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Olivier BAGUELIN, EUREQua

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Strategic interactions in the labor market, self-esteem motivations and socio-demographic disparities

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Abstract

In a previous work, we have developed a model in which agents choose whether to achieve self-esteem through work. When they do, they develop an intrinsic motivation to effort. The analysis was restricted to an employment relation (employers were monopsons). In the present paper, we study the consequences of the latter analysis on labor market outcomes. The model we provide can give an account of many important traits of socio-demographic disparities in the labor market (notably of vertical occupational segregation). We consider how the structure of the labor market conditions this account.

Keywords: Self-esteem motivations, intrinsic motivation, selective hiring, occupational segregation, socio-demographic earnings gaps, oligopsony.

JEL classification: J15, J24, J71, J16, Z13.

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In a previous work, we have posited a theory of socio-demographic disparities in the labor market based on a special kind of occupational segregation. From the characterization of a subset of selective (upon socio-demographic criteria) jobs within the set of jobs¹ we have made explicit *potential* consequences of this occupational segregation on socio-demographic gaps in average earnings. This gap - favorable to the agents who "fit in" the workplace identity - may be a consequence of the fact that, depending on certain conditions, the share of selective jobs is an increasing function of the wage standard under consideration: hiring is more likely to be selective among better-paid jobs. This results from the fact that, all other things equal, a job compensation increases with its degree of demands and that the more demanding a job, the stronger employers' propensity to mobilize intrinsic motivation from workers *i.e.* to arouse the workplace identity. That is precisely on criteria predisposing certain working persons to the workplace identity that selective hiring occurs in our analysis. Our explanation of socio-demographic gaps in average earnings (as a statistical fact) is thus very simple: female workers and black workers would earn less than white male workers because of their concentration among less demanding jobs.

But still, the assumptions upon which we have developed this argument are restrictive (so that we have talked of "potential consequences") and we have provided little discussion of the conditions of its validity. This article, thus, aims at clarifying the channel through which the kind of occupational segregation we have considered impacts on the average earnings of each socio-demographic group within an appropriate framework. In the following, we leave aside the issue of asymmetrical information characterizing the employment relation to study the connection of the trade-off between workplace and out-of-the-workplace identity to market mechanisms. This choice allows us to more directly explore the conditions for our argument to be valid.

Our point is that non-wage differences between various jobs, on the one hand, the heterogeneity of workers' preferences, on the other hand, justify that employers be endowed with a special market power. As a consequence, in this paper, we assume that oligopsony prevails in the labor market. Such an assumption allows us to reproduce and clarify the case considered in previous works and to study the impact of competition between employment offers on our argument regarding socio-demographic gaps in average earnings.

The model of this paper allows us to stress the very nature of our argument: that is a macroeconomic argument. The gap in average earnings favorable to the workers of the "dominant" group does not trivially derive from their predisposition to the workplace identity. This predisposition may actually lead them to accept "low" pay relatively to the degree of demands of the job under consideration, such pay level that a worker of the "dominated" group would not have accepted. By providing an example for which the gap in average earnings is favorable to the dominated group, we put forward the role of the pool of jobs composition in socio-demographic disparities to emerge.

This article includes two parts. The first is devoted to the introduction of our model. We briefly situate it among labor market models mobilizing workers preferences on employment conditions: we show that our approach exhibits a double differentiation, both vertical and horizontal. We then introduce a model

¹Jobs are considered as pairs (ϕ, ψ) , that is (non-wage gratification opportunities, degree of demands).

of labor market with workers manifesting self-esteem motivations. The second part explores, through a simple numerical example, the conditions for our argument to be valid as well as the mechanisms ruling the average earnings of various social groups when competition exists between employers in the labor market. In this second step, we particularly study the impact of labor market rationing on welfare.

1 Employment conditions and strategic interaction in the labor market

This section is devoted to the introduction of a model of oligopsony in the labor market

1.1 Workers' preferences on employment conditions and labor market functioning

The role of workers' preferences on employment conditions are mainly apprehended by the theory of compensating differentials. The idea is that, all other things equal, higher wages compensate less satisfying employment conditions. Perfectly competitive labor market models predictions - in particular the law of single wage - should be understood taking into account employment conditions heterogeneity. Competition between firms, on one side, between workers, on the other, level out utilities attached to various jobs. This theory lies on *objective* differences between jobs: differentiation is vertical. In other words, workers' preferences on jobs characteristics are similar.

But workers' preferences on jobs characteristics can also be heterogeneous. A given job may provide different utility levels depending on the worker under consideration. In such a case, there is an horizontal differentiation of jobs. This affects labor market functioning since labor supply puts to a particular firm is then defined (contrary to what prevails under perfect competition). Oligopsony models of labor market specify and illustrate the way heterogeneous preferences on labor conditions may affect labor market functioning.

In the model of the employment relation provided in early works, the form of workers' preferences is endogenous: it is defined in equilibrium, adjusting in particular to the employment conditions offered by the employer. We have identify employment relations with a pair (ϕ, ψ) where ψ is the degree of demands of the job while the component ϕ is a measure of the non-wage gratification opportunities it provides. The important point is that this second component only enters as an argument of the utility function of a worker if he holds the workplace identity. Hence, for a similar degree of demands, two agents, the one holding the out-of-the-workplace identity, the other the workplace identity, may attach the corresponding job different levels of utility. This justifies, in the spirit of compensating differential, different compensation. There is thus heterogeneous employment conditions as well as heterogeneous workers.

For a given type (and/or identity), jobs are vertically differentiated. Some jobs are demanding, others are less: earning gaps reflect "objective" differences: vertical differentiation lies on ψ . Between workers

of different types, when equilibrium identity differ, jobs are horizontally differentiated. Workers holding the identity A are sensitive to non-wage gratification opportunities provided by their job, others are not: horizontal differentiation lies on the hedonic valuation, or not, of variable ϕ . Employers take this heterogeneity into account.

1.2 A model of strategic interaction in the labor market

We consider agents who choose to achieve self-esteem either through work (identity A) or through other activities outside one's working life (identity B). This choice conditions the form of their utility function. Effort is *perfectly observed* ex post by employers so that a contract only stipulates a transfer² $w \geq 0$ when the agent exerts effort (otherwise, employer can punish the agent). We denote \mathcal{J} the set of available jobs. Jobs are indexed by $j \in \mathcal{J}$, job j being characterized by $(S_j, \phi_j, \psi_j) \in \mathbb{R}_+^3$. The profit of employer j is simply $S_j - w_j$. Let $n_\theta > 0$ be the number of agents with trait $\theta \in \{0, 1\}$. When working in the job j paid w_j , an agent with trait θ 's utility writes

$$U_c(j, w_j; \theta) = \begin{cases} (1 + \gamma_w) w_j - \psi_j + I_A(\phi_j; \theta) & \text{if } c = A \\ w_j - \psi_j + I_B & \text{if } c = B \end{cases}$$

where $I_A(\phi_j; \theta) = \phi_j - \gamma_w w_A - \gamma_\theta (1 - \theta)$, $\theta \in \{0, 1\}$.

Information and timing. The timing of the market game is as follows. 1) The composition of the pool of labor suppliers (n_0 and n_1) is observed by potential employers. 2) Each firm j makes a *single* hiring offer, that is, makes a take-it or leave-it wage offer.³ 3) A first employment applicant is randomly drawn from the pool of labor suppliers, he perfectly observes all hiring offers and choose whether to remain outsider or to become the employee of a firm j ; in that case, job j is subtracted from the set of hiring offers. 4) A second employment applicant is randomly drawn among remaining labor suppliers; it is his turn to choose whether to accept one of the remaining offers or to be an outsider. 5) The process go one until: all the jobs are filled and/or all the applicants have been drawn.

Behaviors, employment conditions and utility. Firms offering a job have two ordered concerns: first, to fill the job and make a positive profit on it; second, when it is filled, to minimize the required transfer *i.e.* to hire the agent who will do the job for the least pay. By choosing to leave the job unfilled, she guarantees a null profit. Hence, job j is filled if and only if $w_j \leq S_j$ and we can restrict employer j 's strategy set to $[0, S_j]$.⁴

For all $j \in \mathcal{J}$ and $\theta \in \{0, 1\}$ let

$$U_j(w_j; \theta) := \max \{ (1 + \gamma_w) w_j - \psi_j + I_A(\phi_j; \theta); w_j - \psi_j + I_B \}$$

this function represents the utility of an agent of type θ taken into account his capacity to adjust his identity to offered employment conditions. The fact that this function be indexed by j should not lead

²The assumption that transfers must remain positive amounts to assuming that jobs degree of demand ψ is large enough.

³An hiring offer is not allowed to be contingent upon non-productive traits (such as once gender).

⁴Any strategy $w_j > S_j$ is strictly dominated.

to confusions: j refers to offered employment conditions (ψ_j, ϕ_j) . The form of agents' preferences being defined but for the trait θ , we are mainly interested in the utility level some employment conditions can arouse for a given pay, which we consider to the pair (j, w_j) . By staying at home, an agent gets a utility $I_B > 0$ so that an agent with trait θ accepts the hiring offer (j, w_j) if and only if $U_j(w_j; \theta) \geq I_B$. Let $w_j(U; \theta) = \min \left\{ \frac{\psi_j + U - I_A(\phi_j; \theta)}{1 + \gamma_w}; \psi_j + U - I_B \right\}$: for all $\theta \in \{0, 1\}$, $w_j(\cdot; \theta)$ denotes the inverse function of $U_j(\cdot; \theta)$.

Note that, by putting aside the issue of effort observability, we abolish the difference between strongly and weakly fulfilling jobs - see Baguelin (2005a). Our typology is restricted to two classes: unfulfilling and fulfilling jobs. Let us indicate within this new context conditions of selective hiring (upon socio-demographic criteria). A job is selective if it is fulfilling for agents with trait $\theta = 1$. It is non-selective if unfulfilling to for agents with trait $\theta = 1$ (a fortiori, it is then unfulfilling to agents with trait $\theta = 0$).

We provide in the appendix a general definition of labor market equilibrium and prove its existence. Yet, up to now, we cannot provide general results. Failing that, we examine, in the next step, the main mechanisms and implications of our model as regards socio-demographic disparities in earnings on the basis of simple example.

2 Market mechanisms and sociodemographic disparities

The purpose of this section is to explore from a simple numerical example, our model mechanism and to draw some consequences as regards socio-demographic disparities.

2.1 The elements of the example

Let us present the specific assumptions of our example.

2.1.1 Three jobs with distinct characteristics

Let us assume $\gamma_w = \gamma_\theta = \frac{1}{2}$ and $w_A = 1$, and consider three jobs $\mathcal{J} = \{1, 2, 3\}$. These jobs are characterized by

$\psi \backslash \phi$	1	2
1	Job 1	Job 3
2	Job 2	

Job 1 is little demanding but does not provide many non-wage gratification opportunities. Job 2 is demanding without providing better non-wage gratification opportunities than the job 1. By contrast, the job 3 is both little demanding and source of many non-wage gratification opportunities. Job 2 will usually obviously give rise to higher compensations than the two others (compensating differential). For this very reason, the assignment of this job to an agent with trait 0 or 1 will play a crucial role in resulting socio-demographic gap in average earnings.

All through this illustration, we make the following hypothesis.

Hypothesis *It is assumed that for all $j \in \{1, 2, 3\}$, $S_j \geq w_j(I_B; 0)$.*

This latter hypothesis guarantees that each firm can make a wage offer meeting the participation constraint of an agent with trait 0 - a fortiori that of an agent with trait 1 since $w_j(I_B; 0) \geq w_j(I_B; 1)$. This hypothesis is particularly important when agents with trait 1 are scarce.

Previous values imply

$$\begin{aligned} w_1(U; \theta) &= \min \left\{ \frac{2}{3}U + \frac{1}{3}(2 - \theta); 1 + U - I_B \right\} \\ w_2(U; \theta) &= \min \left\{ \frac{2}{3}(U + 1) + \frac{1}{3}(2 - \theta); 2 + U - I_B \right\} \\ w_3(U; \theta) &= \min \left\{ \frac{2}{3}(U - 1) + \frac{1}{3}(2 - \theta); 1 + U - I_B \right\} \end{aligned}$$

Each employer is either indifferent between types 0 and 1 workers or strictly prefer type 1 workers. In that case corresponding job will be said selective.

2.1.2 Expected average earnings per socio-demographic groups

The average earnings of a given socio-demographic group depends on its distribution between different jobs. This distribution depends itself on the relative frequency of types 0 and 1. The algorithm we have chosen to account for the impact of this relative frequency (on the distribution of socio-demographic groups between the jobs) generally implies that the order in which types 0 and 1 workers apply matters. We want to neutralize that. To do it, we propose to consider expected average earnings given the relative frequency of the types 0 and 1. The point is about weighting average earnings resulting from each configuration by the probability of this configuration occurrence.

Let $p(\theta_1, \theta_2, \theta_3)$ denote the probability that the configuration "job 1 is filled by an agent with trait θ_1 , job 2 by an agent with trait θ_2 , and job 3 by an agent with trait θ_3 ", θ_1, θ_2 and $\theta_3 \in \{0, 1\}$, and $\hat{w}^\theta(\theta_1, \theta_2, \theta_3)$ the corresponding average wage within the population of workers with trait θ . When no agent of one group is employed, $\theta_1 = \theta_2 = \theta_3$, we fix the average wage for this group to 0 - the reservation wage. Hence, we can always compute the expected average wage \hat{w}^θ for workers with trait θ . It is given by

$$\hat{w}^\theta = \sum_{(\theta_1, \theta_2, \theta_3) \in \{0, 1\}^3} p(\theta_1, \theta_2, \theta_3) \hat{w}^\theta(\theta_1, \theta_2, \theta_3)$$

Probability $p(\theta_1, \theta_2, \theta_3)$ derives from a random successive draw of agents among the work force. It is assumed that, in case of indifference, between available jobs from a drawn agent, the latter chooses each of them with identical probability.

2.1.3 The situation of reference and the steps of the analysis

We successively consider three cases. The first one corresponds to the absence of a significant competition between employers: this is our reference situation. Within a framework where labor supply and demand are both heterogeneous, the case of a simple juxtaposition of three monopsons allows us to focus on the choice, for each job, of a particular type of applicant *independently* from the choice of other employers.

This situation is precisely the one we considered earlier. The second case corresponds to the shortage of type 1 agents (agent with trait $\theta = 1$). In addition to the impact of this shortage on the gap in average pay of groups 0 and 1, we examine consequences in terms of efficiency. To this extent, it offers an echo to the analysis of the employment relation we provide in an early work where we have underlined the impact of the intrinsic motivation on the efficiency of the employment relation. The third case deals with a global shortage of agents (*i.e.* of both types 0 and 1). Which group benefits the most of the competition between employers? The point is about echoing to the crowding hypothesis - Bergmann (1971).

2.2 No significant competition between employers (in the labor market)

Assume that the number of applicants of each type, n_0 and n_1 , be large enough so that labor demand is immune from significant competition. For $n_\theta \geq 3$ whatever θ , firms can *a priori* behave as monopsons and bind applicants' participation constraint. Whether they be of type 0 or 1, an employee's utility is then I_B and wages

$$w_1(I_B; \theta) = \min \left\{ \frac{2}{3} I_B + \frac{1}{3} (2 - \theta); 1 \right\}$$

$$w_2(I_B; \theta) = \min \left\{ \frac{2}{3} (I_B + 1) + \frac{1}{3} (2 - \theta); 2 \right\}$$

$$w_3(I_B; \theta) = \min \left\{ \frac{2}{3} (I_B - 1) + \frac{1}{3} (2 - \theta); 1 \right\}$$

Job j will be selective if and only if $w_j(I_B; 1) < w_j(I_B; 0)$. Under our initial hypothesis, the selective feature of a job only depends on parameter I_B . We have

Selective jobs as a function of I_B				
	$I_B < 1$	$1 \leq I_B < 3/2$	$3/2 \leq I_B < 2$	$2 \leq I_B$
Job 1	selective	non-selective	non-selective	non-selective
Job 2	selective	selective	non-selective	non-selective
Job 3	selective	selective	selective	non-selective

Hence, for $I_B \geq 2$, no employer has a strict preference for type 1 applicants. For given employment conditions, a rise in I_B increases the relative attraction of outside-work, in particular for agents with trait 1. The two types tend to adopt a similar out-of-the-workplace identity which makes them perfect substitutes. By contrast, for $I_B < 1$, outside-work is little attractive, in particular for applicants of type 0. Yet, when the point is about arousing an intrinsic motivation, type 1 applicants have an advantage, jobs are selective.

In the following, we mainly pay attention to the I_B values which make job 2 switch from non-selective to selective. Our point is indeed about illustrating the core argument suggested in Baguelin (2005a): all other things equal (particularly non-wage gratification opportunities), the most demanding jobs switch to the set of selective jobs before others. But these jobs are precisely those which require the highest pay. In our example, in the absence of a significant competition between employers, the gap in average pay favorable to type 1 applicants derives from the proportion of fulfilling jobs (to type 1 workers) in

the economy. When fulfilling jobs are the minority, agents of type 0 benefit of an expected average pay strictly higher than that of agents with trait 1.

2.2.1 A majority of unfulfilling jobs

Let us assume $I_B = \frac{3}{2}$. In that case, in the absence of a significant competition between employers, only the job 3 is selective in the equilibrium (firm 3 is the only one who favors applicants of type 1). In the equilibrium, jobs 1 and 2 are unfulfilling whatever the type of their holder. Job 3 is fulfilling for agents of type 1. Let us denote $w = (w_1; w_2; w_3)$.

Equilibrium 1 For $I_B = \frac{3}{2}$, $n_1 \geq 1$ and $n_0 + n_1 \geq 3$, employers do not significantly compete in the labor market - they bind their employee's participation constraint. Labor market equilibrium is given by

$$w = \left(1; 2; \frac{2}{3} + \varepsilon \right)$$

where $\varepsilon = 0$ if $n_1 \geq 3$, $\varepsilon > 0$, $\varepsilon \rightarrow 0$ otherwise. The hiring offer of firm 3 can only be chosen by an agent of type 1 (it violates the participation constraint of type 0 agents).

This equilibrium characterization is provided in the appendix.

As we have seen in chapter 3, filling an unfulfilling job requires a complete compensation of corresponding demands; by contrast, the fulfilling job is paid below its "objective" disutility.

Let us compare average pay between socio-demographic groups. The strict preference of the employer 3 for type 1 workers entails, for all $\theta_1, \theta_2 \in \{0, 1\}$, $p(\theta_1, \theta_2, 0) = 0$. For $n_1 \geq 3$, an employer targeting a type 1 agent has not to worry that her hiring offer meet no demand. For $n_1 = n_0 = 3$ we obtain⁵ $\hat{w}^0 = 1,135 > \hat{w}^1 \simeq 0,961$. Other values of (n_0, n_1) are considered in the appendix.⁶ One can in particular consider the case of a total absence of type 0 agents. This absence turns out to be perfectly painless to the firms: type 1 agent can indeed be substituted. There exists, as we have seen, an asymmetrical substitutability between types. If certain jobs require type 1 workers, such workers are always perfect substitute of type 0 workers. For $1 \leq n_1 < 3$, at least one job is filled with one agent of each type. Firm 3 offers $\frac{2}{3} + \varepsilon$ so that a type 1 agent always favor this offer. For $n_1 = n_0 = 2$ one obtains⁷ $\hat{w}^0 = \frac{3}{2} > 1,135 > 0,961 > \hat{w}^1 = \frac{21}{24} + \frac{3}{4}\varepsilon$. The gap in average earnings favorable to type 0 agents is widen. The relative shortage of type 1 agents leads to a reduction in their average pay. This simply results from the fact that the probability that an agent 1 holds jobs 1 or 2 - the better paid - is reduced while job 3 is held by a type 1 agent with probability 1. Contrary to usual intuition, here, the growing shortage of type 1 applicants amplifies an expected gap in average earnings favorable to type 0 workers.

As long as the most demanding jobs remain unfulfilling to agents of type 1, the average pay of agents of type 0 can be higher than that of type 1 agents: the predisposition of type 1 agent for the workplace identity plays negatively on (expected) average earnings within group 1. Indeed, employer 3 relies on this predisposition to charge lower pay. *Group 1*, however, enjoy a "guaranteed" access to employment while

⁵See the proof in the appendix.

⁶In this first step however, we favor the case of an equal representation of both types applicants in the labor market.

⁷See the proof in the appendix.

group 0 does not - at least so far the number of 1 is large enough. Besides, in the case $n_1 < 3$, type 1 workers enjoy a utility level strictly higher than that of type 0 workers.

The main conclusion of this step (expected average earnings gap favorable to the group 0) is called into question when the more demanding job becomes fulfilling.

2.2.2 A majority of fulfilling jobs to a type 1 worker

Let us consider the case $I_B = \frac{3}{2} - \epsilon$ where $\epsilon > 0$, $\epsilon \rightarrow 0$. In this case, employers 2 and 3 strictly prefer agents of type 1.

Equilibrium 1' For $I_B = \frac{3}{2} - \epsilon$, $n_1 \geq 2$ and $n_0 + n_1 \geq 3$, employers do not significantly compete - they bind their employee's participation constraint. Labor market equilibrium is given by

$$w = \left(1; 2 - \frac{2}{3}\epsilon + \epsilon; \frac{2}{3} - \frac{2}{3}\epsilon + \epsilon \right)$$

where $\epsilon = 0$ if $n_1 \geq 3$, $\epsilon > 0$, $\epsilon \rightarrow 0$ otherwise. Hiring offers of firms 2 and 3 can only be chosen by type 1 applicants (they violate the participation constraint of type 0 agents).

This equilibrium characterization is provided in appendix.

This marginal reduction in agents' reservation utility does not identically affect the surplus of each firm. All other things equal, only firms 2 and 3 benefit from this reduction. Let us compare expected average earnings of each group. For $n_0 = n_1 = 3$, we obtain⁸ $\hat{w}^0 = \frac{3}{4} < \hat{w}^1 = \frac{47}{36} - \frac{11}{18}\epsilon$. The expected average earnings gap becomes favorable to type 1 agents. This remains true as $n_0 = n_1 = 2$: $\hat{w}^0 = 1 < \hat{w}^1 = \frac{4}{3} + \frac{4}{3}\epsilon - \frac{2}{3}\epsilon$.

The fact that the job 2 (the better paid) switch in the set of selective jobs before the job 1 (the more poorly paid) is obviously not accidental. This results from the fact that job 2 is the most demanding (all other things equal). Indeed, the more demanding a job the better paid and the more likely arousing the workplace identity will be profitable.

Here is illustrated the very idea of the explanation suggested in Baguelin (2005a) of the socio-demographic gaps in average earnings: these gaps result from the overrepresentation of the most demanding jobs among selective ones.⁹ Indeed, an employer appeal to an intrinsic motivation is, for given non-wage gratification opportunities ϕ , all the more likely that the job under consideration is demanding. A job compensation being an increasing function of its degree of demands, its ranking among fulfilling jobs is, all other things equal, all the more likely that it is well-paid and hence an earning gap favorable to agents of type 1.

From now on, we would like to evaluate the impact of a readjustment of market powers (on the labor market) on previous argument. How does this readjustment (favorable to labor supply) affect socio-demographic gaps in average earnings? We have seen below that the relative shortage of type 0 agents (for a total number of applicants higher than 3) was painless to the firms. We could have also underline

⁸See the proof in the appendix.

⁹Fulfilling to type 1 agents.

that this shortage has no consequences regarding efficiency. In the remaining, we successively consider: the case of a shortage of agents of type 1; the case of a global shortage of agents.

2.3 Competition between firms hiring offers

The main issue raised in the following is about the effects of a shortage of labor supply on the socio-demographic gap in average earnings. Does this shortage increase or reduce the gap between average earnings?

We distinguish between two kinds of situations. The first assumes an asymmetrical shortage: type 1 applicants are scarce but not those of type 0. Employers targeting type 1 agents in the absence of any shortage have to decide whether to maintain this option (which supposes to increase their wage offer) or to make do with type 0 employees. In that case, they must decide whether arousing an intrinsic motivation. We examine the impact of type 1 agents shortage on welfare. The second situation assumes a shortage of both types agents. In a case of equal shortage, how is the gap in average earnings between groups changed? The examination of this question is the occasion to consider the impact of labor return on the gap in average earnings of groups 0 and 1.

2.3.1 A shortage of agents of type 1

We continue the analysis in the going about a new step. The shortage does not concern type 0 agents, we thus assume $n_0 \geq 2$. We show below that the shortage of type 1 agents is favorable, in terms of average earning, to the group 0. The global well-being yet is affected. We consider two configurations: in the one, the renunciation to hire type 1 worker goes with a renouncement to stimulate an intrinsic motivation; in the other, firm under consideration makes the most of the capacity of type 0 agents to develop an intrinsic motivation.

Type 0 workers are confined to unfulfilling jobs. As in equilibrium 1', let us assume $I_B = \frac{3}{2} - \epsilon$, $\epsilon > 0$, $\epsilon \rightarrow 0$. Here, the shortage of type 1 agents involves $n_1 = 1$.

Equilibrium 2 For $I_B = \frac{3}{2} - \epsilon$, $\epsilon > 0$, $\epsilon \rightarrow 0$, $n_1 = 1$ and $n_0 \geq 2$, employers 2 and 3 compete to hire a type 1 agent. Employer 3 takes the advantage so that 1 and 2 both hire a type 0 applicant. Labor market equilibrium is thus given by

$$w = \left(1; 2; \frac{2}{3} + \frac{2}{3}\epsilon \right) \text{ where } \epsilon > 0, \epsilon \rightarrow 0$$

The hiring offer of firm 3 can only be chosen by an agent of type 1.

This equilibrium is characterized in the appendix.

Job 2 is fulfilling¹⁰ to an agent of type 1... not to an agent of type 0. The configuration (0, 0, 1) prevails with probability 1, so that $\hat{w}^0 = \frac{3}{2} > \hat{w}^1 = \frac{2}{3}(1 + \epsilon)$. In terms of earnings, the shortage of agents

¹⁰See equilibrium 1'.

of type 1 is then favorable to the group 0. The shortage of type 1 applicants, makes job 2 (the better paid) available to type 0 agents.

But still, the shortage of type 1 applicants is globally prejudicial: it leads to a loss in efficiency.¹¹ This situation is an echo of the discussion provided in Baguelin (2005a) on the gains in efficiency due to the workplace identity. Grafting the argument suggested in Baguelin (2005a) on this discussion leads us to the welfare consequences of a shortage of type 1 agents: a "free" labor (the work done by intrinsically motivated workers) is subtracted from the system.

Fulfilling jobs are shared. We have mentioned in Baguelin (2005a) the advantages of a policy focusing on the development of fulfilling jobs from the perspective of "dominated" groups working persons. The next situation illustrates these advantages in terms of efficiency.

Equilibrium 2' For $I_B = 1 - \epsilon$, $\epsilon > 0$, $\epsilon \rightarrow 0$, $n_1 = 1$ and $n_0 \geq 3$, employers 1, 2 and 3 compete to hire a type 1 worker. The latter is indifferent between firms 2 and 3 hiring offers but strictly prefer them to that of firm 1. Labor market equilibrium leads to

$$w = \left(1; 2 - \frac{2}{3}\epsilon; \frac{2}{3} - \frac{2}{3}\epsilon \right)$$

Hiring offers of these three firms could each be chosen by a type 0 agent.

This equilibrium is characterized in the appendix.

For¹² $n_0 = 3$ and $n_1 = 1$, expected average pays are given by¹³ $\hat{w}^0 = \frac{32}{27} - \frac{10}{27}\epsilon > \hat{w}^1 = \frac{8}{9} - \frac{4}{9}\epsilon$. When type 1 agents are scarce, fulfilling jobs (here jobs 2 or 3) become available to type 0 applicants. The capacity of type 0 agents to develop an intrinsic work motivation is used: this moderates the loss in efficiency highlighted in the previous case.

This case is characterized by a gap between the well-being of type 1 workers and type 0 workers. In jobs 2 and 3, worker 1 benefits from the pay which would have been offered to a 0 although these jobs are intrinsically fulfilling: $U_1 = \frac{3}{2} - \epsilon \gg U_0 (= 1 - \epsilon)$. The shortage of type 1 agents may look unfavorable in terms of pay, this latter situation highlights the benefits they draw from it in terms of well-being.

One should note that previous configuration did not mobilized S_j values beyond our initial hypothesis *i.e.* $S_j \leq w_j(I_B; 0)$ for all j . This is due to the fact that hiring a type 0 agent is an option always available to the firms. How does employers' exposure to an intensified competition affect the gap in average earnings between groups?

2.3.2 The case of a global shortage of the labor force.

We content ourselves with the case $n_0 = n_1 = 1$ and assume, as in equilibrium 1, $I_B = \frac{3}{2}$. The role of the gross surpluses S_j becomes critical. Indeed, one of the three firms is then excluded from labor market:

¹¹Our reference is the equilibrium 1'. Let us denote Ω_1 , the collective well-being - obtained by summing the utilities of employees and the surplus of firms - in equilibrium 1' and Ω_2 collective well-being in equilibrium 2. We have $\Omega_1^{n_1=2} > \Omega_2$ and $\Omega_1^{n_1 \geq 3} > \Omega_2$ for $\epsilon > \epsilon$.

¹²Here, the assumption that competition is centered on type 1 applicants requires $n_0 \geq 3$.

¹³See the derivation in appendix.

the less offering.

Numerical assumptions. The initial hypothesis involves: $S_1 \geq 1$, $S_2 \geq 2$ and $S_3 \geq 1$. The fact that job 3 offers large non-wage gratification opportunities compared to the two others endows employer 3 with special market power. In this section, we would like to explore the effect of a gradual readjustment of the balance regarding market power (in favor of employers 1 and 2) on the average earnings gap between groups. To do that, we maintain the gross surplus of job 3 constant while considering a parallel increase of the gross surplus of jobs 1 and 2. Our choice as regards numerical assumptions, thus, aims at: i) illustrating the evolution of the gap between earnings as firms 1 and 2 intensify the competitive pressure on firm 3; ii) illustrating the reversal of the earnings gap as a consequence of the crowding out of employer 3. Yet, in this choice, we have cared about maintaining a common scale for the surpluses of firms 2 and 3: the gross surplus of firm 2 *per unity of demands* gradually increases up to exceeding job 3 surplus. Moreover, most of the configurations likely to occur (see the appendix) are considered.

What is at stake here is simply having clearer ideas of the mechanisms likely to play in the case of a global shortage of labor. We consider the six following cases.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
S_1	$\frac{28}{20}$	$\frac{31}{20}$	$\frac{33}{20}$	$\frac{34}{20}$	$\frac{36}{20}$	$\frac{44}{20}$
$S_2 (= \frac{3}{2}S_1)$	$\frac{84}{40}$	$\frac{93}{40}$	$\frac{99}{40}$	$\frac{102}{40}$	$\frac{108}{40}$	$\frac{132}{40}$
S_3	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$

That is, in terms of gross surplus per unity of demands:

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
$\frac{S_1}{\psi_1} \simeq$	1,40	1,55	1,65	1,70	1,80	2,2
$\frac{S_2}{\psi_2} \simeq$	1,05	1,16	1,23	1,27	1,35	1,65
$\frac{S_3}{\psi_3} \simeq$	1,33	1,33	1,33	1,33	1,33	1,33

Results and discussion. The reference situation (Ref.) is the equilibrium 1 where employers do not compete. This is the relevant reference to the extent that gross surplus do not have a part beyond the initial hypothesis: whatever the case under consideration (1 to 6), the single difference to equilibrium 1 actually lies on applicants shortage. Results of the analysis are presented in the next table.

Impact of the competition between firms hiring offers ($I_B = \frac{3}{2}$)

	$n_0 + n_1 \geq 3$		$n_0 = n_1 = 1$				
	Ref.	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
w_1	1	$\frac{132}{120} + \varepsilon$	$\frac{159}{120} + \varepsilon$	$\frac{177}{120} + \varepsilon$	$\frac{186}{120} + \varepsilon$	$\frac{200}{120} + \varepsilon$	
w_2	2	n. f. ⁽¹⁾	n. f.	n. f.	n. f.	$\frac{320}{120} + \varepsilon$	
w_3	$\frac{2}{3} + \varepsilon$	$\frac{92}{120} + \varepsilon$	$\frac{119}{120} + \varepsilon$	$\frac{137}{120} + \varepsilon$	$\frac{146}{120} + \varepsilon$	n. f.	
$\frac{w_1 - w_1^{\text{Ref.}}}{w_1^{\text{Ref.}}} \simeq$	0	0, 10	0, 32	0, 47	0, 55	0, 66	
$\frac{w_2 - w_2^{\text{Ref.}}}{w_2^{\text{Ref.}}} \simeq$	0	n. d. ⁽²⁾	n. d.	n. d.	n. d.	0, 33	
$\frac{w_3 - w_3^{\text{Ref.}}}{w_3^{\text{Ref.}}} \simeq$	0	0, 15	0, 48	0, 71	0, 82	n. d.	
\hat{w}^0	[1; 2]	$\frac{132}{120} + \varepsilon$	$\frac{159}{120} + \varepsilon$	$\frac{177}{120} + \varepsilon$	$\frac{186}{120} + \varepsilon$	$\frac{200}{120} + \varepsilon$	
\hat{w}^1	$[\frac{80}{120} + \varepsilon; \frac{160}{120} + \frac{1}{2}\varepsilon]$	$\frac{92}{120} + \varepsilon$	$\frac{119}{120} + \varepsilon$	$\frac{137}{120} + \varepsilon$	$\frac{146}{120} + \varepsilon$	$\frac{320}{120} + \varepsilon$	
$\frac{\hat{w}^1 - \hat{w}^0}{\hat{w}^0} \simeq$	-	-0, 30	-0, 25	-0, 22	-0, 21	0, 60	

(1) non-filled job; (2) non-defined.

The job 2 is obviously excluded from the market in the cases 1 to 4: this is the more demanding job. Unless its gross surplus be particularly high, firms offering the most demanding jobs are the first to suffer as a result of a global shortage of applicants.

Let us start by discussing cases 1 to 4. The shortage obviously results in an increase in earnings. Previous results allow to observe that the pace of this increase differ depending on whether one considers the job 1 (filled with a type 0 agent) or 3 (filled with a type 1 agent). We have computed the relative gap to the reference situation for these jobs. It turns out that (see the appendix) competition is focused on the sole employer 2 exclusion which is, as we said earlier, particularly interested in hiring a type 1 agent. Employer 2, thus, exerts a stronger pressure on the employer 3 than on the employer 1 and hence the asymmetry as regards the pace of the increases in equilibrium wages 1 and 3. The evolution of the earning gap between the groups - favorable to type 0 workers (see equilibrium 1) - reflects previous observation: the increase of labor return in job 2 is relatively more favorable to agents of type 1 than to agent of type 0.

The reversal occurring between cases 4 and 5 underlines the observation we made by distinguishing equilibria 1 and 1'. The earnings gap favorable to the agents of type 1 is mostly due to their prevalence among the most demanding jobs. That is why the crowding out of the firm 3 by employer 2 involves an (average) earnings gap favorable to type 1 workers.

Figures 1 and 2 illustrate the main aspects of the analysis.

The important point of this analysis seems the following. For comparable gross surplus, competition

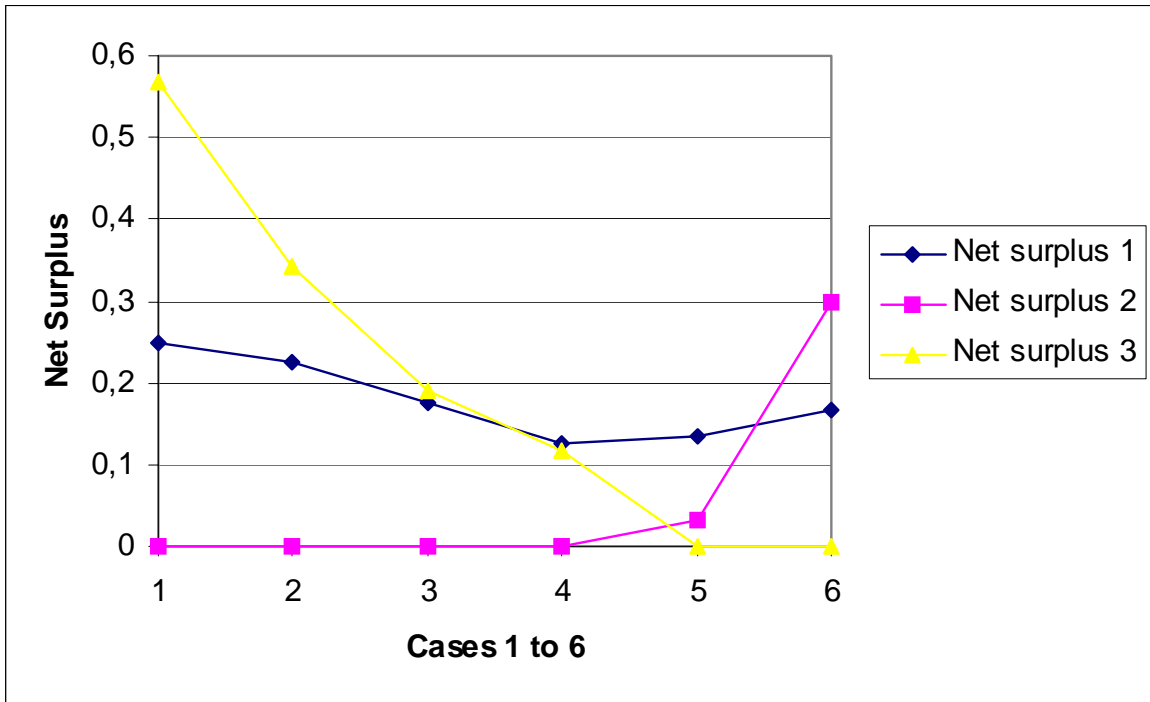


Figure 1: Net surplus for jobs 1, 2, and 3.

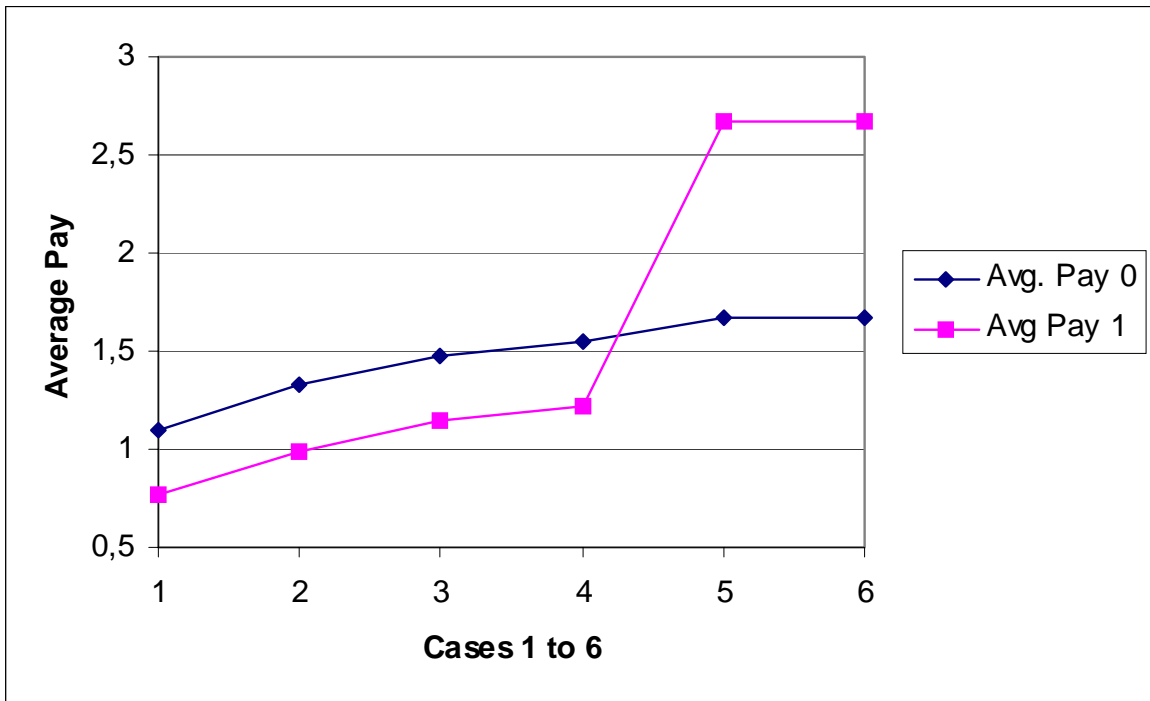


Figure 2: Average earnings for workers of type 0 and 1.

is unfavorable to the employers offering the most demanding jobs. Corresponding firms, hence, tend to be crowd out from the labor market. Yet, the pressure their existence exerts on operating firms is asymmetric. Indeed, firms offering the most demanding jobs are also the most prone to solicit type 1 agents. Maintaining outside the market is thus more costly to the firms employing type 1 workers than to others.

Summary and conclusion

In this paper, we have considered a series of configurations illustrating how our argument of self-esteem achievement through work interplays with labor market functioning.

The case of no significant competition between employers corresponded to the situation examined in Baguelin (2005a). We have established that, if the "dominant" socio-demographic *group* could benefit from a guaranteed access to employment, nothing in our micro model assumptions predetermine a gap in average earnings favorable to this group. The average earnings gap favorable to agents of type 1 does not trivially derives from their predisposition to the workplace identity. Indeed, this predisposition may lead them to accept "low" pays relatively to the degree of demands of the jobs under consideration, pay levels that a 0 agent would not have accepted. In the example considered above, when unfulfilling jobs were the majority, group 1 has an expected average pay lower than that of the group 0. This is only when fulfilling jobs are a majority that the earnings gap becomes favorable to the group 1. The example shows that, all other things equal, a job is all the more likely to require the workplace identity (and then to be selective) that it is more demanding and then *better paid*.

We have also explored the consequences of a shortage of type 1 agents. A first situation (where type 0 agents keep holding unfulfilling jobs) allows us to illustrate some consequences in terms of efficiency, of an under-utilization of the capacity of employees to develop an intrinsic work motivation. A second situation illustrated the policy prescription made in Baguelin (2005a) conclusion: designing jobs which are fulfilling from the point of view of agents belonging to the "dominated" group. We could have shown that this allows to moderate the loss in efficiency resulting from the shortage of type 1 agents.

The issue of a special "shortage" of type 0 agents has been briefly raised. In fact, we have highlighted that the asymmetric substitutability of applicants 0 and 1 (agent 1 is always substitutable to an agent 0 but the reverse is false) excluded that a "shortage" of type 0 workers be detrimental to any employer in the absence of a global shortage of applicants. The last step of our analysis precisely dealt with this case. This led us to see the relative benefits drawn by each group from its shortage. The increases in average pay obtained by type 1 workers are always higher than that of type 0 workers. In other words, a global shortage in labor supply seem to be more beneficial to the workers of the "dominant" group. This results from the fact that competition tends to push the most demanding jobs aside. The competitive pressure exerted by this outsider seems asymmetrical: since corresponding employers are mostly interested by agents of the "dominant" group, they impose their actual employers higher wage increases.

Previous observations remain temporary conclusions. Definitive results require the analytical char-

acterization of the labor market equilibrium in the general case, which still needs to be achieved. The general model provided in appendix allows to make oneself ideas as regards to the obstacle we meet.

A last remark deserves attention which deal with the structure of the market game introduced above. Our assumption as regards the sequence characterizing the match of labor supply and demand may appear as dissatisfying: it is at least clearly arbitrary. Yet, we believe the observation we derived from this assumption should not be strongly affected. In our point of view, this assumption has a functional rather than substantial scope: it is rather an algorithm allowing the characterization of an equilibrium matching. We could have used more sophisticated algorithms such as that of Gale and Shapley usually used in matching models. The important point is to obtain stable equilibria which we believe is the case here.

References

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Appendix

In current appendix, one will find the calculation of expected average earnings \hat{w}^0 and \hat{w}^1 in the different cases considered above, the characterization of equilibria 1 to 3.6 and an attempt of general model.

3 The calculation of expected average earnings of groups 0 and 1

This calculation comprises two steps. The first consists in determining the equilibrium earnings for each employment configuration; the second step is to calculate their respective probability. The procedure to calculate the probability of the different configurations occurrence derives from the conditions of the match of labor offer and demand. It exhibits little variation from one case to the other but deserves attention for qualitative results depending on this calculation. We present though the calculation in the main cases.

3.1 No significant competition between employers

We successively consider the cases in which unfulfilling jobs are the majority and the minority.

3.1.1 A majority of unfulfilling jobs.

Employment configurations depend on the number of agents for each type.

The case $n_1 \geq 3$. Average pays in each employment configurations are given by

	$\hat{w}^\theta(1, 1, 1)$	$\hat{w}^\theta(1, 0, 1)$	$\hat{w}^\theta(0, 1, 1)$	$\hat{w}^\theta(0, 0, 1)$
$\theta = 0$	0	2	1	$\frac{3}{2}$
$\theta = 1$	$\frac{11}{9}$	$\frac{5}{6}$	$\frac{4}{3}$	$\frac{2}{3}$

The point now is about establishing the probability of the various configurations when $n_1 \geq 3$. Under this condition, type 1 agents are indifferent between the three jobs, type 0 agents, by contrast, are only indifferent between jobs 1 and 2. The hiring offer of employer 3 violates their participation constraint.

- The set of draws compatible with $(\theta_1, \theta_2, \theta_3) = (1, 1, 1)$ comprises: $(1, 1, 1, \dots)$ and $(1, 1, 0, \dots)$. The first of these draws lead to $(1, 1, 1)$ with probability 1. The second only leads to it if neither the first drawn agent nor the second do choose the job 3. This occurs with probability $\frac{1}{3}$. As a consequence:

$$p(1, 1, 1) = \frac{n_1}{n_0 + n_1} \frac{n_1 - 1}{n_0 + n_1 - 1} \frac{n_1 - 2}{n_0 + n_1 - 2} + \frac{1}{3} \frac{n_1}{n_0 + n_1} \frac{n_1 - 1}{n_0 + n_1 - 1} \frac{n_0}{n_0 + n_1 - 2}$$

$$p(1, 1, 1) = \frac{n_1}{n_0 + n_1} \frac{n_1 - 1}{n_0 + n_1 - 1} \left(\frac{n_1 - 2 + \frac{1}{3}n_0}{n_0 + n_1 - 2} \right)$$

- The set of draws compatible with the configuration $(1, 0, 1)$ are $(1, 0, 0, \dots)$, $(0, 1, 0, \dots)$, $(0, 1, 1, \dots)$, $(1, 1, 0, \dots)$ and $(1, 0, 1, \dots)$. The first draw leads to configuration $(1, 0, 1)$ with probability $\frac{1}{3}$, the second with probability $\frac{1}{4}$, the third with probability $\frac{1}{2}$, the last with probability $\frac{1}{3}$. Hence

$$p(1, 0, 1) = \frac{1}{3} \frac{n_1}{n_0 + n_1} \frac{n_0}{n_0 + n_1 - 1} \frac{n_0 - 1}{n_0 + n_1 - 2} + \frac{1}{4} \frac{n_0}{n_0 + n_1} \frac{n_1}{n_0 + n_1 - 1} \frac{n_0 - 1}{n_0 + n_1 - 2}$$

$$+ \frac{1}{2} \frac{n_0}{n_0 + n_1} \frac{n_1}{n_0 + n_1 - 1} \frac{n_1 - 1}{n_0 + n_1 - 2} + \frac{1}{3} \frac{n_1}{n_0 + n_1} \frac{n_1 - 1}{n_0 + n_1 - 1} \frac{n_0}{n_0 + n_1 - 2}$$

$$+ \frac{1}{2} \frac{n_1}{n_0 + n_1} \frac{n_0}{n_0 + n_1 - 1} \frac{n_1 - 1}{n_0 + n_1 - 2}$$

$$p(1, 0, 1) = \frac{7}{12} \frac{n_1}{n_0 + n_1} \frac{n_0}{n_0 + n_1 - 1} \frac{n_0 - 1}{n_0 + n_1 - 2} + \frac{4}{3} \frac{n_1}{n_0 + n_1} \frac{n_1 - 1}{n_0 + n_1 - 1} \frac{n_0}{n_0 + n_1 - 2}$$

- The draws compatible with $(0, 1, 1)$ are $(1, 0, 0, \dots)$, $(0, 1, 0, \dots)$, $(1, 0, 1, \dots)$, $(0, 1, 1, \dots)$, $(1, 1, 0, \dots)$. The first leads to configuration $(0, 1, 1)$ with probability $\frac{1}{3}$, the second with probability $\frac{1}{4}$, the third with probability $\frac{1}{3} + \frac{1}{3} \frac{1}{2}$, the fourth with probability $\frac{1}{2}$, the last with probability $\frac{1}{3} \frac{1}{2} + \frac{1}{3} \frac{1}{2} = \frac{1}{3}$. Hence

$$p(0, 1, 1) = \frac{1}{3} \frac{n_1}{n_0 + n_1} \frac{n_0}{n_0 + n_1 - 1} \frac{n_0 - 1}{n_0 + n_1 - 2} + \frac{1}{4} \frac{n_0}{n_0 + n_1} \frac{n_1}{n_0 + n_1 - 1} \frac{n_0 - 1}{n_0 + n_1 - 2}$$

$$+ \frac{1}{2} \frac{n_1}{n_0 + n_1} \frac{n_0}{n_0 + n_1 - 1} \frac{n_1 - 1}{n_0 + n_1 - 2} + \frac{1}{2} \frac{n_0}{n_0 + n_1} \frac{n_1}{n_0 + n_1 - 1} \frac{n_1 - 1}{n_0 + n_1 - 2}$$

$$+ \frac{1}{3} \frac{n_1}{n_0 + n_1} \frac{n_1 - 1}{n_0 + n_1 - 1} \frac{n_0}{n_0 + n_1 - 2}$$

$$p(0, 1, 1) = \frac{7}{12} \frac{n_1}{n_0 + n_1} \frac{n_0}{n_0 + n_1 - 1} \frac{n_0 - 1}{n_0 + n_1 - 2} + \frac{4}{3} \frac{n_1}{n_0 + n_1} \frac{n_0}{n_0 + n_1 - 1} \frac{n_1 - 1}{n_0 + n_1 - 2}$$

• The draws compatible with configuration $(0, 0, 1)$ are $(1, 0, 0, \dots)$, $(0, 1, 0, \dots)$, $(0, 0, 1, \dots)$, $(0, 0, 0, \dots)$. The first leads to configuration $(0, 0, 1)$ with probability $\frac{1}{3}$, the second with probability $\frac{1}{2}$, both the third and fourth with probability 1. Hence

$$p(0, 0, 1) = \frac{1}{3} \frac{n_1}{n_0 + n_1} \frac{n_0}{n_0 + n_1 - 1} \frac{n_0 - 1}{n_0 + n_1 - 2} + \frac{1}{2} \frac{n_0}{n_0 + n_1} \frac{n_1}{n_0 + n_1 - 1} \frac{n_0 - 1}{n_0 + n_1 - 2} \\ + \frac{n_0}{n_0 + n_1} \frac{n_0 - 1}{n_0 + n_1 - 1} \frac{n_1}{n_0 + n_1 - 2} + \frac{n_0}{n_0 + n_1} \frac{n_0 - 1}{n_0 + n_1 - 1} \frac{n_0 - 2}{n_0 + n_1 - 2} \\ p(0, 0, 1) = \frac{11}{6} \frac{n_1}{n_0 + n_1} \frac{n_0}{n_0 + n_1 - 1} \frac{n_0 - 1}{n_0 + n_1 - 2} + \frac{n_0}{n_0 + n_1} \frac{n_0 - 1}{n_0 + n_1 - 1} \frac{n_0 - 2}{n_0 + n_1 - 2}$$

• One can easily check that

$$\sum_{(\theta_1, \theta_2, \theta_3) \in \{0, 1\}^3} p(\theta_1, \theta_2, \theta_3) = 1$$

The probability of occurrence of the various configurations are given by $p(1, 1, 1) = \frac{n_1(n_1-1)}{D} (\frac{1}{3}n_0 + n_1 - 2)$, $p(1, 0, 1) = p(0, 1, 1) = \frac{n_1 n_0}{D} (\frac{7}{12}n_0 + \frac{4}{3}n_1 - \frac{23}{12})$ and $p(0, 0, 1) = \frac{n_0(n_0-1)}{D} (n_0 + \frac{11}{6}n_1 - 2)$ where $D = (n_0 + n_1)(n_0 + n_1 - 1)(n_0 + n_1 - 2)$. For $n_0 = n_1 = 3$: $p(1, 1, 1) = \frac{40}{400}$, $p(1, 0, 1) = p(0, 1, 1) = \frac{115}{400}$, $p(0, 0, 1) = \frac{130}{400}$ and hence $\hat{w}^0 = 1, 135 > \hat{w}^1 \simeq 0, 961$. Expected average earnings are calculated for some values of n_0 and n_1 in the next table.

$n_0 \backslash n_1$	3	4	5
0	(0; 1, 222)	(0; 1, 222)	(0; 1, 222)
1	(1; 1, 129)	(0, 983; 1, 229)	(0, 666; 1, 160)
2	(1, 25; 1, 030)	(1, 1; 1, 069)	(0, 976; 1, 347)
3	(1, 35; 0, 961)	(1, 242; 1, 007)	(1, 142; 1, 040)
4	(1, 4; 0, 914)	(1, 321; 0, 960)	(1, 269; 1, 097)
5	(1, 428; 0, 879)	(1, 369; 0, 924)	(1, 305; 1, 093)

The case $1 \leq n_1 < 3$. Average pay in each configuration are given by

$$\begin{array}{cccc} \hat{w}^\theta(1, 0, 1) & \hat{w}^\theta(0, 1, 1) & \hat{w}^\theta(0, 0, 1) & \\ \theta = 0 & 2 & 1 & \frac{3}{2} \\ \theta = 1 & \frac{5}{6} + \frac{1}{2}\varepsilon & \frac{4}{3} + \frac{1}{2}\varepsilon & \frac{2}{3} + \varepsilon \end{array}$$

The probabilities of various configurations when $n_1 < 3$. Configuration $(1, 1, 1)$ cannot occurs since, at most, two type 1 applicants are available: its probability is zero.

• The draws compatible with configuration $(1, 0, 1)$ are $(1, 1, 0, \dots)$, $(1, 0, 1, \dots)$, $(0, 1, 1, \dots)$. the first leads to configuration $(1, 0, 1)$ with probability $\frac{1}{2}$, the second with probability $\frac{1}{2}$, the third with the probability $\frac{1}{2}$. Hence

$$p(1, 0, 1) = \frac{3}{2} \frac{n_0}{n_0 + n_1} \frac{n_1}{n_0 + n_1 - 1} \frac{n_1 - 1}{n_0 + n_1 - 2}$$

the reasoning is the same as regards configuration $(0, 1, 1)$ and one obtains $p(0, 1, 1) = p(1, 0, 1)$.

• The draws compatible with $(0, 0, 1)$ are $(0, 0, 0, \dots)$, $(1, 0, 0, \dots)$, $(0, 1, 0, \dots)$, $(0, 0, 1, \dots)$. The first leads to $(0, 0, 1)$ with probability 1, as well as all other draws. Hence

$$p(0, 0, 1) = \frac{n_0}{n_0 + n_1} \frac{n_0 - 1}{n_0 + n_1 - 1} \frac{n_0 - 2}{n_0 + n_1 - 2} + 3 \frac{n_1}{n_0 + n_1} \frac{n_0}{n_0 + n_1 - 1} \frac{n_0 - 1}{n_0 + n_1 - 2}$$

• The probabilities of these configurations to occurs are given by $p(1, 0, 1) = \frac{3}{2} \frac{n_0 n_1 (n_1 - 1)}{D}$, $p(0, 1, 1) = \frac{3}{2} \frac{n_0 n_1 (n_1 - 1)}{D}$ and $p(0, 0, 1) = (n_0 + 3n_1 - 2) \frac{n_0 (n_0 - 1)}{D}$.

For $n_0 = n_1 = 2$: $p(0, 0, 1) = \frac{1}{2}$, $p(1, 0, 1) = p(0, 1, 1) = \frac{1}{4}$.

The reasoning remains the same for the other cases.

3.1.2 A majority of fulfilling jobs.

Two cases must be distinguished.

For $n_1 \geq 3$. Under this assumption $\varepsilon = 0$ so that average pays take the following values

$$\begin{array}{ccc} \hat{w}^\theta(1, 1, 1) & \hat{w}^\theta(0, 1, 1) & \\ \theta = 0 & 0 & 1 \\ \theta = 1 & \frac{11}{9} - \frac{4}{9}\varepsilon & \frac{4}{3} - \frac{2}{3}\varepsilon \end{array}$$

For all $\theta_1, \theta_2, \theta_3 \in \{0, 1\}$, $p(\theta_1, \theta_2, 0) = p(\theta_1, 0, \theta_3) = p(\theta_1, 0, 0) = 0$. The probabilities of remaining configurations are

$$\begin{aligned} p(1, 1, 1) &= \frac{n_1 (n_1 - 1) (n_1 - 2)}{D} + \frac{2}{3} \frac{n_0 n_1 (n_1 - 1)}{D} + \frac{1}{3} \frac{n_0 n_1}{(n_0 + n_1) (n_0 + n_1 - 1)} \\ p(0, 1, 1) &= \frac{n_0}{n_0 + n_1} + \frac{2}{3} \frac{n_0 n_1}{(n_0 + n_1) (n_0 + n_1 - 1)} + \frac{1}{3} \frac{n_0 n_1 (n_1 - 1)}{D} \end{aligned}$$

For $n_0 = n_1 = 3$: $p(1, 1, 1) = \frac{1}{4}$, $p(0, 1, 1) = \frac{3}{4}$.

For $n_1 = 2$. Under this assumption $\varepsilon > 0$ and $p(0, 1, 1) = 1$. The average pays for each group are

$$\begin{array}{ccc} \hat{w}^\theta(0, 1, 1) & & \\ \theta = 0 & 1 & \\ \theta = 1 & \frac{4}{3} + \frac{4}{3}\varepsilon - \frac{2}{3}\varepsilon & \end{array}$$

The fact that there is only two applicants of type 1 guarantees that an agent of type 0 be employed and hence a reduced earning gap.

3.2 A shortage of agents of type 1

Where fulfilling jobs are shared.

Possible configurations are $(0, 1, 0)$, $(0, 0, 1)$ and, if $n_0 \geq 3$, $(0, 0, 0)$.

• The draws compatible with the configuration $(0, 0, 0)$ - if this configuration is possible - are $(0, 0, 1, \dots)$ and $(0, 0, 0, \dots)$. The first draw leads to $(0, 0, 0)$ with probability $\frac{1}{3}$, the second with probability 1. One obtains

$$p(0, 0, 0) = \begin{cases} \frac{1}{3} \frac{1}{n_0+1} + \frac{n_0-2}{n_0+1} & \text{if } n_0 \geq 3 \\ 0 & \text{otherwise} \end{cases}$$

• The draws compatible with the configuration $(0, 1, 0)$ are $(1, 0, 0, \dots)$, $(0, 1, 0, \dots)$ and $(0, 0, 1, \dots)$. The first draw leads to configuration $(0, 1, 0)$ with probability $\frac{1}{2}$, the second with probability $\frac{1}{2}$, the third with probability $\frac{1}{3}$. Hence

$$p(0, 1, 0) = \frac{4}{3} \frac{1}{n_0+1}$$

The reasoning is similar as regards configuration $(0, 0, 1)$ and one obtains $p(0, 0, 1) = p(0, 1, 0)$.

4 Characterization of equilibria in the presence of competitive pressures

We consider successively the case of a shortage of type 1 agents and that of a global shortage of applicants.

4.1 Shortage of agents of type 1

Workers of type 0 are led to substitute to type 1 lacking applicants. This can be accompanied or not by the renouncement from the employers to stimulate an intrinsic motivation.

4.1.1 The case $I_B = \frac{3}{2} - \epsilon$, $\epsilon > 0$, $\epsilon \rightarrow 0$.

In this case, the shortage of type 1 agents involves $n_1 = 1$. Previous step indicate that firms 2 and 3 will compete, that is the firms which would have favored type 1 agents if their reservation utility had been I_B . If she had to renounce hiring a type 1 agent, each employer $j \in \{2, 3\}$ would fill her job with a type 0 agent. This type being abundant, j could limit her wage offer to the level binding a 0 participation constraint. Minimal required pay would then be

$$\begin{aligned} w_2(I_B; 0) &= \min \left\{ \frac{2}{3}(I_B + 1) + \frac{2}{3}; 2 \right\} = 2 \left(> w_2(I_B; 1) = 2 - \frac{2}{3}\epsilon \right) \\ w_3(I_B; 0) &= \min \left\{ \frac{2}{3}(I_B - 1) + \frac{2}{3}; 1 \right\} = 1 - \frac{2}{3}\epsilon \left(> w_3(I_B; 1) = \frac{2}{3} - \frac{2}{3}\epsilon \right) \end{aligned}$$

Hence, each firm j will raise the bidding so long $w_j \leq w_j(I_B; 0)$. Let \bar{U}_{1j} denote the utility experienced by an agent of type 1 by the highest pay offer employer j might make. Previous discussion allows to state that \bar{U}_{1j} is defined by $w_j(\bar{U}_{1j}; 1) = w_j(I_B; 0)$. The fact that it is beneficial to firm j to target the type 1 involves the arousing of the workplace identity. As a consequence, $w_2(\bar{U}_{12}; 1) = \frac{2}{3}(\bar{U}_{12} + 1) + \frac{1}{3}$ and $w_3(\bar{U}_{13}; 1) = \frac{2}{3}(\bar{U}_{13} - 1) + \frac{1}{3}$ and hence¹⁴ $\bar{U}_{12} = \frac{3}{2} < \bar{U}_{13} = 2 - \epsilon$. Employer 3 is assured to prevail. She contents herself with guaranteeing to the agent of type 1 a utility level $U_1 = \bar{U}_{12} + \epsilon$, $\epsilon > 0$, $\epsilon \rightarrow 0$,

¹⁴One can check that for $\bar{U}_{12} = \frac{3}{2}$ and $I_B = \frac{3}{2} - \epsilon$, it is true that $\frac{2}{3}(\bar{U}_{12} + 1) + \frac{1}{3} < 2 + \bar{U}_{12} - I_B$.

that is $U_1 = \frac{3}{2} + \varepsilon$. The pay offer made by employer 3 is then $w_3 (U_1; 1) = \frac{2}{3} (\frac{1}{2} + \varepsilon) + \frac{1}{3} = \frac{2}{3} + \frac{2}{3}\varepsilon$. The fall back position of firm 2 is to hire a type 0 agent for a wage $w_2 (I_B; 0) = 2$. Workplace identity is not aroused: job 2 is then unfulfilling!

4.1.2 The case $I_B = 1 - \varepsilon$, $\varepsilon > 0$, $\varepsilon \rightarrow 0$.

If type 1 applicants were abundant, the three employers would favor them over type 0 agents. With this new value of I_B , one obtains following values

	$w_j (I_B; 0)$	$w_j (I_B; 1)$	$\bar{U}_{1j} = U_j (w_j (I_B; 0); 1)$
Emploi 1	1	$1 - \frac{2}{3}\varepsilon$	1
Emploi 2	$2 - \frac{2}{3}\varepsilon$	$\frac{5}{3} - \frac{2}{3}\varepsilon$	$\frac{3}{2} - \varepsilon$
Emploi 3	$\frac{2}{3} - \frac{2}{3}\varepsilon$	$\frac{1}{3} - \frac{2}{3}\varepsilon$	$\frac{3}{2} - \varepsilon$

When participation constraint is binding: job 1 is unfulfilling to agents of type 0 but is now fulfilling to agents of type 1; job 2 is fulfilling whatever the type of its holder, as well as job 3. The examination of the utilities brought to an agent of type 1 by the best offer of each firm shows: firstly, that firm 1 is pushed aside (for agents 1 hiring); secondly, that firms 2 and 3 do not succeed in differentiating themselves. Their competition to attract agent of type 1 leads them to make wage offers to this type of applicant such that they are just indifferent between hiring type 0 or type 1 workers.

For $n_0 = 3$ and $n_1 = 1$, $p(0, 1, 0) = p(0, 0, 1) = p(0, 0, 0) = \frac{1}{3}$. Average pays in each configuration are given by

	$\hat{w}^\theta (0, 1, 0)$	$\hat{w}^\theta (0, 0, 1)$	$\hat{w}^\theta (0, 0, 0)$
$\theta = 0$	$\frac{5}{6} - \frac{1}{3}\varepsilon$	$\frac{3}{2} - \frac{1}{3}\varepsilon$	$\frac{11}{9} - \frac{4}{9}\varepsilon$
$\theta = 1$	$2 - \frac{2}{3}\varepsilon$	$\frac{2}{3} - \frac{2}{3}\varepsilon$	0

and hence the expected average earnings we obtain in the bodytext.

4.2 A global shortage of applicants ($I_B = \frac{3}{2}$)

Competition can be polarized or not. The best wage offer each firm can make to a type $\theta \in \{0, 1\}$ agent is $w_j = S_j$. Corresponding utility levels are

$$\begin{aligned}
 U_1 (S_1; \theta) &= \max \left\{ \frac{3}{2}S_1 + \frac{1}{2}\theta - 1; S_1 + \frac{1}{2} \right\} \\
 U_2 (S_2; \theta) &= \max \left\{ \frac{3}{2}S_2 + \frac{1}{2}\theta - 2; S_2 - \frac{1}{2} \right\} \\
 U_3 (S_3; \theta) &= \max \left\{ \frac{3}{2}S_3 + \frac{1}{2}\theta; S_3 + \frac{1}{2} \right\}
 \end{aligned}$$

Let U_θ denote the utility obtained by the type θ agent in labor market equilibrium.

$$\begin{aligned} w_1(U_\theta; \theta) &= \min \left\{ \frac{2}{3}U_\theta + \frac{1}{3}(2 - \theta); U_\theta - \frac{1}{2} \right\} \\ w_2(U_\theta; \theta) &= \min \left\{ \frac{2}{3}(U_\theta + 1) + \frac{1}{3}(2 - \theta); U_\theta + \frac{1}{2} \right\} \\ w_3(U_\theta; \theta) &= \min \left\{ \frac{2}{3}(U_\theta - 1) + \frac{1}{3}(2 - \theta); U_\theta - \frac{1}{2} \right\} \end{aligned}$$

Polarized competition: cases 1, 2, 3 and 4. Following cases correspond to a balance of powers such that competition is polarized on the exclusion of a single firm from the labor market.

Case 1 implies $U_3(S_3; \theta) > U_1(S_1; \theta) > U_2(S_2; \theta)$ whatever $\theta \in \{0, 1\}$. Job 2 does not exhibit a large enough gross surplus to allow to the corresponding firm to make appealing hiring offers. Yet, this firm exerts a pressure on the two other firms as a potential entrant: its presence raise the utility reservation of each type of agent. For $U_{(0)} = U_2(S_2; 0) (= \frac{8}{5})$ and $U_{(1)} = U_2(S_2; 1) (= \frac{33}{20})$, $0 \succ_1 1$, since $w_1(U_{(0)}; 0) = \frac{11}{10} < \frac{23}{20} = w_1(U_{(1)}; 1)$, and $0 \prec_3 1$ since $w_3(U_{(0)}; 0) = \frac{16}{15} > \frac{23}{30} = w_3(U_{(1)}; 1)$. Employers 1 and 3 do not directly compete with each other - their plans are mutually compatible. In labor market equilibrium, types 0 and 1 utilities are hence $U_0 = U_{(0)} + \varepsilon = \frac{8}{5} + \varepsilon$ and $U_1 = U_{(1)} + \varepsilon = \frac{33}{20} + \varepsilon$.

Equilibrium 3.1 For $I_B = \frac{3}{2}$, $n_0 = n_1 = 1$ employers 1, 2 and 3 compete. Assuming $(S_1, S_2, S_3) = (\frac{28}{20}, \frac{84}{40}, \frac{4}{3})$ employer 2 is pushed outside the market. Employer 1 favors the type 0 agent, employer 3, the agent of type 1. Labor market equilibrium is the characterized by

$$w = (w_1, w_3) = \left(\frac{11}{10} + \varepsilon; \frac{23}{30} + \varepsilon \right) \text{ where } \varepsilon > 0, \varepsilon \rightarrow 0$$

As in equilibrium 1, jobs 1 and 3 are respectively filled by agents of type 0 and 1. Let us compare the impact of competition on the compensation of each job - and then of each agent type. The pay in job 1 is increased by almost 10%, that of job 3 almost 15%.

The case 2 involves $U_3(S_3; 1) > U_1(S_1; 1) > U_2(S_2; 1)$ but $U_1(S_1; 0) > U_3(S_3; 0) > U_2(S_2; 0)$. for $U_{(0)} = U_2(S_2; 0) (= \frac{73}{40})$ and $U_{(1)} = U_2(S_2; 1) (= \frac{159}{80})$ then $0 \succ_1 1$, as a matter of facts, $w_1(U_{(0)}; 0) = \frac{53}{40} < \frac{119}{80} = w_1(U_{(1)}; 1)$, and $0 \prec_3 1$ since $w_3(U_{(0)}; 0) = \frac{73}{60} > \frac{119}{120} = w_3(U_{(1)}; 1)$. Employers 1 and 3 do not compete with each others: for all θ , $U_\theta = U_{(\theta)} + \varepsilon$.

Equilibrium 3.2 For $I_B = \frac{3}{2}$, $n_0 = n_1 = 1$ employers 1, 2 and 3 compete. Assuming $(S_1, S_2, S_3) = (\frac{31}{20}, \frac{93}{40}, \frac{4}{3})$ employer 2 is pushed outside the market. Employer 1 favors the agent of type 0, employer 3, the agent of type 1. Labor market is then characterized by

$$w = (w_1, w_3) = \left(\frac{53}{40} + \varepsilon; \frac{119}{120} + \varepsilon \right) \text{ where } \varepsilon > 0, \varepsilon \rightarrow 0$$

previous observations as regards the relative impact of competition on each job compensation is confirmed here.

The case 3 involves $U_3(S_3; 1) > U_2(S_2; 1) > U_1(S_1; 1)$ but $U_1(S_1; 0) > U_3(S_3; 0) > U_2(S_2; 0)$. Let us recall that the first concern of an employer is to make a positive profit. Employer 1 can at least impose herself in the competition for type 0 employees, employer 3 in the competition for the agent of type 1. Employer 2, once again, will be pushed outside the market. For $U_{(0)} = U_2(S_2; 0) (= \frac{79}{40})$ and $U_{(1)} = U_2(S_2; 1) (= \frac{177}{80})$, $0 \succ_1 1$, as a matter of facts, $w_1(U_{(0)}; 0) = \frac{59}{40} < \frac{137}{80} = w_1(U_{(1)}; 1)$, and $0 \prec_3 1$ since $w_3(U_{(0)}; 0) = \frac{79}{60} > \frac{137}{120} = w_3(U_{(1)}; 1)$. Employers 1 and 3 do not directly compete: for all θ , $U_\theta = U_{(\theta)} + \varepsilon$.

Equilibrium 3.3 For $I_B = \frac{3}{2}$, $n_0 = n_1 = 1$ employers 1, 2 and 3 compete. Assuming $(S_1, S_2, S_3) = (\frac{33}{20}, \frac{99}{40}, \frac{4}{3})$ employer 2 is pushed outside the market. Employer 1 favors the type 0 agent, employer 3, the type 1 applicant. Labor market is then characterized by

$$w = (w_1, w_3) = \left(\frac{59}{40} + \varepsilon; \frac{137}{120} + \varepsilon \right) \text{ where } \varepsilon > 0, \varepsilon \rightarrow 0$$

Previous observations can be renewed here.

The case 4 involves $U_3(S_3; 1) > U_2(S_2; 1) > U_1(S_1; 1)$ but $U_1(S_1; 0) > U_2(S_2; 0) > U_3(S_3; 0)$. Firm 2 remains outside the labor market. Its potential entry imposes firms 1 and 3 reservation utilities $U_2(S_2; 0) = \frac{41}{20}$ and $U_2(S_2; 1) = \frac{93}{40}$. Though, we have $w_1(U_2(S_2; 0); 0) = \frac{31}{20} < w_1(U_2(S_2; 1); 1) = \frac{73}{40}$ i.e. firm 1 prefers agents of type 0; and $w_3(U_2(S_2; 0); 0) = \frac{41}{30} > w_3(U_2(S_2; 1); 1) = \frac{73}{60}$ i.e. firm 3 prefers agents of type 1. Firms 1 and 3 do not directly compete with each other.

Equilibrium 3.4 For $I_B = \frac{3}{2}$, $n_0 = n_1 = 1$ employers 1, 2 and 3 compete. Assuming $(S_1, S_2, S_3) = (\frac{34}{20}, \frac{102}{40}, \frac{4}{3})$ employer 2 is pushed aside the market. Employer 1 favors type 0 agent, employer 3, applicant of type 1. Labor market is characterized by

$$w = (w_1, w_3) = \left(\frac{31}{20} + \varepsilon; \frac{73}{60} + \varepsilon \right) \text{ where } \varepsilon > 0, \varepsilon \rightarrow 0$$

All the cases above represent a polarized competition: the point is just about neutralizing employer 2 but firms 1 and 3 do not exert any pressure toward each others. Cases 5 and 6 involve multipolar competition.

Multipolar competition: cases 5 and 6. The case 5 involves $U_2(S_2; 1) > U_3(S_3; 1) > U_1(S_1; 1)$ but $U_1(S_1; 0) > U_2(S_2; 0) > U_3(S_3; 0)$. This is then employer 3 who is pushed aside from labor market. For $U_{(0)} = U_3(S_3; 0) = 2$ and $U_{(1)} = U_3(S_3; 1) = \frac{5}{2}$, $0 \succ_1 1$ - since $w_1(U_{(0)}; 0) = \frac{3}{2} < 2 = w_1(U_{(1)}; 1)$, but also $0 \succ_2 1$ - since $w_2(U_{(0)}; 0) = \frac{5}{2} < \frac{8}{3} = w_2(U_{(1)}; 1)$. Employers 1 and 2 compete directly to obtain the agent of type 0. Employer 2 is prepared to offer up to $\frac{8}{3}$ to obtain this agent which corresponds to utility $U_2(\frac{8}{3}; 0) = \frac{13}{6}$, while the maximal wage offer of employer 1 is 2 which corresponds to utility $\frac{5}{2} > \frac{13}{6}$. Employer 1 prevails. It is enough to offer wage $w_1 = \min \{ \frac{2}{3}(U_2(\frac{8}{3}; 0) + \varepsilon) + \frac{1}{3}(2 - 0), U_2(\frac{8}{3}; 0) + \varepsilon - \frac{1}{2} \} =$

$\frac{5}{3} + \varepsilon$. Employer 2's fallback position is to hire a type 1 agent to which she offers $w_2 = \frac{8}{3} + \varepsilon$. Hence, $U_1 = U_{(1)} + \varepsilon$ but $U_0 = U_2(w_2(U_{(1)}; 1); 0) + \varepsilon > U_{(0)} + \varepsilon$.

The case 6 involves $U_2(S_2; 1) > U_1(S_1; 1) > U_3(S_3; 1)$ but $U_2(S_2; 0) > U_1(S_1; 0) > U_3(S_3; 0)$. This change does not anything¹⁵ to the case 5.

Equilibria 3.5 and 3.6 For $I_B = \frac{3}{2}$, $n_0 = n_1 = 1$ employers 1, 2 and 3 compete. Assuming $(S_1, S_2, S_3) \in \{(\frac{36}{20}, \frac{108}{40}, \frac{4}{3}), (\frac{44}{20}, \frac{132}{40}, \frac{4}{3})\}$ employer 3 pushed aside from the labor market. Employer 1 favors agents of type 0, employer 2, the agent of type 1. Labor market is characterized by

$$w = (w_1, w_2) = \left(\frac{5}{3} + \varepsilon; \frac{8}{3} + \varepsilon \right) \text{ where } \varepsilon > 0, \varepsilon \rightarrow 0$$

5 An attempt for a general formulation of labor market equilibrium

In this last section, we propose a definition of the labor market equilibrium and show its existence. Such equilibrium must specify for each job: 1) whether it is filled or not, 2) if it is, the type of the agent who hold it, 3) the level of its compensation.

5.1 The building of a best response function

Let us assume four jobs are available: (S_j, ϕ_j, ψ_j) , $j \in \{1, 2, 3, 4\}$. Three agents search for a job: one with trait $\theta = 0$, and two with trait $\theta = 1$. Let us build the reaction function of firm j . Let $j, j', j'', j^{(3)} \in \{1, 2, 3, 4\}$ be such that $j \neq j' \neq j'' \neq j^{(3)}$ while $U_{j'}(w_{j'}; 1) \geq U_{j''}(w_{j''}; 1) \geq U_{j^{(3)}}(w_{j^{(3)}}; 1)$.¹⁶ This latter

¹⁵Firm 3 is again excluded from the labor market and hence $U_3(S_3; 0) = 2$ and $U_3(S_3; 1) = \frac{5}{2}$. We get $w_1(U_3(S_3; 0); 0) = \frac{3}{2} < w_1(U_3(S_3; 1); 1) = 2$ and $w_2(U_3(S_3; 0); 0) = \frac{5}{2} < w_2(U_3(S_3; 1); 1) = \frac{8}{3}$ i.e. firms 1 and 2 compete to hire an agent of type 0. Maximal pay offers of firms 1 and 2 to an agent of type 0 are respectively: 2 and $\frac{8}{3}$. These offers provide to agent 0 utilities $U_1(2; 0) = \frac{5}{2}$ and $U_2(\frac{8}{3}; 0) = \frac{13}{6} < \frac{5}{2}$. Then, it is the firm 1 who prevails, limiting her offer to $w_1(U_2(\frac{8}{3}; 0) + \varepsilon; 0)$.

¹⁶Nota:

$$U_{j'}(w_{j'}; 1) \geq U_{j''}(w_{j''}; 1) ; U_j(w_{j'}; 0) \geq U_{j''}(w_{j''}; 0)$$

Take for instance the case $b'' > b'$, $b'' > a''_1 = a''_0 + \gamma_h$ and $a'_1 = a'_0 + \gamma_h > b' > a'_0$. then $\max\{a'_1, b'\} \geq \max\{a''_1, b''\}$ but $\max\{a''_0, b''\} \geq \max\{a'_0, b'\}$.

condition is about the indexes j' to $j^{(3)}$: they adjust so that it keeps holding.¹⁷

$$r_j(w_{-j}) = \begin{cases} \min \{w_j(I_B; 0), w_j(I_B; 1)\} = w_j(I_B; 1) & \text{if } \leq S_j \text{ and } \forall k \in \{j', j'', j^{(3)}\}, \\ & w_k(I_B; 1) > w_k. \\ \min \{w_j(I_B; 0), w_j(I_B; 1)\} = w_j(I_B; 1) & \text{if } \leq S_j, \\ & w_{j'}(I_B; 0) > w_{j'} \geq w_{j'}(I_B; 1) \\ & \forall k \in \{j'', j^{(3)}\}, w_k(I_B; 1) > w_k. \\ \min \left\{ \begin{array}{c} w_j(I_B; 0) \\ w_j(U_{j''}(w_{j''}; 1) + \varepsilon; 1) \end{array} \right\} & \text{if } \leq S_j, \forall k \in \{j', j''\}, \\ & w_k(I_B; 0) > w_k \geq w_k(I_B; 1) \\ & \text{but } w_{j^{(3)}}(I_B; 1) > w_{j^{(3)}}. \\ \min \left\{ \begin{array}{c} w_j(I_B; 0) \\ w_j(U_{j''}(w_{j''}; 1) + \varepsilon; 1) \end{array} \right\} & \text{if } \leq S_j \text{ and } \forall k \in \{j', j'', j^{(3)}\}, \\ & w_k(I_B; 0) > w_k \geq w_k(I_B; 1). \\ \min \left\{ \begin{array}{c} w_j \left(\max_{k \in \{j', j'', j^{(3)}\}} U_k(w_k; 0) + \varepsilon; 0 \right) \\ w_j(U_{j''}(w_{j''}; 1) + \varepsilon; 1) \end{array} \right\} & \text{if } \leq S_j, \forall k \in \{j', j'', j^{(3)}\}, \\ & w_k \geq w_k(I_B; 1), \text{ but} \\ & w_k(I_B; 0) \geq w_k. \\ S_j & \text{otherwise} \end{cases}$$

where $\varepsilon > 0, \varepsilon \rightarrow 0$. Put in a more compact writing

$$r_j(w_{-j}) = \min \left\{ \begin{array}{c} S_j \\ \max \left\{ w_j(I_B; 0), w_j \left(\max_{k \in \{j', j'', j^{(3)}\}} U_k(w_k; 0) + \varepsilon; 0 \right) \right\} \\ \max \{w_j(I_B; 1), w_j(U_{j''}(w_{j''}; 1) + \varepsilon; 1)\} \end{array} \right\}$$

Let the equilibrium of the labor market $w = (w_1, w_2, w_3, w_4)$ be such that 1 hires the agent with trait 0, 2 and 3 agents with trait 1 while job 4 remains unfilled. This equilibrium is such that

$$\begin{aligned} w_1 &= \max \left\{ \begin{array}{c} w_1(I_B; 0) \\ w_1(\max_{k \in \{2, 3, 4\}} U_k(w_k; 0) + \varepsilon; 0) \end{array} \right\} \leq \max \left\{ \begin{array}{c} w_1(I_B; 1) \\ w_1(\max_{k \in \{2, 3, 4\}} U_k(w_k; 1) + \varepsilon; 1) \end{array} \right\} \\ w_2 &= \max \left\{ \begin{array}{c} w_2(I_B; 1) \\ w_2(\max_{k \in \{1, 4\}} U_k(w_k; 1) + \varepsilon; 1) \end{array} \right\} \leq \max \left\{ \begin{array}{c} w_2(I_B; 0) \\ w_2(\max_{k \in \{1, 4\}} U_k(w_k; 0) + \varepsilon; 0) \end{array} \right\} \\ w_3 &= \max \left\{ \begin{array}{c} w_3(I_B; 1) \\ w_3(\max_{k \in \{1, 4\}} U_k(w_k; 1) + \varepsilon; 1) \end{array} \right\} \leq \max \left\{ \begin{array}{c} w_3(I_B; 0) \\ w_3(\max_{k \in \{1, 4\}} U_k(w_k; 0) + \varepsilon; 0) \end{array} \right\} \\ w_4 &= S_4 \leq \max \left\{ \begin{array}{c} w_4(I_B; 0) \\ w_4(\max_{k \in \{1, 2, 3\}} U_k(w_k; 0) + \varepsilon; 0) \end{array} \right\}, S_4 \leq \max \left\{ \begin{array}{c} w_4(I_B; 1) \\ w_4(\max_{k \in \{1, 2, 3\}} U_k(w_k; 1) + \varepsilon; 1) \end{array} \right\} \end{aligned}$$

where for all $k \in \{1, 2, 3\}, w_k \leq S_k$. Note that $U_2(w_2; 1) = U_3(w_3; 1)$.

¹⁷Note that, for any k , the condition $w_k(I_B; 0) > w_k \geq w_k(I_B; 1)$ is equivalent to $U_k(w_k; 1) \geq I_B > U_k(w_k; 0)$ so that clauses expressed below are all consistent with $U_{j'}(w_{j'}; 1) \geq U_{j''}(w_{j''}; 1) \geq U_{j^{(3)}}(w_{j^{(3)}}; 1)$.

5.2 Labor market equilibrium

For any vector $w_{-j} = (w_k)_{k \in \mathcal{J} - \{j\}}$, for all $\theta \in \{0, 1\}$, let $k_\theta(j; w_{-j}) \in \mathcal{J} - \{j\}$ index any firm such that

$$\begin{aligned} \# \{k \in \mathcal{J} - \{j\} \mid U_k(w_k; \theta) > U_{k_\theta(j; w_{-j})}(w_{k_\theta(j; w_{-j})}; \theta)\} &< n_\theta \\ \# \{k \in \mathcal{J} - \{j\} \mid U_k(w_k; \theta) \geq U_{k_\theta(j; w_{-j})}(w_{k_\theta(j; w_{-j})}; \theta)\} &\geq n_\theta \end{aligned}$$

Whatever w_{-j} and $\theta \in \{0, 1\}$, firms belonging to $\mathcal{J} - \{j\}$ can be ranked by decreasing $U(w; \theta)$. Firm $k_\theta(j; w_{-j})$'s hiring offer simply represents the n_θ -teenth best offer (that of firm j being excluded) made to agent with trait θ .

One can check that, in the equilibrium described above, $k_0(1; w_{-1}) \in \arg \max_{k \in \{2, 3, 4\}} U_k(w_k; 0)$, $k_0(2; w_{-2}) = k_0(3; w_{-3}) = k_0(4; w_{-4}) = 1$ while $k_1(2; w_{-2}) = k_1(3; w_{-3}) \in \arg \max_{k \in \{1, 4\}} U_k(w_k; 1)$ and $k_1(1; w_{-1}) = k_1(4; w_{-4}) \in \{2, 3\}$.

Assuming it is profitable for j to hire an agent with trait θ , her offer has just to be strictly preferred to that of $k_\theta(j; w_{-j})$ to attract an agent with trait θ . Given w_{-j} firm j 's reaction function¹⁸ $r_j(\cdot)$ is given by

$$r_j(w_{-j}) = \min \left\{ \begin{array}{c} S_j \\ \max \{w_j(I_B; 0), w_j(U_{k_0(j; w_{-j})}(w_{k_0(j; w_{-j})}; 0) + \varepsilon; 0)\} \\ \max \{w_j(I_B; 1), w_j(U_{k_1(j; w_{-j})}(w_{k_1(j; w_{-j})}; 1) + \varepsilon; 1)\} \end{array} \right\}$$

where $\varepsilon > 0, \varepsilon \rightarrow 0$.

Note that, since a job left closed entails a null profit, it is equivalent to assume that corresponding employer spends S_j to obtain surplus S_j and hence $r_j(w_{-j}) = S_j$ when minimal required transfer is strictly higher than S_j .¹⁹

Definition An equilibrium of the labor market is a vector w such that for all $j \in J : w_j \in r_j(w_{-j})$.

We now show this equilibrium exists.

Proposition 7 If, for all $\theta \in \{0, 1\}$, $n_\theta \geq 1$, labor market equilibrium exists.

Proof 7 We follow the standard Nash equilibrium existence proof.²⁰ For all $j \in J$, let us define $\rho_j : \times_{k \in \mathcal{J}} [0, S_k] \rightarrow [0, S_j]$ by $\rho_j(w) = r_j(w_{-j})$. Define the correspondence $\rho : \times_{j \in \mathcal{J}} [0, S_j] \rightrightarrows \times_{j \in \mathcal{J}} [0, S_j]$ to be the cartesian product of the ρ_j . A fixed point of ρ is a w such that $w \in \rho(w)$, so that, for each firm, $w_j \in \rho_j(w)$. Thus a fixed point of ρ is a labor market equilibrium. From Kakutani's theorem the following are sufficient conditions for ρ to have a fixed point: (i) $\times_{j \in \mathcal{J}} [0, S_j]$ is compact, convex, nonempty subset of a (finite-dimensional) euclidean space; (ii) $\rho(w)$ is nonempty for all w ; (iii) $\rho(w)$ is convex for all w ; (iv) $\rho(\cdot)$ has a closed graph.

Since $\forall j \in J, S_j \geq 0$, $\times_{j \in \mathcal{J}} [0, S_j]$ is indeed compact, convex and nonempty; furthermore it is a subset of \mathbb{R}^J with $J = \#J < \infty$ so that (i) is clearly satisfied. For all $\theta \in \{0, 1\}$, assuming $n_\theta \geq 1$,

¹⁸Maybe this function should be understood as capturing an algorithm leading to labor market equilibrium rather than as a reaction function.

¹⁹One can interpret this as reflecting the assumption that the employer choose to do the job himself.

²⁰See for instance Fudenberg & Tirole (1996, chapter 1, p.29).

$\forall j \in J, \forall w_{-j} \in \times_{k \in \mathcal{J} - \{j\}} [0, S_k], k_\theta(j; \mathbf{w}_{-j})$ exists. Hence, $\forall \theta \in \{0, 1\}, U_{k_\theta(j; \mathbf{w}_{-j})}(w_{k_\theta(j; \mathbf{w}_{-j})}; \theta)$ is well defined and $\forall j \in J, \forall w_{-j} \in \times_{k \in \mathcal{J} - \{j\}} [0, S_k], r_j(\mathbf{w}_{-j})$, and hence $\rho_j(\mathbf{w})$, is nonempty which entails (ii). If $\rho(\mathbf{w})$ were not convex, there would exist $w' \in \rho(\mathbf{w})$ and $w'' \in \rho(\mathbf{w})$, and $\lambda \in]0, 1[$ such that $\lambda w' + (1 - \lambda) w'' \notin \rho(\mathbf{w})$. For all firm j ,

$$S_j - (\lambda w'_j + (1 - \lambda) w''_j) = \lambda (S_j - w'_j) + (1 - \lambda) (S_j - w''_j)$$

so that if both w'_j and w''_j are best responses to w_{-j} , then so is their weighted average. This verifies (iii).

Assume that (iv) is violated so there is a sequence $(\mathbf{w}^n, \hat{\mathbf{w}}^n) \rightarrow (\mathbf{w}, \hat{\mathbf{w}})$ such that $\hat{w}^n \in \rho(\mathbf{w}^n)$ but $\hat{w} \notin \rho(\mathbf{w})$. Then $\hat{w}_j \notin \rho_j(\mathbf{w})$ for some j . Thus there is an $\epsilon > 0$ and a w'_j such that $S_j - w'_j > S_j - \hat{w}_j + 3\epsilon$. Since $(\mathbf{w}^n, \hat{\mathbf{w}}^n) \rightarrow (\mathbf{w}, \hat{\mathbf{w}})$, for n sufficiently large

$$S_j - w'_j > S_j - w'_j - \epsilon > S_j - \hat{w}_j + 2\epsilon > S_j - \hat{w}_j^n + \epsilon$$

i.e. w'_j does strictly better against w_{-j}^n than \hat{w}_j^n does, which contradicts $\hat{w}_j^n \in \rho_j(\mathbf{w}^n)$. This verifies (iv).

As compensating differentials claims, it must be possible to show that in the equilibrium, two agents of a given type should benefit the same utility level. The case of no shortage of type 1 agents does not raise particular obstacles: all the agents have for reservation utility I_B , and all the firms succeed in filling their job - at least those satisfying minimal profitability conditions.²¹ The difficulties we meet in other cases echo the multipolar characteristics of competition. Each firms has to control: for potential entries, for the pressure exerted by insiders willing to hire an agent of another type. Our attempts to treat these problems are available upon request.

²¹That is such that $S \geq w(I_B; 1)$.