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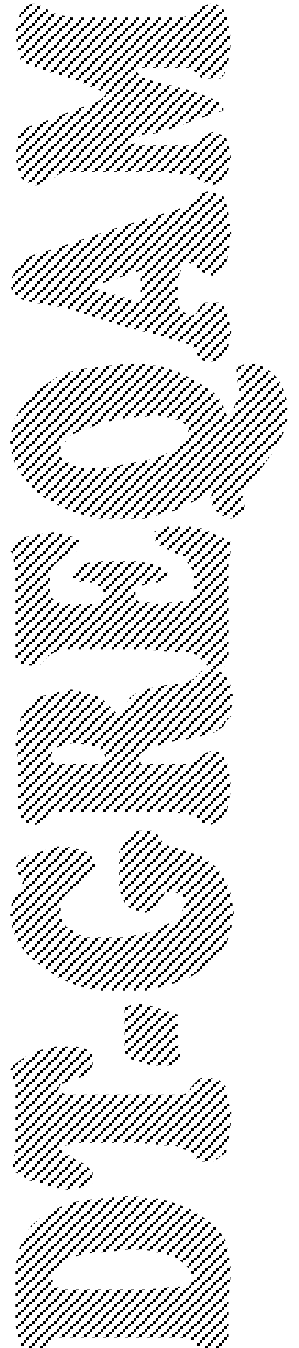
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Analysing CPI inflation by the fractionally integrated ARFIMA-STVGARCH model

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October 2008



Analysing CPI inflation by the fractionally integrated ARFIMA-STVGARCH model*

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October 16, 2008

Abstract

The aim of this paper is to study the dynamic evolution of inflation rate. The model is constructed by extending the ARFIMA-GARCH to ARFIMA with a time varying GARCH model where the transition from one regime to another is evolving smoothly over time. We show by Monte Carlo experiments that the constancy parameter tests perform well. We apply then this new model on eight countries from Europe, Japan and Canada and find that this model is appropriate for six among these countries.

Keywords: Autoregressive fractionally integrated moving average model, Generalised autoregressive conditional heteroskedasticity model, Inflation rate, Long memory process, Nonlinear time series, Time-varying parameter model.

JEL classification: C12, C22, C51, C52, E31

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1 Introduction

There have been a number of studies trying to assess empirically the persistence of inflation, but they appear to reach quite diverging conclusions. In those studies, a popular topic is concerned with the order of integration as the measure of inflation persistence, using unit root tests to classify the inflation process as either an $I(0)$ or $I(1)$ process. Nelson and Schwert (1977), Ball and Cecchetti (1990), Kim (1993), Banerjee et al. (2001) and Banerjee and Russell (2001) find the evidence of a unit root in the inflation rate. Rose (1988) and Grier and Perry (1998) claim that the inflation rate is an $I(0)$ process, while most others, like Kirchgässner and Wolters (1993) obtain mixed results for four European countries. Hassler and Wolters (1995) find that inflation is better characterized as a fractional white noise, while Baillie et al. (1996) suggest that the *ARFIMA-GARCH* models capture well the dynamic properties of inflation in the major industrial countries.

All these studies suggest that the traditional *ARMA* and *ARIMA* specifications are not able to capture the persistence of inflation rate. So, instead of dealing with $I(0)$ or $I(1)$ processes, we use the $I(d)$ process that seem a better modelling of inflation. Although all the previous works have provided the consistent evidence across time periods and countries that inflation rate exhibits long memory property in its first conditional moment, the property of time-dependent heteroskedasticity in the second conditional moment of the inflation rate has not been explored until Baillie et al. (1996); these authors use the *ARFIMA-GARCH* model to describe the inflation dynamics for ten countries and find strong evidence of long memory with mean reverting behaviour in all countries except Japan.

Incorporating the non constancy of the *GARCH* parameters to model the volatility of a process exhibiting a long memory and slow decay in its level has not be attempted before. So our main contribution is to describe the inflation rate of eight European countries, Canada and Japan by an *ARFIMA* process with smooth time varying *GARCH* parameters over time. Several interesting findings emerge from our empirical work. Our first finding is that all the inflation rates have the long memory properties in their first conditional moments. This empirical evidence is consistent with the evidence provided by Baillie et al. (1996) for ten industrial countries except Japan. The second result that comes out from this study is that only six countries are illustrated by an *ARFIMA-STVGARCH* process.

The rest of the paper is organized as follows. In Section 2, we present the class of *ARFIMA-STVGARCH* model and we discuss its properties. Section 3 considers parameter constancy test by using a Lagrange Multiplier (LM) type-test and a Likelihood Ratio test (LR). In Section 4, we propose a

specification strategy and estimation of the model. Section 5 and 6 contains respectively the results of the performance using Monte Carlo experiments and the empirical results for all countries. Section 7 concludes.

2 A general class of ARFIMA with smooth time varying GARCH model

In this section we present the *ARFIMA-GARCH* model, which generates the long memory property in the first conditional moment defined as:

$$\Phi(L)(1-L)^d(y_t - \mu) = \Psi(L)\varepsilon_t \quad (1)$$

where μ is the mean of the process, d is the long memory parameter, $\Phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p$ and $\Psi(L) = 1 + \psi_1 L + \dots + \psi_q L^q$ have all their roots outside the unit circle. $\{\varepsilon_t\}$ is an innovation sequence with the conditional mean $E(\varepsilon_t | \Omega_{t-1}) = \mu$ and a potentially time-varying conditional variance $E(\varepsilon_t^2 | \Omega_{t-1}) = h_t$, where Ω_{t-1} is the information set up to time $t - 1$. That is $\varepsilon_t | \Omega_{t-1} \sim N(0, h_t)$, i.e.:

$$\varepsilon_t = z_t \sqrt{h_t} \quad (2)$$

where $\{z_t\}$ is a sequence of independent standard normal variables with variance 1. The fractional differencing operator $(1-L)^d$ is defined by the binomial expansion,

$$(1-L)^d = 1 - dL - \frac{d(1-d)}{2!}L^2 - \frac{d(1-d)(2-d)}{3!}L^3 + \dots$$

where L is the lag operator. Thus, the process will be covariance stationary and invertible for $-0.5 < d < 0.5$. When $0 < d < 0.5$, the process is a stationary long memory process in the sense that autocorrelations are not absolutely summable and decay hyperbolically to zero. If $0.5 \leq d < 1$, the process does not have a finite variance and is a non-stationary process, but it is still mean reverting. Moreover, when $d = 1$, the process has a unit root and a permanent effect.

To capture the volatility property of the conditional variance process, Bollerslev (1986) introduced the *GARCH(m, r)* (*General Autoregressive Conditional Heteroskedasticity*) model, which defines the conditional variance equation as follows:

$$h_t = \alpha_{00} + \sum_{i=1}^r \alpha_{0i} \varepsilon_{t-i}^2 + \sum_{j=1}^m \beta_{0j} h_{t-j} \quad (3)$$

The parameters in (3) satisfy the restrictions $\alpha_{00} > 0$, $\alpha_{0i} \geq 0$, $\beta_{0j} \geq 0$, $i = 1, \dots, r$ and $j = 1, \dots, m$. These conditions are sufficient to ensure positivity of h_t . In order to characterize smooth changes in the conditional variance¹, we assume that the parameters in (3) vary smoothly over time as follows:

$$h_t = \alpha_{00} + \sum_{i=1}^r \alpha_{0i} \varepsilon_{t-i}^2 + \sum_{j=1}^m \beta_{0j} h_{t-j} + \left(\alpha_{10} + \sum_{i=1}^r \alpha_{1i} \varepsilon_{t-i}^2 + \sum_{j=1}^m \beta_{1j} h_{t-j} \right) F(s_t, \gamma, c) \quad (4)$$

The constraints applied to the parameters to guarantee stationarity and positivity of the conditional variance in (4) are:

$$\begin{cases} \alpha_{00} > 0, \alpha_{00} + \alpha_{10} > 0 \\ \alpha_{0i} \geq 0, \alpha_{0i} + \alpha_{1i} \geq 0, i = 1, \dots, r \\ \beta_{0j} \geq 0, \beta_{0j} + \beta_{1j} \geq 0, j = 1, \dots, m \\ \sum_{i=1}^r (\alpha_{0i} + \alpha_{1i}) + \sum_{j=1}^m (\beta_{0j} + \beta_{1j}) < 1 \end{cases}$$

$F(s_t, \gamma, c)$ is the transition function which is continuous and non-negative bounded between zero and one, governing the movement from one regime to another. This function is such that the parameters of the *GARCH* model change smoothly over time from $(\alpha_{0i}, \beta_{0j})$ to $(\alpha_{0i} + \alpha_{1i}, \beta_{0j} + \beta_{1j})$, $i = 0, 1, \dots, r$ and $j = 1, \dots, m$. A suitable choice for $F(s_t, \gamma, c)$ is the general logistic smooth transition function defined as follows:

$$F(s_t, \gamma, c) = \left(1 + \exp \left\{ -\gamma \prod_{k=1}^K (s_t - c_k) \right\} \right)^{-1}, \gamma > 0, c_1 \leq \dots \leq c_K \quad (5)$$

where $s_t = t/T$ is the transition variable, T is the number of observations and γ is the slope parameter which controls the degree of smoothness of the transition function. When $\gamma \rightarrow \infty$ in (5), the switch to one parameterization to another is abrupt, that is, the process contains structural breaks at the threshold parameters, c_k , $k = 1, \dots, K$. The order K determines the shape of the transition function. In practice², the typical choices of the order are $K = 1$ and $K = 2$. For $K = 1$, the *STVGARCH* model is suitable to describe a process whose dynamics of the conditional variance vary smoothly over time. For our purpose, we choose $K = 1$ that is illustrated in Figure

¹See Hagerud (1996-1997) and González-Rivera (1998) for more details.

²Teräsvirta et al. (2008).

1 where the location parameters c is respectively equal to 0.2, 0.5 and 0.7 and $\gamma = \{5, 10, 50, 100\}$, where the lowest value of γ corresponds to the smoothest function.

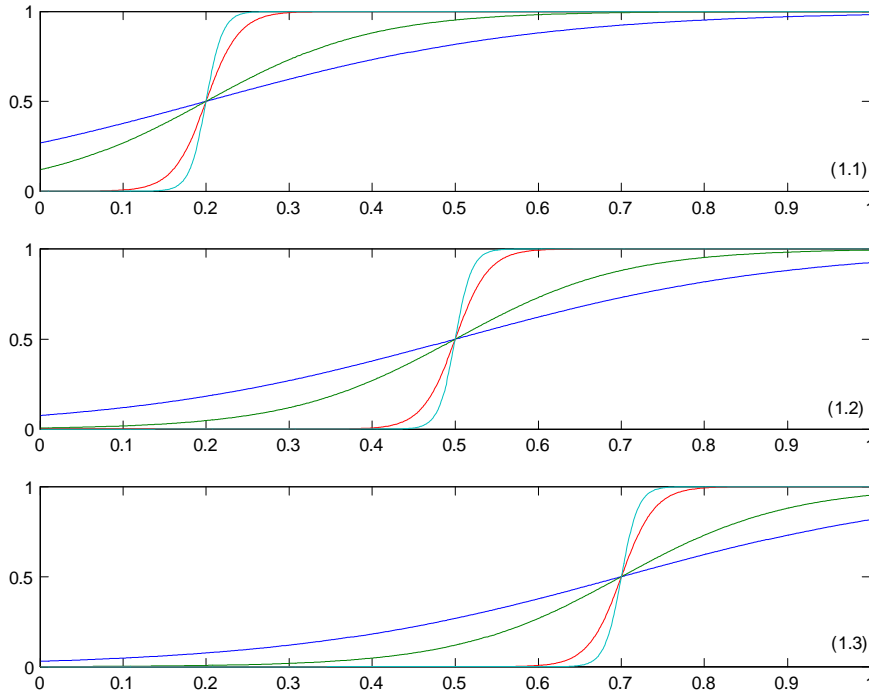


Figure 1. Plots of the logistic transition function

3 Testing parameter constancy

The test of parameter constancy is an important tool for checking the adequacy of *ARFIMA-GARCH* model. If one rejects the parameter constancy against *ARFIMA-STVGARCH* model, one may conclude that the structure of the dynamics of volatility is changing over time. This idea has previously been considered by Lundbergh and Teräsvirta (2002) and Teräsvirta et al. (2008). In order to derive the test statistic, let us rewrite the model as:

$$\begin{cases} \Phi(L)(1-L)^d(y_t - \mu) = \Psi(L)\varepsilon_t, & \varepsilon_t = z_t\sqrt{h_t} \\ h_t = \alpha_{00} + \sum_{i=1}^r \alpha_{0i}\varepsilon_{t-i}^2 + \sum_{j=1}^m \beta_{0j}h_{t-j} \\ \quad + \left(\alpha_{10} + \sum_{i=1}^r \alpha_{1i}\varepsilon_{t-i}^2 + \sum_{j=1}^m \beta_{1j}h_{t-j} \right) F(s_t, \gamma, c) \end{cases} \quad (6)$$

The first step of the test consists in estimating the *ARFIMA*(p, d, q), to obtain the set of residuals $\hat{\varepsilon}_t$ and then testing the null hypothesis of parameter constancy corresponding to $H_0 : \gamma = 0$ against $H_1 : \gamma > 0$. Under the null

hypothesis, c , α_{1i} and β_{1j} for $i = 0, 1, \dots, r$ and $j = 1, \dots, m$ are not identified. This identification problem has been resolved by Luukkonen et al. (1988) by replacing the transition function³ by its first order Taylor approximation around $\gamma = 0$. Thus, the first order Taylor expansion of the logistic transition function around $\gamma = 0$ is given by:

$$T(s_t, \gamma, c) = \frac{1}{4}\gamma(s_t - c) + R(s_t, \gamma, c)$$

where $R(s_t, \gamma, c)$ is a remainder term. Substituting $T(s_t, \gamma, c)$ for $F(s_t, \gamma, c)$ in (6) and after rearranging terms we have:

$$\begin{aligned} h_t = & \alpha_{00}^* + \sum_{i=1}^r \alpha_{0i}^* \varepsilon_{t-i}^2 + \sum_{j=1}^m \beta_{0j}^* h_{t-j} \\ & + \left(\alpha_{10}^* s_t + \sum_{i=1}^r \alpha_{1i}^* s_t \varepsilon_{t-i}^2 + \sum_{j=1}^m \beta_{1j}^* s_t h_{t-j} \right) + R^* \end{aligned} \quad (7)$$

where $\alpha_{0i}^* = \alpha_{0i} - \frac{\gamma}{4}\alpha_{1i}c$, $\beta_{0j}^* = \beta_{0j} - \frac{\gamma}{4}\beta_{1j}c$, $\alpha_{1i}^* = \frac{\gamma}{4}\alpha_{1i}$, $\beta_{1j}^* = \frac{\gamma}{4}\beta_{1j}$ for $i = 0, 1, \dots, r$ and $j = 1, \dots, m$. Therefore, the null hypothesis for parameter constancy becomes: $H_0 : \alpha_{1i}^* = \beta_{1j}^* = 0$. This hypothesis can be tested by a standard LM test or LR test and one can also test the constancy of a subset of parameters. Let $\theta_1 = (\alpha_{0i}^*, \beta_{0j}^*)'$, $\theta_2 = (\alpha_{1i}^*, \beta_{1j}^*)'$ and $\theta = (\theta_1', \theta_2')'$. Under H_0 , the remainder $R^* = 0$, so it does not affect the asymptotic null distribution of the test statistic. Using the reparameterization in (7), the null hypothesis of parameter constancy becomes $H_0 : \theta_2 = 0$. The quasi log-likelihood of the model for observation t is given by:

$$l_t = -\frac{1}{2} \ln 2\pi - \frac{1}{2} \ln h_t - \frac{1}{2} \frac{\varepsilon_t^2}{h_t} \quad (8)$$

3.1 Lagrange Multiplier test

The partial derivatives evaluated under H_0 are given by:

$$\left. \frac{\partial l_t}{\partial \theta} \right|_{H_0} = \frac{1}{2} \left(\frac{\widehat{\varepsilon}_t^2}{\widehat{h}_{0t}} - 1 \right) \left. \frac{\partial \ln h_t}{\partial \theta} \right|_{H_0}$$

Under the null hypothesis, the "hats" indicate the maximum likelihood estimators and \widehat{h}_{0t} denotes the conditional variance estimated at time t . Under

³For the purpose of deriving the test, we replace $F(s_t, \gamma, c)$ by $F(s_t, \gamma, c) - 1/2$, (Teräsvirta et al. (2008)).

H_0 , the LM type statistic is asymptotically distributed as χ^2 with $dim(\theta_2)$ degrees of freedom:

$$LM = \frac{1}{2} \left\{ \sum_{t=1}^T \frac{\partial l_t}{\partial \theta} \Big|_{H_0} \right\}' \left\{ \sum_{t=1}^T \left[\frac{\partial \ln h_t}{\partial \theta} \Big|_{H_0} \right] \left[\frac{\partial \ln h_t}{\partial \theta} \Big|_{H_0} \right]' \right\}^{-1} \left\{ \sum_{t=1}^T \frac{\partial l_t}{\partial \theta} \Big|_{H_0} \right\} \quad (9)$$

In practice this test may be carried out in a straightforward way using an auxiliary least squares regression. Let

$$\begin{aligned} \widehat{W}_t &= (1, \varepsilon_{t-1}^2, \dots, \varepsilon_{t-r}^2, h_{t-1}, \dots, h_{t-m}) \\ \widehat{X}'_{1t} &= \frac{\partial \ln h_t}{\partial \theta_1} \Big|_{H_0} = (\widehat{h}_{0t})^{-1} \left(\widehat{W}_t' + \sum_{j=1}^m \widehat{\beta}_{0j}^* \frac{\partial \widehat{h}_{t-j}}{\partial \theta_1} \right) \\ \widehat{X}'_{2t} &= \frac{\partial \ln h_t}{\partial \theta_2} \Big|_{H_0} = (\widehat{h}_{0t})^{-1} \left((s_t \widehat{W}_t)' + \sum_{j=1}^m \widehat{\beta}_{0j}^* \frac{\partial \widehat{h}_{t-j}}{\partial \theta_1} \right) \end{aligned}$$

Hence, the statistic is computed in a few steps as follows:

- Estimate the parameters of the *ARFIMA* model and obtain the set of residuals $\widehat{\varepsilon}_t$.
- From $\widehat{\varepsilon}_t$, estimate h_{0t} as a *GARCH* model, compute the residuals $\widehat{u}_t = \left(\frac{\widehat{\varepsilon}_t^2}{\widehat{h}_{0t}} - 1 \right)$, $t = 1, \dots, T$, and the sum of squared residuals under the null hypothesis, $SSR_0 = \sum_{t=1}^T \widehat{u}_t^2$.
- Regress \widehat{u}_t on \widehat{X}'_{1t} and \widehat{X}'_{2t} , $t = 1, \dots, T$, and compute the sum of squared residuals, SSR_1 .
- Compute the χ^2 test statistic as:

$$LM = \frac{T(SSR_0 - SSR_1)}{SSR_0}$$

3.2 Likelihood Ratio test

Let ML_u (resp. ML_r) be the maximum log-likelihood value of the unrestricted (resp. restricted) *ARFIMA-STVGARCH*. The number of parameters is reduced in the restricted model based on the assumption $H_0 : \theta_2 = 0$.

Under H_0 the LR type statistic is asymptotically distributed as χ^2 with $\dim(\theta_2)$ degrees of freedom, so the likelihood ratio statistic is given by:

$$LR = 2 [ML_u - ML_r]$$

4 Specification and estimation of the model

In order to describe the specification procedure for *ARFIMA-STVGARCH* models, let us consider the conditional variance h_t defined in (4) such that all parameters are changing smoothly over time. Our strategy contains the following stages:

- The first step is to plot the autocorrelation (ACF) and partial autocorrelation functions (PACF) of the series in order to detect the presence of seasonal phenomenon and long memory behaviour which is characterized by an hyperbolic decay of the autocorrelation function.
- The information criteria (AIC, BIC,...) are not optimal to determine the order of the model in (1), hence we fit an *ARFIMA* model by using the ACF and PACF and we obtain the set of residuals $\widehat{\varepsilon}_t$.
- We use the ACF and PACF functions of the squared estimated residuals $\widehat{\varepsilon}_t^2$ to check the presence of the conditional heteroscedasticity and to identify the orders of the model in (4). The squared standardized errors of the selected *ARFIMA-GARCH* model should be free of serial correlation. Neglected autocorrelation may bias test of parameter constancy.
- The next stage consists on testing the selected *ARFIMA-GARCH* model against the alternative of *ARFIMA-STVGARCH* where just one parameter varying each time⁴. If H_0 is rejected, we keep the varying parameter which provides the highest rejection frequency compared to the other tests parameter constancy. Then, we test *ARFIMA-STVGARCH* model with the changing parameter selected above against the alternative of *ARFIMA-STVGARCH* model where two varying parameters. Once again, if the null hypothesis is rejected, we maintain the two parameters corresponding to the lowest *p-value*. We proceed sequentially by testing the *ARFIMA-STVGARCH* with $s - 1$ changing parameters against the *ARFIMA-STVGARCH*⁵ with s changing parameters until the first non rejection of the null hypothesis.

⁴The changing of parameters occurs only in the GARCH part of the model.

⁵ s is the number of changing parameters.

- Finally, we use the quasi maximum likelihood method to estimate the global *ARFIMA-STVGARCH* and we evaluate the final model by some diagnostic tests. Suppose that y_t is generated by the *ARFIMA-STVGARCH* given in (6).

Let $\lambda = (\mu, d, \Phi', \Psi', \delta'_i, \varphi'_i, \gamma, c)'$ where $\delta_i = (\alpha_{i0}, \dots, \alpha_{ir})'$, $\varphi_i = (\beta_{i1}, \dots, \beta_{im})'$ for $i = 0, 1$, $\Phi = (\phi_1, \dots, \phi_p)'$ and $\Psi = (\psi_1, \dots, \psi_q)'$. The quasi maximum likelihood estimator of the parameter vector λ is obtained by:

$$\hat{\lambda} = \arg \max_{\lambda \in \Lambda} \frac{1}{T} \sum_{t=1}^T l_t(\lambda) \quad (10)$$

where $l_t(\lambda)$ is the quasi log-likelihood of the model for observation t :

$$l_t(\lambda) = -\frac{1}{2} \ln 2\pi - \frac{1}{2} \ln h_t - \frac{1}{2} \frac{\varepsilon_t^2}{h_t} \quad (11)$$

5 Monte Carlo experiment

5.1 Simulation design

In this section we study the size and power properties of the LM-type and LR tests using Monte Carlo simulation methods. In all simulations, we use sample lengths of 500, 1000, 2000 observations and, for each design, a total of 100 replications. To avoid the initialization effects, the first 1000 observations are discarded. The behaviour of the tests is examined for several data generating processes (DGP's) that can be nested in the following *FI-STVGARCH*(1, 1) specification:

$$\begin{cases} (1-L)^d y_t = \varepsilon_t, & \varepsilon_t = z_t \sqrt{h_t}, & \varepsilon_t \rightsquigarrow N(0, h_t) \\ h_t = \alpha_{00} + \alpha_{01} \varepsilon_{t-1}^2 + \beta_{01} h_{t-1} \\ \quad + (\alpha_{10} + \alpha_{11} \varepsilon_{t-1}^2 + \beta_{11} h_{t-1}) F(s_t, \gamma, c) \end{cases} \quad (12)$$

In all the experiments, the change of regime in volatility occurs in the middle of the sample, i.e: $c = 0.5$, while the smoothness parameter γ is equal to 10. The transition variable is the standardized time variable $s_t = t/T$, for $t = 1 \dots T$ and T is the number of observations.

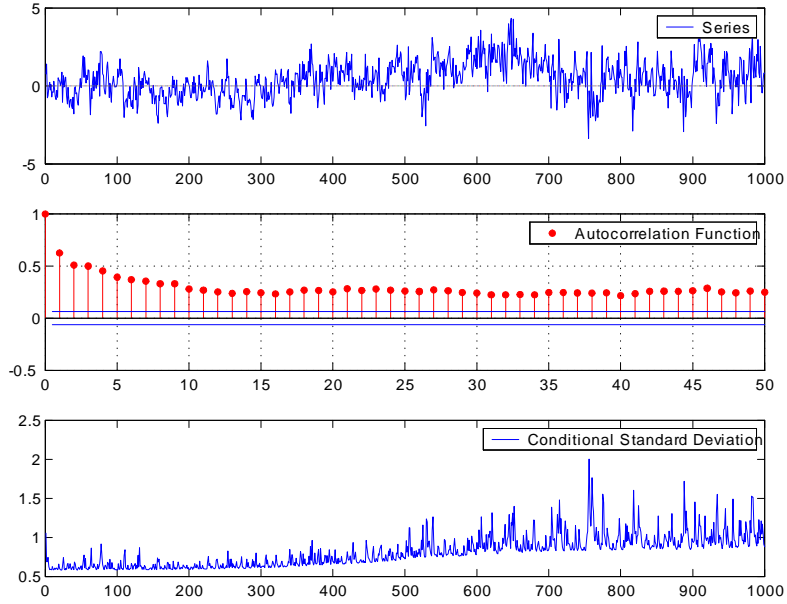


Figure 2. Plots of the series, the autocorrelation function and the conditional standard deviations.

Figure 2 shows the plots of the series, the autocorrelation function and the conditional standard deviations with $T = 1000$, $\alpha_{00} = 0.30$, $\alpha_{01} = 0.10$, $\beta_{01} = 0.10$, $\alpha_{10} = 0.20$, $\alpha_{11} = 0.15$, $\beta_{11} = 0.20$, $d = 0.40$, we use the logistic transition function $F(s_t, 10, 0.5)$. The autocorrelation function exhibits an hyperbolic slow decay that indicates the presence of long memory. Moreover, we notice an increase of the volatility shown by the plots of the series and the conditional standard deviations. This increase is explained by the variation of GARCH parameters.

5.2 Testing parameter constancy

The behaviour of the test statistics is examined for several data generating processes (DGP's) that can be nested in the following *FI-STVGARCH(1,1)* specification:

$$\begin{aligned} (1-L)^d y_t &= \varepsilon_t, \quad \varepsilon_t = z_t \sqrt{h_t}, \quad \varepsilon_t \rightsquigarrow N(0, h_t) \\ h_t &= \alpha_{00} + \alpha_{01} \varepsilon_{t-1}^2 + \beta_{01} h_{t-1} + (\alpha_{10} + \alpha_{11} \varepsilon_{t-1}^2 + \beta_{11} h_{t-1}) F(s_t, \gamma, c) \end{aligned} \quad (13)$$

The data generating processes are as following:

$$DGP \text{ (I)} \quad \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} \\ \alpha_{00} = 0.10, \alpha_{01} = 0.10, \beta_{01} = 0.80. \end{cases}$$

$$DGP \text{ (II)} \quad \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + \alpha_{10}F(s_t, \gamma, c) \\ \alpha_{00} = 0.10, \alpha_{01} = 0.10, \beta_{01} = 0.80, \alpha_{10} = 0.80. \end{cases}$$

$$DGP \text{ (III)} \quad \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + \alpha_{11}\varepsilon_{t-1}^2 F(s_t, \gamma, c) \\ \alpha_{00} = 0.10, \alpha_{01} = 0.10, \beta_{01} = 0.10, \alpha_{11} = 0.70. \end{cases}$$

$$DGP \text{ (IV)} \quad \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + \beta_{11}h_{t-1}F(s_t, \gamma, c) \\ \alpha_{00} = 0.10, \alpha_{01} = 0.10, \beta_{01} = 0.10, \beta_{11} = 0.70. \end{cases}$$

$$DGP \text{ (V)} \quad \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + (\alpha_{10} + \alpha_{11}\varepsilon_{t-1}^2) F(s_t, \gamma, c) \\ \alpha_{00} = 0.10, \alpha_{01} = 0.10, \beta_{01} = 0.10, \alpha_{10} = 0.30, \alpha_{11} = 0.70. \end{cases}$$

$$DGP \text{ (VI)} \quad \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + (\alpha_{10} + \beta_{11}h_{t-1}) F(s_t, \gamma, c) \\ \alpha_{00} = 0.10, \alpha_{01} = 0.10, \beta_{01} = 0.10, \alpha_{10} = 0.30, \beta_{11} = 0.70. \end{cases}$$

$$DGP \text{ (VII)} \quad \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + (\alpha_{11}\varepsilon_{t-1}^2 + \beta_{11}h_{t-1}) F(s_t, \gamma, c) \\ \alpha_{00} = 0.10, \alpha_{01} = 0.10, \beta_{01} = 0.10, \alpha_{11} = 0.30, \beta_{11} = 0.45. \end{cases}$$

$$DGP \text{ (VIII)} \quad \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + (\alpha_{10} + \alpha_{11}\varepsilon_{t-1}^2 + \beta_{11}h_{t-1}) F(s_t, \gamma, c) \\ \alpha_{00} = 0.10, \alpha_{01} = 0.10, \beta_{01} = 0.10, \alpha_{10} = 0.30, \alpha_{11} = 0.30, \beta_{11} = 0.45. \end{cases}$$

The tests of $FI - GARCH(1, 1)$ against $FI - STVGARCH(1, 1)$ are as following:

$$(i) \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + \alpha_{10}F(s_t, \gamma, c) \\ H_0 : \alpha_{10} = 0 \text{ against } H_1 : \alpha_{10} \neq 0 \end{cases}$$

$$(ii) \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + \alpha_{11}\varepsilon_{t-1}^2 F(s_t, \gamma, c) \\ H_0 : \alpha_{11} = 0 \text{ against } H_1 : \alpha_{11} \neq 0 \end{cases}$$

$$(iii) \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + \beta_{11}h_{t-1}F(s_t, \gamma, c) \\ H_0 : \beta_{11} = 0 \text{ against } H_1 : \beta_{11} \neq 0 \end{cases}$$

$$(iv) \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + (\alpha_{10} + \alpha_{11}\varepsilon_{t-1}^2) F(s_t, \gamma, c) \\ H_0 : \alpha_{10} = \alpha_{11} = 0 \text{ against } H_1 : \alpha_{10} \neq 0 \text{ and } \alpha_{11} \neq 0 \end{cases}$$

$$(v) \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + (\alpha_{10} + \beta_{11}h_{t-1}) F(s_t, \gamma, c) \\ H_0 : \alpha_{10} = \beta_{11} = 0 \text{ against } H_1 : \alpha_{10} \neq 0 \text{ and } \beta_{11} \neq 0 \end{cases}$$

$$(vi) \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + (\alpha_{11}\varepsilon_{t-1}^2 + \beta_{11}h_{t-1}) F(s_t, \gamma, c) \\ H_0 : \alpha_{11} = \beta_{11} = 0 \text{ against } H_1 : \alpha_{11} \neq 0 \text{ and } \beta_{11} \neq 0 \end{cases}$$

$$(vii) \begin{cases} h_t = \alpha_{00} + \alpha_{01}\varepsilon_{t-1}^2 + \beta_{01}h_{t-1} + (\alpha_{10} + \alpha_{11}\varepsilon_{t-1}^2 + \beta_{11}h_{t-1}) F(s_t, \gamma, c) \\ H_0 : \alpha_{10} = \alpha_{11} = \beta_{11} = 0 \text{ against } H_1 : \alpha_{10} \neq 0 \text{ and } \alpha_{11} \neq 0 \text{ and } \beta_{11} \neq 0 \end{cases}$$

The parameter of fractional integration varies among $d = \{0.20, 0.45\}$, $c = 0.5$ and $\gamma = 10$.

5.3 Simulation results

From Tables 1-8 we can see that both tests do not have size distortion (the estimated sizes are close to nominal sizes), whatever the specification. Moreover, the simulation results reveal that the LM-type and the LR-type tests provide good results even for small data sets; however, for almost cases, large samples display better size and power than small samples. Finally, results

with $d = 0.20$ are close to those obtained with $d = 0.45$, so increasing the parameter of fractional integration d has no impact on the size and the power of the tests.

6 Empirical Analysis

6.1 The data

We use monthly data of the Consumer Price Index (CPI) inflation rate obtained from the *IMF (International Financial Statistics)*⁶. The data range from 1970:01 to 2004:12 and cover eight European countries: Belgium, Denmark, Finland, France, Germany, Italy, Portugal, Spain with Canada and Japan. The data are not seasonally adjusted, and generally exhibit the clear pattern of slow decay and persistence. The inflation rate is measured by the monthly difference of the log CPI, i.e. $y_t = 100 \cdot \log(CPI_t/CPI_{t-1})$, where CPI_t denotes the Consumer Price Index at month t . Allowing for differencing leaves 419 usable observations.

The summary statistics (Table 9) for the ten inflation rates show that the distributions of the series are skewed to the right with heavy tails. The Jarque-Bera test is in line with this evidence since it strongly rejects normality for the distribution of the inflation rates. The Ljung-Box test applied to the series and squared series, provides clear evidence against the hypothesis of serial independence of observations and indicates the existence of ARCH effect.

The unit root tests of Augmented Dickey-Fuller (1979) (henceforth ADF), Phillips and Perron (1988) (henceforth PP) and Kwiatkowski et al. (1992)⁷ (henceforth KPSS) for the levels and the first differences of the ten inflation series are presented in Table 10. For all the inflation series based on the KPSS test the null hypothesis of stationarity is rejected. However, the ADF test rejects the unit root hypothesis for Canada, Belgium, Germany, Japan and Portugal but it does not reject it for the other countries at 1% and 5% significance levels. Finally, the PP test rejects the unit root hypothesis for all the countries except Denmark. In conclusion, the application of these tests yields contradictory results⁸. These unit root tests are merely suggestive as mentioned in Lee and Amsler (1997) who show that the KPSS statistic cannot distinguish consistently between nonstationary long memory and unit

⁶From CD of March 2005.

⁷Contrary to ADF test, the KPSS test considers the stationarity under the null hypothesis, and the alternative hypothesis is the presence of unit root.

⁸See also Baillie et al. (1996) and Conrad (2005b).

root. Hence for all the countries, there is evidence that inflation may not be generated by an $I(0)$ or $I(1)$ process and is at least indicative of fractional integration.

6.2 Estimation results of the ARFIMA-STVGARCH model

Baillie et al. (1996) suggests that the *ARFIMA-GARCH* specification describes the inflation series of several industrial countries quite well. Within this framework, we use *STVGARCH* structure instead of the *GARCH* to capture the persistence in the volatility of the inflation rate. Estimates of the *ARFIMA-STVGARCH* model for the ten countries are given in Table 12; they are obtained by quasi-maximum likelihood estimation method (QMLE); following Baillie et al. (1996a). We notice that this type of model is general; so it may include *ARFIMA-GARCH* or simple *ARFIMA* model. To check robustness of our estimates, we use a range of starting values to ensure that the estimation procedure converges to a global maximum. To identify the order of the *ARFIMA* part of the model, we follow the procedure explained in section 4. The seasonal moving-average parameters, particularly ψ_{12} and ψ_{24} are necessary to take into account the significant seasonality, which is evident for all the countries.

Table 11 contains the LM statistics and the p – values corresponding to the test of an *ARFIMA-GARCH*(1, 1) model against the alternative hypothesis of an *ARFIMA-STVGARCH*(1, 1) model. The test is performed for several combinations of the *GARCH* coefficients as explained in the simulation part. The sequential testing procedure is carried out (see section 4) until the first non-rejection of the null hypothesis (at each step one parameter is added). In many cases, the null hypothesis is strongly rejected (see the p -values that are very close to zero). We do not report the LR-test results, because they are similar to the LM-test results. Among the ten countries, we find that six countries satisfy the assumption of time-varying parameters (not necessary all) of the *GARCH* part, namely Portugal, Spain, Denmark, Finland, Italy and Japan. The monthly inflation rate, the autocorrelation function, the conditional standard deviation and the logistic transition function of those countries, obtained from the model (13), are shown in Figures 3 to 8.

In each Figure, we observe a slow decay in the autocorrelation function which indicates the presence of long memory behaviour. As the case of the conditional standard deviations, we note a decrease of volatility and a smooth change of regimes over time corresponding to the logistic transition function.

Denmark is the only country whose all *GARCH* parameters vary smoothly over time.

Finally, after studying the selection frequencies of various models using the specification procedure discussed in Section 4, we find that the estimated models are nested in the following specification: *ARFIMA*(1, d , 2)(0, 0, 2)₁₂-*STVGARCH*(1, 1), i.e.:

$$\left\{ \begin{array}{l} (1 - L)^d (y_t - \mu) = x_t \\ (1 - \phi_1 L) x_t = (1 - \psi_1 L - \psi_2 L^2) (1 - \psi_{12} L^{12} - \psi_{24} L^{24}) \varepsilon_t \\ \varepsilon_t = z_t \sqrt{h_t}, \varepsilon_t \rightsquigarrow N(0, h_t) \\ h_t = \alpha_{00} + \alpha_{01} \varepsilon_{t-1}^2 + \beta_{01} h_{t-1} \\ \quad + (\alpha_{10} + \alpha_{11} \varepsilon_{t-1}^2 + \beta_{11} h_{t-1}) F(s_t, \gamma, c) \end{array} \right.$$

The estimates of the fractional differencing parameter are clearly significant and $0 < \hat{d} < 0.5$ for all countries implying covariance-stationarity of the inflation processes, except Canada where the estimated value of d is approximately 0.6 which implies that the inflation series for this country has an infinite variance but is still mean reverting. For the countries who reject the hypothesis of parameter constancy (see Table 11), the estimated slope parameter γ is in the range of [5, 20], so the change from one regime to another in volatility is smooth. The estimated threshold parameter \hat{c} is less than 0.5.

To check the goodness of fit of our model, we consider several diagnostic tests on the standardized residuals, the Ljung-Box test for the 12-th order serial autocorrelation and heteroskedasticity, when it is applied on the squared standardized residuals. We use the Jarque-Bera test, the skewness and the kurtosis coefficients to test the normality of standardized residuals. From Table 13, we can see that, for all countries, the hypothesis of uncorrelated standardized and squared standardized residuals is well supported, indicating that there is no statistically significant evidence of misspecification. The Skewness and Kurtosis coefficients indicate that the standardized residuals are still not normally distributed, which is confirmed by the Jarque-Bera test.

7 Conclusion

The results in this paper highlight the possibility of structural change in the inflation's volatility, we have thus an *ARFIMA-STVGARCH* modelling instead of an *ARFIMA-GARCH* modelling. Moreover, we have investigated the empirical power and size of LM and LR tests that allow choosing between *FI-GARCH(1,1)* and *FI-STVGARCH(1,1)* models. The simulations show that both tests generally behave well, even for moderate sample sizes. As illustrations, the suggested modelling was applied to monthly inflation rate data of eight European countries, namely, France, Germany, Belgium, Denmark, Finland, Italy, Spain, Portugal, Canada and Japan. Apart from Canada, which appears non-stationary but mean reverting, the other low-inflation countries have an estimated order of integration between 0.17 and 0.45. The fractional differencing parameter estimates and their t-statistics indicate that the models are significantly different from $I(1)$ and $I(0)$ behaviour. An interesting interpretation of these models is that an inflationary shock will have long memory and persistence, but, eventually, will be mean reverting. Furthermore, six countries (Denmark, Finland, Italy, Spain, Portugal and Japan) show a clear evidence of parameter non constancy in the GARCH part of the model, so the dual persistence in the conditional mean of inflation and in its conditional variance is well described by *ARFIMA-STVGARCH* processes.

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A Appendix: Simulation Results

A.1 Lagrange Multiplier test

Table 1. *Simulated size for the test of FI-GARCH(1,1) against the alternative FI-STVGARCH(1,1) with $d = 0.20$*

T	α	Tests	i	ii	iii	iv	v	vi	vii
500	1%		0.02	0.03	0.02	0.03	0.02	0.03	0.03
	5%		0.10	0.07	0.09	0.09	0.07	0.08	0.06
	10%		0.18	0.15	0.19	0.16	0.16	0.12	0.15
1000	1%		0.02	0.01	0.02	0.02	0.02	0.02	0.01
	5%		0.06	0.02	0.05	0.05	0.06	0.05	0.07
	10%		0.11	0.08	0.11	0.08	0.09	0.07	0.09
2000	1%		0.01	0.01	0.02	0.02	0.02	0.02	0.01
	5%		0.07	0.02	0.03	0.04	0.05	0.03	0.05
	10%		0.09	0.06	0.07	0.05	0.07	0.06	0.07

Table 2. *Simulated size for the test of FI-GARCH(1,1) against the alternative FI-STVGARCH(1,1) with $d = 0.45$*

T	α	Tests	i	ii	iii	iv	v	vi	vii
500	1%		0.01	0.00	0.01	0.00	0.00	0.00	0.00
	5%		0.03	0.04	0.03	0.03	0.05	0.02	0.02
	10%		0.12	0.08	0.12	0.06	0.10	0.08	0.08
1000	1%		0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%		0.02	0.02	0.01	0.00	0.03	0.01	0.00
	10%		0.08	0.06	0.09	0.02	0.05	0.03	0.05
2000	1%		0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%		0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10%		0.02	0.03	0.04	0.00	0.01	0.00	0.02

Note: The Tables 1 and 2 correspond to $d = 0.20$ and $d = 0.45$; they report the rejection frequencies for the tests (i)-(vii) at the three theoretical significance levels $\{1\%, 5\%, 10\%\}$ and for the three sample sizes $\{500, 1000, 2000\}$. The data generating process is given by DGP (I).

Table 3. Simulated power for the test of $FI-GARCH(1, 1)$ against the alternative $FI-STVGARCH(1, 1)$ with $d = 0.20$

T	α	<i>DGP</i>	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)
		<i>Tests</i>	<i>i</i>	<i>ii</i>	<i>iii</i>	<i>iv</i>	<i>v</i>	<i>vi</i>	<i>vii</i>
500	1%		0.81	0.24	0.84	1.00	1.00	0.66	0.99
	5%		0.93	0.58	0.96	1.00	1.00	0.87	1.00
	10%		0.98	0.77	0.99	1.00	1.00	0.93	1.00
1000	1%		0.98	0.66	0.94	1.00	1.00	0.96	1.00
	5%		1.00	0.92	1.00	1.00	1.00	1.00	1.00
	10%		1.00	0.95	1.00	1.00	1.00	1.00	1.00
2000	1%		1.00	0.95	1.00	1.00	1.00	0.96	1.00
	5%		1.00	0.99	1.00	1.00	1.00	0.99	1.00
	10%		1.00	1.00	1.00	1.00	1.00	0.99	1.00

Table 4. Simulated power for the test of $FI-GARCH(1, 1)$ against the alternative $FI-STVGARCH(1, 1)$ with $d = 0.45$

T	α	<i>DGP</i>	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)
		<i>Tests</i>	<i>i</i>	<i>ii</i>	<i>iii</i>	<i>iv</i>	<i>v</i>	<i>vi</i>	<i>vii</i>
500	1%		0.82	0.30	0.84	0.99	1.00	0.74	1.00
	5%		0.94	0.72	0.98	1.00	1.00	0.95	1.00
	10%		0.97	0.98	0.98	1.00	1.00	0.98	1.00
1000	1%		0.99	0.62	0.96	1.00	1.00	0.94	1.00
	5%		1.00	0.90	1.00	1.00	1.00	0.99	1.00
	10%		1.00	0.95	1.00	1.00	1.00	1.00	1.00
2000	1%		1.00	0.88	1.00	1.00	1.00	0.98	1.00
	5%		1.00	0.97	1.00	1.00	1.00	0.99	1.00
	10%		1.00	0.98	1.00	1.00	1.00	0.99	1.00

Note: The Tables 3 and 4 correspond to $d = 0.20$ and $d = 0.45$; they report the rejection frequencies for the tests $(i)-(vii)$ at the three theoretical significance levels $\{1\%, 5\%, 10\%\}$ and for the three sample sizes $\{500, 1000, 2000\}$. The data generating processes are given by DGP's $\{II, III, IV, V, VI, VII, VIII\}$.

A.2 Likelihood Ratio test

Table 5. *Simulated size for the test of FI-GARCH(1,1) against the alternative FI-STVGARCH(1,1) with $d = 0.20$*

T	α	<i>Tests</i>	<i>i</i>	<i>ii</i>	<i>iii</i>	<i>iv</i>	<i>v</i>	<i>vi</i>	<i>vii</i>
500	1%		0.02	0.02	0.02	0.01	0.02	0.00	0.01
	5%		0.07	0.11	0.10	0.04	0.06	0.05	0.06
	10%		0.15	0.19	0.14	0.13	0.12	0.12	0.13
1000	1%		0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%		0.05	0.08	0.07	0.01	0.03	0.02	0.04
	10%		0.11	0.15	0.10	0.09	0.08	0.07	0.09
2000	1%		0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%		0.02	0.05	0.04	0.00	0.00	0.00	0.01
	10%		0.07	0.10	0.07	0.05	0.04	0.03	0.05

Table 6. *Simulated size for the test of FI-GARCH(1,1) against the alternative FI-STVGARCH(1,1) with $d = 0.45$*

T	α	<i>Tests</i>	<i>i</i>	<i>ii</i>	<i>iii</i>	<i>iv</i>	<i>v</i>	<i>vi</i>	<i>vii</i>
500	1%		0.02	0.01	0.03	0.02	0.02	0.02	0.02
	5%		0.10	0.08	0.08	0.06	0.08	0.07	0.06
	10%		0.12	0.18	0.12	0.15	0.14	0.16	0.14
1000	1%		0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%		0.06	0.05	0.04	0.03	0.04	0.02	0.03
	10%		0.07	0.13	0.07	0.11	0.09	0.11	0.10
2000	1%		0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%		0.02	0.01	0.01	0.00	0.01	0.00	0.00
	10%		0.04	0.06	0.02	0.06	0.05	0.06	0.06

Note: The Tables 5 and 6 correspond to $d = 0.20$ and $d = 0.45$; they report the rejection frequencies for the tests (i)-(vii) at the three theoretical significance levels $\{1\%, 5\%, 10\%\}$ and for the three sample sizes $\{500, 1000, 2000\}$. The data generating processes is given by DGP (I).

Table 7. Simulated power for the test of $FI-GARCH(1, 1)$ against the alternative $FI-STVGARCH(1, 1)$ with $d = 0.20$

T	α	<i>DGP</i>	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)
		<i>Tests</i>	<i>i</i>	<i>ii</i>	<i>iii</i>	<i>iv</i>	<i>v</i>	<i>vi</i>	<i>vii</i>
500	1%		0.95	0.97	0.99	1.00	1.00	0.66	0.99
	5%		0.98	1.00	0.99	0.99	0.90	0.90	1.00
	10%		0.98	1.00	0.99	0.99	0.90	0.93	1.00
1000	1%		1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5%		1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10%		1.00	1.00	1.00	1.00	1.00	1.00	1.00
2000	1%		1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5%		1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10%		1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 8. Simulated power for the test of $FI-GARCH(1, 1)$ against the alternative $FI-STVGARCH(1, 1)$ with $d = 0.45$

T	α	<i>DGP</i>	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)
		<i>Tests</i>	<i>i</i>	<i>ii</i>	<i>iii</i>	<i>iv</i>	<i>v</i>	<i>vi</i>	<i>vii</i>
500	1%		0.93	0.96	0.96	0.98	0.90	0.88	0.99
	5%		0.96	1.00	0.97	0.98	0.90	0.93	1.00
	10%		0.96	1.00	0.99	0.99	0.90	0.95	1.00
1000	1%		1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5%		1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10%		1.00	1.00	1.00	1.00	1.00	1.00	1.00
2000	1%		1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5%		1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10%		1.00	1.00	1.00	1.00	1.00	1.00	1.00

Note: The Tables 7 and 8 correspond to $d = 0.20$ and $d = 0.45$; they report the rejection frequencies for the tests $(i)-(vii)$ at the three theoretical significance levels $\{1\%, 5\%, 10\%\}$ and for the three sample sizes $\{500, 1000, 2000\}$. The data generating processes are given by DGP's $\{II, III, IV, V, VI, VII, VIII\}$.

B Appendix: Empirical Results

Table 9. *Summary statistics*

	μ	σ	k	sk	JB	$Q(12)$	$Q^2(12)$
<i>Portugal</i>	0.92	1.16	11.39	2.07	$1.52 \cdot 10^3$	343.56	48.17
<i>Spain</i>	0.67	0.68	6.02	1.25	268.42	799.82	111.06
<i>Denmark</i>	0.43	0.58	10.30	1.22	$1.03 \cdot 10^3$	349.05	56.19
<i>Finland</i>	0.47	0.57	6.14	1.25	280.79	745.57	140.06
<i>Italy</i>	0.65	0.57	4.93	1.32	187.79	$1.88 \cdot 10^3$	200.10
<i>France</i>	0.43	0.39	3.18	0.64	29.52	$2.32 \cdot 10^3$	270.87
<i>Belgium</i>	0.34	0.37	3.41	0.68	36.07	875.08	168.43
<i>Germany</i>	0.26	0.32	6.98	0.32	261.45	228.01	48.06
<i>Canada</i>	0.39	0.40	4.90	0.64	92.54	753.27	83.62
<i>Japan</i>	0.27	0.68	7.47	1.60	526.08	425.36	110.85

Notes: μ denotes the average inflation rate and σ its standard deviation. k is the Kurtosis, sk is the Skewness coefficient, JB is the Jarque-Bera normality test, $Q(12)$ and $Q^2(12)$ are respectively the 12-th order Ljung-Box tests for serial correlation in the standardized and squared standardized residuals.

Table 10. *Unit Root Test*

		<i>Portugal</i>	<i>Spain</i>	<i>Denmark</i>	<i>Finland</i>	<i>Italy</i>
<i>ADF</i>	y_t	-3.858	-2.191	-2.069	-2.703	-2.259
	Δy_t	-11.31	-13.64	-5.926	-12.38	-10.8
<i>Result</i>		<i>I(0)</i>	<i>I(1)</i>	<i>I(1)</i>	<i>I(1)</i>	<i>I(1)</i>
<i>PP</i>	y_t	-14.79	-13.22	-1.681	-15.28	-7.924
	Δy_t	-47.76	-49.29	-1.681	-59.52	-40.72
<i>Result</i>		<i>I(0)</i>	<i>I(0)</i>	<i>I(1)</i>	<i>I(0)</i>	<i>I(0)</i>
<i>KPSS</i>	y_t	3.178	4.505	3.114	4.386	3.921
	Δy_t	0.028	0.055	0.012	0.022	0.012
<i>Result</i>		<i>I(1)</i>	<i>I(1)</i>	<i>I(1)</i>	<i>I(1)</i>	<i>I(1)</i>

		<i>France</i>	<i>Belgium</i>	<i>Germany</i>	<i>Canada</i>	<i>Japan</i>
<i>ADF</i>	y_t	-1.804	-5.526	-4.865	-8.555	-12.30
	Δy_t	-11.06	-12.15	-11.46	-25.25	-25.68
<i>Result</i>		<i>I(1)</i>	<i>I(0)</i>	<i>I(0)</i>	<i>I(0)</i>	<i>I(0)</i>
<i>PP</i>	y_t	-7.038	-19.31	-13.94	-14.31	-14.89
	Δy_t	-59.52	-54.74	-38.62	-49.99	-45.17
<i>Result</i>		<i>I(0)</i>	<i>I(0)</i>	<i>I(0)</i>	<i>I(0)</i>	<i>I(0)</i>
<i>KPSS</i>	y_t	4.603	0.750	1.659	3.516	3.408
	Δy_t	0.032	0.007	0.014	0.014	0.012
<i>Result</i>		<i>I(1)</i>	<i>I(1)</i>	<i>I(1)</i>	<i>I(1)</i>	<i>I(1)</i>

Note: The unit root tests are Dickey-Fuller Augmented (ADF), Phillips and Perron (PP) and Kwiatkowski et al. (KPSS) tests. For ADF and PP tests, the 1% and 5% critical values are -3.455 and -2.871, respectively. For KPSS test, the 1%, and 5% critical values are 0.739 and 0.463, respectively.

Table 11 : *Testing ARFIMA-GARCH (1,1)*
against ARFIMA-STVGARCH (1,1)

<i>Tests</i>	<i>Portugal</i>	<i>Spain</i>	<i>Denmark</i>	<i>Finland</i>	<i>Italy</i>
<i>i</i>	33.61 [6 10 ⁻⁹]	2.25 [0.13]	10.43 [1 10 ⁻³]	7.63 [5 10 ⁻³]	24.74 [6 10 ⁻⁷]
<i>ii</i>	10.62 [1 10 ⁻³]	7.44 [6 10 ⁻³]	9.86 [1 10 ⁻³]	4.64 [0.03]	20.46 [6 10 ⁻⁶]
<i>iii</i>	—	16.98 [6 10 ⁻⁵]	12.68 [3 10 ⁻⁴]	8.12 [4 10 ⁻³]	28.21 [1 10 ⁻⁷]
<i>iv</i>	1.74 [0.18]	—	—	—	—
<i>v</i>	—	—	10.04 [1 10 ⁻³]	8.27 [4 10 ⁻³]	1.47 [0.22]
<i>vi</i>	—	0.095 [0.75]	9.53 [2 10 ⁻³]	0.89 [0.34]	2.29 [0.13]
<i>vii</i>	—	—	12.35 [4 10 ⁻⁴]	—	—

<i>Tests</i>	<i>France</i>	<i>Belgium</i>	<i>Germany</i>	<i>Canada</i>	<i>Japan</i>
<i>i</i>	—	1.77 [0.18]	0.11 [0.73]	0.02 [0.88]	6.93 [8 10 ⁻³]
<i>ii</i>	—	1.67 [0.19]	0.05 [0.81]	0.03 [0.84]	13.76 [2 10 ⁻⁴]
<i>iii</i>	—	1.63 [0.20]	0.05 [0.81]	0.007 [0.92]	16.15 [5 10 ⁻⁵]
<i>iv</i>	—	—	—	—	—
<i>v</i>	—	—	—	—	1.04 [0.30]
<i>vi</i>	—	—	—	—	5 10 ⁻⁷ [0.99]
<i>vii</i>	—	—	—	—	—

Notes: The first column gives the tests specification used in simulations. The numbers in brackets are the p-values.

Table12 : *Estimated ARFIMA-STVGARCH models for countries CPI inflation*

	<i>Portugal</i>	<i>Spain</i>	<i>Denmark</i>	<i>Finland</i>	<i>Italy</i>
\hat{d}	0.22 (6.48)	0.20 (3.48)	0.17 (2.21)	0.45 (4.25)	0.35 (12.36)
$\hat{\mu}$	0.02 (0.32)	0.62 (2.08)	0.14 (1.98)	-0.06 (-1.07)	-0.03 (0)
$\hat{\phi}_1$	-	-0.84 (-3.44)	-0.67 (-5.61)	-0.38 -3.96	-
$\hat{\psi}_1$	-	0.83 (3.51)	0.66 (5.55)	-0.13 (-2.65)	-
$\hat{\psi}_2$	-	0.83 (3.51)	-0.10 (-2.01)	-	-
$\hat{\psi}_{12}$	0.32 (6.57)	0.36 (3.91)	0.45 (3.45)	0.31 (5.99)	0.26 (5.78)
$\hat{\psi}_{24}$	0.23 (5.91)	0.29 (3.64)	0.18 (2.17)	0.15 (3.42)	0.23 (5.10)
$\hat{\alpha}_{00}$	2.68 (31.39)	0.11 (1.11)	0.04 (1.88)	0.26 (2.75)	0.28 (5.35)
$\hat{\alpha}_{01}$	0.12 (1.99)	0.12 (1.04)	$8.4 \cdot 10^{-18}$ (0)	0.05 (1.23)	0.06 (1.98)
$\hat{\beta}_{01}$	-	0.66 (3.60)	0.98 (7.92)	0.88 (14.29)	-
$\hat{\alpha}_{10}$	-2.61 (-30.93)	-0.10 (-0.70)	-0.02 (-1.69)	-0.24 (-2.62)	-0.26 (-2.02)
$\hat{\alpha}_{11}$	-	-	0.02 (1.79)	-	-
$\hat{\beta}_{11}$	-	0.04 (0.03)	-0.32 (-5.93)	-0.34 (-17.58)	-
$\hat{\gamma}$	15.71 (5.19)	12.40 (1.99)	6.93 (3.35)	8.34 (4.47)	19.15 (2.48)
\hat{c}	0.31 (11.29)	0.39 (3.83)	0.41 (4.01)	0.01 (0.47)	0.28 (7.78)

	<i>France</i>	<i>Belgium</i>	<i>Germany</i>	<i>Canada</i>	<i>Japan</i>
\hat{d}	0.34 (8.42)	0.29 (4.68)	0.25 (5.45)	0.60 (7.46)	0.28 (2.33)
$\hat{\mu}$	0.43 (2.83)	0.29 (2.61)	0.24 (3.08)	0.92 (1.92)	-0.05 (-0.34)
$\hat{\phi}_1$	-0.61 (-6.10)	—	—	0.06 (0.62)	0.17 1.69
$\hat{\psi}_1$	0.75 (8.96)	-0.07 (-1.22)	—	-0.55 (-9.89)	-0.38 (-10.52)
$\hat{\psi}_2$	—	-0.14 (-2.47)	—	—	-0.21 (-3.95)
$\hat{\psi}_{12}$	0.22 (4.04)	0.24 (4.65)	0.22 (4.04)	0.15 (3.14)	0.37 (7.09)
$\hat{\psi}_{24}$	0.16 (3.58)	0.14 (3.12)	—	0.12 (2.81)	0.19 (4.72)
$\hat{\alpha}_{00}$	—	—	—	0.02 (2.41)	0.08 (2.29)
$\hat{\alpha}_{01}$	—	0.08 (2.09)	0.06 (1)	0.27 (3.28)	0.04 (0.61)
$\hat{\beta}_{01}$	—	0.78 6.97	0.81 (4.57)	0.58 (6.50)	0.92 (6.17)
$\hat{\alpha}_{10}$	—	—	—	—	—
$\hat{\alpha}_{11}$	—	—	—	—	—
$\hat{\beta}_{11}$	—	—	—	—	-0.88 (-5.04)
$\hat{\gamma}$	—	—	—	—	5.72 (2.03)
\hat{c}	—	—	—	—	0.39 (3.52)

Notes: For each series, Table 12 reports QML parameter estimates for the *ARFIMA-STVGARCH* model. The numbers in parentheses are t-statistics.

Table 13 : *Diagnostic tests*

	<i>Portugal</i>	<i>Spain</i>	<i>Denmark</i>	<i>Finland</i>	<i>Italy</i>
$\log L$	-373.34	-207.09	-190.06	-165.87	-95.96
$Q(12)$	16.56 [0.17]	15.31 [0.22]	12.20 [0.43]	5.49 [0.93]	12.59 [0.40]
$Q^2(12)$	6.87 [0.86]	7.81 [0.79]	7.84 [0.79]	10.08 [0.60]	12.34 [0.41]
JB	22.38 [1 10 ⁻⁵]	30.58 [2 10 ⁻⁷]	153.62 [0]	14.36 [7 10 ⁻⁴]	22.76 [1 10 ⁻⁵]
sk	0.44	0.51	0.39	0.33	0.22
k	3.76	3.90	5.96	3.66	4.08

	<i>France</i>	<i>Belgium</i>	<i>Germany</i>	<i>Canada</i>	<i>Japan</i>
$\log L$	-46.29	-55.38	-54.08	-107.02	-232.65
$Q(12)$	12.28 [0.42]	5.62 [0.93]	4.31 [0.97]	8.38 [0.75]	14.14 [0.29]
$Q^2(12)$	12.15 [0.43]	11.96 [0.49]	6.55 [0.88]	5.69 [0.93]	8.78 [0.72]
JB	73.96 [1 10 ⁻¹⁶]	6.23 [0.04]	615.82 [0]	162.15 [0]	95.43 [0]
sk	0.48	0.26	0.09	0.52	0.49
k	4.89	3.33	9.30	5.97	5.21

Notes: $\log L$ denotes the maximum value of the log likelihood. $Q(12)$ and $Q^2(12)$ are respectively the 12-th order Ljung-Box tests for serial correlation in the standardized and squared standardized residuals. JB is the Jarque-Bera normality test, sk is the Skewness coefficient and k is the Kurtosis. The numbers in brackets are the p -values.

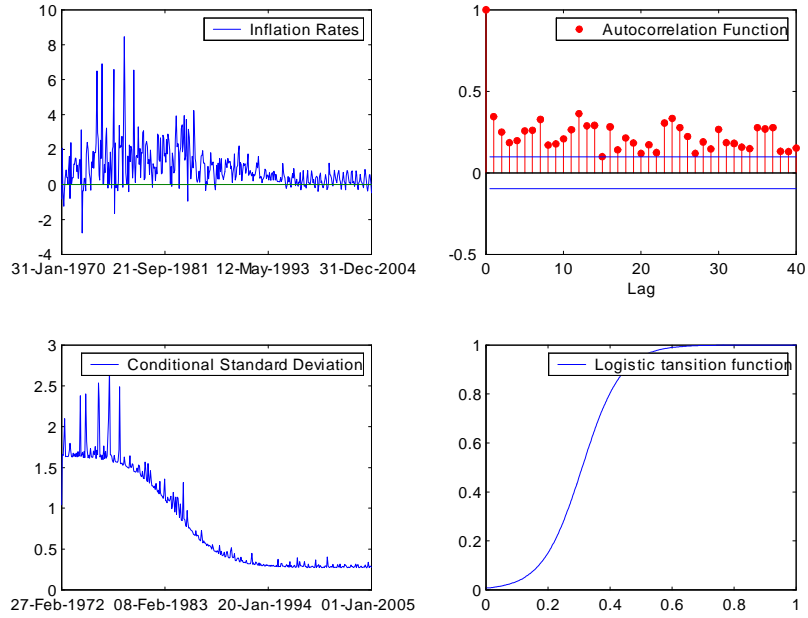


Figure 3. Plots of the inflation rates, the autocorrelation function, the conditional standard deviation and the logistic transition function of Portugal.

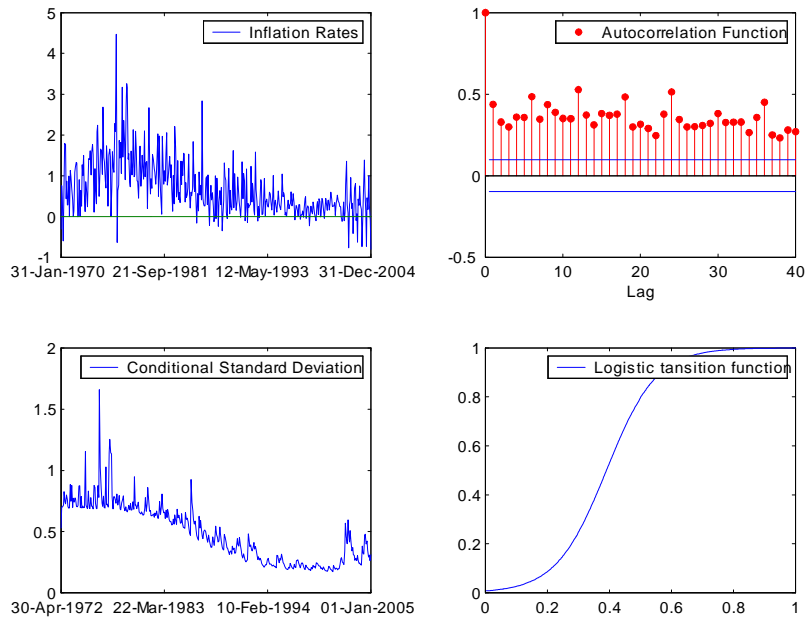


Figure 4. Plots of the inflation rates, the autocorrelation function, the conditional standard deviation and the logistic transition function of Spain.

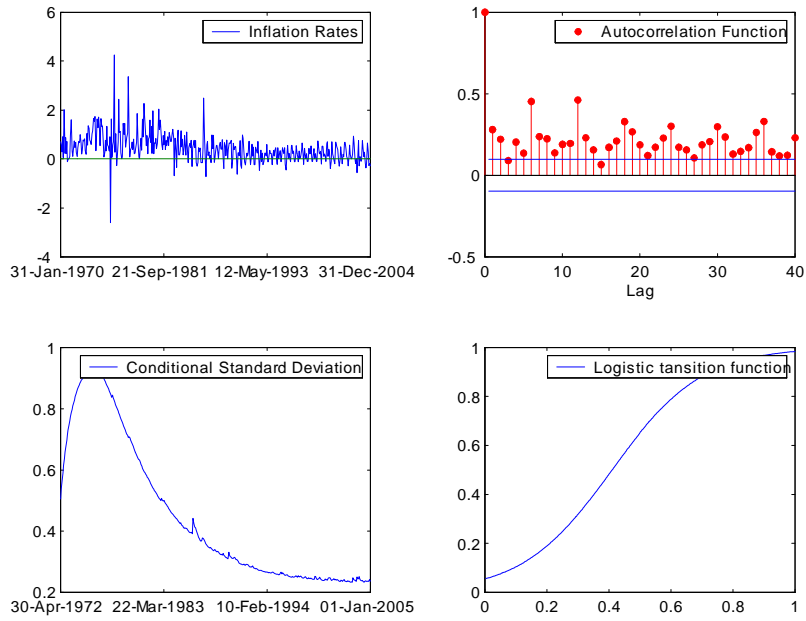


Figure 5. Plots of the inflation rates, the autocorrelation function, the conditional standard deviation and the logistic transition function of Denmark.

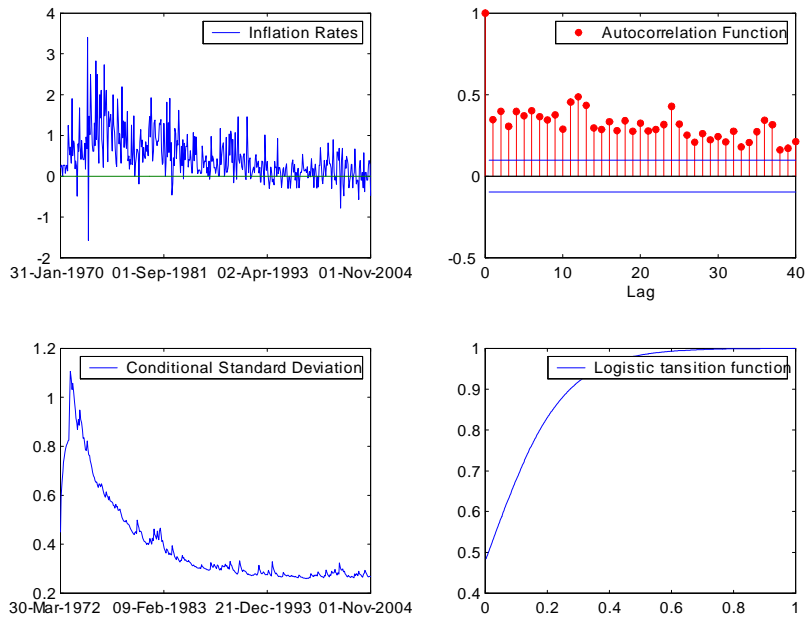


Figure 6. Plots of the inflation rates, the autocorrelation function, the conditional standard deviation and the logistic transition function of Finland.

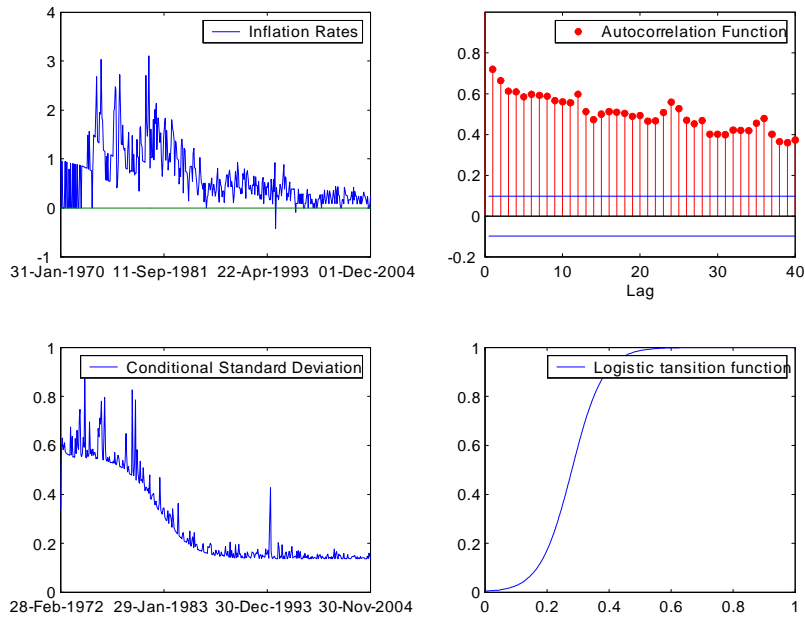


Figure 7. Plots of the inflation rates, the autocorrelation function, the conditional standard deviation and the logistic transition function of Italy.

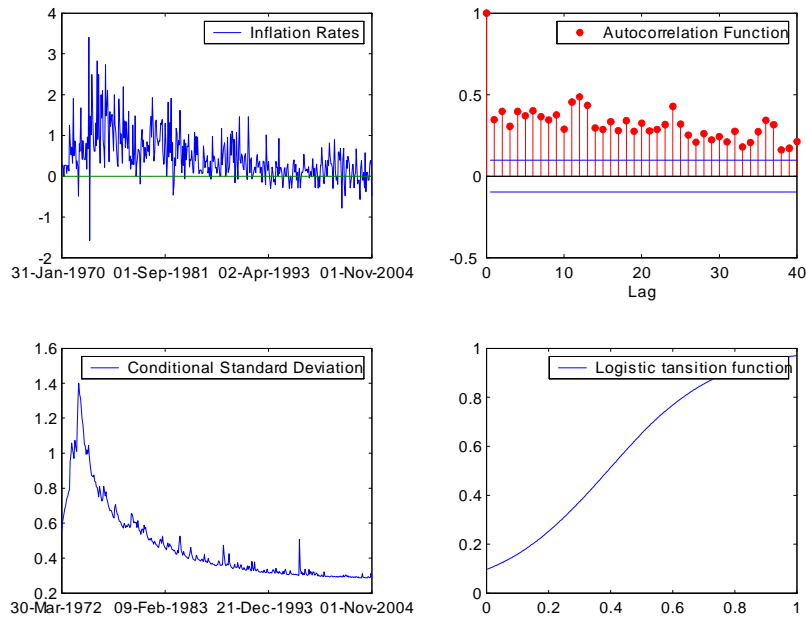


Figure 8. Plots of the inflation rates, the autocorrelation function, the conditional standard deviation and the logistic transition function of Japan.