On Sanskrit commentaries dealing with mathematics (Vth-XIIth century)

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Abstract

This paper explores the diversity of past readings on Sanskrit commentaries dealing with mathematics. The study concentrates on texts composed between the Vth and the XIIth century A. D. It describes how their collection in libraries, the way they were consequently studied by historians of mathematics, and a close reading of the commentaries themselves opens new perspectives on the diversity of mathematical texts produced according to different practices as much as on the history of positonnal arithmetics in India.

Cet article examine les différentes types de lectures qui ont été portée, par le passé, sur les commentaires en langue Sanskrite rédigés entre le VIIème et le XIIème siècle après J. -C, portant sur les mathématiques. Nous nous demanderons comment ces textes ont été collectés dans des bibliothèques puis comment ils ont été étudiés par des historiens des mathématiques. Nous achèverons cette étude sur les perspectives nouvelles que l'étude de la relation du commentaire à ce qu'il commente ouvre tant pour l'histoire des différentes pratiques mathématiques et textuelles en Inde que pour l'histoire de l'arithmétique positionnelle en Inde.

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Introduction

A renewed interest for contextualization in indological studies[1], is but slowly affecting publications on Indian mathematics. This remoteness of history of mathematics within the general field of indology is partly due to a lively historiographical trend of technical and patriotic history of mathematics which remains oblivious to social science. However, there is indeed little information on the context in which mathematics and astronomy were practiced in India in the past. To overcome this problem some historians of science have turned to periods (XVIth-XIXth century) and places where institutions, libraries and many texts help us contextualize the mathematical and astronomical ideas produced in these places[2].

I would like to argue that a focus on the kind of texts produced by astronomers and mathematicians of the Indian subcontinent and the history of how they were transmitted to us can also help us contextualize the knowledge they disclose. Taking in account the diverse textual forms that were produced, trying to understand the function they had, their self-proclaimed aim as well as what they show us of their own conceptions of mathematical practices and ideas can provide information on who produced mathematical texts, why they were produced and how they were used. This approach can also be fruitful for the history of mathematical practices and conceptions in India[3]. These questions can be addressed to texts of earlier periods for which very little background information is known.

Such a focus on text implies a clarification of what manuscripts can be found in libraries today, as they are the sources of editions and studies. Springs here many questions[4]: Who copied the texts we now have at our disposal? Who had them copied? How were these texts used and read? How where these texts collected into libraries? A set of questions that C. Minkowski and D. Raina address more directly in their contributions to this volume.

This paper examines the specific case of commentaries on mathematical subjects written in Sanskrit between the Vth and the XIIth century A. D.[5] with the overall aim to understand the function assigned to commentaries on mathematics. The focus of this study will therefore be on how commentaries were related to the texts they were commenting. This study scrutinizes the point of view of different sets of actors: those who wrote the commentaries, those who had them copied and those who analyzed them as historians of mathematics. The question of whether there was a continuity between these different attitudes will be raised. Were commentaries on mathematics considered as an adherent whole text? Were they thought of as re-disposable pieces of a puzzle to be used when useful, in understanding an algorithm or an idea? In other words, we will try to grasp the kind of readings a commentary was thought to require, overtime in India.

[1]Proemenintly in the project headed by Sheldon Pollock, the "Sanskrit Knowledge Systems at the Eve of Colonialism" (SKEC). See http://www.columbia.edu/itc/mealac/pollock/sks, (Pollock, 2002) and the outcome for literature (Pollock, 2006).

[2]Among the publications on history of science, produced within Pollock's SKEC, see the works of Christopher Minkowski and Dominik Wujastyk, listed at http://www.columbia.edu/itc/mealac/pollock/sks/papers/index.html

[3]K. Chemla has extensively published on the question. Her latest synthesis is (Chemla, 2004).

[4]A similar set of questions addressed to another kind of texts, colonial archives, can be found in (Stoler, 2002).

[5]Precisely: commentaries written between the VIIth and XIIth century on texts composed between the Vth and VIIIth century.

A first part will examine what is known today on commentaries on mathematics of the Vth-XIIth century, attempting to situate them in the larger context of the litterature on astral science (*jyotiṣa*)[6]. Then a description of the manuscripts at our disposal will provide information on who copied them as much as how such texts were copied. A study of certain histories of mathematics in India from the XIXth century onwards, giving a special attention to the story of the rediscovery and edition of the works of Āryabhaṭa (ca. 499 [Billard 512][7]), Brahmagupta (ca. 628) and Śrīdhara (ca. 950), will look at how commentaries on mathematics were read in more recent times. Finally, a close look at algorithms to extract square roots, will serve as an illustration of an analysis of a mathematical text focussing on how commentaries relate to the text they comment, providing insights on the way mathematicians conceived and used the decimal place value notation.

1 Commentaries on mathematics from the Vth to the XIIth century

Indologists are overwhelmed by the number of manuscripts that are at their disposition today to make new editions. The case of astronomy and mathematics is quite exceptional in this respect, since a census has been undertaken, which enables us to evaluate the number of manuscripts and published editions for the specific field of *jyotiṣa*. Indeed, David Pingree's *Census of the Exact Sciences in Sanskrit* (CESS)[8] lists most of what we know today of existing manuscripts on astronomy and mathematics in Sanskrit.

The near exhaustivity of the CESS, can thus enable us to reflect quantitatively on manuscripts which have been collected and preserved for astronomical, astrological or mathematical texts. Since the census not only gives manuscript references but has also tracked published editions, it can help us evaluate how many of these texts couched in manuscripts have been edited and studied in the last two hundred years. A close look at the census[9] shows that mathematical texts in Sanskrit have been over-studied when looked at in the farther landscape of texts on *jyotiṣa*. Indeed, texts on mathematics are comparatively

[6] *Jyotiṣa* (lit. "the ⟨sky's⟩ luminaries"), which we conveniently translate as "astronomy" or more exactly as "astral science" is a field which includes in fact horoscopy and mathematics together with observational and computational astronomy. See (Pingree, 1981, Introduction and Table of contents).

[7] See (Billard, 1971).

[8] (Pingree, 1970-1995). D. Pingree started in 1955 (see CESS I, preface.) a survey of manuscripts on *jyotiṣa* that was still left unfinished when he died fifty years later in november 2005. The CESS is in 5 volumes, volume 6 is still pending. For authors that have not yet been treated in the published CESS, one can refer to (Sen, Bag, & Sarma, 1966) (which also includes text in Persian) and to individual library catalogs. Despite it's name, the CESS lists texts in all sorts of languages of the Indian subcontinent and is not restricted to Sanskrit. David Pingree cast a large net when undertaking his census, considering texts that may refer to some part of *jyotiṣa* in passing. If remaining manuscripts certainly exist in private collections, or are un- or mis- classified in libraries, we can still estimate that most of the manuscripts known are included in his census.

[9] I counted manually the entries, as I had no electronic version of the text: the evaluation is probably fraught with errors. It can however give us a general idea of the proportions involved. The CESS has since then been partly digitalized (volumes 1, 2 and 4) on http://books.google.com/. I have counted texts that were manuscripts, and have thus excluded references to authors for which we do not have any remaining text, or texts for which we have XXth century published editions by XXth century authors and no manuscripts.

few when compared to the other censussed texts[10], but those in Sanskrit have been much more edited, studied and translated than the texts solely concerned with astronomy or astrology[11].

Commentaries on mathematical texts, in this context, have been thus very much studied, eventhough they do not represent much of the transmitted textual tradition[12]. However, as we will see, studies have often treated them in bits and pieces, or looked at them independently from the text they comment. Furthermore, if commentaries have been edited, they have often not been entirely translated.

This specific treatment of commentaries dealing with mathematics, when looked at from the sea of all astral texts known in Sanskrit, can be found, on a lower lever, when looking closely at the texts on mathematics known for the period we are interested in : the Vth-XIIth century.

1.1 A limited number of known texts on mathematics.

All the presently identified texts on mathematics of the Vth to VIIth century[13], for which we have manuscripts are enumerated in Table 1.1.

[10]Indeed, for the 3686 texts I have recorded in the first five volumes of the CESS only 102 (2,7 %) are clearly devoted at least partly to mathematics (*gaṇita*).

[11]I did not, when I was counting, list which texts were edited and translated. However, remarking that among the 102 texts listed, an important number are in vernacular languages (oriya, tamil, notably) and have seldom been edited or even translated, I think it is safe to consider that an important part of the texts in Sanskrit concerning *gaṇita* have been edited and translated.

[12]Under 2972 authors, and 3686 texts that I have recorded devoted to *jyotiṣa*, 816 (22%) are commentaries, and 646 authors (21,7 %) are commentators. Before starting this article, I actually thought that commentaries on mathematics were altogether neglected in the historiography of science in India, and that they were on the contrary an important part of the past tradition. I indeed, mentioned this in the introduction to my book, (Keller, 2006). This was noted and criticized quite rightly by S. R. Sharma. (S. R. Sarma, 2006, p. 144).

[13]Some mathematical commentaries of this period are lost to us for now at least, such as Prabhākara's commentary on the *Āryabhaṭīya* (ca.VIth century) (CESS 4 227 a), and Balabadhadra's (fl. VIIIth century) commentary to the *Brahmasphuṭasiddhānta* (CESS 4 255 a), but we have not taken them into account here.

Table 1: A list of known texts on mathematics, in Sanskrit, Vth-XIIth century.

Dates (A.D.)	Author	Title	Abbreviation
499	Āryabhaṭa	*Āryabhaṭīya* (Chapter 2)	Ab
628	Brahmagupta	*Brahmasputasiddhānta* (Chapter XII)	BSS
629	Bhāskara	*Āryabhaṭīyabhāṣya* (Chapter 2)	BAB
VIIth-Xth century	unknown	*Bhakshālī* manuscript	BM
ca 864	Pṛthudakśvamin	*Vāsanābhāṣya* on the BSS (verses of Chapter XII)	PBSS
850-950	Śrīdhara	*Pāṭīgaṇita*	PG
idem	idem	*Triśatika*[a]	T
Xth century	Mahāvīra	*Gaṇitasārasaṃgraha*	GSS
c. 1039 A.D.	Śrīpati	*Gaṇitatilaka*	GT
ca. 1040	Someśvara	on the *Āryabhaṭīya* (chapter 2)	SAB
ca. 1150	Bhāskara II	*Līlāvatī*	L
idem	idem	*Bijagaṇita*	BG
[IXth-XIIth century	Āryabhaṭa II	*Mahāsiddhānta* (Chapters XV and XVIII)	MS][b]
ca. 1200	Sūryadeva Yajvan	on the *Āryabhaṭīya* (Chapter 2)	SYAB
[Unknown[c]	Unknown	commentary on the *Pāṭīgaṇita*	APG]

[a] Admitting with [Hayashi 1995] that we do not have any extant manuscript of his *Gaṇitapañcaviṃśi*.

[b] For André Billard ((Billard, 1971, 161)), the *Mahāsiddhānta* contains observational data that was made in the first half of the XVIth century. Generally however, Āryabhaṭa II is considered to have lived between Śrīdhara and Bhāskara II (CESS I-II53). Thus the text is optionally added to this list.

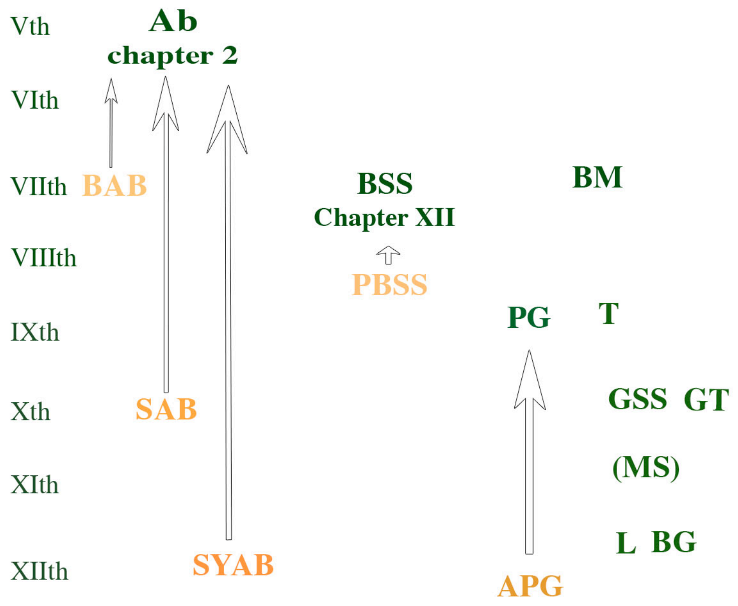
[c] Shukla who edited the text, considers that the commentary bears features of texts of the time span we have chosen, such as the Bhakshālī Manuscript and the BSS. (K. S. Shukla, 1959, xxviii-xxxiv).

The VIIth to the XIIth century is the beginning of an expanding mathematical and astronomical tradition, which will permeate not only the Indian subcontinent but extend in the East to China and in the West to the Arabic peninsula[14].

If the number of mathematical texts transmitted is limited, they strike by their diversity. This maybe partly due to the tense autonomy of mathematics (*gaṇita*, "computation") in respect to astronomy. Indeed, a number of preserved texts on mathematics belong to astronomical treatises. Mathematics seems to have been, to a certain extent, and for certain authors, a sub-discipline of astronomy. A common acception of *gaṇita* in this context is "computational astronomy". However, some Sanskrit

[14] We have chosen as an upper boundary the time before the works of Bhāskara II (ca.1114-1183) started to have an impact (which has enabled us to include Sūryadeva Yājvan's commentary, which dates from after Bhāskara II, but is ignorant of it) and after the vedic period, which had its own specific mathematical tradition or style. The period we are considering thus ranges thus from 499 AD to 1200 precisely.

Figure 1: Texts on mathematics in Sanskrit, VIIth-XIIth century



authors of astronomical texts insisted that mathematics had an existence outside of astronomy[15]. This would explain why a certain number of procedures with no application in astronomy were stated in such chapters. Additionally, independent texts on mathematics, from the vedic time onwards, have been preserved. Accordingly, mathematical texts in an astronomical context, were different from autonomous mathematical texts. The way they differ in respect to one another needs still to be specified. Here, in the first and second part, these texts of different kind will be collected together focussing on the fact that they concentrate on a same subject matter, which is given a specific name, *gaṇita*. In the third part, this difference will emerge again.

Table 1.1 gives a list of 15 texts. It includes two sets of texts. "Primary texts" : treatises and all the texts that stand alone; and "secondary texts", those that need another to exist : commentaries. All the texts on mathematics known for this period are summarized graphically in Figure 1, stressing primary and secondary texts.

Texts so far identified as being commentaries, with *gaṇita* as subject matter, written during our chosen period[16] and for which we have existing manuscripts are thus, in a chronological order:

- Bhāskara's commentary on the second chapter of the *Āryabhaṭīya* (629 A. D.; hereafter abbreviated for the treatise into Ab and for the commentary into BAB, implicitly referring to chapter 2 when quoted in this way),

- Pṛthudakśvamin's mathematical commentary on the XIIth chapter of the *Brahmasphuṭasiddhānta*

[15]See for instance (Keller, 2007), (Plofker, forthcomming).

[16]Commentaries on the treatises enumerated here have been written after our period but are not listed here. We will come back to this bellow.

of Brahmagupta (the treatise is of 628 A. D. and the commentary of ca 864 A. D. ; hereafter abbreviated into BSS for the treatise and PBSS for the commentary, implicitly referring to chapter XII when quoted in this way),

- Someśvara's commentary on the second chapter of the *Āryabhaṭīya* (ca. 1040, hereafter abbreviated as SAB, implicitly referring to chapter 2),
- Sūryadeva Yajvan's mathematical commentary on the *Āryabhaṭīya* (Sūryadeva Yajvan is thought to have been born in 1191 A. D., his commentary is hereafter abbreviated into SYAB, implicitly referring to chapter 2).

To this we can maybe add:

- the anonymous and undated commentary on the *Pāṭīgaṇita* of Śrīdhara (fl. 850-950 A.D , date unknown for the commentary[17]; hereafter abbreviated into PG for the treatise and APG for the commentary.).

The following discussion will especially concentrate on the edited commentaries of this list: BAB, SYAB and APG.

Let us return to the texts these commentaries gloss.

The *Āryabhaṭīya* has been extensively commented upon.

Thus K. S. Shukla and K. V. Sarma count 19 commentators of the *Āryabhaṭīya*[18], 12 of which are in Sanskrit. Half of them are from after the XVth century.

For Brahmagupta's BSS, on the other hand, two commentators are known. Furthermore, only seven manuscripts have been transmitted of this text. It concerns only one commentary, PBSS. Only two manuscripts, contain commentaries of Chapter XII. Four manuscripts out of the 34 remaining manuscripts of the BSS additionally provide an anonymous commentay on it[19].

Finally, as far as I know, the PG is known through a unique manuscript, containing a similarly unique anonymous commentary on this text.

In other words, early treatises dealing with mathematics have not always come down to us with a great number of commentaries explaining them.

All primary texts evoked here have been entirely edited and translated into English. The Ab, BSS, BM, GSS, PG, L and BG have been edited and translated into English[20]. All the other texts have been edited[21]. The BSS is, for this period, the only non-extensively edited treatise containing a mathematical

[17] K. S. Shukla, who edited the text, considers that the commentary bears features of texts of the chosen time span. He especially draws similarities with the Bhakshālī Manuscript and the BSS. (K. S. Shukla, 1959, pp. xxviii-xxxiv).

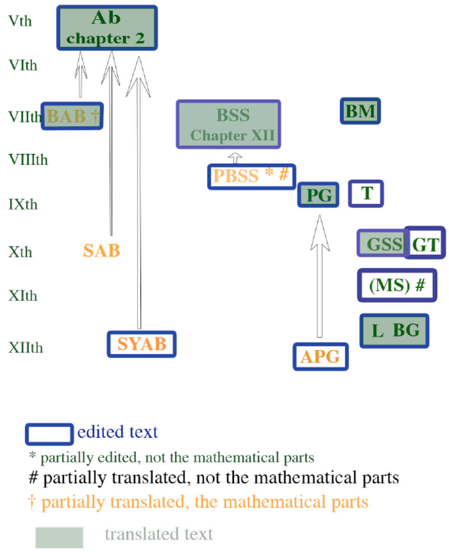
[18] (K. V. Shukla & Sharma, 1976, p. xxv-lviii), we have included in this account, Prabhākara for which no extant commentary is known, although he is quoted by Bhāskara.

[19] CESS IV 255 b; V 239 b.

[20] See (K. V. Shukla & Sharma, 1976), (M. S. Dvivedin, 1902), (Hayashi, 1995), (K. S. Shukla, 1959), (Rangacarya, 1912). There has been many printed editions of L and BG two texts, which are noted in CESS 4 308 a and 311 b; we can note the translation given in (Brahmagupta; Bhāskarācārya; Colebrooke, 1817).

[21] The T was edited in (S. Dvivedin, 1899). The GT was edited with a later commentary (Kāpadīā, 1937). The MS was edited by Sudhākara Dvivedin in 1910 ((S. Dvivedin, 1910)), and partially translated into English by S. R. Sarma, (S. R.

Figure 2: Edited and translated texts on mathematics, Vth-XIIth century



part. It is thus an exception: it has been but partially translated in English, in bits and pieces. Its special situation is probably due to the fact that no extant ancient commentaries in extant manuscripts are known for this text, part of which thus remains hard to understand.

Concerning commentaries, two (PBSS and BAB) have been partially translated into English[22], only one translated for its mathematical part (the BAB). A portion of SAB was edited with BAB.[23].

The situation of these texts concerning editions and translations is given in Table 1.1, and Figure 2.

The case of BSS and PBSS set aside, notice that, strikingly, commentaries have been edited, but usually not translated.

Sarma, 1966).

[22]Part of PBSS's astronomical commentary has been studied, edited and translated by Setsuro Ikeyama ((Ikeyama, 2003)). Pṛthudakṣvamin's commentary on the XIIth chapter of the BSS, is found in a manuscript at the Indian Office and in what appears as a copy of this manuscript used by S. Dvivedi in Benares (CESS A. IV. 221 b, (Ikeyama, 2003, S5-S7)). It has not been edited entirely, especially concerning the mathematical part, probably because the one recension is at times quite difficult to understand.

[23](K. S. Shukla, 1976).

Table 2: A list of the editorial situation of texts on mathematics, in Sanskrit, Vth-XIIth century

Abbreviation	Treatise, Commentary, Others	Edited	Translated
Ab	Versified treatise	Yes	Yes
BSS	Versified treatise	Partly	Partly
BAB	Prose commentary	Yes	Partly
BM	Fragmentary text (versified rules and examples; prose resolutions)	Yes	Yes
PBSS	commentary	Partly	Partly
PG	Versified treatise	Yes	Yes
T	Treatise	Yes	No
GSS	Treatise	Yes	Yes
GT	Treatise	Yes	Partly (not the mathematical parts)
L	Treatise	Yes	Yes
BG	Treatise	Yes	Yes
[MS	Treatise	Yes	Partly][a]
SYAB	Commentary	Yes	No
[APG	Commentary	Yes	No]

[a] For André Billard ((Billard, 1971, Billard 1971, p. 161)), the *Mahāsiddhānta* contains observational data that was made in the first half of the XVIth century. Generally however, Āryabhaṭa II is considered to have lived between Śrīdhara and Bhāskara II ([CESS, Vol 1-2, p. 53]). Thus the text is optionally added to this list.

This editorial situation can be taken as a symptom of the special treatment of commentaries as texts. It raises many questions: Were commentaries less translated because historians of science were not interested in them? Or was this a consequence of the state of manuscripts? These questions lead us to the way ancient collectors of texts, and maybe the authors of the commentaries themselves, thought of the text they were composing.

1.2 Reading and collecting commentaries on mathematical subjects

Who collected manuscripts? Who had them made? Who copied them? As C. Minkowski and D. Raina's articles in this volume underline, we have, to this date, only sporadic information enabling partial answers, varying from library to library, region to region, collection to collection. While restricting ourselves to our chosen corpus, information is especially sparse. We usually do not know the dates of the manuscripts nor their stories, but it is reasonable to believe that they are mostly results of the copying frenzy of the late modern period described by C. Minkowski in his article of this volume.

In certain cases a great number of ancient hand made copies of a given text have been preserved. Thus the CESS counts for Ab [24] 149 manuscripts of which 47 are with commentaries (more or less 1/3rd).

[24] CESS 1 51a-52b; 2 15b; 3 16a; 4 27b.

These manuscripts can be found in 28 different libraries. They are mostly of unknown origin and are either paper or palm leaf.

Comparatively, extant manuscripts of commentaires are less important. Bhāskara's commentary is known through 6 manuscripts, 5 of which are in the same library in Kerala. They are all incomplete[25]. Similarly Sūryadeva's commentary has been transmitted to us, through 8 south Indian recensions and copies[26], while SAB is known through only one copy[27]. The BSS is known through 34 manuscripts[28] and PBSS is known in 7 manuscripts, two of which are fairly recent copy of two others, none of which are extant[29]. Only two contain the commentary on chapter XII which is explicitly devoted to *gaṇita*[30]. Finally the edited PG is known through one manuscript the one containing APG[31].

This is summarized in Table 3.

Table 3: Number of manuscripts for commentaries and treatises on mathematics, Vth-XIIth century

Text	Treatise	Commentary	Number of remaining manuscripts
Ab	x		47
BAB		x	6
SAB		x	1
BSS	x		24
PBSS		x	7
PG	x		1
APG		x	1

Therefore, in these collections, commentaries were much less numerous then treatises, and not always complete. Is this due to the hazards of preservation? Does it reflect a trend in the conception of commentaries when the texts were copied or collected sometime during the XVIIIth or XIXth century? Did the authors of commentaries themselves think of their commentaries as fragmentary texts? Does this explain the way commentaries were subsequently treated in the historiography? To answer these questions with precision, we need to delve a bit more into the complexity of each text, and how it has been transmitted to us.

Before looking at how these commentaries were transmitted, let us specify what kind of texts this analysis is concerned with.

[25](K. S. Shukla, 1976), CESS IV 297b.

[26](K. V. Sarma, 1976, p. xvii to xxv).

[27](Sen et al., 1966, p. 202), CESS I-II 51a.

[28]CESS IV 254 b-255b, CESS V 239 b- 240 a.

[29]CESS IV 221 a, CESS V 224 a .

[30](Ikeyama, 2003, p. S7).

[31](K. S. Shukla, 1959). As noted previously (Sen et al., 1966, p. 204) note a second incomplete manuscript of the PG *ṭīkā* in the Descriptive catalogue of the Oriental Mss in the Mackenzie Collection, compiled by H. H . Wilson in Madras in 1882, which would be in 54 folios.

1.3 A first rough characterization of commentaries on mathematics

There is something tricky in attempting to individualize mathematical commentaries among other texts on mathematics in Sanskrit, since the aim of this article is precisely to underline that much stills needs to be done.

Titles of texts, and expressions used in them, testifies that the Sanskrit scholarly tradition distinguished between treatises (*śāstra* , *tantra*, *siddhānta*) and commentaries (*vyakhyā*, *bhāṣya, ṭīkā*). This was also the case in the field of *jyotiṣa* and *gaṇita*[32]. All sorts of other kinds of texts were composed in this field such as handbooks for making calendars (*karaṇa*), almanacs (*pañcāṅga*), compendiums (*nibandha*), etc[33]. With this diversity in mind, we can thus wonder if certain kinds of text were preferred to others in the past. Did such preferences vary over time, in different places, according to certain sub-fields? Does this tell us something of the conception the different actors had of what an astronomical or mathematical text was to be? Of how such disciplines were practiced? Did historians of science, discovering these texts, focus on one kind of text over another?

Concerning the texts of the Vth-XIIth century we are interested in, the Ab is referred to as a *tantra* by Bhāskara[34] and as a *śāstra* by Sūryadeva Yajvan. APG refers to the text it comments as a *śāstra*[35].

Similarly a dirth of names are used by authors to evoke the commentaries they are writing. Thus, Bhāscara calls his commentary a *vyākhyā*[36]. The tradition, as reflected in the title given to his commentary, evokes however a *bhāṣya*. Sūryadeva Yajvan from time to time refers to his own text as a *vyākhyā*, and, more commonly, as a *prakāśa*, "light"[37]. The APG calls itself a *ṭīkā*[38].

We do not know then if these names are just synonyms or if they tell us something of the kind of commentary, authors and readers had in mind, when they read or gave them such names.

[32] Concerning commentaries, many different technical names are recorded either in the titles or by D. Pingree in the CESS. Keeping in mind that the numbers should be subject to much caution, preeminently *ṭīkā*s (549- 67,2% of all commentaries), but also *vyākhyā*s (85-10,4%), *vivṛtti*s (50-6,1%) and *bhāṣya*s (34-4,1%) among *avacūrṇi*s, *vārtikka*s, *tippaṇaṇi*s, *vivāraṇa*'s are noted. This diversity raises the question of how titles were given to texts. We seldom encounter here the names of patrons, as in the cases examined by C. Minkowski. Do authors refer to these texts with such titles? Were they given by later scholars? By those who copied the texts? This set of questions can be extended to all the texts of the census. I have counted only 205 (5,5%) texts bearing *siddhānta* in their title. I count 18 *śāstra*s and *tantra*s. Titles can express either that they bestow knowledge 141 (*saṅgraha, jñana*; 3, 8 %) or the idea of providing an essence of something 118 (*phala, sāra*; 3,2%); sometimes both (*sārasaṅgraha* being quite a common title compound). The metaphor of light (*dīpika*, *prakāśa*) is used also (289; 7,8%), for commentaries also (28).

[33] (Pingree, 1981). I have counted in the CESS 130 (3,5 %) texts bearing the name *karaṇa* or associated titles and 66 (1,7%) almanacs (*pacāṅgas*) and texts explaining how to make them. The CESS notes a number of non-standard texts, such as the *Aparājitapṛcchā* of Bhuvanadeva (fl. XII-XIIth century) whose text on architecture is written in dialogue (CESS V 264 a.).

[34] To be more specific, it is the three last chapters of the *Āryabhaṭīya* which are referred to in this way, when concluding the commentaries to each of these chapters. (K. V. Sarma, 1976, p. xxv), (K. S. Shukla, 1976, p. 171; p. 239; p. 288).

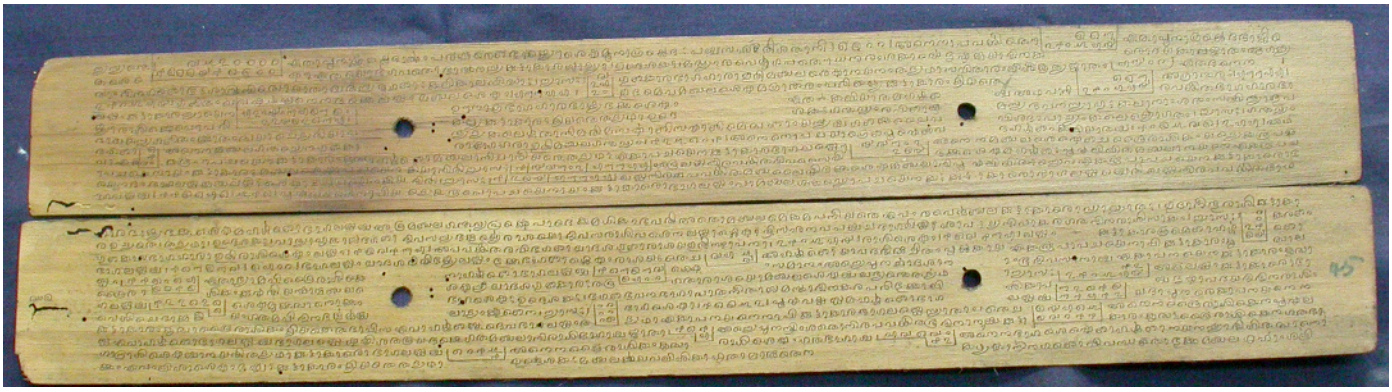
[35] (K. S. Shukla, 1959, p.1).

[36] See the *maṅgalācaraṇam* of BAB.2 : *vyākhyānaṃ gurupādalabdham adhunā kiñcin mayā likhyate*, (K. S. Shukla, 1976, p. 43).

[37] Thus as the end of the introduction which begins his commentary he writes *evam upodghātaṃ pradarśya śastraṃ vyākhyāyate.* (K. V. Sarma, 1976, p. 7) At the end of that chapter's commentary, he refers to his text as a *prakāśa.* (K. V. Sarma, 1976, p. 32, p. 79 (note 11); p. 117, p. 185). The end of SYAB.2 uses also again the verbal root *vyākh-*.

[38] (K. S. Shukla, 1959, p.1).

Figure 3: Palm leaf manuscript of BAB in a copy of the Kerala University Oriental Manuscripts Library



A commentary, by definition, is a secondary text, a deuteronomic text, a text that needs another to exist[39]. Is this definition however sufficient to characterize it as a text? Sanskrit commentaries are very diverse. This is somewhat reflected in the mathematical field. Some commentaries respect the original order of the text, when others do not[40]. Thus BAB, SAB, SYAB and APG[41] respect the order of the text whereas PBSS does not[42]. SYAB develops a long introduction (*upodghata*) before the gloss of the text, this is not the case in the other commentaries considered here. So that if all commentaries have in common the fact that they comment a text, the way they do comment is quite variable. One needs to specify the relation between a commentary and the treatise it comments.

All the commentaries considered here quote entirely the text they comment[43]. Nonetheless, finding a stylistic criteria separating treatises from commentaries is a difficult task. As soon as a principle of separation is brought up, a counter-example can be provided almost immediately.

For instance, manuscripts, did not always graphically distinguish treatises from their commentaries. Thus, as illustrated in Figure 3, the treatises seems to be an undifferentiated part of the commentary. In some cases, such as in Figure 4, the commented part was colored or graphically disposed separately from the treatise, but this was not a systematical rule[44]. Therefore, even thought titles suggest a difference, the material culture does not always follow it. Does this mean that the copied texts were not made to

[39]On the question of "secondary texts" in the history of mathematics, see (Netz, 1998), (Chemla, 1999), (Bernard, 2003) and also Chemla in this volume.

[40]See (Bronkhorst, 1990).

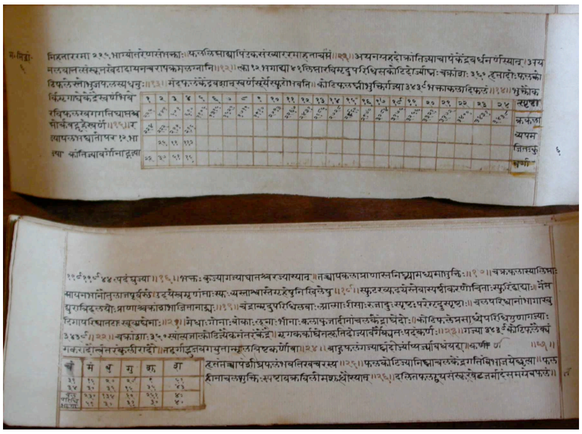
[41]One should recall however that PG as given in Shukla's edition is known in this unique recension only (K. S. Shukla, 1959).

[42]According to CESS IV 221b the order is, for chapters noted in roman numbers: I 1-3; XXI 1-XXII 3; I 4-II 68; XV 1-9; III 1-XIV 55; XV 1à-XX 19; and XXII 4-XXIV 13. The question being then, what is the logic found by PBSS for this order.

[43]However, they all quote also other texts, although not completely. Thus SYAB quotes often PG, and APG, BSS. SYAB often paraphrases BAB. So that these commentaries are in fact composite texts made of parts of previously composed texts, that are sometimes rewritten (paraphrase), intertwined with their own original compositions. The versified examples commentaries share with each other can be seen as a form of quotation as well.

[44]One can wonder if this does not spring from a modern (XVIIIth-XIXth century) scribal tradition which may very well have been influenced by the new presence of European texts.

Figure 4: Paper manuscript with colored highlights of MS in a copy of the Mumbai University Library



be read? That those who copied texts in such manners, did not make a difference? We see here how a historical study of traditions of reading and copying in the Indian subcontinent could shed a much needed light[45].

The opposition of treatises and commentaries has often been described as the written reflection of the opposition, in an oral tradition, of what was to be known by heart (the treatise) and the explanations that were given subsequently (the commentary). The treatise being an oral composition, the commentary a written text[46]. This is rooted in what commentators write. Thus Bhāskara insists on an opposition between Āryabhaṭa who states orally *āh-* the commented treatise, and Bhāskara's own writing, *likh-*. It is not certain however how much what is firmly opposed for us, was indeed a real opposition for the authors we are concerned with. Thus, Hayashi has shown that the word *likh-* is used as a synonym for *vac-* in the BM[47]. In this case, there appears to be no difference between a written text and an oral one.

Even the opposition between versified and prose text does not always hold its way. Indeed, the treatises we are concerned with here are all versified. Thus chapter 2 of the AB, chapter XII of the BSS and the PG are composed in *ārya* verses. The commentaries all include prose sections, which can present a great diversity : they can set up dialogs for instance, or grammatical analysis. But all include versified parts. Thus BAB contains versified tables. All commentaries contain versified examples[48].

The existence of solved examples may be a unique feature of mathematical commentaries in Sanskrit, together with the fact that they contain non discursive parts, such as numerical tables and drawings. Only further systematic study of the characteristics of Sanskrit commentaries, especially of Sanskrit commentaries in the field of *jyotiṣa*, may provide additionnal elements.

What characteristises commentaries and treatises appears as a complex and difficult question, when considered in a general case as much as when restricting ourselves to the case of commentaries whose topic is mathematics.

[45]Note also that a strong stylistic criteria seperating commentaries from treatises could help philologers to determine, while editing manuscripts, what belongs to one and the other.

[46]See for instance (K. S. Shukla, 1976, Introduction).

[47](Hayashi, 1995, p. 85).

[48]In his edition of PG and APG, Shukla seems to hesitate: are the examples part of the treatise, or part of the commentary? Editions of L, all consider examples as part of the treatise.

Internal evidence may tell us something then of how texts were conceived in relation to one another by authors of commentaries. Questioning why and how texts were copied, with what aim may also yield some evidence, of a different kind, providing elements for a historical evolution in the conception of such texts. The history of text copying, collection and reading can additionally give us contextual information on who had them collected and copied, as we will now see.

Let us turn then to how specific manuscripts can tell us something on who had them made, and maybe how they were read and copied.

1.4 An example: the manuscripts of the editions of SYAB

K. V. Sarma has given a description of the codex- that is the bundlle in which sometimes several text are collected within a library, indicating that they have either been copied together or had been collected together at the time they were integrated in the library- in which the SYAB is found in Kerala[49]. It shows us a first set of manuscripts were part of a collection made by scholarly astronomers.

Indeed, the codex containing Manuscript A, Codex C. 224-A , was made for the royal family of Eḍapaḷḷī of Kerala, in 1753. It included, the *Bhaṭaprakāśikā* of Sūryadeva Yajvan (SYAB) and also the *Laghubhāskarīya* and *Mahābhāskarīya* of Bhāskara I (VIIth century), the *Tantrasaṇgraha* of Nīlakaṇṭha Somayājin (fl. 1450), the *Sūryasiddhānta* (Xth century), the *Goladīpikā* of Parameśvara (XIVth) and the *Siddhāntaśekhara* of Śrīpati (fl. 1050). All these texts have in common their subject matter: astronomy. Similarly manuscript D, also originating from a codex (C. 22475-A) belonging to the same royal family of Kerala, contains both SYAB and the first chapter of Ab, alone.

Manuscript E of the same edition belongs to a codex (C.2121 C and D) which originally came from the "library of a family of astronomical scholars, the Maṅgalappaḷḷī Illam, at Āranmuḷa, in southern Kerala"[50]. The codex contains with SYAb, the 3 last chapters of the Ab, T and also the *Mahābhāskarīya* of Bhāskara I, a *Mahābhāskarīyavyakhyā* (an anonymous commentary on the *Mahābhāskarīya*), the *Laghumānasakaraṇa* of Muñjāla (fl. 932) with an anonymous commentary, a commentary on chapter II and parts of chapter III of the *Sūryasiddhānta*, the whole of the *Sūryasiddhānta*, and an anonymous prose *Rāmāyaṇa*. Setting aside the prose version of the famous Indian Epic, once again such a codex concentrates essentially on astronomical and mathematical lore. Here, only part of a commentary on the *Sūryasiddhānta* is copied, while the treatise itself is extensively copied, separately. Ab, similarly, is known to have been transmitted in two separate parts, the first chapter being copied separately (as in the codex for Manuscript A) from the 3 other parts (as in the codex of Manuscript E).

Commentaries then were not copied together with other commentaries, but a codex could contain a treatise alone, and a commentary of a couple of chapters by a given author. Unlike the Chinese tradition, texts of important treatises could be copied and collected without any commentaries. Different commentaries of a same text were usually copied separately, one did not record in the same manuscript different commentaries of a same verse, as for the Chinese *Nine Chapters*.

Another codex, shows us a text copied for another kind of context: that of a brahmin cast of Kerala,

[49](K. V. Sarma, 1976, Introduction, xvii-xx).

[50](K. V. Sarma, 1976, Introduction, p.xix)

the *nampūtiri*, whose priests follow vedic rituals (*śrauta*). The codex which contains SYAB, C. 2320-A, contains also a text describing the horse sacrifice and detailed accounts of expenses for a ceremony carried out in 1535. The codex seems a copy of a manuscript dating from 1536. Indeed, Sūryadeva understood Āryabhaṭa's text in relation to *śrauta*, being himself a performer of such rituals[51]. This small codex is of the kind of *śrauta* collection C. Minkowski describes for the Toro family, although the copy of this *siddhāntic* text, may seem surprising. Was the use of *siddhāntic* astronomy by ritualist families a usual phenomena?

On the contrary, Manuscript B of Sarma's edition was given by a scholarly family coming from the *vaṭṭapaḷḷi maṭham* (e.g. a religious complex) near the southernmost tip of the Indian sub-continent, *kanyākumārī*, in Tamil Nadu. With the *Bhaṭaprakāśikā* (SYAB) the codex contains an *Āṣṭādhyāyīsūtrānukramaṇi*, eg an alphabetical index of Pāṇini's *sūtras*, in other words a text of grammatical lore[52].

In this case, SYAb seems to have been copied as part of a more general endeavor to collect general Sanskrit scholarship.

The provenance of manuscripts, then, underlines an already well known posterity of *Āryabhaṭa* in Kerala. SYAB seems to have been used in three different, although probably not separate, social contexts: that of scholarly astronomers, that of priests performing vedic rituals, the more general scholarly atmosphere of south Indians monasteries.

We have however but little information on how these texts were integrated into the Kerala University Oriental Manuscript Library were they are now to be found. The library was originally created by the government of Travencore (one of the autonomous states within the British Raj) in 1908, aiming at the preservation of their own heritage[53]. More on the history of this library needs to be investigated, but the structure of the codex in which SYAB is to be found recalls the kind of collection work C. Minkowski described.

The context of preservation of other manuscripts on other mathematical texts of this period yields but little of information as rich as those of SYAB, maybe because manuscripts have seldom been examined with such a detail by sufficiently knowledgable people. The BAB manuscripts preserved at the KUOML are not given within a codex, for instance.

Existing manuscripts of the PBSS tell us more about their recent collecting, then how the ancient tradition has transmitted them to us existing manuscripts. Thus, the two manuscripts which explicitly contain chapter XII of the PBSS, are those that served respectively for the translation Colebrooke made of this text and the edition that S. Dvivedin made in Benares[54].

Treatises on mathematics then have been more often studied and translated than commentaries on

[51]SYAB states this clearly in its general introduction to the Ab ((K. V. Sarma, 1976, xxv-xxvi, 2-4)). This is an originality of SYAB, Āryabhaṭa's text alone has so little to do with it, that some modern historiographers have even imagined him to be a materialist.

[52](K. V. Sarma, 1976, xviii).

[53]The "index of manuscripts" of the library notes that "In 1940 it possessed 3000 manuscripts, 142 publications in sanskrit, 63 in Malayalam. Travancore University (which became the University of Kerala) organized after its establishment (1938) a manuscript preservation and collection department. Both were amalgamated (sic) in 1940. In 1958 there was 28 000 Codices in Sanskrit; 5 000 in malayalam."

[54]See (Pingree, 1970-1995, op.cit), (Ikeyama, 2003, op.cit) and (M. S. Dvivedin, 1902).

mathematics, although mathematics as a discipline has attracted far more scholarship in the last two centuries than other parts of *jyotiṣa* lore. Tools to differentiate and understand how commentaries and treatises relate to each other, when each individual text seems to have an individual story of preservation/transmission have been difficult to find. We have seen that in the early modern period in South India, royal fammilies with a marked interest in astronomy, ritualist families and religious groupings had copies made of texts on mathematics. Those who copied commentaries in south India in the early modern period, seemed to have considered them not as extant texts.

Did the authors themselves consider commentaries as whole texts on their own? And were treatises also considered such? A close look at the texts themselves may help us yield some answers. One may also wonder how much of this attitude of copiers was subsequently reflected by the orientalists who looked at them. Indeed, orientalists, and pandits working for them, had also mathematical texts copied at the end of the XVIIIth century, begining of the XIXth century. Let us then now turn to how XIXth century and XXth century scholars treatment of Sanskrit commentaries in the mathematical field.

2 Rediscovering Ab, BSS, PG and their commentaries in the historiography of Indian mathematics

This section is a first attempt to highlight textual aspects of the historiography of mathematics in India. How historians of mathematics looked at the Ab, the BSS and the PG and their VIIth-XIIth century commentaries will be analyzed. No simple trend can be found unifying how different historians dealt with commentaries. The key may be whether historians were mathematicians or philologers. The more the historian was a mathematician, the less there seems to have been a sensitivity to text, the more he was a philologer, the more the commentary was given an important place. However, this does not mean that commentaries were always seen as secondary texts by mathematicians and important texts by philologers. As the XXth century came to a close, the general historiographical trend has been to pay more and more attention to the mathematical contents of commentaries. We cannot afford here to look closely at the shifting attitudes of all different actors, but we will try to draw out some of the characteristics of Colebrooke, Datta & Sing and K. S. Shukla's attitude towards commentaries.

As early as the XVIIth century a certain number of people in Europe knew of the existence of Sanskrit astronomical treatises, by the testimonies of travelers, academic envoys and jesuit missionaries, as D. Raina's article in this volume explains[55]. It took time for the Europeans curious of them, to get hold of the texts and be able to study them. By the early XIXth century, the first translations of Sanskrit texts on mathematics in European languages (especially English) were made. Thus in 1812, in London, Stratchey translated for the first time the BG in English, in 1816 Taylor translated the BG in Mumbai, followed by Colebrooke's translation of L and BG together with the mathematical chapters of Brahmagupta's BSS which was published in London in 1817[56].

[55] See also (Raina, 2003), (Raina, 1999).

[56] (Brahmagupta; Bhāskarācārya; Colebrooke, 1817).

2.1 Colebrooke and commentaries

Colebrooke's publication proved to be a landmark. He was the first serious well established indologist interested in mathematical texts from the Indian subcontinent. He was at the time a former director of the Asiatic Society of Bengal, a recognized specialist of Hindu Law, Vedic ritual and Indian languages.

His translations of these mathematical texts was preceded by a general "dissertation", followed by "notes and illustrations", aiming at placing these texts in a general history of mathematics which would provide for India a name it had until then lacked[57]. His introduction takes a strong position on the antiquity of the Indian tradition in mathematics, especially in algebra, in an ongoing controversy which it will heighten[58]. The quality of his translations, generally made in close collaboration with pandits, has made it an enduring reference.

Colebrooke included portions of commentaries his translations' footnotes. Thus Gaṅgadhāra (fl. 1420), Sūryadasa (1541), Gaṇeṣa's (fl.1520/1554) and also Ramakṛṣṇa's (? date unknown maybe 1687[59]) commentaries are partially included in L[60]'s translation; Kṛṣṇa's (ca.1615), Rangunāṭha's (?) and Ramakṛṣṇa's commentaries are used in BG's translation. Finally, PBSS (noted as "CA", an abbreviation of *Caturdeva*, a part of his name) is used (and quoted) to translate the BSS. Furthermore, although the original text is not known to him or to his English readers, Colebrooke's introduction refers and discusses Āryabhaṭa's works[61]. Āryabhaṭa was indeed criticized by Brahmagupta; his verses were discussed and quoted by diverse commentators. Colebrooke also refers to a work of Śrīdhara, which is not however our PG[62]. Thus Colebrooke's readers came to know of Āryabhaṭa, Brahmagupta and Śrīdhara.

Colebrooke aimed at describing the mathematical tradition of India. This led him to consider only the mathematical part of Brahmagupta's treatise, the BSS. And consequently only a portion of PBSS. By doing so, was he prolonging an attested tradition of copying (and thus showing a separate interest) of mathematical chapters of astronomical treatise that existed in the Sanskrit tradition? This selection of BSS's chapters in a printed edition, may also mark the beginning of a long enduring historiographical trend noted in the introduction: mathematical subjects have been more treated than astronomy or astrology in the study of old *jyotiṣa* texts.

In Colebrooke's introduction, commentators first appear as proofs of the authenticity and antiquity of the texts he is concerned with. He writes[63]:

> "The genuineness of the text is established with no less certainty [than its date] by numerous commentaries in Sanskrit, besides a Persian version of it. Those commentaries comprise a perpetual gloss, in which every passage of the original is noticed and interpreted : and every word of it is repeated and explained, a comparison of them authenticates the text where they agree; and would serve, where they did not, to detect any alterations of it that might have taken place, or variations, if any had crept in, subsequent to the composition of the earliest of them. A careful collation of

[57] *Op.cit.*; p. xvi.
[58] (Kejariwal, 1988, 111-112).
[59] CESS V 453 a.
[60] *Op.cit.* Note A p. xxv and p.xxvii
[61] *Op.cit.* Introduction, sections G to I pp. xxxvii-xiv.
[62] *Op.cit.* p.v he writes that he has a copy of "Srìdhara's compendium of arithmetic", which is probably the *Triśatika*.
[63] (Brahmagupta; Bhāskarācārya; Colebrooke, 1817, iii)

> several commentaries, and of three copies of the original work, has been made, and it will be seen in the notes to the translation how unimportant are the discrepancies. "

Commentaries are thus useful and necessary when one wants to edit a text, they are philological tools. However the way they are integrated in Colebrooke's translations, pin points to the fact that commentaries were far from being just that. They were used to understand the treatises. They were stimulating mathematically with their examples and proofs. However, a commentary was not treated as a text in itself. It was given in bits and pieces. Selected.

Look at the translation, as seen in Figure 5. The commentary appears typographically as a secondary text, written in a smaller font. It is fragmentary, given in different footnotes. However, it literally spills over and eats the space which is meant for the treatise. The commentary's importance for Colebrooke is visually explicit.

Recalling that codex's sometimes include only portions of commentaries, we may wonder how much this attitude, consisting in considering commentaries in bits and pieces, reflects the way the pandits that trained Colebrooke, and then perhaps helped him in collating and understanding the texts, worked. A history of how commentaries were thought of and read in the Indian subcontinent would be very helpful here. A precise description of how Colebrooke (and other orientalists) worked with pandits in relation to texts would be useful as well[64].

2.2 An Indian scholarship with commentaries: Datta & Singh and K. S. Shukla

During the XIXth century, the history of mathematics in India slowly opened officially to Indian scholars, who elaborate a scholarship that is as much directed towards an inner audience, as an answer and a discussion with European interlocutors. Their presence is first seen in articles discussing authorship of texts, before being felt in a series of editions and translations of texts on mathematics. Thus, the names of Datta, Sengupta and Dvivedin appear in an initial period of confusion were the Vth century Āryabhaṭa was confused with his XIth century namesake, and the VIIth century Bhāskara I was confused with his XIIth century namesake.

By the end of the XIXth century the movement, which started at the begining of the century, of edition, translation and analysis of texts on mathematics in Sanskrit comes to a peak. Kern in 1874[65] edits for the first time [66] the Ab. His edition is printed with Parameśvara (XIVth century) 's commentary. His introduction evokes SYAB, which is quoted and referred to by Parameśvara; but not BAB. In 1879, chapter 2 of the Ab is translated and analyzed into French by Leon Rodet[67]. In 1896, Dikshit gives an edition of the BSS with his own commentary[68]. These editions are followed by a number of translations in English and the first studies in this language. Thus in 1907 and 1908 G. R. Kaye publishes his

[64]The latter has been studied partially in (Kejariwal, 1988), (Aklujkar, 2001), (Bayly, 1996) and others .

[65](Kern, 1874).

[66]The tradition of copying manuscripts can of course also be seen as an editorial tradition of classical India, but we are referring here to printed books.

[67](Rodet, 1879)

[68](Dikshit, 1896).

Figure 5: The BSS and PBSS in Colebrooke's translation

GAṄITÁD'HYÁYA, ON ARITHMETIC;

THE TWELFTH CHAPTER OF THE

BRAHME-SPHUṬA-SIDD'HÁNTA,

BY BRAHMEGUPTA;

WITH SELECTIONS FROM THE COMMENTARY ENTITLED

VÁSANÁ-BHÁSHYA,

BY CHATURVÉDA-PRĬT'HÚDACA-SWÁMÍ.

CHAPTER XII.

ARITHMETIC.

SECTION I.

1. HE, who distinctly and severally knows addition and the rest of the twenty logistics, and the eight determinations including measurement by shadow,[1] is a mathematician.[2]

2. Quantities, as well numerators as denominators, being multiplied by

[1] Addition, subtraction, multiplication, division, square, square-root, cube, cube-root, five [should be, six] rules of reduction of fractions, rule of three terms [direct and inverse,] of five terms, seven terms, nine terms, eleven terms, and barter, are twenty *(paricarman)* arithmetical operations. Mixture, progression, plane figure, excavation, stack, saw, mound, and shadow, are eight determinations *(vyavahára)*. CH.

For topics of Algebra, see note on § 66.

[2] *Gaṅaca*, a calculator; a proficient competent to the study of the sphere. CH.

the opposite denominator, are reduced to a common denomination. In addition, the numerators are to be united.[1] In subtraction, their difference is to be taken.[2]

3. Integers are multiplied by the denominators and have the numerators added. The product of the numerators, divided by the product of the denominators, is multiplication[3] of two or of many terms.[4]

4. Both terms being rendered homogeneous,[5] the denominator and nu-

[1] SCANDA-SÉN-ÁCHÁRYA, who has exhibited addition by a rule for the summation of series of the arithmeticals, has done so to show the figure of sums; and he has separately treated of figurate quantity *(cshétra-ráśi)*, to show the area of such figure in an oblong. But, in this work, addition being the subject, sum is taught; and the author will teach its figure by a rule for the summation of series (§ 19). In this place, however, sum and difference of quantities having like denominators are shown: and that is fit. CH.

[2] Example of addition:* What is the sum of one and a third, one and a half, one and a sixth part, and the integer three, added together?

Statement: $1\frac{1}{3}$ $1\frac{1}{2}$ $1\frac{1}{6}$ 3. Or reduced $\frac{4}{3}$ $\frac{3}{2}$ $\frac{7}{6}$ $\frac{3}{1}$.

The numerator and denominator of the first term being multiplied by the denominator of the second, 2, and those of the second by that of the first, 3, they are reduced to the same denominator ($\frac{8}{6}$ $\frac{9}{6}$; and, uniting the numerators, $\frac{17}{6}$). With the third term no such operation can be, since the denominator is the same: union of the numerators is alone to be made; $\frac{24}{6}$, which abridged is) 4. So with the fourth term: and the addition being completed, the sum is 7.

Subtraction is to be performed in a similar manner; and the converse of the same example may serve. CH.

[3] *Pratyutpanna*, product of two proposed quantities.—CH. See a rule of long multiplication, § 55.

[4] Example: Say quickly what is the area of an oblong, in which the side is ten and a half, and the upright seventy sixths.

Statement: $10\frac{1}{2}$ $11\frac{4}{6}$. Multiplying the integers by the denominators, adding the numerators, and abridging, the two quantities become $\frac{21}{2}$ and $\frac{35}{3}$. From the product of the numerators 735, divided by the product of the denominators 6, the quotient obtained is $122\frac{1}{2}$. It is the area of the oblong.

Others here exhibit an example of the rule of three terms, making unity stand for the argument or first term. For instance, if one *pala* of pepper be bought for six and a half *paṅas*, what is the price of twenty-six *palas*? Answer: 169 *paṅas*.] CH.

[5] The method of rendering homogeneous has been delivered in the foregoing rule (§ 3) "Integers are multiplied by the denominators," &c.—CH. It is reduction to the form of an improper fraction.

* It is not quite clear whether the examples are the author's or the commentator's. The metre of them is different from that of the rules; and they are not comprehended, either in this or in the chapter on Algebra, in the summed contents at the close of each. They are probably the commentator's; and consigned therefore to the notes.

controversial *Notes on Indian Mathematics*, part two of which is devoted to Āryabhaṭa[69]. Sengupta publishes a first English translation of the Ab[70], followed by Clark in 1930[71]. B. Datta and S. N. Singh publish in 1937 the enduring classic "Hindu Mathematics"[72]. Their main aim is to provide a general description of all the different ways Hindu mathematicians practiced elementary and sometimes higher mathematics, each author adding a stone to this description. But they also want to answer G. R. Kaye's claims on the Arabic or European origin of Indian mathematics in general and Ab's mathematics in particular. To do so, part of their effort consists in comparing the history of mathematics in Europe with what has been discovered about the history of mathematics in the Indian subcontinent. Datta & Singh write then, to a certain extent, a history of mathematics in India with its great authors and treatises. As trained mathematicians, their focus is essentially on the mathematical contents of the texts[73]. B. Datta and A. N. Singh consider mathematical commentaries essentially as mathematical texts. Thus Bhāskara I, whose text was known but was not published, is referred to several times as an astronomer sometimes dealing with mathematics[74]. His commentary is at times quoted to explain or give an interpretation of Ab's verses[75], sometimes even in footnotes like in Colebrooke's text[76], often together with other commentators of Ab such as Nīlakaṇṭha (add date), none of whose commentaries were at the time edited texts. Essentially, however, BAB is refered to for its mathematical contents, outside of its relation with Ab[77]. It is mingled at times with the contents of Bhāskara's other astronomical texts[78].

With the independence and the creation of institutions for the history of science in India, a new wave of editions is set forth. K. S. Shukla was an important actor of this movement. In 1959, he publishes, at the University of Lucknow, an edition of PG with APG, together with an English translation of APG[79]. He then turns to Bhāskara's work, first editing and translating his treatises, which can be seen as elaborations of Āryabhaṭa's astronomy, the *Laghubhāskarīya* and the *Mahābhāskarīya*[80]. In 1976, an edition and translation of the AB, with editions of BAB and SYAB within the Indian National Science Academy are conjointly published by K. S. Shukla and S. R. Sharma[81].

By the 80's a new generation will take up the study of commentaries with mathematical subjects. On the one hand, publications on the Mādhava school of mathematics will call the attention of historians of mathematics on scholiasts of Āryabhaṭa; on the other hand the Japanese students of D. Pingree, T.

[69] (Kaye, March 1908).
[70] (Sengupta, 1927).
[71] (Clark, 1930).
[72] (Datta, 1935).
[73] But they also embody a dying tradition of mastering Sanskrit texts on *jyotiṣa*. After all, B. Datta was at the end of his life addressed as *pandit*. Furthermore the enduring quality of their translations attest this mastering. "Hindu Mathematics" is certainly a blend of the two traditions, *jyotiṣa* lore and modern historiography of mathematics. A detailed scrutiny of Datta & Singh's works could probably yield much information on the first.
[74] (Datta, 1935, Volume I, 125).
[75] (Datta, 1935, Volume I, 66-67; 196; 211. Volume II, 93-95).
[76] (Datta, 1935, Volume I, 170). SYAB also such as in *op. cit.*[Volume II, 91, footnote 4].
[77] (Datta, 1935, Volume I, 80, 82, 87, 130, 204, 239; Volume II, 87, 238).
[78] Thus the entire part devoted to the *kuṭṭaka* in Volume 2, quotes all the different texts of Bhāskara I, sometimes to present his own algorithms, some other times as explaining Ab's algorithm.
[79] (K. S. Shukla, 1959).
[80] (K. S. Shukla, 1963), (K. S. Shukla, 1960)
[81] (K. V. Shukla & Sharma, 1976), (K. S. Shukla, 1976), (K. V. Sarma, 1976).

Hayashi in particular, will start publishing articles on the mathematical contents of different commentaries on Ab, BG and L. We can add to this enumeration the publications of F. Patte's PhD and my own, to testify of the growing interest for mathematical commentaries in Sanskrit as the XXth century came to an end[82].

Shukla's editions uses commentaries in three separate ways: commentaries are first used, as in Colebrooke's case, as a philological assessment of the original text he wants to edit. They are also used to explain the text. Thus APG is not translated but Shukla refers to it, in the written comments that escort his translation of PG. Shukla sometimes seems to consider commentators as giving a peculiar interpretation of the treatise, and at other times as explicitating the contents. In his co-edition and translation of the Ab, commentators are summoned to add a mathematical depth to algorithms. They are at times quoted for their conflicting interpretations. Finally in some instances, especially concerning BAB, commentaries appear as a mathematical text in their own right[83]. The trend that sees commentaries not only as philological aids, but also as independent mathematical texts, has been growing. This has no doubt to do with a general trend in the history of mathematics at large, as such approaches seem to have been characteristic of the Chinese corpus as well.

Commentaries then were used, read and analyzed but seldom translated by modern historians of mathematics. This made quite sense when commentaries where thought of, as often they were, as philological tools to edit and understand the texts they commented. The abscence of translation remains quite surprising when commentaries are considered as texts on mathematics in their own right. Furthermore, if we see commentaries and treatises as ballroom dancing couples, it is as if the focus had been essentially on the treatise without its partner, sometimes on the commentary without its partner, and sometimes on details of how their steps follow each other. However, the global picture of how they danse together has not been drawn, or even aimed at.

3 The relation of commentaries with their treatises: the example of the extraction of square roots

In studies on the history of mathematics in India, the mathematical relation of commentaries to the text they comment has been left, in general, unresolved. Reflections on the relation of one text to another, usually restrict themselves to the grid of a "right or wrong" interpretation of the treatise by the commentary. Scholarship on the mathematical contents of commentaries, has focussed on the interesting

[82]How much have these attitudes towards commentaries been linked to more general developments in the field of Indology? Indeed, Indology developed a special focus on the study of treatises and the contents of important commentaries, somewhat neglecting to reflect on the commentary as a specific kind of text. The last five, ten years has seen however a renewed interest in this kind of texts, as testified by the conference "forms and uses of the commentary in the Indian world", which was held in Pondicherry in february 2005, see http://www.ifpindia.org/Forms-and-Uses-of-the-Commentary-in-the-Indian-World.html.

[83]Before publishing his edition of BAB, Shukla published a number of analysis, pinpointing the mathematical importance of the text. (K. S. Shukla, n.d.)

mathematical ideas they contained, and thus essentially their proofs[84]. Implicitely then a relation of the commentary as providing the mathematical justification of the treatise is drawn. We would like to show that the relation is more complex.

In the following, rules to extract square roots, which are found in Ab and PG, but not in BSS, will be examined.

3.1 Extracting square roots along different lines

The square root extraction procedures during this period use the decimal place-value notation as a basis. The algorithms given by our authors are not "useful" procedures to extract square roots, such as the interpolations usually described in astronomical (parts of) treatises. They all suppose that one extracts the root of a perfect square[85]. The idea is to recognize in a number written with the decimal place-value notation the hidden development of a square expansion. To say it in other words, this would mean to recognize in a number written as $a_{2n} \times 10^{2n} + a_{2n-1} \times 10^{2n-1} + \ldots + a_1 \times 10^2 + a_0 \times 10 + c^2$ the development of a square of the kind $(b_n \times 10^n + \ldots + b_i \times 10^i + \ldots + c)^2$ $(i < n)$.[86] Crucial then to this algorithm is to distinguish between powers of ten that are also squares (the even powers of ten), and those that are not.

Let us note that Ab's algorithm provides directly the square root. PG's algorithm gives first a procedure to extract a double square root, and then says that it should be halved. In other words, although they are grounded on the same idea, these two procedures differ in their intermediary steps. We will not expose their respective algorithms here, concentrating on what what treatises tell us that commentaries do not and vice-versa.

This is how Ab gives the rule to extract square roots[87]:

> **Ab.2.4. One should divide, constantly, the non-square ⟨place⟩ by twice the square root|**
> **When the square has been subtracted from the square ⟨place⟩, the quotient is the root in a different place||**

Without a commentary the algorithm itself is hard to understand. Part of the difficulty springs from the fact that the verse states the algorithm, starting from its middle, emphazing its iterative aspect. Another difficulty rises from the fact that Āryabhaṭa uses a pun which makes us confuse which of the

[84] (Srinivas, 1990), (Patte, 2004). Strangely enough little reflection has been published on the comments contained in commentaries on the field of *gaṇita* itself. Such reflections could however explain why chapters on *gaṇita* containing algorithms with little astronomical applications, where included in treatises on astronomy. See (Pingree, 1981), (Keller, 2007), (Plofker, forthcomming).

[85] Why then were such procedures given? In a mathematical tradition where the correction of an algorithm is sometimes verified by inverting it, and finding the initial input, such a procedure which inverts he squaring procedure may have seemed useful. Solved examples in the commentaries to these rules, all revert the squarings that where illustrated in the squaring procedure.

[86] For general explanations on the different methods see (Datta, 1935, Volume I, 170-171) and (Bag, 1979, 78-79).

[87]

> *bhāgaṃ hared avargān nityaṃ dviguṇena vargamūlena|*
> *vargād varge śuddhe labdhaṃ sthānāntare mūlam||*

See (K. S. Shukla, 1976, 36-37) for an explanation of the algorithm.

digits are squares (*varga*) and which of the digits are noted on "square positions". This pun however highlights exactly the mathematical idea behind the root extraction: "square places" of the decimal place value notation are the places where we get hints to which digits have been squared in order to produce the number we are dealing with.

BAB and SYAB, will lift the difficulty raised by Ab's pun, by introducing a new grid. Instead of considering square and non-square places, even and odd places are used. This is done by a simple commentarial act, the substitution of one word by another. Thus, BAB states:

> In this computation (*gaṇita*), the square (*varga*) is the odd (*viṣama*) place.

And SYAB[88]:

> In the places where numbers are set-down, the odd places are square places. The even places are non-square places.

We will come back to the different grids placed on the decimal place value notation. The commentarial act of substituting a name given in the treatise by another has two functions in this case: first, explaining the literal meaning of the verse; second, pointing to the mathematical meaning of the pun. More, this word substitution points to how, according to the commentators, the *sūtra* was composed: using a mathematical pun that pins down the mathematical idea behind the algorithm. If Ab gives the core mathematical idea of the algorithm, BAB and SYAB highlight it.

Ab's confusing pun was not taken up by PG[89]:

> **PG.24. Having removed the square from the odd term, one should divide the remainder by twice the root that has trickled down to the place|**
> **⟨And⟩ dispose the remainder on a line (*paṅkti*)||**
> **PG.25. Having subtracted the square of that, one should divide the previous result|**
> **That has been doubled. Thus again and again, ⟨finally⟩ one should halve twice the square.||**

PG's formulation of the algorithm avoids all puns on the words that would underline a mathematical idea behind the given algorithm: compared to Ab it is not so dense or confused. It however gives precise elements hinting to the concrete carrying out of the algorithm on a working surface. A line (*paṅkti*) is evoked, and the movement of trickling down as a drop of water (*cyuta*) is used to characterize the apparition of a partial, double root, digit by digit.

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[88](K. V. Sarma, 1976, p. 36, line 15).

[89]See (K. S. Shukla, 1959, 18 for the Sanskrit, 9-10 of the part in English for an explanation of the procedure as described in APG)

> *viṣamāt padas tyaktyvā vargaṃ sthānacyutena mūlena|*
> *dviguṇena bhajec cheṣaṃ labdhaṃ viniveśayet paṅktau ||*
> *tadvargaṃ saṃśodhya dviguṇaṃ kurvīt purvaval labdham|*
> *utsārya tato vibhajec śeṣaṃ dviguṇīkṛtaṃ dalayet ||*

This is further developed by APG, which multiplies the "setting down" of partial roots, and evokes repeatedly operations carried above (*uparita*), below (*adhas*) and the fact that the partial double root extracted slides like a snake (*sarpaṇa*, *sarpita*) to the next position[90]. The procedure then is described in the details of how practically it is to be carried out in a working surface. The intricate way the positional system works is quite precisely indicated by APG. However no reference is made to the idea we find in Ab's treatise evoking partial squares.

We see then the commentaries help us unveil how they perceived the different nature of Ab and PG as treatises. The emphasis is, on one side, on the idea behind the procedure (in Ab), on the other, on expressing all the different steps of the algorithm, including the fact that the double square is obtained on a separate line (for PG). And indeed, two different kind of treatises are involved here. Ab is a theoretical astronomical treatise while PG is explicitly devoted to practical mathematics (*vyavahāra*). Commentaries differ then according to the type of texts they comment. If they all include illustrated examples, APG is the only one to follow precisely how the different intermediary operations are carried out, even evoking the possibility for a doubled number to become bigger than 10, during the steps of the algorithm[91]. On the other hand, neither BAB nor SYAB insist on these intermediary steps, highlighting on the contrary the essential idea behind the algorithm. PG concentrates on whole numbers, whereas the emphasis in SYAB and BAB is on the fact that the square root of fractions is the fraction of the square roots. The fact that commentaries adhere to the kind of text they are commenting, giving only what is appropriate in such circumstances, is especially clear in the case of SYAB: SYAB has read PG. SYAB actually quotes PG in this very verse commentary, not for the details of how the procedure is carried out on whole numbers, nor for the positioning of digits during the procedure, but for a rule concerning the square-root of fractions.

Thus commentaries are not systematically practical, detailed explanations of the general cases formulated in the treatises. Commentaries follow closely what they deem is the aim of the treatise, explaining it, sometimes linking it to other considerations, but not going into details which would not be appropriate for the kind of treatise at hand. A commentary of a a theoretical treatise will not detail the carrying out of a procedure, and reversely a commentary of a practical text, will not reflect on ideas, even if the authors of commentaries know better. Furthermore, eventhought the non textual practice of tabular computations seems to be the realm of the disposition and resolution parts of solved examples in commentaries, we have seen that PG alludes to it : treatises can also testify of these practices, although probably not in detail. So that style of commentaries depend on styles of treatises, or more precisely on how commentaries read treatises. An analysis of how commentaries, by word substitutions, has read the treatises can furthermore give us insights onto practices and thoughts that have until now not been analyzed: that of the use of the decimal place value as a formal notation to which we will now turn.

[90](K. S. Shukla, 1959, p.18-19 of the Sanskrit, p. 9-10 of the English version to see how the numbers are disposed and change during the algorithm, according to APG).

[91]*op. cit*[18, line 15-16]

3.2 Positional notation and extracting square roots

If the history of mathematics in the Indian subcontinent, has long insisted on showing that the decimal place value notation came from India, it reflected little on the concept different authors had of this system, and in particular how they thought and used the idea of position. And indeed, for BAB, SYAB and APG the decimal place value is a conventional notation for which places where digits are noted to make a number are an ordered set on a horizontal line[92]. The important idea stressed with a great continuity by one author after another, is that a "place" (*sthāna*) does not exist alone, but only in relation to another, in an ordered relation. This is how a "place" then becomes a "position", although there is no new Sanskrit word that expresses this conceptual change. Consequently, is this positional notation thought of as a positional *system*? A close look at the way commentators treat the extraction of square root procedures enables us to approach more closely their conception of position.

Let us recall how Ab gives the rule to extract square roots[93]:

> **Ab.2.4. One should divide, constantly, the non-square ⟨place⟩ by twice the square root|**
> **When the square has been subtracted from the square ⟨place⟩, the quotient is the root in a different place||**

We have noted previously that one of the difficult aspects of this verse comes from a pun which makes us confuse which of the digits are squares (*varga*) and which of the digits are noted on "square positions". Ab considers the decimal place-value notation as an ordered line of places, for increasing powers of ten. To this reading, he adds a new grid to qualify the places, distinguishing the powers of ten which are squares from those that are not squares. Ab's readings of positions are concerned simultaneously with the mathematical dimension of the decimal place value notation and with the mathematical idea on which the algorithm rests. BAB followed by SYAB together help us understand Ab's verse by giving new names to these places. They both start from the decimal place value notation, but consider it outside of its mathematical signification. They count the different places were digits are noted, starting on the right, from the lowest power of ten, and continuing on the left. All the even numbers of this enumeration indicate "even places", and odd numbers, "odd places". Since, 10^0 starts this enumeration, what Ab calls "square places", BAB and SYAB call "uneven place", and what Ab calls "non-square places", BAB and SYAB call "even place":

[92] (Keller, forthcomming).

[93]

> *bhāgaṃ hared avargān nityaṃ dviguṇena vargamūlena|*
> *vargād varge śuddhe labdhaṃ sthānāntare mūlam||*

See (K. S. Shukla, 1976, 36-37) for an explanation of the algorithm.

10^5	10^4	10^3	10^2	10^1	10^0
non-square place (*avarga*)	square place (*varga*)	non-square place	square place	non-square place	square place
6	5	4	3	2	1
even place (*sama*)	odd place (*viṣama*)	even place	odd place	even place	odd place

Commentators then add to the ordered list of places that define the decimal place value notation their own grid. This grid considers the notation outside of its mathematical content, as a tabular form with a numbered list of items on a line. They apply a mathematical assessment, with odd and even numbers[94], to this formal way of looking at the decimal place value notation. This mathematical assessment is not directly related to the algorithm, until commentators link this grid to the one used by Ab, simply by substituting one word to another. The name substitution they make is synthesized in Table 3.2.

Table 4: Names of places in the algorithm to extract square roots

Texts	Even powers of ten	Uneven powers of ten
Ab	*varga*	*avarga*
BAB	*viṣama*	*sama*
PG	*viṣama*	nihil
APG	*viṣama*	*sama*
SYAB	*viṣama*	*sama*

These places are used and qualified in different classifications: some of these classifications underline their values, others their positions in an ordered line, others again pinpointing their mathematical qualities (as squares). The multiplication of these classifications points to the fact that all the authors considered here, do indeed use the decimal place value as a system of positions which can be qualified in as many different ways an algorithms requires. We had seen previously that the commentarial act of substituting a name given in the treatise not only explains the literal meaning of the verse but also highlights the mathematical meaning of the pun. It also creates a new grid, a new system of positions, linking it to the one it has unravelled for the treatise.

PG uses a vocabulary that is the same as BAB[95]:

[94] BAB brings it in through a linguistic analysis of the term *avarga*, noting:

> Since a non-square ⟨takes place⟩ when oddness is denied, by means of ⟨the affix⟩ *nañ* ⟨the expression refers to⟩ an even (*sama*) place, because, indeed, a place is either odd or even.

[95] See (K. S. Shukla, 1959, 18 for the Sanskrit, 9-10 of the part in English for an explanation of the procedure as described in APG)

> *viṣamāt padas tyaktyvā vargaṃ sthānacyutena mūlena*|

PG.24. Having removed the square from the odd term, one should divide the remainder by twice the root that has trickled down to the place|

⟨And⟩ dispose the remainder on a line (*paṅkti*)||

PG.25. Having subtracted the square of that, one should divide the previous result|

That has been doubled. Thus again and again, ⟨finally⟩ one should halve twice the square.||

APG comments the verse regarding questions of place, as follows [96]:

> One should subtract a possible square (*saṃbhavinaṃ vargaṃ*), from the *viṣama* ⟨place⟩ of the square quantity, ⟨in other words⟩ from what is called odd (*oja*), that is from the first, third, fifth, or seventh etc. ⟨place⟩, from the places for one, one hundred, ten thousand, or one million, etc., from the *pada*, that is from the last among all other places.

And considering the square root of 188624, it adds[97]:

> In due order from the first place which consists of four, making the names: "odd (*viṣama*), even (*sama*), odd (*viṣama*), even (*sama*)".
>
> Setting down:
>
> | sa | vi | sa | vi | sa | vi |
> |---|---|---|---|---|---|
> | 1 | 8 | 6 | 6 | 2 | 4 |
>
> In this case, the odd terms which are the places one, a hundred, ten thousands, consist of four, six and eight. Therefore the last odd term is the ten thousand place which consists of eight.

---

dviguṇena bhajec cheṣaṃ labdhaṃ viniveśayet paṅktau ||
tadvargaṃ saṃśodhya dviguṇaṃ kurvīt purvaval labdham|
utsārya tato vibhajec śeṣaṃ dviguṇīkṛtaṃ dalayet ||

[96](K. S. Shukla, 1959, 18, line 10-12)

vargarāśer viṣamāt padād ojākhyād ekatṛtīyapañcamasaptamāder ekaśatāyutaprayutādisthānebhyo 'nyatamasthānād antyāt padāt sambhavinaṃ vargaṃ tyajet

[97](K. S. Shukla, 1959, 18, line 19-22)

ānulomyena ekasthānāc catuṣkāt prabhṛti viṣamaṃ samaṃ viṣamaṃ samam iti saṃjñākaraṇam /

sa	sa	sa	vi	sa	vi
1	8	6	6	2	4

atra catuḥṣaḍaṣṭakāni ekaśatāyutasthānāni viṣamapadāni tebhyo 'yutasthānastham aṣṭakam antyaṃ viṣamapadaṃ

APG completes precisely the mathematical background in relation to the decimal place value notation that the treatise just alludes to. Indeed, the anonymous commentator uses different syntactic expressions for the place, its value within a power of ten, the place it has in the row of numbers noted on the line, and the digit which is noted in this place. The values in power of tens that a place stands for are noted within a *tatpuruṣa* compound ending with *sthāna* and incorporating an inner enumerative *dvandva*, which gives the concerned powers of ten in increasing order (*ekaśatāyutasthānāni*, "the places for one, a hundred, ten thousands"). The place it has in the row of numbers is referred to in several ways. APG numbers the places, starting with one for the lowest power of ten and increasing successively. These are enumerated in a *dvandva* giving the ordinals of the concerned places (*ekatṛtīyapañcamasaptamāder*, "for the first[98], the third, the fifth, the seventh, etc.). APG, following PG, reproduces then the *viṣama* (odd) / *sama* (even) terminology we found in BAB and SYAB. But PG and APG insist that the places are used within an ordered set, or *series* of numbers for which there is a first and a last term: PG uses the word *pada*, which is used for the terms of a series, and APG glosses *pada* with the expression *anyatamasthānāt antyāt*, "the last among all the other places". Finally the digits noted are understood as tools to be worked with, within these positions. This is how we can maybe understand the fact that the names of numbers are given ending with the suffix *-ka* (*catuḥṣaḍaṣṭakāni* , "consists of four, six and eight").

Thus a tightly knitted connection between commentary and treatise explains to us all the different ways in which a place becomes a position on an elaborate grid which rests not only upon the decimal place value notation, and on the algorithm to be carried out, but also within the formal system created by the notation itself: a line that can be ordered in many different ways.

To sum it up, we have seen different authors use the places of the decimal notation in different manners: Ab underlining their mathematical powers, BAB and SYAB linking these mathematical meanings to the formal notation of numbers, and using mathematical properties of this formal notation to do so, PG describing only this formal notation and its mathematical properties, and APG linking this formal notation to their meaning in relation to the decimal place value notation. If we recall the different lines on which the procedure to extract square root should be carried out, the different grids described by APG, have now both a meaning vertically and horizontally, and this is used to carry out the algorithm itself. Here then the horizontal expansion of the decimal place value notation is extended into a table, with operations conducted within columns and others within lines, as for other elementary arithmetical operations. A chronological perspective gives us a historical evolution: it seems that over time the decimal place value notation, slowly took on a formal aspect, as a tabular form which could be used not referring systematically to what the positions meant in terms of values of powers of ten.

Conclusion

Reflections on the textual functions of commentaries and treatises in mathematics, can lead to conceptual insights into the practice of the decimal place value notation. A simple word substitution as used in BAB, SYAB and APG, has showed how the decimal place value notation is seen as a horizontal line in a table

[98] *Eka* however is used here and not *prathāma*.

on which formal operations can be carried out.

Viewed with commentator's glasses, two different kind of *śāstra*s have been studied here: one whose emphasis is on mathematical ideas, another on mathematical practices. The first belongs to a chapter of an astronomical treatise, the second belongs to a practical mathematical text. They each have different ways of describing algorithms. Different texts, with different aims do not have the same descriptive practice's. Describing the descriptive practices may enable us to understand the use of texts?

Furthermore, the perceived aim of treatises determines what commentaries should do. Commentaries do not embody "practical" knowledge then: the kind of treatise determines the form of the commentary, whether it will concentrate on ideas or on how the algorithm is carried out on a working surface. Treatises then can hint to "pratical knowledge", that the commentaries can choose to spell out or not. However in all cases, the work of the commentaries is establishing relations, integrating what is hinted in the verse commented into a network of other systems. If we take the ballroom dancing metaphor again then, the treatise seems to lead the danse, but this is what the commentator wants us to believe. Like a virtuose yet discrete partner, it uses all its techniques and knowledge to leave the limelight to the treatise. Such an attitude, which has its counter examples, helps us understand why a stylistic criteria to distinguish commentaries from treatises is so difficult to find.

We have seen that a late tradition may have considered commentaries as fragmentary explanations and not full texts. Looking at the texts themselves, one can wonder wether they were made to be read, verse by verse, verse-commentary by verse-commentary separately. In the case of the three commentaries on root extraction that we have looked at, the decimal place value notation and the rules to carry out elementary operations are used. If these are known, the commentary can be followed as if it was autonomous. However this autonomous verse commentary reading may not hold for all algorithms. Nonetheless, we can imagine different verse commentaries by different authors being used simultaneously, leaving out how a full commentary would provide a more integrated vision of what the treatise was about. But how much of this imagined partial reading of commentaries was originally there when the authors wrote their commentaries and treatises? Can we imagine an oral culture of texts where they are known as an integrated whole but can be quoted and mobilized in fragmentary ways?

The examples we have studied here show us at least a late XVIIth century tradition in South India considering commentaries as fragmentary explanations and not full texts. On the way then, we have gathered some information on who had text copied: in the case of SYAB scholarly astronomers, ritualist families and religious institutions of south India.

Coming back to how commentaries were read, on can consider maybe this hypothetical evolution: This pandit tradition may have seeped into the way Colebrooke and other European orientalists considered these texts. This could have mingled with a European tradition of considering mathematical texts not for their textual characteristics but as just containing mathematical lore. The result being the kind of historiography, prevalent today when studying mathematical commentaries.

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