



# An Agent-Based Simulation of Rental Housing Markets

John Mc Breen, Florence Goffette-Nagot, Pablo Jensen

## ► To cite this version:

John Mc Breen, Florence Goffette-Nagot, Pablo Jensen. An Agent-Based Simulation of Rental Housing Markets. 2009. halshs-00374157v1

**HAL Id: halshs-00374157**

**<https://shs.hal.science/halshs-00374157v1>**

Submitted on 8 Apr 2009 (v1), last revised 16 Jul 2009 (v2)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

**DOCUMENTS DE TRAVAIL - WORKING PAPERS**

W.P. 09-08

**An Agent-Based Simulation of Rental Housing  
Markets**

**John Mc Breen, Florence Goffette-Nagot, Pablo Jensen**

Avril 2009

GATE Groupe d'Analyse et de Théorie Économique  
UMR 5824 du CNRS  
93 chemin des Mouilles – 69130 Écully – France  
B.P. 167 – 69131 Écully Cedex  
Tél. +33 (0)4 72 86 60 60 – Fax +33 (0)4 72 86 60 90  
Messagerie électronique [gate@gate.cnrs.fr](mailto:gate@gate.cnrs.fr)  
Serveur Web : [www.gate.cnrs.fr](http://www.gate.cnrs.fr)

# An Agent-Based Simulation of Rental Housing Markets

John Mc Breen<sup>1,2,3,4</sup>, Florence Goffette-Nagot<sup>1,4,5</sup> and Pablo Jensen<sup>1,2,3,4</sup>

April 7, 2009

<sup>1</sup>Université de Lyon, Lyon, F-69003, France.

<sup>2</sup>CNRS, LET, UMR 5593, Lyon, F-69363 France.

<sup>3</sup>ENS-LYON, Laboratoire de Physique, UMR 5672, Lyon, F-69007, France.

<sup>4</sup>Institut rhônalpin des systèmes complexes, IXXI, Lyon, F-69007, France.

<sup>5</sup>CNRS, GATE, UMR 5824, Ecully, F-69130, France.

**Keywords:** Real estate, Rental markets, Search, Information, Simulation, Multi-agent systems

**JEL classification:** R13, R20, R31, C63, J63

## Abstract

We simulate a closed rental housing market with search and matching frictions, in which both landlord and tenant agents are imperfectly informed. Homogeneous landlords set rents to maximise revenue, using information on the market to estimate the relationship between posted rent and time-on-the-market (TOM). Tenants, heterogeneous in income, engage in undirected search accepting residences based on their idiosyncratic tastes for housing and a disagreement point derived from information on the distribution of offers. The steady state to which the simulation evolves shows price dispersion, nonzero search times and vacancies.

The main results concern the effects of increasing information on either side of the market. When tenants see a greater percentage of the distribution of offers, tenants learn to refuse high rents and so the population rises and tenants' utilities rise as does overall welfare. Conversely, when landlords have less information, their utility can rise as over estimations in best posting rent move the market to higher rents.

# 1 Introduction

Search market frictions are a vitally important phenomenon in the urban housing market as imperfectly-informed agents take time to meet and agree on prices. The market frictions that result from search are responsible for many important phenomena in search markets such as vacancies, price dispersion and positive times on the market. Housing markets are also characterised by idiosyncratic preferences for housing, adjustment of housing consumption by moving, an oligopolistic supply of dwellings and slow adjustment of supply in response to market changes.

The impact of frictions has been extensively studied in search-matching models of the labour market. A survey of search theoretic models of the labour market by (Rogerson and Wright, 2005), presents the various approaches to modelling the main search related frictions: how agents meet and how wages are determined.

The first major contribution to the housing market literature based on this pre-existing labour market literature is that of (Wheaton, 1990). He created a model of the owner-occupier market in which buyers are also sellers and the cost of search effort and its efficiency are defined by an exogenous matching function. The ‘thin’ nature of the housing rental market due to the heterogeneity of housing and tenant’s idiosyncratic tastes has been used to explain vacancies by (Arnott, 1989). The existence of vacancies is shown to have a social function as it expands the choice set of tenants. (Read, 1993, 1997) developed two models of the rental market with rent dispersion and vacancies. (Read, 1993) is a partial equilibrium model with rent dispersion and vacancies where searchers with heterogeneous information engage in directed search. (Read, 1997) is a partial equilibrium model where tenants, who have heterogeneous incomes, engage in stochastic search. The imperfect information of tenants leads to equilibrium rent dispersion. All of these *static* analytical models contain strong simplifying assumptions which ignore important aspects of the housing market. Among these assumptions are the ‘law of one price’, the absence of heterogeneities and perfect information.

We propose a multi-agent model as a basis for relaxing many of these assumptions in order to obtain a more realistic model of urban housing markets, while retaining strong foundations from the existing search market literature. We develop a simulation model of a closed urban housing market focusing on the role of information. Landlords deduce times-on-the-market (TOM) corresponding to posted rents from market signals and optimise their expected utility on this basis. Tenants with heterogeneous incomes have idiosyncratic preferences for housing units and engage in undirected search, selecting their reservation utility using imperfect information on the distribution of posted rents.

Our results also have to be interpreted in light of the consumer search literature that has studied the various effects of search on price dispersion and market outcomes. A survey of search and dispersion is (Baye and Scholten, 2006).

## 2 Related Literature

### 2.1 Empirical

We review here the empirical work on the main phenomena related to frictions in the housing market : agent's information, vacancies, time on the market and price setting, and price dispersion.

The role of information in the evolution of housing markets has received much attention. (Fisher et al., 2003) studied the correlations in prices and liquidity changes over the housing market cycle. It is well known that when prices rise liquidity is high and when prices fall liquidity is low. They attributed this to the differing rates at which agents update their beliefs. For example, when liquidity is low, there are few market signals for sellers to follow, causing them to be slow to update their beliefs. (Clayton et al., 2008) examined a number of possible explanations of the correlations in price and liquidity changes, and found evidence supporting sellers slow rate for updating their beliefs. In our model, the rate at which information arrives affects the quality of landlord's estimation of the market state.

We now turn to the role of vacancies. (Blanck and Winnock, 1953) made a major contribution to the understanding of vacancies' role in determining urban housing price dynamics. They showed that price dynamics appear to be led by changes in the vacancy rate. A more thorough analysis was performed by (Rosen and Smith, 1983) who provided evidence, in a cross city analysis, of the existence of natural vacancy rates that are crucial in determining the strong correlations between fluctuations in the vacancy rate and the evolution of rents. Numerous authors report similar results including (Gabriel and Nothaft, 1988) in the rental housing market, (Shilling et al., 1987, Grenadier, 1995) in the office rental market and (Hwang and Quigley, 2006) in the purchasing housing market.

The price setting behaviour of imperfectly informed agents in search markets - who trade-off higher prices with longer selling times - is an important guide in model building. This has been closely studied, primarily in the residential purchasing market. (Yavas and Yang, 1995, Knight, 2002, Anglin et al., 2003) have looked at the role of posted price on the final price obtained and the time on the market. (Merlo and Ortalo-Magne, 2004) have also looked at posted price changes and found that when reductions in asking price occur, they are generally substantial rather than incremental.

Price dispersion is a near universal aspect of search markets. Clear evidence has been presented by (Leung et al., 2006), for the Hong Kong purchasing market. The duration of residence is another important element in any housing model. Empirical data in various rental housing markets in the USA has been presented by (Deng et al., 2003) and across tenure types by (de Una-Alvarez et al., 2008) for the spanish market.

## 2.2 Simulations

Simulation models have considerable advantages over analytical approaches when modelling dynamic processes and heterogeneities among agents. The difficulty of studying out-of-equilibrium dynamics in current analytical search models with price dispersion has been underlined by (Postel-Vinay and Robin, 2006). (Bradburd et al., 2005, 2006) use agent-based simulations to relax the assumption of a single price with random matching and Nash bargaining, and do not model search behaviour. They use their model to examine the distributional effects of rent controls (Bradburd et al., 2006) and ‘access discrimination’ (Bradburd et al., 2005), modelled as a reduced matching probability. In contrast, our simulation explicitly models search with rent posting, which is more realistic in the rental housing market.

Agent-based modeling is described by (Bradburd et al., 2006) as follows:

Agent-based modeling is analysis of complex systems through simulations that are based on specification of the behavior of individual agents who interact, within some structured space of possible choices, over time consisting of rounds of interaction or cycles in the computer running the simulation. Unlike most simulation exercises with which economists and regional scientists may be familiar, agent-based modeling does not specify structural equations for the entire market. The analysis proceeds instead through specification of relatively simple behavioral rules for each agent, along with the structure of the space within which they interact. The outcome of the exercise is a sequence of decisions that can be summarized and analyzed using the same techniques used for empirical analysis of micro data. Thus an agent-based model is a data-generating process (to borrow from econometric jargon) where the analyst has control over features of the process, and can investigate how changing these features will affect the data that emerge. In this way, agent-based modeling permits formation of testable hypotheses about the likely impacts of comparable changes in actual markets or systems.

The evolution of urban rents is influenced by many factors, including interest rates, conditions of credit, land supply and zoning, the economic environment, incomes, demographics etc. All of these factors cannot be easily integrated into a single model. A notable and very detailed multi-agent model of the 1970’s French labour market was constructed by (Ballot, 2002). Here, we shall attempt to determine the generic effects that result from the fundamental market structure of heterogeneous agents searching in an uncertain market in order to maximise their utilities.

### 3 Model

In housing markets, the state of the market is determined by the interaction of the search procedures of households looking for dwelling space and landlords searching for tenants, and by tenant turnover and institutional and market characteristics specific to each city.

Landlords can relatively easily inform themselves of the approximate going rental rates for their dwellings. In this uncertain market they then have a strategic choice of what rent to ask, which is likely to influence the arrival rate of potential tenants. According to many studies, (Yavas and Yang, 1995), (Merlo and Ortalo-Magne, 2004) and (Anglin et al., 2003), the higher the posted price, the longer the time on the market, with the associated costs for landlords.

Tenants are presented with a market in which the acquisition of information on residences is costly in time and money. There are also considerable costs associated with visiting residences. Tenants must decide at what level of utility a residence they have viewed is likely to be better than the outside opportunity, which is uncertain in quality and associated extra search costs.

The interactions of these two types of agents, which we shall model, are clearly complicated even before we consider the heterogeneity in strategies employed, in agents' preferences, incomes and urgencies of search. The question for the modeller is what set of hypotheses can most faithfully capture this complexity while retaining a simplicity of conception that allows pertinent insights to be obtained from the model.

Our aim is to build a model close enough to the existing search literature for its results to be comparable, while allowing the extension of these existing models in different directions. Two parallel paths have been followed in search-matching models of the labour market. Models aimed at analysing varying unemployment rates have focused on the matching function and search behaviours on the part of searching workers (Pissarides, 1990, Mortensen and Pissarides, 1994). Models aiming to explain the observed dispersion of wages have highlighted the behaviours of wage posting firms, that benefit from the existence of frictions to offer wages below the walrasian wage (Burdett and Mortensen, 1989, 1998, Mortensen, 1990). (Mortensen, 2000) builds a model merging the two approaches. He underlines the contribution of doing so: by allowing for a general equilibrium in which the wage distribution and the unemployment rate interact, this model allows the study of the welfare effect of a minimum wage or an unemployment subsidy policy.

Our model follows the same direction by explicitly modelling landlords that set rents and make take-it-or-leave-it offers to the tenants. These landlords face a trade-off between setting a higher rent and finding a tenant more quickly. Their optimising behavior is based on their knowledge of the market state, both in terms of rent offers and corresponding times-on-the-market. Tenants are supposed to observe a sample of the offer distribution but to visit residences sequentially. They accept offers based on an optimising behaviour that trade-offs a quicker match and therefore reduced search costs against a lower rent.

As will be clearer in the following and in the results, our agent model departs

from theoretical models in that tenant agents do not know perfectly the rent offer distribution. More importantly, landlord agents do not have the knowledge of the acceptance probability of all tenants searching on the market. The discrepancy in the results which ensue from these differences is highlighted.

In the following, we outline more specifically the proposed agent behaviours, the interaction mechanism as well as the dimensions of heterogeneity whose influence on simulation outcomes shall be explored.

### 3.1 Tenants

Tenant agents have heterogeneous incomes which translate into heterogeneous housing budgets for the purpose of this model. In the closed town unhoused tenant agents search every iteration. A searching agent sees one randomly chosen apartment from the distribution of offers, and has knowledge of a percentage of the full distribution of offers. Upon visiting an apartment, they have to decide whether to accept it or keep searching. This will depend upon their idiosyncratic preference for the apartment, the posted rent, their housing budget and the cost of search. Using these elements, searchers choose their reservation utility  $U_{res}$ , that is the minimum utility they are willing to accept from a residence. Each tenant is fully described by his *housing budget*. In this simple model, this represents the maximum price she is willing to pay to rent an apartment (assuming neutral idiosyncratic preferences) and is a fixed percentage of income. Tenants' homogeneous costs of search are equal to the largest housing budget, and are assumed to include all costs of searching in the town. These costs include temporary accommodation costs, transport costs, lost earnings and estate agency fees.

We assume that each tenant has a separable utility function whose housing part is given by

$$U = Y - R + \theta \quad (1)$$

where  $Y$  = housing budget,  $R$  = rent and  $\theta$  = the idiosyncratic preference of an agent for an apartment, discovered by the agent once the apartment is visited.  $\theta \sim N(0, \sigma)$ , where  $\sigma$  is the variance of the normally distributed idiosyncratic preferences,  $\sigma$  is expressed as a percentage of the housing budget, as shown in Table 1.

All unhoused tenants engage in sequential undirected search seeing one random available residence per iteration which they either accept or refuse, having visited it to learn their idiosyncratic preference for the residence following their reservation utility. Their decision variable is their reservation utility  $U_{res}$ , which is optimised to yield the maximum utility per unit time over the expected period of search and residence, knowing that their idiosyncratic preferences for apartments are on average zero. Tenants see a percentage  $S_T$  of residences currently on the market and use this knowledge to recalculate their reservation utility each iteration.

The expected benefit for a given reservation utility is given by



$$E[B(U_{res})] = \frac{1}{X + T(U_{res})} * \left( (Y - C_T) \int_0^{T(U_{res})} \exp(rt) dt + \int_{T(U_{res})}^{T(U_{res})+X} E[U(U_{res})] \exp(rt) dt \right) \quad (2)$$

where  $C_T$  is the cost of searching,  $X$  is the expected duration of residence,  $T(U_{res})$  is the expected search time,  $E[U(U_{res})]$  is the expected utility flow per iteration once housed if the chosen reservation utility is  $U_{res}$  and  $r$  is the discount rate. The constant probability of moving out of a residence in any given iteration is a satisfactory approximation, see (de Una-Alvarez et al., 2008).

The expected probability of accepting the residence seen in any given iteration is simply the number of residences the agent is prepared to accept divided by the total number of residences, both quantities being taken from the residences that the agent sees. The expected search time  $T(U_{res})$  in iterations is equal to the inverse of the probability of accepting a residence at each iteration. The expected utility flow per iteration once housed  $E[U(U_{res})]$ , is given by the average utility of residences which give utility larger than  $U_{res}$  :

$$E[U(U_{res})] = \frac{\sum_{offers} \phi(U(U_{res})) * U}{\sum_{offers} \phi(U(U_{res}))}, \quad (3)$$

where  $\phi(U(a)) = 1$  if  $U > a$  and  $\phi(U(a)) = 0$  if  $U \leq a$ . Note that the instantaneous utility enjoyed is often greater than  $U_{res}$ .

As expected, the probability of accepting an apartment increases when the outside opportunity is less promising. Once a residence is rejected it cannot be revisited, unless it remains vacant and is randomly reselected.

### 3.2 Landlords

Landlords' decision variable is what rent to post. In making this decision, they are assumed to trade-off speed of sale with rent procured. Landlords can be assumed to be generally aware of the market price for their class of apartment and to have a somewhat vaguer idea of the time required to sell an apartment, as this is harder to observe. Research has shown (Yavas and Yang, 1995; Knight, 2002; Anglin et al., 2003) that in the selling market a house with a higher asking price generally takes longer to sell, particularly in thick markets.

Each landlord is fully described by his maintenance cost  $C_L$ . The off-market cost associated with ownership is normalised to zero<sup>1</sup> and hence  $C_L$  is the minimum price for which she is willing to rent her apartment. Here, landlords are homogeneous in their maintenance costs. They never leave the town but

---

<sup>1</sup>This is equivalent to assuming that the expected utility flow in another activity is equal to zero.

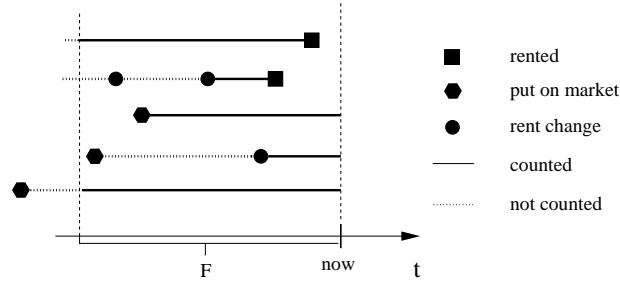


Figure 1: The periods of time-on-the-market known to landlords are shown above.

may withdraw from the market if they do not expect to benefit from participation. Contracts are for fixed rents and have an exogenous probability of being terminated each iteration.

Landlords' perceptions of the market state are characterised by the expected search time required to find a tenant for a given asking rent. Landlords are assumed to have access to information on a certain percentage of residences on the market over the last  $F$  iterations. Concretely, they know for these residences how many iterations they were on the market at their most recent market price, within the last  $F$  iterations, and whether or not they have been rented, as shown in Figure 1. This procedure generates two histograms, one of the cumulative times on the market within each rent interval (of size  $R_I$ , see Table 2) and another of the number of sales within each interval. This allows landlords to calculate the probability per iteration of finding a tenant for a range of price intervals, making the implicit assumption that the probability to sell was constant over the last  $F$  iterations. This probability is simply the number of sales divided by the cumulative times on the market. The assumption of a constant hazard means that the landlords assume an exponential probability distribution for the time-on-the-market (TOM) for a constant rent, see (Kiefer, 1988).

The sale probability per iteration  $\lambda$ , is simply the inverse of the probable time required to rent.

$$E(T) = \int_0^{\infty} t\lambda \exp(-\lambda t) = \frac{1}{\lambda} \quad (4)$$

The landlords then calculate the expected time-on-the-market from the data and estimate the best least-squares fit of the exponential function for the expected time-on-the-market  $T(R)$  as a function of the rent  $R$ ,<sup>2</sup>

$$T(R) = A \exp(DR) \quad (5)$$

<sup>2</sup>Each point  $T(R) = \omega$  is given a weight equal to the natural logarithm of the number of rentals  $N(R)$  in the rent interval centred on  $R$  plus one, that is  $\text{weight} = \ln(N(R) + 1)$ . This weighting as opposed to a linear weighting gives greater importance to rarer information (high accepted rents) and hence causes a reduction in posted rents.

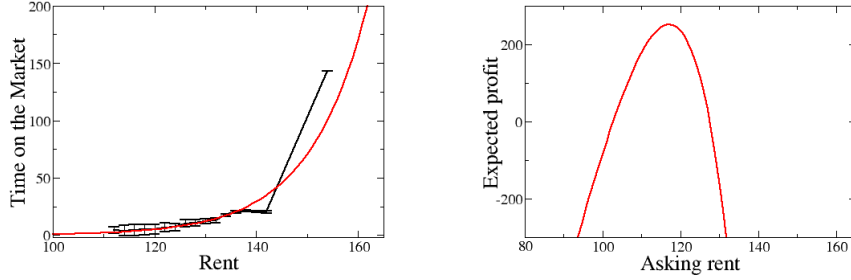


Figure 2: Left: Example of an estimated relation (red line) between asking rent and TOM per rental. The size of the ‘error bars’ is the statistical weight given to each point in the least-squares fit of the exponential. Right: The corresponding expected profits.

where  $A$  and  $D$  are fit parameters.

Landlords then calculate their most advantageous rent, that is the rent that in expectation provides the highest benefit per iteration. The function that gives this expected benefit is  $B(R)$  :

$$B(R) = \frac{1}{X + T(R)} \left( -C_L \int_0^{T(R)} e^{-rt} dt + (L - C_L) \int_{T(R)}^{X+T(R)} e^{-rt} dt \right) \quad (6)$$

where  $r$  is the discount rate,  $X$  is the exogenous expected time a tenant will stay in the apartment and  $C_L$  is the maintenance cost per iteration. To avoid errors by landlords that would lengthen search times for tenant agents, the chosen rent cannot be more than  $I_{max}\%$  greater than the rent of the highest accepted rent seen. The percentage of rent increases equal to the maximum allowed is near 2% for the default value of  $I_{max} = 10$ , Figure 14. Note that the overall time over which the profit is determined varies with  $T(R)$ .

The memory of recent sales is extended when necessary in order to keep the number of recent sales in memory above the minimum of 100 (default value). This means that landlords always have enough information on sale times to perceive the basic relation between times-on-the-market and posted rents.

The behaviour of landlords is not a realistic representation of actual landlord price setting decisions. However, we believe that it is an elegant way to simulate the trade-off made by real landlords who have imperfect information on the market.

### 3.2.1 Rent revision

When a residence remains on the market at the end of an iteration, landlords review their chosen rent with probability  $1/F$ . They repeat the procedure described above and chose the rent that they believe will bring the maximum profit.

### 3.2.2 Withdrawing and returning to the market

If the expected profit estimated for a residence is negative, the residence is withdrawn from the market. Landlords who have withdrawn residences from the market review the market situation with probability  $1/F$  every iteration, and return to the market if the expected profit is positive.

### 3.3 Model Mechanism

Our model repeats the following steps until a steady state is reached:

- Searchers visit a randomly chosen apartment, and accept or reject it.
- A portion of landlords ( $1/F$ ) whose apartments remain vacant decide if they shall change their rent or withdraw from the market.
- A portion of landlords ( $1/F$ ) who have withdrawn from the market decide if they shall return.
- A certain fraction ( $1/X$ ) of tenants, randomly chosen, leave each iteration.
- Landlords of newly empty apartments choose their asking rents.
- The next iteration begins.

### 3.4 Parameters

Tables 1 & 2 along with the form of the distribution of the housing budgets of tenants defines a simulation run.

The default value of  $X$ , the expected length of residence, which can reasonably be assumed as being of the order of 4 years, see (de Una-Alvarez et al., 2008), means that five iterations corresponds to one month. Hence landlords knowledge of recent rentals,  $F$ , stretches three months into the past and they change their rent in expectation every three months. The default discount rate is equivalent to an annual rate of 3%. It should be noted that these are simply default values. The effects of varying the parameters on the steady state configuration are discussed below and in the Appendix.

### 3.5 Initialisation

In order that landlords have some information on the market so that they can set rents, we use the following initialisation procedure. Ten thousand landlords and tenants are created. The landlords all have a maintenance cost  $C_L$  of 100 and an initial asking rent randomly chosen in the interval 100-120. The tenant agents have a uniform distribution of housing budgets between 100 and 198 in 50 discrete groups.

Over the first  $Z$  iterations, tenant agents see five apartments and select the lowest asking rent if it offers the agent a positive utility. This preference for lower rent residences initialises the market in such a way that the information available

Table 1: Dynamic Parameters of the Model

Symbol	Meaning	Default Value of Parameters
Landlords		
$S_L$	% of sales seen	20%
$F$	timescale rent changes (and memory)	15
$I_{max}$	maximum rent increase	10%
$C_L$	maintenance cost per iteration	100
Tenants		
$X$	expected length of residence	240
$\sigma$	Idiosyncrasy of tenants preferences (% Y)	5
$S_T$	percentage of offers seen	5%
$C_T$	search costs	200
$Y$	housing budget	100-198
Other		
$r$	discount rate (default = 3% annual rate)	0.0005

Table 2: Static and Initialisation Parameters of the Model

Symbol	Meaning	Default Value of Parameters
$size$	town size	10000
$Z$	number of initialising iterations	10
$R_I$	estimation rent interval size	2

to landlords indicates that higher rents mean longer waiting times. After the  $Z$  initialisation iterations are complete, the mechanism described in Section 3.3 is implemented, in which searchers see only one residence per iteration.

### 3.6 Theoretical benchmark

What are the intuitive results we can expect for this model? The potential support of offered rents is [100;198] because tenants with the highest income determine the highest possible acceptable rent. At the other extreme, no landlord is willing to offer a rent below 100 due to maintenance costs. To predict the steady-state of our market, it should first be noted that there is a fundamental asymmetry.

Landlords individually post prices which cannot be negotiated, while tenants decide whether or not to accept the offer receive. With this setting, in a perfectly competitive market in which tenants would be able to meet all the offers, any landlord offering a below market rent would immediately attract a tenant. Rents would thus be pushed down to landlords' maintenance costs (Bertrand price competition), i.e. at 100. What drives the market away from this configuration are the frictions due to imperfect information and search costs. We know from existing theoretical labour-market models that in such a model with frictions and homogenous workers, the equilibrium would be with a single wage, equal to the workers reservation wage: firms are able in such a situation to extract the whole surplus from the match (Diamond, 1971). However, if workers differ in their reservation wages, due to heterogenous outside options for instance, a distribution of wages emerges (Albrecht and Axell, 1984, Eckstein and Wolpin, 1990). This ties in with the consumer search literature, where price dispersion has been shown to result from sellers playing mixed strategies against potential buyers that have different reservation rents, see (McMillan and Rothschild, 1994). Furthermore, because workers with high outside options only choose jobs with high wages, the distribution of offered wages differs from the distribution of reservation wages (Eckstein and van den Berg, 2007). In our housing-market model, this would translate into a distribution of rents explained by heterogenous reservation rents. Since tenants with a low income only accept low rents, it is expected that the distribution of rents differs from the distribution of reservation rents.

## 4 Results

### 4.1 Base Case

We present simulation results for a 'city' with an inelastic supply of ten thousand apartments owned by ten thousand different landlords and ten thousand tenant agents wishing to be accommodated. All the parameters values are the default values presented in Tables 1 & 2.

The simulation converges to a steady state as can be seen in Figure 3-left,

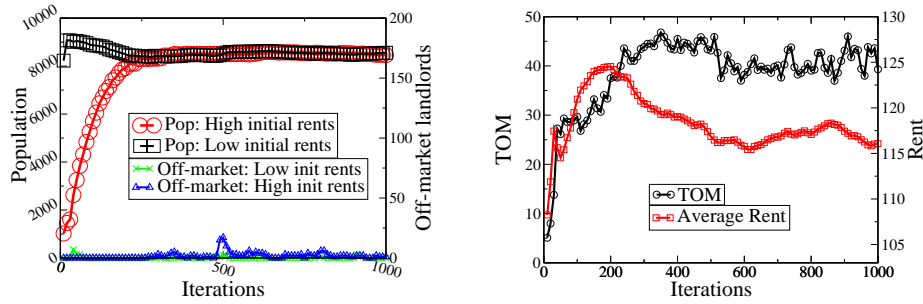


Figure 3: Left: The steady state Population and number of landlords off-the-market. The "Pop : high initial rents" curve is from an initialisation following the procedure in Section 3.5 with initial rents distributed between 180 and 200. The "Pop : low initial rents" curve corresponds to a standard initialisation as described in Section 3.5. Right: The average TOM of residences accepted over the last 15 iterations and their average rent for a standard initialisation.

for any initialisation. The vacancy rate is about 15%, and the percentage of landlords who have withdrawn from the market is negligible.

The dispersion in both accepted and posted rents can be seen in Figure 4-left. The curves are the cumulated distributions of rents set and accepted over 15 iterations. In Figure 4-right we can see that, as expected, lower rents are more likely to be accepted. Figure 4-left shows that most landlords offer rents close to the standard 'going rate'. The few who ask higher rents are less likely to find tenants. We can see that the basic model converges to a reasonable steady-state with a positive vacancy rate, rent dispersion and nonzero search times.

The simulations results show that a dispersion of rents occurs at steady-state. This dispersion results from different appreciations of the state of the market across imperfectly informed landlord and tenant agents. It is also due to the idiosyncratic preferences of tenants for residences and their heterogeneous housing budgets. This is a standard result in consumer search literature (McMillan and Rothschild, 1994) and in recent labour market models (see the review in (Eckstein and van den Berg, 2007)). By analogy with this literature, one can interpret rent dispersion as the result of landlords playing mixed strategies against potential tenants that have different reservation rents.

The distribution of offered rents differs from the uniform distribution of reservation rents: high income agents are likely to accept any rent below their reservation rent. As a result, the maximum offered rent by landlords is lower than the maximum reservation rent, it is below 150 in Figure 4-left. Landlords find that for higher rents it takes longer to find a tenant (because those tenants with low incomes can no longer afford it, and wealthier tenants prefer cheaper accommodation) and so offer relatively low rents. This result is in line with (Bontemps et al., 1999) who demonstrates that with heterogeneous searching workers, the maximum offered wage is lower than the maximum reservation

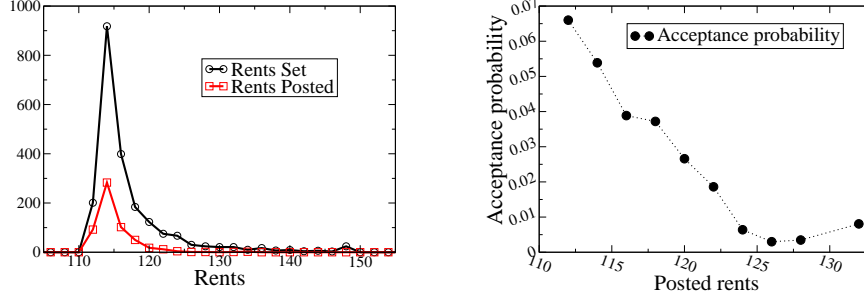


Figure 4: Left: Rents posted and accepted in last 15 iterations at the steady state, after 2000 iterations. Right: The probability of renting as a function of posted rent. The slight increase at the higher rents corresponds to "noise" due to a small number of rentals.

wage. Correspondingly, with homogeneous tenants who would accept any offered rent, landlords would capture all the surplus by setting rents to the tenants' reservation rent.

The lower bound of offered rent in our results also differs from the lower reservation rent: although landlords might get profit with any rent strictly above 100, the minimum offered rent in our simulation is above 110 and the lowest income tenants are homeless. This occurs due to high maintenance costs compared to the expected benefit of having a housing on the market with a low posted rent. This could be different with lower maintenance costs.

## 4.2 Effects of agent's characteristics on the market state

After having shown that the basic model leads to a reasonable steady-state, we shall show its evolution upon changes in the information parameters, the discount rate and tenants' idiosyncratic preferences. The effects of the other parameters (memory length, maximum rent increase, tenants' expected duration of residence, city size) are shown in the Appendix.

### Varying Information Levels

Two very important parameters of this model are the percentages of offers seen by tenants and the percentage of times on the market of recent rentals and of vacant residences seen by landlords. To simplify the measurement of welfare, we present here results obtained for a discount rate equal to zero. Figure 9 shows that the difference between the default value  $r = 3$  and  $r = 0$  is not significant.

#### 4.2.1 Tenants

The population is not greatly affected by alterations in the percentage of offers seen by tenants,  $S_T$  (Figure 5-left). However, some effects at low levels of infor-



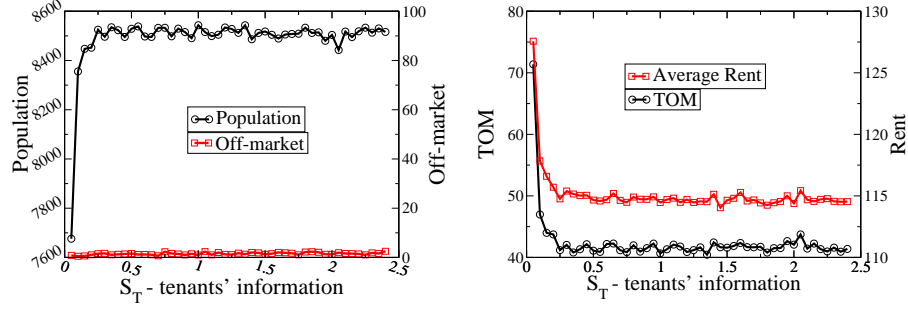


Figure 5: Left: The variations in population and the number of landlords off-the-market as a function of the percentage of offers seen by tenants. Right: The average rent and the average time-on-the-market (TOM).

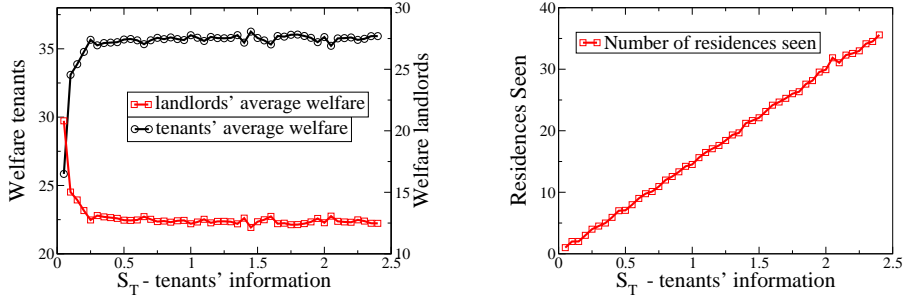


Figure 6: Left: The average welfares of tenant and landlord agents are shown as a function of the percentage of offers seen by tenants. Right: The number of residences seen by tenant agents with respect to the percentage of the distribution that they see.

mation can be seen in Figures 5-right & 6-left. This is because a relatively low number of observations are required for tenant agents to have a good estimation of the real distribution of offers<sup>3</sup>. The percentage of offers seen by tenants ceases to alter the steady state configuration above a threshold value which can be seen to be approximately 0.25% or 5 observations from Figure 6-right.

Increasing tenants' information improves their welfare as they learn to refuse higher rent offers.

#### 4.2.2 Landlords

In contrast, increasing landlords' information decreases their welfare as rents decrease (Figures 7-right&8). The population also increases, as shown in Figure 7-left.

<sup>3</sup>Preliminary tests seem to confirm that reducing agent's information leads to less noise, as suggested by the figure. Further work is needed to understand this feature of the model.

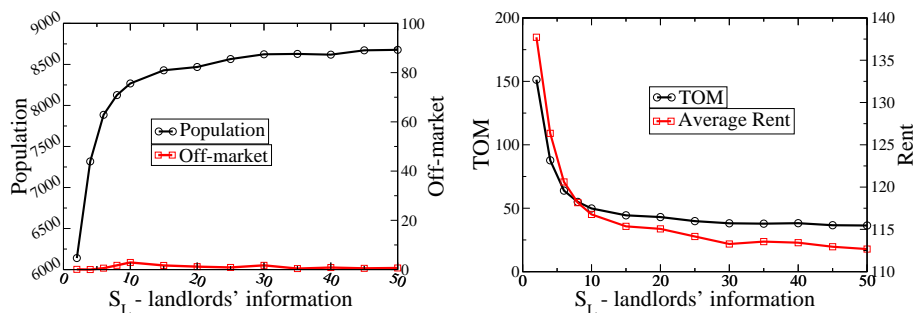


Figure 7: Left: The variations in population and the number of landlords off-the-market as a function of landlords information. Right: The average rent and the average time-on-the-market (TOM).

The dramatic difference in the sensitivity to changes in the two information parameters (of tenants and of landlords) is due to the fundamental asymmetry in the market exposed in Section 3.6. Tenants choose among landlords, while landlords individually post prices which cannot be negotiated. Therefore, a perfectly competitive market should have rents offered at landlords' maintenance costs (Bertrand's price competition). What drives the market away from this configuration are the different frictions due to imperfect information and search costs. Therefore, increasing landlords' information makes them sharper competitors, leading to reduced rents. The search frictions always present in our market prevent the theoretical competitive rent being realised. These frictions are the search costs, which are significant because maintenance costs are high relative to the low gains at rents near 100. In addition, landlords' estimation of search times for different rents is based on a smoothing procedure (the exponential fit) over noisy data. Landlords need accurate two dimensional information to decide their asking rent, that is rents offered and their associated times-on-the-market. The times-on-the-market of many residences in each rent interval are required for accurate estimates of the rent times-on-the-market relationship. Therefore, their information, which also depends on their memory length  $F$ , is not perfect even with  $S_L = 100\%$ .

Errors made by ill informed landlords tend to lead to higher asking rents because they are less likely to see the long waiting times for higher rents, which are rare events likely to disappear from their limited vision. As the landlords are homogeneous, and make the same errors on average, this pushes the market prices upwards. Note that every high posted rent, if refused by tenants, increases their expected search times as tenants' search is undirected. This necessarily affects searchers' optimal reservation utilities, pushing the market towards higher rents.

The observation that a high level of information tends to lead to the competitive outcome is in line with the results of theoretical models of the labour market such as (Burdett and Mortensen, 1998), who show that in the extreme

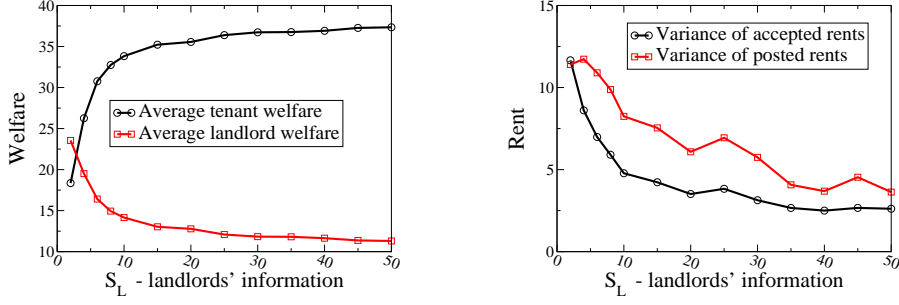


Figure 8: Left: The average welfares of tenant and landlord agents are shown for variations in the level of landlord's information. Right: The average variance of the distributions of posted and accepted rents over 1000 iterations, after convergence to a steady state.

case where frictions disappear (with an instantaneous arrival rate of job offers), the wage is competitive.

#### 4.3 Discount rate - $r$

The discount rate was varied from less than 1% per annum to over 17% per annum. Figures 9 show that increases in the discount rate lead to a reduction in average rents and TOM, and to an increase in population. It should be noted that the effects of changing the discount rate arise from changes in all four terms in the right-hand sides of Equations 2 & 6 and the subsequent market interactions. Overall, increasing the discount rate increases agents' impatience. Therefore, landlords have a tendency to post lower rents, while tenants are willing to accept higher rents. This explains the decrease of TOM seen in Figure 9. It is not obvious which of these contradictory effects should dominate.

For the default values of the parameters, the average rent is lower with a higher discount rate. Correspondingly, the average welfare of tenants is improved and that of landlords dis-improved with increasing discount rates, as can be seen in Figure 10.

However, one may anticipate that the net effect depends on the relative market power of the two kinds of agents. We are going to test this idea in forthcoming simulations.

#### 4.4 Idiosyncratic preferences

We now present the effect of varying tenant's idiosyncratic preferences for residences visited  $\sigma$ . As in the information Section we present results with a discount rate of zero.

The average rent increases with  $\sigma$ , as tenants agree to higher rents for apartments they particularly like Figure 11. More importantly, the level of vacancies

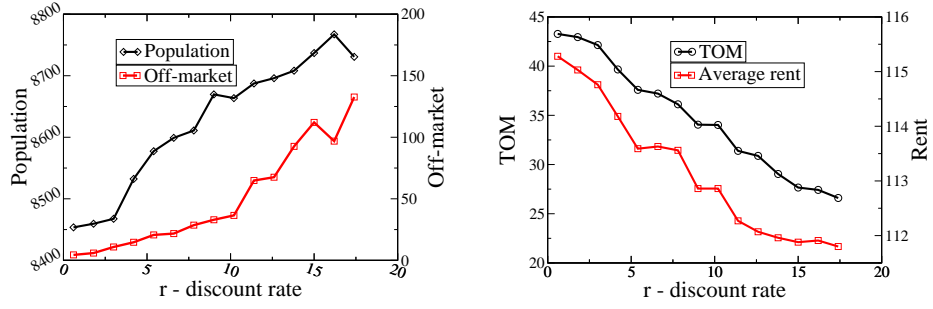


Figure 9: Left: The variations in population and the number of landlords off the market. Right: The average rent and the average Time-on-the-market for residences rented over the last 15 iterations. Both graphs for a variation in the annual discount rate of both agents types.

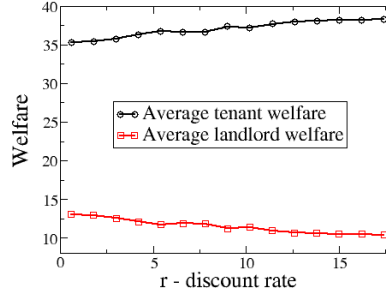


Figure 10: The average welfares of tenant and landlord agents are shown for variations in the annual discount rate.

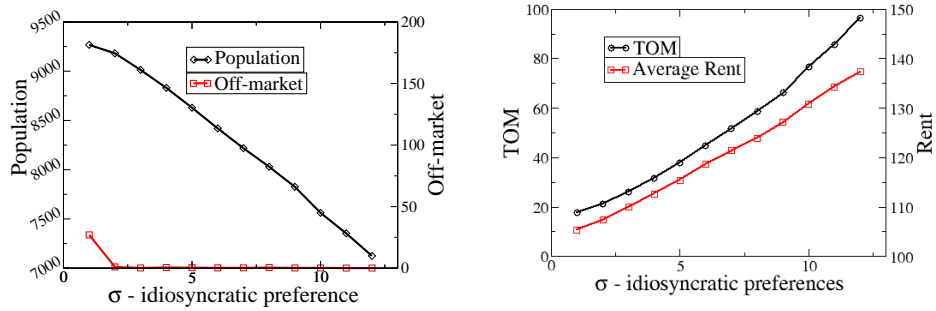


Figure 11: Left: The variations in population and the number of landlords off-the-market. Right: The average rent and the average time-on-the-market (TOM) for residences rented over the last 15 iterations. Both graphs for a variation in the idiosyncratic preferences of tenant agents.

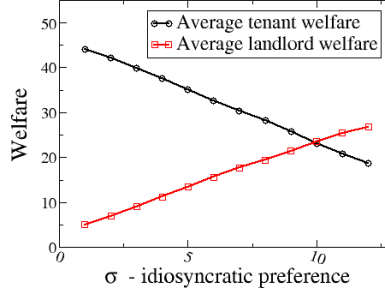


Figure 12: The variance of the average welfares of tenants and landlords for a variation in the idiosyncratic preferences of tenant agents.

strongly increases with  $\sigma$  (the number of residences off the market tends to 0 while the city population decreases) and the average TOM rises. The rise in rents excludes many potential tenants from the market. Hence the same number of landlords, all attracted by the high rents, search among a smaller number of tenants TOM rises.

Tenants actually tend to accept higher rents with larger  $\sigma$  (when they are associated with a positive draw of  $\theta$ ), but also to reject more offers when they draw a negative  $\theta$  and they compare the utility they would have from this housing to their expected utility given the distribution of rents. Figure 12 shows that the increase in rent penalises tenants on average by more than they gain from being able to choose apartments for which they have a greater personal preference. The differential effects across tenants of different incomes remains to be tested.

## 5 Conclusion

Our dynamic model includes imperfect information and heterogeneous interacting agents. It leads to price dispersion, nonzero search times and vacancies, three essential ingredients of any realistic housing model. The matching probability depends endogenously on the posted price of apartments. It is a general equilibrium model that provides a basis for examining policy questions such as rent control and its welfare and distributional effects, the benefits from taxation on vacant housing or the general equilibrium effect of providing social housing.

In our model landlords are Stackleberg leaders who set rents which tenant agents accept or refuse. This is a reasonable representation of the rental market.

Landlords act upon their partial knowledge of the market in order to maximise their profits. Their ‘knowledge’ is in the form of probable times-on-the-market before renting for the full range of possible rents. This allows the calculation of the expected profit and hence rational maximisation. Greater informa-

tion for landlords disimproves their overall utility due to greater competition, see Section 4.2.1. The heuristics of real world agents are simulated here by a regression and profit calculation with a larger number of individual information points than real agents normally know.

Tenants are also partially informed of the state of the market and use their information to decide the minimum utility they are willing to accept from a residence. Their search is undirected, that is they have equal probability of seeing any available offer in a given iteration. Greater information for tenants improves their overall welfare.

Our main aim has been to construct a model that allows hypotheses on the functioning of the urban rental market to be investigated. We believe that a dynamic model based on straightforward micro-economic behaviours with imperfect information is a good approach. We believe we have found robust and simple agent dynamics (or rules) that reproduce the essential features of the rental housing market and results from analytical search models that have been developed to analyse labour as well as housing markets. This is an exploratory research project that should inform other researchers on the capacity of dynamic and disaggregated models to provide deeper insights into urban housing markets.

One of the most interesting possible extensions is to examine further the role of information on both sides of the market. Heterogeneities in the level of information among landlords could represent the difference between large commercial landowners and small private owners.

Making the tenant side of the market open - that is having a constant flow of arrivals of tenant agents - instead of a fixed number of searching tenants would allow the composition of the town to be endogenous. This would also mean that the vacancy rate would represent market frictions only and not a mixture of market frictions and those tenants who are unable to pay market rates as is currently the case. Transferring our model to one where tenant agents engage in directed search is also a promising extension.

The current setup allows the investigation of the distributive effects of policy decisions among tenants agents of varying incomes. Rent control is one possible example, as is the level of information among tenants. These two effects have been examined in static simulation models by (Bradburd et al., 2005, 2006). It would be interesting to test whether the effects are similar in our more realistic model.

There is great scope to complexify the model for both types of agent. Tenants could be differentiated by their work places, housing preferences, household sizes, transportation modes etc.. Landlords could own different numbers of apartments, and apply different rent setting rules. For example, large landlords are likely to be better informed, better able to absorb vacancies (as discussed by (Blanck and Winnock, 1953)), and more able to adapt their rental strategies to market conditions.

The modelling of construction, demolition, depreciation, conversion and refurbishment of buildings would be an important extension for modelling price and liquidity dynamics. Coupling the rental market to the ownership market

is another important possible extension. A simulation model of the ownership market needs to take into account that the presence of the same agents on both sides of the market is a critical difference from the rental market.

## References

- JW Albrecht and B Axell. An equilibrium-model of search unemployment. *JOURNAL OF POLITICAL ECONOMY*, 92(5):824–840, 1984.
- P. M. Anglin, R. Rutherford, and T. M. Springer. The trade-off between the selling price of residential properties and time-on-the-market: The impact of price setting. *JOURNAL OF REAL ESTATE FINANCE AND ECONOMICS*, 26(1):p95–111, 2003.
- R. Arnott. Housing vacancies, thin markets, and idiosyncratic tastes. *JOURNAL OF REAL ESTATE FINANCE AND ECONOMICS*, 2:p5–30, 1989.
- G. Ballot. Modeling the labor market as an evolving institution: model artemis. *JOURNAL OF ECONOMIC BEHAVIOR & ORGANIZATION*, 49(1):51–77, 2002.
- J. Baye, M.R. Morgan and P. Scholten. Information, search, and price dispersion. in *HANDBOOK ON ECONOMICS AND INFORMATION SYSTEMS (ELSEVIER, T. HENDERSHOTT, ED.)*, 2006.
- D.M. Blanck and L. Winnock. The structure of the housing market. *QUARTERLY JOURNAL OF ECONOMICS*, 67:p181–203, 1953.
- C Bontemps, JM Robin, and GJ Van den Berg. An empirical equilibrium job search model with search on the job and heterogeneous workers and firms. *INTERNATIONAL ECONOMIC REVIEW*, 40(4):1039–1074, 1999.
- R Bradburd, S Sheppard, J Bergeron, E Engler, and E Gee. The distributional impact of housing discrimination in a non-Walrasian setting. *JOURNAL OF HOUSING ECONOMICS*, 14(2):p61–91, 2005. doi: 10.1016/j.jhe.2005.06.001.
- R Bradburd, S Sheppard, J Bergeron, and E Engler. The impact of rent controls in non-Walrasian markets: An agent-based modeling approach. *JOURNAL OF REGIONAL SCIENCE*, 46(3):p455–491, 2006.
- K. Burdett and D.T. Mortensen. Equilibrium wage differentials and employer size. Discussion Paper n° 860, Northwestern University, 1989.
- K Burdett and DT Mortensen. Wage differentials, employer size, and unemployment. *INTERNATIONAL ECONOMIC REVIEW*, 39(2):257–273, 1998.
- J Clayton, G MacKinnon, and L Peng. Time variation of liquidity in the private real estate market: An empirical investigation. *JOURNAL OF REAL ESTATE RESEARCH*, 30(2):p125–160, 2008.

- J de Una-Alvarez, R Arevalo-Tome, and MS Otero-Giraldez. Nonparametric estimation of households duration of residence from panel data. *The Journal of Real Estate Finance and Economics*, available online, 2008.
- Y. H. Deng, S. A. Gabriel, and F. E. Nothhaft. Duration of residence in the rental housing market. *JOURNAL OF REAL ESTATE FINANCE AND ECONOMICS*, 26(2-3):p267–285, 2003.
- PA Diamond. Model of price adjustment. *JOURNAL OF ECONOMIC THEORY*, 3(2):156–168, 1971.
- Z. Eckstein and G. J. van den Berg. Empirical labor search: A survey. *JOURNAL OF ECONOMETRICS*, 136(2):531–564, 2007.
- Zvi Eckstein and Kenneth I. Wolpin. Estimating a market equilibrium search model from panel data on individuals. *ECONOMETRICA*, 58(4):783–808, 1990.
- J Fisher, D Gatzlaff, D Geltner, and D Haurin. Controlling for the impact of variable liquidity in commercial real estate price indices. *REAL ESTATE ECONOMICS*, 31(2):p269–303, 2003.
- SA Gabriel and FE Nothhaft. Rental housing markets and the natural vacancy rate. *AREUEA JOURNAL*, 16(4):p419–429, 1988.
- SR Grenadier. The persistence of real-estate cycles. *JOURNAL OF REAL ESTATE FINANCE AND ECONOMICS*, 10(2):p95–119, 1995.
- M Hwang and J M. Quigley. Economic fundamentals in local housing markets: Evidence from US metropolitan regions. *JOURNAL OF REGIONAL SCIENCE*, 46(3):425–453, 2006.
- NM Kiefer. Economic duration data and hazard functions. *JOURNAL OF ECONOMIC LITERATURE*, 26(2):p646–679, 1988.
- JR Knight. Listing price, time on market, and ultimate selling price: Causes and effects of listing price changes. *REAL ESTATE ECONOMICS*, 30(2):p213–237, 2002.
- C. K. Y. Leung, Y. C. F. Leong, and S. K. Wong. Housing price dispersion: An empirical investigation. *JOURNAL OF REAL ESTATE FINANCE AND ECONOMICS*, 32(3):p357–385, 2006.
- J McMillan and M Rothschild. *HANDBOOK OF GAME THEORY WITH ECONOMIC APPLICATIONS VOLUME 2*, chapter Search, pages p905–927. Elsevier, 1994.
- A. Merlo and F. Ortalo-Magne. Bargaining over residential real estate: evidence from england. *JOURNAL OF URBAN ECONOMICS*, 56(2):p192–216, 2004.



- D Mortensen. *Panel Data and Structural Labor Market Models*, chapter Equilibrium Unemployment with Wage Posting. Amsterdam: Elsevier, 2000.
- Dale T. Mortensen and Christopher A. Pissarides. Job creation and job destruction in the theory of unemployment. *THE REVIEW OF ECONOMIC STUDIES*, 61(3):397–415, 1994.
- D.T. Mortensen. *Panel Data and Loabor Market Studies*, chapter Equilibrium wage distributions: a synthesis. Amsterdam: North Holland, 1990.
- Christopher Pissarides. *Equilibrium unemployment theory*. Oxford: Basil Blackwell, 1990.
- F. Postel-Vinay and J-M. Robin. *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress*, chapter Microeconomic Search-Matching Models and Matched Employer-Employee Data, pages p279–310. Cambridge University Press, 2006.
- C Read. Tenants search and vacancies in rental housing markets. *REGIONAL SCIENCE AND URBAN ECONOMICS*, 23(2):p171–183, 1993.
- C Read. Vacancies and rent dispersion in a stochastic search model with generalized tenant demand. *JOURNAL OF REAL ESTATE FINANCE AND ECONOMICS*, 15(3):p223–237, 1997.
- R. Rogerson, R. Shimer and R. Wright. Search theoretic models of the labour market: A survey. *JOURNAL OF ECONOMIC LITERATURE*, 63:p959–988, 2005.
- KT Rosen and LB Smith. The price-adjustment process for rental housing and the natural vacancy rate. *AMERICAN ECONOMIC REVIEW*, 73(4):p779–786, 1983.
- JD Shilling, CF Sirmans, and JB Corgel. Price adjustment process for rental office space. *JOURNAL OF URBAN ECONOMICS*, 22(1):p90–100, 1987.
- W C. Wheaton. Vacancy, search, and prices in a housing market matching model. *THE JOURNAL OF POLITICAL ECONOMY*, 98(6):1270–1292, 1990.
- A Yavas and S Yang. The strategic role of listing price in marketing real-estate - theory and evidence. *REAL ESTATE ECONOMICS*, 23(3):p347–368, 1995.

## 6 Appendix

### 6.1 Reviewing frequency - $F$

Alterations in the steady-state configuration due to changes in the reviewing frequency parameter,  $F$ , are shown in Figure 13.

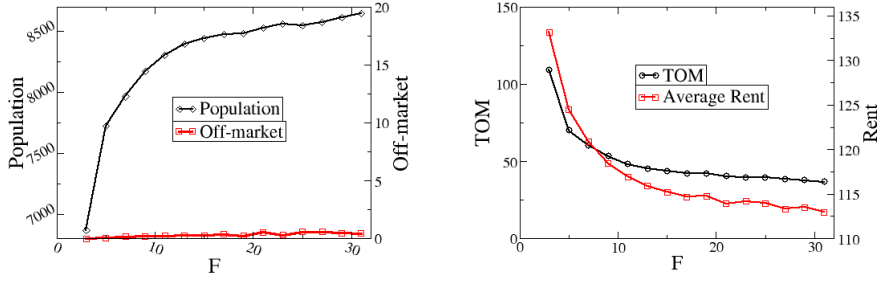


Figure 13: Left: The variations in population and the number of landlords off-the-market as a function of the landlord memory and updating parameter  $F$ . Right: The average rent and the average time-on-the-market (TOM).

As  $F$  rises initially the population increases and rents fall. When landlords review their rents less frequently (higher  $F$ ) they appear less able to extract a surplus from imperfectly informed tenants.

### 6.2 Maximum rent increase - $I_R$

Changes in the maximum allowed rent increase do not have a dramatic effect on the steady-state configuration, see Figure 14. Average rents initially increase slightly, as is to be expected. The percentage of maximum rent increases also decreases.

### 6.3 Occupation duration - $X$

Increases in the expected duration of residence increase the average rent as the profits from higher rents for landlords become greater, see Figure 15. This has the effect of reducing the population as some tenants can no longer afford the market rents.

### 6.4 Size - $S$

The size parameter plays an important role, as the level of information available to agents depends directly on the size of the city. As the city size increases, so does the occupation rate, which is probably due to less errors being made by landlords in setting rents and by tenants in refusing rents that they should

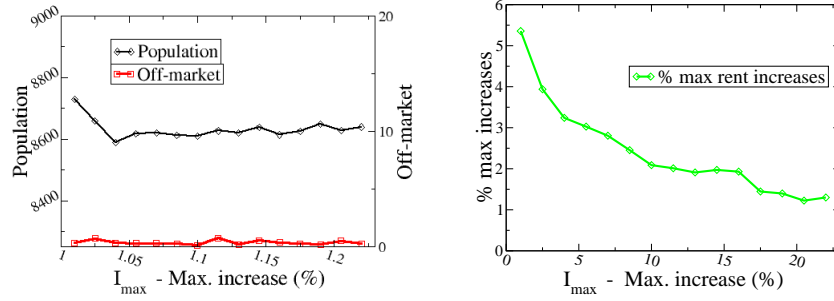


Figure 14: Left: The steady-state levels of population and the number of landlords off-the-market, for changes in the maximum allowed increase of rent. Right: The percentage of maximum rent increases averaged over 1000 iterations.

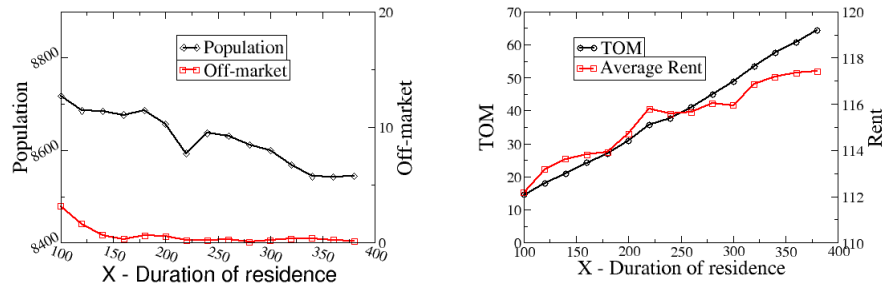


Figure 15: Left: The variations in population and the number of landlords off-the-market as a function of the landlord expected duration of residence parameter  $X$ . Right: The average rent and the average time-on-the-market (TOM).

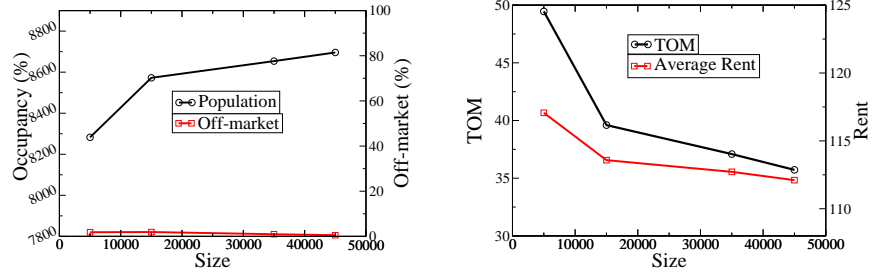


Figure 16: Left: The variations in population and the number of landlords off-market as a function of town size. Right: The corresponding average rent and the average time-on-the-market (TOM).

normally accept. We note that even for a town size of 45,000 agents, the information effect has not yet converged to an asymptotic value. This value is likely to be close to that of the largest urban rental markets supply of close substitutes once questions of type and location are taken into account. We feel that the presence of finite size effects is a positive feature of the model.