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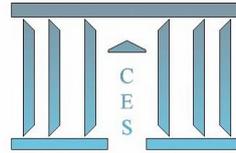
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**Forecasting electricity spot market prices with  
a k-factor GIGARCH process**

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**2007.58**

# Forecasting electricity spot market prices with a k-factor GIGARCH process.

Abdou Kâ Diongue\*, Dominique Guégan<sup>†</sup>, Bertrand Vignal<sup>‡</sup>

## Abstract

In this article, we investigate conditional mean and variance forecasts using a dynamic model following a  $k$ -factor GIGARCH process. We are particularly interested in calculating the conditional variance of the prediction error. We apply this method to electricity prices and test spot prices forecasts until one month ahead forecast. We conclude that the  $k$ -factor GIGARCH process is a suitable tool to forecast spot prices, using the classical RMSE criteria.

*Keywords:* Conditional mean; conditional variance; forecast; Electricity prices; GIGARCH process.

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# 1 Introduction

In finance, commodity price forecasting is a crucial issue, in particular for electricity. Indeed, price forecasts can help evaluating bilateral contracts. For such a commodity, price forecasts can be based on a spot price analysis. Since electricity spot prices have a behaviour which presents specific features, the price forecasts problem becomes more complex. Electricity price volatility has a marked variability in time. We observe both high and low periods of price reaction. Spot price volatility has been recently studied and modelled using a volatility model depending on time. Using such a model, Benini, Maracini, Pelacchi and Venturini (2002) analyze several markets, such as Spain, California, England and Wales and the PJM (Pennsylvania-New Jersey-Maryland) system. Other models have also been considered: a Value-at-Risk model, Dahlgren, Liu and Lawarree (2001), a regression model with Fossa, (2001) and a model mixing deterministic and stochastic components with Barquin (2004). In addition electricity spot prices exhibit long memory, combined with a periodic behaviour. Recent works have taken into account these features using related ARFIMA models for spot data of the Nordic power market, Olsson and Söder (2004), Diongue, Guégan and Vignal (2004) and Koopman, Ooms and Carnero (2007).

After modelling, the forecasting problem arises for these electricity prices. Nogales, Contreras, Conejo and Espinola (2002), treated this problem using ARIMA models and transfer function models, see also Fosso, Gjelsvik, Haustad, Birger, Wangersteen (1999). Misiolek, Truek and Weron (2006) assessed the short term forecasting power of different time series models including the AR, ARX, AR-GARCH, ARX-GARCH and Regime switching models to California Power eXchange (CalPX) system power prices, see also Cuaresma, Hlouskova, Kossmeier, Obersteiner (2004) and Bosco, Parision and Pelagatti (2007) . Forecasting has also been considered using non

parametric methods, we can cite the works of Ramsay and Wang (1997) and Conejo, Contreras, Espinola and Plazas (2005) with neural networks, Szkuta, Sanabria and Dillon (1999) with Fourier Transform .

In this paper, in order to provide robust forecasts for spot electricity prices, we propose a new approach based on the  $k$ -factor GIGARCH process which allows taking into account a lot of stylized facts observed on the electricity spot prices, in particular stochastic volatility, long memory and periodic behaviours. The probabilistic study of this model was introduced and developed in Guégan (2000, 2003). Diongue and Guégan, (2004) introduced the parameter estimation of the  $k$ -factor GIGARCH process. Here, we provide the expression of the forecasts using the  $k$ -factor GIGARCH process and give their properties. Then we apply these results on the German market providing forecasting prices up until a one month-ahead. These results are totally new in the sense that, in most published papers, the previsions concern mostly the on day-ahead horizon. We consider two models and discuss their capability in forecasting, using the RMSE criteria.

This paper is organized as follows. In the next section, we specify some notations and recall the  $k$ -factor GIGARCH process. Section three presents new theoretical results on forecasting in mean and in variance using this model. In the fourth Section, we provide forecasts for the German spot prices data set. The last section is dedicated to conclusions.

## 2 Notations and model

Assume that  $(\xi_t)_{t \in \mathbb{Z}}$  is a white noise process with unit variance and mean zero. Let  $(X_t)_{t \in \mathbb{Z}}$  be a  $k$ -factor GIGARCH process defined by the following

equations:

$$\Phi(B) \prod_{i=1}^k (I - 2\nu_i B + B^2)^{d_i} (X_t - \mu) = \Theta(B) \varepsilon_t, \quad (2.1)$$

where

$$\varepsilon_t = \sqrt{h_t} \xi_t \quad \text{with } h_t = a_0 + \sum_{i=1}^r a_i \varepsilon_{t-i}^2 + \sum_{j=1}^s b_j h_{t-j}. \quad (2.2)$$

In (2.1)-(2.2),  $\mu$  represents the mean of the process  $(X_t)_{t \in \mathbb{Z}}$ ,  $d = (d_1, \dots, d_k)$ , the memory parameters and  $\nu = (\nu_1, \dots, \nu_k)$  the frequency location parameters with  $0 < d_i < \frac{1}{2}$  if  $|\nu_i| < 1$  or  $0 < d_i < \frac{1}{4}$  if  $|\nu_i| = 1$  for  $i = 1, \dots, k$ .  $\Phi(B)$  and  $\Theta(B)$  are the autoregressive and the moving average polynomials defined by  $\Phi(B) = I - \sum_{i=1}^p \hat{\phi}_i B^i$  and  $\Theta(B) = I - \sum_{j=1}^q \theta_j B^j$ , where  $p, q$  are integers. We suppose that all the roots of the polynomials  $\Phi(B)$  and  $\Theta(B)$  are outside the unit circle. Finally,  $B$  is the backshift operator defined by  $B^n X_t = X_{t-n}$ ,  $n \geq 0$ . Assume that  $a_0 > 0, a_1, \dots, a_r, b_1, \dots, b_s \geq 0$ , and  $\sum_{i=1}^r a_i + \sum_{j=1}^s b_j < 1$ . We know that the  $k$ -factor GIGARCH process with the previous assumptions is well defined, we can use it to forecast electricity spot prices. The interest of a heteroskedastic model is that it allows emphasizing the optimal predictor for conditional variance. A basic property for the  $k$ -factor GARMA process with time varying innovations following a GARCH( $r, s$ ) model (we call it the  $k$ -factor GIGARCH process), is that the long memory part provides a way to model the conditional mean whereas the heteroscedastic part provides a way to model the conditional variance. From a forecasting point of view, the  $k$ -factor GIGARCH model and the  $k$ -factor GARMA model will give the same optimal predictor for the conditional mean, but the difference between these two forecasting methods appears mainly in the confidence interval. For more details we refer to Guégan (2000, 2003).

### 3 Forecasting equation for the $k$ -factor GIGARCH process

In this section, we provide new results concerning the predictions both in mean and in variance, using a  $k$ -factor GIGARCH process defined by equations (2.1)-(2.2). In this definition, we only assume that the process  $(\varepsilon_t)_{t \in \mathbb{Z}}$  is a strong white noise.

#### 3.1 Forecasting in mean

Observing  $X_1, \dots, X_T$ , using the stationary and invertible  $k$ -factor GIGARCH process defined by equations (2.1)-(2.2), we compute the  $h$ -time-step-ahead predictor, denoted  $\hat{X}_t(h)$ ,  $h > 0$ . Our approach follows the work of Ferrara and Guégan (2000, 2001). The predictor is obtained by minimizing the mean squared error between forecasts and observations. We assume that the parameters of the model (2.1)-(2.2) have been estimated used Whittle method, see Diongue and Guégan (2004). Under the regular conditions specified in Section 2, this model can be rewritten in its infinite moving average form:

$$X_t = \sum_{j=0}^{\infty} \hat{\beta}_j(d, \nu, \gamma) \varepsilon_{t-j}, \quad (3.1)$$

with

$$\hat{\beta}_j(d, \nu, \gamma) = \hat{\psi}_j(d, \nu) + \sum_{i=1}^{\min(j,p)} \hat{\phi}_i \hat{\beta}_{j-i}(d, \nu, \gamma) - \sum_{i=1}^{\min(j,q)} \hat{\theta}_i \hat{\psi}_{j-i}(d, \nu), \quad (3.2)$$

and

$$\hat{\psi}_j(d, \nu) = \sum_{\substack{0 \leq l_1, \dots, l_k \leq j, \\ l_1 + \dots + l_k = j}} C_{l_1}(d_1, \nu_1) \cdots C_{l_k}(d_k, \nu_k). \quad (3.3)$$

The weights  $(C_{l_i}(d, \nu))_{l_i \in \mathbb{Z}}$  are the Gegenbauer polynomials and they can be computed using the following recursion formula:

$$\begin{cases} C_0(d_i, \nu_i) = 1 \\ C_1(d_i, \nu_i) = 2d_i \nu_i \\ C_j(d_i, \nu_i) = 2\nu_i \left( \frac{d_i-1}{j} + 1 \right) C_{j-1}(d_i, \nu_i) - \left( 2\frac{d_i-1}{j} + 1 \right) C_{j-2}(d_i, \nu_i), \quad \forall j > 1. \end{cases}$$

See, for instance, Magnus et al., 1966, Rainville, 1960 or Szegő, 1959 for more details on Gegenbauer polynomials.

We introduce now some notations. We denote the vectorial parameters  $d = (d_1, \dots, d_k)$ ,  $\gamma = (\phi_1, \dots, \phi_p, \theta_1, \dots, \theta_q)$ , and  $\omega = (a_0, a_1, \dots, a_r, b_1, \dots, b_s)$ , then,

**Theorem 3.1** *Let  $h \geq 1$ , if  $(X_t)_{t \in \mathbb{Z}}$  follows a  $k$ -factor GIGARCH process defined by equations (2.1)-(2.2), then the asymptotic  $h$ -time-step-ahead least square predictor  $\hat{X}_t(h)$ , is given by the following expression:*

$$\hat{X}_t(h) = - \sum_{j=1}^{h-1} \hat{\alpha}_j(d, \nu, \gamma) \hat{X}_t(h-j) - \sum_{j \geq 0} \hat{\alpha}_{j+h}(d, \nu, \gamma) X_{t-j}, \quad (3.4)$$

where the weights  $(\hat{\alpha}_j(d, \nu, \gamma))_{j \in \mathbb{Z}}$  follow the expression:

$$\hat{\alpha}_j(d, \nu, \gamma) = \hat{\pi}_j(d, \nu) - \sum_{i=1}^{\min(j,p)} \hat{\phi}_i \hat{\pi}_{j-i}(d, \nu) + \sum_{i=1}^{\min(j,q)} \hat{\theta}_i \hat{\alpha}_{j-i}(d, \nu, \gamma), \quad (3.5)$$

with  $\hat{\pi}_j(d, \nu) = \hat{\psi}_j(-d, \nu)$ , given in equation (3.3).

The proof of Theorem 3.1 can be adapted from the proof of Propositions 2 and 4 in Ferrara and Guégan (2001), when the parameters of the model (2.1)-(2.2) are estimated using the Whittle's method.

### 3.2 Forecasting in variance

Let  $\mathcal{F}_t$  be the  $\sigma$ -algebra generated by  $\varepsilon_0, \dots, \varepsilon_t$ , (the strong white noise which appears in (2.1)). The  $h$ -time-step-ahead prediction error for  $\hat{X}_t(h)$  given in (3.4) is denoted  $e_{t,h}$  for  $h > 0$ , and is equal to:

$$\begin{aligned} e_{t,h} &= X_{t+h} - \hat{X}_t(h) \\ &= \sum_{j=0}^{h-1} \hat{\beta}_j(d, \nu, \gamma) \varepsilon_{t+h-j}, \end{aligned} \quad (3.6)$$

with the weights  $(\hat{\beta}_j(d, \nu, \gamma))_{j \in \mathbb{Z}}$  as given in the equation (3.2). The conditional mean squared error for this predictor has the following expression:

$$E(e_{t,h}^2 | \mathcal{F}_t) = \sum_{j=0}^{h-1} \hat{\beta}_j^2(d, \nu, \gamma) E(\varepsilon_{t+h-j}^2 | \mathcal{F}_t). \quad (3.7)$$

We remark that the conditional mean squared error depends on the future conditional variance of the innovation process  $(\varepsilon_t^2)_{t \in \mathbb{Z}}$ . We provide its expression in the theorem 3.2.

**Theorem 3.2** *Let  $e_{t,h}$  be the  $h$ -time-step ahead prediction error defined in (3.6), then*

1. *The conditional variance of  $e_{t,h}$ , denoted  $\text{var}_t(e_{t,h})$ , is given by:*

$$\begin{aligned} \text{var}_t(e_{t,h}) &= \sum_{i=1}^h \hat{\beta}_{h-i}^2(d, \nu, \gamma) \omega_i \\ &\quad + \sum_{k=1}^h \hat{\beta}_{h-k}^2(d, \nu, \gamma) \left[ \sum_{i=0}^{s-1} \delta_{i,k} h_{t-i} + \sum_{i=0}^{m-1} \rho_{i,k} \varepsilon_{t-i}^2 \right], \end{aligned} \quad (3.8)$$

with  $m = \max(r, s)$ ,  $\omega_h = e_1'(I + \Gamma + \dots + \Gamma^{h-1}) e_1 \hat{a}_0$ ,  $\delta_{j,h} = -e_1' \Gamma^h e_{m+j+1}$  if  $j = 0, \dots, s-1$ ,  $\rho_{j,h} = e_1' \Gamma^h (e_{j+1} + e_{m+j+1})$  if  $j = 0, \dots, s-1$ ,  $\rho_{j,h} = e_1' \Gamma^h e_{j+1}$  if  $j = s, \dots, m-1$ .

Here  $\{e_j\}_{j=1}^{m+s}$  is the standard base of  $\mathbb{R}^{m+s}$  and the matrix  $\Gamma$  is defined such that its first row is equal to  $(a_1 + b_1, \dots, a_m + b_m, -b_1, \dots, -b_s)$ , and the other elements are all equal to zero, except the first one under diagonal elements which are all equal to one.

2. *Now, we assume that  $(\varepsilon_t)_{t \in \mathbb{Z}}$  is a GARCH( $r, s$ ) process with Gaussian innovations  $\xi_t$ . Then*

$$\mathbb{P} \left\{ \frac{|X_{t+h} - \hat{X}_t(h)|}{\sqrt{\text{var}(e_{t,h})}} < t_{\frac{\alpha}{2}} | \mathcal{F}_t \right\} = 1 - \alpha, \quad (3.9)$$

where  $t_{\frac{\alpha}{2}}$  is the quantile of standard Gaussian law and  $\text{var}(e_{t,h})$  is provided in (3.8).

*Proof:*

1. Following the works of Bollerslev (1988), the linear GARCH( $r, s$ ) model defined in (2.2) can be rewritten as

$$\varepsilon_t^2 = a_0 + \sum_{i=1}^m (a_i + b_i) \varepsilon_{t-i}^2 + \nu_t - \sum_{j=1}^s b_j \nu_{t-j}, \quad (3.10)$$

where  $m = \max(r, s)$ ,  $a_i = 0$  if  $i > s$  and  $b_j = 0$  if  $j > r$ . The process  $(\nu_t)_{t \in \mathbb{Z}}$  is equal to  $\nu_t = \varepsilon_t^2 - h_t$ , and is a serially uncorrelated process. The equation (3.10) corresponds to an ARMA ( $m, s$ ) model for the process  $(\varepsilon_t^2)_{t \in \mathbb{Z}}$ , thus the least squares  $h$ -time-step-ahead prediction value for the conditional variance is given by:

$$\begin{aligned} E(\varepsilon_{t+h}^2 | \mathcal{F}_t) &= E(h_{t+h} | \mathcal{F}_t) \\ &= \omega_h + \sum_{j=0}^{s-1} \delta_{j,h} h_{t-j} + \sum_{j=0}^{m-1} \rho_{j,h} \varepsilon_{t-j}^2, \end{aligned} \quad (3.11)$$

with  $\omega_h$ ,  $e_j$ ,  $\Gamma$ ,  $\delta_{j,h}$ , and  $\rho_{j,h}$  given in Theorem 3.2. The conditional mean squared error for the optimal mean predictor of a  $k$ -factor GI-GARCH process is then obtained by combining equations (3.7) and (3.11).

2. If  $(\varepsilon_t)_{t \in \mathbb{Z}}$  has a conditional Gaussian distribution, then so does  $e_{t,h}$  introduced in equation (3.6). Moreover, we can observe that

$$E(e_{t,h} | \mathcal{F}_t) = 0.$$

Thus the expression (3.9) in Theorem 3.2 follows.

## 4 A real case study

In this section, we investigate the German Wholesale spot electricity market EEX (European Energy eXchange) and the associated spot prices. The German spot prices have several key features such as: long memory persistence and non-constant volatility. These stylized facts, (studied in details in

Diongue, Guégan and Vignal (2004)) strongly suggest to use a GIGARCH process in order to make forecast. Thus, we apply two  $k$ -factor GIGARCH processes and test their forecasting performances.

#### 4.1 The data set

We consider the hourly series of the EEX spot prices, denoted  $(S_t)_{t \in \mathbb{Z}}$ , from August, 15, 2000 to December 31, 2003. In order to make stationary the series, we work below with the data set  $(X_t)_t$  such that,  $X_t = \log(S_t)$ . For this electricity spot market, the prices are fixed each day, for each of the 24 hours of the next day. This spot market has two particular interests. On the one hand, it allows a buyer (seller) to supply their bid (load) on an hourly basis (physical market). On the other hand, one can think, that the spot market corresponds to the price reference for the contracts. We build two subsets from the original series. The first one is used for the parameter estimation procedure and goes from August 15, 2000 to December 31, 2002 (see figure 1). The second one is used for validation and goes from January 1, 2003 to December 31, 2003. The first series contains 20856 observations. On figure 1, we can observe the existence of periods with low volatility (Regions 1) following by periods with high volatility (Regions 2). Figure 2 gives a zoom on the evolution of the spot prices, in its logarithmic scale, from October 14, 2000 to October 27, 2000 (two weeks). We observe that the series has two kinds of seasonality: daily and weekly seasonalities.

The negative value of the skewness,  $-2.6$ , indicates that the series has long left hand side tails while the higher kurtosis statistic equal to  $29.67$ , greater than  $3$ , allows us to conclude that the series has fat tails. The autocorrelation function of the EEX spot prices provided in figure 3, exhibits a slow decay pattern at two seasonality lags corresponding to daily and weekly seasonalities. The autocorrelation function does not converge exponentially to

zero in covariance sense, (Guégan, 2005, for discussions on this concept), thus we conclude that the series has a long range dependence behaviour. Moreover, the spectral density represented by the periodogram in figure 4, is unbounded at three frequencies corresponding respectively, to daily, weekly and half daily seasonality. We propose two different models for the spot prices. Indeed, the choice of the number of factors in a  $k$ -factor GIGARCH process is not obvious, however it is crucial from a forecasting point of view, Collet, Guégan and Valdes-Sosa, (2003).

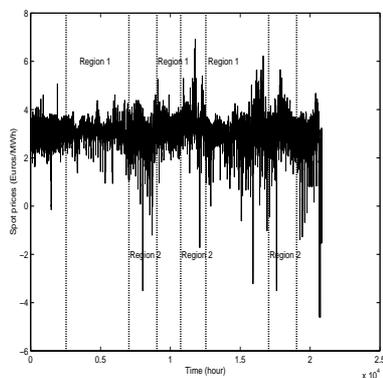


Figure 1: Evolution of the EEX spot market prices in its logarithmic scale from 15/08/2000 to 31/12/2002.

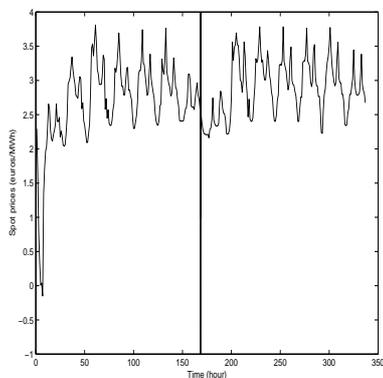


Figure 2: Two weeks zoom concerning the evolution of the EEX spot market prices in its logarithmic scale from october 14, 2000 to october 27, 2000.

## 4.2 Presentation of the models

Following the previous empirical and statistical analysis, we decide to adjust two models and we compare them with respect to their capability to provide robust forecasts. The choice of the models are based on the analysis of the correlation function and of the periodogram.

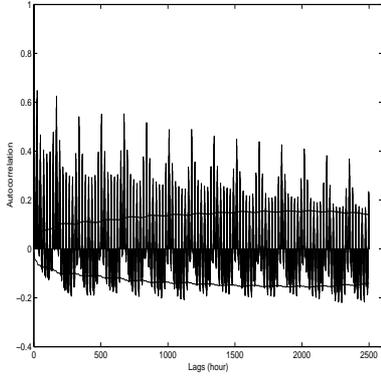


Figure 3: Autocorrelation function for the EEX spot market prices in its logarithmic scale from 15/08/2000 to 31/12/2002.

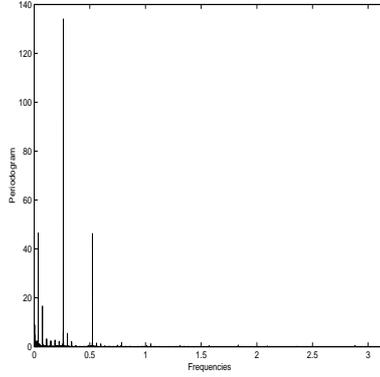


Figure 4: Periodogram of the EEX spot market prices in its logarithmic scale from 15/08/2000 to 31/12/2002.

#### 4.2.1 First model ( $M_1$ )

The autocorrelation function of the EEX spot prices (figure 3) exhibits an exponential rate at the weekly lag. We remove the weekly seasonality by applying the filter  $(I - B^{168})$  to the data set. Then, we suppose that the filtered series can be modeled by a 1-factor GIGARCH process, Guégan (2000, 2003, for more details). The first model  $M_1$  is defined by:

$$(I - B^{168}) \Phi^{(1)}(B) \left( I - 2\nu^{(1)}B + B^2 \right)^{d^{(1)}} (X_t - 2.97) = \Theta^{(1)}(B) \varepsilon_t, \quad (4.1)$$

where  $\mu$  is the mean of the series  $(X_t)_{t=1, \dots, T}$ ,

$$\Phi^{(1)}(B) = I - \phi_{24}^{(1)} B^{24}, \quad (4.2)$$

$$\Theta^{(1)}(B) = I - \theta_{24}^{(1)} B^{24} - \theta_{168}^{(1)} B^{168}, \quad (4.3)$$

and

$$\varepsilon_t = \sqrt{h_t^{(1)}} \eta_t, \text{ with } h_t^{(1)} = a_0^{(1)} + a_1^{(1)} \varepsilon_{t-1}^2. \quad (4.4)$$

The parameter estimation is done using Whittle's parametric method, Diongue and Guégan (2004) and the estimated parameters of this model are given in table 1 with their standard deviations. In order to estimate the G-frequency parameter, we use Yajima's method (1996). We find that the periodogram

of the filtered series is unbounded at the frequency  $\lambda^{(1)} = 0.001$ , therefore  $\nu^{(1)} = \cos(\lambda^{(1)})$ . In table 1, the bold values correspond to the estimated values for the model ( $M_1$ ). This model is satisfactory in the sense that the innovations appear to be white noise. The values of the Portmanteau statistic for  $M = 136$  is equal to  $Q(M) = 1.69910^2$ . This statistic is larger than the corresponding critical value at significance level 0.05, and permits to accept the model.

#### 4.2.2 Second model ( $M_2$ )

The periodogram of the EEX spot prices is unbounded at three frequencies, thus we assume that the series  $(X_t)_{t \in \mathbb{Z}}$  can be modelled by a 3-factor GIGARCH process. Thus, the observed prices, in its logarithmic scale,  $(X_t)_{t=1, \dots, T}$  can be modelled by the following equation

$$\Phi^{(2)}(B) \prod_{i=1}^3 \left( I - 2\nu_i^{(2)}B + B^2 \right)^{d_i^{(2)}} (X_t - 2.97) = \Theta^{(2)}(B) \zeta_t, \quad (4.5)$$

where  $\mu$  is the mean of the series  $(X_t)_{t=1, \dots, T}$ ,

$$\Phi^{(2)}(B) = I - \phi_{24}^{(2)}B^{24} - \phi_{168}^{(2)}B^{168}, \quad (4.6)$$

$$\Theta^{(2)}(B) = I - \theta_{24}^{(2)}B^{24} - \theta_{168}^{(2)}B^{168}, \quad (4.7)$$

and

$$\zeta_t = \sqrt{h_t^{(2)}} \xi_t, \text{ with } h_t^{(2)} = a_0^{(2)} + a_1^{(2)} \zeta_{t-1}^2. \quad (4.8)$$

The parameter estimation is carried out using the parametric Whittle's method assuming that the G-frequencies are known. We retain three different G-frequencies for which the spectral density estimation is unbounded given by  $\nu_1^{(2)} = \cos\left(\frac{2\hat{\pi} \times 869}{20856}\right)$ ,  $\nu_2^{(2)} = \cos\left(\frac{2\hat{\pi} \times 124}{20856}\right)$  and  $\nu_3^{(2)} = \cos\left(\frac{2\hat{\pi} \times 1738}{20856}\right)$ . All estimated parameters are provided in table 1 with their standard deviations. The residuals can be considered as white noise, using Portmanteau test. Indeed, for  $M = 203$ ,  $Q(M) = 2.4410^2$  and this statistic is larger than

Parameter	Estimate	St. dev.
$d_1$	0.135 <b>0.38</b>	0.009 <b>0.017</b>
$d_2$	0.214	0.006
$d_3$	0.141	0.015
$\phi_{24}$	0.863 <b>0.874</b>	0.179 <b>0.342</b>
$\phi_{168}$	0.951	0.221
$\theta_{24}$	0.720 <b>0.706</b>	0.331 <b>0.342</b>
$\theta_{168}$	0.794 <b>0.851</b>	0.298 <b>0.303</b>
$a_0$	0.0141 <b>0.0167</b>	$3.3910^{-4}$ <b><math>3.3610^{-4}</math></b>
$a_1$	0.756 <b>0.594</b>	0.365 <b>0.072</b>

Table 1: Estimated parameters for the model  $M_1$  and  $M_2$ , 15/08/2000 to 31/12/2002, for the EEX spot market prices.

the corresponding critical value at significance level 0.05.

The Portmanteau test indicates that model  $M_1$  is better than model  $M_2$ . However, the variance of the model  $M_2$  (0.0414) is lower than the variance of the model  $M_1$  (0.0432). Thus, in terms of variance comparison, the model  $M_2$  appears better than the model  $M_1$ . Thus, we decide to keep these two models to forecast the prices.

### 4.2.3 Forecasting results

To evaluate the capability of the models (4.5-4.8) and (4.1-4.4) to forecast electricity German prices, we use the forecasting method developed in Section 3. The period from January 1, 2003 to December 31, 2003 is considered as the out-of sample forecasts period. We assess the predictive ability of the two models by considering the Root Mean Squared Error (RMSE) of the forecasts defined by

$$RMSE = \sqrt{\frac{1}{M} \sum_{i=1}^M \left( X_{t+i} - \hat{X}_t(i) \right)^2}, \quad (4.9)$$

where  $M$  is the size of the out-of-sample forecast period and  $\hat{X}_t(i)$  is the predicted value of  $X_{t+i}$ . We use this criteria because we are interested in the measure of the variation between the observed data and the forecast data. Nevertheless, other measures can be used, see Darbellay and Slama (2000) and Bakirtzis, Petridis, Kiartzis, Alexiadis and Maisis (1996).

For each model, forecasts are computed for  $h = 1, \dots, 720$  hours (from one hour to one month). As we use the prices in a logarithmic scale, we need to compute the exponentials of these log-forecastings. In order to get these forecasts, we assume the Gaussianity of the white noise  $(\xi_t)_{t \in \mathbb{Z}}$  in equation (2.2), and the forecasted spot prices at time  $T$  can be computed using the following expression

$$\hat{S}_T(h) = \exp \left\{ \hat{X}_T(h) + \frac{1}{2} var [\hat{e}_{T,h}] \right\}. \quad (4.10)$$

### 4.2.4 Forecasting results

The forecasting performances of the two previous model on EEX spot prices data are given on figures 5-8 and in tables 2 to 4. On figures 5-8, we exhibit the results for two specific weeks. The first one goes from March 16, 2003 to March 22, 2003 and the second one goes from September 8, 2003

to September 14, 2003. On these figures, the continuous line represents the observations and the dotted line represents the forecasts.

We notice that the forecasts obtained with the 3-factor GIGARCH model (model  $M_2$ ) give a better fit than the forecasts obtained with the 1-factor GIGARCH process (model  $M_1$ ) with respect to the RMSE criteria, see tables 2 to 4. In these tables the RMSE is given for specific hours for horizons lying from one day ahead to one month ahead forecasts. The bold numbers correspond to the RMSE obtained using the model  $M_2$ . These RMSE are computed hour-by-hour over all the year 2003. We consider that it has no sense to compare the RMSE for a specific hour of a day, to the RMSE of another one, because the spot prices are given hour by hour. These RMSE values reveal that model  $M_2$  (3-factor GIGARCH process) is better, using this criteria, than predictions provided using the model  $M_1$  (1-factor GIGARCH process).

The RMSE values from the two proposed models, namely those for one day ahead to one month ahead, rise from below 3 euros/MWh to above 8 euros/MWh in 2003 for hour 1 a.m. It rises from 3 euros/MWh to 8 euros/MWh and from 5 euros/MWh to 10 euros/MWh for hour 5 p.m and 11 p.m respectively. The first model allows reducing the RMSE up to 36%, while with the second model we reduce the RMSE up to 40% at 11 p.m. At hour 5 p.m, we note a reduction of 31% on the RMSE with the first model and of 35% with the second model. The same remark is observed at 1 a.m with a reduction of 20% and 31% respectively for the models  $M_1$  and  $M_2$ . We can conclude that the model  $M_2$  (3-factor GIGARCH process) provides more accurate forecasts, for the  $h$ -ahead horizon ( $h = 1$  hour to  $h = 720$  hours), than the model  $M_1$  (1-factor GIGARCH process).

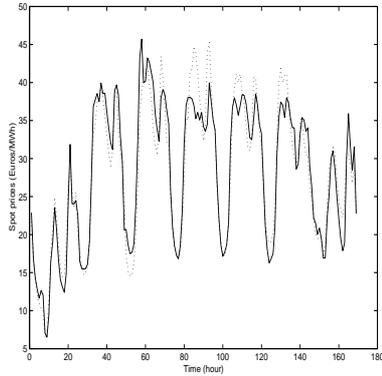


Figure 5: Seven days ahead spot prices forecast for EEX spot market using model  $M_2$ , from March 16, 2003 to March 22, 2003.

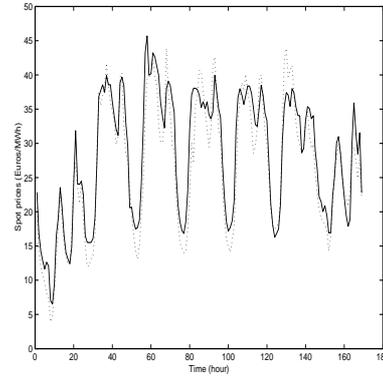


Figure 6: Seven days ahead spot prices forecast for EEX spot market using model  $M_1$ , from March 16, 2003 to March 22, 2003.

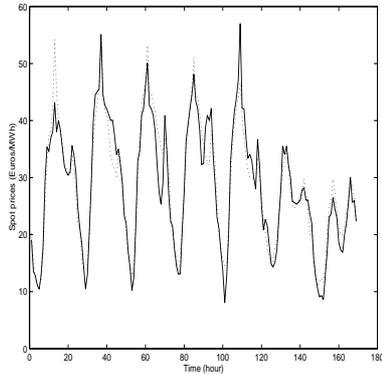


Figure 7: Seven days ahead spot prices forecast for EEX spot market using model  $M_2$ , from September 8, 2003 to September 14, 2003.

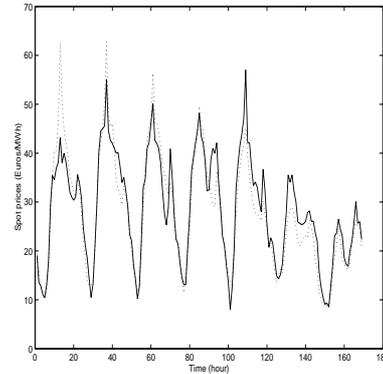


Figure 8: Seven days ahead spot prices forecast for EEX spot market using model  $M_1$ , from September 8, 2003 to September 14, 2003.

## 5 Conclusion

In this paper we have investigated forecasting's method using a stochastic model such as a  $k$ -factor GIGARCH process. We derive a least square predictor and its properties and we characterize the conditional variance error of this predictor. The results provided in section 3 are new and permit to obtain exact predictions using a  $k$ -factor GIGARCH process. This forecasting method is applied to the German hourly electricity spot market prices.

We test two models: a 1-factor GIGARCH model and a 3-factor GIGARCH model. The forecasting results for the year 2003 with the two proposed models are highly convening in the sense of RMSE criteria. We also conclude that model  $M_2$  (3-factor GIGARCH process) provides better forecasts in the sense of RMSE criteria than model  $M_1$  (1-factor GIGARCH process), when modelling EEX prices on the period under study. Empirical studies show that the methodology proposed in this paper can easily be applied to other spot markets having similar characteristics such as Powernext in France or APX in The Netherlands.

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	1 h	3 h	5 h	11 h	17 h	22 h	23 h	00 h
1 jour	5.39	4.44	4.68	16.17	8.97	5.61	5.45	4.16
	<b>3.63</b>	<b>3.72</b>	<b>3.76</b>	<b>15.29</b>	<b>8.47</b>	<b>5.38</b>	<b>5.63</b>	<b>4.48</b>
2 jours	5.17	4.92	4.88	17.08	9.26	5.87	5.83	4.53
	<b>4.88</b>	<b>4.73</b>	<b>4.61</b>	<b>16.94</b>	<b>9.32</b>	<b>6.25</b>	<b>6.50</b>	<b>5.27</b>
3 jours	5.33	5.22	5.08	17.46	9.50	6.09	6.09	4.77
	<b>5.58</b>	<b>5.21</b>	<b>5.03</b>	<b>17.43</b>	<b>9.59</b>	<b>6.47</b>	<b>6.64</b>	<b>5.36</b>
4 jours	5.44	5.39	5.20	17.69	9.67	6.24	6.26	4.96
	<b>5.68</b>	<b>5.30</b>	<b>5.12</b>	<b>17.49</b>	<b>9.61</b>	<b>6.28</b>	<b>6.39</b>	<b>5.18</b>
5 jours	5.54	5.50	5.29	17.83	9.73	6.32	6.37	5.07
	<b>5.51</b>	<b>5.28</b>	<b>5.14</b>	<b>17.49</b>	<b>9.57</b>	<b>6.09</b>	<b>6.16</b>	<b>4.98</b>
6 jours	5.60	5.58	5.33	17.92	9.80	6.40	6.43	5.17
	<b>5.45</b>	<b>5.32</b>	<b>5.16</b>	<b>17.54</b>	<b>9.57</b>	<b>6.05</b>	<b>6.09</b>	<b>4.94</b>
7 jours	5.64	5.67	5.39	17.97	9.85	6.47	6.50	5.26
	<b>5.45</b>	<b>5.37</b>	<b>5.20</b>	<b>17.55</b>	<b>9.63</b>	<b>6.14</b>	<b>6.15</b>	<b>4.98</b>
8 jours	5.70	5.77	5.45	17.60	9.29	6.54	6.41	5.32
	<b>5.44</b>	<b>5.40</b>	<b>5.21</b>	<b>17.10</b>	<b>8.97</b>	<b>6.22</b>	<b>6.12</b>	<b>5.11</b>
9 jours	5.65	5.72	5.44	17.64	9.29	6.52	6.34	5.34
	<b>5.44</b>	<b>5.37</b>	<b>5.23</b>	<b>17.07</b>	<b>8.97</b>	<b>6.27</b>	<b>6.17</b>	<b>5.21</b>
10 jours	5.55	5.68	5.43	17.59	19.22	6.48	6.33	5.32
	<b>5.37</b>	<b>5.35</b>	<b>5.23</b>	<b>16.98</b>	<b>8.87</b>	<b>6.27</b>	<b>6.20</b>	<b>5.21</b>

Table 2: RMSE for the two models associated to the spot prices at some hours each day from one day-ahead spot prices forecasts to ten day-ahead spot prices forecasts (the bold numbers are RMSE provided by model  $M_2$  while the others are provided by model  $M_1$ ).

	1 h	3 h	5 h	11 h	17 h	22 h	23 h	00 h
11 jours	5.46	5.60	5.38	17.48	9.18	6.46	6.31	5.33
	<b>5.34</b>	<b>5.29</b>	<b>5.17</b>	<b>16.89</b>	<b>8.83</b>	<b>6.19</b>	<b>6.09</b>	<b>5.17</b>
12 jours	5.42	5.55	5.39	17.50	9.21	6.47	6.32	5.36
	<b>5.27</b>	<b>5.28</b>	<b>5.18</b>	<b>16.89</b>	<b>8.84</b>	<b>6.11</b>	<b>5.98</b>	<b>5.10</b>
13 jours	5.43	5.57	5.41	17.50	9.20	6.49	6.33	5.40
	<b>5.25</b>	<b>5.29</b>	<b>5.18</b>	<b>16.87</b>	<b>8.82</b>	<b>6.13</b>	<b>5.98</b>	<b>5.11</b>
14 jours	5.46	5.59	5.44	17.39	9.14	6.50	6.36	5.44
	<b>5.29</b>	<b>5.33</b>	<b>5.20</b>	<b>16.75</b>	<b>8.74</b>	<b>6.16</b>	<b>6.05</b>	<b>5.17</b>
15 jours	5.69	5.72	5.60	17.48	9.25	6.63	6.46	5.58
	<b>5.44</b>	<b>5.44</b>	<b>5.33</b>	<b>16.70</b>	<b>8.78</b>	<b>6.29</b>	<b>6.23</b>	<b>5.38</b>
16 jours	5.70	5.75	5.63	17.52	9.28	6.68	6.50	5.64
	<b>5.57</b>	<b>5.50</b>	<b>5.41</b>	<b>16.76</b>	<b>8.79</b>	<b>6.40</b>	<b>6.36</b>	<b>5.51</b>
17 jours	5.74	5.76	5.67	17.57	9.30	6.71	6.53	5.68
	<b>5.66</b>	<b>5.56</b>	<b>5.47</b>	<b>16.84</b>	<b>8.80</b>	<b>6.43</b>	<b>6.38</b>	<b>5.53</b>
18 jours	5.78	5.78	5.70	17.59	9.30	6.74	6.56	5.73
	<b>5.67</b>	<b>5.58</b>	<b>5.50</b>	<b>16.88</b>	<b>8.83</b>	<b>6.41</b>	<b>6.33</b>	<b>5.50</b>
19 jours	5.82	5.78	5.72	17.55	9.32	6.78	6.58	5.75
	<b>5.67</b>	<b>5.58</b>	<b>5.50</b>	<b>16.85</b>	<b>8.83</b>	<b>6.36</b>	<b>6.26</b>	<b>5.45</b>
20 jours	5.85	5.79	5.75	17.55	9.34	6.81	6.60	5.77
	<b>5.63</b>	<b>5.58</b>	<b>5.51</b>	<b>16.83</b>	<b>8.82</b>	<b>6.34</b>	<b>6.24</b>	<b>5.43</b>

Table 3: RMSE of the two models associated to the spot prices at some hours each day from eleven day-ahead spot prices forecasts to twenty day-ahead spot prices forecasts (the bold numbers are RMSE provided by model  $M_2$  while the others are provided by model  $M_1$ ).

	1 h	3 h	5 h	11 h	17 h	22 h	23 h	00 h
21 jours	5.87	5.80	5.78	17.57	9.36	6.84	6.62	5.78
	<b>5.61</b>	<b>5.59</b>	<b>5.54</b>	<b>16.85</b>	<b>8.83</b>	<b>6.36</b>	<b>6.28</b>	<b>5.47</b>
22 jours	6.04	5.90	5.90	17.85	9.56	7.05	6.80	5.91
	<b>5.67</b>	<b>5.66</b>	<b>5.64</b>	<b>17.11</b>	<b>9.03</b>	<b>6.60</b>	<b>6.56</b>	<b>5.70</b>
23 jours	6.04	5.92	5.90	18	9.60	7.10	6.86	5.96
	<b>5.82</b>	<b>5.72</b>	<b>5.71</b>	<b>17.35</b>	<b>9.13</b>	<b>6.72</b>	<b>6.69</b>	<b>5.82</b>
24 jours	6.07	5.95	5.93	18.15	9.62	7.15	6.92	5.99
	<b>5.91</b>	<b>5.79</b>	<b>5.77</b>	<b>17.55</b>	<b>9.16</b>	<b>6.75</b>	<b>6.70</b>	<b>5.83</b>
25 jours	6.09	6.00	5.96	18.20	9.65	7.19	6.95	6.01
	<b>5.91</b>	<b>5.83</b>	<b>5.80</b>	<b>17.55</b>	<b>9.14</b>	<b>6.69</b>	<b>6.64</b>	<b>5.76</b>
26 jours	6.10	6.01	5.96	18.28	9.65	7.23	7.00	6.04
	<b>5.86</b>	<b>5.81</b>	<b>5.78</b>	<b>17.53</b>	<b>9.11</b>	<b>6.58</b>	<b>6.50</b>	<b>5.66</b>
27 jours	6.12	6.04	5.98	18.33	9.66	7.27	7.03	6.05
	<b>5.84</b>	<b>5.81</b>	<b>5.79</b>	<b>17.54</b>	<b>9.09</b>	<b>6.56</b>	<b>6.47</b>	<b>5.65</b>
28 jours	6.13	6.09	6.02	18.37	9.68	7.29	7.05	6.06
	<b>5.83</b>	<b>5.84</b>	<b>5.83</b>	<b>17.62</b>	<b>9.12</b>	<b>6.61</b>	<b>6.51</b>	<b>5.67</b>
29 jours	6.28	6.21	6.16	18.66	10.04	7.52	7.21	6.17
	<b>5.89</b>	<b>5.96</b>	<b>5.95</b>	<b>17.84</b>	<b>9.51</b>	<b>6.86</b>	<b>6.73</b>	<b>5.84</b>
30 jours	6.26	6.24	6.16	18.73	10.07	7.54	7.25	6.20
	<b>5.99</b>	<b>6.02</b>	<b>6.01</b>	<b>17.94</b>	<b>9.62</b>	<b>6.95</b>	<b>6.82</b>	<b>5.90</b>

Table 4: RMSE of the two models associated to the spot prices at some hours each day from twenty one day-ahead spot prices forecasts to one month-ahead spot prices forecasts (the bold numbers are RMSE provided by model  $M_2$  while the others are provided by model  $M_1$ ).