

# Poincaré, Henri (1854–1912)

## French mathematician and scientist

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Poincaré, Jules Henri (b. Nancy, France, 29 April 1854; d. Paris, 17 July 1912), mathematics, celestial mechanics, theoretical physics, philosophy of science. For the original article on Poincaré by Jean Dieudonné see Volume 11.

Historical studies of Henri Poincaré's life and science turned a corner two years after the publication of Jean Dieudonné's original DSB article, when Poincaré's papers were microfilmed and made available to scholars. This and other primary sources engaged historical interest in Poincaré's approach to mathematics, his contributions to pure and applied physics, his philosophy, and his influence on scientific institutions and policy. The result of these new sources and updated historiography is a new Poincaré: if Poincaré's outstanding achievements in mathematics ensure his prominent position in the history of this subject, his corpus and intellectual legacy are known to extend well beyond mathematics proper, touching the core methods of theoretical physics, and central tenets of the philosophy of science. Increased recognition of the breadth of Poincaré's activity, from geodesy and electrical engineering to algebraic topology and the philosophy of space and time, is itself a prime result of Poincaré studies. A brief summary follows of the ways these studies have revised or enriched historical understanding of Poincaré's life and work.

### Early Career

Consider, to begin with, Jean Dieudonné's evaluation of the young Poincaré's spotty education in higher mathematics, gleaned from Poincaré's published correspondence with Lazarus Fuchs and Felix Klein and since then confirmed, on

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one hand, by studies of Poincaré's qualitative theory of differential equations, and on the other hand, by analyses of the manuscripts submitted by Poincaré for the *Grand prix des sciences mathématiques* in 1880, and discovered by Jeremy Gray a century later. These manuscripts, written between 28 June and 20 December 1880, show in detail how Poincaré exploited a series of insights to arrive at his first major contribution to mathematics: the discovery of the automorphic functions of one complex variable (called "Fuchsian" and "Kleinian" functions by Poincaré). In particular, the manuscripts corroborate Poincaré's introspective account of this discovery (1908), in which the real key to his discovery is given to be the recognition that the transformations he had used to define Fuchsian functions are identical with those of non-Euclidean geometry.

The manuscripts of 1880 also shed light on the origins of Poincaré's conventionalist philosophy of geometry. Even at this early point in his career, Poincaré understands geometry to be the study of groups of transformations. Such a view recalls Klein's Erlangen program (1872), but where Klein's approach was projective and hierarchical, that employed by Poincaré in 1880 was entirely metrical, and free of hierarchy, rendering unlikely any direct influence of Klein's Program. The influence of Eugenio Beltrami, however, is apparent in Poincaré's disk model of hyperbolic geometry, a model he later employed in the service of his conventionalist philosophy of geometry.

## **Foundations of Geometry**

Poincaré was elected to the geometry section of the Paris Academy of Sciences on 31 January 1887 (at age thirty-two), and that same year published his first paper on the foundations of geometry. Impressed with the writings of Sophus Lie and Hermann Helmholtz on the so-called Riemann-Helmholtz-Lie-Poincaré problem of space, Poincaré characterized plane geometries by considering intersections of a plane with a certain quadric, and formulating a common set of axioms. In this way, "straights" and "circumferences" of hyperbolic geometry are made to correspond, for example, to straights and circumferences (circles) of Euclidean geometry, and the available theorems depend on the chosen quadric. (He later popularized this insight in terms of a "translation dictionary" for Euclidean and non-Euclidean geometry). Extending his purview to the geometry of space, Poincaré held hyperbolic geometry and Euclidean geometry both to be adequate to the task of describing physical phenomena. Darwinian evolution had provided humans only with the general notion of a group; consequently, if humans regularly employed Euclidean geometry instead of hyperbolic geometry, this was by virtue of the simplicity and convenience of the former, the motion of solids corresponding roughly to the Euclidean group. Experience provided the "occasion" for this choice to be made, in that employment of Euclidean geometry was an outcome

contingent upon the behavior of light rays and solids on Earth.

A few years later, Poincaré explained further that the choice of hyperbolic geometry entailed non-standard laws of physics, thereby rendering his view equivalent in essentials to that of Helmholtz (1876). For Poincaré, the empirical equivalence of the two possible points of view – Euclidean geometry plus ordinary physics, or hyperbolic geometry plus some unspecified, alternative physics – ruled out an empirical foundation of the geometry of physical space. He regretted that Helmholtz – the champion of methodological empiricism – had not made this point clear. Few scientists found compelling Poincaré’s extreme view of the geometry of physical space, but philosophers (including Paul Natorp, Aloys Müller, Moritz Schlick, and Rudolf Carnap) were swayed by the clever arguments advanced in its favor, which shaped later debates on the conventionality of the space-time metric in general relativity.

Poincaré’s contributions to relativity theory have given rise to priority claims on his behalf, because he presented a theory mathematically indistinguishable from that of Albert Einstein, four weeks earlier than Einstein. In fact, Poincaré made a crucial step in 1900 toward Einstein’s 1905 redefinition of physical time and space, when he realized that the validity of the principle of relativity for electromagnetic phenomena depended on a certain definition of the time coordinate in uniformly-moving systems (H.A. Lorentz’s “local time”), realized by an exchange of light-signals between co-moving observers relatively at rest. Like Einstein, Poincaré elevated the principle of relativity to a postulate in 1905; unlike Einstein, he retained the notion of a luminiferous ether (thus obviating the need for Einstein’s light postulate). In addition, he characterized the Lie-algebra of the Lorentz group, and derived the first two Lorentz-covariant laws of gravitation. Along the way, Poincaré provided the four-vectors for Hermann Minkowski’s four-dimensional spacetime theory (1908), but deplored the latter’s Einsteinian view of space and time coordinates, advocating in its place an interpretative convention equivalent to the postulation of Galilean spacetime.

In practice, Poincaré applied the principle of relativity as one prong of a two-pronged method for the discovery of relativistic laws: whatever laws happen to describe the behavior of natural phenomena, these laws are required to be covariant with respect to a certain group of transformations. The first prong left him with an infinite number of candidate laws, all covariant with respect to the Lorentz group. The second prong placed a constraint on the search domain: for velocities that are small with respect to that of light, the relativistic law should correspond to that of classical physics. While Poincaré applied his covariance-plus-correspondence method only to the case of Lorentz-covariant laws of gravitation, his approach is applicable to any group of transformations, and any principle of correspondence. Upon further formal elaborations from Minkowski and Arnold Sommerfeld, Poincaré’s method became a touchstone of theoretical physics.

## Work on Trajectories

Interest in Poincaré's innovative contributions to the theory of dynamical systems has expanded, with the rise of chaos theory in the 1970s, and the archival discovery of the first version of Poincaré's submission for the King Oscar II of Sweden competition to solve the  $n$ -body problem in celestial mechanics (1889). The published version of Poincaré's paper on the restricted three-body problem (1890) contains the first mathematical description of a chaotic trajectory in a dynamical system, or what Poincaré referred to as a doubly-asymptotic (and later, homoclinic) trajectory, but this extraordinary result is absent from the original, prize-winning version of the paper. In its place is a hastily-written corollary concerning unstable periodic solutions of differential equations, the flaw in which Poincaré discovered while preparing his manuscript for publication. In a moment of inattention Poincaré convinced himself of the convergence of the power series expansion of certain periodic solutions, which pointed to well-behaved asymptotic trajectories. In fact, the published version shows that the series in question belongs to the class of asymptotic series he had defined in 1886 (known later as a Poincaré expansion), and that in light of his recurrence theorem, the behavior of the corresponding trajectories becomes quite complicated (or in modern terms, chaotic). The significance of these doubly-asymptotic trajectories was not generally recognized at first, perhaps in part because Poincaré soft-pedaled his discovery, the child of an oversight in the original submission that was a huge embarrassment both for him, and for those who awarded him the prize (Karl Weierstrass, Charles Hermite, and Gösta Mittag-Leffler).

While the fate of doubly-asymptotic trajectories, neglected for decades, is extreme, relative neglect befell other results obtained by Poincaré in algebraic topology and mathematical physics. There were several reasons for this, including an allusive writing style, and a lack of follow-through. As his former student Émile Borel put it in 1909, Poincaré was "more a conqueror than a colonist". A self-made mathematician in the French style, Poincaré took on no doctoral students, formed no school, and delivered lectures incomprehensible to all but a handful of auditors. With the aid of student note-takers, however, he published a complete series of lectures on mathematical physics and celestial mechanics, the technical sophistication and logical coherence of which were much admired; they effectively disseminated Poincaré's mathematical methods and problem-solving style.

## Physics and Philosophy

Significantly for the historical development of electrodynamics, Poincaré's 1890 lectures on James Clerk Maxwell's theory of electromagnetism were the first to appear in German and the second to appear in French (other than translations of

Maxwell's 1873 *Treatise*). Poincaré emphasized the Scottish physicist's abstract and powerful Lagrangian approach and proved his remark that if a phenomenon admits of one mechanical explanation, it admits of an infinity of such explanations. He also transformed Maxwell's single-fluid theory into a two-fluid theory, to the irritation of certain Maxwellians, and misrepresented its notions of charge and current. Nonetheless, his streamlined, inductive approach appealed to physicists in Britain as on the Continent, and furthered the cause of Maxwell's theory and British abstract dynamics.

For the general reader, Poincaré explained the cosmic consequences of his discoveries in the dynamics of systems, for the foundations of the second law of thermodynamics, the question of determinism, and the stability of the Solar system. Starting in the 1890s, Poincaré laid out in dozens of popular articles a coherent scientific worldview informed by contemporary research in mathematics and physics, and a familiarity with the writings of the leading physicist-philosophers: Maxwell, Helmholtz, Ernst Mach, and Heinrich Hertz. To Poincaré's surprise, his epistemological reflections were enrolled by Catholic intellectuals in an effort to undermine scientific authority. This placed him in the delicate position of having to explain the value of a science whose pretension to absolute truth he had systematically destroyed.

Technological advances issuing from fundamental discoveries in physics were readily apparent in Poincaré's time, some of which he celebrated (including wireless telegraphy, and the use of x-ray images in medical diagnostics). Nonetheless, he located the value of science not in its utility, or even in its power to relieve human suffering, but in its capacity to awaken the intellect: "to see," or at least, someday, "to let others see." The unity and progress of science were linked for Poincaré; the unity derived not from any shared method of investigation, a wholly illusory notion, but from the shared mathematical structures of the physical world. Progress, on the other hand, was realized by overcoming the obstacles inherent in the methodological disunity of science. In practice, overcoming such obstacles meant adopting the most general point of view possible of the problem at hand, an approach he employed regularly himself.

While scientific progress in Poincaré's sense is possible, it is by no means inevitable, nor does it lead to absolute truth in the long run. Objectivity itself is a social construct for Poincaré: without discourse, he wrote, there can be no objectivity. All science is not discourse, although some of Poincaré's contemporaries understood him to be a nominalist. Laws describing natural phenomena are elaborated pragmatically, in the sense that scientists' choices remain free, but are guided by experience. Mathematics is a science apart for Poincaré because the structure of the natural numbers is intuitively given (in a non-Kantian sense), while the natural sciences depend on a theory of measurement requiring the real numbers and arithmetical operations. Progress in mathematics is itself possible

due to the principle of induction, which Poincaré understood to be a synthetic a priori judgment. Attempts to provide a logical foundation for mathematics by Bertrand Russell and Alfred North Whitehead were doomed to failure, Poincaré argued, because their constructions necessarily involved circular reasoning.

## Reputation

The brilliance of Poincaré's contributions to mathematics, celestial mechanics, and mathematical physics was recognized by scientific academies across Europe and in the United States, most of which counted Poincaré as a foreign member by the end of the nineteenth century. The mathematical community was unanimous in its recognition of his accomplishments, the combined depth and breadth of which none could reasonably hope to emulate. This intellectual ascendancy had no counterpart in the French institutional domain, where Poincaré was visibly less skilled than some of his peers in securing his objectives. At the height of his scientific authority in 1907, he stood for the position of Perpetual Secretary for the Physical Sciences at the Paris Academy of Sciences, with the backing of members of the physics section. Faced with opposition from the chemistry and mineralogy sections, he withdrew his candidacy, in order to avoid the humiliation of a loss, or a narrow win. In return for his retreat, Poincaré's opponents endorsed his candidacy to join the Académie Française, where he was elected in 1908.

A second setback occurred in 1910. Following a highly-concerted campaign, Poincaré garnered a record number of nominations for the Nobel prize in physics, for the most part in view of his advances in partial differential equations of mathematical physics. Support for his candidacy issued largely from France and Italy, with the British holding back, partly out of concern over a second *de facto* extension of the prize domain, this time to cover work in mathematical physics, only one year after it had been awarded for applied physics (wireless telegraphy). In the end, the Royal Swedish Academy chose to recognize the work of another theorist, Johannes Diderik van der Waals.

While he did not win the Nobel prize, Poincaré cut an authoritative figure among physicists, even in areas of physics he had ignored, such as the theory of black-body radiation. In the fall of 1911, he took an active part in the First Solvay Council, dedicated to the discussion of problems in molecular and kinetic theory, along with twenty of Europe's leading experimental and theoretical physicists. A few weeks after the meeting, inspired by what he had learned, he proved the quantum hypothesis to be a sufficient and necessary condition for Planck's law.

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For a complete bibliography of Poincaré's writings, a calendar of his correspondence (with annotated transcriptions and digitized images), and a list of secondary sources, consult the Henri-Poincaré Papers website.

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