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LABORATOIRE D'ECONOMIE DE LA PRODUCTION  
ET DE L'INTEGRATION INTERNATIONALE

UMR 5252 CNRS - UPMF

## **CAHIER DE RECHERCHE**

N° 11

# **Optimal household energy management and economic analysis : from sizing to operation scheduling**

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# Optimal household energy management and economic analysis: from sizing to operation scheduling

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## **Abstract**

The study presented in this paper takes part in a project aiming to increase the value of solar production for residential application with a medium-term vision where preferential solar energy subsidies will decrease before to disappear. This study is dedicated to propose and develop optimal energy architecture at supply side, a multi-source system based on photovoltaic (PV) solar energy connecting to main electrical network, taking further into account the effectiveness of intelligent demand side management. To investigate this issue, a method of optimal supplying system sizing and household energy management has been developed. This method, which has been formulated employing Mix Integer Linear Programming (MILP), enables the calculation of the appropriate configuration for power supply system and the optimal operation control to be applied. Using a Net Present Value (*NPV*) and Probability Index (*P.I*) basis, the economic analysis allows estimation of the viability of the proposed system under different factors of influence such as renewable energy policies, technology evolutions leading to cheaper installed PV module cost and deregulated electricity market. Simulation results show that, the solution makes it possible for PV power to be significantly valued by the customers without subsidized measures.

*Keywords:* connected-grid PV system, battery storage, sizing optimization, energy management, MILP

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## Nomenclature

### Model parameters

$\eta_B$	Battery storage efficiency
$C_B$	Coefficient cost of storage system, [€/kWh]
$c_g(t)$	Grid energy price, [€/kW]
$c_{gn}$	Coefficient of grid connection fix cost, [€/kW]
$C_{ond}$	Coefficient cost of inverter, [€/kW]
$C_{PV}$	Coefficient cost of peak power rate of PV module, [€/kWp]
$c_s(t)$	Sellback price of local production, [€/kWh]
$i$	Discount rate, [%/year]
$k$	Study year, [year]
$r_{ch}$	Charge coefficient rate, [kW/h]
$r_{dch}$	Discharge coefficient rate, [kW/h]
$SP(t)$	Spot price, [€/kWh]
$t$	Calculation step time, [hour]
$T$	Study period, equal to lifetime of installation, [year]

### Decision variables

$\alpha(t)$	Decision binary variable, $\alpha(t) = 1$ if the battery is in the charging mode, $\alpha(t) = 0$ if the battery is in the discharging mode
$\beta(t)$	Decision binary variable, $\beta(t) = 1$ if the system imports grid energy, $\beta(t) = 0$ if the system purchases its PV production
$\alpha(t)$	Decision variable, used to translate the absolute relation into linear representation
$P_{bin}(t)$	Charge power consign [kW]
$P_{bout}(t)$	Disharge power consign [kW]
$P_g(t)$	Consumed grid power [kW]
$P_L(t)$	Electrical demand prevision [kW]
$P_{LP}(t)$	Consumed power by controllable loads
$P_{NLP}(t)$	Consumed power by non-controllable loads
$P_{PV}(t)$	PV available power [kW]
$P_s(t)$	PV power to be used locally [kW]
$SOC(t)$	Temporal state of charge [kWh]
$z(t)$	PV power to be injected to network [kW]

## 1. Introduction

In Europe, the rapid growth connected-grid PV systems in the residential sector over the last year have been promoted by government supported programs with considerable investment subsidies [1]. By this way, the PV power producers find it currently beneficial to sellback maximum of his solar production to electricity utilities. This is especially convenient for short-time frame when PV penetration rate in electrical network still

remains marginal or to be applied in the small scale applications.

However, actual policies to develop the use of renewable energies would evolve. Feed-in tariffs could be lower to lessen the cost of renewable energies promotion. They would be then disappeared (as in Japan, for example) or replaced by another system less attractive (green certificates or market component as in Spain [2]). Given that context, it will be difficult to justify the real interest of this operation mode of PV connected-grid system as currently.

However, the potential benefits of PV system are quite large. The core idea is that PV applications could impact both supply-side and demand-side issues. Aside being an alternative for the housing power supply having lower environment impacts, PV also serve as a vehicle for triggering efficient energy utilization by influencing consumer awareness of energy saving.

Another observation and analysis of energy markets changing also make connected-grid PV system interesting to study. The liberalization process leads to the end of regulated tariffs (toward 2011 in France). So, market price contracts would be generalized to all consumers, with certainly an indexation on volatile spot prices. A consumer could have incentives and benefits to develop PV generation to limit the price risk volatility. This is, for a household consumer, a mean to hedge it against high prices.

Lastly, energy vulnerability of several European countries and climate change policies imply to develop cleaner production and demand-side management (rationale use of energy, energy positive houses as in Great-Britain where all new houses in 2016 would be zero Greenhouse Gas emission [3]).

These issues at stake ask for investigating the innovative energy architecture for housing applications. It is expected that the building of tomorrow should be a *positive, green and intelligent* element. The project sustaining this paper, MULTISOL, has the ambition to design an economical and technical efficient framework for household energy management. A new architecture is proposed with: at supply side, a PV-based multi-source system; and at demand side, source and load co-management.

## **2. Model framework description**

In this project, the works to design and demonstrate the multi-sources and loads co-management system architecture has been in progress. We suggest separating the components between the house and electric network into two connection boards (Fig. 1), [4]. All production means such as PV generator, battery, network, and possible diesel or others complementary sources, are connected in the "Power production control board" to supply loads via the traditional electric delivery box called "Power delivery control board". A coupling and

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\* STC is specified at condition air mass of 1.5, radiation of 1 kW/m<sup>2</sup>, and ambient temperature of 25°C.

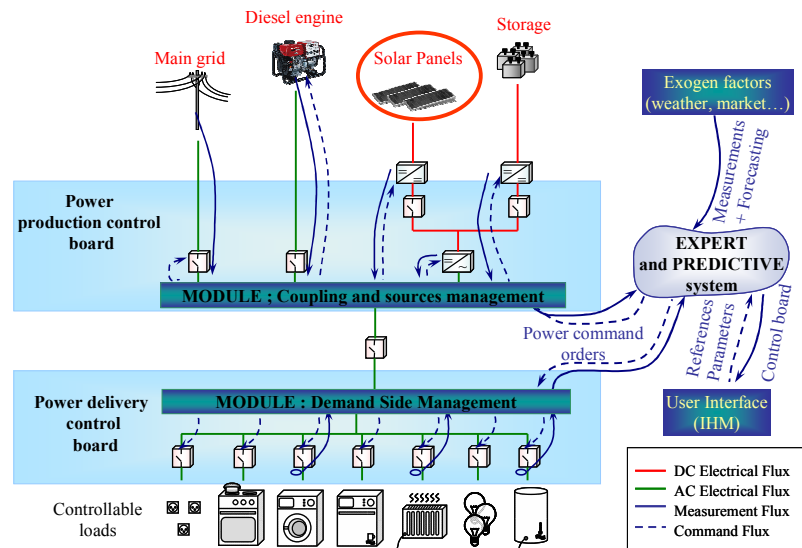


Fig.1. Optimal multi-sources and loads co-management system architecture

multi-sources management module integrated in the production board must allow the optimal control of the power-flows obtaining according as loads. A demand side management module would rather be placed in the electric delivery box in order to facilitate the control - command of loads. Besides, power electronic interfaces (inverters) play an important role in such system because the power-flow control of DC sources is realized by their intermediary. Measurement and telecommunication equipments are also indispensable for control-command system. The "Expert and Predictive System" is the intelligence of system that receives information (weather forecast, electricity market, user behavior, measurements...), calculates the optimal strategy control and sends command order to equipments, etc. By this way, this architecture is modular with standardized interfaces facilitating the interchangeability and the evolution of the components of system.

In the design of such system, the optimal sizing is an important and challenging task

In the literature, sizing problem for connected-grid PV system consists more often in optimizing of PV/inverter ratios as function of inclination, orientation, inverter characteristics and costs ([5], [6]) to reduce energy losses and increase efficiency of system. Others authors [7] proposed a methodology that defines the most appropriate size of the PV generator for building by optimizing the profitability and amortization of system. Fernández-Infantes and al. [8], based on a pre-defined PV system size, seek for a design method evaluating almost influences parameters. These methods all consider connected-grid PV systems in the favorable conditions where feed-tariffs on selling solar energy are applied. Thereby, they divide optimization design into two stages (size procedure and techno-economical analysis) carried out separately where sizing problem is simplified (more often, the PV generator's size is pre-defined and limited by available economic funds or no technique reasons). This makes the global optimal compromise solution may not be achieved.

In almost cases, the energy management has not been considered yet in sizing problem. The sizing problem for applications with PV integration in building requires therefore further investigations. We propose to incorporate in sizing problem the optimal operation management that the main objective is to maximize the

benefit for system's owner. A result analysis method is also given. Once built up, this method will provide a tool for user to automatically dimension his installation and analyze the economic viability of system under different conditions.

### 3. Mathematical problem formulation

Sizing optimization problems presented herein are formulated using a Mix Integer Linear Programming (MILP) algorithm which requires all mathematical representations (objective function and constraints) expressed in standard form, [9]:

Minimize  $f(x)$

Subject to:  $Ax \leq b$

$$A_{eq} \cdot x = b_{eq}$$

$$lb \leq x \leq ub$$

With:  $x$  are the variables (continue, binary or integers)

$A, A_{eq}$  are matrixes

$f, b, b_{eq}$  are vectors

The  $x$  vector (unknown variables) includes:

- *sizing variables* : peak power of PV array ( $P_{PVp}$ ), battery storage capacity ( $S_{max}$ ), grid connection contracted rate power ( $P_{gmax}$ )
- *hourly operation variables*: charging power set-point  $P_{bin}(t)$ , discharging power set-point  $P_{bout}(t)$ , consumed grid power  $P_g(t)$ , surplus power  $z(t)$ , consumed controllable load  $P_{LP}(t)$ , and consumed non-controllable load  $P_{NLP}(t)$
- *decision binary variables*:  $\alpha(t)$  used to distingue charging and discharging mode in battery operation model,  $\beta(t)$  used to define the mode for exchanging energy with the network.

Each variable is limited to its lower ( $lb$ ) and upper ( $ub$ ) bounds.  $A, b, A_{eq}, b_{eq}$  represent the inequality and equality equation constraints of  $x$ .  $f$  is the vector of objective function.

The following flowchart shows the mains steps of optimization algorithm:



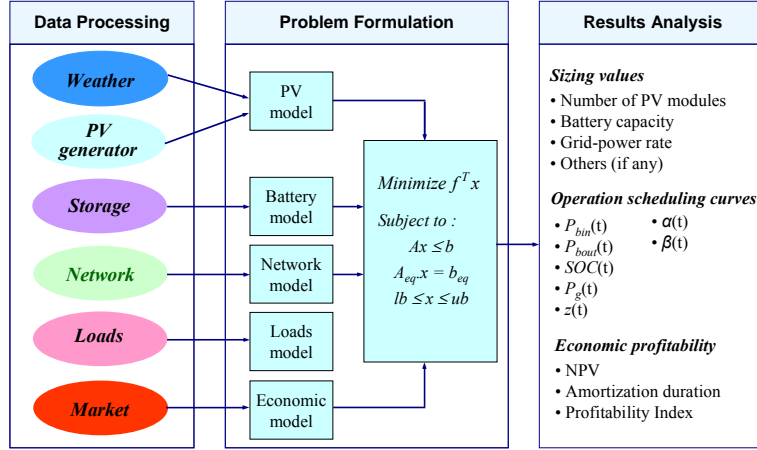


Fig. 1. Overview of sizing and multi-sources management optimization algorithm

### 3.1 Objective function

The main objective of the sizing optimization described in this section is to maximize the benefit of the system. As others studies on economic PV profitability [10], the Net Present Value or Present Value of Net cash flows (*NPV*) method is used. This method expresses how much value an investment will result in by measuring all cash flow overtime back towards the current point in present time. With a particular project, if *NPV* is a positive value, the project could be amortized and add value for investor; if not the project is profitless and subtract the value for investor. This method is especially useful to evaluate the profit of a project and also to compare available investment solutions.

By this way, the objective of sizing problem is to maximize the *NPV* over the lifetime period of installation (*T*) which is always chosen normally as the lifetime of PV panel (20 to 25 years). This function can be formulated as the difference between initial investment and the amount of cash flow of each year discounted back to the initial year as shown in the formula (1):

$$\text{Maximize: } NPV = -I_o + \sum_{j=1}^T \left( \frac{IT_j - CT_j}{(1+i)^j} \right) = -I_o + \sum_{j=1}^T \left( \frac{CF_j}{(1+i)^j} \right) \quad (1)$$

*NPV* includes the following cost items:

- $I_o$  : Initial investment
- $j$  : year of cash flow
- $CT_j$  : running cost of year  $j$
- $IT$ : income of year  $j$
- $CF_j$  : present value of cash flow for year  $j$
- $i$  : annual discount rate

To simplify the calculation and because the operation and maintenance cost of system components (PV, inverter, battery) is negligible before others costs (from 1 - 2%), only the replacements of equipment and the purchasing of grid energy is counted in the running cost.

$$I_o = C_{PV} \cdot P_{PVp} + C_{inv} \cdot P_{PVp} + C_B \cdot S_{max} \quad (2)$$

$$CT_j = C_{gn} \cdot P_{gn} + \sum_{t=1}^{8760} P_g(t) \cdot c_g(t) + \lambda_{inv} \cdot C_{inv} \cdot P_{PVp} + \lambda_B \cdot C_B \cdot S_{max} \quad (3)$$

where:

$\lambda_{inv} = 1$  if replacement of inverter is needed,  $\lambda_{inv} = 0$  otherwise

$\lambda_B = 1$  if replacement of battery is needed,  $\lambda_B = 0$  otherwise

The first term in (3) corresponds to the fix cost of grid connection, the second stands for variable cost of purchasing grid energy, the third and the fourth are the replacement cost of inverter and battery, respectively.

$$IT_j = \sum_{t=1}^{8760} (P_L(t) - P_g(t)) c_g(t) + C_{gn} \cdot \Delta P_{gn} + \sum_{t=1}^{8760} z(t) \cdot c_s(t) \quad (4)$$

The two first terms in (4) are the expected saving on the consumption of grid power and the fix cost related to grid connection. The last term corresponds to the income from selling PV production to network.

A typical cash-flow characteristic of this multi-sources system can be shown in the Fig. 3. The initial investment of the system is completely made at time 0. So, on the lifetime of the installation T, the system is profitable if the sum of future discounted cash-flows is higher than the initial investment. The breaking point indicates that investment has been already amortized and PV installation could now be profitable. The period time from initial point (initial year) to breaking point is the required return on investment duration (or pay-back period). Obviously, the shorter amortization duration is obtained, the more system is economically interesting. The downward jumps on the *NPV* curve are an increase in cost investment because of the necessity to replace part of the installation (batteries, inverters). For the considered year, this replacement decreases the linked cash-flow and thus *NPV*.

Analysis based on *NPV* method has two advantages. Firstly, it clearly points out if project is profitable or not (positive *NPV* is sufficient to uncouple investment). Secondary, it indicates which time initial investment is profitable. But *NPV* rule cannot explicitly show to decision maker how much financial attractiveness of the proposed solution is. So, we complete with another index that attempts to identify the relationship between the *Benefits - Costs* through a ratio, called *Profitability Index (P.I)*, calculated as:

$$P.I = \frac{\sum_{j=1}^T \left( \frac{CF_j}{(1+i)^j} \right)}{I_o} = \frac{NPV + I_o}{I_o} = 1 + \frac{NPV}{I_o} \quad (5)$$

Logically, a ratio of 1.0 is the lowest acceptable measure on the index. The value smaller than 1.0 indicates that the project is profitless ( $NPV < 0$ ). The higher value is, the more profit created by investment. For example, calculation gives  $P.I = 1.5$ , it means  $NPV = 0.5$ , and every unit invested can create a value of 1.5.

### 3.2 Components' models specifications and constraints

#### 3.2.1 PV generation model

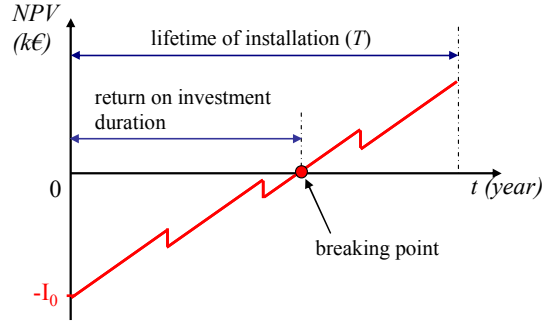


Fig. 2. Typical Net Present Value ( $NPV$ ) of multi-sources supplying system based on solar energy

The available PV energy production is calculated based on the meteorological information for an installation at a specific geographical location and PV panel characteristics. The calculation of PV production is performed in two steps: calculation of DC power generated by each PV module, and calculation of total AC power generated by all installed PV modules.

*Step 1:* Calculation of DC power generated by each PV module

The DC power generated by each PV module is calculated based on the ampere-voltage characteristic provided by manufacturer. Three points are given at Standard Test Conditions(\*) (STC): short-circuit point ( $I_{sc}, 0$ ), open-circuit point ( $0, V_{oc}$ ) and maximum operation condition point ( $I_{mpp}, V_{mpp}$ ). The equivalent circuit current  $I$  can be expressed as a function of the module voltage  $V$  as follows, [11]:

$$I = I_{sc} \cdot \left\{ 1 - k_1 \cdot \left( e^{k_2 \cdot V^m} - 1 \right) \right\} \quad (6)$$

$$k_1 = 0.01175; \quad k_2 = \frac{k_4}{V_{oc}^m}; \quad k_4 = \ln\left(\frac{1+k_1}{k_1}\right); \quad m = \ln\left(\frac{k_3}{k_4}\right) / \ln\left(\frac{V_{mpp}}{V_{oc}}\right); \quad k_3 = \ln\left\{\frac{I_{sc} \cdot (1+k_1) - I_{mpp}}{k_1 \cdot I_{sc}}\right\}$$

The equation (6) is only applicable on a particular irradiance level  $G$  and cell temperature,  $T_c$ . When radiance and ambient temperature change, the change in above parameters can be calculated as follows:

$$\Delta I = \alpha_{scT} \cdot \left( \frac{G}{G_{STC}} \right) \cdot \Delta T_c + \left( \frac{G}{G_{STC}} - 1 \right) \cdot I_{SC,STC} \quad (7)$$

$$\Delta V = -\beta_{ocT} \cdot \Delta T_c - R_s \cdot \Delta I \quad (8)$$

$$\Delta T_c = T_a + \frac{G}{800} \cdot (NOCT - T_{a,ref}) - T_{STC} \quad (9)$$

$$P_{module} = (V_{STC} + \Delta V) \cdot (I_{STC} + \Delta I) \quad (10)$$

where:

- $\alpha_{scT}$  short-circuit current temperature coefficient
- $\beta_{ocT}$  open-circuit current temperature coefficient
- $R_s$  serie resistant in the module equivalent circuit
- $T_a$  ambient temperature

<sup>§</sup> For more informations about research programs on PV, see [www.industrie.gouv.fr](http://www.industrie.gouv.fr) for French programs and <http://www.bulletins-electroniques.com/actualites/43484.htm> for German programs.

*NOCT* Normal Operating Cell Temperature, specified to be 45°C  
(at condition: irradiance  $G = 0.8 \text{ W/m}^2$ , ambient temperature  $T_a = 20^\circ\text{C}$ , wind speed = 1 m/s)

*Step 2*: Calculation of AC power generated by the PV generator

The AC power generated by PV generator is the sum of production of all installed modules.

$$P_{PV} = N_{modules} \cdot P_{module} \cdot \eta_{inverter} \quad (11)$$

where  $\eta_{inverter}$  : efficiency of inverter

By this way,  $P_{PV}$  is the upper limit of the available power produced by PV generator.

### 3.2.2 Battery storage model

The lead-acid batteries are the most used in PV application. The storage system is known as a flexible element of multi-sources system. It can store the total or surplus of production which is not used locally and provide energy when needed. On the other hand, battery storage system constitutes a weak point due to short lifetime period which are strongly influenced by many factors relating to the way it is operated such as: discharge rate, partial cycling, charge factor, temperature ...etc. Therefore, two parts will be considered in storage system modeling: operating model and ageing model.

#### Operation model

The battery storage system is characterized by its energy capacity ( $S_{max}$ ), battery efficiency ( $\eta_B$ ), charging/discharging power capacity ( $P_{bin} / P_{bout}$ ). We also use the battery charging and discharging rate coefficients ( $r_{ch} / r_{dch}$ ) to define the maximal charging/discharging power capacity for each step time. Knowing that charging and discharging are two processes quite independent and cannot be carried out at the same time, it is necessary when the battery charges the discharging power must be null and contrarily. A new variable  $\alpha(t)$  is added, [12]:

$$\begin{cases} 0 \leq P_{bin}(t) \leq \alpha(t) \cdot \frac{S_{max} \cdot r_{ch} \cdot \eta_B}{\Delta t} \\ -\frac{S_{max} \cdot r_{dch} \cdot \frac{1}{\eta_B}}{\Delta t} \cdot (1 - \alpha(t)) \leq P_{bout}(t) \leq 0 \end{cases} \quad (12)$$

$$\Rightarrow \begin{cases} \text{if } \alpha(t) = 1 \rightarrow \left\{ \begin{array}{l} 0 \leq P_{bin}(t) \leq \frac{S_{max} \cdot r_{ch} \cdot \eta_B}{\Delta t} \\ P_{bout}(t) = 0 \end{array} \right\} \rightarrow \text{battery is charging} \\ \text{if } \alpha(t) = 0 \rightarrow \left\{ \begin{array}{l} P_{bin}(t) = 0 \\ -\frac{S_{max} \cdot r_{dch} \cdot \frac{1}{\eta_B}}{\Delta t} \leq P_{bout}(t) \leq 0 \end{array} \right\} \rightarrow \text{battery is discharging} \end{cases}$$

The relationship between storage level (*SOC*) and power flow in or out of the battery at any step time is:

$$SOC(t+1) = SOC(t) + P_{bin}(t) - P_{bout}(t) \quad (13)$$

To ensure the security, battery must be operated in a specific rang defined by maximal and minimal limit of charge or discharge ( $SOC_{max}$  and  $SOC_{min}$ ):

$$SOC_{min} \leq SOC(t) \leq SOC_{max} \quad (14)$$

### Ageing model

The cumulative ampere-hour (Ah) throughput or Ah throughput method is used as basis for calculating battery ageing. This method [13-16] assumes that there is a fixed amount of energy (called *Ah throughput*) can be cycled through a battery before it requires replacement (regardless of the deep of individual cycles or any other specific parameters to the way the energy is stored in or left out of the battery). The estimated Ah throughput is derived from the depth of discharge versus cycles to failure curve provided by manufacturer:

$$Ah \text{ throughput} = Average \left\{ (S_{max} \cdot DoD_k) \cdot CF_k \right\}_{x\%}^{y\%} \quad (15)$$

where:

$k$ : battery operating zone (specified depth of charge ranges between  $x\%$  to  $y\%$ )

$DoD_k$ : depth of charge  $k$

$CF_k$ : cycles to failure (or number of cycles) if battery operates always at specific depth of charge  $k$

An approximation is made by making the assumption that the product of the number of cycles by the depth of discharge is constant. So the program can use the cycle life at a particular DOD, such as 50% to calculate the Ah throughput cycled through the battery.

From these underlying assumptions, we can deduce the calculation of Ah throughput by:

$$Ah \text{ throughput} = S_{max} \cdot DoD_{50\%} \cdot CF_{50\%} \quad (16)$$

For example, a cycle life of 1050 of a 2.1 kWh battery at 50% DoD means that whenever 1102.5 ampere-hour cycled through the battery, the battery is considered as used and needed to be replaced.

We notice that this simplistic approach minimizes the complexity of the problem considerably. Battery life could then be estimated by the only accounting of exchanged energy thus avoiding the detection and the counting of the effective cycles.

### 3.2.3 Grid model

The grid is modelled in the sizing optimization as a power source which is theoretically available constantly, and represents an attribute of energy purchasing and selling policy. As the system is applied for household application, the consumed grid power is only limited by the contracted power limit denoted  $P_{gmax}$ .

$$P_g(t) \leq P_{gmax} \quad (17)$$

In the connection architecture with one connecting point with the network, only excess PV power will be sold to network. When the house consumes grid energy it cannot sell his solar energy. Conversely, when local production (PV and battery) can satisfy the demand, surplus can be exported to network. So that:

$$z(t) \cdot P_g(t) = 0 \quad (18)$$

Similarly to (12) and by introducing a decision binary variable  $\beta(t)$ , the nonlinearity in (18) can be transformed into linear form:

$$\begin{cases} 0 \leq z(t) \leq P_{PV}(t) \cdot \beta(t) \\ 0 \leq P_g(t) \leq P_{gmax} \cdot (1 - \beta(t)) \end{cases} \quad (19)$$

$$\Rightarrow \begin{cases} \text{if } \beta(t) = 1 \rightarrow \begin{cases} 0 \leq z(t) \leq P_{PV}(t) \\ P_g(t) = 0 \end{cases} \rightarrow \text{the house exports the surplus to grid} \\ \text{if } \beta(t) = 0 \rightarrow \begin{cases} z(t) = 0 \\ 0 \leq P_g(t) \leq P_{gmax} \end{cases} \rightarrow \text{the house consumes grid power} \end{cases}$$

### 3.2.4 Controllable and no controllable loads model

Independently from electricity price paid, the consumption is composed by no controllable loads and controllable loads. The load management possibility is considered in sizing optimization as follows:

It is supposed that the end users don't mind the power consumption patterns if the purpose to use the service is satisfied. For example, user expects that the service  $d$  is achieved at  $t = a_d$ , the consumed energy  $e_{chP}^d$  required should be maintained in an appropriate prescribed period  $\tau = [a_d - \delta_d : a_d]$  but consumed power would be deferred.

$$\sum_{u=(a_d - \delta_d)}^{a_d} P_L^d(u) = e_L^d \quad (20)$$

where:

$\delta_d$  is the time to realize the service  $d$

However, the energy consumption must be the same as expected in the case without load management:

$$\sum_t P_L(t) = \sum_t (P_{LP}(t) + P_{NLP}(t)) \quad (21)$$

## 3.3 Results analysis method

For this study, the problem is solved using the solver CPLEX implemented in Java environment. The solution defines the the sizing values ( $N_{modules}$ ,  $S_{max}$ ,  $P_{gmax}$ ), the operation plan for sources and loads, and the economical analysis value ( $NPV$ ,  $P.I$ , the amortization duration) in function of the scenario given by problem parameters as input data.

Sizing problem is to be solved by decision maker once before installation of system in order to determine the optimal architecture, operation strategy and profit hope over its lifetime period. These calculations have thus a great meaning in the feasibility and acceptability of the proposed solution. As said previously, the obtained results depend on the scenario's parameters which could be sensitive and influenced by many exogenous aspects. It is necessary for investor to identify the most important factors of influence and then, to quantify their impacts on the solution. Several ones are cited, for exogenous factors as the renewable energy support policy, the possible technology evolutions, the electricity market

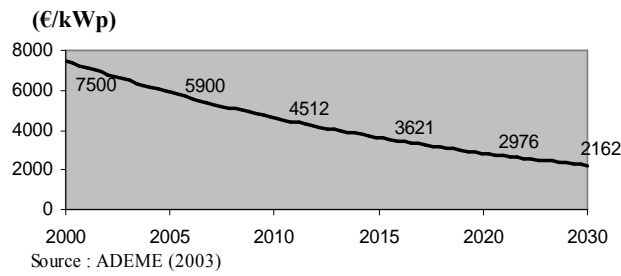


Fig. 4. Trend of installed kWp PV cost (€/kWp)

changes, climate changes, etc; and for endogenous factors as user's profile changes, load management possibilities, etc. In what follows, we choose analyzing three main factors of the development and profitability of the proposed system: reduction of subsidies, technology cost evolution, price evolution with deregulation process.

- The first one has the impact on investment in charge of installation owner, reduces the return on investment and the profitability.
- The second one also has direct impact on initial investment. Many efforts have been carried out to develop cheaper components' cost technology for PV system (cheaper solar cell, less cost inverter, higher efficiency and longer lifetime battery or other technologies beyond lead-acid battery, etc) (Fig.4). Besides, as in Germany and Japan, the PV generation quickly develops, the production on large scales of PV cells could induce a decrease in costs (scale economies). Moreover, in several countries (France, Germany), some research program<sup>s</sup> of about ten or hundred millions euros have been adopted by decision makers (public or private) to increase the performance of PV cells and to move more quickly towards the next generation. These will make installed module substantially less costly in the coming year (near 0.5€/Wp for the third generation of solar cell in the period of 2020-2030, [17]).
- Lastly, the third one describes the impacts of deregulation process directly on sells and purchases of PV electricity. The reduced investment in generation, consumption characteristics (as the increase use of air conditioning) and difficulties to extend electrical grid lead to more often long peak period and pressure on offer/demand equilibrium. The increase in fossil fuels prices (natural gas and oil) and climate change policies (European emission trading system) impact electric production costs. As prices are (or will) set on a pool or power exchange, prospective studies predict higher electricity prices on the market [18]. Furthermore, the oligopolistic organization of electric industry and interactions between operators affect the market price that is sensitive to industry organization (number of operators, strategies, nature of trading, etc.). Given this context, higher market prices can result in the sells of PV electricity more profitable. If we use the prevision of electric prices and PV cost evolution of Fig. 5 [17], we can conclude that PV generation could be profitable in peak period.

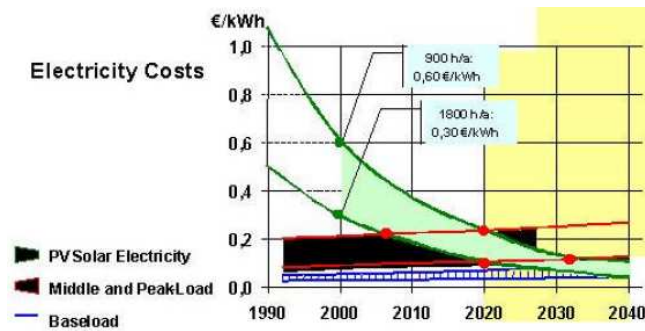


Fig. 5. Cost of PV generation and electricity price evolution [17]

The profitability increases and the learning investment decreases with the drop in PV duration of utilization. With a supply contract based on market prices, consumer could decrease its purchases from suppliers to consume PV electricity. These behaviors imply a decrease in electricity bill, consuming less kWh and taking a cheaper package, and a minimization of an increase of price risk with contract renegotiation\*\*.

A complete analysis on impacts of these aspects for system sizing is thus needed and will be demonstrated in the next section.

#### 4. Study case and discussions

The study case is a residential house of about 100 m<sup>2</sup>, located in France at a sunny place with mean daily radiation of 5 kWh/m<sup>2</sup>, and having about 50 m<sup>2</sup> of surface available for PV installation. The house presents high energy demand increments especially in summer due to air-conditioning loads and smaller demand in winter (assuming that the heater in use is not electrical power device). Controllable loads represent about 26.8% and 32.5% of daily energy consumed of the house in winter and summer season, respectively. The load curves are given in the Fig. 6 and mean radiation curves at site are shown in the Fig. 7.

We suppose that evolution of electrical demand and electricity prices are about 5%/year and 3%/year, respectively. The discount rate is set to 4%/year. Installed PV module cost is currently about 5.5 €/W<sub>p</sub>, lead-acid battery costs about 150 €/kWh, subsidies (if any) may be up to 30% - 60% of the initial investment (except battery cost). The solar energy sold back to electricity utility is set to 30 c€ for each kWh injected with government support and to electricity market price if no subsidy granted.

Based on description of electrical demand of loads, electrical utility proposed, in mode of supplying only by network, a grid-power rate of 9 kW and double-rate tariff (with 8 off-peak hours per day, from 22h to 7h of the following day).

\*\* Operators usually propose to household consumers contracts with a fixed price per kWh for a year. After this year, the price is revisited with indexation or renegotiation clause based on market price.



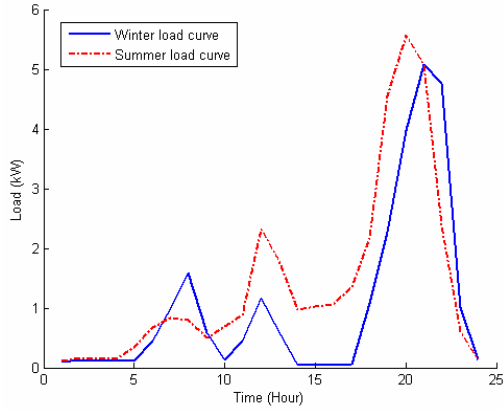


Fig. 6. Load consumption curves

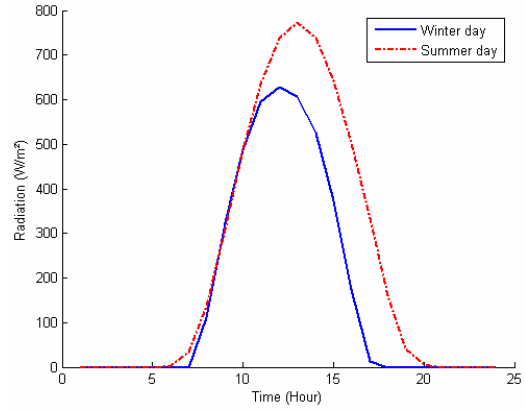


Fig. 7. Mean radiation at site

#### 4.1 With subsidy consideration

It is observed that, with subsidy on material investment cost, the real investment in charge of householder is not very high. Also, with feed-in tariff, it would be better to export the totality of solar production to electricity network to make benefits. The more PV modules are installed the further benefits it takes. The best solution in this case is to put the maximum of modules within the limit of available surface. The recommended number for this house is of 50 panels of 80 W corresponding to a peak power rate of 4 kWp.

Local consumption must be then entirely ensured by grid power in complement with battery storage system. Grid energy is cheaper in the off-peak hours, the presence of a small size battery is necessary to help better use of grid power (by storing in the off-peak hours and discharging later in the peak hours) and to facilitate the loads rescheduling. As shown in the long time operation curves of different sources (Fig. 11) controllable loads and battery's charging are rescheduled to the night while battery's discharging is programmed at the consumption's peak hours. By this way, the household can *smooth* his consumption curve to limit the required grid-power rate to 3 kW (instead of 9 kW) hence economics saving in the grid fix cost as well as grid-energy consumption cost. Sizing optimization routine returns the required battery's size of about 3.73 kWh, just enough for the denoted purposes.

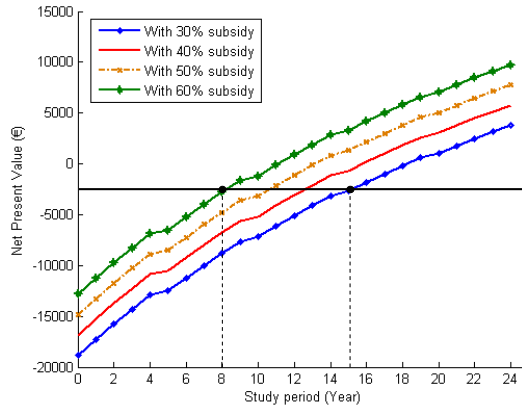


Fig. 8. Net Present Value

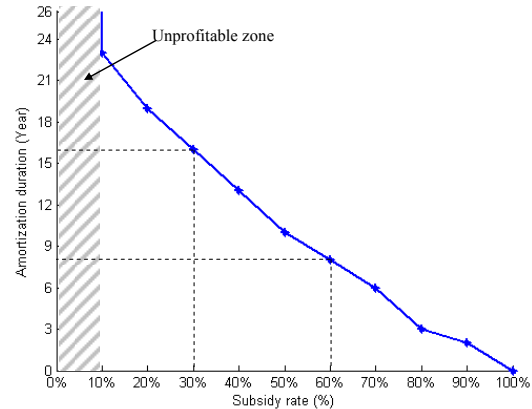


Fig. 9. Pay-back period

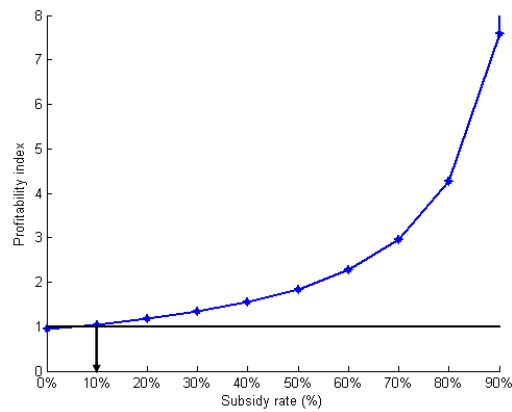


Fig. 10. Profitability index

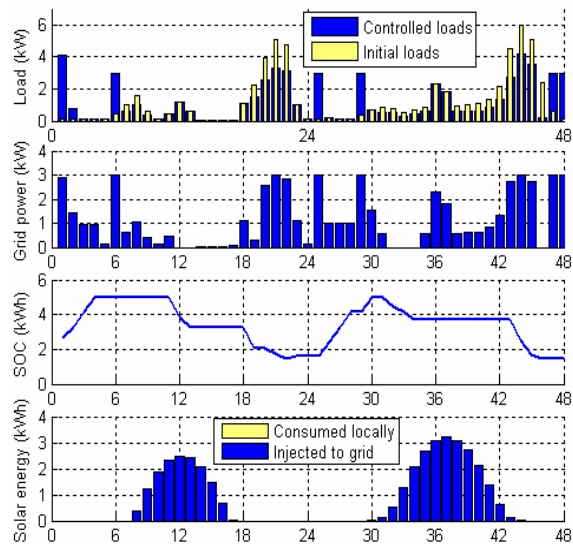


Fig. 11. Operation plan for installation with 40% subsidies

Under our assumptions and with investment subsidies, a proposed system could be profitable between 8 to 15 years. This return on investment duration is a decreasing function of subsidies. New installation brings benefits with only 10% of subsidies on investment (Fig. 10).

#### 4.2 No subsidy consideration

The results presented in the previous paragraph highlighted the fact that subsidy support makes the connected grid PV system economically viable. Decreasing subsidy rate drags out the amortization period of installation, so less advantageous even unprofitable for the owner of system. In the following analyses, we investigate in the impacts of two important aspects that support PV system so that it becomes economically self-sufficient. The first one regards in the technology evolutions involving a potentially cheaper cost for a kWp of PV installed. The second one is interested in impacts of opened electricity market to private user in residential sector leading to the growth of energy price. A combination analysis of these two impacts gives an interesting overview in progress of multi-sources and load co-management application in the coming years.

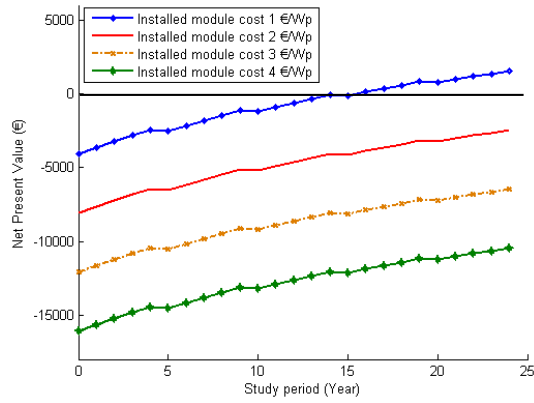


Fig. 12. Net Present Value

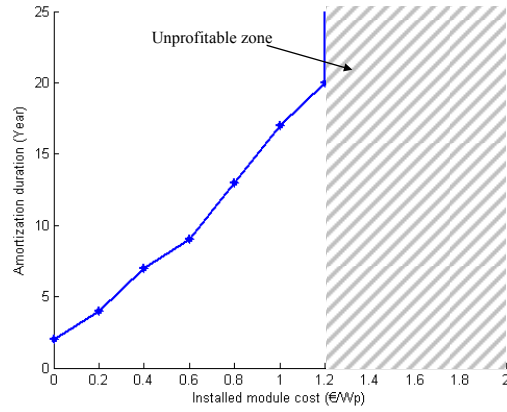


Fig. 13. Pay-back period

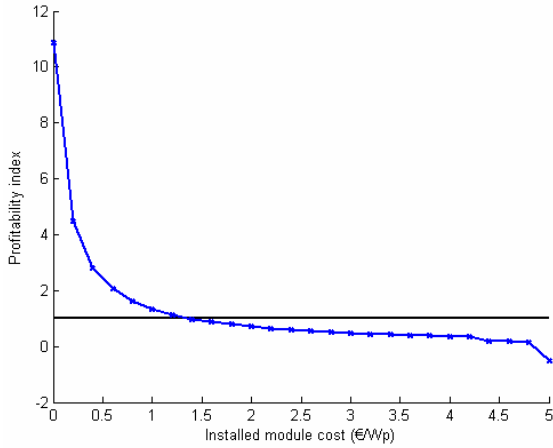


Fig. 14. Profitability index

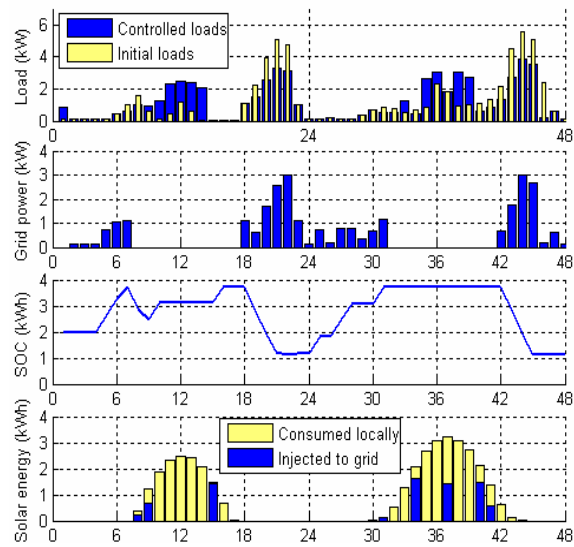


Fig. 15. Operation plan (for installed module cost of 1.2 €/Wp)

#### 4.2.1 Impact of technology evolutions in PV cells fabrication

We consider the proposed system without subsidies. Injected solar energy price is set up to electricity market, in this case, equal to energy cost of grid-energy. Battery and others costs are held constant. Analysis with different installed module costs is shown in the following figures.

Obtained results on *NPV* in different scenarios without subsidy shows that it would not interesting to invest in this project if each installed kWp of PV costs more than 1.2 €. Because of too high investment rate, system could not be amortized if module cost overpasses this threshold. This can also be seen in the Fig. 14 for which the *Profitability Index* drops-off when PV module cost steps up.

The changed circumstance opens up new possibilities for the customers to optimize their system operation. For all viable cases (installed module cost is not greater than 1.2 €/kWp), optimization routine shows importance of using solar energy locally.

Although a part of controllable loads needs to be rescheduled to the night to profit the cheaper grid-energy tariff in off-peak hours, the more important part is better to be removed to midday hours (from 10 am to 15 pm) where solar energy is prospered. Since injected energy selling price is equal to grid-energy buying tariff, there is no interest to storing solar energy to resell latter. As long as it is not consumed locally, surplus will be sold to electrical network. In this configuration, battery plays clearly the role of flexible element of system by charging grid-energy as well as a limited solar energy just for load management purpose. For this reason, grid-power subscription is also limited at 3 kW and obtained battery's size is 3.73 kWh.

As selling of the surplus of solar energy brings more benefit for system to accelerate the amortization duration. It is recommended to install as much modules as possible in the limit of available surface (here, 50 modules equivalent to 4 kWp).

We can conclude that if investment costs of PV are lower and with the assumption that electric PV generation is sold to the grid at the market price, the proposed system could show profit for a household consumer. In the system operation consideration, we can see that user can take further benefit if PV electricity is more consumed locally.

#### 4.2.2 *Impacts of the deregulated electricity market*

Impact of opened electricity market to user of residential sector is analyzed in this paragraph. For this purpose, installed modules and others cost take values as in current situation (c.f. 4.1). Energy prices vary from 10 c€/kWh (current grid-energy price applied for residential user) to 1 €/kWh. Injected solar energy sold to network at the same tariff of grid-energy cost.

Analysis on *NPV* and *P.I* indicates that in the current condition (10 c€/kWh), the project is completely unprofitable. As shown in Fig. 16, if energy prices are lower, consumer has not incentives to make investment on the proposed system. It does not produce with the system, so costly, and consumes grid energy. Thus, cash-flows are negatives and the *NPV* is always decreasing. The system is unprofitable. This confirms once again the importance of actual government's subsidy for connected-grid PV system to promote the development of solar energy. However, if energy cost increases (greater than 25 c€/kWh), the system can be self-amortized in spite of the high investment rate.

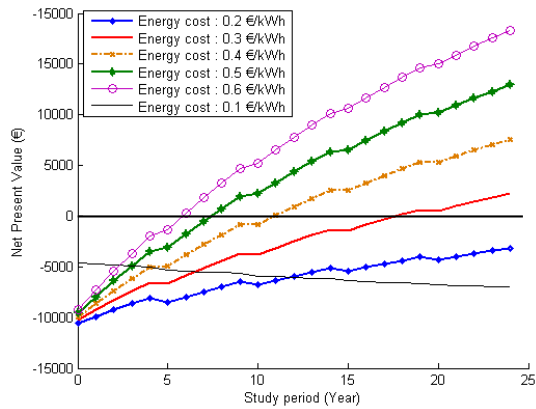


Fig.16. Net Present Value

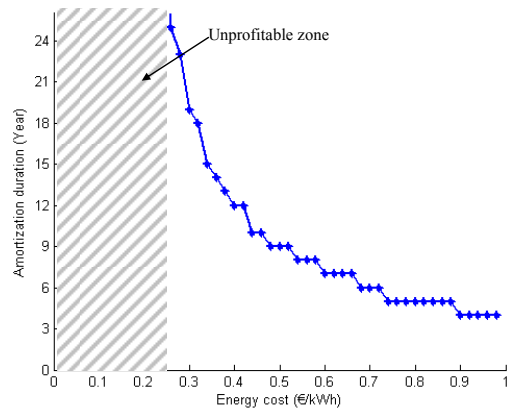


Fig. 17. Pay-back period

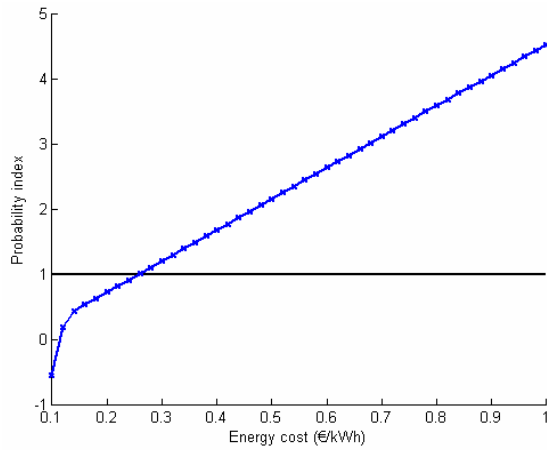


Fig. 18. Profitability index

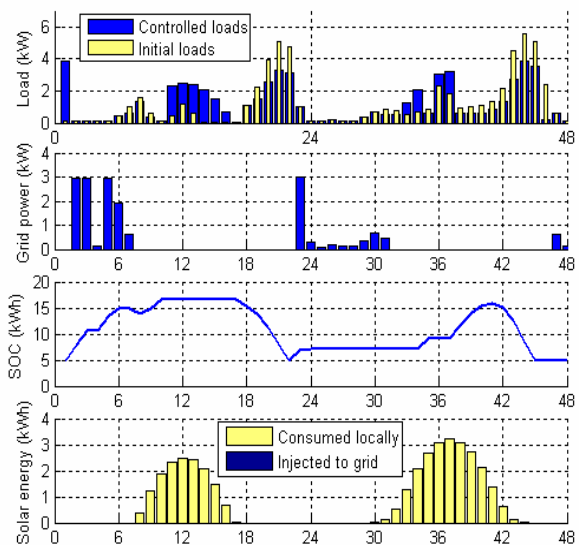


Fig. 19. Operation plan for energy cost of 30 c€/kWh

In the operation plan given in the Fig. 19, when energy cost grows up, the selling energy to electrical network is not more to the fore for householder but optimizing the consumption of all their power obtaining. It seems to be better for householder to consume all available solar production locally. Results show that controllable loads are rescheduled in optimal way to profit the maximum of grid-energy in off-peak hours and all available solar energy, consumption of grid-energy is therefore minimized. To operate the system in this way, battery is sized to optimize these purposes. The co-relation between battery capacity and PV peak rate is shown in the Fig. 20.

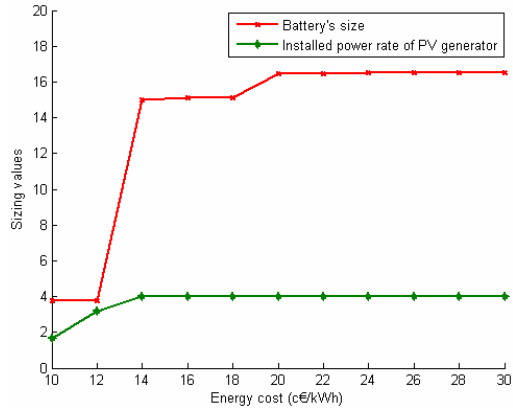


Fig. 20. Correlation between PV generator and battery size

With the assumption that electric PV generation is sold to the grid at the market price and an increasing electricity market price, a proposed system is profitable for a household consumer. Analysis in this scenario indicate, in the consumer point of view, choosing solar energy for his consumption, in the context of deregulated electricity market, is also resulted of an economic reason.

#### 4.2.3 Impacts of the deregulated electricity market with technology evolutions

Deregulated electricity market and contribution of technology development are two main factors on profitability of system and they are continuously in progress. Analyzing the impacts of both of these factors will allow describing a panorama of the new contexts of development for PV system in the coming years (Fig. 21 and 22). Obtained results show that, by either reducing the installed module cost or by the fact that energy cost could be higher, or both, the proposed multi-sources and load co-management will be economical viable and interesting solution for household consumer. If we project in the 2020 - 2030 period [17] for energy cost of 20 to 30 c€/kWh and installed module cost about 2 €/Wp, the return on investment of system can be estimated in about 7 to 14 years (This is equivalent to the results expected as if the system received subsidies).

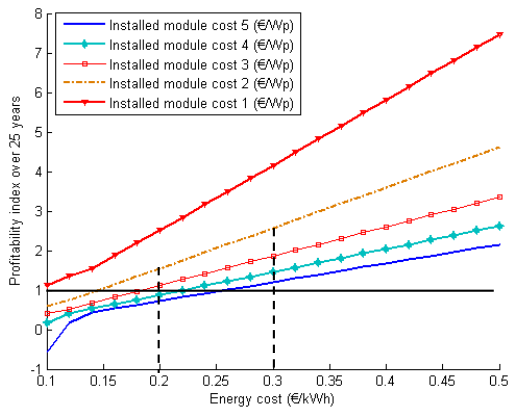


Fig. 21. Profitability index

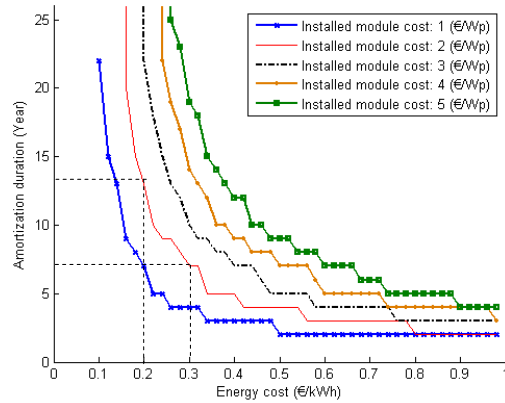


Fig. 22. Pay-back period

## 5. Conclusions

We are in a context of solar energy managing. We come out of the current idea that, with feed-in tariffs and obligatory purchases, a consumer has interest to invest in the maximum capacity of photovoltaic production and then, he sells all produced quantities to the network utilities.

Here, we assume a future context in which feed-in tariffs and purchase obligations, are finished. All photovoltaic generation is consumed or sold on the market at the market price. Household consumers sign market price contract, with indexation and renegotiations in prices. So, the importance of global household energy in general and management strategies of photovoltaic electricity in particular appear. The objective of this paper is to propose a new method of optimal supplying system sizing and household energy management. The main advantages of this method are:

- optimizing, based on MLP the components' size of connected-grid supplying multi-source system,
- optimizing the operation of system, (source and load management)
- providing a optimization tool, based on *NPV* and *P.I* methods, to analyze the economic viability of PV system as function of scenarios defined by problem parameters

The obtained results could give an overview on the PV system development towards the near and medium-term future. It is shown that, in the current period, the incentives policies are necessary to promote renewable energies because the return on investment duration is very long (or do not exist) if we only consider current market price and investment costs. So, interest to build a PV system without these policies, do not exist. But, with the previsions of market price evolutions, market price contracts negotiation and contribution of technology developments, the problematic will change. Under assumptions close to previsions of PV development and electricity market design that we anticipate up to the period 2020-2030, it is estimated that the system should be profitable with a relative independent regarding public subsidy. This becomes a good assessment for household customer to shift their consumption more intelligent and to be willing to invest in lower environment impacts energy productions.

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