t^{(1)}, & \text{if } Y_{t-l} \leq c: \quad \text{regime 1} \\ (1-B)^{d'} Y_t = \varepsilon_t^{(2)}, & \text{if } Y_{t-l} > c: \quad \text{regime 2.} \end{cases} \tag{6}$$

This model assumes long memory behaviors on each state. it has been introduced and discussed by Dufrénot, Guégan and Peiguin-Feissolle (2003), see also references therein.

One of the interest of SETAR processes lies on their predictability. This problematic has been developed recently by De Goojier and De Bruin (1999) and Clements and Smith (1999).

3.2 The Switching models

The mean stationary switching model $(Y_t)_t$ is defined by the following equations, $\forall t$:

$$Y_t = \phi_{0,s_t} + \phi_{1,s_t} Y_{t-1} + \varepsilon_t, \tag{7}$$

where the non-observed process $(s_t)_t$ is an ergodic Markov chain and $(\varepsilon_t)_t$ is a classical noise. The associated transition's probability to the process $(s_t)_t$ is defined by:

$$P[s_t = j | s_{t-1} = i] = p_{ij}, \tag{8}$$

with $0 < p_{ij} < 1$ and $i, j = 1, 2$ (if we consider only two states). In that latter case, if $s_t = 1$, the process $(Y_t)_t$ follows the regime $\phi_{0,1} + \phi_{1,1} Y_{t-1} + \varepsilon_t$ and if the variable $s_t = 2$, the process $(Y_t)_t$ follows the regime $\phi_{0,2} + \phi_{1,2} Y_{t-1} + \varepsilon_t$. In the two-regime case, it is possible to compute the non-conditional probabilities associated to the process $(Y_t)_t$. They are equal to:

$$P[s_t = 1] = \frac{1 - p_{22}}{1 - p_{11} + 1 - p_{22}} = \pi,$$

and

$$P[s_t = 2] = 1 - \pi.$$

We can generalize these processes, imposing the existence of states on the variances, then we get the AR-SWGARCH processes introduced by Hamilton and Susmel (1994). These processes are defined by the following equations (using the same notations as above), $\forall t$:

$$\begin{aligned} Y_t &= \phi_{0,s_t} + \phi_{1,s_t} Y_{t-1} + u_t & (9)\\ u_t &= \sigma_t.\nu_t & (10)\\ \sigma_t^2 &= a_{0,s_t} + a_{1,s_{t-1}} u_{t-1}^2 + \delta_{s_{t-1}} \sigma_{t-1}^2. & (11) \end{aligned}$$

Here, $(\nu_t)_t$ is a white noise process. Other extensions permit to introduce different scale parameters, see for instance Krolzig (1997). State space representations including these models have been introduced and developed for instance by Kim and Murray (2002). Details and statistical properties on switching models can be found in Guégan (2003b) and Guégan and Rioublanc (2003) with a lot of references therein.

To make predictions with the Switching processes is difficult. These models permit mainly to describe the existence of regimes inside data sets. They provide an estimation of the probability for the existence of regimes. This last result does not permit to forecast the regimes. As we are not interested by prediction in this chapter, in the following, we will use this last class of models to investigate inter-dependence between national economies in terms of expansion and recession. This will provide a framework to examine the possibility of contamination between the financial and real spheres.

4 Non parametric framework

In this Section, we introduce different nonparametric measures which give information on the inter-dependence between two or more economies or markets. The measures that we have retained are the linear correlation's coefficient, the conditional correlation's coefficient, the τ of Kendall and the tail dependence index. We describe now these tools.

4.1 Second-order non parametric measures

1 - The linear correlation's coefficient. The linear correlation's coefficient between two random variables X and Y is defined as:

$$\rho(X,Y) = \frac{cov(X,Y)}{\sigma(X)\sigma(Y)}.$$

Recall that correlation is the canonical measure in the world of multivariate Gaussian distributions, and more generally for spherical and elliptical distributions. It is mainly a measure of linear dependence. In a nonlinear setting, correlation leads to misinterpretations. The popularity of linear correlation's coefficient can be explained in different ways. It is straightforward to calculate. For bivariate distributions, it is simple matter to calculate second moments and hence to derive the linear correlation's coefficient. Nevertheless the variances of the distributions can be infinite, then the linear correlation's coefficient is not defined. Random variables can present strong dependence, even if they have a linear correlation's coefficient equal to zero. Finally this coefficient is not invariant under strictly increasing transformations. Thus, its use is very limited. For more details, we refer to Embrechts, McNeil and Straumann (1999).

2 - The conditional correlation's coefficient. To better understand whether the increase in the volatility of returns varies together with an increase in sampling correlations even when the true correlations are constant, the conditional correlation's coefficient relative to a specific information set has been proposed. The choice of this information set can be used to characterize the periods of calm and turmoil on two markets for instance. For a bivariate Gaussian vector (X, Y) this coefficient is equal to

$$cor(X, Y|X \in A) = \rho_A = \rho(\rho^2 + (1-\rho^2)\frac{var(X)}{var(X|X \in A})^{-1/2}, \qquad (12)$$

where ρ represents the linear correlation's coefficient between X and Y and A, the information set. For different values of ρ one can compute analytically the conditional correlation's coefficient as soon as the set A is specified.

Generally, one uses the sets defined by the deciles of the distribution function of the random variable X. It is quite natural that the variance of the points which belong to the first decile set $var(X|X \in D_1)$ and to the last decile set $var(X|X \in D_{10})$ are higher than the others, because we are considering the tails of the distributions to take into account the influence of shocks. Thus, it seems that the coefficient ρ_A can be used to try to understand the behavior of the volatility. Nevertheless, some works have shown that this property can provide miss-interpretation concerning the behavior of the volatility, see Loretan and English (2000), Forbes and Rigodon (2002) and Avouyi-Dovi, Guégan and Ladoucette (2002), for instance.

Empirically, the coefficient ρ_A is calculated using the following expression, for an information set A:

$$\rho_A(X, Y) = \frac{\mathrm{tr}(\sum_{\mathbf{XY|A}})}{\sqrt{\mathrm{tr}(\sum_{\mathbf{XX|A}})\mathrm{tr}(\sum_{\mathbf{YY|A}})}},$$

where tr(A) represents the trace of a matrix.

3 - The Kendall's tau. The Kendall's tau, between two random variables Z_1 and Z_2, is defined as:

$$\tau(Z_1, Z_2) = \mathbb{P}[(Z_1 - Z_1')(Z_2 - Z_2') > 0] - \mathbb{P}[(Z_1 - Z_1')(Z_2 - Z_2') < 0],$$

where $(Z'_1, Z'_2)^T$ is an independent copy of the vector $(Z_1, Z_2)^T$. Hence, Kendall's tau is simply the probability of concordance minus the probability of discordance. Recall that $-1 \leq \tau(Z_1, Z_2) \leq 1$. It is invariant under strictly increasing transformations: that is, if f and g are strictly increasing functions then $\tau(f(Z_1), g(Z_2)) = \tau(Z_1, Z_2)$. This property does not hold for linear correlation's coefficient. Now, $\tau = 1$ $(= -1)$ if and only if $Z_2 = f(Z_1)$ for some monotonic increasing (or decreasing) function. The coefficient τ is null if Z_1 and Z_2 are independent. If we denote F the joint distribution function of the random vector $(Z_1, Z_2)^T$, then:

$$\tau = \tau(Z_1, Z_2) = 4E[F(Z_1, Z_2)] - 1. \tag{13}$$

(A^T represents the transpose of the matrix A).

4.2 Copulas' method

Here, we present a tool which permits to investigate the multivatiate distribution of n random variables which can represent economies or financial markets. We specify first the notion of copula, then we focus on the notion of tail dependence index.

4.2.1 Definition of a copula

Let $X_1, X_2, \cdots, X_n$, be n random variables whose margins are denoted $F_1, F_2, \cdots,$ F_n, then the function C defined by : $\forall x_1, x_2, \cdots, x_n \in \mathbb{R}$:

$$F(x_1, x_2, \cdots, x_n) = C(F_1(x_1), F_2(x_2), \cdots, F_n(x_n)) \tag{14}$$

is the copula associated to the joint distribution function F of the n random variables. When the margins are continuous, the representation (14) holds for a unique copula C. If $F_1, F_2, \cdots, F_n$ are not all continuous it can still be shown that the joint distribution can always be expressed as in (14), although in this case the function C is no longer unique and we refer to it as a possible copula of F. The representation (14) suggests that we interpret a copula associated with $(X_1, X_2, \cdots, X_n)^T$ as being the dependence structure of this n vector.

There exist different ways to estimate this copula C, from data sets. We refer to Caillault and Guégan (2003) for presentation of two different methods and references therein.

4.2.2 The tail dependence

Now, if we are particularly concerned with extreme values, which is the characteristic of any shock, we can use the concept of tail dependence. An asymptotic measure of tail dependence can be defined for pairs of random variables X and Y. If the marginal distributions of these random variables are continuous, then this dependence measure is also a function of their copula. Indeed, the shock which produces propagation which characterizes contagion's phenomena corresponds generally to an explosion inside the data. Then, it can be assimilated to an extreme value which appears in the tail of the distribution. We introduce now this notion for two random variables.

The upper and lower tail dependence parameters of a vector $(Z_1,\ Z_2)^T$ with continuous marginal distributions functions F_1 and F_2, denoted respectively λ_U and λ_L, are defined by:

$$\lambda_U = \lim_{u\uparrow 1} \mathbb{P}[Z_2 > F_2^{-1}(u) | Z_1 > F_1^{-1}(u)], \tag{15}$$

and

$$\lambda_L = \lim_{u\downarrow 0} \mathbb{P}[Z_2 < F_2^{-1}(u) | Z_1 < F_1^{-1}(u)]. \tag{16}$$

We can remark that $\mathbb{P}[Z_2 > F_2^{-1}(u)|Z_1 > F_1^{-1}(u)]$ can be written as

$$\frac{1 - \mathbb{P}[Z_1 \leq F_1^{-1}(u)] - \mathbb{P}[Z_2 \leq F_2^{-1}(u)] + \mathbb{P}[Z_1 \leq F_1^{-1}(u), Z_2 \leq F_2^{-1}(u)]}{1 - \mathbb{P}[Z_1 \leq F_1^{-1}(u)]}.$$

Then, the quantities λ_U and λ_L can be expressed in terms of copulas and their expressions are given in the following definition:

Definition 4.1 *If a bivariate copula C is such that*

$$\lim_{u\uparrow 1} \overline{C}(u,\ u)/(1-u) = \lambda_U$$

exists, with $\overline{C}(u,\ u) = 1 - 2u + C(u,\ u)$, then the copula C has an upper tail dependence *if $\lambda_U \in (0,\ 1]$ and no upper tail dependence if $\lambda_U = 0$. Moreover if a bivariate copula C is such that*

$$\lim_{u\downarrow 0} C(u,\ u)/u = \lambda_L$$

exists, we will say that the copula C has a lower tail dependence *if $\lambda_L \in (0,\ 1]$ and no lower tail dependence if $\lambda_L = 0$.*

We can investigate the behavior in the tails for different classes of copulas like the Archimedean and the Elliptical ones for instance. For the Gaussian and Student copulas which are Elliptical copulas, the tail dependence is respectively, null for the Gaussian copula ($\lambda_U = \lambda_V = 0$) and, $\lambda_{U_{t_\nu}}$ is an increasing function of ρ for the Student copula. The Archimedean copulas C_θ depend on a parameter θ. They are characterized by a generator function ϕ_θ, then the expressions (15) and (16) depend also on this generator function such as:

$$\lambda_U = 2 - 2\lim_{t\downarrow 0} \phi_\theta^{-1'}(2t)/\phi_\theta^{-1'}(t), \tag{17}$$

and

$$\lambda_L = 2 \lim_{t\to\infty} \phi_\theta^{-1'}(2t)/\phi_\theta^{-1'}(t), \tag{18}$$

when these limits exist. Thus, if one knows $\phi_\theta^{-1'}$, one can calculate the tail dependence of any Archimedean copula. There exists also simple relationship between the parameter θ of a copula C_θ and the Kendall's tau. Thus, this provides an easy way to estimate the parameter θ when the copula is known, (see Joe (1997) for details and Tables 1 and 2 for some examples).

Family	ϕ_θ	$C(u,\ v)$	*Domain of* θ
G_θ	$(-\log t)^\theta$	$\exp\{-((-\tilde{u})^\theta + (-\tilde{v})^\theta)^{1/\theta}\}$	$[1,\ \infty[$
F_θ	$-\log \dfrac{e^{-\theta t}-1}{e^{-\theta}-1}$	$-\dfrac{1}{\theta}\log\left(1+\dfrac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right)$	$]-\infty,\ +\infty[/\{0\}$
A_θ	$\log\dfrac{1-\theta(1-t)}{t}$	$\dfrac{uv}{1-\theta\overline{u}\overline{v}}$	$[-1,\ 1[$

Table 1: Generators ϕ_θ and analytical expressions $C(u,\ v)$ for three Archimedean copulas: the Gumbel copula, G_θ, the Frank copula, F_θ and the Ali-Mikhail-Haq copula, A_θ.

Family	$\phi_\theta^{-1'}$	λ_U	λ_L	τ
G_θ	$\dfrac{-t^{1/\theta-1}e^{-t^{1/\theta}}}{\theta}$	$2-2^{\frac{1}{\theta}}$	0	$\dfrac{\theta-1}{\theta}$
F_θ	$-\dfrac{(1-e^{-\theta})e^{-t}}{\theta(1-(1-e^{-\theta})e^{-t})}$	0	0	$1+\dfrac{4(D_1(\theta)-1)}{\theta}$
A_θ	$\dfrac{(\theta-1)e^t}{e^t-\theta}$	0	0	$\dfrac{3\theta^2-2\theta-2(1-\theta)^2\log(1-\theta)}{3\theta^2}$

Table 2: Lower and upper tail coefficients as Kendall's tau for the Archimedean Copulas presented in Table 1. $D_1(\theta) = \frac{1}{\theta}\int_0^\theta \frac{t}{e^t-1}dt$.

To illustrate these properties, we present on Figure 1 the Gumbel and the survival Gumbel copulas, using Gaussian marginals. A survival copula C_S of a copula C is defined as: $C_S(u,v) = u+v-1+C(1-u,1-v)$. We can observe specific behaviors on these graphes. The Gumbel copula is upper tail dependent and the survival Gumbel copula is lower tail dependent. This means that if two random variables are explained by a Gumbel copula, they move in the same sense for high positive values.

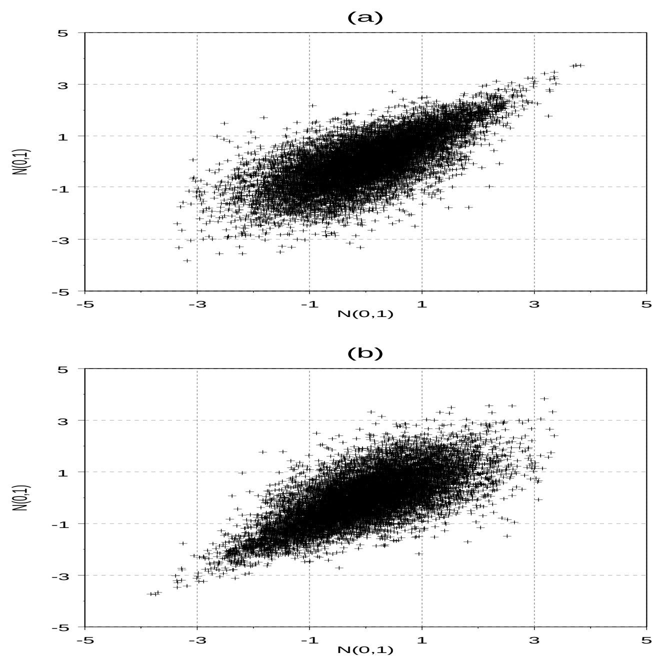

Figure 1: Representation of the Gumbel copula in (a) and the survival Gumbel copula in (b), using Gaussian marginals N(0,1). The sample size is $N = 5000$.

In the following, these different tools are used to measure the dependence between economies and markets and to specify existence of co-movements.

5 Applications

In this Section, we propose two applications. The first one concerns the evolution of national economies. We investigate the possible inter-dependence between these national economies using the different tools presented in the previous sections. In a second part, we measure the possible contagion between a national economy and the financial market. In both cases, we use non parametric tools and parametric models.

5.1 Inter-dependence between national economies

Here, we employ GDP data as a basic broad-based measure of economic activity for five countries, (France, Great Britain, Germany, Italy and United States), to study the possible inter-dependence between these national economies. Data are quarterly from 1st of January, 1970 to 1st of April, 2002 (130 points). They come from Teleco basis. These data presented some linear trend (see Guégan , (2003b) for details). We make them stationary using some linear transformations. If $(X_t)_t$, represents the observed series, the stationary series $(Y_t)_t$ is defined by:

$$\forall t, Y_t = \ln X_t - \ln X_{t-1}. \tag{19}$$

Table 3 displays some descriptive statistics for the returns $(Y_t)_t$ of the five GDP returns. The third and the fourth moments show that the returns' non conditional density functions are non Gaussian. This is confirmed by the value obtained for the Jarque-Bera test (whose probability is always less than 0.05). All the returns' autocorrelation function exhibit short memory behavior, but their scatter plots indicate that they are non linear, see Guégan (1994, 2003a). All the graphs can be found in Guégan (2003b). Thus, in order to adjust a model on these data sets, we will use a non linear process.

	RDE	RFR	RIT	RUK	RUS
Mean	0.006	0.006	0.006	0.006	0.007
Median	0.006	0.006	0.005	0.004	0.0078
Maximum	0.044	0.019	0.031	0.042	0.038
Minimum	-0.020	-0.016	-0.021	-0.033	-0.021
Standard-Deviation.	0.006	0.006	0.008	0.011	0.009
Skewness	0.259	-0.496	0.151	0.336	-0.180
Kurtosis	4.182	4.019	4.347	5.330	4.345
Jarque-Bera	8.948	10.888	10.247	31.613	10.429
Probability	0.011	0.004	0.006	0	0.005
Observations	129	129	129	129	129

Table 3: Descriptive statistics for the GDP returns defined in (19): German (RDE), French (RFR), Italian (RIT), English (RUK) and American (RUS) returns, on the period 1st January, 1970 to 1st April, 2002.

We will consider also in the following, the series of the quarterly returns of the GDP data, denoted $(Z_t)_t$, and defined by:

$$\forall t, \quad Z_t = \ln X_t - \ln X_{t-4}. \tag{20}$$

The descriptive statistics of the series $(Z_t)_t$ are given in Table 4.

	D4DE	D4FR	D4IT	D4UK	D4LOGUS
Mean	0.022	0.024	0.024	0.023	0.030
Median	0.022	0.023	0.024	0.024	0.035
Maximum	0.067	0.056	0.094	0.103	0.083
Minimum	-0.032	-0.015	-0.037	-0.041	-0.029
Standard-Deviation	0.020	0.016	0.022	0.023	0.023
Skewness	-0.077	-0.154	0.514	-0.318	-0.524
Kurtosis	2.851	2.523	4.401	4.614	3.199
Jarque-Bera	0.241	1.691	15.851	15.799	5.983
Probability	0.886	0.429	0.001	0.001	0.050
Observations	126	126	126	126	126

Table 4: Descriptive statistics of the quarterly returns of the GDP indexes defined in (20): German (D4DE), French (D4FR), Italian (D4IT), English (D4UK) and American (D4US) returns, on the period: 1st January, 1970 to 1st April, 2002.

5.1.1 Nonparametric methods

Here, we consider four different measures to characterize the possible inter-dependence between these economies: the linear correlation's coefficient, the conditional correlation's coefficient, the tau of Kendall and the tail dependence index.

1 - Linear correlation's coefficient and Kendall 's tau. Table 5 provides the linear correlation's coefficient and the Kendall's tau for GDP returns, defined respectively in (19) and (20) with respect to the American GDP return.

	τ for $(Y_t)_t$	ρ for $(Y_t)_t$	τ for $(Z_t)_t$	ρ for $(Z_t)_t$
US/UK	0.136	0.251	0.42	0.59
US/FR	0.137	0.232	0.24	0.35
US/IT	0.026	0.136	0.25	0.32
US/DE	0.138	0.191	0.30	0.43

Table 5: Kendall's tau and Linear correlation's coefficient for the returns $(Y_t)_t$ and $(Z_t)_t$, for each European economy, compared with respect to the American economy.

The linear correlation's coefficients between the American economy and the different European economies are very small. They vary between 0.191 and 0.251. Nevertheless the values are greater when we use the quarterly GDP returns. Thus, the investigation of a possible inter-dependence between these economies will be more relevant using the latter data set. The Kendall's tau is small for the $(Y_t)_t$ returns and its value is greater when we use the quarterly GDP returns $(Z_t)_t$. These results show that the European economies are correlated with the American economy, with a probability equal to 0.191 (at least) if we use the $(Y_t)_t$ data and 0.32 if we use the $(Z_t)_t$ data, on the period under investigation.

2 - Conditional correlation's coefficient. Here, the information set A is the set defined by the quantiles of the distribution of the quarterly American GDP returns, (see (12)). Following the results given in Table 5, we consider, in this paragraph, the returns $(Z_t)_t$ which present the highest correlation. In Table 6 we provide the conditional correlation's coefficient for different pairs of economies.

	Interval		(a)	(b)	(c)	(d)
D1	-0,029	-0,005	0,66	0,59	0,23	-0,50
D2	-0,004	0,008	0,41	0,10	-0,01	-0,53
D3	0,012	0,023	0,15	-0,06	0,12	0,05
D4	0,023	0,027	0,02	-0,12	0,09	-0,15
D5	0,028	0,033	-0,06	0,43	0,03	0,27
D6	0,034	0,039	-0,35	-0,17	-0,07	-0,01
D7	0,039	0,041	0,31	-0,02	0,23	0,36
D8	0,041	0,044	0,21	0,14	0,28	0,41
D9	0,045	0,055	-0,10	-0,28	-0,40	0,05
D10	0,056	0,083	-0,07	-0,16	-0,37	0,27

Table 6: Conditional correlation's coefficient for the quarterly GDP returns of the European economies with respect to the quantiles of the empirical distribution of the American economy : (a) $\rho_{D_i}(US, DE)$, (b) $\rho_{D_i}(US, IT)$, (c) $\rho_{D_i}(US, FR)$, (d) $\rho_{D_i}(US, UK)$.

For all these economies, we never obtain a U-shape for this conditional correlation. Thus, there is no specific "pattern" which permits to explain the evolution of the pairs of economies. In particular, we do not detect specific pattern in the tails (D_1 and D_{10} deciles). Recall that this statistic has been proposed to measure existence of some contagion effect between different markets in turmoil periods based on the presence of shocks. Here, this coefficient does not provide any information concerning a possible change of behavior due to any shock.

3 - Tail dependence index. First, we investigate the bivariate distribution function between each pair of economies. These economies are not independent, thus, we chose to characterize their bivariate distribution function using a copula function. We adjust it on the previous data sets, using the Akaike criteria (for details on the method, we refer to Caillault et Guégan (2003) and Breymann, Dias and Embrechts (2003)). We use a panel of eight copulas: the Gaussian, the Student, the Gumbel, the survival Gumbel, the Clayton, the survival Clayton, the Ali-Mikhail-Haq and the Frank ones. We give detailed results for the GDP returns $(Y_t)_t$ in Table 7. We provide the estimate value of the parameter θ, its standard deviation and the value of the AIC criteria obtained at the end of the maximum likelihood procedure. The best copula corresponds to the copula adjusted with the minimum value of the Akaike criteria.

Pairs of Econ.	Copulae	$\hat{\theta}$	s.d.	AIC
	Gaussian	0.259	0.087	-5.947
	t	0.242 9.745	0.049 5.946	-4.400
	Gumbel	1.158	0.063	-8.273
(US/FR)	Surv Gumbel	1.173	0.065	-9.799
	Clayton	0.335	0.137	-6.020
	Surv Clayton	0.281	0.146	-3.658
	Amh	0.589	0.220	-4.150
	Frank	1.292	0.569	-3.551
	Gaussian	0.208	0.101	-3.045
	t	0.222 5.367	0.076 3.325	-3.542
	Gumbel	1.147	0.069	-5.206
(US/DE)	Surv Gumbel	1.165	0.066	-8.560
	Clayton	0.305	0.154	-4.346
	Surv Clayton	0.215	0.155	-0.937
	Amh	0.560	0.213	-3.824
	Frank	1.331	0.610	-3.714
	Gaussian	0.072	0.111	1.416
	t	0.044 3.358	0.095 2.676	-1.765
	Gumbel	1.098	0.060	-1.723
(US/IT)	Surv Gumbel	1.120	0.061	-4.379
	Clayton	0.157	0.121	-0.335
	Surv Clayton	0.044	0.132	1.816
	Amh	0.154	0.333	1.707
	Frank	0.289	0.641	1.734
	Gaussian	0.243	0.085	-4.946
	t	0.242 144.213	0.116 100.213	-2.917
	Gumbel	1.126	0.058	-3.405
(US/UK)	Surv Gumbel	1.169	0.060	-9.512
	Clayton	0.351	0.117	-7.129
	Surv Clayton	0.186	0.122	-0.393
	Amh	0.614	0.187	-4.800
	Frank	1.270	0.541	-3.496

Table 7: Estimation of the parameter θ (which characterizes each copula), its standard deviation (s.d.), and Akaike's criteria (AIC) for the GDP returns $(Y_t)_t$.

The results given in Table 7 suggest that the survival Gumbel copula is the best copula adjusted on each pair of economies using the GDP returns $(Y_t)_t$. For the quarterly GDP returns $(Z_t)_t$, the same method has been used and different copulas have been retained for the four pairs of economies, for details see Guégan (2003b). We summarized the results in Table 8.

Pairs of Econ.	Best copula
(US/FR)	Gaussian
(US/DE)	Clayton
(US/IT)	Survival Gumbel
(US/UK)	Frank

Table 8: Best copula adjusted for each pair of economies using Akaike criteria, for the quarterly GDP returns $(Z_t)_t$, .

Now, examining the behavior of the adjusted copulas in the tails, we can detect existence of co-movements between the different economies, . Indeed, looking at the results given in Table 7, we observe that all the European economies co-move in the same sense as the American economy in presence of negative shocks. This means that if a recession is announced in the United States, then, this one attains quickly the four European countries. This approach does not permit to give a precise "datation" for the recession, but it says that it is nearly instantaneous.

Looking at the results summarized in Table 8, the interpretation cannot be the same. Indeed, the Gaussian and the Frank copulas are not tail dependent. This means that the European countries like France and Great Britain are not concerned by the recession and expansion periods in United States, when we use the quarterly DGP returns, whereas Germany and Italy seem to be sensitive to recessions in United States.

This exercise shows the great influence of the choice of the data sets and the importance of any transformation on the data (to achieve stationarity). Even if it seems more interesting to study the quarterly returns, because of the great value of the linear correlation's coefficient, the analysis concerning the inter-dependence between these economies is more relevant with the $(Y_t)_t$ returns. Moreover, we show also, with this latter data set, that the nonparametric method based on the bivariate distribution is relevant. This also indicates the poor ability of the linear correlation's coefficient concerning the evolution of the previous pairs of data sets.

5.1.2 Multivariate Switching models

The univariate switching models can permit to detect cycles inside the data, but if we want to analyse existence of contagion between these different data sets, we need to work with a multivariate representation of these models. Here, for the previous economies, we try to detect the existence of a common behavior, in terms of recession and expansion. We consider the multivariate representation of the model (7). We apply it to the five quarterly GDP returns. With the parametric models, we get a better adjustment with the $(Z_t)_t$ than with the $(Y_t)_t$ returns. So, the results are given only for $(Z_t)_t$ returns, see Figure 2.

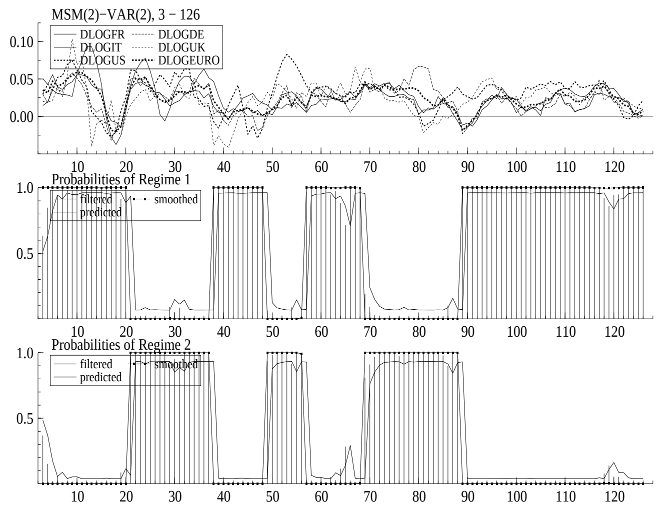

Figure 2: Smoothed probabilities for the quarterly GDP returns investigated together using a multivariate swithching model (7).

We observe that the five economies have the probability 0.64 to be at the same time in the regime 1 and the probability 0.36 to be in the regime 2. The dates to enter in the regime 2 are: March 1975 (20th point), January 1983 (50th point) and January 1988 (70th point). The entry in the regime 1 is at the following dates: January 1971 (3rd point), March 1980 (40th point), April 1984 (57th point) and January 1993 (90th point). We observe that, on the period 1993 - 2002, no change are proposed with this model! The residual noise is close to a white noise, but it is non Gaussian, see graphes in Guégan (2003b). Table 9 summarizes the transition probabilities to be in the regimes 1 or 2, the number of observations in each regime, the conditional probabilities associated to each regime and the duration in mean to stay in a regime.

5.1.3 Remarks

With this exercise, we explicit the co-mouvement between the European economies using the parametric approach and their inter-dependence with

Tans. Proba.	Regime1	Regime2	
Regime1	0.9619	0.0381	
Regime2	0.0672	0.9328	
	Nb. of Obs.	Noncond. Prob.	Duration
Regime1	79.0	0.6385	26.26
Regime2	45.0	0.3615	14.87

Table 9: Multivariate adjustment for all quarterly GDP returns $(Z_t)_t$. Transition probabilities. Number of observations in each state. Non conditional probabilities. Duration (in mean) in each state.

the American economy using the non parametric approach. These two complementary approaches seem relevant to detect existence of inter-dependence between these national economies.

We also note that the copula's method is more consistent when we use the $(Y_t)_t$ returns instead of the parametric models which are more relevant when we use the $(Z_t)_t$ returns. We observe that the non parametric method is very sensitive to the transformation of the data sets, and more we achieve stationary for the data, more the parametric models seem adequate. But in that latter case, the information set is poorer and here data overlapped.

5.2 Contagion between the financial and real spheres

We now study the evolution of French industrial production index (using the IP index) and the CAC40 index. We examine their possible role in relation with the contagion's phenomena between financial market and national economy. The data used are monthly data from 17th of July, 1987 to 19th of September, 2002. The data come from Bloomberg data basis. Their descriptive statistics are given in Table 10.

As in the previous subsection, to investigate the contagion's phenomena, we use both methods based on non parametric tools and on multivariate switching models.

1 - Second order measures. The linear correlation's coefficient between these two returns is equal to 0.10. Thus, it is very small. In table 11, we give the conditional correlation's coefficient for the two series. We condition using the quantiles of the empirical distribution of each series. We do not observe a U-shape. It seems that no relevant information is obtained from this statistic concerning a possible contamination of the volatility behavior from one series to the other one, in presence of shocks.

2 - The copula's approach. In order to get a global information on the

	DLOGCAC	DLOGIP
Mean	0.177936	0.062406
Median	0.547035	0.037231
Maximum	9.508118	1.422586
Minimum	-11.2883	-1.32375
Standard Deviation	2.838764	0.439028
Skewness	-0.52261	0.056257
Kurtosis	4.298993	4.017816
Jarque-Bera	21.19635	7.99564
Probability	0.000025	0.018356
Observations	183	183

Table 10: Descriptive statistics for the monthly returns of the French IP index and of the CAC40 index on the period 1987 - 2002.

	Interval		$\rho_{D_i}(CAC, IP)$
D1	-11,29	-4,07	-0,15
D2	-4,01	-2,75	0,44
D3	-2,70	-2,02	0,03
D4	-2,01	-1,51	0,19
D5	-1,46	-0,66	-0,05
D6	-0,63	0,01	0,02
D7	0,01	0,52	-0,14
D8	0,55	0,93	-0,21
D9	1,03	1,36	0,04
D10	1,37	1,81	0,21
	Interval		$\rho_{D_i}(IP, CAC)$
D1	-1,32	-0,57	-0,55
D2	-0,57	-0,33	0,01
D3	-0,32	-0,22	0,28
D4	-0,22	-0,13	0,48
D5	-0,13	-0,04	0,10
D6	-0,04	0,00	-0,53
D7	0,00	0,04	-0,72
D8	0,04	0,07	0,05
D9	0,08	0,14	-0,23
D10	0,15	0,24	0,17

Table 11: Conditional Correlation's coefficient between the two indexes IP and CAC.

evolution of these two data sets, we use an adjustment with copulas. We retain seven copulas and use the maximum likelihood approach to chose the best one. The results are given in Table 12.

The best adjustment based on Akaike criteria is obtained with the Gumbel copula. This copula is upper tail dependent. This means, that for these two indexes, if there exists some positive shock in the economy or on the financial

	θ	s.d	AIC
Gaussian	0,157	0,071	-2,169
t	0,156 14,394	0,098 126,703	-0,784
Gumbel	1,152	0,047	-11,791
Survival Gumbel	1,087	0,058	-2,255
Clayton	0,141	0,111	0,321
Survival Clayton	0,197	1,076	-3,486
Frank	0,903	0,431	-2,003

Table 12: For each pair (IP, CAC): estimation of the copula's parameter, its standard deviation (s.d.), and the Akaike's criteria (AIC).

market, these two indexes co-move in the same sense. These two series do not react, simultaneously, if the shock is negative. Thus a period of expansion in a national economy can influence the financial market, but not a recession.

3 - Multivariate switching models. Now, we use the multivariate parametric approach and try to see if these series co-move when we investigate them with a multivariate switching model. After an univariate analysis of these indexes whose details can be found in Guégan (2003b), we decide to use a multivariate model which analyses the simultaneous behavior in variance of these indexes. We use a multivariate representation of the model (7). The results are given on Figure 3.

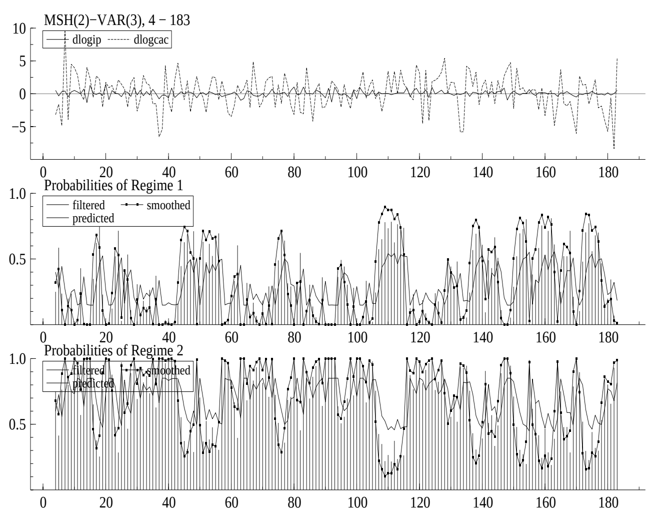

Figure 3: Smoothed probabilities of the returns of the IP and of the CAC using a multivariate version of the model (7), on the period 1984 - 2002, with monthly data.

The detection of two cycles is not straightforward. Thus, this approach does not permit to distinguish precisely when these two indexes co-move when we investigate their volatility's behavior. This method does not permit to study the inter-dependence between the national French economy and the financial market. Nevertheless, we observe that these two indexes react strongly to the 1990's events, same reaction is observed in February 1996 and May 1997.

On Figure 4, we give the graphs which correspond to the impulse response to different shocks on these two indexes. Recall that the impulse response for a process $(Y_t)_t$, at an horizon h, which is submitted to a shock (whose amplitude is δ), coming from another process $(V_t)_t$ is given by:

$$\begin{aligned} I_Y(\delta, h) = \\ E[Y_{t+h}|V_t = \delta, V_{t+1} = 0, .., V_{t+h} = 0] - E[Y_{t+h}|V_t = \quad 0, V_{t+1} = 0, .., V_{t+h} = 0]. \end{aligned} \tag{21}$$

Here, for each index, we look at its behavior when it is submitted to a shock which comes from the other index. On the left graph, on Figure 4, we make a shock IP on the two indexes IP and CAC. Below, the response of the CAC is small. On the right graphs, we apply a CAC shock on the two indexes. The reaction of the IP index is stronger. Thus, we observe that a shock on the financial market provokes a reaction on the national economy, but a shock on the national economy seems to have no effect on the financial sphere.

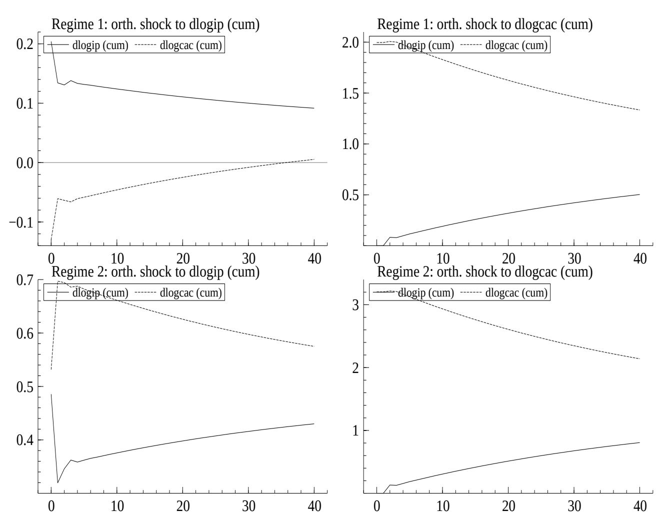

Figure 4: Impulse response between the IP index and the CAC index.

This last analysis permits to make robust the result obtained with the copula's method. Indeed, a "intermediary-specific" shock seems to influence the real economy, but a "real" shock does not contaminate the financial sphere. Thus, we can say that there exists a contagion's effect from the financial sphere to the real sphere, in presence of positive shocks, when we use these two data sets on the mentioned period. We see, with this last exercise, the strength of both methods (copula's approach and switching models) when we use them in a complementary way.

6 Conclusion

In this paper, we focus on different methods to measure the inter-dependence which characterizes the evolution of recession and expansion's periods inside national economies and the co-movements between the real sphere and the financial markets. We review briefly some statistical tools to measure this kind of interaction. We show also specific sensitivity of the different tools with respect to some transformations made on the data sets. This work suggests the necessity to integrate both kinds of tools in any approach. Other extensions can be considered: the use of the copula's approach for more than two economies or financial markets and their use in a dynamical context. At the same time, this work provides a framework to control the risks generated by this kind of phenomena thanks to the use of the multivariate distributions, see for instance Caillault and Guégan (2004).

References

[1] F. Allen, D. Gale (2000) Financial contagion, Journal of Political Economy, 108, 1 - 33.

[2] S. Avouyi-Dovi, Guégan D., Ladoucette S. (2002) What is the best approach to measure the interdependence between the different markets?, NER 95, Banque de France, Paris.

[3] O. de Bandt, P. Hartman (2000) Systemic risk: a survey, Working paper 35, European Central Bank, Francfort.

[4] W. Breyman, A. Dias, P. Embrechts (2003) Dependence structures for multivariate high frequency data in finance, To appear in Quantitative Finance.

[5] G. Bry, C. Boshan (1971) Cyclical analysis of time series: Selected procedures and computer programs, NBER, New York.

[6] A. F. Burns, W.C. Mitchell (1946) Measuring business cycles, NBER, N.Y., U.S.A.

[7] C. Caillault, D. Guégan (2003) Empirical estimation of tail dependence using copulas: application to asian Markets, Note de recherche MORA-IDHE 05-2003, ENS-Cachan, France.

[8] C. Caillault, D. Guégan (2004) Dynamical VaR and Other Measures of Risks, Note de recherche MORA-IDHE 01-2004, ENS-Cachan, France.

[9] M.P. Clements, J. Smith (1999) A Monte Carlo study of the forecasting performance of empirical SETAR models, Journal of Applied Econometrics, 14, 123 - 141.

[10] C. Croux, M. Forni, L. Reichlin (2001) A measure of comovement for economic variables: theory and empirics, The review of Economics and Statistics, 83, 232 - 241.

[11] G. Dufrénot, D. Guégan, A. Peguin-Feissolle (2003) A SETAR Model with Long Memory Dynamics, Note de Recherche, MORA - IDHE 11-2003, ENS-Cachan, France.

[12] P. Embrechts, A. McNeil, D. Straumann (1999) Correlation and dependence in risk managment: properties and pitfalls, Preprint ETHZ, Switzerland.

[13] K. Forbes, R. Rigobon (2002) No contagion, only interdependence measuring stock market co-movements, The journal of Finance, LVII, 2223 - 2261.

[14] J.G. de Goojier, P.T. de Bruin (1999) On forecasting SETAR processes, Statistics and Probability Letters, 37, 7 - 14.

[15] D. Guégan (1994) Séries chronologiques non linéaires à temps discret, Economica, Paris.

[16] D. Guégan (2003a) Les chaos en finance: approche statistique, Economica, Paris.

[17] D. Guégan (2003b) Point de vue personnel sur le problème de contagion en économie et l'intéraction entre cycle réel et cycle financier, Note de recherche, MORA-IDHE, 06-2003, ENS-Cachan, France.

[18] D. Guégan, S. Rioublanc (2003) Study of Regime Switching Models. Do they provide Long memory Behavior? An empirical Approach, Note de recherche, MORA-IDHE, 13-2003, ENS-Cachan, France.

[19] J.D. Hamilton (1988) Rational Expectations Econometric Analysis of Change in Regime: An Investigation of the Term Structure of Interest Rates, Journal of Economic Dynamics and Control, 12, 385-423.

[20] J.D. Hamilton, R. Susmel (1994) Autoregressive conditional heteroscedasticity and changes in regimes, J. of Econometrics, 64, 307 - 353.

[21] H. Joe (1997) Multivariate models and dependence concepts, Chapman and Hall, London.

[22] R. Kaiser, A. Marawall (2001) Measuring business cycles in economic time series, Lecture Notes in Statistics, 154, Springer.

[23] C.J. Kim, C.J. Murray (2002) Permanent and transitory components of recessions, Empirical Economics, 27, 163 - 183.

[24] L.E. Kodres, M. Pritsker (2000) A rational expectations model of financial contagion, FEDS W.P., 1998 - 48, The federal Reserve bank.

[25] H.M. Krolzig (1997) Markov-Switching vector Autoregression: Modelling statistical inference and application to business cycle analysis, Springer, Berlin.

[26] M. Loretan, W.B. English (2000) Evaluating 'correlation breakdowns' during periods of market volatility, Board of Governors of the Federal Reserve System, 658, U.S.A.

[27] S,N. Neftçi (1982) Optimal predictions of cyclical downturns, Journal of Economic Dynamics and Control, 4, 307-327.

[28] S,N. Neftçi (1984) Are economic time series asymmetric over the business cycle, J. of Political Economy, 92, 307 - 328.

[29] G.A. Pfann, P.C. Schotman, R. Tchernig (1996) Nonlinear interest rate dynamics and implication for the term structure, Journal of Econometrics, 74, 149 - 176.

[30] R.E. Quandt (1958) The Estimation of Parameters of Linear Regression System Obeying Two Separate Regime, Journal of the American Statistical Association, 55, 873-880.

[31] T. Terasvirta, H.M. Anderson (1992) Characterising nonlinearities in business cycles using smooth transition autoregressive models, Journal of Applied Econometrics, 7, S119 - S136.

[32] H. Tong (1990) Non-linear time series: a dynamical approach, Oxford Scientific Publications, Oxford.

[33] H. Tong, K.S. Lim (1980) Threshold autoregression, limit cycles and cyclical data, JRSS, B, 42, 245-292.