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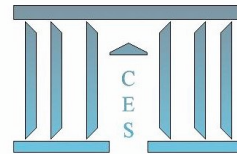
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Power Distribution and Endogenous Segregation

Catherine BROS

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Power Distribution and Endogenous Segregation

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Abstract

The aim of this paper is to provide a detailed analysis of the process of segregation formation. The claim is that segregation does not originate from prejudice or exogenous psychological factors. Rather it is the product of strategic interactions among social groups in a setting where one group has captured power. While using a model featuring random matching and repeated games, it is shown that whenever one group seizes power, members of other groups will perceive additional value in forging long term relationships with the mighty. They will systematically cooperate with the latter either because it is in their interest to do so or because they do not have other choice. The mighty natural response to this yearning to cooperate is to refuse intergroup relationships. The dominated group will best reply to this new situation by in turn rejecting the relationships and a segregation equilibrium emerges. Segregation stems from the systematic cooperation by one group with another. However, not all societies that have experienced power captures converge towards segregation. It is shown that the proportion of individuals that are actually powerful within the mighty group determines convergence towards segregation.

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1 Introduction

Inequalities defined as an unequal distribution of valued attributes such as wealth or power have been often seen as the origin of prejudice and its corollary: discrimination. Becker (1957) finds that discrimination is rooted in an exogenous taste for discrimination, or we could say prejudice, that springs from the cultural arena. Arrow (1973) finds that discrimination makes up for a lack of information regarding individuals' capacities. As an employer may not be able to test individually candidates' skills, he would approximately assess them at the average skills' level of the candidate's group or by resorting to cognitive dissonance when his perception is at odds over reality. In these two theories and their later developments, inequalities give rise to prejudice that, in turn create discrimination. However, the very origin of prejudice has rarely been delved into by economists. The latter have often given up investigating this question, treating prejudice as a disconcerting phenomenon that belongs to a peripheral field : a society specific culture. It seems that one step is missing in the argument that is to demonstrate how inequalities can generate such a large cleavage within a society that antagonism and rejection among groups and as a result prejudice appear. We intend to show in this article how inequalities and more specifically unequal power distribution across groups may give rise to a segregation equilibrium. Our assumption is that prejudice and discrimination are the logic outcome of identity phenomena, that stems from a deep cleavage in society whose utmost form is segregation. Therefore, through the study of the creation of segregation, we may be able to understand the forming of prejudice.

Our work aims at showing that segregation may pre-exist prejudice. Works in the wake of Schelling's (1978) segregation theory have shown that a segregationist equilibrium may appear even in the absence of discriminatory dispositions, or even when agents have pro-integrationist preferences. Such an equilibrium is the product of individuals' strategic decisions, who do not perceive the general effect of the sum of their microdecisions. This impact may often widely differ from their individual motives. In this article, we significantly concur on this view by considering that segregation is an equilibrium produced by the junction of individual rational decisions that have nothing to do with discriminatory intentions. Discrimination here should be understood as a negative treatment an individual receive due to its social or ethnic origin, its sex, etc.

We need to make clear what is understood here by segregation. This term has often been reduced to its spatial definition in economics works. Authors have mainly focussed on the fact for example that blacks and whites do not live in the same neighborhood, do not attend the same schools, etc. However, we argue that this term should be taken here in a broader sense. We define segregation as a refusal by members of different groups or communities to interact. Such a refusal may manifest itself through marriage decisions or by choosing a neighborhood or friends' company. Spatial segregation is only a specific case of this refusal to interact with members of another social or ethnic group.

It is striking that authors who investigated segregation phenomena addressed only three kinds of issues: (i) how is segregation perpetuated and why doesn't

it collapse? (ii) How could segregation be measured, and what point has it reached? How has it evolved over time especially in the United States? (iii) What are the properties of segregation equilibria? Works on segregation may be grouped in three kinds along these three types of questions. Studies that intend to explain why do agents choose strategies that allow them to avoid contacts with members of another communities are few.

As far as the first issue is concerned, a couple of economists have tried to show how a society could move from a segregation equilibria to a more mixed one and vice versa. Akerlof in his 1976 and 1980 papers, identified some of the factors that prevent the caste system from collapsing but did not look into the forming of the caste institution. Lundberg and Starz in a 1998 paper presented a model that aim at explaining how segregation reinforce itself through prejudice. Chaudhuri and Sethi (2003) showed how neighborhood effects perpetuate segregation, while Loury (2000) emphasized the concept of social capital. Most of these works do raise the question of the persistence of segregation but not of its origin. Sethi and Somanathan did approach this issue in a paper published in 2004. They found that segregation "can be stable when racial income disparities are either very great or very small, but instable in some intermediate range. And small income disparities can give rise to multiple equilibria, with segregation and integration both being stable". Even though these authors pointed out factors that make a society move from a segregation equilibrium to integration, this work exhibit one major drawback: it only depict movements of an already segregated society. The model fails to explain how segregation is produced ex nihilo but show how a society may return to its original situation. These types of analysis focus on societies that have segregation conditions already embedded in them and provide little help in explaining the occurrence of these conditions. However, these studies are helpful at establishing relationships between levels of segregation and other variables such as income distribution within and across groups.

The second kind of issue regards segregation measurement and calls for empirical studies. These works have mainly focused on either spatial segregation or labour market discriminations since these are the most tangible forms of segregation and almost the only viable source of data. Such pieces of work are legion.

The third group of studies do not address in our view the issue of the segregation origin but rather look into the efficiency of segregation equilibria. Studies undertook by Cutler, Glaeser and Vigdor (1997, 2007) are good examples. The work undertook by Eeckhout (2006) on which our model is based, does push the analysis quite far but finally fails to explain why do agents adopt segregationist strategies. In his article segregationist strategies are exogenous. The author is most concerned with the stability and Pareto efficiency features of a segregation equilibrium.

Schelling's works are an interesting exception to the three groups mentioned above. Schelling brilliantly managed to show how a segregation equilibrium may be produced by the junction of decisions made by individuals who do have only very mild preferences regarding whom they interact with. Nevertheless,

such preferences need to be specified in his model. The question as to why do individuals prefer a certain proportion of their likes in their neighborhood is left open. Again there seems to be a missing link in our search for the very origin of segregation. We firmly share Schelling's view that segregation is generated by individual strategic decisions that did not intend to be discriminatory and that segregation may preexist prejudice. Indeed the objective of this article is to show that inequalities create a cleavage (through a process that will be laid out in this article). Such a cleavage generates identity responses and prejudice that in turn will lead to discriminatory practices as described by either Becker or Arrow.

As our stand is that segregation is produced by individual strategic decisions, game theory models seem appropriate tools to analyze the conditions under which this equilibrium appears. The model used in this paper relates to the literature on random matching and repeated games. In this research field, the main question is how do individuals create and maintain long term relationships that are at odds with selfish individual motives. This problem is center to the study of segregation, as the latter may be viewed as a systematic rejection of any form of cooperation among two social groups. One solution to this problem provided by repeated game theory is that cooperation arises from a punishment threat such as reputation loss or social norms (Rosenthal and Landau 1979). This paper aims at showing that norms such as ostracism are the consequences and not the origins of a particular equilibrium, contrary to Lundberg and Startz position (2004). Ellison (1994), Kranton (1996), Ghosh and Ray (1996) provide solutions to the cooperation emergence puzzle. Their models show that cooperative equilibrium may arise despite the absence of information flows among individuals and provided that players are sufficiently uncertain about their partners' valuation of future transactions, and that cooperation payoffs increase over time. These main features are accounted for in the model used in this paper. The originality of this paper lies in the fact that we do not assume that prejudice, discrimination or to some extent hierarchy exist prior to the formation of the segregation equilibrium.

Although the model presented here is close to the one used in Eeckhout (2006), it differs by the fact that agents' choices between segregationist and non discriminatory strategies are endogenized. The main example that will be used in the article is the caste system in India, while it is believed that the analysis could be applied to many other segregated societies such as colonial societies for example. Indeed many anthropologists support the theory according to which the formation of one of the most perfect form of segregation, the caste system in India, arose from the invasion by the Aryan in India some 3 600 years ago who progressively took over most of the sub-continent's resources. Relationships between individuals considered here are analyzed through marriage decisions while it could be generalized to many other types of relationships.

Section 2 gives an account of the model's main features and depicts equilibrium conditions assuming that social group membership has influence neither on strategies nor on pay offs. In section 3 it is assumed that one group captures most of the power. The other group will perceive additional benefits in entering

into long-term relationships with them. Members of the powerless group will start systematically cooperating when matched with a member of the powerful group and the latter will best reply to this new situation by systematically rejecting any cooperation with the powerless. A stable segregation equilibrium will appear. Section 4 addresses the following question: why do some societies converge towards a segregation equilibrium while other move towards mixed equilibrium? It is argued that power distribution within the powerful group triggers segregation. Section 5 discusses the findings and provides selected historical examples. Section 6 concludes.

2 The model

2.1 Basic hypothesis

Consider a society with a continuum of infinitely lived agents. In each period a mass one of the agents is born. Time is discrete. At the beginning of each t period, agents are either single or matched in pairs. Two social groups are present in society: Ls and Hs and no individual can possibly hide his membership to one group that is costlessly observable. The proportion of Hs in the society is denoted π . The model is symmetric prisoner's dilemma. In this section we suppose that social group membership has no impact on the game payoffs. The importance of group memberships is analyzed in the following section. Individuals are randomly matched in pairs. Each individual while being in a relationship with another can choose between the action C which implies cooperation, or D , non-cooperation. When making the decision of action, each individual is unaware of his partner's simultaneous action decision. The action couple $\{C; C\}$ means that both partners cooperate, while $\{C; D\}$, means that the non cooperative individual who played D free rides his cooperative partner. The $\{D; D\}$ action couple means that both partner refuse to cooperate. Hence the prisoner's dilemma action space is $A = \{C; D\} \times \{C; D\}$. The stage game pay off function $\gamma : A \rightarrow \mathbb{R}^2$ is as follows:

	C	D
C	$1, 1$	$-l, 1 + g$
D	$1 + g, -l$	$0, 0$

where $g > 0, l > 0$. We assume $g - l \leq 1$.

The action couple $\{D; D\}$ with payoffs $(0, 0)$ is the only Nash and dominant strategy equilibrium. In this section, given that social group membership does not have any impact on the payoffs, it will be assumed that individuals will not condition their action decisions on their partners' social group, name or type. This situation will be investigated in section 3.

2.2 Game lay-out

The game has numerous stages. The first phase could be identified as the "meeting period" where both individuals make their action decision without knowing the partner's decision. The outcome is observed by both individuals at the end of each stage of the game. Depending on the outcome, they individually decide whether they terminate the relationship. If at least one partner decides to terminate the match, the partnership is dissolved and a new match is formed in the second period. It is assumed that if one partner has played D , the match is terminated by the other partner. Hence a match could only go on if both partners have played C at each stage of the game. The pay off of any s strategy over the repeated game is denoted by $v(s)$ and players discount the future at a common factor $\delta < 1$. The objective of the player is to maximize the normalized sum

$$v(s) = E(1 - \delta) \sum \delta^t \gamma(s)$$

The normalization allows us to compare the repeated game pay off directly with the stage game pay off.

2.3 Equilibrium condition

We assume that individuals are not allowed to condition their actions on their historical actions in past partnerships and have no information about their partners actions in previous matches. Therefore, should a partner deviate and play D , the match is dissolved and the deviator can not be punished for its deviation beyond the termination of the match. Individuals will therefore be tempted to adopt a systematically non-cooperative strategy (systematically play D) and no cooperation can occur. In order to reach a certain level of cooperation a no-deviation constraint must be placed on strategies that is to say, the pay off from deviating should be inferior to the cooperation pay off.

2.4 No deviation constraint definition

Given an infinite horizon, pay off from cooperation is 1. pay off from deviation (v^D) can be written as

$$v^D = (1 + g)(1 - \delta) + \delta v(s),$$

In a first period the deviator free rides his partner and receives $1 + g$ discounted by the factor $(1 - \delta)$ and has to start a partnership anew whose pay off will be $v(s)$ discounted back by the factor δ . The no deviation constraint requires the pay off from deviation (v^D) to be lower than the pay off from eternal cooperation that is to say 1. The non deviation constraint can be written thus:

$$v^D \leq 1 \text{ or } v(s) \leq 1 - g \frac{1-\delta}{\delta}$$

In other words, a s strategy may be considered as an equilibrium strategy provided its pay off $v(s)$ is lower than $1 - g\frac{1-\delta}{\delta} = V$ i.e. provided that the no-deviation constraint is respected. Should a strategy pay off be greater than V , then any player would systematically deviate from the equilibrium strategy that is play C until a deviation occurs. A player who adopts a strategy s such as $v(s) > V$ will systematically play D and no equilibrium can occur.

2.5 Equilibrium strategies definitions

Suppose players i and j enter into a relationship.

Let's note for player i the pay off from playing C by $F_i(s)$ at one stage of the game and the pay off from playing D by $G_i(s)$, where:

$$F_i(s) = \sigma_j + (1 - \sigma_j)[(1 - \delta)(-l) + \delta v(s)]$$

As $F_i(s)$ is considered, player i plays C with certainty. His partner j plays C with a probability σ_j . In this case, the outcome of the stage game would be $\{C, C\}$ and player i will receive a pay off of 1. Hence the first term on the right side of the equation (σ_j). Should partner j choose to play D with a probability $(1 - \sigma_j)$, the outcome of the stage game would be $\{C, D\}$ and player i would be free ridden by his partner j (j is not cooperative while i is). Player i would receive in a first stage $-l$ discounted at a factor $(1 - \delta)$. Given that D had been played, a new match would be formed in the future whose pay off will be $v(s)$ discounted at the rate δ .

$$G_i(s) = \sigma_j[(1 - \delta)(1 + g) + \delta v(s)] + (1 - \sigma_j)\delta v(s)$$

Similarly, player i is now assumed to play D . His partner j plays C with a probability σ_j , in which case, i would free ride j and receive $(1 + g)$ in a first stage discounted at a factor $(1 - \delta)$ and a new partnership would have to be formed given that the outcome is $\{C, D\}$. If partner j decides on playing D (probability $1 - \sigma_j$), then the outcome would be $\{D, D\}$, and player i would receive 0 in a first stage and a new partnership would have to be formed in the future which pay off will be $v(s)$ discounted at the δ rate.

Then the expected pay off for a newly matched individual i at the beginning of the game is

$$v(s) = \sigma_i F(s) + (1 - \sigma_i)G(s)$$

where σ_i is the probability that individual i will choose to play C .

Best equilibrium strategies s^* are found by solving the following problem

$$\begin{aligned} s^* \in \arg \quad & \max v(s), \\ \text{s.t. } & v(s) \leq V \end{aligned}$$

and in addition, supposing indifference between the pay off of playing C and D if $\sigma_j \in (0, 1)$. Note that if the partner j plays C with certainty, i would no longer be indifferent between playing C and D and will choose D with certainty.

We suppose that when at equilibrium, social group membership does not make a difference in the choice of strategies. This equilibrium will be named a "group-blind equilibrium". Specifically, the probability for a player to play C or D does not depend on the social origin of the partner.

3 Formation of a segregation equilibrium

3.1 Introduction of an attractive feature of group H

So far, social group has had no incidence on partnership formation. Let's assume from now on that one group, the Hs, monopolize all the economic resources. For example, the Hs may invade the Ls' country and seize most of the land or political power. Entering into a relationship such as marriage with an H may bring additional opportunities to an L. Ls will perceive additional value in a long-term relationship with Hs. In the context of India for example, a low caste father may be eager to have his daughter married to a higher caste man as such an alliance may give his own family, or if not his direct family his lineage, access to better opportunities. This may be true beyond the context of marriage. International trade literature has shown that less developed countries are more eager to trade with developed countries as these relationships give them access to products of different quality. This additional value perceived is symbolized by parameter z in the pay off matrix. It is worth noticing that this additional parameter only appears on the L side in a H-L relationship. Intragroup payoffs and Hs payoffs are unchanged. Payoffs for a H-L match are represented in the matrix below:

	pay off to the L	
pay off to the H	C	D
C	$1, 1 + z$	$-l, 1 + g$
D	$1 + g, -l$	$0, 0$

3.2 Impact of the parameter z on relationships payoffs

Let's note $F_{H/L}$ the pay off from playing C to a member of the H group when matched to a partner drawn from the L group. Similarly $G_{L/H}$ is the pay off from playing D to a member of the L group when matched to a partner from the H group. Please note that the introduction of the parameter z only modifies the pay off from playing C to an L who meets an H i.e. $F_{L/H}$. All other payoffs remain identical as in section 2.5. For the sake of clarity the four possible cases are laid out below.

3.2.1 Pay off to an H individual in a H-L relationship

$$F_{H/L} = \sigma_{L/H} + (1 - \sigma_{L/H})[(1 - \delta)(-l) + \delta v_H(s)]$$

Where $F_{H/L}$ is the pay off from playing C to an H who meets an L.

$\sigma_{L/H}$ is the probability that the L partner plays C given the fact that he is matched to an H. Should this case occur it means that both partners would have played C , pay off to the H would be 1 and the partnership is never dissolved. Should the L partner choose to play D (probability $1 - \sigma_{L/H}$), the H would be free ridden by the L, earn $-l$ in the first period (hence the $(1 - \delta)$ discount factor), the relationship would be terminated and a new match formed whose pay off to the H would be $v_H(s)$ discounted at a factor δ .

Symmetrically, pay off from playing D to an H who meets an L is

$$G_{H/L}(s) = \sigma_{L/H}[(1 - \delta)(1 + g) + \delta v(s)] + (1 - \sigma_{L/H})\delta v_H(s)$$

$\sigma_{L/H}$ the probability that the L plays C when matched to an H. In this event, H would have played D and L, C . H would earn $1 + g$ in the first period and the match would break up. If the L individual chooses to play D (probability $(1 - \sigma_{L/H})$), the action couple $\{D, D\}$ would appear. In this case no one would receive any payment in the first period, the relationship would be terminated and the H would start a new match whose discounted pay off is $\delta v_H(s)$.

The expected pay off for an H that is newly matched to an L is therefore

$$v_{H/L}(s) = \sigma_{H/L}F_{H/L}(s) + (1 - \sigma_{H/L})G_{H/L}(s)$$

where $\sigma_{H/L}$ is the probability that the H chooses to play C given the fact that the other player is drawn from the L group.

3.2.2 Pay off from intragroup relationships

Intragroup relationship payoffs are derived in the same way as above

For an H meeting an H

$$\begin{aligned} F_{H/H}(s) &= \sigma_{H/H} + (1 - \sigma_{H/H})[(1 - \delta)(-l) + \delta v_H(s)] \\ G_{H/H}(s) &= \sigma_{H/H}[(1 - \delta)(1 + g) + \delta v_H(s)] + (1 - \sigma_{H/H})\delta v_H(s) \\ v_{H/H}(s) &= \sigma_{H/H}F_{H/H}(s) + (1 - \sigma_{H/H})G_{H/H}(s) \end{aligned}$$

For an L meeting an L

$$\begin{aligned} F_{L/L}(s) &= \sigma_{L/L} + (1 - \sigma_{L/L})[(1 - \delta)(-l) + \delta v_L(s)] \\ G_{L/L}(s) &= \sigma_{L/L}[(1 - \delta)(1 + g) + \delta v_L(s)] + (1 - \sigma_{L/L})\delta v_L(s) \\ v_{L/L}(s) &= \sigma_{L/L}F_{L/L}(s) + (1 - \sigma_{L/L})G_{L/L}(s) \end{aligned}$$

3.2.3 Pay off to L individual in an H-L relationship

Pay off from playing C to an L member matched to an H is somewhat different due to the introduction of the parameter z .

$$F_{L/H} = \sigma_{H/L}(1 + z) + (1 - \sigma_{H/L})[(1 - \delta)(-l) + \delta v_L(s)]$$

If the L partner chooses to play C and the H partner is also cooperative (probability $\sigma_{H/L}$) then a long-term relationship may be set up and the L individual will gain 1 from the partnership plus all the additional opportunities such a partnership may provide him with. The value of these additional opportunities is estimated by the parameter z . Note that this parameter plays a role only if the action couple $\{C; C\}$ is the outcome.

Pay off from playing D to an L matched to an H is similar to those occurring in intragroup matches.

$$G_{L/H}(s) = \sigma_{H/L}[(1 - \delta)(1 + g) + \delta v_L(s)] + (1 - \sigma_{H/L})\delta v_L(s)$$

The expected pay off for an L who is newly matched to an H is therefore

$$v_{L/H}(s) = \sigma_{L/H}F_{L/H}(s) + (1 - \sigma_{L/H})G_{L/H}(s)$$

We mentioned in section 2 that at the group blind equilibrium, i.e. prior to the introduction of the parameter z , players do not condition their strategies on the social origin of their partner. Therefore $\sigma_{L/H} = \sigma_{L/L}$. Now that z has been introduced in the pay off matrix, it can be argued that L players will modify their probability to play C when matched to an H.

3.3 Systematic cooperation from the Ls leads to a segregation equilibrium

It is quite obvious that the introduction of the z parameter increases Ls' potential payoffs. In the event of a match with an H they do not only benefit from the value of the match itself but also from additional social opportunities such a relationship may bring them. The intuition is as follows: Ls individuals have a strong interest in building long-term relationships with H members. They are enticed to cooperate when matched to an H in order to maximize the chance of having a long term relationship. The probability to play C for an L meeting an H increases up to 1 as a consequence of the introduction of the parameter z . Now that the H individual knows for certain that the L partner will cooperate, he has no incentive anymore to cooperate. He would be better off by systematically free riding his cooperative partner. In this case, the only equilibrium strategy will be for the H to systematically reject any cooperation with an L. The final equilibrium situation is the one where a guaranteed cooperation from one group leads the other group to systematically reject any relationship with this group. Perfect segregation has emerged and in the aftermath prejudice that reinforces segregation. It is rather counter intuitive to think that segregation germinates from the yearning by one group to cooperate with another. Several historical examples support this argument. It is striking that Portuguese colonies contrary to the French and British ones, were not segregated. While the Portuguese mainly set up trading posts, the French and the British led an active power and wealth takeover policy. In the french and british empires, natives had to be cooperative with the new masters to secure social opportunities, while in Portuguese colonies trade enabled locals to make social progress

on their own. In the first case, segregationist ideologies appeared while in the second a more integrated situation prevailed. More historical examples will be discussed in section 3.3.2. A formal demonstration of the argument is provided below.

Proposition 1 *The introduction of the parameter z increases the probability for an L to play C when matched to an H up to 1 that is $\sigma_{L/H} = 1$*

We mentioned that an equilibrium condition must be the indifference between the pay off of playing C or D that is $F(s) = G(s)$. If this condition was not satisfied, a player would modify his probability to play C . For example, if the pay off from playing C increases comparatively to the pay off from playing D , ($F(s) > G(s)$) a player would adjust upward his propensity σ to play C , until the equality condition between $F(s)$ and $G(s)$ is restored. Let's note $\Phi = F(s) - G(s)$. We assume that $\frac{\partial \sigma}{\partial \Phi} > 0$.

The introduction of the parameter z alters the pay off from playing C for an L who is matched to an H . The L individual will therefore alter his probability to play C ($\sigma_{L/H}$) consequently. Let's note $\Phi_{L/H} = F_{L/H}(s) - G_{L/H}(s)$ then $\frac{\partial \sigma_{L/H}}{\partial \Phi_{L/H}} > 0$. and the following equation can be written:

$$\frac{\partial \sigma_{L/H}}{\partial z} = \frac{\partial \sigma_{L/H}}{\partial \Phi_{L/H}} \frac{\partial \Phi_{L/H}}{\partial z}$$

where we know that the first term on the right hand side of the equation is positive. The second term of the equation is also positive since the inclusion of the parameter z has unilaterally increased $F_{L/H}(s)$ without altering $G_{L/H}(s)$.

Introducing the parameter z increases $\sigma_{L/H}$ i.e. the probability for an L who meets an H to cooperate. An increase in $\sigma_{L/H}$ will alter the pay off for the H matched to an L to play C comparatively to playing D . $\Phi_{H/L}$ will be modified and $\sigma_{H/L}$ will therefore be adjusted. A movement in $\sigma_{H/L}$ will modify in turn $\sigma_{L/H}$, and so on. So the total effect on $\sigma_{L/H}$ due to the introduction of the parameter z can be written:

$$\frac{\partial \sigma_{L/H}^t}{\partial z} = \frac{\partial \sigma_{L/H}^i}{\partial z} + \frac{\partial \sigma_{L/H}^i}{\partial z} \sum_{j=1}^n \left(\frac{\partial \sigma_{H/L}}{\partial \sigma_{L/H}} \frac{\partial \sigma_{L/H}}{\partial \sigma_{H/L}} \right)^j = \frac{\partial \sigma_{L/H}^i}{\partial z} (1 + n)$$

Where $\frac{\partial \sigma_{L/H}^i}{\partial z}$ is the initial impact of z on $\sigma_{L/H}$ and $\frac{\partial \sigma_{L/H}^t}{\partial z}$ is the total effect of the introduction of z . The introduction of z has two impacts:

- one with the direct increase of $\sigma_{L/H}$ via the increase of $\Phi_{L/H}$
- one with the cumulative effect of a positive shock on $\sigma_{L/H}$

n is the number of periods in the game needed for players to adapt their action strategies. It is argued that there exists n^* , so that $\sigma_{L/H}$ will move from its group blind equilibrium level ($\sigma_{L/H} = \sigma_{L/L}$) to a new equilibrium level $\sigma_{L/H} = 1$. n^* is defined by the following equation:

$$\frac{\partial \sigma_{L/H}^i}{\partial z} (1 + n^*) dz = 1 - \sigma_{L/L}$$

Proposition 2 *Whenever $\sigma_{L/H} = 1$ the only possible equilibrium for an H-L partnership is given by $\sigma_{H/L} = 0$*

Recall that in order for a strategy s to be an equilibrium strategy, the no deviation constraint must be respected that is $v(s) \leq 1 - g \frac{(1-\delta)}{\delta}$.

If $\sigma_{L/H} = 1$,

$$F_{H/L} = \sigma_{L/H} + (1 - \sigma_{L/H})[(1 - \delta)(-l) + \delta v_H(s)] = 1$$

$$G_{H/L}(s) = \sigma_{L/H}[(1 - \delta)(1 + g) + \delta v_H(s)] + (1 - \sigma_{L/H})\delta v_H(s)$$

$$G_{H/L}(s) = (1 - \delta)(1 + g) + \delta v_H(s)$$

$$v_{H/L}(s) = \sigma_{H/L}F_{H/L}(s) + (1 - \sigma_{H/L})G_{H/L}(s) \leq 1 - g \frac{(1-\delta)}{\delta}$$

$$v_{H/L}(s) = \sigma_{H/L} + (1 - \sigma_{H/L})[(1 - \delta)(1 + g) + \delta v_H(s)] \leq 1 - g \frac{(1-\delta)}{\delta}$$

After rearranging,

$$\sigma_{H/L}[\delta - g(1 - \delta) - \delta v_H(s)] \leq \delta - g(1 - \delta) - \delta v_H(s) - g \frac{(1-\delta)}{\delta}$$

Recall that $V = 1 - g \frac{(1-\delta)}{\delta}$, then $\delta V = \delta - g(1 - \delta)$

$$\sigma_{H/L}[\delta][V - v_H(s)] \leq [\delta][V - v_H(s)] - g \frac{(1-\delta)}{\delta}$$

Recall that V is the non deviation constraint i.e. the highest attainable pay off for any equilibrium strategy. Therefore any player tries to minimize the difference $V - v_H(s)$ under the constraint $[\delta][V - v_H(s)] - g \frac{(1-\delta)}{\delta} \geq 0$. Indeed, $\sigma_{H/L}[\delta][V - v_H(s)]$ cannot be negative so $0 \leq \sigma_{H/L}[\delta][V - v_H(s)] \leq [\delta][V - v_H(s)] - g \frac{(1-\delta)}{\delta}$. If this constraint is respected then $[\delta][V - v_H(s)]$ lowest level is $g \frac{(1-\delta)}{\delta}$. Minimizing $[\delta][V - v_H(s)]$ is equivalent to replacing the expression by its minimum $g \frac{(1-\delta)}{\delta}$ in the equation:

$$\sigma_{H/L}[g \frac{(1-\delta)}{\delta}] \leq 0$$

$$[g \frac{(1-\delta)}{\delta}] > 0 \text{ by definition}$$

$$\sigma_{H/L} \geq 0 \text{ by definition.}$$

The only strategy that maximizes the pay off while respecting the no deviation constraint is $\sigma_{H/L} = 0$ when $\sigma_{L/H} = 1$. However, whenever $\sigma_{H/L} \in [0; 1[$, an equilibrium is attainable provided that $\sigma_{L/H} \in [0; 1[$. In other words, if a partner cooperates for sure, the only strategy that guarantees both an equilibrium and the other partner's pay off maximization is a systematic non-cooperative behavior.

Once $\sigma_{H/L} = 0$, i.e. the Hs play with certainty D when matched to an L, Ls maximize their pay off in such a match by setting their probability to play C to 0 i.e. $\sigma_{L/H} = 0$. Indeed, if $\sigma_{H/L} = 0$

$$F_{L/H} = (1 - \delta)(-l) + \delta v_L(s)$$

$$G_{L/H}(s) = \delta v_L(s)$$

$$v_{L/H}(s) = \sigma_{L/H}F_{L/H}(s) + (1 - \sigma_{L/H})G_{L/H}(s)$$

$$v_{L/H}(s) = \sigma_{L/H}(1 - \delta)(-l) + \delta v_L(s)$$

Given that $\sigma_{L/H}(1 - \delta)(-l) \leq 0$, $v_{L/H}(s)$ is maximized by setting $\sigma_{L/H} = 0$

It is worth noticing that whenever one of the partner plays D with certainty, the other partner maximizes his pay off while respecting the no-deviation constraint by setting his probability to play C at 0.

3.3.1 Equilibrium stability

The equilibrium obtained here is a segregation equilibrium where members from two groups systematically refuse to cooperate when matched to each other. The path to the segregation equilibrium is as follows:

- one social group captures more power or economic resources. The other group (the Ls) perceive additional benefits from entering into long-term relationships with the previous group (the Hs) through additional opportunities or may not have much choice but to cooperate to survive. The parameter z is introduced in the matrix.

- the introduction of the parameter z increases the propensity of L group members to cooperate with Hs, which increases the likelihood of a cooperation from the Hs, which in turn raises the probability for Ls to play C , and so on. At the end of n^* periods, Ls cooperate with certainty with Hs. At this time, although the probability for Hs to play C has reached quite high levels, as soon as $\sigma_{H/L} = 1$, Hs have no equilibrium strategy available but to be systematically non-cooperative with Ls, thus $\sigma_{L/H} = 0$.

- Whenever $\sigma_{H/L} = 0$, Ls maximize their payoffs by choosing never to cooperate whenever matched to an L, thus $\sigma_{L/H} = 0$

At the end of the n^* periods a segregation equilibrium prevails: members of the two groups will systematically refuse to cooperate when matched to each other. This result is rather interesting when thinking about the ambiguity one may find in the dominated groups attitudes towards dominant groups. Dominated often wishes to conform to dominants (translating a will to cooperate) while depreciating them when they feel they're being rejected by the mighty. As the fox in Aesop fable who can not reach grapes hanging high up on a vine, that he eyed greedily retreats and says: "The grapes are sour anyway!".

As mentioned above, as soon as one partner sets his probability to play C to 0, the other partner will reciprocate by setting his probability to play C to 0 and cannot be better off. Therefore $\sigma_{L/H} = 0, \sigma_{H/L} = 0$ is a stable equilibrium on the L-H market. The path that leads to a segregation equilibrium is rather interesting. Assuming that one group is deprived of power, it has not much choice but to cooperate with another. We may conclude that given this systematic cooperation, the latter will refuse any interaction with the first one. Finally the deprived group's best response will be to refuse any relationship too.

3.3.2 Segregation origin: economic circumstances and the dominated group

Two interesting points stand out from the demonstration above. Firstly, segregation is not the direct product of psychological and exogenous factors such as prejudice or a taste for discrimination. Such a situation arises from a socio-economic context in which one group has taken over much of the power, be this power economic, political or to some extent symbolic. Indeed, many authors have considered that segregation is the consequence of psychological factors or social prejudice that are most of the time exogenous and in any case beyond

the field of economics. We have tried to show that an unequal economic opportunities distribution triggers social strategies that generate a stable segregation equilibrium. Secondly, it has been shown that segregation does not originate from a favored group's intention to put a deprived group at its mercy. On the contrary, it is the systematic yearning or necessity by the latter to build long-term relationship with the mighty that leads to a rejection by the favored group of any relationship with the powerless. Although that result is in conflict with the generally accepted idea that segregation comes from the favored group, there are many good historical examples, especially related to colonization processes. Amartya Sen recounts in "The Argumentative Indian : Writings on Indian History, Culture and Identity" an interesting story. At the time when British trading posts were created in India in the XVIIth and early XVIIIth centuries, some British actually settled into the sub-continent, married Indian women and somewhat managed to mingle with the local society. Then came the time of the proper colonization process in the XIXth century, when the Honourable East India Company and later the British government took over power and wealth in the sub-continent. The locals had then to deal with the new masters to secure social opportunities. At that time, a new ideology started to appear in the Raj according to which India and its civilization were backward and needed to be patronized. Indian and British communities were set aside and any admitted relationship between the two was to be on a teaching mode. The power takeover by the British led to a segregation between Europeans and Indians that is well illustrated by novels from that time.

Although the demonstration above sheds an interesting light on the process of segregation formation it fails to explain why some societies exhibit high levels of segregation, as it is the case in India through the caste system, and other societies lower levels of segregation. Indeed, in any society at least one group is more dominant than the others but all societies do not move towards a segregation equilibrium. The next section will try to identify factors that trigger a segregation equilibrium. Power distribution within the favored group appears to be the determining factor for a segregation equilibrium to occur

4 Segregation versus non segregation equilibria

Let's assume that Ls have access to additional social opportunities only when establishing long-term relationships with powerful Hs who are in proportion $\theta \in [0, 1]$ in the H population and the Ls receive the same kind of pay off as in other relationship when matched to powerless Hs. The probability to gain z for an L matched to an H depends on the proportion of powerful individuals in the H population i.e. θ . The intuition tells us that the lower this probability, the less inclined will be the L individuals to adjust upwards their probability to cooperate. In other words, the lower θ , the more limited the increase in $\sigma_{L/H}$ due to the introduction of z . It will take a longer time for $\sigma_{L/H}$ to reach its maximum value of 1. The time needed is called the "adaptation phase". In the meantime, mixed H-L couples may be formed. We may realistically assume

that a mighty H who forge a long term relationship with an L loses its power, but remains an H. Indeed, let's take the example of an aristocrat who marries a commoner. The odds are that he or his children will still be considered as members of the nobility but somewhat of a second rate. Therefore as mixed matches are made, the proportion θ of powerful H decreases over time. Should the adaptation phase be long enough, which is the case if the initial θ is small, the proportion of powerful H declines and converges to 0 before the segregation equilibrium is triggered, i.e. before $\sigma_{L/H} = 1$. At this point in time, there is virtually no powerful H any more and the H group becomes similar to the L group. Members of the latter do not perceive additional value in building long term relationships with the Hs and the situation is back to the group blind equilibrium. By intuition, we understand that the initial proportion of powerful H is the determinant of the final degree of segregation. A more formal argument is presented in the following sections.

4.1 The impact of z is conditional upon the proportion of powerful individuals within the H group.

Proposition 3 *the impact of the parameter z on $\sigma_{L/H}$ is an increasing function of the initial θ (the proportion of powerful H within the H population).*

Let's note t the time period, θ_t the proportion of powerful H at time t , $\sigma_{L/H;t}$ the probability an L plays C when matched to an H at time t , $F_{L/H;t}$ the pay off from playing C for L matched to an H at time t , $v_{L;t}$ the strategy total pay off for a newly matched L at time t .

The pay off from playing C for an L matched to an H at time t can be written

$$F_{L/H;t} = \sigma_{H/L;t}[(1+z)\theta_{t-1} + (1-\theta_{t-1})] + (1-\sigma_{H/L;t})[(1-\delta)(-l) + \delta v_{L;t}(s)]$$

An L is matched to an H at time t . L plays C with certainty. If H plays C as well (probability $\sigma_{H/L;t}$) and is powerful (probability θ_{t-1} which is the proportion of powerful H within the H group at the start of the period hence the subscript $t-1$), a long term relationship starts and the L receives $1+z$. In the case the L is matched to a powerless H and they start a long term relationship, he receives the same pay off as in a L-L relationship. In the event H plays D, a new partnership must be formed.

The pay off from playing D for an L matched to an H at time t can be written

$$G_{L/H;t} = \sigma_{H/L;t}[(1-\delta)(1+g) + \delta v_{L;t}(s)] + (1-\sigma_{H/L;t})[\delta v_{L;t}(s)]$$

Total strategy pay off for an L who is newly matched to an H at time t is

$$v_{L/H;t}(s) = \sigma_{L/H;t}F_{L/H;t} + (1-\sigma_{L/H;t})G_{L/H;t}$$

The strategy total pay off for a newly matched L is at time t

$$v_{L;t}(s) = \pi_t v_{L/H;t}(s) + (1-\pi_t)v_{L/L;t}(s)$$

where π_t is the proportion of H in the society at period t , $v_{L/H}(s)$ the strategy pay off for an L matched to an H and $v_{L/L}(s)$ the strategy pay off for an L matched to another L for the period t .

Let's assume that the parameter z is introduced during the first period i.e.; for $t = 1$.

$$\frac{\partial v_{L;1}}{\partial z} = \pi_1 \frac{\partial v_{L/H;1}}{\partial z} + (1 - \pi_1) \frac{\partial v_{L/L;1}}{\partial z} = \pi_1 \frac{\partial v_{L/H;1}}{\partial z}$$

Note that the strategy pay off for an L matched to an L is independent from z , therefore $\frac{\partial v_{L/L}}{\partial z} = 0$. As for the impact of z on the strategy pay off for an L matched to an H is as follows (it is assumed that in a first phase $\sigma_{L/H;1}$ is not dependent on z as $\sigma_{L/H;1}$ will be modified in the subsequent periods)

$$\begin{aligned} \frac{\partial v_{L/H;1}}{\partial L} &= \sigma_{L/H;1} \left[\frac{\partial F_{L/H;1}}{\partial L} - \frac{\partial G_{L/H;1}}{\partial L} \right] + \frac{\partial G_{L/H;1}}{\partial L} \\ \frac{\partial F_{L/H;1}}{\partial L} &= \sigma_{H/L;1} \theta_0 + (1 - \sigma_{H/L;1}) (\delta) \left(\frac{\partial v_{L;1}(s)}{\partial L} \right) \\ \frac{\partial G_{L/H;1}}{\partial L} &= \delta \frac{\partial v_{L;1}(s)}{\partial L} \\ \frac{\partial v_{L/H;1}}{\partial L} &= \sigma_{L/H;1} \sigma_{H/L;1} (\theta_0 - \delta \frac{\partial v_{L;1}(s)}{\partial L}) + \delta \frac{\partial v_{L;1}(s)}{\partial L} \\ \frac{\partial v_{L/H;1}}{\partial L} &= \sigma_{L/H;1} \sigma_{H/L;1} (\theta_0 - \delta \pi_1 \frac{\partial v_{L/H;1}}{\partial L}) + \delta \pi_1 \frac{\partial v_{L/H;1}}{\partial L} \\ \frac{\partial v_{L/H;1}}{\partial L} &= \frac{\theta_0 \sigma_{L/H;1} \sigma_{H/L;1}}{1 - \delta \pi_1 [1 - \sigma_{L/H;1} \sigma_{H/L;1}]} \geq 0 \end{aligned}$$

Please note that the increase in the strategy pay off due to the introduction of z is now a function of θ_0 , the proportion of powerful Hs at the outset of the game. The higher this probability, the larger the impact of z on the pay off and arguably the larger the increase in the probability to play C for an L matched with an H. Note that it is only the proportion θ at the beginning of the period when z is introduced that matters and not subsequent values of θ . Indeed subsequent increases in $\sigma_{L/H}$ are only related to movements in $\sigma_{H/L}$ that are linked to the initial move in $\sigma_{L/H}$ due to the introduction of z . Hence only the initial value of θ (i.e. θ_0) matters for the size of the increases in $\sigma_{L/H}$.

To estimate the total impact of z on the probability that an L will play C when matched to an H, we first estimate the impact of z on $\sigma_{L/H}$ without taking into account the cumulative effect of an increase in $\sigma_{L/H}$ on $\sigma_{H/L}$, and so on.

We can write

$$\begin{aligned} \frac{\partial \sigma_{L/H;1}^i}{\partial z} &= \frac{\partial \sigma_{L/H;1}^i}{\partial \Phi_{L/H;1}} \frac{\partial \Phi_{L/H;1}}{\partial z} \\ \Phi_{L/H;1} &= F_{L/H;1} - G_{L/H;1} \\ \frac{\partial \Phi_{L/H;1}}{\partial z} &= \sigma_{H/L;1} \theta_0 + (1 - \sigma_{H/L;1}) \delta \frac{\partial v_{L;1}}{\partial z} - \delta \frac{\partial v_{L;1}}{\partial z} \\ \frac{\partial \Phi_{L/H;1}}{\partial z} &= \sigma_{H/L;1} \left[\theta_0 - \delta \pi_1 \frac{\partial v_{L/H;1}}{\partial z} \right] \\ \frac{\partial \Phi_{L/H;1}}{\partial z} &= \sigma_{H/L;1} \left[\theta_0 - \delta \pi_1 \frac{\theta_0 \sigma_{L/H;1} \sigma_{H/L;1}}{1 - \delta \pi_1 [1 - \sigma_{L/H;1} \sigma_{H/L;1}]} \right] \\ \frac{\partial \Phi_{L/H;1}}{\partial z} &= \sigma_{H/L;1} \theta_0 \left[\frac{1 - \delta \pi_1}{1 - \delta \pi_1 [1 - \sigma_{L/H;1} \sigma_{H/L;1}]} \right] \\ \frac{\partial \sigma_{L/H;1}^i}{\partial z} &= \frac{\partial \sigma_{L/H;1}^i}{\partial \Phi_{L/H;1}} \left[\sigma_{H/L;1} \theta_0 \left[\frac{1 - \delta \pi_1}{1 - \delta \pi_1 [1 - \sigma_{L/H;1} \sigma_{H/L;1}]} \right] \right] \end{aligned}$$

The same scheme as in section 3 applies. Any move in $\sigma_{L/H}$ will trigger a change in $\sigma_{H/L}$ that in turn will modify $\sigma_{L/H}$, and so on. So the total effect of the introduction of z at period t can be written

$$\frac{\partial \sigma_{L/H;t}^i}{\partial z} = \frac{\partial \sigma_{L/H;1}^i}{\partial z} + \frac{\partial \sigma_{L/H;1}^i}{\partial z} \sum_{j=1}^n \left(\frac{\partial \sigma_{H/L}}{\partial \sigma_{L/H}} \frac{\partial \sigma_{L/H}}{\partial \sigma_{H/L}} \right)^j = \frac{\partial \sigma_{L/H;1}^i}{\partial z} (1+t)$$

$$\frac{\partial \sigma_{L/H;t}^i}{\partial z} = \frac{\partial \sigma_{L/H;1}^i}{\partial \Phi_{L/H;1}} \sigma_{H/L;1} \theta_0 \left[\frac{1-\delta\pi_1}{1-\delta\pi_1[1-\sigma_{L/H;1}\sigma_{H/L;1}]} \right] (1+t)$$

The larger θ_0 , that is to say the initial proportion of mighty individuals within the H group, the larger the impact of z on $\sigma_{L/H}$. This result means that if the proportion of powerful individuals within the H group is relatively small, then the increase in $\sigma_{H/L}$ will be quite limited and it will take a rather long time before it reaches its maximum value of 1. In the meantime, mixed long term relationships may be formed.

4.2 Evolution of the proportion θ

As mentioned earlier, it is assumed that a powerful H who enters into a long term relationships with an L loses its power but not his H membership. Therefore π the proportion of H in the population does not vary over time: $\pi_t = \pi$. During the period t , the proportion of mighty who are matched to L individuals is $\frac{1}{2}\theta_{t-1}\pi(1-\pi)$, where $\theta_{t-1}\pi$ is the proportion of mighty individuals in the society. The so matched L-H couple decide to cooperate with the probability $\sigma_{L/H;t}\sigma_{H/L;t}$. The proportion of mighty among the H at period t (i.e. θ_t) is the proportion of powerful among the H for the previous time period minus the proportion of powerful H who forged relationships with Ls, formally

$$\theta_t = \theta_{t-1} \left[1 - \frac{1}{2}\pi(1-\pi)\sigma_{L/H;t}\sigma_{H/L;t} \right]$$

Let's note

$$p_t = \left[1 - \frac{1}{2}\pi(1-\pi)\sigma_{L/H;t}\sigma_{H/L;t} \right]$$

and rewrite

$$\theta_t = \theta_0(p_1)(p_2)(p_3)\dots(p_t)$$

Given that p_t is lower than 1, θ_t decreases over time but less and less rapidly as $\sigma_{L/H;1}$ increases over time. θ_t decreases up to the time $\sigma_{L/H;t}$ reaches 1, at which point segregation occurs and no mixed long term relationship may be forged anymore. Note that when $\sigma_{L/H;t}$ reaches 1, θ_t may still be quite large if θ_0 was large enough or θ_t may be close to 0 if θ_0 was small enough (this point will be shown in the next subsection). If θ_t becomes sufficiently close to 0, Hs are not more powerful than the L any more and the two groups become similar. The Ls do not perceive additional value any more in building relationships with the Hs, that is to say that the parameter z virtually disappears and the situation moves back to a "group blind equilibrium".

4.3 Comparative evolution of the proportion θ_t , when θ_0 is large or small

Recall that the smaller θ_0 , the smaller the impact of z on $\sigma_{L/H}$, the longer the time until $\sigma_{L/H}$ reaches 1. Let's consider two situations: one in which θ_0 is small and noted θ'_0 .

If θ'_0 is small, then in virtue of proposition 3, we know that $\sigma_{L/H;t}$ will be small and will be noted $\sigma'_{L/H;t}$. Let's note n' the number of periods necessary

for $\sigma'_{L/H;t}$ to reach 1. n' will be large.

In another case, let's note θ''_0 the large initial proportion of mighty individuals among the Hs. $\sigma''_{L/H;t}$ will be large and n'' the number of periods necessary for $\sigma''_{L/H;t}$ to reach 1 small.

Proposition 4 *if the initial proportion of powerful individuals within the H population is small, i.e. θ_0 small, then by the time $\sigma_{L/H}$ reaches 1, which occurs at period n , θ_n may be close to 0. If θ_n is close to 0, then the society moves back to a "group blind equilibrium".*

Let's note

$$p'_t = [1 - \frac{1}{2}\pi(1 - \pi)\sigma'_{L/H;t}\sigma'_{H/L;t}]$$

$$p''_t = [1 - \frac{1}{2}\pi(1 - \pi)\sigma''_{L/H;t}\sigma''_{H/L;t}]$$

$$\frac{\theta'_{n'}}{\theta''_{n''}} = (\frac{\theta'_0}{\theta''_0})(\frac{p'_1}{p_1})(\frac{p'_2}{p_2})(\frac{p'_3}{p_3})\dots(\frac{p'_n}{p_n})(p'_{(n+1)})(p'_{(n+2)})(p'_{(n+3)})\dots(p'_{(n')})$$

Let's note

$$U_{n'} = (p'_{(n+1)})(p'_{(n+2)})(p'_{(n+3)})\dots(p'_{(n')})$$

and rewrite

$$\frac{\theta'_{n'}}{\theta''_{n''}} = (\frac{\theta'_0}{\theta''_0})[(\frac{p'_1}{p_1})(\frac{p'_2}{p_2})(\frac{p'_3}{p_3})\dots(\frac{p'_n}{p_n})](U_{n'})$$

From proposition 3, we know that

$$\sigma'_{L/H;t}\sigma'_{H/L;t} < \sigma''_{L/H;t}\sigma''_{H/L;t}$$

Hence

$$(\frac{p'_t}{p_t}) > 1$$

We also know that $U_{n'}$ converges towards 0. For n' sufficiently large relatively to n'' (which is the case when θ'_0 is small and θ''_0 is large), $U_{n'}$ is close to 0. Thus, for θ'_0 small and θ''_0 large, $\frac{\theta'_{n'}}{\theta''_{n''}}$ is close to 0. This may occur in two ways: either $\theta''_{n''}$ has become infinitely large, or $\theta'_{n'}$ is infinitely small. The first case may not occur as θ'_t is a decreasing sequence whose maximum initial value is 1. Therefore, $\theta'_{n'}$ has become very small.

To put it all in a nutshell, if θ'_0 is sufficiently small, by the time $\sigma'_{L/H}$ reaches 1 that ensures a segregation equilibrium, which occurs at the period n' , $\theta'_{n'}$ will be close to 0, meaning that virtually no H will be powerful. The Hs will not differ from the Ls any more and the latter will not perceive additional value in building relationships with them. The parameter z would disappear and the situation would reach a group blind equilibrium. Segregation would not occur. On the other hand, should θ''_0 be large enough, the society may be trapped in a segregation equilibrium.

4.4 Characterization of the comparative evolution of the proportion θ , when θ_0 is large or small

This section illustrates the evolution of θ over time. We take as a base case $\theta_0 = 1$ and decline it for different values of θ_0 . Values are assigned to key parameters such as π , $\frac{\partial \sigma_{L/H;1}^i}{\partial z}$, $\frac{\partial \sigma_{H/L}}{\partial \sigma_{L/H}}$ and $\sigma_{L/H;0}$, $\sigma_{H/L;0}$ randomly. Note that for the declined cases, $\frac{\partial \sigma_{H/L}}{\partial \sigma_{L/H}}$, $\sigma_{L/H;0}$ and $\sigma_{H/L;0}$ are left unchanged (indeed variations in θ_0 do not affect the values of these parameters). By virtue of proposition 3, we know that if for $\theta_0 = 1$, $\frac{\partial \sigma_{L/H;1}^i}{\partial z} = x$, then for a case where $\theta_0 = y$, then $\frac{\partial \sigma_{L/H;1}^i}{\partial z} = xy$. Therefore, $\frac{\partial \sigma_{L/H;1}^i}{\partial z}$ is adapted in this manner for every variation of θ_0 made. The table below presents the values assigned to key parameters in the base case:

Base Case: $\theta_0 = 1$		
Variable	Value given ¹	Requirements
π	0.3	$\in [0; 1]$
$\frac{\partial \sigma_{L/H;1}^i}{\partial z} dz$	0.06	$\in [0; 1 - \sigma_{L/H;0}]$
$\frac{\partial \sigma_{H/L}}{\partial \sigma_{L/H}} d\sigma_{L/H}$	0.4	$\in [0; 1 - \sigma_{H/L;0}]$
$\sigma_{L/H;0}$	0.5	$\in [0; 1]$ and indifference between playing C and D
$\sigma_{H/L;0}$	0.5	$\in [0; 1]$ and indifference between playing C and D

Table below presents summary calculations for the case $\theta_0 = 1$. Segregation in this case is triggered at period $t = 9$ and at this time θ still represents 66%, meaning that power is still largely shared by members of the H group.

Case 1: $\theta_0 = 1$			
Time period	Proportion of powerful individuals	Probability to cooperate for an	
		L matched to an H	H matched to an L
t	$\theta_t = \theta_{t-1} [1 - \frac{1}{2}\pi(1 - \pi)\sigma_{L/H;t}\sigma_{H/L;t}]$	$\sigma_{L/H;t}$	$\sigma_{H/L;t}$
0	100.00%	0.50	0.50
1	96.92%	0.56	0.52
2	93.46%	0.62	0.55
3	89.64%	0.68	0.57
....
7	76.33%	0.92	0.67
8	71.40%	0.98	0.69
9	66.32%	0.00	0.00

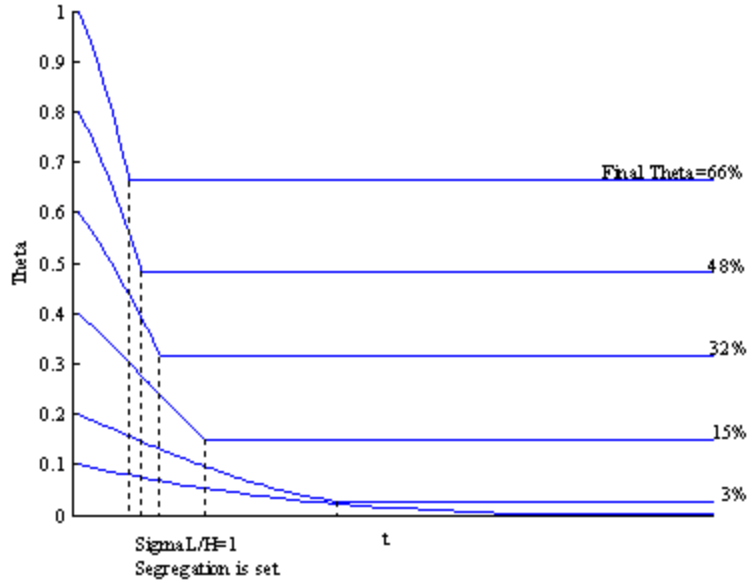
The table below illustrates the case for a small initial value of θ . Following our argument, the time needed for $\sigma_{L/H}$ to reach 1 is much larger (83 periods versus 9) and at the time segregation is triggered, the proportion of powerful individuals is so low that the H group doesn't differ anymore from the L group. A group blind equilibrium emerges.

¹Detailed calculations and additional simulations are available upon request

Case 2: $\theta_0 = 0.1$			
Time period	Proportion of powerful individuals	Probability to cooperate for an	
		L matched to an H	H matched to an L
t	$\theta_t = \theta_{t-1}[1 - \frac{1}{2}\pi(1 - \pi)\sigma_{L/H;t}\sigma_{H/L;t}]$	$\sigma_{L/H;t}$	$\sigma_{H/L;t}$
0	10.00%	0.500	0.500
1	9.73%	0.506	0.502
2	9.47%	0.512	0.505
3	9.21%	0.518	0.507
....
80	0.20%	0.980	0.692
81	0.19%	0.986	0.694
82	0.18%	0.992	0.697
83	0.16%	0.00	0.00

The graph below depicts the evolution of the proportion of powerful individuals within the H group for every period of time i.e. θ_t . For example in our base case of $\theta_0 = 1$, it is worth noticing that $\sigma_{L/H}$ reaches 1 and segregation is triggered when $\theta_t = 0.66$. On the other hand, when θ_0 is small enough, say 20% or 10%, $\sigma_{L/H}$ takes such a long time to reach 1, that θ_t becomes close to 0 and a group blind equilibrium is triggered. Indeed, as θ approaches 0, the H group does no longer differ from the L group and the situation is back to a group blind equilibrium.

Evolution of θ depending on its initial value



This figure illustrates our reasoning by showing that the initial proportion of powerful individuals within the H group is the main determinant of a segregation

equilibrium. This does not mean that segregation may be uprooted by changing power distribution within the H group. It has been shown in a previous section that once the segregation equilibrium is in place it is stable and robust. Our argument only helps at explaining why do some societies exhibit high degrees of segregation and other lower degrees of segregation. The reason advanced is the following: when a group takes over most of the power within a society, *provided* that this power is uniformly distributed within the dominant group a segregation equilibrium is triggered. However, *at the time of the takeover*, if only a minority captures this power within the dominant group, a group blind equilibrium appears. For segregation to be uprooted, intuition tells us that power need to be redistributed across the H and L group. We have not provided any proof of this as we leave this question to future research. This may explain why revolutions are the most common way to overthrow segregation.

5 Discussion and selected examples

Power distribution within one group determines whether a society moves toward or away from a segregation equilibrium. The term power should be taken in a broad acceptance, that is the ability for someone to give another person additional opportunities that are otherwise unreachable. We have shown that the process through which segregation occurs starts with a dependence of one group on another to gain access to additional opportunities. The deprived group will start looking for long-term relationships with the more powerful group who in turn will start setting up barriers. The caste system in India is a good example of the move from one equilibrium to another from many standpoints.

When the Aryans settled in the sub continent some 4 000 years ago they brought along a large corpus of religious texts: the Vedas. The first text codifying the caste system is known as Manu's law and it is considered by tradition as part of the Vedas. Many experts support the idea that Manu's code was written in a much later form of Sanskrit and is therefore more recent. Some anthropologists allege that such a segregationist frame was brought forward by the Aryans once their power over most of what is now India was totally secured. Therefore it seems that power grasp was prior to the set up of a segregationist ideology. This thesis remains nevertheless much controversial, especially as the text in question is still very much revered and considered as a revelation.

Much less controversial is the theory according to which the Aryans assigned a very low status to the submitted Indian natives, whose descendants are the actual outcasts. Indeed as the natives did not have the same religion as the invaders they were not included in the caste system (therefore labelled "outcasts") and assigned humiliating statuses and occupations. It seems that through this invasion, conquerors took over most of the powers and generated a segregated society. Some sociologists such as Dehejia (1993) support the idea that the caste system in India was quite flexible until the Medieval age when land granting replaced cash salaries. Land was progressively granted mostly to the higher castes, starting a feudalization process that made lower caste dependent on the

higher-ups. "Concurrent with the feudalization of India was the ossification of the once flexible caste system".

Another good example is the jāti fission process that has been widely observed and commented. Indian society is made of a very large number of groups the jātis, whose features are the prohibition of relationships especially strict endogamy rules, hierarchy and professional specialization. When questioned about his caste any Indian will refer to his jāti. One is born in a jāti and cannot leave it. It has been observed that as soon as a fringe of one jāti becomes wealthy or more powerful, this sub-group will adopt a number of customs including rules in allowed relationship that aim at improving its status. This new group will ban any relationship with the group they want to escape from, takes a new name and adopt customs that are more adequate to their desired status. This process is referred to as "Sanskritization" by sociologists. It seems that a decisive improvement in the scope of opportunities leads to a rejection of those who do not benefit from the same kind of perspectives.

The Sanskritization process could be considered as the materialization of the yearning by one group to gain association with another and as Srinivas (1955) noted, this can lead to a severe rejection and its corollary discrimination. "Discrimination against the Smiths occurs everywhere in peninsular India, possibly as a result of their attempts in the past to rise high in the caste hierarchy by means of a thorough Sanskritization of their customs". This confirms our result that segregation originates from the desire of one group to associate with another.

6 Concluding remarks

This paper aims at providing an analytical framework of a phenomenon, namely segregation, that is often considered as exogenous and intricate. We have tried to break up the process of segregation formation, drawing from this analysis two interesting points.

Firstly, segregation is not the result of psychological factors and does not come of the mighty group. Rather, it stems from a socioeconomic context in which one group seizes power. Power should be understood in a broad sense, that is to say the ability to give someone else additional opportunities that are otherwise unreachable. Dominated groups members are given little choice but to systematically cooperate with the mighty. A natural response to a potential partner's ensured cooperation is to reject the relationship. This is what the powerful group will do. The dominated group will best reply to this situation by in turn rejecting cooperation. A stable segregation equilibrium is set. Such an equilibrium is the product of strategic "microdecisions", as Schelling put it, that have little to do with prejudice, discrimination and other psychological factors. It is our belief that discrimination arises from segregation and not the opposite.

Secondly, segregation equilibria are triggered by power distribution within the mighty group at the time of power capture. It has been shown that whenever

power is widely shared by members of the dominant group, society may be trapped in a segregation equilibrium. On the contrary, when power is grasped by only a small fraction of the powerful group, society may converge towards a mixed equilibrium. Please note that an economic definition of power has been taken so far, although it is believed that the analysis could apply to any kind of power such as religious and symbolic ones.

One point should be made clear. One of the points made in this article is to explain why do some societies move towards a segregation equilibrium and others do not. Power distribution within the dominant group *at the time of power takeover* appears as the trigger. Once segregation is set, it is a remarkably stable equilibrium. Variations in power distribution within the dominant group as well as in the perception by the Ls of the value of relationships with Hs will not modify the equilibrium. No group will have any interest in raising its probability to be cooperative, knowing that the other group's probability is null. As a consequence, it may be quite difficult to explain what makes a society move from a segregation trap to a mixed equilibrium. This question is left open.

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