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Louis Bachelier
On the centenary of Théorie de la Spéculation

J.-M. Courtault, Yu. Kabanov,
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1 Centenary of mathematical finance

The date March 29, 1900, should be considered as the birthdate of mathematical finance: on this day French postgraduate student Louis Bachelier successfully defended at Sorbonne his thesis *Théorie de la Spéculation.* As a work of exceptional merit, strongly supported by Henri Poincaré, Bachelier's supervisor, it has been published in *Annales scientifiques de l'École Normale Supérieure*, one of the most influential French scientific journals. This pioneering study on the analysis of the stock and option markets contains several ideas of enormous value in both finance and probability. In particular, the theory of Brownian motion, one of the most important mathematical discoveries of the twentieth century, was initiated and used for the mathematical modeling of price movements and evaluation of contingent claims in financial markets.

The thesis of Louis Bachelier, together with his subsequent works, influenced deeply the whole development of stochastic calculus and mathematical finance. As a testimony of his great contribution, the newly created international Bachelier Finance Society is named after him. The centenary of the famous thesis is widely celebrated as a landmark event in the history of modern science. On this occasion we present to the reader's attention a brief account of our research on Louis Bachelier. In spite of his remarkable contributions, he remained, until recently, in obscure darkness, as one of the most mysterious figures in mathematics of the twentieth century. Only a few facts about him could be found in the literature. There is enormous public interest in his scientific biography, which can be explained by the amazingly fast development of mathematical finance in the last two decades together with a deeper understanding of the fundamental role of Brownian motion.

We believe that our short note brings some new light on Louis Bachelier, a human person, a mathematician, and a philosopher.

2 Dates of biography

11 March 1870	Louis Jean-Baptiste Alphonse Bachelier was born in Le Havre
October 1888	Graduated from secondary school at Caen
11 January 1889	Father's death
7 May 1889	Mother's death
1889 - 1991	He is the head of *Bachelier fils*
1891 - 1892	Military service
1892	Student at Sorbonne
October 1895	Bachelor in sciences at Sorbonne
July 1897	Certificate in mathematical physics
29 March 1900	Bachelier defended his thesis
1909 – 1914	Free lecturer at Sorbonne
1912	The book *Calcul des Probabilités*
1914	The book *Le Jeu, la Chance et le Hasard*
9 September 1914	Drafted as a private in the French army
31 December 1918	Back from the army
10 December 1919	A member of the French Mathematical Society
1919 – 1922	Assistant professor in Besancon
14 September 1920	Married Augustine Jeanne Maillot; she died soon
1922 – 1925	Assistant professor in Dijon
1925 – 1927	Associated professor in Rennes
January 1926	Blackballed in Dijon
1 October 1927	Professor in Besançon
1937	Professor Emeritus
1 October 1937	Retired
1941	The last publication
28 April 1946	Died in Saint-Servan-sur-Mer; buried in Sanvic near Le Havre
1996	The Bachelier Finance Society is founded

Louis Bachelier was born in a respectful bourgeois family known in Le Havre for its cultural and social traditions. His father, Alphonse Bachelier, was not just a wine merchant but also the vice-consul of Venezuela at Le Havre and an amateur-scientist. Louis' mother, Cecile Fort-Meu, was a banker's daughter; her grandfather, an important person in the financial business, was known also as the author of poetry books. Unfortunately, at the age of eighteen just after graduation from the secondary school at Caen and getting the French bachelor degree *baccalauréat es sciences*, Louis Bachelier lost his parents and was forced to interrupt his studies to continue his father's business and take care of his sister and three year old brother.

These dramatic events had far reaching consequences for his academic career; in particular, they explain why Bachelier did not follow any *grande école* with the French scientific elite, a weak point in his curriculum. However, quite probably, as the head of the family enterprise (officially registered as *Bachelier fils*) he got acquainted with the world of financial markets (one can find some hints of personal experiences in his works).

After the military service, having in total a four-year interruption of his studies (an enormous handicap in a forming mathematician!), Louis arrived in Paris to continue his university education at Sorbonne. Scarce information is available about these years. He followed lectures of Paul Appell, Joseph Boussinesq, and Henri Poincaré. Apparently, he was not among the best students. For example, Bachelier's marks in mathematics in the 1895 register were largely below those of his classmates Langevin and Liénard. But in spite of a huge delay in his career, Bachelier's development as a scientist, was fast enough and he wrote his celebrated thesis *Théorie de la Spéculation* on the application of probability to stock markets. This was historically the first attempt to use advanced mathematics in finance and witnessed the introduction of Brownian motion was. In accordance with the tradition of the epoch, he also defended a second thesis on a subject chosen by the faculty, namely, on mechanics of fluids. Its title (as well as the names of his professors) may reflect Bachelier's background: *Résistance d'une masse liquide indéfinie pourvue de frottements intérieurs, régis par les formules de Navier, aux petits mouvements variés de translation d'une sphère solide, immergée dans cette masse et adhérente à la couche fluide qui la touche.*

3 The thesis and Poincaré's report

The first part of Bachelier's thesis contains a detailed description of products available in the French stock market of the epoch, like forward contracts and options. Their specifications were quite different from the corresponding products in the American market, see comments in [2]. For example, all payments were related to a single date and one had no need to think about the discounting or change of *numéraire*. After financial preliminaries, Bachelier starts the mathematical modeling of the stock price movements and formulates the principle that "the expectation of the speculator is zero". Obviously, he understands here by expectation the conditional expectation given the past information. In other words, he implicitly accepts as an axiom that the market evaluates assets using a martingale measure. The further hypothesis is that the price evolves as a continuous Markov process, homogeneous in time and space. Bachelier shows that the density of one-dimensional distributions of this process satisfies the relation known now as the Chapman–Kolmogorov equation and notes that the Gaussian density with the linearly increasing variance solves this equation. The question of the uniqueness is not discussed, but Bachelier provides some further arguments to confirm his conclusion. He arrives at the same law by considering the price process as a limit of random walks. Bachelier also observes that the family of distribution functions of the process satisfies the heat equation: the probability diffuses or "radiates."

The results in these dozen pages are spectacular, but this is not the end! The model is applied to calculate various option prices. Having in mind American and path-dependent options, Bachelier calculates the probability that the Brownian motion does not exceed a fixed level and finds the distribution of the supremum of the Brownian motion.

One hundred years after publishing the thesis it is quite easy to appreciate the importance of his ideas. It can be viewed as the origin of mathematical finance as well as several important branches of stochastic calculus such as the theory of Brownian motion, Markov processes, diffusion processes, and even weak convergence in functional spaces. Of course, the reasoning of Bachelier was not rigorous but, basically, on the intuitive level, correct. This is really astonishing, because at the beginning of the century the mathematical foundations of probability did not exist. A.Markov started his studies on what are now called Markov chains only in 1906, and the concept of conditional expectations with respect to an arbitrary random variable or σ-algebra was developed only in the 1930's.

Poincaré's report (signed also by P. Appell and J. Boussinesq) is a remarkable document showing that Bachelier's thesis was highly appreciated by the outstanding mathematician. It contains a deep analysis not only of the mathematical results but also an insight into market laws. In contrast to the legend that the evaluation note "*honorable*" means somehow that the examinators were skeptical about the thesis, it seems that it was the highest note which could be awarded for a thesis which was essentially outside mathematics and which had a number of arguments far from being rigorous. The excellent note was usually assigned to memoirs containing the solution of a challenging problem in a well-established mathematical discipline. In our opinion, the report shows clearly that Henry Poincaré was an extremely attentive and benevolent reader, and his mild criticism was positive. The expressed regret that Bachelier did not study in detail the discovered relationship of stochastic processes with equations in partial derivatives can be interpreted that he was really intrigued, seeing here further perspectives. Poincaré's report and the conclusion to publish the thesis in the most prestigious journal of the epoch seems to be in contradiction with the disappointing note "*honorable*." One may guess that Bachelier was not awarded the note "*très honorable*" because of a weaker presentation of his second memoir (the corresponding report of P. Appell is very short: Bachelier has a good command of works of M.Boussinesq on movements of a sphere in a fluid...).

Needless to say, the innovative ideas of Bachelier were much above the prevailing level of the existing financial theory. Nevertheless, they were noticed. In the book by de Montessus [9] of 1908 containing several chapters on applications of probability to insurance, artillery, etc., there is a whole chapter devoted to probabilistic methods in finance and based upon Bachelier's thesis.

4 Further studies

Louis Bachelier remained quite active in the period of 1900–1914. He continued to develop the mathematical theory of diffusion processes in a series of papers published in reputed French journals. In his memoir of 1906 he defined classes of stochastic processes now called processes with independent increments and Markov processes, and he derived the distribution of the Ornstein–Uhlenbeck process (see [10]). His approach can be considered as the development of the theory of stochastic differential equations using the language and concepts of gambling. Two functions playing essential roles in the paper, "relative expectation" and "relative instability," can be identified with the drift and diffusion coefficients of the modern vocabulary.

We could not find any traces of his employment during the whole decade. However, he received quite regularly scholarships to continue his studies. In the period of 1909–1914 Bachelier gave lectures at Sorbonne as "free professor" (he was paid starting only from 1913). In particular, he gave the lecture course "probability calculus with applications to financial operations and analogies with certain questions from physics."

Apparently, Bachelier was aware of the importance of his contributions. He wrote in his curriculum vitae ("Notice" of 1924) that his book *Calcul des probabilités* was the first that surpassed the great treatise by Laplace. The efforts to advertise his approach were not always successful: e.g., the demand to get fundings for a monograph on his studies in probability was rejected because the committee considered that "the results in the suggested book are not essentially different from those published yet in the journal of M. Jordan and *Annales de l'École Normale Supérieure*". Nevertheless, he published in 1914 *Le Jeu, la Chance et le Hasard* a book which had a great public success: more than 6 thousand copies were sold. Bachelier considered that his principal achievement was the systematic use of the concept of continuity in probabilistic modeling: the continuous distributions are the fundamental objects correctly describing the very nature of many random phenomena and not just mathematical inventions simplifying a work with discrete distributions.

In 1914 the Council of the Paris University decided to make Bachelier's appointment permanent, but the war destroyed this plan. World War I began and he was drafted as a soldier; the army service ended only on December 31, 1918. His first regular academic position was in Besançon, replacing professor C.-E. Traynard on leave. Return of the latter in 1922 forced him to move to Dijon where he replaced R. Baire and, afterwards, to Rennes. At last, in 1927 he got his permanent professorship in Besançon where he worked until his retirement in 1937. At the end of his academic career Bachelier published the résumés of his early works, trying to attract public attention to his work on the theory of Markov processes. The final chapter of his scientific career was a note in *Comptes-rendus de l'Académie des Sciences* of 1941 on distributions of functionals of Brownian motion. Remarkably, the importance of his results in mathematical finance, for instance, the pricing of barrier options, was revealed relatively recently (see [3]).

5 Blackballed in Dijon

One of the most dramatic events in Bachelier's life was his attempt to get a position in Dijon in 1926[1]. It was really a shame for the Council of the Faculty of Sciences to classify him as the second candidate (after G. Cerf), a shame not because of the result but the pretext: Professor M. Gevrey blamed Bachelier for an error in the paper of 1913 readily confirmed by a letter of Paul Lévy, professor of *École Polytechnique*. Bachelier was furious. There exists a typed letter of him where he exposes the details of this sad story:

> Due to a sequence of incredible circumstances... I have found myself at the age of 56 in a situation worse than I had during the last six years; this is after twenty six years with the doctor degree, six years of teaching as free professor at Sorbonne, and six years of official replacement of a full professor. My carrier happened to be blocked in a deplorable and extremely injust way without a minor reproach which could be given to me.
>
> ...
>
> The critique of M. Lévy is simply ridiculous: he accuses me to use... continuous formulae for the case where they are, in fact, discontinuous, but a large number of experiments allows for an asymptotic approach.
>
> ...
>
> M. Lévy noted that the first formula of the accused memoir is the formula of Brownian motion. Exactly, but why is it criticized since this is myself who presents it?
>
> ...
>
> M. Lévy pretends not to know my other five large memoirs published in *Annales de l'École normale* and in *Journal de Mathématiques pures et appliquées* as well as various notes published elsewhere.
>
> He has written a work of 300 pages on probability without even opening my book on the same subject, the book which is, in some respects, above the book by Laplace and which contains a lot of new results.
>
> He does not know my work on scientific philosophy, which has the forth edition available today (seven thousand copies).
>
> Briefly, M. Lévy did not know my studies, completely, while writing the letter to M. Gevrey.
>
> He did not know also the works of M. Cerf, as he himself confessed. Nevertheless, he recommended, explicitly, M. Cerf for the first rank.

Paul Lévy, apparently, was quite sincere in his report, misled by an erroneous interpretation of Bachelier's notation. The latter defined the Brownian motion as a limit, as $\tau \to 0$, of random functions which are linear on each interval $[n\tau, (n+1)\tau]$ with derivatives v or $-v$ taken at random with equal probabilities. The dependence of v on τ (as $c\tau^{-1/2}$) was omitted but always assumed by Bachelier. Of course, with a constant v the reasoning cannot be correct and Lévy no doubt had to write the extremely negative conclusion. To his surprise, several years later in the fundamental

[1] Not in Geneva in 1927 as was claimed in [1].

paper by Kolmogorov, whose mathematical genius he highly appreciated, he found references to Bachelier works, including the thesis. Lévy also made a revision and observed that a number of properties of the Brownian motion have been discovered by Bachelier several decades ago. To the honor of Paul Lévy, he wrote a letter with excuses and the two reconciled. But Lévy could not appreciate Bachelier's contribution to financial theory. In his notebook with abstracts of the most important works, he gave the following comment on the thesis "Too much on finance!"

6 The Bachelier heritage

There is a traditional public view according to which Louis Bachelier was a mathematical prodigy whose remarkable insight had no impact on the development of the theory of stochastic processes, whose results were totally ignored by the probabilistic and financial communities, and who was underestimated by his superiors, perhaps because the title of his thesis was not attractive to mathematicians. Unfortunately, certain historical research follows this legend. But our study of documents shows that this simplified story is far from being correct.

As we mentioned earlier, his thesis was published in one of the most prestigious mathematical journal by recommendation of Poincaré as well as in a separate book. Very quickly, the results appeared in a book on applied probability [9]. This fact is quite remarkable, because in that period there were few books on probability. He successfully published his "Calcul des Probabilités" (it was not just a coincidence that the format was *in-quarto* as the famous treatise of Laplace).

Did his ideas influence further studies? Our answer is yes, without any doubt. The key paper of Kolmogorov [7] on diffusion processes seems to be, at least partially, inspired by Bachelier's work. Roughly speaking, Kolmogorov, who was at this time at the zenith of his mathematical power, just after suggesting the axioms of probability which made it really a mathematical discipline, did the work suggested in Poncaré's comment: he developed in full generality the analytical approach for continuous Markov processes.

It is worth noting that in spite of the fact that in modern English textbooks Brownian motion is traditionally referred to as the Wiener process, the original terminology suggested by W. Feller in his famous two-volume treatise "An Introduction to Probability Theory and its Applications" was the Wiener-Bachelier process. Another clear message about Bachelier's importance can be found on the very first page of another outstanding book: "Diffusion Processes and their Sample Paths" by K. Ito and H. McKean. One can also find references to Bachelier in the early literature written by economists (see, e.g., [6]). The book [1] tells a story about how Bachelier's thesis and books were discovered in the U.S. by the pioneers of modern financial theory, see also [8].

7 Bachelier's works

Books

1. *Théorie de la spéculation*, 1900, Gauthier-Villars, 70 p.
2. *Calcul des probabilités*, 1912, Gauthier-Villars, Tome 1, 516 p.
3. *Le Jeu, la Chance et le Hasard*, Bibliothèque de Philosophie scientifique, 1914, E. Flammarion, 320 p.
4. *Les lois des grands nombres du Calcul des Probabilités*, 1937, Gauthier-Villars, v-vii, 36 p.
5. *La spéculation et le Calcul des Probabilités*, 1938, Gauthier-Villars, v-vii, 49 p.
6. *Les nouvelles méthodes du Calcul des Probabilités*, 1939, Gauthier-Villars, v-viii, 69 p.

Papers

1. Théorie de la spéculation, *Annales scientifiques de l'École Normale Supérieure*, 1900, p. 21-86.
2. Théorie mathématiques du jeu, *Annales scientifiques de l'École Normale Supérieure*, 1901, p. 143-210.
3. Théorie des probabilités continues, *Journal de Mathématiques pures et appliquées*, 1906, p. 259-327.
4. Étude sur les probabilités des causes, *Journal de Mathématiques pures et appliquées*, 1908, p. 395-425.
5. Le problème général des probabilités dans les épreuves répétées, *Comptes-rendus des séances de l'Académie des Sciences*, Séance du 25 Mai 1908, p. 1085-1088.
6. Les probabilités à plusieurs variables, *Annales scientifiques de l'École Normale Supérieure*, 1910, p. 339-360.
7. Mouvement d'un point ou d'un système matériel soumis à l'action de forces dépendant du hasard, *Comptes-rendus des séances de l'Académie des Sciences*, Séance du 14 Novembre 1910, présentée par M. H. Poincaré, p. 852-855.
8. Les probabilités cinématiques et dynamiques, *Annales scientifiques de l'École Normale Supérieure*, 1913, p. 77-119.

9. Les probabilités semi-uniformes, *Comptes-rendus des séances de l'Académie des Sciences*, Séance du 20 Janvier 1913, présentée par M. Appell, p. 203-205.

10. La périodicité du hasard, *L'enseignement mathématique*, 1915, p. 5-11.

11. Sur la théorie des corrélations, *Comptes-rendus des séances de la Société Mathématique de France*, Séance du 7 Juillet 1920, p. 42-44.

12. Sur les décimales du nombre π, *Comptes-rendus des séances de la Société Mathématique de France*, Séance du 7 Juillet 1920, p. 44-46.

13. Le problème général de la statistique discontinue, *Comptes-rendus des séances de l'Académie des Sciences*, Séance du 11 Juin 1923, présentée par M. d'Ocagne, p. 1693-1695.

14. Quelques curiosités paradoxales du calcul des probabilités, *Revue de Métaphysique et de Morale*, 1925, p. 311-320.

15. Probabilités des oscillations maxima, *Comptes-rendus des séances de l'Académie des Sciences*, Séance du 19 Mai 1941, p. 836-838

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