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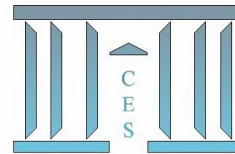
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**Which is the best model for the US inflation rate : A  
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# Which is the best model for the US Inflation rate: A structural changes model or a long memory process?

Lanouar Charfeddine <sup>\*†</sup>      Dominique Guégan <sup>‡</sup>

## Abstract

This paper analyzes the dynamics of the US inflation series using two classes of models: structural changes models and Long memory process. For the first class we use the Markov Switching (MS-AR) model of Hamilton (1989) and the Structural CHange (SCH-AR) model using the sequential method proposed by Bai and Perron (1998, 2003). For the second class, we use the ARFIMA process developed by Granger and Joyeux (1980). Moreover, we investigate whether the observed long memory behavior is a true behavior or a spurious behavior created by the presence of breaks in time series.

Our empirical results provide evidence for changes in mean, breaks dates coincide exactly with some economic and financial events such Vietnam War and the two oil price shocks. Moreover, we show that the observed long memory behavior is spurious and is due to the presence of breaks in data set.

JEL classification: C13;C32;E3

Keywords: Structural breaks models, Long range dependance, Inflation series

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# 1 Introduction

One of the most examined macro-economic series in the empirical literature is the inflation time series. The importance of this variable comes from the central role that plays in economic policies. Changes in the inflation time series has an important implication on the private behavior of investment and on the competitiveness of the economics. Moreover, inflation behavior has important effects on the performance of many financial and economic models.

The existent empirical econometric literature shows that inflation time series can be modelled by different processes and until now no consensus has been reached about the true behavior of this variable. Many researchers find evidence for  $I(0)$  behavior, whereas several other studies argue that this series follows an  $I(1)$  process. Recently, some empirical works provide evidence for long range dependence and show that the estimated fractionally integrated parameter  $d$  is significantly different from 0 and lies generally inside  $(0, \frac{1}{2})$ . This last approach suggests that shocks on inflation have a long-lasting effect, for more details see Hassler and Wolters (1995), Baillie, Chung and Tieslau (1996), Bos, Franses and Ooms (1999), and Lee (2005) among others.

With respect to the presence of jumps inside the inflation series an interesting approach is to use models with changes in mean. In that case, breaks can be caused by international events such as war, oil-price shocks, changes in monetary policies and so on. Among the available switching models in the literature a special attention is attributed to two kinds of models: The Markov switching model developed by Hamilton (1989, 1990) and the Structural CHange Auto-Regressive SCH-AR model introduced in the literature by Quandt (1958, 1960) and developed recently by Bai and Perron (1998, 2003).

The importance of the Markov switching model lies on the fact that we can allow to the mean and the variance to change over states. Applications of this model in economic and finance have been extensively increased in recent years, see for instance Garcia and Perron (1996) which applied this model to the US inflation rate and which support the presence of three states in both mean and variance. In the other hand, structural changes (SCH-AR) model have the advantage of allowing for multiple and unknown dates of breaks.

An important problem that encounter researchers, when they try to use these two classes of models for real data, lies on the possibility of confusing between long memory process and models with shifts in mean. Discrimination between these two classes of models is problematic because spurious long memory behavior can be detected in time series known to be theoretic-

cally short memory with changes in means. Several theoretical and empirical works have dealt with this problem and until now no statistical test or empirical strategy are available to solve this problem, see for instance Diebold and Inoue (2001), Granger and Hyung (2004), Hsu and Kuan (2000) and Charfeddine and Guégan (2006) among others.

In this paper we address two main objectives. First, we propose a model that describes very well the dynamics of the US inflation time series. Then, we investigate the nature of the observed long memory behavior. To select a model that fit very well the US inflation series, we examine in the empirical application three models: i) the structural change models as developed by Bai and Perron (1998, 2003), ii) the Markov switching model of Hamilton (1989) and iii) the ARFIMA long memory process of Granger and Joyeux (1980). Our empirical results establish instability inside the inflation time series. The selected Markov switching model shows that the US inflation rate switches between four regimes. The first regime detects outliers in the US inflation series. The three others describe the dynamics of the inflation series. Moreover, we show that the observed long memory behavior is spurious and is due to the presence of breaks inside the inflation series.

The remaining of the paper proceeds as follows. Section 2 describes some tests which discriminate between the different switching models. Section 3 recalls the methods that we use to detect the presence of long memory behavior. Section 4 presents the main empirical results for the US inflation time series. Finally, section 5 concludes.

## 2 Tests for structural breaks

Many economic and financial time series exhibit sudden changes, great depression, irreversibility time, and so on. Modeling these series using models with constant coefficients lead to a wrong specification which induces problems in forecasting and in the analysis of policy changes. Thus, in the last 50 years we have assisted to an important development of the literature concerning tests that detect the presence of changes inside the data sets. Quandt (1958,1960) discusses the problem of testing the null hypothesis of constant coefficients against the alternative that a structural change occurred at unknown time. Implementation of this procedure has been hindered by the lack of a distribution theory<sup>1</sup>. Chernoff and Zacks (1964) and Karder and Zacks (1966) suggest to use partial sums of demeaned data to analyze structural changes. Brow, Durbin, and Evans (1975) proposed the CUSUM test based on recursive residuals. Kim and Siegmund (1989) used likelihood ratio tests

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<sup>1</sup>Quandt shows basis on empirical analysis that the  $\chi^2$  distribution is a poor approximation to the true distribution.

to detect a structural change in a simple linear regression. They tabulated the critical values using Monte Carlo methods. Andrews (1993) and Andrews and Ploberger (1994) suggest to use the *SupLM* test and a class of average exponential LM, Wald and LR tests. Andrews and Ploberger (1995) show that the Sup LR test is asymptotically admissible.

Garcia (1998) developed the Sup LR test to detect switches coming from Markov switching model. Gong and Mariano (1997) developed two tests in the frequency domain: the Difference Test  $DT_N$  and the *LM* test. They derive their exact asymptotic null distributions under the condition of unidentified nuisance parameters. Recently, Carrasco et al (2004) studied the *SupTS* test which is 'asymptotically equivalent to Garcia's test in the sense that both are close to likelihood ratio tests and hence they are expected to have similar powers'<sup>2</sup>. In an interesting paper, Bai and Perron (1998, 2003) proposed a sequential procedure that allows to detect the presence of multiple structural changes which occurs at unknown time.

## 2.1 Garcia's test

In this paper, we focus our analysis on models that have only changes in mean. The proposed Sup LR test of Garcia (1998) is the most robust test constructed to recognize a Markov Switching model under the alternative. As shown in empirical investigation this test has a good power to detect switches in regimes whatever the origin of the switch. Garcia (1998, appendix.3, p.785) shows that the Sup LR test has the same distribution under the null when the alternative is driven from one of the three following models: the MS-AR, the SETAR or the Structural CHange Auto-Regressive SCH-AR models without auto-regressive component.

The testing procedure proposed by Garcia (1998) is briefly presented in this subsection. We start by presenting the general form of the Markov switching model as reported in Hamilton (1989). Let  $(y_t)$  the process

$$(y_t - \mu_{s_t}) = \phi_1(y_t - \mu_{s_{t-1}}) + \phi_2(y_t - \mu_{s_{t-2}}) + \dots + \phi_p(y_t - \mu_{s_{t-p}}) + e_t. \quad (1)$$

where,  $e_t = N(0, \sigma_{s_t}^2)$  and,

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<sup>2</sup>When nuisance parameters are present under the alternative hypothesis the usual statistics test the Likelihood Ratio (LR), the Lagrange Multiplicateur (LM) and the Wald (W) tests have not their standard null distribution, this is the so-called Davies problem. This problem occur in the case of TAR, MSAR and structural change model when we test the null hypothesis of linear model against the alternative of several states, see also Charfeddine and Guégan (2005).

$$\mu_{s_t} = \mu_1 s_{1t} + \mu_2 s_{2t} + \dots + \mu_n s_{nt},$$

$$\sigma_{s_t}^2 = \sigma_1^2 s_{1t} + \sigma_2^2 s_{2t} + \dots + \sigma_n^2 s_{nt},$$

with  $s_{jt} = 1$ , if  $s_t = j$ , and  $s_{jt} = 0$ , otherwise,  $j= 1, \dots, n$  and  $p_{ij} = Pr[s_t = j | s_{t-1} = i]$  and  $\sum_{j=1}^n p_{ij} = 1$ .

To test the null hypothesis ( $H_0$ ) of  $l$  states or regimes against the alternative hypothesis ( $H_1$ ) of  $l + 1$  regimes Garcia (1998) proposes to use the Sup LR test:

$$LR = 2[L(\hat{\beta}) - L(\tilde{\beta})]. \quad (2)$$

where  $L(\cdot)$  represents the log-likelihood function,  $\hat{\beta}$  and  $\tilde{\beta}$  are respectively the maximum likelihood estimators of the parameters of interest under the alternative hypothesis of  $l + 1$  regimes and the null hypothesis of  $l$  regimes. To avoid the possibility of local maxima we use a lot of starting values in order to be sure that the maximum obtained is a global one.

## 2.2 Bai and Perron's test

This subsection is devoted to describe the sequential method as proposed in Bai (1997) and Bai and Perron (1998, 2003). The authors proposed a test which considers the date of breaks as unknown and which allows for multiple structural changes.

We consider the following multiple linear regression with  $l$  breaks ( $l + 1$  regimes):

$$\begin{aligned} y_t &= x_t' \beta + z_t' \delta_1 + u_t & \text{if } 1 \leq t \leq T_1 \\ y_t &= x_t' \beta + z_t' \delta_2 + u_t & \text{if } T_1 < t \leq T_2 \\ & \vdots & \\ & \vdots & \\ & \vdots & \\ y_t &= x_t' \beta + z_t' \delta_{l+1} + u_t & \text{if } T_l < t \leq T. \end{aligned} \quad (3)$$

In this model,  $y_t$  is the observed dependent variable,  $x_t$  ( $p \times 1$ ) and  $z_t$  ( $q \times 1$ ) are vectors of covariates, and  $\beta$  and  $\delta_j$  ( $j = 1, \dots, l + 1$ ) are the corresponding vectors of coefficients,  $u_t$  is the disturbance. The break points, are explicitly treated as unknown. When  $\beta$  is not subject to shifts, the model is called a partial structural change model and by imposing  $p = 0$  we obtain the pure structural change model in which all the coefficients vary with the break points.

To test for the presence of structural changes we need to estimate the unknown coefficients  $(\beta^0, \delta_1^0, \dots, \delta_{l+1}^0, T_1^0, \dots, T_l^0)$ . These parameters are obtained by minimizing the sum of squared residuals (SSR) from (3). Then, to determine if structural changes occur, Bai and Perron (1998) suggest to use the following two statistics: the  $UD_{\max} = \max_{1 \leq l \leq L} \sup F(l)$  statistic, where  $L$  denote the maximum number of breaks allowed and the  $WD_{\max} = \max_{1 \leq l \leq L} \omega_l \sup F_T(l)$ , where the weights are such that the marginal  $p$ -values are equal across values of  $l$ . Moreover, the authors propose a Sup F type test of no structural change ( $l = 0$ ) against the alternative hypothesis of ( $l = i$ ) with  $i = 1, \dots, l$  breaks.

To determine exactly the number of breaks, Bai and Perron (1998, 2003) propose a sequential procedure which is based on the three tests presented above and a  $SupF(l+1/l)$  test for the null hypothesis of  $l$  states against the alternative of  $l+1$  states, for more details see Bai and Perron (1998, 2003).

### 3 Procedures to detect long memory behavior

In the last two decades a varieties of methods that estimate the fractional long memory parameter  $d$  have been proposed. In this section, we describe in a first part the ARFIMA model. Then, we present the GPH method proposed by Geweke and Porter-Hudak (1983) and the Exact Local Whittle (ELW) of Shimotsu & Phillips (2005).

#### 3.1 The ARFIMA model

The first long memory process introduced in the literature is the popular Auto-Regressive Fractionally Integrated Moving Average (ARFIMA) model developed independently by Granger and Joyeux (1980) and Hosking (1981).

We say that a process  $\{y_t\}_1^T$ , with  $t \in Z$ , follows an  $ARFIMA(p, d, q)$  process if it takes the form,

$$\Phi(B)(I - B)^d(y_t - \mu) = \Theta(B)u_t, \quad (4)$$

where  $\Phi(\cdot)$  and  $\Theta(\cdot)$  are the autoregressive and the moving average polynomials of order  $p$  and  $q$  respectively, whose roots lie outside the unit circle and  $\mu$  is an unknown mean.  $(u_t)_t$  is a Gaussian strong white noise  $N(0, \sigma_u^2)$  and  $B$  is the lag operator. If  $d \in (-\frac{1}{2}, 0)$  the  $ARFIMA(p, d, q)$  model (4) is an invertible stationary process with intermediate memory and if  $d \in (0, \frac{1}{2})$  the model (4) is stationary and invertible and has an autocorrelation function  $\rho(k)$  which exhibits a slow decay when the lag  $k$  increases, see Beran (1994). In this latter case we say that we are in presence of a stationary long memory behavior.



The testing procedure for the presence of a long memory behavior consists on testing the null hypothesis

$$H_0 : d = 0 \quad \text{against the alternative} \quad H_1 : d \neq 0, \quad (5)$$

where  $d$  is the long memory parameter introduced in (4). Thus, under the null hypothesis ( $H_0$ ) we have a short memory behavior and a long range dependence under the alternative ( $H_1$ ).

### 3.2 The GPH technique

The first semi-parametric method that we use to estimate the long memory parameter is the GPH technique. This method is based on the log-periodogram. For frequency near zero,  $d$  can be consistently estimated from the least squares regression

$$\ln\{I(w_j)\} = a - d \ln\{4\sin^2(w_j/2)\} + \epsilon_t, \quad j = 1, \dots, m. \quad (6)$$

For consistency it is required that  $m$  grows slowly with respect to the sample size. It is suggested to set  $m = T^r$  with  $r = 0.5$ ,  $m \in N$  see Banerjee and Urga (2004).  $I(w_j)$  is the periodogram of the process  $(Y_t)_t$  at frequency  $w_j = 2\pi j/T$ . The ordinate least-square estimator of  $d$  is asymptotically normal with standard error equal to  $\pi(6m)^{-1/2}$ , see Geweke and Porter-Hudak (1983) and Robinson (1995). Agiakloglou *et al.* (1992) show that this method is biased and inefficient when the error term is an  $AR(1)$  or an  $MA(1)$  and in addition this estimator does not possess good asymptotic properties. Note that this method is only robust for  $|d| < 1/2$ .

### 3.3 The Exact Local Whittle (ELW)

Another semi-parametric method That we want to use in this paper is the Exact Local Whittle (ELW) method proposed by Shimotsu and Phillips (2005). This method avoids some approximation in the derivation of the Local Whittle estimator proposed by Künsch (1987) and Yajima (1989). The exact local Whittle (ELW) approach is attractive because the ELW estimator has interesting asymptotic properties under reasonable assumptions. The estimated value  $\hat{d}_{ELW}$  is obtained as follows:

$$\hat{d}_{ELW} = \text{Arg} \min_{d \in [d_1, d_2]} R(d), \quad (7)$$

where  $d_1$  and  $d_2$  are the lower and upper bounds of the admissible values of  $d$  such that  $-\infty < d_1 < d_2 < \infty$  and,

$$R(d) = \ln G(d) - 2d \frac{1}{m} \sum_{j=1}^m \ln(\omega_j), \quad (8)$$

where  $m$  is the truncation parameter, and  $G(d) = \frac{1}{m} \sum_{j=1}^m I_{\Delta^{d_y}}(\omega_j)$  where  $I_{\Delta^{d_y}}(\omega) = \frac{1}{2\pi T} \left| \sum_{t=1}^T \Delta^d y_t e^{it\omega} \right|^2$  is the periodogram of  $\Delta^d y_t = (1 - L)^d y_t$ .

Under certain assumptions<sup>3</sup> the ELW estimator  $\hat{d}_{ELW}$  satisfies

$$\sqrt{m}(\hat{d}_{ELW} - d) \rightarrow_d N(0, 1/4), \quad \text{when } T \rightarrow \infty. \quad (9)$$

## 4 US Inflation Application

In this section we analyze the behavior of the US inflation time series by using the following three models: the Markov switching model, the structural change model and finally the ARFIMA long memory process. Our first object is to select the model that fit better the inflation behavior. Then, we investigate the nature of the observed long memory behavior.

### 4.1 Motivation and Data

We use a monthly data for the U.S inflation rate over the period January 1957 to June 2006,  $T=594$  points. The dataset is provided by Datastream Base. Inflation rates are constructed by computing  $y_t = 12 * 100 * [\log(p_t) - \log(p_{t-1})]$ , where  $p_t$  is the Consumer Price Indices (CPI). We use a seasonally adjusted data set. Graphical representation and autocorrelation function for the US inflation rate time series are given in figure 1 (a) and (b).

Figure (b) shows a slowly decaying of the autocorrelation function which means that a long memory process can be used to model this time series. In an other hand, the trajectory of the series, figure (a), shows the presence of changes between some regimes.

Table 1 shows descriptive statistics for the US inflation series where the normal distribution is rejected using the Jarque-Bera test.

Table 1: Descriptive statistics for the US inflation time series.

	Mean	Med.	Std. Dev.	Skw.	Kur.	J-B
US Inflation	4.024	3.571	3.632	0.913	4.768	160.014

### 4.2 Results

In this subsection, we perform in a first part two switching models: the Markov switching model of Hamilton (1989) and the structural change model

<sup>3</sup>For consistency and asymptotic normality see assumptions (1-5) and (1'-5') in Shimotsu and Phillips (2005)

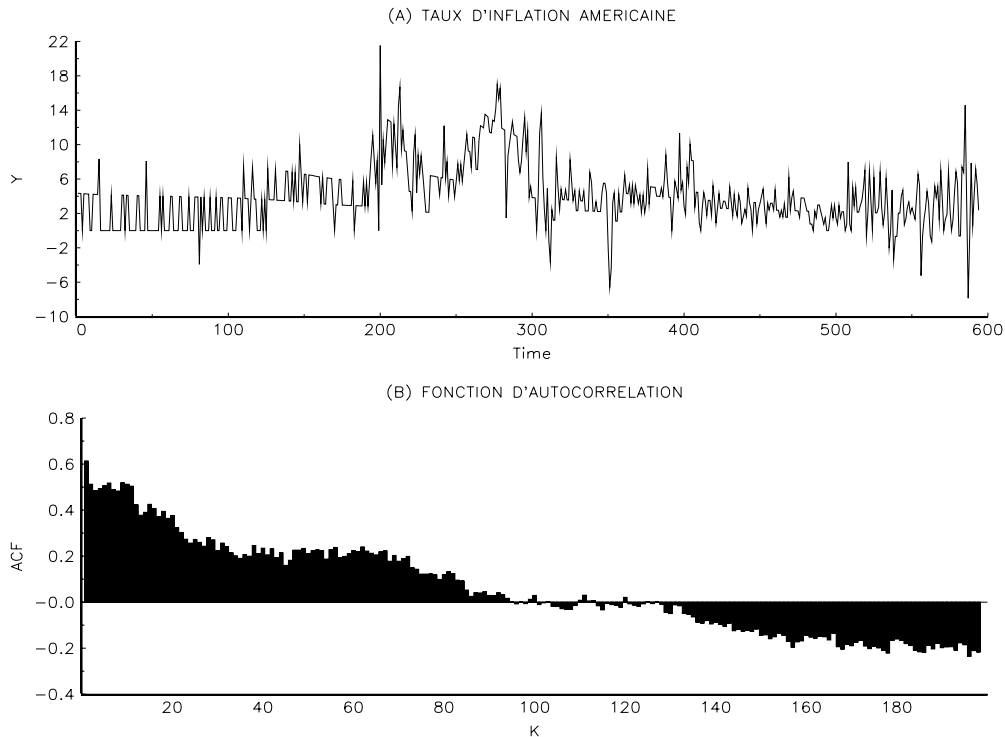


Figure 1: (A) and (B) are respectively the trajectory and the ACF of the US inflation time series.

using the sequential procedure proposed by Bai and Perron (1998, 2003). Then, in the second part we perform an ARFIMA model.

#### 4.2.1 MSAR models

As noted in previous section a special problem that researchers encounter when they estimate the Markov switching model is the selection of the exact number of regimes. This problem is due to the presence of nuisance parameters under the null hypothesis. Only few works have dealt with this problem, see for instance Hansen (1992), Garcia (1998), Carrasco (2004) and Charfeddine and Guégan (2005). Indeed, the usual tests (LR, LM and Wald) do not have the standard asymptotic distribution. Garcia (1998) has tabulated critical values for some Markov switching models and shows that they depend on the autoregressive parameter.

In our empirical application we use the following Markov switching specification, where  $y_t$  is the observed inflation rate, then

$$(y_t - \mu_{s_t}) = \phi(y_{t-1} - \mu_{s_{t-1}}) + e_t. \quad (10)$$

where  $e_t \sim N(0, \sigma^2)$ , and,

$$\mu_{s_t} = \mu_1 s_{1t} + \mu_2 s_{2t} + \mu_3 s_{3t} + \mu_4 s_{4t} + \mu_5 s_{5t},$$

where  $s_{jt} = 1$ , if  $s_t = j$ , and  $s_{jt} = 0$ , otherwise,  $j= 1, 2, 3, 4, 5$  and  $p_{ij} = Pr[s_t = j | s_{t-1} = i]$  and  $\sum_{j=1}^5 p_{ij} = 1$ .

Table 2: Estimates of MS-AR(1) models of the U.S inflation time series (Monthly data)

Par.	AR(1)	MS-2S-AR(1)	MS-3S-AR(1)	MS-4S-AR(1)	MS-5S-AR(1)
$\mu_1$	1.552*** (8.813)	3.014*** (11.496)	4.839*** (11.235)	-3.390*** (-3.385)	-3.475*** (-3.749)
$\mu_2$	-	10.989*** (15.441)	2.338*** (10.782)	2.196*** (10.540)	2.183*** (10.484)
$\mu_3$	-	-	11.375*** (17.689)	4.525*** (13.164)	4.314*** (12.720)
$\mu_4$	-	-	-	11.359*** (19.010)	5.316*** (8.161)
$\mu_5$	-	-	-	-	11.546*** (18.397)
$\phi$	0.615*** (18.927)	0.382*** (10.792)	0.236*** (6.019)	0.256*** (7.029)	0.243*** (6.339)
$\sigma^2$	8.248	5.908	5.318	4.665	4.636
$p_{11}$	-	0.986	0.967	0.466	0.467
$p_{12}$	-	-	0.010	0.000	0.000
$p_{13}$	-	-	-	0.402	0.402
$p_{14}$	-	-	-	-	0.000
$p_{21}$	-	-	0.011	0.000	0.000
$p_{22}$	-	0.895	0.989	0.993	0.992
$p_{23}$	-	-	-	0.008	0.004
$p_{24}$	-	-	-	-	0.004
$p_{31}$	-	-	0.085	0.019	0.009
$p_{32}$	-	-	0.030	0.006	0.008
$p_{33}$	-	-	-	0.944	0.981
$p_{34}$	-	-	-	-	0.002
$p_{41}$	-	-	-	0.000	0.043
$p_{42}$	-	-	-	0.000	0.000
$p_{43}$	-	-	-	0.1287	0.000
$p_{44}$	-	-	-	-	0.817
$p_{51}$	-	-	-	-	0.000
$p_{52}$	-	-	-	-	0.000
$p_{53}$	-	-	-	-	0.000
$p_{54}$	-	-	-	-	0.147
Log-L	<b>-1465.922</b>	<b>-1417.379</b>	<b>-1396.542</b>	<b>-1382.981</b>	<b>-1373.64</b>

Note: t-stat are in parentheses.

The estimation of the Markov switching models are reported in Table 2. To check for possible mis-specification we calculate the LR statistic as proposed by Garcia (1998). The value of the Likelihood Ratio statistic of the null hypothesis of linear AR(1) model against the alternative of two

Table 3: Estimates of MS-AR(1) models of the U.S inflation time series (Monthly data)

Par.	AR(1)	MS-2S-AR(1)	MS-3S-AR(1)	MS-4S-AR(1)	MS-5S-AR(1)
AIC	4.954	4.800	4.747	4.725	4.724
HQ	4.963	4.818	4.779	4.777	4.801
SC	4.976	4.845	4.829	4.858	4.924
J-B	295.155	89.000	105.905	24.728	40.295
Q(12)	109.55	53.514	17.46	18.766	19.166
Q(24)	173.33	85.514	36.16	35.049	35.956

states Markov switching model MS-2S-AR(1) is equal to 97.086. This value is largely higher than the 5% and 1% critical values as reported in Garcia (1998). Thus, we accept the two-states Markov switching specification. The values of the LR statistic of the hypothesis of  $l$  states against the alternative hypothesis of  $l + 1$  (for  $l = 2, 3$  and 4) states Markov switching model are respectively equals to 41.674, 27.122 and 18.682<sup>4</sup>. As we have noted, no critical values of the LR test are tabulated for more than two states. Thus, to select the appropriate Markov switching model we use the properties of the residuals innovations.

Using the Garcia's test we reject the null hypothesis of linear model. As shown in table 3, the four states specification have the best residuals properties. The MS-4S-AR(1) model has the minimum values of Akaike information criterion (AIC) and Hannan-Quinn (HQ) criteria. Also, the four states Markov switching specification have the minimum value of the J-B test<sup>5</sup> and no autocorrelation is detected in the residuals as shown by the Ljung-Box test, Q(12) and Q(24). Based on in these results, we selected the MS-4S-AR(1) model for the US inflation Time series. This choice is confirmed by some economic and financial events that have influenced the evolution of US inflation rate.

In order to assess whether our selected model fit very well the US inflation rate, we use the filtered and the smoothed probabilities to determine the dates of shifts. Then, we compared it with some events that have marked the US economy. The filtered probabilities of the estimated MS-4S-AR(1) model indicate that inflation switches between four regimes. The first regime detects outliers and the others three regimes describe the evolution of the inflation rate series. The filter probabilities show that inflation rate remains on the low state until May 1967. From May 1967 and January 1991 inflation oscillates between the middle and the high regime. Finally, the inflation rate comes back to the low state until the end of the sample were the series

<sup>4</sup>Note that the first two values 41.674 and 27.122 are largely higher than the critical value reported in Garcia (1998).

<sup>5</sup>The normality hypothesis of the residuals can be accepted if we filter completely the outlier detected at July 1973. In that case  $J-B=5.313 < \chi^2(2) = 5.99$ .

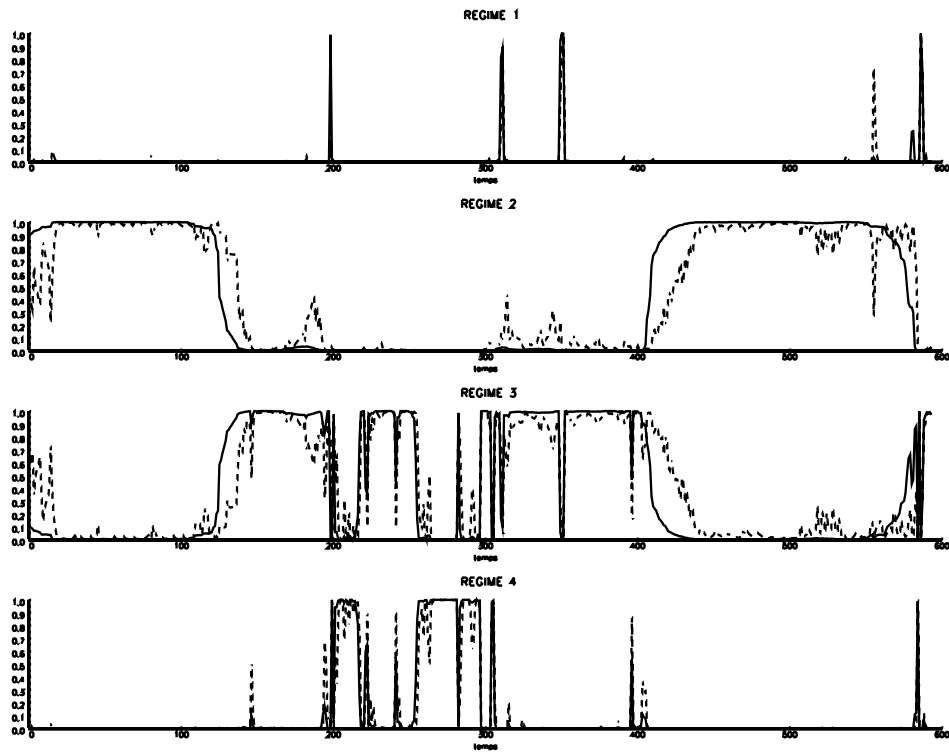


Figure 2: Filtered probabilities (dash line) and Smooth probabilities (solid line).

jumps to the middle state. Figure 3 shows exactly the dates of breaks as selected by the MS-4S-AR(1) model. The dates coincide with the Vietnam War 1967, the beginning and the end of the first oil price shock (1973-1975), the beginning and the end of the second oil crisis (1978-1982), the Iraqien War (1991) and finally the last increase of the oil price (2005).

The first regime which detects outliers inside the US inflation rate gives an idea concerning the volatility of this variable. From the filtered and the smoothed probabilities, it appears that the latter part of 1970s, the beginning of the 1980s, during the 1986 and 1992 years, and the end of 2005 year are characterized by a high volatility. The first two periods are due to changes in the monetary base and to the recession at 1981-82 years. The two others high volatility periods correspond to the Plaza Accord Agreement (1986) and the collapse of the ERM, September 1992. Finally, the high volatility at the end of 2005 year corresponds to the last increase of the oil price.

#### 4.2.2 The Structural CHange model

In order to determine the number of breaks in a time series. BP (1998, 2003) suggest to use the following sequential strategy. First, they propose to use

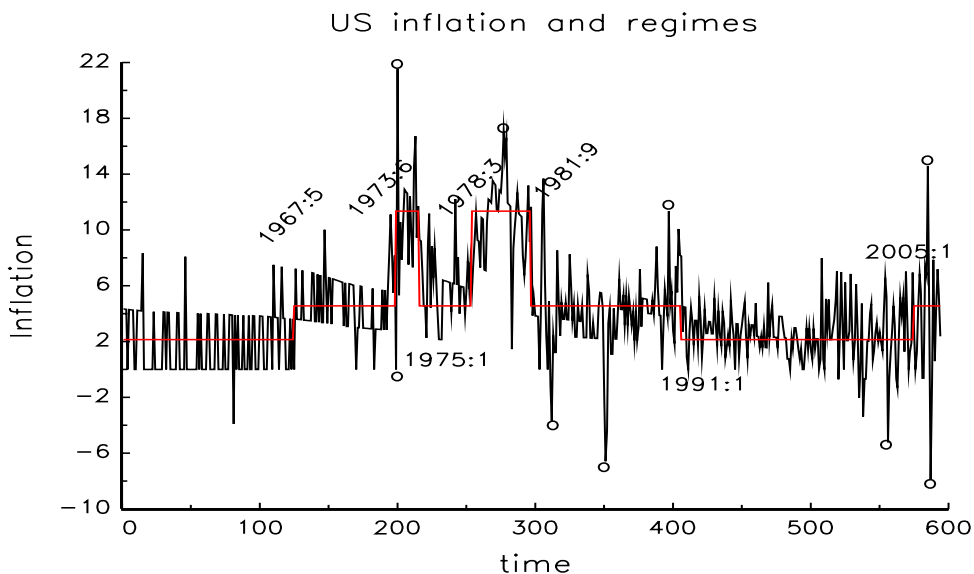


Figure 3: Solid line represent Regimes detected by the MS-4S-AR(4) specification and only the negative outliers "o" are detected by the first regime.

the Sup F, the UDmax and the WDmax tests to verify if at least one break exists in the data. Then, if the results are positive the authors suggest to use the sequential Sup F(1+1|1) to select the number of breaks. This strategy can be completed by using the Bayesian Information Criterion (BIC) or the modified Schwarz' Criterion (LWZ) due to Liu, Wu and Zidek (1997).<sup>6</sup>

The result concerning the Sup F, the UDmax and the WDmax tests for the US inflation time series is reported in tables 4-5. All tests provide evidence for structural breaks. The sequential method and the BIC criteria (not reported here) support the existence of three breaks in the US inflation rate. The results of the estimated structural change model are reported in table 7. Figure.4 shows the dates of breaks as selected by the sequential method:

Table 4: Sup F tests against a fixed number of breaks.

Test \ l	1	2	3	4	5
$Sup(0 l)$	8.115*	20.646***	18.476***	14.910***	12.117***

\* and \*\*\* Indicate that the corresponding values of the Sup F are significant at the 10% and 1% levels.

The estimated dates of breaks correspond to the 124 points which is equivalent to April 1967, the second break occurs at the 191 point which corresponds to January 1973 and the third break at the 296 point equivalent to July 1981. The first date corresponds to the Vietnam war, April 1967,

<sup>6</sup>We refer reader to BP (1998, 2003) for more details on the sequential procedure.

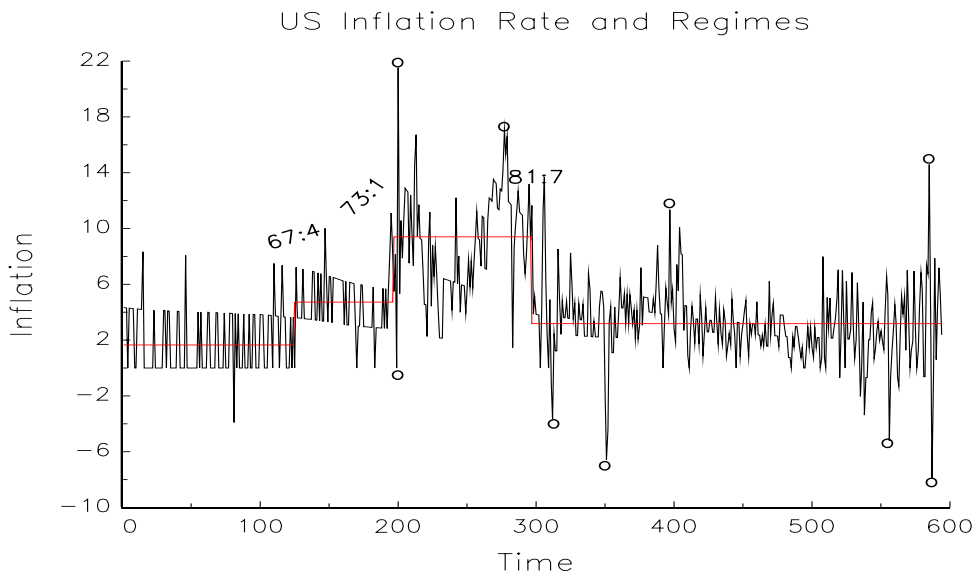


Figure 4: Solid line represent Regimes detected by the SCH-AR(6) model.

Table 5:  $W_{max}$  tests against an unknown number of breaks.

Test	10%	5%	1%
$WD_{max}$	22.511	24.580	28.526

The critical value at the 1%, 5% and 10% levels are respectively 14.53, 10.39 and 8.63.

which exerts a positive effect on prices. The second date corresponds to the first oil crisis January 1973 and finally the third break (July 1981) corresponds to the end of the second oil price shock. The first regime covers the period between January 1957 and April 1967 during which the inflation rate is stable at a low level with a mean equal to 1.653%. The second regime covers the period from April 1967 to January 1973 which is characterized by a high mean, 4.72%. The third period is characterized by a very high mean of 9.408%. During this latter period many shocks have exert an affect on the level of inflation such as the end of the Bretton Woods accord, the first and the second oil price crisis. The last period shows a low mean level of 3.175%. In this later regime inflation is more stable than in the two previous subperiods.

To test the adequacy of the estimated SCH-AR(6) model we examine whether the residuals series are white noise, see table 8. One need to test for residual autocorrelation, then for homoscedasticity and finally for normality. Compared to the US inflation series properties the residuals series of the SCH-AR(6) estimated model have a mean equal to zero, a small skewness and high kurtosis. Ljung-Box (LB) test for residual autocorrelation shows that  $Q_{LB}(10) = 13.885$  which is less than the critical value of a  $\chi^2(10)$ .



Table 6: Sequential of Sup F(1+1|l) tests.

(1+1 l)	Sup F(1+1 l) tests	Date of new break	corresponding events
(1 0)	21.313*	196 (January 73)	First oil price shock
(2 1)	48.612*	296 (July 81)	End of second oil crisis
(3 2)	44.241*	124 (April 1967)	Vietnam War
(4 3)	9.243 $\diamond$	407	Not significant break
(5 4)	0.733 $\diamond$	505	Not significant break

\* This indicates that the corresponding value of the  $Sup F(l+1|l)$  test is significant at 10% the level.  
 $\diamond$  this indicates that the corresponding value of the  $Sup F(l+1|l)$  test is not significant even at the 10% the level.

Table 7: The estimation of the structural change model SCH-AR(1) with 4 breaks as selected by the sequential method.

Models		AR(6)		SCH-AR(6)	
Paramter	Coefficient	Std. Error	Coefficient	Std. Error	
$\delta_1$	0.583	(0.189)	0.873	(0.252)	
$\delta_2$	-	-	2.491	(0.391)	
$\delta_3$	-	-	4.970	(0.534)	
$\delta_4$	-	-	1.683	(0.228)	
$\phi_1$	0.357	(0.041)	0.284	(0.039)	
$\phi_2$	0.076	(0.043)	-	-	
$\phi_3$	0.067	(0.043)	-	-	
$\phi_4$	0.098	(0.043)	-	-	
$\phi_5$	0.104	(0.043)	0.071	(0.040)	
$\phi_6$	0.154	(0.041)	0.099	(0.040)	
$\sigma^2$	7.075	(-)	6.543	(-)	

The  $R^2 = 0.513$ , the  $DW = 2.013$  and the estimated date of breaks are respectively 124, 196 and 296 for the SCH-AR(6) model. For the linear AR(6) model have an  $R^2 = 0.473$  and a value of  $DW = 2.021$ .

The Jarque-Bera test rejects the hypothesis of Gaussian distribution. The rejection of normality hypothesis is due to the existence of outliers in the US inflation series, see graphical trajectory of the residual series figure (3).

Table 8: Descriptive statistics for the US inflation time series.

	Mean	Med.	Std. Dev.	Skw.	Kur.	J-B	$Q_{LB}(10)$
Residus	-3.23 $10^{-10}$	-0.297	2.544	0.285	6.432	296.5	13.885

To summary, using the sequential method we accept a structural change model with four states and a four states Markov switching specification using residuals analysis and Garcia's test. In each estimated model the breaks dates coincide with specific economic events. Contrary to the SCH-AR(6) model, the MS-4S-AR(1) model shows that the second and third regime

occur more than once.<sup>7</sup> Also, the first regime in the MS-4S-AR(1) model detects outliers at special events and coincides with epochs with high volatility. Based on these results, we suggest that the MS-4S-AR(1) specification fits better the US inflation time series.<sup>8</sup> This contradicts the result given in Perron (2003) which suggests that the structural change model can describe better the US inflation time series.

As shown by the autocorrelation function this time series exhibits a long memory behavior. Thus, In the following subsection we model the US inflation series using long memory process. Then we propose a general strategy which discriminates between the true long memory behavior and the spurious one.

### 4.2.3 Testing for long memory behavior

The autocorrelation function of the US inflation time series shows a slow decaying at long lags. This means that this time series exhibit a persistent behavior and one shall model it using long memory process. Before estimating the long memory process one need to use some statistical tests to confirm the presence of such behavior. In the theoretical literature many statistical tests and methods have been proposed. From the available tests we use the R/S and the V/S statistics with different value of the truncation parameter  $q$ , and from the semi parametric methods we use the GPH and the ELW techniques. The empirical results of the testing procedures are reported in table 9 and 10 below:

Table 9: The results of the R/S and V/S long memory tests.

tests\q	0	1	2	5	10	20	30
R/S(q)	6.376	5.0128	4.338	3.319	2.558	1.939	1.67
V/S(q)	4.4666	2.767	2.067	1.21	0.718	0.412	0.307

Note: 1%, 5% and 10% critical value for the R/S and the V/S are 2.001, 1.747 and 1.620 for the R/S statistic and 0.2685, 0.1869 and 0.1518 for the V/S statistic.

Table 10: Estimated values of the fractional long memory parameter  $d$  using the ELW and the GPH methods.

m	$T^{0.5}$	$T^{0.6}$	$T^{0.7}$	$T^{0.8}$
$\hat{d}_{ELW}$	0.89	0.707	0.461	0.408
t-stat	(8.787)	(9.606)	(8.620)	(7.629)
$\hat{d}_{GPH}$	0.925	0.802	0.520	0.388
t-stat	(7.595)	(8.151)	(6.435)	(6.688)

Note: t-values are in parentheses.

<sup>7</sup>As shown in figure 2, regime 2 and 3 occur twice and regime 3 three times.

<sup>8</sup>We have fitted a Markov switching model by allowing to the mean and variance to depend on the Markov chain like in Garcia and Perron (1996) but the results are not so robust like in the select specification.

The R/S and V/S statistics provide evidence for long memory, at 5% level. The values of these two statistics are larger than the 5% critical value of 1.747 and 0.1869 except for the R/S statistic when we use a larger value of the truncation parameter ( $q=30$ ). The two semi parametric methods, the GPH and the ELW techniques, confirm this evidence of long range dependence and strong persistence. We use a value of the frequency  $m$  equal to  $T^{0.5}$ ,  $T^{0.6}$ ,  $T^{0.7}$  and  $T^{0.8}$  for the two methods. For all values of  $m$  the estimated parameter  $\hat{d}$  is significantly different from zero and the corresponding t-statistics confirm that results.

Now, we suppose that the observed long memory behavior is a real behavior and we perform a long memory process for the US inflation rate series. Then, we compare the estimated residuals of this model to the two previous structural change models. The results of the Exact Maximum Likelihood method are reported in table 11. We retain an ARFIMA (2,d,3) with  $\hat{d} = 0.449$ .

Table 11: The estimated ARFIMA(2,d,3) for the US inflation rate.

Parameter	Coef.	St-Err
$\hat{d}$	0.449	0.048
$\phi_1$	0.446	0.033
$\phi_2$	-0.953	0.038
$\theta_1$	-0.572	0.091
$\theta_2$	1.017	0.049
$\theta_3$	-0.173	0.082
$\sigma^2$	6.174	(-)

This means that the US inflation series has a persistent behavior. This model suggests that shocks on this variable have a long lasting effects. The residuals of the estimated model are correlated and the normality hypothesis is rejected (J-B= 410.34). Compared to the two previous structural change models the estimated residuals of this ARFIMA(2,d,3) specification has the higher value of J-B test and autocorrelated residuals. Then, compared to the Markov switching specification the ARFIMA(2,d,3) fit less better the US inflation series. Thus, we conclude that this model does not perform the results obtained from the MS-4S-AR(1) model.

Finally, we test using the residuals of the three estimated models: the MS-4S-AR(1), the SCH-AR(6) and the ARFIMA(2,d,3) specifications the presence of any remaining ARCH effects . For each model, we compute the F-statistic using a regression based on three-order autoregressive structure for the squared residuals. The results are reported in table.12. For the Markov switching specification no remaining ARCH effects is detected. For the two other specifications we reject the absence of remaining ARCH effects at the 1% level. We conclude that the US inflation rate is better fitted using the Markov switching specification.

Table 12: Tests for Remaining ARCH effects.

Parameter	MS-4S-AR(1)	SCH-AR(6)	ARFIMA(2,d,3)
constant	0.825 (0.089)	4.712 (0.712)	5.412 (0.815)
Resid $^2_{t-1}$	0.024 (0.041)	0.235*** (0.042)	0.122*** (0.041)
Resid $^2_{t-2}$	0.057 (0.041)	0.087** (0.042)	0.115*** (0.041)
Resid $^2_{t-3}$	0.019 (0.042)	-0.054 (0.042)	-0.020 (0.041)
R $^2$	0.004	0.07	0.031
F-stat	0.838	14.61	6.298
Prob(F)	0.473	0.000	0.000

Note: Standard errors are in parentheses,  
 \*\*\* and \*\* indicate that the variable is significative at 1% and 5%.

### 4.3 Long Memory versus Structural Change

The long memory behavior and the changes in mean have different implications on monetary policies decisions and on the forecasting of the inflation rate. Thus, it is important to propose a test or an empirical strategy that can discriminate the true long memory behavior from the spurious one. In this subsection, we try to solve the problem of confusing between these two behaviors by using the following strategy:

i) First, we start by estimating the fractional long memory parameter for the observed time series  $y_t$  by using methods proposed in section 3. We call this value  $\hat{d}$ .

ii) Second, we perform for the same series ( $y_t$ ) a model with changes in mean.<sup>9</sup>

iii) Third, we filter all the breaks and we get the series,  $\tilde{y}_t$ . On this series we estimate again the parameter  $d$ , we call it  $\tilde{d}$ .

Three possible results can be observed:

a) If the new estimated value  $\tilde{d}$  is not significantly different from zero then we conclude that the observed long memory behavior is created by the presence of breaks.

b) Now, if  $\tilde{d}$  is different from  $\hat{d}$  but remains significantly different from zero then we conclude that the observed long memory behavior is a

<sup>9</sup>In our empirical application we use the Markov switching model proposed by Hamilton (1989) and the structural change model following the Bai and Perron (1998, 2003) sequential procedure.

true behavior but it is amplified by the presence of breaks in the real data.

c) Finally, if  $\tilde{d}$  is very close to  $\hat{d}$  then we conclude that the presence of breaks in the series has no impact on the estimated long memory parameter.<sup>10</sup>

In order to assess if the detected long memory behavior in the US inflation time series is due to the presence of breaks or it is a true behavior generated by the data mechanisms we propose to use the previous strategy. We estimate the fractional long memory parameter  $d$  by using the US inflation series after filtering out the breaks detected by the MS-3S-AR(1) and the SCH-AR(6) models. Then, we compare these values to those obtained from the original series (subsection 4.2.3). We use the same values of  $m$  as before. The results are reported in table 13 and 14 below:

Table 13: The results of the R/S and V/S long memory tests using residuals series from the SCH-AR(6) and the MS-4S-AR(1) models.

tests\q	0	1	2	5	10	20	30
R/S(q) <sub>SCH</sub>	1.550	1.555	1.553	1.536	1.466	1.298	1.258
V/S(q) <sub>SCH</sub>	0.093	0.096	0.094	0.091	0.083	0.065	0.061
R/S(q) <sub>MSAR</sub>	1.386	1.393	1.419	1.402	1.284	1.200	1.160
V/S(q) <sub>MSAR</sub>	0.120	0.121	0.126	0.123	0.103	0.089	0.084

Note: 1%, 5% and 10% critical value for the R/S and the V/S are 2.001, 1.747 and 1.620 for the R/S statistic and 0.2685, 0.1869 and 0.1518 for the V/S statistic.

From table 13, we can reject the presence of long-range dependence in the residuals for each estimated switching model. Neither the R/S statistic nor the V/S statistic accept the hypothesis of a persistence in the filtered series. These results are confirmed by the two semi-parametric GPH and ELW methods. As shown in table.14 the values of the estimated fractional parameter are close to zero for all cases except for the SCH-AR(6) model using the GPH technique. The corresponding t-stats of these methods are less than the 5% critical value of 1.96 except for the mentioned three cases. Based on these results we can suggest that the observed long memory behavior is spurious and is due to the presence of breaks in this time series.

## 5 Conclusion

In this paper, we have studied the US inflation time series using three models: Two switching models and a long memory process. The results show that the MS-4S-AR(1) model gives the better fit for this series. Indeed, this last model uses 4 states, detects 7 breaks and determines the outliers inside the first states. The residuals analysis confirms the best adequacy of the

<sup>10</sup>If this result occurred, it contradict the empirical simulations finding given in Granger and Hyung (2004), Diebold and Inoue (2001) and Charfeddine and Guégan (2006).

Table 14: Estimated values of the fractional long memory parameter  $d$  using the ELW and the GPH methods After filtering out the breaks.

m	GPH		ELW	
	MS-4S-AR(1)	SCH-AR(6)	MS-4S-AR(1)	SCH-AR(6)
$T^{0.5}$	0.088 (0.371)	0.282 (2.303)	0.113 (1.115)	0.327 (3.228)
$T^{0.6}$	0.145 (1.079)	0.197 (2.610)	0.228 (3.098)	0.209 (2.839)
$T^{0.7}$	0.060 (0.686)	0.141 (1.976)	0.117 (2.187)	0.048 (0.897)
$T^{0.8}$	-0.055 (-0.976)	0.06 (1.228)	0.046 (1.183)	0.05 (1.286)

Note: t-stat are in parentheses.

MS-4S-AR(1) model on the data set. Finally, this model permits to well understand some economic events on the period under study.

Comparing to the SCH-AR(6) and the ARFIMA(2,d,3) models, the MS-4S-AR(1) model has the low value for the J-B test, a better residuals and no remaining ARCH effect. In conclusion, we select this model to describe the evolution of US inflation from January 1957 to June 2006.

Using this approach, we show that trying to explain the evolution of the US inflation with a long memory model appears confusing, indeed the long memory effect is a spurious one. This idea of long memory comes from existence of breaks. This empirical study is confirmed by the works of several authors showing that existence of breaks can provoke spurious long memory behavior, we refer for instance to Charfeddine and Guégan (2006), Granger and Hyung (2004), and Diebold and Inoue (2001) .

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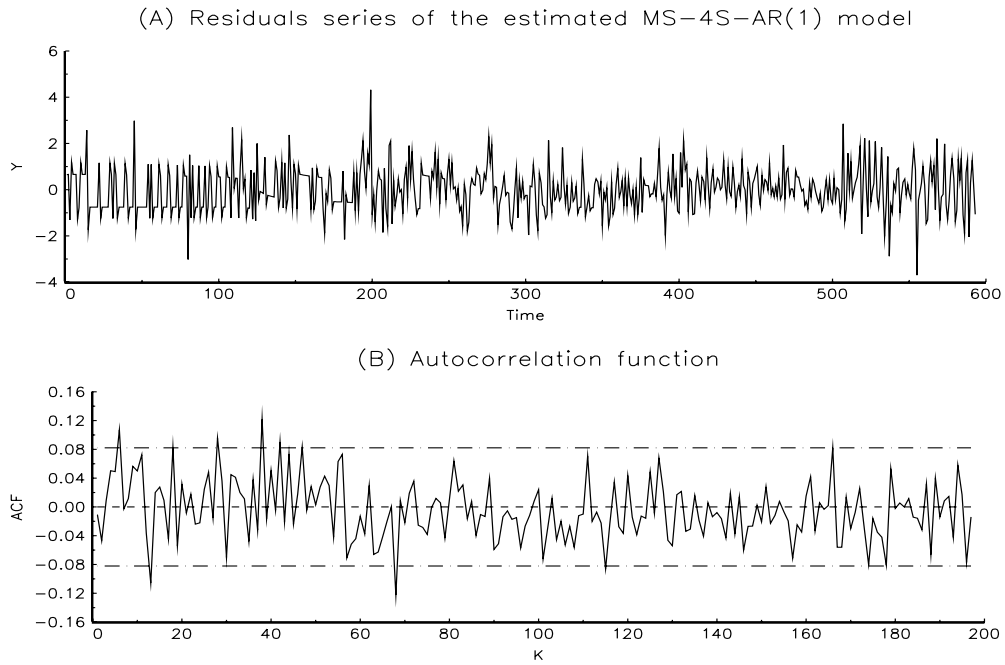


Figure 5: (A) and (B) are respectively the trajectory and the ACF of the estimated residuals series of the MS-4S-AR(1) model.

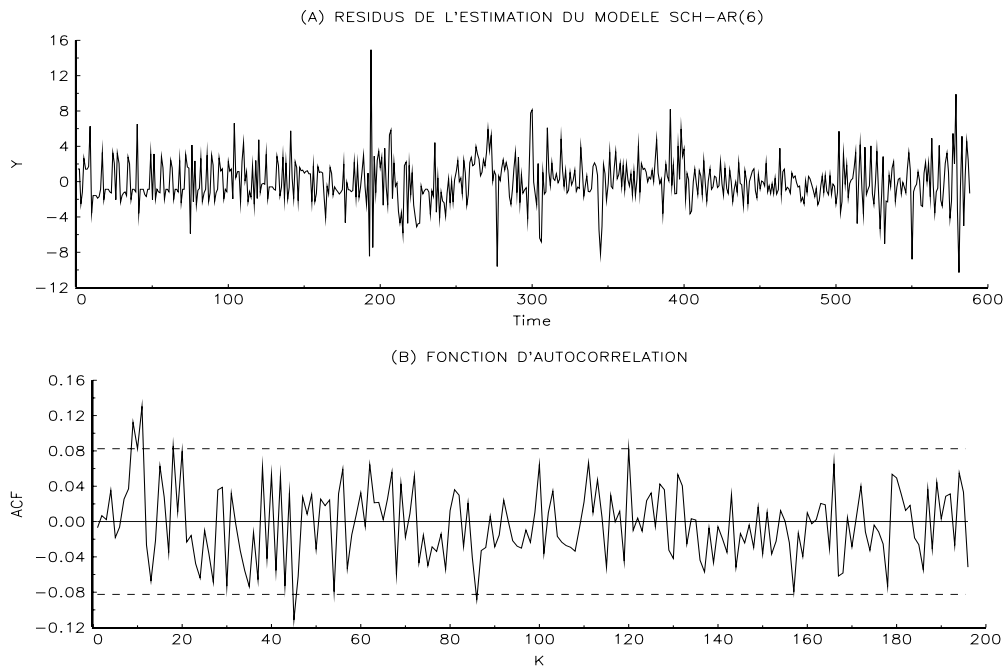


Figure 6: (A) and (B) are respectively the trajectory and the ACF of the residual series of the estimated SCH model.