Alternative methods for forecasting GDP

Dominique Guegan, Patrick Rakotomarolahy

▶ To cite this version:

Dominique Guegan, Patrick Rakotomarolahy. Alternative methods for forecasting GDP. 2010. halshs-00505165

HAL Id: halshs-00505165

https://shs.hal.science/halshs-00505165

Submitted on 22 Jul 2010

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Documents de Travail du
Centre d'Economie de la Sorbonne

CES Working Papers

Alternative methods for forecasting GDP

Dominique GUEGAN, Patrick RAKOTOMAROLAHY

2010.65

Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13
http://ces.univ-paris1.fr/cesdp/CES-docs.htm
ISSN : 1955-611X

Alternative methods for forecasting GDP

Dominique Guégan[*], Patrick Rakotomarolahy[‡]

Abstract

An empirical forecast accuracy comparison of the non-parametric method, known as multivariate Nearest Neighbor method, with parametric VAR modelling is conducted on the euro area GDP. Using both methods for nowcasting and forecasting the GDP, through the estimation of economic indicators plugged in the bridge equations, we get more accurate forecasts when using nearest neighbor method. We prove also the asymptotic normality of the multivariate k-nearest neighbor regression estimator for dependent time series, providing confidence intervals for point forecast in time series.

Keywords: Forecast - Economic indicators - GDP - Euro area - VAR - Multivariate k nearest neighbor regression - Asymptotic normality.

JEL: C22 - C53 - E32.

1 Introduction

Forecasting macroeconomic variables such as GDP and inflation play an important role for monetary policy decisions and for assessment of future state of the economics. Policy makers and economic analysts either adapt their theoretical analysis of economic conditions according to the macroeconomic variable forecasts or even probably use them as a support and a justification of their theoretical analysis. Better forecast performance for macroeconomic variables will lead to

[*] Paris School of Economics, CES-MSE, Université Paris 1 Panthéon-Sorbonne, 106 boulevard de l'Hopital 75647 Paris Cedex 13, France, e-mail: dguegan@univ-paris1.fr.

[†] CES-MSE, Université Paris 1 Panthéon-Sorbonne, 106 boulevard de l'Hopital 75647 Paris Cedex 13, France, e-mail: rakotopapa@yahoo.fr.

[‡] This paper has been presented at the following conferences: 3^{rd} CFE in october 2009 Limassol Cyprus, 1^{st} ISCEF in february 2010 Sousse Tunisia and 9^{th} GOCPS in mars 2010 Leipzig Germany. We would like to thank the organizers of these conferences and participants for their remarks.

better decisions.

The objective of this chapter is to propose a new way to forecast GDP reducing the forecasting errors for this macroeconomic variable. We work with a non-parametric technique, say the nearest neighbors method, and we compare our methodology with classical well known parametric methods.

Considering forecasting issues, we can identify two kinds of methologies in the literature : methods based on parametric modelling, and methods based on non-parametric techniques. The former method includes the linear Autoregressive models (Box and Jenkins, 1970), the non-linear SETAR-STAR and Markov switching models, (Tong, 1990) and (Pena *et al.*, 2003)) among others. The latter one includes kernels method, nearest neighbors method, neural network and wavelet methods, (Silverman, 1986) and (Härdle *et al.*, 2004) for instance. The former method has had great consideration in economic forecasting due to the huge development of theoretical results concerning consistent, asymptotic properties and robustness of the parameters' modellings, although different problems raised concerning the strong hypotheses on model specification, estimation and asymptotic properties of the estimated parameters among others. The latter method has overcomed some of these problems avoiding an a priori specification on the modelling and the distribution of residuals. Indeed, this methodology is based on the fact that it lets the data speak to themselves. Hence it avoids the subjectivity of choosing a specific parametric modelling before looking at the data. However there is the cost of more complicated mathematical arguments such as the selection of smoothing parameters. Nevertheless recent studies help to avoid these problems and also the speed of computers that can develop search algorithms from appropriate selection criteria, Devroye and Gyorfi (1985), and Becker *et al.* (1988).

The forecasting of GDP has been studied for along starting from the growing use of linear autoregressive models. Indeed, in 1980, Sims forecasts American GDP using linear VAR model, then Litterman (1986) extends this work using the Bayesian VAR aiming on reducing VAR's parameters estimation. Engle and Granger (1987) point out possible cointegration between US GDP and monetary aggregate M2 using Vector Error Correction modelling; this approach has been recently used by Gupta (2006) to forecast South African GDP. Besides to the linear modelling for forecasting GDP, we observe the development of non-linear methods for forecasting GDP using Markov Switching models, Hamilton (1989) or SETAR models, Clements and Krolzig (1998). Another approach combines linearity and aggregation, for instance through the Bridge Equations method, Baffigi, Golinelli and Parigi (2004), and Diron (2008). In that last case, the author wants also to diminuish the number of economic indicators. Alternatively, factor models based on a great numbers of indicators have been proposed by Stock and Watson (2002) with a dynamic extension developped by Bernanke and Boivin (2003), Forni *et al.* (2005), and Kapetanios and Marcellino (2006). Recently forecasting GDP based on microeconomic foundation appears with the so called dynamic stochastic general equilibrium models, Smets and Wouters (2004). Nevertheless the linear univariate ARIMA or VAR models remain the benchmarks in the literature.

Alternatively the non-parametric techniques have not been considered a lot in economic literature for forecasting: an interesting review is Yatchew (1998). Concerning the use of these techniques to forecast GDP, in our knowledge very fews works exist. We can cite, Tkacz and Hu (1999), and Blake (1999) who use neural networks to forecast Canadian GDP or Ferrara *et al.* (2010) and Guégan and Rakotomarolahy (2010) who use nearest neighbors, and radial basis function methods to forecast euro area GDP. The kernel method which is one of the well known non-parametric method is rarely used in forecasting because it lies on important restrictions that we recall in the next Section. In this paper we focus on the nearest neighbors method because we obtain robust theoretical results for the nearest neighbors estimates which permit to build confidence intervals: these results do not exist for radial basis function in a simple way, nor with the neural networks method, and the kernel approach. Concerning these non-parametric techniques, some references are Prakasa Rao (1983), Donoho and Johnstone (1992), Kuan and White (1994), Friedman (1988), and Mack (1981) for instance.

Predicting a time series needs to estimate the conditional mean and variance of this time series in some period, given an information set based on the past. Thus, our work - based on nearest neighbors method - first concerns the estimate of the conditional mean, which permits to get the point forecast. To build the confidence intervals, we will need to estimate the conditional variance.

In order to estimate the conditional mean associated to a time series, we proceed in the following way. Given a time series $(X_n)_n$, we consider the following representation for the regression function $m(\cdot)$ associated to this time series:

$$m(x) = E[X_{n+1}|X_n = x]. \tag{1.1}$$

Model (1.1) has the type of a nonlinear regression problem for which many smoothing methods can be used for estimating purpose, Hart (1997).

In this paper, we reconstruct the function $m(\cdot)$ using multivariate k-nearest neighbors method. Among the non-parametric techniques, we focus on this method because apart from its advantage away from risk of model miss-specification or some strong hypotheses of parametric method, it handles the non-linearity of the data sets by the embedding. Indeed, working in a multivariate environment allows us to discover and takes into account the structural behavior which cannot always be discerned on a path. The other advantages of the method are for practicians due to the fact that it permits to use a small set of parameters, it is easiest to implement and it is not time consuming. Finally it is the alone non-parametric method for which robust theoretical results are available permitting to build confidence intervals which permit to compare its accuracy with classical parametric modellings. Its use for forecasting GDP growth is fundamental as soon as nonlinearity has already been detected, Hamilton (1989). On the other hand, the interest of the nearest neighbors methods has already been pointed to take into account the non-linear features of the financial data sets, Mizrach (1992), Nowman and Saltoglu (2003), and Guégan and Huck (2005) among others. Finally, working in a multivariate setting is not too constrained as soon as recent results have made available a method for selecting the number of neighbors within a given space (Ouyang *et al.*, 2006).

Our theoretical result is a contribution to the general problem concerning the non-parametric estimate of the regression function $m(.)$ with k-NN method, extending well known results obtained for independent and identically distributed random variables, Stone (1977), Mack (1981), Devroye (1982), and Stute (1984). In case of dependent variables, Collomb (1984) provides piecewise convergence for univariate variables, and Yakowitz (1987) gets the quadratic mean error for uniformly weighted k-NN estimates for univariate samples. Here, working with multivariate time series, we control the bias of a general multivariate k-NN estimate, using several weights, and we establish the asymptotic normality of this estimate from which we can construct confidence intervals.

In this paper, we use our theoretical result to propose a new way to estimate and forecast the macroeconomic indicators used to build the GDP. For that, we follow the work developed by Diron (2008) which is used a lot in Central Banks. Her method is based on a limited number of economic indicators which are plugged in eight linear equations from which an estimate of GDP is obtained. Her method associates bridge equations and forecasts combinations incorporating a large number of economic activities including different single forecasts based on production sectors, survey datas, financial variables and leading index constructed from large number of economic indicators. Her method is competitive with methods allowing a huge number of indicators and we do not compare here these different modellings. Considering one specific parametric modelling (bridge equations), we compare our forecasting errors based on non-parametric techniques with the same methodology based on linear modelling, Runstler and Sedillot (2003), and Darne (2008). Our estimation procedure is different of theirs in the sense that we estimate the economic indicators with multivariate nearest neighbors method.

The paper is organized as follows. In Section 2, we establish our theoretical result: the asymptotic normality of the multivariate k-NN regression estimate for mixing time series. In Section 3, an empirical forecast exercise is provided. It permits to compare non-parametric and parametric approaches for monthly indicators and their impact in the final GDP estimate. Section 4 concludes and Section 5 is devoted to the proofs.

2 Theoretical result

We consider a real time series $(X_n)_n$, and we transform the original data set by embedding it in a space of dimension d, building $\underline{X}_n = (X_{n-d+1}, \cdots, X_n) \in \mathbb{R}^d$. The embedding concept is important because it allows to take into account some characteristics of the series which are not always observed on the trajectory in $\mathbb{R}$.

Considering the expression (1.1), we are interested on getting an estimate of $m(\underline{x})$, $\underline{x} \in \mathbb{R}^d$, using the k closest vectors to $\underline{X}_n = \underline{x}$ inside the training set $S = \{\underline{X}_t = (X_{t-d+1}, \cdots, X_t) \mid t = d, ..., n-1\} \subset \mathbb{R}^d$. We define a neighborhood around $\underline{x} \in \mathbb{R}^d$ such that $N(\underline{x}) = \{i \mid i = 1, \cdots, k(n)$ whose $\underline{X}_{(i)}$ represents the i^{th} nearest neighbor of $\underline{x}$ in the sense of a given distance measure$\}$. Then the k-NN regression estimate of $m(\underline{x})$, $\underline{x} \in \mathbb{R}^d$ is given by:

$$m_n(\underline{x}) = \sum_{\underline{X}_{(i)} \in S, i \in N(\underline{x})} w(\underline{x} - \underline{X}_{(i)}) X_{(i)+1}, \tag{2.1}$$

where $w(\cdot)$ is a weighting function associated to neighbors and it is noteworthy that the parameter k has to be estimated. A general form for the weights is:

$$w(\underline{x} - \underline{X}_{(i)}) = \frac{\frac{1}{nR_n^d} K(\frac{\underline{x} - \underline{X}_{(i)}}{R_n})}{\frac{1}{nR_n^d} \sum_{i=1}^{n} K(\frac{\underline{x} - \underline{X}_{(i)}}{R_n})}, \tag{2.2}$$

where R_n corresponds to the distance between $\underline{x}$ and the further neighbors, and $K(\cdot)$ is a given weighting function. Two weighting functions have been mainly used, the exponential function $K(\frac{\underline{x} - \underline{X}_{(i)}}{R_n}) = exp(-||\underline{x} - \underline{X}_{(i)}||^2)$, and the uniform function $K(\frac{\underline{x} - \underline{X}_{(i)}}{R_n}) = \frac{1}{k}$.

The result established in Theorem 2.1 proves the asymptotic convergence of the NN regression estimate belonging to $\mathbb{R}^d$ for dependent variables: this result extends Yakowitz' work (1987). This result is interesting because, in practice, it is not necessary to filter the observed data to make them independent working with this approach. In another hand, this result guarantees the consistence of the estimate $m_n(\cdot)$, and therefore the conditional mean forecast will asymptotically coincide with the expected true value. Indeed, the knowledge of the bias and speed rate of the variance of the estimates provides consistent estimates, and their asymptotic normality provides confidence intervals. The building of confidence intervals can be used to compare the quality of point forecasts obtained from different methods, and enhances comparison of several methods (parametric and non-parametric methods), beyond point forecast. Indeed, no rigorous test is available to discuss the choice between the parametric and the non-parametric approaches, and predictive methodology can be used for that objective.

To establish our main result, we assume that the time series $(X_n)_n$ is strictly stationary and is characterized by an invariant measure with density f. Moreover, the random variable $X_{n+1} \mid (\underline{X}_n = \underline{x})$ has a conditional density $f(y \mid \underline{x})$, and the invariant measure associated to the embedded time series $(\underline{X}_n)_n$ is h.

Theorem 2.1. *Assuming that $(X_n)_n$ is a stationary time series, and that the following assumptions are verified:*
(i) $(X_n)_n$ is ϕ-mixing.
(ii) $m(\underline{x}), f(y \mid \underline{x})$ and $h(\underline{x})$ are p continuously differentiable.
(iii) $f(y \mid \underline{x})$ is bounded,
(iv) the sequence $k(n) < n$ is such that $\sum_{i=1}^{k(n)} w_i = 1$,
then k-NN regression function $m_n(\underline{x})$ defined in (2.1) verifies:

$$\sqrt{n^Q}(m_n(\underline{x}) - Em_n(\underline{x})) \to_{\mathcal{D}} \mathcal{N}(0, \sigma^2), \tag{2.3}$$

with

$$E[(m_n(\underline{x}) - m(\underline{x}))^2] = O(n^{-Q}), \tag{2.4}$$

where $0 \leq Q < 1$, $Q = \frac{2p}{2p+d}$, and

$$\sigma^2 = \gamma^2(Var(X_{n+1} \mid \underline{X}_n = \underline{x}) + B^2),$$

with $B = O(n^{-\frac{(1-Q)p}{d}})$, and γ a positive constant which is equal to 1 when we use uniform weights.

The proof of this theorem is postponed to the end of the article.

Some points can be mentioned:
1- As soon as the number of neighbors k is different from one, we remark that $\forall u$, $0 < w_i(u) < 1$, whatever the weighting function is used: uniform or exponential function.
2- The main difference between k-NN method and kernel method (Silverman, 1986) lies on the information set that we use to estimate the function $m(\cdot)$ at a given point $\underline{x}$. In the latter case the information set is fix and in the former case, it is flexible with respect to the choice of the number of neighbors k. In this case, such a flexibility has an impact on the values of the weights. Indeed, when the number of neighbors k increases the weights $(w_i)_{i=1}^k$ decrease, then the product $(k.w_i)_{i=1}^k$ turn around a constant γ which belongs to $\mathbb{R}$. For uniform weights, $w_i = \frac{1}{k}$ and $\gamma = 1$. This last property implies that the asymptotic variance of the estimate $m_n(\cdot)$ does not depend on

the true density nor on the quantity $\int w^2(u)du$. This asymptotic property is not verified when we work with the kernel method, details are provided in Section 5.

3- The mixing conditions characterize different behaviors of dependent variables. Parametric processes like the bilinear models including ARMA models, the related GARCH processes and the Markov switching processes are known to be mixing, Guégan (1983) and Carrasco and Chen (2002). Thus, in practice this condition is not too restrictive.

4- The condition (iv) in theorem 2.1 is verified in particular for the weights introduced in equation (2.2). The parameter γ introduced before entails the correlations between the vectors $\underline{X}_n$. Finally the theorem (2.1) providing asymptotic normality for the estimate $m_n(\underline{x})$ under regular conditions permits to build confidence interval whose expression is given in the following corollary.

Corollary 2.1. *Under the assumptions of theorem 2.1, a general form for the confidence interval around $m(\underline{x})$, for a given risk level $0 < \alpha < 1$, is:*

$$m(\underline{x}) \in [m_n(\underline{x}) - B - \frac{\hat{\sigma} z_{1-\frac{\alpha}{2}}}{\sqrt{k}}, m_n(\underline{x}) + B + \frac{\hat{\sigma} z_{1-\frac{\alpha}{2}}}{\sqrt{k}}] \tag{2.5}$$

where $z_{1-\frac{\alpha}{2}}$ is the $(1-\frac{\alpha}{2})$ quantile of the Student law, $\hat{\sigma}$ is an estimate for σ and B is such that:

1. *B is negligible, if $\frac{k(n)}{n} \to 0$, as $n \to \infty$,*
2. *If not, $B = O(r^p)$, with $r = \left(\frac{k(n)}{(n-d)\hat{h}(\underline{x})c}\right)^{\frac{1}{d}}$ where $c = \frac{\pi^{d/2}}{\Gamma((d+2)/2)}$, and $\hat{h}(\underline{x})$ is an estimate for the density $h(\underline{x})$.*

The proof of this corollary is postponed at the end of the article.

3 Forecasting Euro-area GDP

Information on the current state of economic activity is a crucial ingredient for policy making. Economic policy makers, international organisations and private sector forecasters commonly use short term forecasts of real gross domestic product (GDP) growth based on monthly indicators. There exists many studies proposing real-time modelling in order to take into account some complexity inherent to the computation of the GDP which are: the number of economic indicators, the modelling for GDP and the impact of data revisions, Koenig *et al.* (2003), Baffigi *et al.* (2004), and Schumacher and Breitung (2008), among others. The first cited paper focuses on the choice of vintage data and found a substantial improvement of GDP forecast when using real-time vintage data; the second paper deals on the modelling for GDP by comparing euro area GDP forecasts from linear ARIMA and VAR models with bridge equations and concludes that the latter provides better forecasts; the last paper suggested the use of large factor models for mixed-frequency datas supported by experiment on monthly German GDP, and concludes on a minor impact of the data revision in forecasting performance. In the present exercise we show that beyond the model chosen for GDP forecasts, the estimates of the monthly economic indicators is crucial and can lead to non negligible errors if there are not properly estimated. Thus our approach is slightly new and different comparing with most of the published works on this subject.

In order to illustrate our proposal, we restrict to the modeling of GDP using the Bridge equations modelling, Runstler and Sedillot (2003). We consider the eight equations introduced in the paper of Diron (2008), each equation providing a model of GDP, denoted $Y_t^i, i = 1, \cdots, 8$. They are finally aggregated to provide a final value of GDP, denoted Y_t. Each equation is calculated from thirteen monthly economic indicators, denoted $X_t^i, i = 1, \cdots, 13$. We focus here in the forecasting of these thirtheen indicators. We estimate and forecast these indicators from two models: the unrestricted VAR modelling and the multivariate NN approach. For the latter method we distinguish forecasts obtained without embedding data sets (d $= 1$) from forecasts obtained when d> 1. The thirteen economic indicators that we consider are listed in Table 2.

For this exercise, we use the real-time data base provided by EABCN through their web site[1]. The real-time information set starts in January 1990 when possible (exceptions are the confidence indicator in services, that starts in 1995, and EuroCoin, that starts in 1999) and ends in November 2007. The vintage series for the OECD composite leading indicator are available through the OECD real-time data base [2]. The EuroCoin index is taken as released by the Bank of Italy. The vintage data base for a given month takes the form of an unbalanced data set at the end of the sample. To solve this issue, we apply the two previous methodologies to forecast the monthly variables in order to complete the values until the end of the current quarter for

[1] www.eabcn.org

[2] http://stats.oecd.org/mei/

GDP nowcasts and until the end of the next quarter for GDP forecasts, then we aggregate the monthly data to quarterly frequencies.

Alternatively, we use a stationary VAR (Vector Autoregressive) methodology for forecasting economic indicators and combine later with bridge equations proposed by Diron (2008) to get estimates for GDP, making first all the data stationary using first difference. Among the thirteen indicators used in Diron equations, three indicators (Economic Sentiment Indicator (ESI), Composite Leading Indicator (CLI) and EuRoCoin (ERC)) appear redundant in the sense that they are built from the ten others. Thus, in order to avoid variables repetition in the model which could produce extra contribution on the variance through correlations, we consider only ten indicators as endogenous variables in the VAR modelling, building 10-variates VAR for these remaining ten indicators. Finally, using AIC and Schwartz criteria for order selection we retain a 10-variates VAR(1), Akaike (1974) and Schwartz (1978). Nevertheless, we need estimates for the previous three indicators to finalize the computation of GDP with the Diron equations. We adjust a specific ARIMA modelling for each three variables using AIC criterion for determining the orders p and q . Finally, we retain ARIMA(3,1,0), ARIMA(10,1,0) and ARIMA(1,1,0) respectively for ESI, CLI and ERC indicators. In all parametric models the parameters are estimated by least squares method. We use recursive forecasts when computing the forecast beyond one step ahead.

Regarding the method of NN, d being given, we determine the number of neighbors k by minimizing the mean square error criterion (RMSE):

$$\sqrt{\frac{1}{n-k-d}\sum_{t=k+d+1}^{n}||\hat{X}_{t+1}^{i}-X_{t+1}^{i}||^{2}} \quad i=1,\cdots,13 \tag{3.1}$$

where n is the sample size, $\hat{X}_{t+1}^{i}$ is the estimate of the i-th economic indicator X_{t+1}^{i} calculated from the expression (2.1). The number $1 \leq k \leq 5$ of nearest neighbors determined by this criterion at the horizon h=1 is used to calculate the forecasting capabilities for $h > 1$.

In the case of the multivariate approach ($d > 1$), we describe below the algorithm used to determine the embedding dimension d and the number of neighbors k used to obtain the best predictor for X_{n+h}^{i} in the sense of the previous RMSE. We present the method for all indicators, and thus for simplicity we drop the index i in the algorithm. All the data sets have been made stationary with the same transformation than the one used for VAR modelling. Thus, now we assume that we observe a stationary data set $X_1, ..., X_n$ in $\mathbb{R}$.

1. We embed this data set in a space of dimension d, $2 \leq d \leq 10$, getting a sequence of vectors in $\mathbb{R}^d$: $\{\underline{X}_d, \underline{X}_{d+1}, ..., \underline{X}_n$, where $\underline{X}_i = (X_{i-d+1}, ..., X_i)\}$.
2. Ranking the vectors, we determine the k nearest vectors of $\underline{X}_n$. Denoting $r_i = \|\underline{X}_n - \underline{X}_i\|$, $i = d, d{+}1, ..., n{-}1$, the distance between these vectors, we build the sequence $r_d, r_{d+1}, ..., r_{n-1}$ ordered in an increasing way: $r_{(d)} < r_{(d+1)} < ... < r_{(n-1)}$, which provides the k nearest vectors $\underline{X}_{(j)}$ corresponding to these $r_{(j)}$, $j = d,\ d+1, ...,\ d+k-1$.
3. The one step ahead forecast $m_n(\underline{X}_n) = \hat{X}_{n+1}$, is obtained from:
$$\hat{X}_{n+1} = \sum_{j=d}^{k+d-1} w(\left\|\underline{X}_n - \underline{X}_{(j)}\right\|)X_{(j)+1}. \tag{3.2}$$
4. Considering now the new information set: $X_1, ..., X_n, \hat{X}_{n+1}$, redo step 1 to step 4, we get the two steps ahead forecast. We obtain the forecast of third step ahead in a similar way as for the two steps ahead forecast. And so on $\cdots$. We limit the choice of the embeddings d to ten in order to get enought data when we make the embedding.

We consider exponential weighting function since it reflects the local behavior of nearest neighbors method giving more weight to closest neighbors. We favor this kind of weights rather than the uniform weights which give the same importance to all neighbors. Now, for each indicator $X_t^i, i = 1, \cdots, 13$, the best pair (d, k) is determined by minimizing again the RMSE criteria defined in (3.1). Once the pair (d, k) is found, it is used for all prediction horizons.

As soon as we get the estimates for the monthly indicators with the two previous methods (VAR and k-NN methods), we compute the GDP flash estimates that were released in real-time by Eurostat from the first quarter of 2003 to the third quarter of 2007 using the previous forecasts of the monthly indicators. According to this scheme, the monthly series have to be forecast for an horizon h varying between 3 and 6 months in order to complete the data set at the end of the sample. Recall that the h-step-ahead predictor for $h > 1$ is estimated recursively starting from the one-step-ahead formula.

Using five years of vintage data, from the first quarter 2003 to the third quarter 2007, we provide RMSEs for the Euro area flash estimates of GDP growth $\hat{Y}_t$ in genuine real-time conditions. We have computed the RMSEs for the quarterly GDP flash estimates, obtained with the forecasting methods used to complete adequately in real-time the monthly indicators, that is VAR modelling and k-NN methods ($d = 1$ and $d > 1$). More precisely, we provide the RMSEs of the combined forecasts based on the arithmetic mean of the eight Diron equations. Thus, for a given forecast horizon h, we compute $\hat{Y}_t^j(h)$ which is the predictor stemming from these equations $j = 1, \cdots, 8$, in which we have plugged the forecasts of the monthly economic indicators, and we compute the final estimate GDP at horizon h: $\hat{Y}_t(h) = \frac{1}{8}\sum_{j=1}^{8} \hat{Y}_t^j(h)$. The RMSE criterion used for the final GDP is

$$RMSE(h) = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{Y}_t(h) - Y_t)^2}, \tag{3.3}$$

where T is the number of quarters between Q1 2003 and Q2 2007 (in our exercise, $T = 18$) and Y_t is the Euro area flash estimate for quarter t. The RMSE errors for final GDP are provided in table 1 and comments follow.

h	VAR	k-NN(1)	k-NN(d>1)
6	0.225	**0.198**	0.214
5	0.224	0.203	**0.192**
4	0.214	0.202	**0.196**
3	0.192	0.186	**0.177**
2	0.181	0.176	**0.177**
1	0.173	0.174	**0.171**

Table 1: RMSE for the estimated mean quarterly GDP Y_t computed from equation (3.3), using VAR(p) modelling (column 2) and k-NN predictions ($d = 1$ (column 3), and $d > 1$ (column 4)) for the monthly economic indicators $X_t^i, i = 1, \cdots, 13$, h is the monthly forecast horizon. Values in boldface correspond to the smallest error for a given forecast horizon.

For both methods, VAR modelling and k-NN method, the RMSE becomes lower when the forecast horizon reduces from $h = 6$ to $h = 1$, illustrating the accuracy of the nowcasting and forecasting which increases as soon as the information set becomes more and more efficient, thanks to the released monthly data. This is the strengthen of GDP forecasting based on monthly economic indicators, instead of considering only GDP itself since each month, new true values of economic indicators are available. We remark that few days before the publication of the flash estimate (around 13 days with $h = 1$), the lowest RMSE is obtained with the multivariate k-NN method (RMSE=0.171).

Looking at forecast errors by comparing column 2 on one side with columns 3 and 4 on the other side, we find that forecast errors are always lower with the method of NN rather than with VAR modelling (except at horizon $h = 1$ where VAR modelling gives better forecast error than univariate k-NN). One source of such gain comes from the use of nearest neighbor method which is adapted even with small samples, which is not the case when working with VAR modelling requiring large samples to be robust.

Lastly, if we focus on nearest neighbor method, we obtain smaller errors when working with multivariate setting $d > 1$ than with univariate one $d = 1$. This result shows the gain of the method developed in a space of higher dimension. Indeed, we expect that in terms of predictions, any method developed in higher dimension improves the forecast accuracy. This is confirmed when we compare, for the same method, the forecast errors obtained only in $\mathbb{R}$ with the error calculated from a treatment in $\mathbb{R}^d$: in this latter case the errors are always smaller (e.g. comparing columns 3 and 4 of Table 1). This idea has already developed in other empirical works considering multivariate methods, Kapetanios and Marcellino (2006) with factor models, and Guégan and Rakotomarolahy (2010) with multivariate non-parametric techniques.

To see the evolution of the trajectory of both forecasting parametric and non-parametric methods, we provide, in figures 1 and 2, the graphs of the observed and estimated GDP growth from k-NN methods and VAR modeling for forecast horizons varying from one to six quarters. Some points can be mentioned from these graphs. The trajectories of GDP forecasts from both methods are very close for horizons $h \leq 3$ and are able to follow the "true" trajectory. In addition, they permit to detect also some declines of euro area GDP for example in 2003Q2. In another hand, for forecast horizons $h > 3$, the VAR modelling provides forecasts for GDP which converge to the sample mean when the forecast horizon increases. Conversely, the k-NN modelling provides forecasts which follow the observed GDP trajectory.

4 Conclusion

Knowing the importance of the nowcast and the forecast of macroeconomic variables (such as GDP or inflation) when analysing the current state of the economics and setting policy for the future economic conditions, we suggest in this paper alternative methods based on non-parametric multivariate nearest neighbor to improve the accuracy of GDP forecasts.

We focus on detecting the best predictor for economic indicators using a RMSE criterion, working in an embedded space of dimension d, and focusing on the relevant set of data permitting to solve this specific criterion, Han et al. (1997) and Hoover and Perez (1999).

Our application used a new theoretical result which extends, for the multivariate nearest neighbors estimation method, the L^2 consistence result obtained in Yakowitz (1987) with uniform weighting.

Some opened questions arise. We cite some: the use of the aggregated monthly economic indicators to match quarterly GDP; a specific test to decide a good strategy between parametric and non-parametric modellings; the trade-off between stationarity and non-linearity when we work with non-parametric techniques.

References

Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transaction on Automatic Control,* AC.19.

Baffigi, A.R., R. Golinelli and G. Parigi (2004). Bridge models to forecast the euro area GDP. *International Journal of Forecasting 20,* 447-460.

Becker, R.A., J.M. Chambers and A.R. Wilks (1988). *The new S language.* Chapman and Hall: New York.

Blake, A.P.(1999). An Artificial Neural Network System of Leading Indicators. *National Institute of Economic and Social Research, unpublished paper.*

Bernanke, B.S. and J. Boivin (2003). Monetary policy in a data-rich environment. *Journal of Monetary Economics 50,* 525-546.

Box, G.E.P. and G.M. Jenkins (1970). *Time Series Analysis: Forecasting and Control.* Holden Day: New York.

Carrasco, M. and X. Chen (2002). Mixing and moment properties of various GARCH and Stochastic Volatility models. *Econometric Theory 18,* 17-39.

Clements M.P. and H.M. Krolzig (1998). A comparison of the forecast performance of Markov-switching and threshold autoregressive models of US GNP. *Econometrics Journal 1,* C47-C75.

Collomb, G. (1984). Nonparametric time series analysis and prediction: Uniform almost sure convergence of the window and k-NN autoregression estimates. *Math Oper. Stat., Ser. Statistics 1984.*

Darne, O. (2008). Using business survey in industrial and services sector to nowcast GDP growth: The French case. *Economics Bulletin 3*, No 32, 1-8.

Devroye, L.P. (1982). Necessary and sufficient conditions for the pointwise convergence of nearest neighbor regression function estimates. *Z. Wahrscheinlichkeneitstheorie Verw Gebiete 61*, 467-481.

Devroye L.P. and L. Györfi (1985). *Nonparametric Density Estimation: the L1 View.* Wiley: New York.

Diron, M. (2008). Short-term forecasts of Euro area real GDP growth: an assessment of real-time performance based on vintage data. *Journal of Forecasting 27,* 371-390.

Donoho, D.L. and I.M. Johnstone (1992). Minimax estimation via wavelet shrinkage. Technical report 402, Dept. Stat. Stantford university.

Engle R.F. and C.W.J. Granger (1987). Co-integration and error correction: Representation, Estimation, and Testing. *Econometrica 55,* 251-276.

Ferrara, L., D. Guégan and P. Rakotomarolahy (2010). GDP nowcasting with ragged-edge data: a semi-parametric modeling. *Journal of Forecasting 29*, 186-199.

Forni, M., D. Giannone, M. Lippi and L. Reichlin (2005). Opening the black box: structural factor models with large cross-sections. *European Central Bank WP, No 571.*

Friedman, J.H. (1988). Multivariate adaptative regression splines (with discussion). *Annals of Statistics 19,* 1-141.

Guégan, D. (1983). Une condition d'ergodicité pour des modèles ilinéaires à temps discret. *CRAS Série 1, 297-301.*

Guégan, D. and N. Huck (2005). On the use of nearest neighbors in finance. *Revue de Finance 26,* 67-86.

Guégan, D. and P. Rakotomarolahy (2010). A Short Note on the Nowcasting and the Forecasting of Euro-area GDP Using Non-Parametric Techniques. *Economics Bulletin 30,* No.1, 508-518.

Gupta, R. (2006) . Forecasting the South African economy with VARs and VECMs. *South African Journal of Economics 74,* 611–628.

Hamilton, J.D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica 57,* 357–384.

Han, J., Y. Fu, W. Wang, K. Koperski and O. Zaine (1997). DMQL: a data mining query language for relational databases. WP Simon Fraser University, B.C. Canada.

Hardle, W., Muller M., Sperlich S. and Werwatz A. *Non-Parametric and Semi-Parametric Models.* Springer Verlag, 2004.

Hart, J.D. (1994). *Nonparametric smoothing and lack-of-fit tests.* Springer verlag, New York.

Hoover, K.D. and S.J. Perez (1999). Data mining reconsidered: encompassing and the general-to-specific approach to specification search. *Econometrics Journal 2,* 167–191.

Kapetanios, G. and M. Marcellino (2006). A parametric estimation method for dynamic factors models of large dimensions. *IGIER WP, No 305.*

Koenig E.F., S. Dolmas and J. Piger (2003). The use and abuse of real-time data in economic forecasting. *The Review of Economics and Statistics 85,* 618-628.

Kuan, C.M. and H. White (1994). Artificial neural networks : an econometric perspective. *Econometric Reviews 13,* 1-91.

Litterman R.B. (1986). Forecasting with Bayesian vector autoregressions, five years of experience. *Journal of Business Economic Statistics 4,* 25-38.

Mack, Y.P. (1981). Local properties of k-NN regression estimates. *SIAM Journal on Algebraic and Discrete Methods 2,* 311-323.

Mizrach, B. (1992). Multivariate Nearest-Neighbour Forecasts of EMS Exchange Rates. *Journal of Applied Econometrics 7,* Supplement: Special Issue on Nonlinear Dynamics and Econometrics (Dec., 1992), S151-S163.

Nowman, B. and B. Saltoglu (2003). Continuous time and nonparametric modelling of U.S. interest rate models. *International Review of Financial Analysis 12,* 25-34.

Ouyang, D., D. Li and Q. Li (2006). Cross-validation and nonparametric k nearest neighbor estimation. *Econometrics Journal 9,* 448–471.

Pagan A. and A. Ullah (1999). *Nonparametric Econometrics.* Cambridge University Press: Cambridge.

Peligrad, M. and S.A. Utev (1997). Central limit theorem for Linear Processes. *The Annals of Probability 25,* 443-456.

Pena, D., G.C. Tiao and R.S. Tsay (2003). *A course in time series analysis.* Wiley Series in Probability and Statistics: New York.

Prakasa Rao, B.L.S. (1983). *Nonparametric functional estimation.* Orlando FL, Academic press.

Rünstler, G. and F. Sedillot (2003). Short-term estimates of Euro area real GDP by means of monthly data. *European Central Bank WP, No 276.*

Schwartz, G. (1978). Estimating the dimension of ARIMA models. *Annals of Statistics 6,* 461-464.

Silverman, B.W. (1986). *Density estimation for statistics and data Analysis.* Chapmann and Hall: London.

Sims C.A. (1980). Macroeconomics and reality. *Econometrica 48*, 1–48.

Smets, F. and R. Wouters (2004). Forecasting with a Bayesian DSGE model: An application to the Euro area. *Journal of Common Market Studies 42,* 841–867.

Stock, J.H., and M.W. Watson (2002). Macroeconomic Forecasting Using Diffusion Indexes. *Journal of Business & Economic Statistics 20,* 147–162.

Stone, C. (1977). Consistent non parametric regression. *Annals of Statistics 5,* 595-645.

Stute, W. (1984). Asymptotic normality of nearest neighbor regression function estimates. *Annals of Statistics 12,* 917-926.

Tkacz, G. and S. Hu (1999). Forecasting GDP growth using artificial neural networks. *Bank of Canada WP, No 99.*

Tong, H. (1990). *Non-linear time series: a dynamical system approach.* Oxford University Press: Oxford.

Webb R.H. (1995). Forecasts of Inflation from VAR Models. *Journal of Forecasting 14,* 267-285.

Yakowitz, S. (1987). Nearest neighbors method for time series analysis. *Journal of Time Series Analysis 8,* 235-247.

Yatchew, A.J. (1998). Nonparametric regression techniques in economics. *Journal of Economic Literature 36,* 669-721.

5 Proofs of Theorem 2.1 and Corollary 2.1

We start giving the proof of theorem 2.1. We first establish a preliminary lemma.

Lemma 5.1. *Under the hypotheses of theorem 2.1, either the estimate $m_n(\underline{x})$ is asymptotically unbiased or*

$$E[m_n(\underline{x})] = m(\underline{x}) + O(n^{-\beta}) \tag{5.1}$$

with $\beta = \frac{(1-Q)p}{d}$.

Proof 5.1. *We denote* $B(\underline{x}, r_0) = \{z \in \mathbb{R}^d, \|\underline{x} - z\| \leq r_0\}$ *the ball centered at* $\underline{x}$ *with radius* $r_0 > 0$. *We characterize the radius* r *insuring that* $k(n)$ *observations fall in the ball* $B(\underline{x}, r)$*; indeed, since the function* $h(.)$ *is* $p-$*continuously differentiable, for a given* i *the probability* q_i *of an observation* $\underline{x}_i$ *to fall in* $B(\underline{x}, r)$ *is:*

$$\begin{aligned} q_i &= P(\underline{x}_i \in B(\underline{x}, r)) && (5.2) \\ &= \int_{B(\underline{x},r)} h(\underline{x}_i) d\underline{x}_i = h(\underline{x}). \int_{B(\underline{x},r)} d\underline{x}_i + \int_{B(\underline{x},r)} (h(\underline{x}_i) - h(\underline{x})) d\underline{x}_i && (5.3) \\ &= h(\underline{x}) c r^d + o(r^d), && (5.4) \end{aligned}$$

where c *is the volume of the unit ball and* $\underline{x} = dx_1 dx_2 \cdots dx_d$. *Thus,* $q_i - q_j = o(r^d)$ *for all* $i \neq j$. *We consider now the k-NN vectors* $\underline{x}_{(k)}$ *and we denote* q *the probability that they are in the ball* $B(\underline{x}, r)$*, that is* $q = P(\underline{x}_{(k)} \in B(\underline{x}, r))$*, then :*

$$q_i = q + o(r^d). \tag{5.5}$$

Being given $N(r, n)$*, the number of observations falling in the ball* $B(\underline{x}, r)$*, for a given* $r > 0$*, we characterize* r *such that* $k(n)$ *observations fall in* $B(\underline{x}, r)$. *We proceed as follows. We denote* $\mathcal{S}_i^n$ *all non ordered combinations of the* $i-$*uple indices from* $(n - d)$ *indices, then:*

$$\begin{aligned} E[N(r,n)] = \sum_{i=0}^{n-d} i P(N(r,n) = i) &= \sum_{i=0}^{n-d} i \sum_{(j_1,\cdots,j_i) \in \mathcal{S}_i^n} \prod_{j=j_1}^{j_i} q_j \prod_{\substack{\ell=1 \\ \ell \notin \{j_1,\cdots,j_i\}}}^{n-d} (1 - q_\ell) \\ &\geq \sum_{i=0}^{n-d} i \sum_{(j_1,\cdots,j_i) \in \mathcal{S}_i^n} \underline{q}^i (1 - \overline{q})^{n-d-i} = \sum_{i=0}^{n-d} i \begin{pmatrix} n-d \\ i \end{pmatrix} \underline{q}^i (1 - \overline{q})^{n-d-i} && (5.6) \\ &= \underline{q}(n-d)(1 + \underline{q} - \overline{q})^{n-d}, \end{aligned}$$

where $\underline{q}$ *and* $\overline{q}$ *are respectively the smallest and largest probabilities* q_i $i = 1, \cdots, n - d$. *Thus, we obtain a lower bound for* $E[N(r, n)]$. *If* $E[N(r, n)] = k(n)$*, using (5.4) - (5.6), we obtain:*

$$r \leq \left(\frac{k(n)}{(n-d)} \right)^{\frac{1}{d}} D(\underline{x}), \tag{5.7}$$

with $D(\underline{x}) = \left(\frac{1}{h(\underline{x})c} \right)^{\frac{1}{d}}$.

Now, using the relationship (2.1), we get:

$$E[m_n(\underline{x})] \;=\; \sum_{i\in N(\underline{x})} E[w(\underline{x}-\underline{X}_{(i)})Y_i], \tag{5.8}$$

where $Y_i = X_{(i)+1}$. *We can remark that* $E[w(\underline{x}-\underline{X}_{(i)})Y_i] = \int_{\mathbb{R}^d}\int_{\mathbb{R}} w(\underline{x}-\underline{x}_i)y_i f(y_i,\underline{x}_i)d\underline{x}_i dy_i$. *Since* $f(y_i,\underline{x}_i) = f(y_i \mid \underline{x}_i)h(\underline{x}_i)$, *then we obtain* $E[w(\underline{x}-\underline{X}_{(i)})Y_i] = \int_{\mathbb{R}^d}\int_{\mathbb{R}} w(\underline{x}-\underline{x}_i)y_i f(y_i \mid \underline{x}_i)h(\underline{x}_i)d\underline{x}_i dy_i$. *Thus, as soon as the weighting function* $w(\cdot)$ *is vanishing outside the ball* $B(\underline{x},r)$*:*

$$E[w(\underline{x}-\underline{X}_{(i)})Y_i] \;=\; \int_{B(\underline{x},r)} w(\underline{x}-\underline{x}_i)\bigg(\int_{\mathbb{R}} y_i f(y_i \mid \underline{x}_i)dy_i\bigg)h(\underline{x}_i)d\underline{x}_i \tag{5.9}$$

$$\;=\; \int_{B(\underline{x},r)} w(\underline{x}-\underline{x}_i)m(\underline{x}_i)h(\underline{x}_i)d\underline{x}_i. \tag{5.10}$$

To compute the bias we need to evaluate: $E[m_n(\underline{x})] - m(\underline{x})$. *We begin to evaluate :*

$$\sum_{i\in N(\underline{x})}\int_{B(\underline{x},r)} w(\underline{x}-\underline{x}_i)m(\underline{x})h(\underline{x}_i)d\underline{x}_i = m(\underline{x})E[\sum_{i\in N(\underline{x})} w(\underline{x}-\underline{X}_{(i)})] = m(\underline{x}). \tag{5.11}$$

Then,

$$E[m_n(\underline{x})] - m(\underline{x}) \;=\; \sum_{i\in N(\underline{x})}\int_{B(\underline{x},r)} w(\underline{x}-\underline{x}_i)(m(\underline{x}_i)-m(\underline{x}))h(\underline{x}_i)d\underline{x}_i. \tag{5.12}$$

The equation (5.12) holds because $\sum_{i\in N(\underline{x})}\int_{B(\underline{x},r)} w(\underline{x}-\underline{x}_i)h(\underline{x}_i)d\underline{x}_i = 1$, *(Assumption (iv) in Theorem 2.1). Then,*

$$|E[m_n(\underline{x})] - m(\underline{x})| \leq \sum_{i\in N(\underline{x})}\int_{B(\underline{x},r)} w(\underline{x}-\underline{x}_i)a\,\|\underline{x}_i-\underline{x}\|^p\, h(\underline{x}_i)d\underline{x}_i. \tag{5.13}$$

We get this last expression since the constant a *is known and* $m(\cdot)$ *is* $p-$*continuously differentiable. The inequality (5.13) implies that:*

$$|E[m_n(\underline{x})] - m(\underline{x})| \leq ar^pE[\sum_{i\in N(\underline{x})} w(\underline{x}-\underline{X}_{(i)})]. \tag{5.14}$$

The relationship in (5.14) holds because $\|\underline{x}_i-\underline{x}\|^p < r^p$, *as soon as* $\underline{x}_i \in B(\underline{x},r)$. *Now, both cases be considered:*

1. *When* r *is very small, than the bias is negligible and* $E[m_n(\underline{x})] = m(\underline{x})$ *.*

2. *If the bias is not negligible, using (5.7) and (5.14), we get:*

$$|E[m_n(\underline{x})] - m(\underline{x})| \leq a\bigg(\frac{k(n)}{(n-d)}\bigg)^{\frac{p}{d}} D(\underline{x})^p. \tag{5.15}$$

   If we choose $k(n)$ *as in integer part of* n^Q, *and knowing that* $\frac{k}{n-d} \sim \frac{k}{n}$, *then* $|E[m_n(\underline{x})] - m(\underline{x})| = O(n^{-\beta})$ *with* $\beta = \frac{(1-Q)p}{d}$.

The proof of lemma 5.1 is complete.

Now, we prove theorem 2.1.

Proof 5.2. *1. We begin to establish the relationship (2.4). In the following, we denote* $Y_i = X_{(i)+1}$*. We rewrite the left part of equation (2.4) as follows:*

$$E[(m_n(\underline{x}) - m(\underline{x}))^2] = Var(m_n(\underline{x})) + (E[m_n(\underline{x})] - m(\underline{x}))^2. \tag{5.16}$$

We first compute the variance of $m_n(\underline{x})$*, considering two cases:*

a) First case: The weights w_i*,* $i = 1, ..., k$*, are independent of* (X_n)*. In that case the variance of* $m_n(x)$ *is equal to:*

$$Var(m_n(\underline{x})) = A + B, \tag{5.17}$$

where $A = \sum_{i=1}^{k(n)} w_i^2 Var(Y_i)$ *and* $B = \sum_{i=1}^{k(n)} \sum_{j \neq i} w_i w_j cov(Y_i, Y_j)$*. Using the assumption (ii) of theorem 2.1, we get* $|B| \leq \sum_{i=1}^{k(n)} \sum_{j \neq i} |cov(Y_i, Y_j)|$*. This last term is negligible due to Yakowitz' result (1987) on the sum of covariances. Now ,* $A = \frac{1}{k(n)^2} \sum_{i=1}^{k(n)} (k(n)w_i)^2 (v(\underline{x}) + (E[Y_i] - m(\underline{x}))^2)$*. Using the fact that the weights are decreasing with respect to the chosen distance,* $w_k \leq \cdots \leq w_1$*, we get:*

$$\frac{1}{k(n)^2} \sum_{i=1}^{k(n)} (k(n)w_k)^2 (v(\underline{x}) + (E[Y_i] - m(\underline{x}))^2) \leq A \leq \frac{1}{k(n)^2} \sum_{i=1}^{k(n)} (k(n)w_1)^2 (v(\underline{x}) + (E[Y_i] - m(\underline{x}))^2). \tag{5.18}$$

As soon as $k(n) \to \infty$ *the product* $k(n)w_i$ *converges to one in case of uniform weights, and can be bounded for exponential weights for all* i *and for all* n*, thus there exist two positive constants* c_0 *and* c_1 *such that (5.18) becomes :*

$$\frac{c_1^2}{k(n)^2} \sum_{i=1}^{k(n)} (v(\underline{x}) + (E[Y_i] - m(\underline{x}))^2) \leq A \leq \frac{c_0^2}{k(n)^2} \sum_{i=1}^{k(n)} (v(\underline{x}) + (E[Y_i] - m(\underline{x}))^2). \tag{5.19}$$

where $v(\underline{x}) = Var(X_{n+1} \mid \underline{X}_n = \underline{x})$*. Using the assumption (iv) of Theorem 2.1, we remark that* $E[Y_i] = E[m_n(\underline{x})]$*. Now again, if* $k(n) = [n^Q]$ *where* $[\cdot]$ *corresponds to the integer part of a real number, then* $A = O(n^{-Q})$ *from lemma 5.1 when* $n \to \infty$*. It follows that the relationship (5.17) becomes:*

$$Var(m_n(\underline{x})) = O(n^{-Q}), \tag{5.20}$$

and

$$(E[m_n(\underline{x}) - m(\underline{x})])^2 = O(n^{-2\beta}). \tag{5.21}$$

Plugging equations (5.20) and (5.21) inside equation (5.16), we get $2\beta = Q$ *or* $Q = \frac{2p}{2p+d}$ *and the proof is complete.*

b) Second case: the weights w_i, $i = 1, ..., k$, *depend on* (X_n). *We use again the relationship (5.17) with* $A = \sum_{i=1}^{k(n)} Var(w(x - \underline{X}_{(i)})Y_i)$ *and* $B = \sum_{i=1}^{k(n)} \sum_{j \neq i} cov(w(x - \underline{X}_{(i)})Y_i, w(x - \underline{X}_{(j)})Y_j)$. *Remarking that* $(w(x - \underline{X}_{(j)})Y_j)$ *are* ϕ*-mixing since* (X_j) *and* (Y_j) *are* ϕ*-mixing Pagan and Ullah (1999), then* B *is negligible from Yakowitz' result (1987). We remark also that* $A = \sum_{i=1}^{k(n)} (E[(w(x - \underline{X}_{(i)})Y_i)^2] - (E[w(x - \underline{X}_{(i)})Y_i])^2)$, *then*

$$A = \sum_{i=1}^{k(n)} [\int_{\mathbb{R}^d} \int_{\mathbb{R}} w(\underline{x} - \underline{x}_i)^2 y_i^2 f(y_i, \underline{x}_i) d\underline{x}_i dy_i - (\int_{\mathbb{R}^d} \int_{\mathbb{R}} w(\underline{x} - \underline{x}_i) y_i f(y_i, \underline{x}_i) d\underline{x}_i dy_i)^2]. \tag{5.22}$$

When k *increases, the weights* w_i *decrease, and* $k(n)w_i \sim \gamma$ *where* γ *is a real constant, then*

$$\begin{aligned} A &= \frac{\gamma^2}{k(n)^2} \sum_{i=1}^{k(n)} [\int_{\mathbb{R}^d} \int_{\mathbb{R}} y_i^2 f(y_i, \underline{x}_i) d\underline{x}_i dy_i - (\int_{\mathbb{R}^d} \int_{\mathbb{R}} y_i f(y_i, \underline{x}_i) d\underline{x}_i dy_i)^2] & (5.23) \\ &= \frac{\gamma^2}{k(n)^2} \sum_{i=1}^{k(n)} (E[Y_i^2] - E[Y_i]^2). & (5.24) \end{aligned}$$

Under stationary conditions for (X_n) *and recalling that* $Y_i = X_{(i)+1}$, *then equation (5.24) is equivalent to* $A = \frac{\gamma^2}{k(n)} (E[X_1^2] - E[X_1]^2)$ *and* $A = \frac{\gamma^2}{k(n)} Var(X_1)$. *Finally expression (5.17) becomes:*

$$Var(m_n(\underline{x})) = \frac{\gamma^2}{k(n)} Var(X_1). \tag{5.25}$$

Moreover, when we take $k(n) = n^Q$, *thus equation (5.25) is equal to:*

$$Var(m_n(\underline{x})) = O(n^{-Q}). \tag{5.26}$$

Plugging equations (5.26) and (5.21) in equation (5.16), gives $2\beta = Q$, *and* $Q = \frac{2p}{2p+d}$, *and the proof is complete.*

2. *We prove now the asymptotic normality of* $m_n(\underline{x})$. *We assume that the variance* $\sigma_n = var[m_n(\underline{x})]$ *exists and is non null, thus:*

$$\frac{m_n(\underline{x}) - Em_n(\underline{x})}{\sigma_n} = \sum_{i=1}^{k(n)} \frac{w_i Y_i - E w_i Y_i}{\sigma_n}. \tag{5.27}$$

To establish the asymptotic normality of $m_n(\underline{x})$, we distinguish three cases corresponding to the choice of the weighting functions.

i) The weights are uniform, $w_i = \frac{1}{k(n)}$, then equation (5.27) becomes:

$$\frac{m_n(\underline{x}) - Em_n(\underline{x})}{\sigma_n} = \sum_{i=1}^{k(n)} \frac{1}{k(n)} Z_i, \tag{5.28}$$

where $Z_i = \frac{Y_i - EY_i}{\sigma_n}$. The asymptotic normality of equation (5.28) is obtained using theorem 2.2 in Peligrad and Utev (1997) . To compute the variance, we follow Yakowitz's work (1987): $var(m_n(\underline{x})) = \frac{1}{k(n)^2} var(\sum_{i=1}^{k(n)} Y_i) = \frac{1}{k(n)}[var(Y \mid \underline{X} = \underline{x}) + O(n^{-\frac{2(1-Q)p}{d}})]$, then equation (5.28) becomes,

$$\frac{m_n(\underline{x}) - Em_n(\underline{x})}{\sigma_n} = \sqrt{n^Q} \sum_{i=1}^{k(n)} \frac{w_i Y_i - Ew_i Y_i}{\sigma}, \tag{5.29}$$

when $k(n) = [n^Q]$ and $\sigma^2 = var(Y \mid \underline{X} = \underline{x})$, and the proof is complete.

ii) The weights w_i are real numbers and do not depend on $(X_n)_n$, then

$$\frac{m_n(\underline{x}) - Em_n(\underline{x})}{\sigma_n} = \sum_{i=1}^{k(n)} w_i Z_i, \tag{5.30}$$

where $Z_i = \frac{Y_i - EY_i}{\sigma_n}$. Now, applying again the theorem 2.2 in Peligrad and Utev (1997), we get the asymptotic normality remarking that $E[\sum_{i=1}^{k(n)} w_i Z_i] = 0$ and $Var[\sum_{i=1}^{k(n)} w_i Z_i] = 1$. To compute $\sigma_n^2 = Var[m_n(x)]$, we use the stationary condition of the time series $(X_n)_n$, thus:

$$Var[m_n(\underline{x})] = \sum_{i=1}^{k(n)} w_i^2 Var[Y_i] = \sum_{i=1}^{k(n)} w_i^2 [Var[Y_{n+1} | \underline{X}_n = \underline{x}] + B^2],$$

where B is given in lemma 3.1. Remarking that $\frac{1}{k(n)^2} \sum_{i=1}^{k(n)} (k(n) w_i)^2 < \infty$, then $\sum_{i=1}^{k(n)} w_i^2 < \infty$ and

$$Var[m_n(\underline{x})] = [Var[Y_i | \underline{X}_i = \underline{x}] + B^2] \sum_{i=1}^{k(n)} w_i^2.$$

As soon as $\sum_{i=1}^{k(n)} w_i^2 \sim \frac{\gamma^2}{k(n)}$, and $k(n) = [n^Q]$, we get the result.

iii) Finally, we assume that $w_i = \frac{w(\underline{x} - \underline{X}_{(i)})}{\sum_{i=1}^{K} w(\underline{x} - \underline{X}_{(i)})}$ where $w(.)$ is a given function. In that latter case, the weights depend on the process $(X_n)_n$. In the following, we denote by $N(i)$ the order of the i^{th}

neighbor. We rewrite the neighbor indices in an increasing order such that $M(1) = min\{N(i), 1 \leq i \leq K\}$ *and* $M(k) = min\{N(i) \notin \{M(j), \forall j < k\}, 1 \leq i \leq K\}$ *for* $2 \leq k \leq K$*, and* $K = k(n)$ *is the number of neighbors. We introduce a real triangular sequence* $\{\alpha_{Ki}, 1 \leq i \leq K$ *and* $\alpha_{Ki} \neq 0$ $\forall i\}$ *such that*

$$Sup_{K} \sum_{i=1}^{K} \alpha_{Ki}^2 < \infty \quad and \quad \max_{1 \leq i \leq K} |\alpha_{Ki}| \underset{n\to\infty}{\longrightarrow} 0. \tag{5.31}$$

Now using the sequences $M(j), j = 1, \cdots, K$ *and* $(\alpha_{Ki}), 1 \leq i \leq K$*, we can rewrite expression (5.27) as:*

$$\frac{m_n(\underline{x}) - Em_n(\underline{x})}{\sigma_n} = \sum_{i=1}^{K} \alpha_{Ki} S_i, \tag{5.32}$$

with $S_i = \frac{w_{M(i)} X_{M(i)+1} - Ew_{M(i)} X_{M(i)+1}}{\alpha_{Ki}\sigma_n}$*. The sequence* (S_i^2) *is uniformly integrable and* S_i *is function only of* $(X_j, j \leq M(i)+1)$*, thus if we denote* $\mathcal{F}_i$*,* $\mathcal{G}_i$*,* $\mathcal{F}_i^j$ *and* $\mathcal{G}_i^j$*, the sigma algebras generated by* $\{X_r\}_{r \leq i}$*,* $\{S_r\}_{r \leq i}$*,* $\{X_r\}_{r=i}^{j}$ *and* $\{S_r\}_{r=i}^{j}$ *respectively, then* $S_i \in \mathcal{F}_{M(i)+1}$*, and* $\mathcal{G}_i \subset \mathcal{F}_{M(i)+1}$*. For a given integer* ℓ*, we have also* $\mathcal{G}_{n+\ell}^{\infty} \subseteq \mathcal{F}_{n+M(\ell)+1}^{\infty}$ *since* $M(1) < M(1) + 1 \leq M(2) < \cdots \leq M(n+\ell) < M(n+\ell) + 1 \leq M(n+\ell+1)$*. Then:*

$$\sup_{\ell} \; Sup_{A \in \mathcal{G}_1^{\ell}, B \in \mathcal{G}_{n+\ell}^{\infty}, P(A) \neq 0} |P(B \mid A) - P(B)| \leq \sup_{\ell} \; Sup_{A \in \mathcal{F}_1^{M(\ell)+1}, B \in \mathcal{F}_{n+M(\ell)+1}^{\infty}, P(A) \neq 0} |P(B \mid A) - P(B)|. \tag{5.33}$$

Under the $\phi-$*mixing assumption on* $(X_n)_n$*, the right hand part of the expression (5.33) tends to zero as* $n \to \infty$ *and the lelf hand part of (5.33) converges to zero, hence the sequence* $(S_i)_i$ *is* $\phi-$*mixing. Moreover, for all* i*:*

$$S_i \quad is \quad centered \quad and \quad var(\sum_{i=1}^{K} \alpha_{Ki} S_i) = var(\frac{m_n(\underline{x})}{\sigma_n}) = 1. \tag{5.34}$$

Then, using expressions (5.31) - (5.34), and the theorem 2.2 in Peligrad and Utev (1997), we get:

$$\frac{m_n(\underline{x}) - Em_n(\underline{x})}{\sigma_n} \to_{\mathcal{D}} \mathcal{N}(0,1) \tag{5.35}$$

The variance of $m_n(x)$ *is given by the relation (5.25). The proof of the theorem 2.1 is complete.*

We provide now the proof of Corollary 2.1.

Proof 5.3 (Proof of corollary 2.1)**.** *From theorem 2.1, a confidence interval, for a given* α *can be computed, and has the expression:*

$$-z_{1-\frac{\alpha}{2}} \leq \frac{m_n(\underline{x}) - Em_n(\underline{x})}{\hat{\sigma}_n} \leq z_{1-\frac{\alpha}{2}} \tag{5.36}$$

where $z_{1-\frac{\alpha}{2}}$ is the $(1-\frac{\alpha}{2})$ quantile of Student law. Previoulsy, we have established that the estimate $m_n(\underline{x})$ can be biased, thus the relationship (5.36) becomes:

$$m_n(\underline{x}) + B - \hat{\sigma}_n z_{1-\frac{\alpha}{2}} \leq m(\underline{x}) \leq m_n(\underline{x}) + B + \hat{\sigma}_n z_{1-\frac{\alpha}{2}} \tag{5.37}$$

When the bias is negligible, the corollary is established. If the bias is not negligible, we can bound it. The bound is obtained using expressions (5.7) and (5.38):

$$B = O\Big(\Big(\frac{k(n)}{(n-d)\hat{h}(\underline{x})c}\Big)^{\frac{p}{d}}\Big) \tag{5.38}$$

with $c = \frac{\pi^{d/2}}{\Gamma((d+2)/2)}$, $\hat{h}(\underline{x})$ being an estimate of the density $h(\underline{x})$. Introducing this bound in expression (5.37) completes the proof.

6 APPENDIX

6.1 Euro Area Monthly Indicators

We provide in table 2 the list of the monthly economic indicators used in this study for the computation of the GDP using the bridge equations.

6.2 The bridge equation

We specify the bridge equations we use, details can be found in Diron (2008). Let us define Y_t as: $Y_t = (\log GDP_t - \log GDP_{t-1}) \times 100$, where GDP_t is the GDP at time t. The final GDP Y_t used in the paper is the mean of the eight values computed below.

1. EQ1. $Y_t^1 = a_0^1 + a_1^1(\log X_t^1 - \log X_{t-1}^1) + a_2^1(\log X_t^2 - \log X_{t-1}^2) + a_3^1 X_{t-1}^3 + \varepsilon_t$.

2. EQ2.

   $$Y_t^2 = a_0^2 + a_1^2(\log X_t^1 - \log X_{t-1}^1) + a_2^2(\log X_t^2 - \log X_{t-1}^2) + a_3^2(\log X_t^4 - \log X_{t-1}^4) + a_4^2(\log X_t^5 - \log X_{t-1}^5) + \varepsilon_t.$$

3. EQ3. $Y_t^3 = a_0^3 + a_1^3 X_t^7 + a_2^3 X_{t-1}^7 + \varepsilon_t$.

4. EQ4. $Y_t^4 = a_0^4 + a_1^4(X_t^6 - X_{t-1}^6) + a_2^4 X_t^3 + \varepsilon_t$.

5. EQ5. $Y_t^5 = a_0^5 + a_1^5(X_t^6 - X_{t-1}^6) + a_2^5 X_t^9 + a_3^5 X_t^8 + \varepsilon_t$.

6. EQ6. $Y_t^6 = a_0^6 + a_1^6(\log X_{t-2}^{10} - \log X_{t-3}^{10}) + a_2^6(\log X_{t-1}^{11} - \log X_{t-2}^{11}) + \varepsilon_t$.

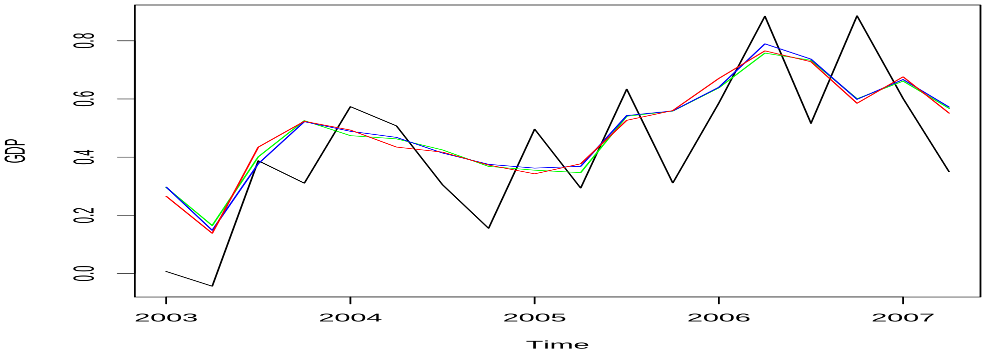
Short Notation	Notation	Indicator Names	Sources	Period
X^1	IPI	Industrial Production Index	Eurostat	1990-2007
X^2	CTRP	Industrial Production Index in Construction	Eurostat	1990-2007
X^3	SER-CONF	Confidence Indicator in Services	European Commission	1995-2007
X^4	RS	Retail sales	Eurostat	1990-2007
X^5	CARS	New passenger registrations	Eurostat	1990-2007
X^6	MAN-CONF	Confidence Indicator in Industry	European Commission	1990-2007
X^7	ESI	European economic sentiment index	European Commission	1990-2007
X^8	CONS-CONF	Consumers Confidence Indicator	European Commission	1990-2007
X^9	RT-CONF	Confidence Indicator in retail trade	European Commission	1990-2007
X^{10}	EER	Effective exchange rate	Banque de France	1990-2007
X^{11}	PIR	Deflated EuroStock Index	Eurostat	1990-2007
X^{12}	OECD-CLI	OECD Composite Leading Indicator, trend restored	OECD	1990-2007
X^{13}	ERC	EuroCoin indicator	Bank of Italy	1999-2007

Table 2: Summary of the thirteen economic indicators of Euro area used in the eight GDP bridge equations.

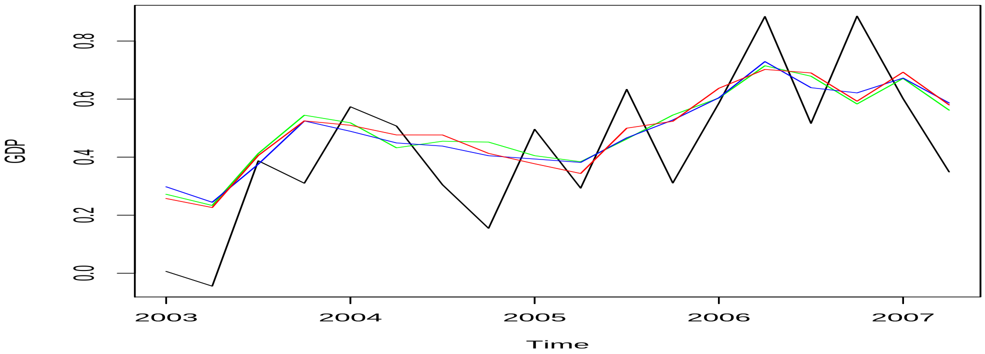
7. EQ7. $Y_t^7 = a_0^7 + a_1^7(\log X_t^{12} - \log X_{t-1}^{12}) + a_2^7(\log X_{t-2}^{12} - \log X_{t-3}^{12}) + a_3^7 Y_{t-1}^7 + \varepsilon_t$, and

8. EQ8.$Y_t^8 = a_0^8 + a_1^8 X_t^{13} + \varepsilon_t$.

(a) GDP growth rate forecast at horizon H=1 using VAR and k-NN methods.



(b) GDP growth rate forecast at horizon H=2 using VAR and k-NN methods.



(c) GDP growth rate forecast at horizon H=3 using VAR and k-NN methods.

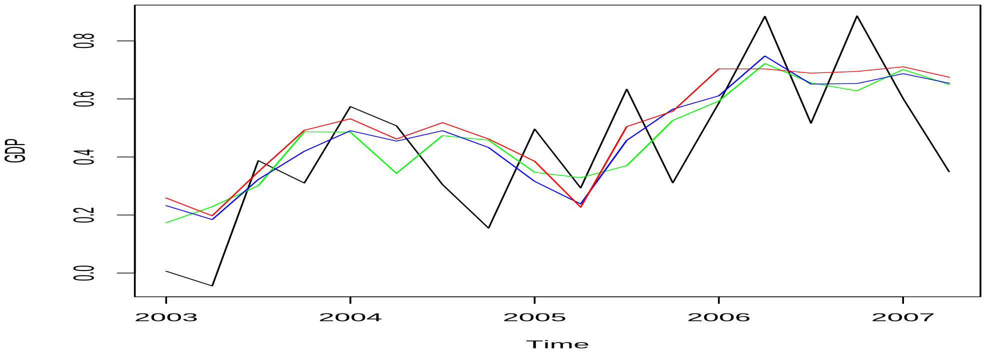
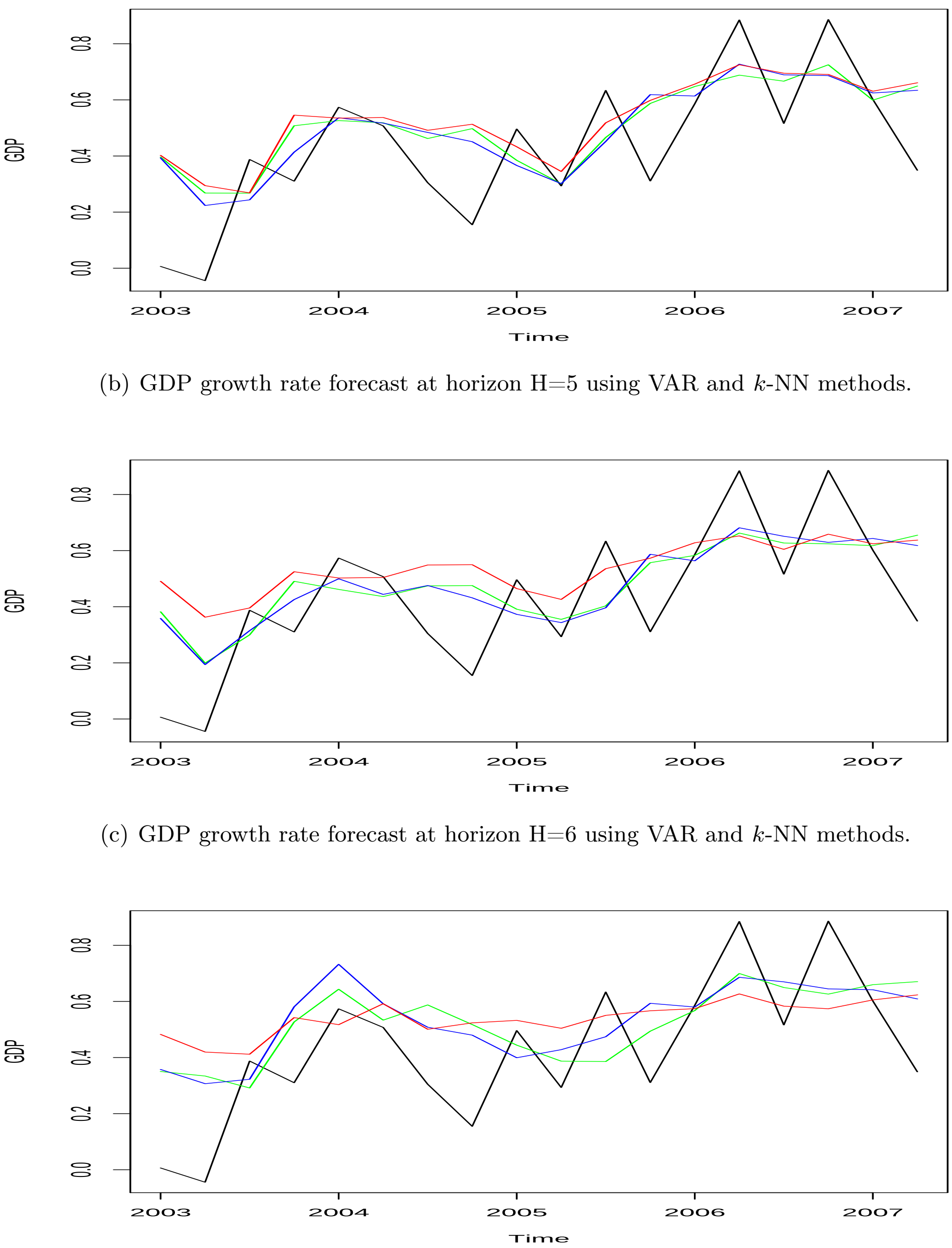


Figure 1: Quarterly observed (in black) and forecasted GDP growth rate computed from k-NN with d=1 (in green), k-NN with d>1 (in blue) and VAR (in red) models between 2003Q1 and 2007Q2 for different forecast horizons in panel: (a) for horizon $H = 1$, (b) for horizon $H = 2$ and (c) for horizon $H = 3$.



(a) GDP growth rate forecast at horizon H=4 using VAR and k-NN methods.

(b) GDP growth rate forecast at horizon H=5 using VAR and k-NN methods.

(c) GDP growth rate forecast at horizon H=6 using VAR and k-NN methods.

Figure 2: Quarterly observed (in black) and forecasted GDP growth rate computed from k-NN with d=1 (in green), k-NN with d>1 (in blue) and VAR (in red) models between 2003Q1 and 2007Q2 for different forecast horizons in panel: (a) for horizon $H = 4$, (b) for horizon $H = 5$ and (c) for horizon $H = 6$.