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Information and the Dispersion of Cross-Border Equity Holdings.

COVRIG Vicentiu M., FONTAINE Patrice,

JIMENEZ-GARCES Sonia, SEASHOLES Mark S





## Information and the Dispersion of Cross-Border Equity Holdings

Vicentiu M. Covrig Sonia Jimenez-Garcès Patrice Fontaine Mark S. Seasholes

This Version 18-Mar-2009\*

#### Abstract

This paper studies, both theoretically and empirically, dispersion in cross-border equity holdings. We present a multi-asset rational expectations equilibrium model in which agents have information about asset-specific components of payoffs and/or information about components that affect many stocks' payoffs. The model produces closed-form solutions for investors' holdings (positions) and stock prices. A numerical analysis can generate large dispersion in cross-border holdings. The last section of the paper analyzes cross-border mutual fund holdings of 5,781 stocks from 21 developed countries. We create a proxy variable for asset-specific information advantages and another proxy for common-component information advantages. Regression analysis and double sorts using our proxy variables produce ownership patterns (across stocks) that match those implied by the model.

JEL Classification: D82, G11, G12, G15

Keywords: Information Economics, REE Models

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This paper studies, both theoretically and empirically, dispersion in cross-border equity holdings. We present a multi-asset rational expectations equilibrium model in which agents have information about asset-specific components of payoffs and/or information about components that affect many stocks' payoffs. The model produces closed-form solutions for investors' holdings (positions) and stock prices. A numerical analysis can generate large dispersion in cross-border holdings. The last section of the paper analyzes cross-border mutual fund holdings of 5,781 stocks from 21 developed countries. We create a proxy variable for asset-specific information advantages and another proxy for common-component information advantages. Regression analysis and double sorts using our proxy variables produce ownership patterns (across stocks) that match those implied by the model.

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#### 1 Introduction

Why do institutional investors hold large fractions of some foreign equities while eschewing others? Consider a quick study of aggregate mutual fund positions in 467 German stocks (using data from Thomson Financial). From the perspective of the average German stock, foreign funds own 3.66% of the shares outstanding. For 25% of the stocks, the same funds hold less than 0.01% of the equity in aggregate. For the upper 25% of stocks, funds hold at least 4.35% of the equity. Foreign ownership dispersion of this magnitude is typical when looking at non-French mutual fund positions in French stocks, non-UK fund positions in UK stocks, and so on. Cross-border ownership of equities is low, but varies considerably across assets. The goal of this paper is to understand ownership dispersion and we ask: which stocks do mutual fund managers choose when they invest overseas? In particular, we are interested in the ability of information structures to explain the large dispersion in cross-border equity holdings.

We start by providing a one-period, two-date, rational expectations equilibrium model with multiple assets, multiple common factors, and multiple groups of investors. Economists have long been interested in how markets aggregate and transmit disperse pieces of information. We contribute to understanding the role of information in determining asset prices by considering three different components of each asset's payoff (dividend). The first component is asset-specific and investors may be differentially informed about this component. The second component is a function of common (global) factors and investors may also be differentially informed about these common components. The third component is called "residual uncertainty" and no investor has information about this part of an asset's payoff. We envisage a generalized information structure that allows an agent to have information about all, some, or none of the asset-specific components. The same agent may also have information about all, some, or none of the common components.

The first contribution of this paper is theoretical and involves the introduction of generalized information asymmetries about the common components of assets' payoffs. Our approach allows us to link a rich variety of information structures with investor portfolios and asset prices. Consider three groups of investors with asset-specific information about three, different, and non-overlapping sets of stocks. The first group has no information about common components. The second group has information about a common component that only affects a small subset of stocks. The third group has information about a second common component that affects all stocks. While stylized, such a situation re-

<sup>&</sup>lt;sup>1</sup>While our model starts with a standard rational expectations set-up, the addition of information about a generalized factor structure sets the paper apart from existing work. See Admati (1985) for a note about the difficulty in extending her model to one with information about common factors. We contrast our paper with recent work in Section 1.1.

flects how information is distributed globally. No one group of investors is better informed about all stocks nor do common components affect all stocks in the same manner.

The model produces closed-form solutions for investors' holdings. An investor's holdings are proportional to the ratio of two uncertainty matrices.<sup>2</sup> The first (inverse) matrix represents the investor's uncertainty. The second matrix represents the market's uncertainty about stocks' future payoffs. In general, the higher the investor's uncertainty, the lower his holdings. Correlations complicate the analysis. An investor with little information about a given asset may still have high demand for the asset if he has precise information about a given asset may have low demand if others have more precise information about that asset or information about other, highly correlated assets. Adding a generalized information structure about common components leads equilibrium holdings to become particularly complex. Investors with extremely precise information about the asset-specific component of dividends may still avoid a given stock if a common component plays big part in determining the stock's payoff (i.e., if the stock has a high factor loading) and if another group of investors has precise information about the common component.<sup>3</sup>

The second contribution of this paper is to numerically analyze equilibrium holdings from the perspective of a given stock. In this section, we show that the existence of common (cross-border) components and different factor loadings lead to a large dispersion in cross-border holdings. To make the analysis concrete, we focus on an example with three-countries (France, USA, and Japan), three-assets (one from each country), and two common components. We assume agents receive superior information about the asset-specific component of their home country's asset. We next assume that institutional investors in the USA and Japan each have superior information about one of the common components. American and Japanese investors are typically located in major financial center such as New York City and Tokyo. Analysts working for large investment banking houses or mutual funds synthesize and produce information about economic variables such as short-term interest rates, commodity prices, and global shipping costs. Information about these economic variables plays an important role when estimating the future payoffs of many different assets. Fund managers with access to this research use the information when choosing their portfolios.

<sup>&</sup>lt;sup>2</sup>Matrix division is not formally defined. We use the term "ratio of matrices" to succinctly indicate the product of an inverse matrix with another (non-inverse) matrix.

<sup>&</sup>lt;sup>3</sup>Our model also produces closed-form solutions for equilibrium prices. When all agents have full information about everything except the residual uncertainty, we obtain a form of the capital asset pricing model (CAPM), expressed in terms of prices rather than returns, and adjusted for the supply uncertainty present in our model. When not all agents have full information, an asset's price today equals its expected future payoff minus the CAPM (full information) discount and minus an additional discount related to information uncertainty. Hence, we call this second discount the "information price discount" and we show it is related to both uncertainty about the asset-specific component of payoffs and uncertainty about common components.

We focus on cross-border holdings of the French stock's shares in our comparative static analysis. When the French stock does not load on either of the common components, and there is no asset-specific information advantages, cross-border investors hold 6.67% of the French shares. As the French investors' asset-specific information advantage (about the French asset) goes up, cross-border holdings are reduced to 2.59% of shares outstanding. For a given level of asset-specific information advantage, cross-border holdings go up as the French asset increases its loading on the common components. This increase stems from the fact that American investors have information about one component and Japanese investors have information about the other component.

The third contribution of our paper is to empirically analyze cross-border, mutual fund holding data. Again, we focus on holdings from the perspective of a listed stock. The data come from the Thomson Financial International Mutual Fund Holdings dataset and consist of US\$788 billion of cross-border holdings (positions) in 5,781 stocks from 21 developed countries. The average cross-border holding of an individual stock is 2.76% of shares outstanding. The interquartile range is 0.14% to 3.22%. Our goal is to explain this dispersion.

We create two proxy variables to test broad implications of our model. The first proxy represents the asset-specific information advantage that domestic investors have. The second proxy represents the common-component information advantage that global (cross-border) investors have. We then show that cross-border holdings increase as asset-specific information advantages decrease and as information advantages about common components increase. Our results continue to hold after controlling for variables that have, in the past, been used as proxies for familiarity (the size of the company and the number of equity analysts following the company). Our results also hold after controlling for a firm's leverage—another variable that has been found to explain cross-border holdings. A number of robustness checks give consistent results.

We sort stocks into quartiles based on both our proxy variables. When the proxy for common-component information advantage is high (top 25%), stocks with a low value of the asset-specific proxy have average cross-border holdings of 4.90% while stocks with a high value of the asset-specific proxy have average cross-border holdings of 2.23%. The dispersion in holdings is 2.67% (equal to 4.90% - 2.23%). Likewise, when the asset-specific component is low (bottom 25%), stocks with a low value of the common component proxy have average cross-border holdings of 2.93%. The dispersion of cross-border holdings is 1.97% (equal to 4.90% - 2.93%). The sort results and additional empirical tests are motivated by implications of our model. Our results highlight the fact that both asset-specific information advantages and common-component information advantages play an economically and statistically important role in understanding cross-border holdings.

#### 1.1 Literature Review

Our paper is related to theoretical work on information structures, investor holdings, and risk premia. Easley and O'Hara (2004) present a multi-asset model that focuses on the role of public and private signals in determining a firm's cost of capital. Private signals in their model are received only by a group of informed investors as in Grossman and Stiglitz (1980). In our model, it is possible for different groups of investors to have information about different groups of the securities. In this way, investors can be asymmetrically informed without introducing a strict information hierarchy.<sup>4</sup> Bacchetta and van Wincoop (2006) argue in favor of structures with a "[broad] dispersion of information."

In a paper similar in spirit to ours, Hughes, Liu, and Liu (2007) model two groups of investors. Each informed investor effectively observes a global signal "s" and these signals are perfectly correlated across investors. Unlike our paper, investors in the Hughes, Liu, and Liu (2007) paper cannot separate the asset-specific and global components. Additionally, information about the components is not differentially dispersed across investors. Similarly, Kodres and Pritsker (2002) offer a model that contains an underlying factor structure. However, there are no information asymmetries regarding the factors.

Our paper has both important differences from, and certain similarities to, a recent paper by Albuquerque, Bauer, and Schneider (2006). Their paper contains multiple stocks and multiple time periods. The payoff of a given stock is equal to the sum of three terms: a constant, a local component, and a single global factor. There are public and private signals about both the local components and the global factor. Only one group of investors (from the USA) receives private information about the global factor. Our model contains multiple, global factors, each of which may be known by a different group of investors. Moreover, their model uses a single factor loading (equal to one) for all assets. Our model is more general and allows for different common factor loadings—both across assets and across factors. Most importantly, American investors in their model have the same informational advantage vis-a-vis each foreign stock. This assumption implies that their model does not generate cross-border holding dispersion based on common-factor information. Our model is more general and produces large differences in cross-border holdings. In our model, this dispersion is tied directly to informational differences about common factors.

Empirically, our paper is best viewed in terms of a line of research that studies cross-border ownership patters from the perspective of a listed company. Kang and Stulz (1997) look

<sup>&</sup>lt;sup>4</sup>Our model incorporates an aspect of models which endow all agents with small pieces of information about risky assets payoffs—see Grossman (1976), Hellwig (1980), and Admati (1985). Coval (1997) uses diffuse information in a manner similar to our paper. Van Nieuwerburgh and Veldkamp (2006a) study information acquisition and dynamic learning.

at foreign holdings of Japanese stocks. Dahlquist and Robertsson (2001) study relations between Swedish firm characteristics and foreign ownership. Covrig, Lau, and Ng (2006) conduct a cross-country analysis of fund manager preferences for stock characteristics. Finally, Ferreira and Matos (2007) document preferences of institutional investors.

Studying ownership patters from the perspective of a listed company is motivated by the well-known and extensive "home bias" literature. Information models relating asymmetries to home bias are well studied. As such, our paper speaks to this literature as well. Gehrig (1993) presents a related two-country model. Brennan and Cao (1997) study investment flows (changes in holdings) and information asymmetries. In their model, investors with less information (foreigners) update priors about future payoffs more heavily than investors with more information (locals). Van Nieuwerburgh and Veldkamp (2006a) use a rational expectations equilibrium model to justify a persistent preference for home country assets when investors initially have a small information advantage.

The paper proceeds as follows. Section 2 presents our model, notation, and assumptions. We focus on closed-form solutions for equilibrium holdings. Section 3 numerically analyzes holdings as a function of the model's parameters. Section 4 empirically studies cross-border mutual fund holdings. The final section concludes.

#### 2 Model

The model has I investors indexed  $i=1,\ldots,I$  who trade at date 0 and consume at date 1. Each agent i can invest his initial wealth,  $w_i^0$ , in a riskless asset and J risky assets indexed  $j=1,\ldots,J$ . The riskless interest rate is denoted  $r_f$  and we define  $R\equiv (1+r_f)$ . For simplicity, we normalize the price of the riskless asset to one. Each risky asset j pays a liquidating dividend  $\tilde{P}_j^1$  at date 1. The vector of final payoffs  $\tilde{P}^1=(\tilde{P}_1^1,\ldots,\tilde{P}_J^1)'$  is generated by a K-factor linear process:

$$\tilde{P}^1 = \tilde{\theta} + \mathbf{B}\tilde{f} + \tilde{\varepsilon} \tag{1}$$

The vector  $\tilde{\theta} = (\tilde{\theta}_1, \dots, \tilde{\theta}_J)'$  is the asset-specific component of payoffs, the vector  $\tilde{f} = (\tilde{f}_1, \dots, \tilde{f}_K)'$  contains the K common factors, and  $\mathbf{B}$  is a  $J \times K$  matrix of factor loadings. The remaining part of each asset's final payoff,  $\tilde{\epsilon} = (\tilde{\epsilon}_1, \dots, \tilde{\epsilon}_J)'$ , is unknown to all investors and referred to as residual uncertainty. We assume that  $\tilde{\theta}$ ,  $\mathbf{B}\tilde{f}$ , and  $\tilde{\epsilon}$  are jointly multivariate normal and independent. We further assume that  $\tilde{f}$  and  $\tilde{\epsilon}$  have mean zero. Since  $\tilde{\theta}$  is the asset-specific component, we assume its covariance matrix is diagonal and

denoted  $\Sigma_{\theta}$ .<sup>5</sup> For tractability, we assume that the covariance matrix of  $\tilde{f}$  is the identity matrix. The covariance matrix of  $\mathbf{B}\tilde{f}$  is  $\mathbf{B}\mathbf{B}'$ . Finally, the covariance matrix of  $\tilde{\varepsilon}$  is denoted  $\Sigma_{\varepsilon}$ .

The per-capita supply of risky assets is defined as the realization of a random vector  $\tilde{z}$ . The vector  $\tilde{z}$  is independent and jointly normally distributed along with the other variables in the model and has a covariance matrix denoted  $\Sigma_z$ . The assumption of random net supply is standard in rational expectations models. As Easley and O'Hara (2004) write "one theoretical interpretation is that this approximates noise trading in the market. A more practical example of this concept is portfolio managers current switch toward using float-based indices from shares-outstanding indices." In order to insure the existence and uniqueness of the date 0 equilibrium price vector,  $\tilde{P}^0$ , we assume that  $\Sigma_\varepsilon$ ,  $\Sigma_\theta$ , and  $\Sigma_z$  are regular matrices.

We assume all agents have an exponential utility function:  $U(\tilde{w}_i^1) = -e^{-a\tilde{w}_i^1}$ , where  $\tilde{w}_i^1$  is the wealth of investor i on date 1. The utility function has a constant absolute risk aversion with coefficient a > 0 which is the same for all agents. The choice of utility functions is also common in rational expectations equilibrium models and ensures that an investor's demand for the risky asset is independent of his initial wealth. Let  $X_i$  be investor i's vector of holdings of the risky assets. Investor i's final wealth is:

$$\tilde{w}_i^1 = w_i^0 R + X_i' (\tilde{P}^1 - R\tilde{P}^0) \tag{2}$$

#### 2.1 Information Structure and Notation

To facilitate linking our model to international holdings data, we partition investors, assets, and common factors into groups. A group of investors can be thought of as a nationality (French investors, Japanese investors, etc.) The I investors in our model are partitioned into N non-overlapping groups labeled n = 1, ..., N. Each group of investors represents a fraction,  $\lambda_n$ , of the total number of investors (I) in the market such that  $\sum_{n=1}^{N} \lambda_n = 1$ .

Asset-Specific Information: The J securities are partitioned into N non-overlapping groups. A group of securities can be thought of as comprising a country's equities (French stocks, Japanese stocks, etc.) We define the set of all assets as S. The set of assets in group n contains  $J_n$  risky assets and is denoted  $S_n$ . Thus,  $\bigcup_{n=1}^N S_n = S$  and  $\forall (n_a, n_b)$ ,  $n_a \neq n_b$ ,  $S_{n_a} \cap S_{n_b} = \emptyset$ . We assume there is an equal number (N) of securities groups and investors groups to ensure that each security has at least one investor with specific

<sup>&</sup>lt;sup>5</sup>This assumption is not necessary to solve the model. However, it enables us to distinguish information that affects a single asset from common factors that affect two or more assets.

information about that security. A single investor i in group n knows the realization of the asset-specific component,  $\theta_j$ , of each asset j in the set  $S_n$ . For any asset j not in  $S_n$ , investor i only knows the distribution of  $\tilde{\theta}_j$  but he does not know its realization.

Common Component Information: We assign the K common factors into N groups denoted  $F_n$ , with n = 1, ..., N. The set  $F_n$  contains  $K_n$  common factors. A single investor i in group n knows the realization of each common factor  $\tilde{f}_k$  in the set  $F_n$ . For any factor not in  $F_n$ , the investor only knows the distribution of  $\tilde{f}_k$  but not its realization. For tractability purposes of the model, we assume that two groups of investors do not have information about the same common factor.

Chen et al. (1986) document nine macroeconomic risk factors affecting stock returns. We therefore envisage the number of common factors to be much less than the number of assets,  $K \ll J$ . If the number of common factors is less than the number of investor groups, then K < N, some  $F_n$  sets will not contain any common factors ( $K_n = 0$ ), and the corresponding investor group will not be informed about any of the common factors.

**Notation**: The information structure of our model implies that investors belonging to the same group n possess the same private information (for asset-specific components and for common factors), they face the same optimization problem, and they optimally choose identical portfolios. In this sense they can be said to be identical. We use the following terms interchangeably (and a bit loosely): "investor i from group n", "investor group n", and "investor n". Furthermore, to simplify the notation, we write the payoffs of the risky assets as:

$$\tilde{P}^1 = \mathbf{C}\tilde{\eta} + \tilde{\varepsilon} \tag{3}$$

Where,  $\tilde{\eta} = (\tilde{\theta}' \quad \tilde{f}')'$  is a J + K column vector and  $\mathbf{C}$  is a  $J \times (J + K)$  block-diagonal matrix consisting of a  $J \times J$  identity matrix,  $\mathbf{I}_J$ , and the matrix  $\mathbf{B}$ . The variance-covariance matrix of  $\tilde{\eta}$  is  $\mathbf{Q} = \begin{pmatrix} \Sigma_{\theta} & 0 \\ 0 & \mathbf{I}_K \end{pmatrix}$  where  $\mathbf{I}_K$  is the identity matrix of order K.

**Definition 2.1.** For each investor n, we define the diagonal matrix  $\mathbf{D}_n$  of order J+K with  $\mathbf{D}_n(j,j)=1$  if investor n knows the realization of the  $j^{th}$  random variable in  $\tilde{\eta}$  and  $\mathbf{D}_n(j,j)=0$  otherwise. The  $j^{th}$  random variable represents an asset-specific component of stock j's payoffs if  $j \leq J$ , and a common factor otherwise.

**Definition 2.2.** We define  $\mathbf{D} \equiv \sum_{n=1}^{N} \lambda_n \mathbf{D}_n$ . The matrix  $\mathbf{D}$  plays an important role in our model as each element on the main diagonal represents the proportion of investors who know the realization of the corresponding random variable in the vector  $\tilde{\eta}$ .

**Definition 2.3.** For each investor group n, the matrix  $\mathbf{M}_n$  is obtained by eliminating all the null rows of  $\mathbf{D}_n$ . Consequently, the number of rows of  $\mathbf{M}_n$  is equal to  $J_n + K_n$ , which represents the number of asset-specific and common factors about which investor n is informed. If investor n does not receive any private information,  $\mathbf{D}_n$  becomes the null matrix and  $\mathbf{M}_n$  cannot be defined. It is straightforward that  $\mathbf{M}'_n\mathbf{M}_n = \mathbf{D}_n$  and  $\mathbf{M}_n\mathbf{M}'_n = \mathbf{I}_{J_n+K_n}$ , where  $\mathbf{I}_{J_n+K_n}$  is the identity matrix of order  $J_n + K_n$ .

Under these definitions, the private information received by investor n consists of the realization of the random vector  $\mathbf{M}_n\tilde{\eta}$ . As in Admati (1985), equilibrium prices also reveal some information to investors beyond their own private information. Consequently, each investor n maximizes his expected utility of consumption conditional on the realization of his private information and on the observation of the public information in the form of prices at date 0.

#### 2.2 Equilibrium Solution

We seek a closed-form solution for holdings and prices at date 0 within the class functions that are linear in our information variable  $\tilde{\eta}$  and supply variable  $\tilde{z}$ . The form of the solution implies investors assume prices are a linear function of private signals and noise. In equilibrium, this hypothesis is verified. The date 0 price vector is:

$$\tilde{P}^0 = A_0 + \mathbf{A}_1 \tilde{\eta} - \mathbf{A}_2 \tilde{z} \tag{4}$$

where the dimensions of the vector  $A_0$  is  $J \times 1$ , the matrix  $\mathbf{A}_1$  is  $J \times (J + K)$ , and the matrix  $\mathbf{A}_2$  is  $J \times J$ . We suppose that  $\mathbf{A}_2$  is regular. Under these assumptions, investor n's demand is:

$$\tilde{X}_n = a^{-1} \mathbf{V}_n^{-1} \left( E_n \left[ \tilde{P}^1 \right] - R \tilde{P}^0 \right) \tag{5}$$

Equation (5) gives an expression for agent n's holdings at date 0—please see Appendix A for additional details. The expression  $E_n[\tilde{P}^1] = E[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0]$  gives the expected prices of the risky assets at date 1 from investor n's point of view (i.e. conditional on his information set).  $\mathbf{V}_n = Var[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0]$  represents the conditional return variance of  $\tilde{P}^1$  from investors n's point of view. By equating the supply and the aggregate demand of the N groups of investors,  $\left(\sum_{n=1}^N \lambda_n \tilde{X}_n = \tilde{z}\right)$ , it follows:

$$\sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} \left( E_n \left[ \tilde{P}^1 \right] - R \tilde{P}^0 \right) - a \tilde{z} = 0$$
 (6)

Joint normality implies that the distribution of prices, conditional on investor n's private and public information, is also multi-variate normal with the following expectation:

$$E_{n}\left[\tilde{P}^{1}\right] = E\left[\tilde{P}^{1}|\mathbf{M}_{n}\tilde{\eta},\tilde{P}^{0}\right]$$

$$= B_{0n} + \mathbf{B}_{1n}\mathbf{M}_{n}\tilde{\eta} + \mathbf{B}_{2n}\tilde{P}^{0}$$
(7)

where the dimension of  $B_{0n}$  is  $J \times 1$ ,  $\mathbf{B}_{1n}$  is  $J \times (J_n + K_n)$ , and  $\mathbf{B}_{2n}$  is  $J \times J$ . Equations (4), (6), and (7) imply the system to be solved is (please see Appendix B):

$$a\mathbf{A}_{2}^{-1}A_{0} = \sum_{n=1}^{N} \lambda_{n} \mathbf{V}_{n}^{-1} B_{0n}$$

$$a\mathbf{A}_{2}^{-1}\mathbf{A}_{1} = \sum_{n=1}^{N} \lambda_{n} \mathbf{V}_{n}^{-1} \mathbf{B}_{1n} \mathbf{M}_{n}$$

$$a\mathbf{A}_{2}^{-1} = \sum_{n=1}^{N} \lambda_{n} \mathbf{V}_{n}^{-1} (R\mathbf{I}_{J} - \mathbf{B}_{2n})$$
(8)

As shown in Appendix C, the matrices  $\mathbf{B}_{1n}$ ,  $\mathbf{B}_{2n}$  and  $\mathbf{V}_n$  can be written as functions of the matrices  $\mathbf{A}_1$  and  $\mathbf{A}_2$ . The system of equations in (8) represents a fixed point problem in a  $2J^2 + JK + J$  Euclidian space.

Closed-Form Solution: To obtain a closed-form solution for  $\tilde{P}^0$ , we define the matrix  $\mathbf{U} \equiv \mathbf{A}_2^{-1} \mathbf{A}_1$ . We also introduce the function  $g(\mathbf{G}) = \sum_{n=1}^{N} \mathbf{D}_n \mathbf{G} \mathbf{D}_n$ , where  $\mathbf{G}$  is a matrix of order J + K. The function  $g(\cdot)$  transforms a matrix  $\mathbf{G}$  into a N-block diagonal matrix whose block elements are the same as the elements of the matrix  $\mathbf{G}$ .

**Definition 2.4.** We define a "g-matrix" to be any square matrix G of order J + K which satisfies g(G) = G. This means that G is an N-block diagonal matrix, the size of block n is equal to the number of specific and common factors known by investor n.

Define  $\Psi \equiv Var\left[\tilde{\eta}|\tilde{P}^0\right]$  i.e., the variance-covariance matrix of  $\tilde{\eta}$  conditional on observing the equilibrium price vector at date 0. The matrix  $\Psi$  is endogenously defined and represents the variance of  $\tilde{\eta}$  from the point of view of an investor who does not possess any private information but only observes the equilibrium price vector. The following lemma gives an analytical solution for U.

**Lemma 2.1.** If  $(\Psi^{-1} + \mathbf{C}'\Sigma_{\epsilon}^{-1}\mathbf{C})$  is a *g*-matrix, then the closed-form solution for  $\mathbf{U}$  is:

$$\mathbf{U} = a^{-1} \mathbf{\Sigma}_{\varepsilon}^{-1} \mathbf{CD} \tag{9}$$

**Proof:** See Appendix D.

For the particular case of Lemma (2.1),  $\mathbf{U}$  is not a function of the coefficients  $B_{0n}$ ,  $\mathbf{B}_{1n}$ , and  $\mathbf{B}_{2n}$ . Therefore, to determine  $A_0$ ,  $\mathbf{A}_1$ , and  $\mathbf{A}_2$ , we must first compute the matrix  $\mathbf{\Psi}$  as a function of  $\mathbf{U}$ . In this way, the variance-covariance matrix of any investor group,  $\mathbf{V}_n$ , can be written as a function of  $\mathbf{\Psi}$ :

$$\mathbf{V}_n = \mathbf{\Sigma}_{\varepsilon} + \mathbf{C}\mathbf{\Psi}\mathbf{C}' - \mathbf{C}\mathbf{\Psi}\mathbf{M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{\Psi}\mathbf{C}' \tag{10}$$

Where  $\Psi_n = \mathbf{M}_n \Psi \mathbf{M}'_n$ . Also,  $\Psi = \mathbf{Q} - \mathbf{Q} \mathbf{U}' \mathbf{M}^{-1} \mathbf{U} \mathbf{Q}$  and  $\mathbf{M} = \mathbf{U} \mathbf{Q} \mathbf{U}' + \Sigma_z$ . The following theorem gives a closed-form solution for the equilibrium price vector at date 0.

**Theorem 2.1.** Under the conditions of Lemma (2.1), there exists a closed-form solution for Equation (6) within the class of linear functions of  $\tilde{\eta}$  and  $\tilde{z}$ . The solution can be written as,  $\tilde{P}^0 = A_0 + \mathbf{A}_1 \tilde{\eta} - \mathbf{A}_2 \tilde{z}$ , where  $\mathbf{A}_2$  is a regular matrix and:

$$A_0 = \frac{1}{R} \Big( (\mathbf{C} - R\mathbf{A}_1) E[\tilde{\eta}] + (R\mathbf{A}_2 - a\mathbf{V}_N) E[\tilde{z}] \Big)$$
 (11)

$$\mathbf{A}_{1} = \frac{1}{R} \left( \mathbf{CQC'} + \mathbf{\Sigma}_{\varepsilon} - \mathbf{V}_{N} \right) \left( \mathbf{CDQC'} \right)^{-1} \mathbf{CD}$$
 (12)

$$\mathbf{A}_{2} = \frac{1}{R} a \left( \mathbf{CQC'} + \mathbf{\Sigma}_{\varepsilon} - \mathbf{V}_{N} \right) \left( \mathbf{CDQC'} \right)^{-1} \mathbf{\Sigma}_{\varepsilon}$$
 (13)

**Proof:** See Appendix E.

The matrix  $\mathbf{V}_N = (\sum_{n=1}^N \lambda_n \mathbf{V}_n^{-1})^{-1}$  represents the variance-covariance matrix of  $\tilde{P}^1$  for the "average" investor in the market. The precision matrix  $\mathbf{V}_N^{-1}$  equals the weighted mean of each group's precisions where the weights are proportional to the number of agents in each group. From Equation (10), it is straightforward to show that  $\mathbf{V}_N$  can be written as:

$$\mathbf{V}_{N} = (\mathbf{\Sigma}_{\varepsilon} + \mathbf{C}\mathbf{\Psi}\mathbf{C}')(\mathbf{I}_{J} + \mathbf{\Sigma}_{\varepsilon}^{-1}\mathbf{C}\mathbf{D}\mathbf{\Psi}\mathbf{C}')^{-1}$$
(14)

Thus we have provided a closed-form solution for prices at date 0. The solution takes the form shown in (4) with constant values shown in (11), (12), and (13). Holdings of investors in group n are given in Equation (5).

#### 2.3 Investor Holdings (Portfolios)

We analyze the relationship between model parameters  $\{r_f, a, \lambda_1, \dots, \lambda_N, \mathbf{B}, \Sigma_{\varepsilon}, \Sigma_{\theta}, \Sigma_z\}$  and ex-ante equilibrium holdings.<sup>6</sup> We take expectations of Equations (5) and expecta-

<sup>&</sup>lt;sup>6</sup>The approach involves taking expectations over the random variables in the model  $\{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\}$ . An alternative methodology involves drawing a set of random variables  $\{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\}$  and calculating holdings at date 0. Repeating

tions of the market clearing condition in Equation (6). Rearranging terms, we get the following expression for investor n's ex-ante holdings:

$$E[X_n] = a^{-1}\mathbf{V}_n^{-1} \left( E\left[\tilde{P}^1\right] - RE\left[\tilde{P}^0\right] \right)$$

$$= \mathbf{V}_n^{-1}\mathbf{V}_N E\left[\tilde{z}\right]$$
(15)

In a single-stock world with no common factors, investor n's holdings depends on the ratio of the market's uncertainty about the future payoff  $(\mathbf{V}_N)$  to his own uncertainty about the same payoff  $(\mathbf{V}_n)$ . The higher the investor's uncertainty relative to the market, the lower the ratio, and the lower the weight of the asset in his portfolio.

In a multi-asset framework with uncorrelated residual uncertainty and no common factors, the matrices  $(\mathbf{V}_N)$  and  $(\mathbf{V}_n)$  are diagonal. The term  $\mathbf{V}_n^{-1}\mathbf{V}_N$  represents a series of uncertainty ratios. The same intuition described in the paragraph above holds.

In a multi-asset model with correlated residual uncertainties and/or a factor structure of payoffs, thinking about  $\mathbf{V}_n^{-1}\mathbf{V}_N$  as a ratio of two uncertainty measures provides rough intuition only. However, the ratio of two matrices includes covariance terms relating to uncertainty about assets' payoffs. Investor n's holdings of a specific asset now depends on his uncertainty about the asset's payoffs, his uncertainty about other assets' payoffs, and other investors' uncertainty about all assets (including the asset in question). These uncertainties can arise from information asymmetries about the asset-specific components of payoffs and/or common components. Section 3 numerically analyzes holdings in an effort to better understand the role of the covariance terms.

#### 3 Numerical Analysis of Holdings

This section provides a comparative static analysis of equilibrium stock holdings from the perspective of a listed stock. We focus on international portfolio choice and a measure of cross-border holdings. Our goal is to understand the net effect of covariance terms in Equations (5) and (15).<sup>7</sup>

The numerical analysis considers a setting with three assets (a French stock, an American stock, and a Japanese stock), two common factors, and three groups of investors in equal numbers (French, American, and Japanese people). In terms of the model parameters

this process converges to the same expected values as the number of draws goes to infinity. Our methodology solves for holdings before agents receive private information. As such, solutions are sometimes referred to as ex-ante. Appendix F provides closed-form solutions for ex-ante prices at date 0.

<sup>&</sup>lt;sup>7</sup>Covariance terms also play a role in the equilibrium prices. An explicit expression for prices, as well as economic interpretations, are given in Appendix F.

from the previous section, J=3; K=2; and N=3. Each investor group has asset-specific information about their home country's stock. All investors have a risk aversion coefficient of a=1.00. Payoffs of the French (fr) stock follow from Equation (1):

$$\tilde{P}_{fr}^{1} = \tilde{\theta}_{fr} + \beta_{1,fr}\tilde{f}_{1} + \beta_{2,fr}\tilde{f}_{2} + \tilde{\varepsilon}_{fr}$$

Similar expressions hold for the American asset (subscript "am") and Japanese asset ("jp"). The expected asset-specific component of all payoffs is  $E[\tilde{\theta}_{fr}] = E[\tilde{\theta}_{am}] = E[\tilde{\theta}_{jp}] = 1.00$ . We assume that the variance-covariance matrices  $\Sigma_z$  and  $\Sigma_\varepsilon$  are both equal to the identity matrix. The variance-covariance matrix  $\Sigma_\theta$  is proportional to the identity matrix. We vary the degree of asst-specific information by varying the diagonal elements from 0 to 10 in the  $\Sigma_\theta$  matrix.<sup>8</sup>

We start by endowing each group of investors asset-specific information about their home country's asset. Such an assumption is standard in the home bias literature. For this numerical analysis, all three groups of investors simultaneously have the same information advantage about their respective asset-specific component. We next endow investors in the large financial centers with common-component information. The realization of the first common component (factor) is known only by the Americans. The realization of the second common component is known only by the Japanese. In the calibration, the expected factor realization is  $E[\tilde{f}_1] = E[\tilde{f}_2] = 0$  and the variance is  $Var[\tilde{f}_1] = Var[\tilde{f}_2] = 1$ . The degree of information advantage about the common component is proportional to the variance of  $\beta_1 \tilde{f}_1$  and  $\beta_2 \tilde{f}_2$ . We simultaneously vary both loadings  $(\beta_{1,fr}, \beta_{2,fr})$  of the French asset from 0 to 4.

In order to highlight the different effects of that asset-specific information and common-component information have on cross-border holdings, we set the factor loadings for the American and Japanese assets  $(\beta_{1,am}, \beta_{2,am}, \beta_{1,jp}, \beta_{2,jp})$  to be half those of the French asset. Our choices highlight dispersion, but are not critical for the results. Similar analysis, with different parameter choices (i.e., different factor loadings) are available from the authors upon request. To scale the results, we assume the generated "data" represent 10% of world-wide holdings.

The fraction of the French company owned by cross-border investors equals the number of the French stock's shares in the American investor's portfolio  $(Shrs_{fr}^{am})$  plus the number of

<sup>&</sup>lt;sup>8</sup>Fundamentally, the information asymmetry/advantage about an asset should be measured by the corresponding element of the matrix  $\Psi$ . However, as seen in the model section,  $\Psi$  is an endogenous matrix. All else being equal, an increase in the matrix  $\Sigma_{\theta}$  corresponds to an increase in the matrix  $\Psi$ , and vice-versa. This is due to the fact that an increase in the variance of the asset-specific component corresponds to an increase in the asymmetric information surrounding this asset. We note that in this section the matrix U is obtained numerically and not by using the expression in Equation (9).

French shares in the Japanese investor's portfolio  $(Shrs_{fr}^{jp})$  all divided by the total number of French shares outstanding  $(Shrs_{fr}^{total})$ .

$$\Omega_{fr} \equiv \frac{Shrs_{fr}^{am} + Shrs_{fr}^{jp}}{Shrs_{fr}^{total}}$$
(16)

Figure 1 plots  $\Omega_{fr}$  for different levels of asset-specific information advantages and different levels of information advantage about the common components. The bottom (red) line shows cross-border holdings when the French asset does not load on either of the common factors. When asset-specific information advantages are zero, cross-border holdings are 6.67% of the stock's shares. As asset-specific information advantages increase (along the x-axis), cross-border holdings fall eventually reaching 2.59% on the lower right-hand corner of the graph. The dispersion (6.67% - 2.59%) is 4.08% and due to asset-specific information advantages.

#### [Insert Figure 1 About Here]

As the French stock loads more and more heavily on the common components, cross-border holdings increase. Consider the top line (marked with circles) which represents  $\beta_{1,f}=4$  and  $\beta_{2,f}=4$ . Cross-border holdings range from 8.39% on the upper-left to 5.24% on the lower-right. Cross-border holdings are high because the French stock loads (heavily) on  $\{f_1, f_2\}$  and foreign investors have information about these components.

Figure 1 also shows large dispersion in cross-border holdings even when the asset-specific information advantages are constant across assets. Consider the value of four (4) on the x-axis. Moving from bottom to top, cross-border holdings increase from 3.47%, to 4.69%, to 5.69%, to 6.20%. Again, the increases in cross-border holdings come from increased factor loadings on the common factors.

We conclude by noting that our numerical analysis generates significant dispersion in cross-border holdings. As discussed in Section 1.1 most other papers do not allow investors to have information about common factors. The one paper that does all this feature—Alburquerque, Bauer, and Schneider (2006)—endows the American investors with the same informational advantage vis-a-vis each foreign stock. In our model, cross-border ownership dispersion stems from differences in asset-specific information advantages, differential factors loadings, or both.

#### 4 Data and Empirical Analysis

We empirically analyze a cross-section of international mutual fund holdings. The unit of analysis is ownership of a publicly listed company. We measure the fraction of shares outstanding held by cross-border investors. This quantity is often referred to as "foreign holdings" when viewed from the perspective of a listed company.

Holdings Data: We obtain international mutual fund holdings on 31-Dec-2002 from Thomson Financial. These cross-sectional data are from the same source used by Chan, Covrig, and Ng (2005). The data contain holdings of stocks from 21 developed (target) countries.<sup>9</sup> For a single security, the data consist of the number of shares held by domestic mutual funds and the number of shares held by cross-border (foreign) mutual funds. Approximately 71% of the mutual funds are located in Canada and the United States. Since we are interested in cross-board holdings, we exclude stocks from these two countries.

#### [Insert Table 1 About Here]

Table 1, Panel A provides the number of stocks from each country. The main dataset contains 10,292 different securities which are identified by Sedol number. More stocks are Japanese (2,676) than from any other country. There are 1,973 stocks from the U.K., 744 from Germany, down to 56 stocks from Portugal. For each stock in our sample, we calculate the fraction of total shares held by foreign mutual funds in our dataset. The value of the holdings is US\$ 788 billion as of 31-Dec-2002.

We create a measure, such that if funds hold the world market portfolio, our measure will be equal across stocks. Consider a fund that wants to hold 1% of the world market portfolio—at market capitalization weights. The fund can simply buy 1% of each company's shares outstanding. As prices adjust, this passive strategy continues to hold 1% of the world market portfolio at the correct weights. The fund only needs to buy or sell shares to match changes in firms' capital structures. The measure below is analogous to the measure shown in Equation (16).

$$\Omega_j = \frac{\text{# Shares of Stock } j \text{ Held by Foreign Funds}}{\text{# of Shares of Stock } j \text{ Outstanding}}$$
(17)

We also calculate a domestic measure of holdings. Using this domestic measure allows us to calculate the observed difference between cross-border and domestic holdings:

<sup>&</sup>lt;sup>9</sup>The target countries are: Australia, Austria, Belgium, Denmark, Ireland, Finland, France, Germany, Greece, Hong Kong, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Singapore, Spain, Sweden, Switzerland, and the United Kingdom. The data we obtain are aggregated at the stock level and do not include holdings of US and Canadian stocks.

$$\Omega_{j}^{D} = \frac{\text{\# Shares of Stock } j \text{ Held by Domestic Funds}}{\text{\# of Shares of Stock } j \text{ Outstanding}}$$

$$\Omega_{j}^{*} = \Omega_{j} - \Omega_{j}^{D} \tag{18}$$

Note that  $\Omega_j + \Omega_j^D < 1$  in our data as the Thomson Financial dataset contains a fraction of total holdings.

Return Data: We obtain up to 60 months of individual stock return data from Datastream (dividends included) starting July 1997 and ending June 2002. Returns are lagged by at least six months (from the Dec-2002 holdings date) in order to separate the holdings measure from information proxy variables (described in Section 4.1 below). The return series may be denominated in a currency other than US dollars (USD). Therefore, we also obtain monthly exchanges rates in order to convert to a base currency (USD). Datastream has available Sedol numbers and sufficient return data for 7,553 stocks as shown in the second column of Table 1, Panel A. For each stock, Datastream includes an associated sector code. There are 38 sectors which are listed in Table 1, Panel B. For each sector, we obtain the monthly return of a US dollar index. We also obtain the monthly returns of the MSCI World Market Index denominated in US dollars.

Firm Characteristics: For each stock, as of December 31, 2002, we obtain the number of shares outstanding, the share price, the number of analysts covering the stock, sales, and the ratio of the book value of debt to total assets. Together with the holdings and return data, our final sample consists of 5,781 stocks. The stocks in our final sample are tabulated by country (Table 1, Panel A) and by sector (Panel B).

#### 4.1 Proxy Variables

Asset-Specific Information Advantages: We create a proxy variable related to the asset-specific information advantages that domestic investors have about their home country's stocks. The following time-series regression is estimated for each stock j in our sample using up to five years of monthly returns. The data start July 1997 and end June 2002 and we require at least 20 months of returns.<sup>10</sup>

<sup>&</sup>lt;sup>10</sup>Section 4.3 considers an alternative proxy for asset-specific information based on sales (not return data.)

$$r_{j,t} = \alpha + \beta_w r_{w,t} + \beta_k r_{k,t} + \beta_s SMB_t + \beta_h HML_t + \beta_x r_{x,t} + \varepsilon_{j,t}$$
(19)

$$\textit{Asset-Specific}_{j} \equiv 1 - R_{j}^{2}$$

Above,  $r_{j,t}$  is the return on stock j in month t,  $r_{w,t}$  is the return on the world market portfolio,  $r_{k,t}$  is the return from a global sector index where k is determined by the sector of stock j,  $SMB_t$  is the Fama-French small minus big factor,  $HML_t$  is the Fama-French high minus low book-to-market factor, and  $r_{x,t}$  is the return of currency x which is determined by stock j's quoted prices. Our proxy variable Asset- $Specific_j$  is defined as one minus the fit from Equation (19)—see Durnev, Morck, and Yeung (2004) for a similar example. Note that co-linearity between right-hand side variables such as between  $r_{w,t}$  and  $r_{k,t}$  should not affect our measure of  $R_j^2$  since we are interested in fit and not slope coefficients. A high value of Asset- $Specific_j$  indicates domestic investors have an advantage in knowing the asset-specific component of a stock's payoff. A low value of Asset- $Specific_j$  indicates that domestic investors do not have an information advantage. Using variants of Equation (19) do not materially affect our results.

Common Component Information Advantages: We create a proxy variable for the information advantages global mutual fund managers may have about the common components of stocks' payoffs. Fund managers in our sample are primarily located in North America. We note the industry (k) for each stock j in our sample. We calculate the fraction (weight) each industry k represents of total market capitalization of all *United States* stocks. The fraction (denoted *Industry Wgt<sub>k</sub>*) is used to proxy for information advantages that U.S. fund managers may have about a sector of the economy. The implicit assumption is that larger industries in the United States generate more information about U.S. stocks in that industry and about the industry itself. Greater levels of industry information help managers analyze stocks overseas.

$$Common_{j,k} \equiv 100 \times Industry \ Wgt_k \times \hat{\beta}_{j,k}^2$$
 (20)

Above,  $\hat{\beta}_{j,k}^2$  comes from estimating:  $r_{j,t} = \alpha + \beta_{j,k} r_{k,t} + \varepsilon_{j,t}$  where  $r_{j,t}$  is the return of stock j in month t and  $r_{k,t}$  is the return of industry k. The time-series regression is estimated for each stock j in our sample using up to five years of monthly returns. The data start July 1997 and end June 2002 and we require at least 20 months of returns. Our measure is created without using the price levels or market capitalizations of the 5,781 stocks in our sample. The sector weights are from U.S. stocks which are not in our sample.

<sup>&</sup>lt;sup>11</sup>U.S. stock data come from Datastream. We use only U.S. securities with a primary listing on one of the three major U.S. exchanges (New York Stock Exchange, American Stock Exchange, and Nasdaq).

Most importantly, Equation (20) includes the loading  $(\beta_{j,k})$  of stock j on its industry (k) index. We are interested in knowing if stock j's loading is large in magnitude (positively or negatively). Understanding the outlook for an industry does not help analyze the outlook of a firm if the firm's  $\beta_{j,k} = 0$ .

The model in Section 2 suggests using the  $\beta^2$  term in Equation (20). To see why this is the case, note that the variance of  $\mathbf{B}\tilde{f}$  in the model is  $\mathbf{B}'\mathbf{B}$ . The  $\mathbf{B}$  matrix enters holdings via the  $\mathbf{C}$  matrix in Equation (3). Section 2.2 and 2.3 show that investor *i*'s portfolio weights are proportional to his conditional variance of payoffs. Intuitively, information about the common component is valuable whenever a stock loads heavily on the component (i.e., when the loading is large positively or negatively). The  $\beta^2$  term in Equation (20) reflects this intuition.

Overview Statistics: Table 2 provides overview statistics of the variables used in this paper. Panel A shows that the average stock has  $\Omega_j = 2.76\%$  of its shares held by foreign mutual funds from our dataset. The  $25^{th}$  percentile of holdings is 0.14% and  $75^{th}$  percentile is 3.22%. The difference between the number of stock j's shares held by foreign and domestic mutual funds (normalized by shares outstanding) is denoted  $\Omega_j^*$ , has a - 2.00% average value and a [-3.40%, 0.39%] interquartile range. Section 4.3 explains how we use  $\Omega_j^*$  in robustness checks.

The average value of Asset- $Specific_j$ —the average value of  $1 - R_j^2$  from Equation (19)—is 0.85 with a [0.79, 0.95] inter-quartile range. The average value of  $Common_{j,k}$  is 1.28 with a [0.16, 1.46] inter-quartile range.

Market capitalizations are highly skewed. The average is USD 2.56 billion with a [0.04, 0.47] inter-quartile range. For this reason, we use the natural log of market capitalization in our cross-sectional regressions. The average log of market capitalization is 18.75 with a [17.41, 19.97] inter-quartile range. The average number of analysts covering the foreign stocks is 4.16 and the average book leverage is 0.57.

#### [ Insert Table 2 About Here ]

Table 2, Panel B shows correlation coefficients for the variables used in this paper. The level of a stock's foreign ownership  $(\Omega_j)$  is negatively correlated with the asset-specific information proxy (-0.15 coefficient), but positively correlated with the common information proxy (0.14 coefficient), natural log of market capitalization (0.33 coefficient), and number of analysts (0.46 coefficient). Asset-Specific<sub>j</sub> is negatively cross-sectionally correlated with the natural log of market capitalization (-0.28 correlation coefficient). Asset-Specific<sub>j</sub> is

also negatively correlated with number of analysts (-0.27 coefficient) indicating that it is likely to be a good proxy for information not observed by mutual fund managers in our dataset.  $Common_{j,k}$  is positively correlated with the natural log of market capitalization (0.15 coefficient) and with the number of analysts (0.24 coefficient).

#### 4.2 Cross-Border Holdings and Double Sort Results

We compare the empirical relationship between mutual fund holdings and information proxies with the theoretical relationship shown in Figure 1. We test if cross-border holdings decrease as asset-specific information advantages increase and if holdings increase as common component information advantages increase. We sort the 5,781 stocks into quartiles using our proxy variable for asset-specific information. We also sort the stocks into quartiles using our proxy variable for common component information.

Table 3 shows the average foreign holdings for the four combinations where the sort variable is either "low" (bottom 25%) or "high" (upper 25%). When there are low-levels of information advantages about common components and high-levels of asset-specific information advantages, the average cross-border holding is 0.0139 of shares outstanding. See the lower-right hand side of Figure 1 for a graphical example.<sup>12</sup>

#### [Insert Table 3 About Here]

When there are high-levels of information advantages about common components and low-levels of asset-specific information advantages, the average cross-border holdings is 0.0490 of shares outstanding. See the upper-left hand side of Figure 1 for a graphical example. When asset-specific information advantages are "low", an increase in common component information advantages leads holdings to increase from 0.0293 to 0.0490. The increase is 0.0197 with a 3.51 t-statistic.

Table 3 shows that when information advantages about common components are "high", an increase in asset-specific information leads to a decrease of 0.0267 in cross-border holdings (with a 7.39 t-statistic). When information advantages about common components is "low", an increase in asset-specific information leads to an increase of 0.0154 in cross-border holdings (with a 2.99 t-statistic). The increases are both statistically and economically significant.

Sorting stocks by our information proxy variables produces large dispersions in observed cross-border holdings. The dispersions (or differences) shown in Table 3 range from 0.84%

<sup>&</sup>lt;sup>12</sup>If the mutual funds in our dataset held the world market portfolio, we should measure similar values of  $\Omega_j$  across all stocks and thus report similar values of  $\bar{\Omega}$  for each of the four bins shown in Table 3.

to 2.67%. In fact, if we compare the largest ownership group (4.90%) in Table 3 with the lowest group (1.39%), the dispersion is 3.51%. We conclude that sorting by our proxy variables produces holding dispersions that are large and economically significant.

#### 4.3 Cross-Border Holdings and Regression Results

We use regression analysis to examine the relationship between our proxy variables for information advantages and holdings. Table 4 reports results of a regression of holdings on  $Asset-Specific_j$  and other variables known to influence cross-border holdings. Table 5 repeats the same regressions but first partitions stocks into those with high and low values of our proxy variable for factor information. The basic regression is:

$$\Omega_j = \gamma_0 + \gamma_1 \left( Asset\text{-}Specific_j \right) + \nu_j$$

The coefficient of interest is  $\gamma_1$ . Table 4, Regression 1 shows the estimated value of  $\gamma_1$  is -0.0569 with a -7.87 t-statistic. We use robust (White) standard errors to compute t-statistics. Stocks with high levels of asset-specific information (i.e., stocks that move less with world and sector indices) have lower levels of cross-border holdings. This finding of a negative slope coefficient matches the downward sloping graph lines in Figure 1.

#### [Insert Table 4 About Here]

We expand the basic regression to include variables that have previously been linked to cross-border holdings. The variables include the natural log of equity market value  $(lnMC_j)$ , and the number of analysts following the stock  $(\# of Analysts_j)$ .

$$\Omega_j = \gamma_0 + \gamma_1 \left( Asset\text{-}Specific_i \right) + \gamma_2 \left( lnMC_i \right) + \gamma_3 \left( \# \text{ of } Analysts_i \right) + \nu_j$$

In Table 4, Regression 2 shows the results after including two explanatory variables in the cross-sectional regression. The coefficient on  $Asset-Specific_j$  ( $\gamma_1$ ) is -0.0224 with a -3.12 t-statistic. The fit of the regression is 0.11 which is higher than the fit in Regression 1.<sup>13</sup>

The remainder of Table 4 is a series of robustness checks. We test different regression specifications and check if the  $\gamma_1$  coefficient remains significantly negative. In Regression 3, the left-hand side variable is changed to  $\Omega_j^*$  from Equation (18). If a stock is particularly attractive to all institutional investors (as opposed to just cross-border investors),  $\Omega_j^*$  will

<sup>&</sup>lt;sup>13</sup>The fit shown at the bottom of Table 4 is an adjusted  $R^2$  from the cross-sectional regression—not to be confused with the fit from the time series regression in (19) used to construct  $Asset-Specific_i$ .

be low. However, the coefficients in Regression 3 are broadly similar to those in the first two regressions except that number of analysts is now significantly positive. Regression 4 includes book leverage as an explanatory variable. The coefficient is 0.0029 with a 10.35 t-statistic. We conclude that controlling for possibly (unobserved) characteristics that may be attractive to institutional investors does not affect our results.

A Sales-Based Measure of Asset-Specific<sub>j</sub>: We calculate a second measure of Asset-Specific<sub>j</sub> based on sales data as opposed to stock market data. For each company, we obtain a history of annual sales (in US dollars) and calculate annual sales growth. We next calculate each industry's annual sales growth as the equal-weighted average of company sales growths. The number of firms per industry is given in Table 1, Panel B. The new measure of Asset-Specific<sub>j</sub> (Sales) is simply one minus the correlation of stock j's annual sales growth with its industry's annual sales growth. We require a company to have six years of sales growth data to be included which reduces our sample to 3,905 stocks.

In Table 4, Regression 5, the  $\gamma_1$  coefficient is -0.0602 with a -13.69 t-statistic. We continue to use robust (White) standard errors when calculating t-statistics. In Regression 6, we include country fixed effects (dummy variables). The coefficient on  $Asset-Specific_j(Sales)$  is -0.0076 with a -1.98 t-statistic. Controlling for country fixed effects addresses the possibility that unobserved country-level differences are driving results. If, however, country-level differences are related to information structures, such a regression would not make sense in light of our model. We report results for completeness.

In the final column, Regression 7, we control for the large number of Japanese and UK firms in our sample. We randomly choose 155 of the 2,108 Japanese stocks in our sample and 155 of the 721 UK stocks in the sample (note that 155 is the average number of stocks from other countries in our sample.) The sample size is 1,354 as we have dramatically reduced the number of Japanese and UK stocks used. In addition, some of the 155 stocks that were randomly chosen may not have six years of sales growth data. Regression 7 shows the coefficient on  $Asset-Specific_j(Sales)$  is -0.0186 with a -3.15 t-statistic.

Our regressions show that cross-border holdings decrease as asset-specific information advantages become more important for a given stock's returns. We measure the level of asset-specific information advantages that domestic investors have about own country's stocks using both stock return data and sales growth data.

#### 4.4 Information about Common Factors and Regressions

We end the empirical analysis by combining our proxy for information advantages about common components with the regression analysis. Table 5, Regression 1a considers only stocks with a low degree of information advantage about common factors (bottom 25%). We regress our cross-border holdings  $(\Omega_j)$  on a constant and our proxy for asset-specific information (Asset-Specific<sub>j</sub>). The coefficient on Asset-Specific<sub>j</sub> is -0.0331 with a -3.70 t-statistic. Regression 1b uses stocks with a high degree of information advantage about common factors (upper 25%). The coefficient on Asset-Specific<sub>j</sub> is -0.0741 with a -3.40 t-statistic.

#### [Insert Table 5 About Here]

The regression results shown in Table 5 parallel the double sort results shown in Table 3. In Table 5, Regressions 1a and 1b, stocks with a high degree of information advantage about common factors are more sensitive to our proxy for asset-specific information (e.g., the coefficient is more negative: -0.0331 > -0.0741). In Table 3, and for stocks with a high common information proxy, stocks with high asset-specific information have 2.67% lower holdings that stocks with low asset-specific information.

Also in Table 5, stocks with a high degree of information advantage about common factors have higher levels of cross-border holdings (when the asset-specific proxy is zero)—see the estimated constant term  $\gamma_0$  and note that 0.0478 < 0.0821. An F-test for differences in the asset-specific coefficients (only) has a 0.0029 p-value.

Table 5, Regressions 2a and 2b use  $\Omega_j^*$  as the dependent variable. Both specifications give the same general picture even after controlling for stock size and leverage. The  $\gamma_1$  coefficients become increasingly negative when moving from stocks with low information advantage about the common factor to stocks with high information advantage (-0.0523 > -0.1001). The F-test for differences in the asset-specific coefficient rejects the hypothesis of coefficient equality at the 3%-level.

#### 5 Conclusion

This paper proposes a multi-asset, rational expectations equilibrium model in which agents are asymmetrically informed about asset-specific and common components of payoffs. Our model allows agents to have asset-specific information or information about common components or both or neither. The model produces closed-form solutions for holdings of individual agents as well as asset prices. We apply the model to a study of international portfolio choice.

When all variables in our model are uncorrelated, an investor's holdings of a given stock

is straightforward to analyze—the holdings are related to the ratio of the market's uncertainty about the stock's future payoff to the investor's uncertainty. When asset payoffs are correlated (possibly through exposure to common components) or when supply shocks are correlated, analysis of holdings is much more complicated. An investor's position is related to the ratio of two matrices (the market's uncertainty and his own uncertainty).

An investor may have high demand for a stock for which he has valuable private information. He may also have high demand for the same stock if he has valuable private information about a stock with highly correlated payoffs. When considering common components, the investor may have high demand for a stock (even if he does not have stock-specific information) provided he has information about a common component of the stock's payoffs. Of course, having information about a common component is not sufficient to determine if the investor will have high demands for certain stocks. Stocks must load heavily on the factor and the net effect of the loading must be sufficient to outweigh private, stock-specific information that other investors' may have.

To better understand holdings when asset payoffs are correlated, we conduct a numerical analysis of our model. We consider three stocks, from three countries, and two factors. We focus on cross-border holdings of the first (assumed to be French) stock. Our model generates large dispersion in cross-border holdings ranging from 2.59% of shares outstanding to 8.39%. This dispersion can result from different levels of asset-specific information advantages. The dispersion can also result from differential loadings on the model's common components.

We end the paper with an empirical analysis of international portfolio choice. We create two proxy variables. The first measures the degree of information advantage about the asset-specific component of a stock's payoffs. The second measures the degree of information advantage about common components. We show that both a decrease in the proxy for asset-specific information advantage and an increase in the proxy for common information lead to greater levels of cross-border holdings. In regression analysis, our information proxies explain holding levels even after including variables that have previously been linked to cross-border holdings (e.g., the market capitalization of a firm's equity, the number of analysts following the firm, and the firm's leverage.)

There are a number of potential avenues for future research. First, one could try to extend our model to multiple periods. This would provide expressions for net trading as in Brennan and Cao (1997) as well as suggest empirical tests based on trading (as opposed to holdings) data. Second, one could work to devise methods of empirically identifying different information structures. While no small task, structures could then be used to test relative asset prices using expressions in this paper. Third, our model may be adapted

to better understanding partially segmented markets. In such cases, information is the "friction" that segments markets. One may be able to model groups of investors who face low frictions only when trading securities from their home country, groups of investors who face low frictions when trading securities in a contiguous block of countries (a geographic region), or groups of investors who face low frictions when investing in any global security. None of the three extensions is likely to be easy—all are potentially interesting.

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#### Appendix A

The information set of investor n (formally, investor i in group n) consists of the realization of private signals  $\mathbf{M}_n\tilde{\eta}$  and of equilibrium prices  $\tilde{P}^0$ . The equilibrium price vector  $\tilde{P}^0$  is a linear function of the information  $\tilde{\eta}$  and the supply  $\tilde{z}$  with  $\tilde{P}^0 = A_0 + \mathbf{A}_1\tilde{\eta} - \mathbf{A}_2\tilde{z}$ . Since,  $\tilde{w}_n^1 = w_n^0 R + X_n'(\tilde{P}^1 - R\tilde{P}^0)$  and  $\tilde{P}^1$  is a linear function of  $\tilde{\eta}$  and  $\tilde{\varepsilon}$ , it follows that  $\tilde{w}_n^1$  joins the multivariate normal distribution of  $(\tilde{\eta}, \tilde{\varepsilon}, \tilde{z})$ . Consequently,  $\tilde{w}_n^1$  is a normal random variable conditional on  $\mathbf{M}_n\tilde{\eta}$  and  $\tilde{P}^0$ . Properties of normal distributions imply that investor n's expected utility can be written as:

$$\begin{split} E\left[U(\tilde{w}_{n}^{1})|\mathbf{M}_{n}\tilde{\eta},\tilde{P}^{0}\right] &= U\left\{E\left[\tilde{w}_{n}^{1}|\mathbf{M}_{n}\tilde{\eta},\tilde{P}^{0}\right] - \frac{a}{2}Var\left[\tilde{w}_{n}^{1}|\mathbf{M}_{n}\tilde{\eta},\tilde{P}^{0}\right]\right\} \\ &= U\left\{E\left[w_{n}^{0}R + X_{n}'(\tilde{P}^{1} - R\tilde{P}^{0})|\mathbf{M}_{n}\tilde{\eta},\tilde{P}^{0}\right] - \frac{a}{2}Var\left[w_{n}^{0}R + X_{n}'(\tilde{P}^{1} - R\tilde{P}^{0})|\mathbf{M}_{n}\tilde{\eta},\tilde{P}^{0}\right]\right\} \end{split}$$

Since the utility function is exponential, maximizing this expected utility is identical to maximizing:

$$\begin{aligned} \max_{X_n} \left\{ E\left[ w_n^0 R + X_n' (\tilde{P}^1 - R\tilde{P}^0) | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0 \right] - \frac{a}{2} Var\left[ w_n^0 R + X_n' (\tilde{P}^1 - R\tilde{P}^0) | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0 \right] \right\} \\ &= \max_{X_n} \left\{ X_n' E\left[ \left( \tilde{P}^1 - R\tilde{P}^0 \right) | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0 \right] - \frac{a}{2} X_n' Var\left[ (\tilde{P}^1 - R\tilde{P}^0) | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0 \right] X_n \right\} \end{aligned}$$

The equation to be solved is:

$$0 = E\left[ (\tilde{P}^1 - R\tilde{P}^0) | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0 \right] - aVar\left[ (\tilde{P}^1 - R\tilde{P}^0) | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0 \right] X_n$$
 (21)

This implies that investor n's demand vector is:

$$X_n = a^{-1} V a r^{-1} \left[ \tilde{P}^1 | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0 \right] \times \left( E \left[ \tilde{P}^1 | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0 \right] - R P^0 \right)$$
 (22)

#### Appendix B

From Equations (4), (6), and (7), we have:

$$0 = \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} \Big( B_{0n} + \mathbf{B}_{1n} \mathbf{M}_n \tilde{\eta} + (\mathbf{B}_{2n} - R\mathbf{I}_J) (A_0 + \mathbf{A}_1 \tilde{\eta} - \mathbf{A}_2 \tilde{z}) \Big) - a\tilde{z}$$

By canceling the  $\tilde{z}$ ,  $\tilde{\eta}$ , and constant terms, it is straightforward to show that:

$$a\mathbf{A}_{2}^{-1}A_{0} = \sum_{n=1}^{N} \lambda_{n} \mathbf{V}_{n}^{-1} B_{0n}$$

$$a\mathbf{A}_{2}^{-1}\mathbf{A}_{1} = \sum_{n=1}^{N} \lambda_{n} \mathbf{V}_{n}^{-1} \mathbf{B}_{1n} \mathbf{M}_{n}$$

$$a\mathbf{A}_{2}^{-1} = \sum_{n=1}^{N} \lambda_{n} \mathbf{V}_{n}^{-1} (R\mathbf{I}_{J} - \mathbf{B}_{2n})$$
(23)

#### Appendix C

The vector  $(\tilde{P}^{1'} \mathbf{M}_n \tilde{\eta}' \tilde{P}^{0'})'$  is normally distributed and its var-cov matrix is:

$$Var\left[\begin{pmatrix} \tilde{P}^{1'} & \mathbf{M}_{n}\tilde{\eta}' & \tilde{P}^{0'} \end{pmatrix}'\right] = \begin{pmatrix} \mathbf{C}\mathbf{Q}\mathbf{C}' + \mathbf{\Sigma}_{\epsilon} & \mathbf{C}\mathbf{Q}\mathbf{M}_{n}' & \mathbf{C}\mathbf{Q}\mathbf{A}_{1}' \\ \mathbf{M}_{n}\mathbf{Q}\mathbf{C}' & \mathbf{M}_{n}\mathbf{Q}\mathbf{M}_{n}' & \mathbf{M}_{n}\mathbf{Q}\mathbf{A}_{1}' \\ \mathbf{A}_{1}\mathbf{Q}\mathbf{C}' & \mathbf{A}_{1}\mathbf{Q}\mathbf{M}_{n}' & \mathbf{A}_{1}\mathbf{Q}\mathbf{A}_{1}' + \mathbf{A}_{2}\mathbf{\Sigma}_{z}\mathbf{A}_{2}' \end{pmatrix}$$
(24)

The conditional expectation is:

$$E_{n}\left[\tilde{P}^{1}\right] = E\left[\tilde{P}^{1}|\mathbf{M}_{n}\tilde{\eta},\tilde{P}^{0}\right]$$

$$= E[\tilde{P}^{1}] + Cov\left[\tilde{P}^{1}; \begin{pmatrix} \mathbf{M}_{n}\tilde{\eta} \\ \tilde{p}^{0} \end{pmatrix}\right] \times Var^{-1}\left[\begin{array}{c} \mathbf{M}_{n}\tilde{\eta} \\ \tilde{p}^{0} \end{array}\right] \times \left(\begin{pmatrix} \mathbf{M}_{n}\tilde{\eta} \\ \tilde{p}^{0} \end{pmatrix} - E\left[\begin{array}{c} \mathbf{M}_{n}\tilde{\eta} \\ \tilde{p}^{0} \end{array}\right]\right)$$

Normal distributions give  $E\left[\tilde{P}^1|\mathbf{M}_n\tilde{\eta},\tilde{P}^0\right]=B_{0n}+\mathbf{B}_{1n}\mathbf{M}_n\tilde{\eta}+\mathbf{B}_{2n}\tilde{P}^0$ . Hence,

$$\begin{pmatrix} \mathbf{B}_{1n} & \mathbf{B}_{2n} \end{pmatrix} = Cov \begin{bmatrix} \tilde{P}^1; \begin{pmatrix} \mathbf{M}_n \tilde{\eta} \\ \tilde{P}^0 \end{pmatrix} \end{bmatrix} \times Var^{-1} \begin{bmatrix} \mathbf{M}_n \tilde{\eta} \\ \tilde{P}^0 \end{bmatrix}$$
 (25)

$$\begin{pmatrix} \mathbf{CQM}_n' & \mathbf{CQA}_1' \end{pmatrix} = \begin{pmatrix} \mathbf{B}_{1n} & \mathbf{B}_{2n} \end{pmatrix} \begin{pmatrix} \mathbf{M}_n \mathbf{QM}_n' & \mathbf{M}_n \mathbf{QA}_1' \\ \mathbf{A}_1 \mathbf{QM}_n' & \mathbf{A}_1 \mathbf{QA}_1' + \mathbf{A}_2 \mathbf{\Sigma}_z \mathbf{A}_2' \end{pmatrix}$$
(26)

The variance of returns conditional on n's information is:

$$\mathbf{V}_{n} = Var \left[ \tilde{P}^{1} | \mathbf{M}_{n} \tilde{\eta}, \tilde{P}^{0} \right]$$

$$= Var \left[ \tilde{P}^{1} \right] - Cov \left[ \tilde{P}^{1}; \begin{pmatrix} \mathbf{M}_{n} \tilde{\eta} \\ \tilde{P}^{0} \end{pmatrix} \right] \times Var^{-1} \left[ \begin{pmatrix} \mathbf{M}_{n} \tilde{\eta} \\ \tilde{P}^{0} \end{pmatrix} \right] \times Cov \left[ \begin{pmatrix} \mathbf{M}_{n} \tilde{\eta} \\ \tilde{P}^{0} \end{pmatrix}; \tilde{P}^{1} \right]$$
(27)

We use Equation (25) to get:

$$\mathbf{V}_{n} = Var \left[ \tilde{P}^{1} \right] - \left( \mathbf{B}_{1n} \mathbf{B}_{2n} \right) Cov \left[ \left( \mathbf{M}_{n} \tilde{\eta} \right) ; \tilde{P}^{1} \right]$$

Because 
$$Cov\left[\begin{pmatrix} \mathbf{M}_{n}\tilde{\eta} \\ \tilde{p}^{0} \end{pmatrix}; \tilde{P}^{1}\right] = \begin{pmatrix} \mathbf{M}_{n}\mathbf{Q}\mathbf{C}' \\ \mathbf{A}_{1}\mathbf{Q}\mathbf{C}' \end{pmatrix}$$
 we get:
$$\mathbf{V}_{n} = \mathbf{C}\mathbf{Q}\mathbf{C}' + \mathbf{\Sigma}_{\varepsilon} - \mathbf{B}_{1n}\mathbf{M}_{n}\mathbf{Q}\mathbf{C}' - \mathbf{B}_{2n}\mathbf{A}_{1}\mathbf{Q}\mathbf{C}'$$
(28)

#### Appendix D

In order to determine a closed-form solution for U, we solve the second equation from the system shown in Equation (8):

$$a\mathbf{A}_{2}^{-1}\mathbf{A}_{1} = a\mathbf{U} = \sum_{n=1}^{N} \lambda_{n} \mathbf{V}_{n}^{-1} \mathbf{B}_{1n} \mathbf{M}_{n}$$

$$(29)$$

The following properties apply to matrices  $\mathbf{D}_n$  and  $\mathbf{M}_n$ . Below we use  $n_a$  and  $n_b$  to denote two different groups of investors such that  $n = \{1, 2, 3, \ldots, n_a, \ldots, n_b, \ldots, N\}$ :

P1: 
$$\sum_{n=1}^{N} \mathbf{D}_n = \mathbf{I}_J$$

P2:  $\forall n_a \neq n_b$ :  $\mathbf{D}_{n_a} \mathbf{D}_{n_b} = \mathbf{0}_{J+K}$  where  $\mathbf{0}_{J+K}$  is the null matrix of order J+K;

P3:  $\mathbf{M}_{n_a}\mathbf{M}'_{n_b} = \mathbf{0}_{Jn_a+Kn_a,Jn_b+Kn_b}$  where  $\mathbf{0}_{Jn_a+Kn_a,Jn_b+Kn_b}$  is the null matrix;

P4:  $\mathbf{D}_n \mathbf{D}_n = \mathbf{D}_n$  and  $\mathbf{M}_n \mathbf{M}_n^{-1} = \mathbf{I}_{Jn+Kn}$ 

P5: 
$$\forall \mathbf{G_1}, \mathbf{G_2}$$
:  $g(\mathbf{G_1})g(\mathbf{G_2}) = \sum_{n=1}^{N} \mathbf{D}_n \mathbf{G_1} \mathbf{D}_n \mathbf{G_2} \mathbf{D}_n$ 

P6: 
$$\forall$$
 **G**:  $g(GD) = g(G)D = Dg(G)$ 

There are three matrices key to obtaining a closed-form solution for U:

$$\mathbf{M} = \mathbf{U}\mathbf{Q}\mathbf{U}' + \mathbf{\Sigma}_{z}$$

$$\mathbf{\Psi} = Var\left[\tilde{\eta}|\tilde{P}^{0}\right] = \mathbf{Q} - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q}$$

$$\mathbf{\Psi}_{n} = \mathbf{M}_{n}\mathbf{\Psi}\mathbf{M}'_{n}$$

We first solve Equation (26) for  $\mathbf{B}_{1n}$  and  $\mathbf{B}_{2n}$ . Note, we do not assume  $\mathbf{A}_1$  is invertable to get the following results (in fact,  $\mathbf{A}_1$  is not square). The two equations to be solved are:

$$\mathbf{B}_{1n}\left(\mathbf{M}_{n}\mathbf{Q}\mathbf{M}_{n}^{\prime}\right) + \mathbf{B}_{2n}\left(\mathbf{A}_{1}\mathbf{Q}\mathbf{M}_{n}^{\prime}\right) = \mathbf{C}\mathbf{Q}\mathbf{M}_{n}^{\prime} \tag{30}$$

$$\mathbf{B}_{1n} \left( \mathbf{M}_{n} \mathbf{Q} \mathbf{A}_{1}^{\prime} \right) + \mathbf{B}_{2n} \left( \mathbf{A}_{1} \mathbf{Q} \mathbf{A}_{1}^{\prime} + \mathbf{A}_{2} \Sigma_{z} \mathbf{A}_{2}^{\prime} \right) = \mathbf{C} \mathbf{Q} \mathbf{A}_{1}^{\prime}$$
(31)

Using  $\mathbf{M} = \mathbf{U}\mathbf{Q}\mathbf{U}' + \mathbf{\Sigma}_z$ , we obtain  $\mathbf{A}_1\mathbf{Q}\mathbf{A}_1' + \mathbf{A}_2\mathbf{\Sigma}_z\mathbf{A}_2' = \mathbf{A}_2\mathbf{M}\mathbf{A}_2'$ . Starting with

Equation (31), we get:

$$\begin{array}{rcl} \mathbf{B}_{1n} \left( \mathbf{M}_{n} \mathbf{Q} \mathbf{A}_{1}' \right) + \mathbf{B}_{2n} \left( \mathbf{A}_{1} \mathbf{Q} \mathbf{A}_{1}' + \mathbf{A}_{2} \boldsymbol{\Sigma}_{z} \mathbf{A}_{2}' \right) & = & \mathbf{C} \mathbf{Q} \mathbf{A}_{1}' \\ & \mathbf{B}_{1n} \left( \mathbf{M}_{n} \mathbf{Q} \mathbf{A}_{1}' \right) + \mathbf{B}_{2n} \left( \mathbf{A}_{2} \mathbf{M} \mathbf{A}_{2}' \right) & = & \mathbf{C} \mathbf{Q} \mathbf{A}_{1}' \\ & \left( \mathbf{C} \mathbf{Q} \mathbf{A}_{1}' - \mathbf{B}_{1n} \mathbf{M}_{n} \mathbf{Q} \mathbf{A}_{1}' \right) \left( \mathbf{A}_{2} \mathbf{M} \mathbf{A}_{2}' \right)^{-1} & = & \mathbf{B}_{2n} \\ & \left( \mathbf{C} - \mathbf{B}_{1n} \mathbf{M}_{n} \right) \mathbf{Q} \left( \mathbf{A}_{2}^{-1} \mathbf{A}_{1} \right)' \mathbf{M}^{-1} \mathbf{A}_{2}^{-1} & = & \mathbf{B}_{2n} \\ & \left( \mathbf{C} - \mathbf{B}_{1n} \mathbf{M}_{n} \right) \mathbf{Q} \mathbf{U}' \mathbf{M}^{-1} \mathbf{A}_{2}^{-1} & = & \mathbf{B}_{2n} \end{array}$$

In a second step, we solve Equation (30):

$$\begin{array}{rcl} \mathbf{B}_{1n}(\mathbf{M}_{n}\mathbf{Q}\mathbf{M}_{n}') + (\mathbf{C} - \mathbf{B}_{1n}\mathbf{M}_{n})\mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{A}_{2}^{-1}(\mathbf{A}_{1}\mathbf{Q}\mathbf{M}_{n}') & = & \mathbf{C}\mathbf{Q}\mathbf{M}_{n}'\\ & \mathbf{B}_{1n}\mathbf{M}_{n}(\mathbf{Q}\mathbf{M}_{n}' - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q}\mathbf{M}_{n}') & = & (\mathbf{C}\mathbf{Q} - \mathbf{C}\mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q})\mathbf{M}_{n}'\\ & \mathbf{B}_{1n}\mathbf{M}_{n}(\mathbf{Q} - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q})\mathbf{M}_{n}' & = & \mathbf{C}(\mathbf{Q} - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q})\mathbf{M}_{n}'\\ & \mathbf{B}_{1n}\mathbf{M}_{n}\mathbf{\Psi}\mathbf{M}_{n}' & = & \mathbf{C}\mathbf{\Psi}\mathbf{M}_{n}'\\ & \mathbf{B}_{1n}\mathbf{\Phi}_{n} & = & \mathbf{C}\mathbf{\Psi}\mathbf{M}_{n}'\\ & \mathbf{B}_{1n} & = & \mathbf{C}\mathbf{\Psi}\mathbf{M}_{n}'\mathbf{\Psi}_{n}^{-1} \end{array}$$

We have thus demonstrated that:

$$\mathbf{B}_{1n} = \mathbf{C}\Psi \mathbf{M}_n' \Psi_n^{-1}$$

$$\mathbf{B}_{2n} = (\mathbf{C} - \mathbf{B}_{1n} \mathbf{M}_n) \mathbf{Q} \mathbf{U}' \mathbf{M}^{-1} \mathbf{A}_2^{-1}$$
(32)

By substituting  $\mathbf{B}_{1n}$  and  $\mathbf{B}_{2n}$  into Equation (28) we obtain the variance-covariance matrix  $\mathbf{V}_n$  as a function of  $\Psi$ 

$$\mathbf{V}_{n} = \mathbf{C}\mathbf{Q}\mathbf{C}' + \mathbf{\Sigma}_{\varepsilon} - \mathbf{B}_{1n}\mathbf{M}_{n}\mathbf{Q}\mathbf{C}' - \mathbf{B}_{2n}\mathbf{A}_{1}\mathbf{Q}\mathbf{C}' 
\mathbf{V}_{n} = \mathbf{C}\mathbf{Q}\mathbf{C}' + \mathbf{\Sigma}_{\varepsilon} - \mathbf{C}\mathbf{\Psi}\mathbf{M}'_{n}\mathbf{\Psi}_{n}^{-1}\mathbf{M}_{n}\mathbf{Q}\mathbf{C}' - (\mathbf{C} - \mathbf{C}\mathbf{\Psi}\mathbf{M}'_{n}\mathbf{\Psi}_{n}^{-1}\mathbf{M}_{n})\mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{A}_{2}^{-1}\mathbf{A}_{1}\mathbf{Q}\mathbf{C}' 
\mathbf{V}_{n} = \mathbf{\Sigma}_{\varepsilon} + \mathbf{C}(\mathbf{Q} - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q})\mathbf{C}' - \mathbf{C}\mathbf{\Psi}\mathbf{M}'_{n}\mathbf{\Psi}_{n}^{-1}\mathbf{M}_{n}\mathbf{Q}\mathbf{C}' + \mathbf{C}\mathbf{\Psi}\mathbf{M}'_{n}\mathbf{\Psi}_{n}^{-1}\mathbf{M}_{n}\mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q}\mathbf{C}' 
\mathbf{V}_{n} = \mathbf{\Sigma}_{\varepsilon} + \mathbf{C}\mathbf{\Psi}\mathbf{C}' - \mathbf{C}\mathbf{\Psi}\mathbf{M}'_{n}\mathbf{\Psi}_{n}^{-1}\mathbf{M}_{n}\mathbf{\Psi}\mathbf{C}'$$
(33)

We use Equation (29) to determine **U** by right-multiplying by  $\mathbf{M}'_n$ . Note that  $\mathbf{M}'_n$  concerns investor group n. Property P3, from the start of this appendix, shows we need only carry out the multiplication with terms from the same investor group.

Thus, we obtain  $\lambda_n \mathbf{V}_n^{-1} \mathbf{B}_{1n} = a \mathbf{U} \mathbf{M}_n'$ . Also from P3,  $\mathbf{M}_{n_a} \mathbf{M}_{n_b}' = \mathbf{0}$ . We then multiply

this last expression by  $V_n$  on the left and we replace  $B_{1n}$  with its value from (32):

$$\lambda_n \mathbf{C} \mathbf{\Psi} \mathbf{M}_n' \mathbf{\Psi}_n^{-1} = a \mathbf{V}_n \mathbf{U} \mathbf{M}_n' \tag{34}$$

If we multiply (34) by  $\mathbf{M}_n$  on the right and if we sum for n = 1, ..., N, we obtain Equation (29). We conclude that Equation (29) is equivalent to Equation (34) for all n = 1, ..., N. If we multiply Equation (34) by  $\mathbf{\Psi}_n$  and  $\mathbf{M}_n$  on the right and if we replace  $\mathbf{V}_n$  with its value in Equation (33) we then obtain:

$$\lambda_n \mathbf{C} \mathbf{\Psi} \mathbf{D}_n = a \left( \mathbf{\Sigma}_{\varepsilon} + \mathbf{C} \mathbf{\Psi} \mathbf{C}' - \mathbf{C} \mathbf{\Psi} \mathbf{M}_n' \mathbf{\Psi}_n^{-1} \mathbf{M}_n \mathbf{\Psi} \mathbf{C}' \right) \mathbf{U} \mathbf{M}_n' \mathbf{\Psi}_n \mathbf{M}_n$$

If we now sum for n = 1, ..., N we obtain:

$$\sum_{n=1}^{N} \lambda_n \mathbf{C} \mathbf{\Psi} \mathbf{D}_n = a \left( \mathbf{\Sigma}_{\varepsilon} \sum_{n=1}^{N} \mathbf{U} \mathbf{M}_n' \mathbf{\Psi}_n \mathbf{M}_n + \mathbf{C} \mathbf{\Psi} \mathbf{C}' \sum_{n=1}^{N} \mathbf{U} \mathbf{M}_n' \mathbf{\Psi}_n \mathbf{M}_n - \sum_{n=1}^{N} \mathbf{C} \mathbf{\Psi} \mathbf{M}_n' \mathbf{\Psi}_n^{-1} \mathbf{M}_n \mathbf{\Psi} \mathbf{C}' \mathbf{U} \mathbf{M}_n' \mathbf{\Psi}_n \mathbf{M}_n \right)$$

which is equivalent to:

$$\mathbf{C}\Psi\mathbf{D} = a\left(\mathbf{\Sigma}_{\varepsilon}\mathbf{U}\sum_{n=1}^{N}\mathbf{D}_{n}\Psi\mathbf{D}_{n} + \mathbf{C}\Psi\mathbf{C}'\mathbf{U}\sum_{n=1}^{N}\mathbf{D}_{n}\Psi\mathbf{D}_{n} - \mathbf{C}\Psi\sum_{n=1}^{N}\mathbf{M}_{n}'\Psi_{n}^{-1}\mathbf{M}_{n}\Psi\mathbf{C}'\mathbf{U}\mathbf{M}_{n}'\Psi_{n}\mathbf{M}_{n}\right)$$

By introducing the function  $g(\cdot)$ , we obtain the expression below. The reader can easily check that (35) is equivalent to (29).

$$\mathbf{C}\mathbf{\Psi}\mathbf{D} = a\mathbf{\Sigma}_{\varepsilon}\mathbf{U}g(\mathbf{\Psi}) + a\mathbf{C}\mathbf{\Psi}\mathbf{C}'\mathbf{U}g(\mathbf{\Psi}) - a\mathbf{C}\mathbf{\Psi}\left(\sum_{n=1}^{N}\mathbf{M}_{n}'\mathbf{\Psi}_{n}^{-1}\mathbf{M}_{n}\mathbf{\Psi}\mathbf{C}'\mathbf{U}\mathbf{M}_{n}'\mathbf{\Psi}_{n}\mathbf{M}_{n}\right)$$
(35)

To prove Lemma 2.1, we start by assuming  $\mathbf{U} = a^{-1} \mathbf{\Sigma}_{\varepsilon}^{-1} \mathbf{CD}$ . We then substitute this expression for  $\mathbf{U}$  into (35) and check that the following equality holds:

$$\mathbf{C}\mathbf{\Psi}\mathbf{D} = \mathbf{C}\mathbf{D}g(\mathbf{\Psi}) + \mathbf{C}\mathbf{\Psi}\mathbf{C}'\mathbf{\Sigma}_{\varepsilon}^{-1}\mathbf{C}\mathbf{D}g(\mathbf{\Psi}) - \mathbf{C}\mathbf{\Psi}\left(\sum_{n=1}^{N}\mathbf{M}'_{n}\mathbf{\Psi}_{n}^{-1}\mathbf{M}_{n}\mathbf{\Psi}\mathbf{C}'\mathbf{\Sigma}^{-1}\mathbf{C}\mathbf{D}\mathbf{M}'_{n}\mathbf{\Psi}_{n}\mathbf{M}_{n}\right)$$
(36)

Under the assumption that  $\Psi^{-1} + \mathbf{C}' \Sigma_{\varepsilon}^{-1} \mathbf{C}$  is a g-matrix, we can write  $g(\Psi^{-1} + \mathbf{C}' \Sigma_{\varepsilon}^{-1} \mathbf{C}) = \Psi^{-1} + \mathbf{C}' \Sigma_{\varepsilon}^{-1} \mathbf{C}$ . We then replace  $\mathbf{C}' \Sigma_{\varepsilon}^{-1} \mathbf{C}$  by  $g(\Psi^{-1} + \mathbf{C}' \Sigma_{\varepsilon}^{-1} \mathbf{C}) - \Psi^{-1}$  in the right hand side of Equation (36) and confirm the equality. We conclude  $\mathbf{U} = a^{-1} \Sigma_{\varepsilon}^{-1} \mathbf{C} \mathbf{D}$  represents a solution.

#### Appendix E

We replace  $\mathbf{B}_{2n}$  in the third equation of (8) with its value given in (32). We then obtain  $\mathbf{A}_2$ . We obtain  $\mathbf{A}_1$  directly from the expression for  $\mathbf{U}$ . In order to determine  $A_0$ , we substitute the following expression for  $B_{0n}$  into the first equation of (8).

$$B_{0n} = (\mathbf{C} - \mathbf{B}_{1n} \mathbf{M}_n - \mathbf{B}_{2n} \mathbf{A}_1) E\left[\tilde{\eta}\right] - \mathbf{B}_{2n} \left(A_0 - \mathbf{A}_2 E\left[\tilde{z}\right]\right)$$

To check that  $\mathbf{A}_2$  is a regular matrix, we start with Equation (13). Note that matrices  $\mathbf{C}$ ,  $\mathbf{D}$ ,  $\mathbf{Q}$  and  $\mathbf{\Sigma}_{\varepsilon}$  are, by definition, regular matrices. Moreover,  $\mathbf{C}\mathbf{Q}\mathbf{C}' + \mathbf{\Sigma}_{\varepsilon} - \mathbf{V}_N$  is a positive definite matrix, thus regular. The positive-definiteness can be seeing by noting that  $\mathbf{C}\mathbf{Q}\mathbf{C}' + \mathbf{\Sigma}_{\varepsilon}$  is the variance-covariance of a totally uninformed investor who does not even observe equilibrium prices. Matrix  $\mathbf{V}_N$ , on the other hand, is the variance-covariance matrix of the "average investor" who has some private information and observes prices.

#### Appendix F

We analyze the relations between model parameters  $\{r_f, a, \lambda_1, \ldots, \lambda_N, \mathbf{B}, \Sigma_{\varepsilon}, \Sigma_{\theta}, \Sigma_z\}$  and ex-ante equilibrium prices. We do this by taking expectations over the random variables in the model  $\{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\}$ . The analysis in this section is similar to the analysis of holdings shown in Section 2.3.

When all investors are informed about all asset-specific components and common components, prices reduce to a form of the Capital Asset Pricing Model (CAPM), expressed in term of prices, and adjusted for supply uncertainty. To see this, subtract R times Equation (4) from Equation (3) and take expectations to get:

$$E\left[\tilde{P}^{1}\right] - RE\left[\tilde{P}^{0}\right] = (\mathbf{C} - R\mathbf{A}_{1})E\left[\tilde{\eta}\right] + R\mathbf{A}_{2}E\left[\tilde{z}\right] - RA_{0}$$

Equations (11), (12), and (13) enable us to write:

$$R\mathbf{A}_1 = (\mathbf{CQC'}) + \mathbf{\Sigma}_{\varepsilon} - \mathbf{V}_N)(\mathbf{CDQC'})^{-1}\mathbf{CD} = \mathbf{C}$$
  
 $R\mathbf{A}_2 = a(\mathbf{CQC'}) + \mathbf{\Sigma}_{\varepsilon} - \mathbf{V}_N)(\mathbf{CDQC'})^{-1}\mathbf{\Sigma}_{\varepsilon} = a\mathbf{\Sigma}_{\varepsilon}$   
 $RA_0 = (\mathbf{C} - R\mathbf{A}_1)E[\tilde{\eta}] + (R\mathbf{A}_2 - a\mathbf{V}_N)E[\tilde{z}] = 0$ 

Combining these results gives the CAPM expressed in prices:

$$E\left[\hat{P}^{0}\right] = \frac{1}{R} \left( E\left[\tilde{P}^{1}\right] - a\Sigma_{\varepsilon}E\left[\tilde{z}\right] \right) \tag{37}$$

We can express the same result in terms of covariance. When all investors are informed, they know the realization of  $\tilde{\eta}$  is  $\eta$ . Therefore,  $\tilde{P}^1 = \mathbf{C}\eta + \tilde{\varepsilon}$  and  $Var\left[\tilde{P}^1\right] = \Sigma_{\varepsilon}$ :

$$a\Sigma_{\varepsilon}E\left[\bar{z}\right] = aVar\left[\tilde{P}^{1}\right]E\left[\bar{z}\right] = aCov\left[\tilde{P}^{1},\tilde{P}^{1}\right]E\left[\bar{z}\right] = aCov\left[\tilde{P}^{1},\left(\tilde{P}^{1}\right)'E\left[\bar{z}\right]\right]$$
$$= aCov\left[\tilde{P}^{1},\tilde{P}^{1}_{m}\right]$$

Here,  $\tilde{P}_m^1$  is the payoff of the market portfolio (the one that contains all the assets) divided by the number of investors (since  $\tilde{z}$  has been defined as the supply per investor). As the supply is unknown by the agents in the market, we consider the expectations of the supply, rather than the supply itself. Using Equation (37) and the above result gives:  $E[\tilde{P}^1] - RE[\tilde{P}^0] = a\Sigma_{\varepsilon}E[\tilde{z}] = aCov[\tilde{P}^1, \tilde{P}_m^1]$ . For asset j, we get:

$$E\left[\tilde{P}_{j}^{1}\right]-RE\left[\tilde{P}_{j}^{0}\right]=aCov\left[\tilde{P}_{j}^{1},\tilde{P}_{m}^{1}\right]$$

#### Model with Disperse Information:

Rearranging Equation (6) gives a general expression for prices at date 0:

$$E\left[\tilde{P}^{0}\right] = \frac{1}{R}\left(E\left[\tilde{P}^{1}\right] - a\mathbf{V}_{N}E\left[\tilde{z}\right]\right) \tag{38}$$

Equation (38) shows that asset prices at date 0 are less than the value of expected future payoffs.<sup>14</sup> The total price discount (risk premium) is given by the expression  $a\mathbf{V}_N E\left[\tilde{z}\right]$ . The price discount depends on risk aversion (a) and the market's "average" uncertainty about future payoffs ( $\mathbf{V}_N$ ).

**Information Price Discount:** We define the "information price discount" (or "*IPD*") as the difference between the price discounts shown in Equations (38) and (37). The *IPD* represents the amount an asset's price at date 0 is below its expected future value due to agents not having full information about future payoffs.

<sup>&</sup>lt;sup>14</sup>Assuming assets are expected to be in positive net supply:  $E[\tilde{z}] > 0$ 

$$IPD \equiv a\mathbf{V}_N E\left[\tilde{z}\right] - a\mathbf{\Sigma}_{\varepsilon} E\left[\tilde{z}\right]$$

$$= a(\mathbf{V}_N - \mathbf{\Sigma}_{\varepsilon}) E\left[\tilde{z}\right]$$
(39)

In a single-asset model with no factor structure, the information price discount is proportional to the difference between the market's average uncertainty about future payoffs  $(\mathbf{V_N})$  and residual uncertainty about the same payoffs  $(\Sigma_{\varepsilon})$ . This difference is a signal-to-noise measure. When the difference is small, investors have a lot of information about future payoffs, the IPD is low, and prices are high. Note that  $IPD \geq 0$  as the market is always bounded in its assessment of future payoffs by  $\Sigma_{\varepsilon}$ .

In a multi-asset model with uncorrelated residual uncertainties and no factor structure, the single-asset intuition discussed in the paragraph above continues to hold. The diagonal matrix  $(\mathbf{V}_N - \Sigma_{\epsilon})$  represents a series of signal-to-noise differences.

In a multi-asset model with correlated residual uncertainties and/or a factor structure, the information price discount can be driven by both the asset-specific components of payoffs and common factors. The matrix  $(\mathbf{V}_N - \mathbf{\Sigma}_{\varepsilon})$  can still be roughly interpreted as signal-to-noise differences. However, the matrix is no longer diagonal which means that covariance terms affect the IPD.

Numerical Analysis: The numerical analysis in Section 3 can easily be extended to include asset prices. As such, the analysis can help shed light on general properties of equilibrium prices in our model. For example, an asset is less risky from the perspective of a single agent if he has precise information about its future payoffs. The asset is more risky if he must glean information from equilibrium prices. When common components of payoffs are considered, a single asset may no longer be viewed as low-risk even if the agent has information about the asset-specific component of the stock's payoff. As the common components become more prominent in payoffs, asset-specific information becomes less valuable. Thus, the price of a given asset is sensitive to how many agents have information about the asset-specific component of payoffs, how many agents have information about common components, how sensitive the asset's payoffs are to the common components, and what other information the agents have. Graphs showing the price discounts for different levels of asset-specific information advantages and different levels of information advantages about the common components are available from the authors upon request.

Table 1 Sample Size

The table shows the number of stocks in our data sample. Panel A sorts stocks by country. Panel B sorts the final sample of 5,781 stocks by industry. The table is based on cross-border holdings on December 31, 2002. Holdings data are from Thompson Financial. Price data are from Datastream.

Panel A: Number of Stocks by Country

|     | Country       | Holdings<br>Data | Holdings and<br>Price Data | Holdings and<br>Firm Char. Data        | Holdings, Prices,<br>Firm Char. Data |
|-----|---------------|------------------|----------------------------|--|--------------------------------------|
| -   | Australia     | 695              | 587                        | 307                                    | 293                                  |
| 7   | Austria       | 91               | 78                         | 99                                     | 09                                   |
| 3   | Belgium       | 191              | 166                        | 66                                     | 95                                   |
| 4   | Denmark       | 162              | 124                        | ************************************** | 26                                   |
| 5   | Ireland       | 72               | 48                         | 37                                     | 34                                   |
| 9   | Finland       | 143              | 122                        | 109                                    | 76                                   |
| 7   | France        | 989              | 567                        | 497                                    | 448                                  |
| ∞   | Germany       | 744              | 642                        | 507                                    | 467                                  |
| 6   | Стеесе        | 299              | 242                        | 117                                    | 110                                  |
| 10  | Hong Kong     | 196              | 160                        | 157                                    | 150                                  |
| 111 | Italy         | 302              | 243                        | 219                                    | 189                                  |
| 12  | Japan         | 2,676            | 2,370                      | 2,216                                  | 2,108                                |
| 13  | Netherlands   | 198              | 137                        | 117                                    | 109                                  |
| 14  | New Zealand   | 79               | 70                         | 41                                     | 41                                   |
| 15  | Norway        | 178              | 106                        | 107                                    | 88                                   |
| 16  | Portugal      | 26               | 46                         | 34                                     | 30                                   |
| 17  | Singapore     | 274              | 233                        | 214                                    | 200                                  |
| 18  | Spain         | 289              | 121                        | 118                                    | 105                                  |
| 19  | Sweden        | 326              | 226                        | 209                                    | 174                                  |
| 20  | Switzerland   | 264              | 198                        | 184                                    | 165                                  |
| 21  | United Kindom | 1,973            | 1,067                      | 794                                    | 721                                  |
|     | TOTAL STOCKS  | 10,292           | 7,553                      | 6,259                                  | 5,781                                |

Table 1 Sample Size

Panel B: Industry Break-Down

|          |                     | Num. of |          |                     | Num. of |
|----------|---------------------|---------|----------|---------------------|---------|
| Industry | £.                  | Stocks  | Industry | î.                  | Stocks  |
| 11       | Aerospace & Defense | 23      | 20       | Industrial Metals   | 100     |
| 7        | Auto & Parts        | 160     | 21       | Industrial Trans.   | 149     |
| ĸ        | Banks               | 225     | 22       | Leisure Goods       | 70      |
| 4        | Beverages           | 76      | 23       | Life Insurance      | 23      |
| 5        | Chemicals           | 234     | 24       | Media               | 219     |
| 9        | Construction        | 372     | 25       | Mining              | 64      |
| 7        | Electricity         | 54      | 26       | Mobile Telecom.     | 33      |
| ∞        | Electronic Equip.   | 315     | 27       | Nonlife Insur.      | 99      |
| 6        | Equity Investments  | 55      | 28       | Oil & Gas Producers | 51      |
| 10       | Fixed Line Telecom. | 26      | 29       | Oil Equip. & Srvcs  | 18      |
| Ξ        | Food & Drug Retail  | 79      | 30       | Personal Goods      | 190     |
| 12       | Food Producers      | 224     | 31       | Pharm. & Biotech.   | 154     |
| 13       | Forestry & Paper    | 47      | 32       | Real Estate         | 252     |
| 7        | General Financial   | 197     | 33       | Software Services   | 474     |
| 15       | General Indus.      | 110     | 34       | Support Services    | 253     |
| 16       | General Retailers   | 293     | 35       | Tech. Equipment     | 246     |
| 17       | Health Equipment    | 110     | 36       | Tobacco             | ∞       |
| 18       | Household Goods     | 159     | 37       | Travel & Leisure    | 230     |
| 19       | Industrial Engin.   | 384     | 38       | Utilities           | 38      |
|          |                     |         |          |                     |         |

Total Number of Stocks: 5,781

# Table 2 Overview Statistics

correlation coefficients. "Foreign Holdings ( $\Omega_j$ )" is the number of shares held by foreign funds divided by shares outstanding. "Foreign – Domestic Holdings ( $\Omega_j$ )" is the number of shares held by foreign funds minus shares held by domestic funds all divided by shares outstanding. We have two proxy variables for information—one is asset specific and one relates to common components. The table is based on cross-border holdings on 31-Dec-2002. Holdings data are The table shows the overview statistics for the main variables in our empirical analysis. Panel A shows each variable's cross-sectional mean, standard deviation, 25th, 50th, and 75th percentiles for the 5,781 stocks. Panel B shows correlation coefficients of the variables. P-Values are shown below the from Thompson Financial. Price data are from Datastream.

Panel A: Cross-Sectional Statistics

|                                   | Units  | Mean   | Stdev | 25th Ptile | 50th Ptile | 75th Ptile |
|-----------------------------------|--------|--------|-------|------------|------------|------------|
| Foreign Hold ( $\Omega_{\rm j}$ ) | %      | 2.76   | 5.20  | 0.14       | 0.59       | 3.22       |
| For-Dom Hold ( $\Omega^*_{j}$ )   | %      | (2.00) | 8.43  | (3.40)     | (0.36)     | 0.39       |
| Asset Specific Info Proxy j       | •      | 0.85   | 0.13  | 0.79       | 68.0       | 0.95       |
| Common Info Proxy j.k             | *      | 1.28   | 2.18  | 0.16       | 0.57       | 1.46       |
| Market Capitalization j           | \$ bn  | 2.56   | 79.29 | 0.04       | 0.11       | 0.47       |
| In( Market Cap <sub>j</sub> )     | In(\$) | 18.75  | 1.96  | 17.41      | 18.55      | 19.97      |
| Leverage j                        | 1      | 0.57   | 0.36  | 0.39       | 0.57       | 0.73       |
| Num. of Analysts i                | ŀ      | 4.16   | 6.44  | 0.00       | 1.00       | 5.00       |
|                                   |        |        |       |            |            |            |

Table 2 - Continued

Panel B: Correlation of Variables

|   | $\Omega_{_{ m j}}$ | .G    | Asset Spec. j Common jik MktCap j | Соттоп ј.к | MktCap <sub>j</sub> | $ln(MC_{\rm J})$ | Lev <sub>j</sub> | #Analyst j |
|---|--------------------|-------|-----------------------------------|------------|---------------------|------------------|------------------|------------|
| Foreign Hold ( $\Omega_{ m j}$ )        | 1.00               |       |                                   |            |                     |                  |                  |            |
| For-Dom Hold ( $\Omega^{\star}_{\ j}$ ) | 0.52               | 1.00  |                                   |            |                     |                  |                  |            |
| Asset Specific Info Proxy j             | -0.15              | -0.14 | 1.00                              |            |                     |                  |                  |            |
| Common Info Proxy j.k                   | 0.14               | 0.06  | -0.31                             | 1.00       |                     |                  |                  |            |
| Market Capitalization j                 | 0.03               | 0.03  | -0.05                             | 0.03       | 1.00                |                  |                  |            |
| ln( Market Cap <sub>j</sub> )           | 0.33               | 0.15  | -0.28<br>(0.00)                   | 0.15       | 0.12 (0.00)         | 1.00             |                  |            |
| Leverage j                              | -0.01              | 0.03  | 0.03                              | 0.02       | 0.00                | 0.04             | 1.00             |            |

Table 3
Cross-Border Holdings and Double Sort Results

uses our proxy for the information advantage of foreign investors with respect to common components of asset payoffs. "Low" indicates a proxy variable is in the lower 25% of its distribution. The table is based on cross-border holdings on The table shows average cross-border holdings as a fraction of a stock's shares outstanding for four different groups of stocks. We sort stocks into quartiles along two dimensions. The first sort uses our proxy for the information advantage about the asset-specific component of a stock's returns. The second sort December 31, 2002. Holdings data are from Thompson Financial.

Asset Specific Info Proxy

| ·          |        | Low    | High   | Diff   | T-stat |
|------------|--------|--------|--------|--------|--------|
| Common     | High   | 0.0490 | 0.0223 | 0.0267 | 7.39   |
| Info Proxy | Low    | 0.0293 | 0.0139 | 0.0154 | 2.99   |
|            | Diff   | 0.0197 | 0.0084 |        |        |
|            | T-stat | 3.51   | 2.97   |        |        |

Table 4
Regression Results and Asset-Specific Information Proxy

funds, all divided by shares outstanding. "Asset Specific," is our proxy for the information advantage about the asset specific component of a stock's returns. "Asset Specific, (Sales)" is based on sales growth data instead of returns (methodology described in the text.) The table is based on cross-border holdings on December 31, 2002. Holdings data are from Thompson Financial. Price data are from Datastream. The table shows cross-sectional regression results. The dependent variable in Regressions 1 and 2 is  $\Omega_i$  which is the ratio of shares held by foreign funds divided by shares outstanding. The dependent variable in Regressions 3-7 is  $\Omega_i^*$  which is the difference between shares held by foreign funds and domestic

| •                               | Reg. 1            | Reg. 2             | Reg. 3            | Reg. 4            | Reg. 5                           | Reg. 6            | Reg. 7               |
|---------------------------------|-------------------|--------------------|-------------------|-------------------|----------------------------------|-------------------|----------------------|
| Depend. Var.                    | Ω                 | i                  | , c               | O,                | Ω,<br>i                          | Ω,<br>,           | $\Omega_{j}^{\star}$ |
| Asset Specific <sub>j</sub>     | -0.0569<br>927]   | -0.0224<br>[3.12]  | -0.0701           | -0.0570           |                                  |                   |                      |
| Asset Spec <sub>j</sub> (Sales) |                   |                    |                   |                   | -0.0602<br>[13.69]               | -0.0076<br>[1.98] | -0.0186<br>[3.15]    |
| In(MktCap <sub>j</sub> )        |                   | 0.0083             | 0.0050            | -0.0009           | -0.0011<br>[0.95]                | 0.0017<br>[1.56]  | 0.0016<br>[1.03]     |
| Num. Analysts <sub>j</sub>      |                   | -0.0031<br>[0.98]  | 0.0074            | 0.0068            | 0.0110                           | 0.0051<br>[1.16]  | 0.0045               |
| Leverage <sub>j</sub>           |                   |                    |                   | 0.0029<br>[10.35] | 0.0032<br>[9.27]                 | 0.0016            | 0.0025               |
| Constant                        | 0.0693<br>[12.34] | -0.1107<br>[11.90] | -0.0660<br>[4.95] | 0.0234<br>[1.36]  | 0.0159<br><i>[</i> 0.75 <i>]</i> | Country<br>F.E.   | -0.0410              |
| sqO#                            | 5,780             | 5,780              | 5,780             | 5,780             | 3,095                            | 3,095             | 1,354                |
| H.                              | 0.02              | 0.11               | 0.03              | 90.0              | 0.13                             | 0.46              | 0.08                 |

Table 5
Regression Results, Asset Specific Proxy, and Common Component Proxy

The table shows pairs of cross-sectional regression results. The first regression in the pairs considers stocks with low information advantages vis-à-vis the common factor (bottom 25%). The second regression in the pair considers stocks with high information advantages (upper 25%). The dependent variable in Regressions 1a and 1b is Q, which is the ratio of shares held by foreign funds divided by shares outstanding. The dependent variable in Regressions 2a and 2b is  $\vec{\Omega}_i^*$  which is the difference between shares held by foreign funds and domestic funds, all divided by shares outstanding. The table is based on cross-border holdings on December 31, 2002. Holdings data are from Thompson Financial. Price data are from Datastream.

Reg. 2a & 2b

Reg. 1a & 1b

| Low     High     Low $\Omega_1$ $\Omega_1^*$ $\Omega_1^*$ -0.0331     -0.0741     -0.0523 $[3.70]$ $[3.40]$ $[3.78]$ $[3.70]$ $[3.40]$ $[3.78]$ 0.044 $[4.26]$ 0.0192 $[7.34]$ $[4.76]$ $[3.05]$ 0.01     0.03     0.03       0.01     0.03     0.03   |  | Common                | Common Info Proxy | Common  | Common Info Proxy |
|--|--|-----------------------|-------------------|---------|-------------------|
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |  | Low                   | High              | Low     | High              |
| -0.0331 -0.0741 -0.0523 [3.70] [3.40] [3.40] [3.78]  0.0044 [4.26]  0.0478 0.0821 -0.0758 [7.34] [4.76] [3.24]  1,445 1,445  0.01 0.03  0.03  4.68   | Depend. Var.                           | $\Omega_{\mathrm{j}}$ | Ω                 | Δ,*     | * <sub>i</sub> Ω  |
| (3.70)     (3.40)     (3.78)       0.0044     (4.26)       (4.26)     (0.0192       (7.34)     (4.76)     (3.05)       (7.34)     (4.76)     (3.24)       (7.34)     (4.76)     (3.24)       (7.34)     (4.76)     (3.24)       (7.34)     (4.76)     (3.24)       (7.34)     (4.76)     (3.24)       (7.34)     (4.76)     (3.24)       (6.01)     (0.03)     (0.03)       (6.02)     (6.03)     (6.03) | Asset Specific j                       | -0.0331               | -0.0741           | -0.0523 | -0.1001           |
| 0.0044<br>[4.26]<br>0.0192<br>[3.05]<br>0.0478 0.0821 -0.0758<br>[7.34] [4.76] [3.24]<br>1,445<br>1,445<br>0.01 0.03 0.03  | (T-stat)                               | [3.70]                | [3.40]            | [3.78]  | [4.53]            |
| 0.0478 0.0821 0.0192<br>[3.05]  0.0478 0.0821 -0.0758  [7.34] [4.76] [3.24]  1,445 1,445  0.01 0.03 0.03   | In(MktCap <sub>i</sub> )               |                       |                   | 0.0044  | 0.0044            |
| 0.0192<br>(3.05)<br>0.0478 0.0821 -0.0758<br>(7.34) (4.76) (3.24)<br>1,445 1,445 1,445<br>0.01 0.03 0.03   | (T-stat)                               |                       |                   | [4.26]  | [4.74]            |
| 6.0478 0.0821 -0.0758<br>[7.34] [4.76] [3.24] [3.24] [3.24] [4.45] [1,445 1,445 0.01 0.03 0.03   | Leverage                               |                       |                   | 0.0192  | 0.0045            |
| 0.0478 0.0821 -0.0758<br>[7.34] [4.76] [3.24]<br>1,445 1,445 1,445<br>0.01 0.03 0.03<br>0.03<br>0.03<br>0.03   | (T-stat)                               |                       |                   | [3.05]  | [0.64]            |
| [7.34] [4.76] [3.24] 1,445 1,445 1,445 0.01 0.03 0.03 0.03 0.03 0.03 0.03  | Constant                               | 0.0478                | 0.0821            | -0.0758 | -0.0294           |
| 1,445 1,445 1,445 0.03 0.03 0.03 8.87 4.68 0.03 0.03 0.03  | (T-stat)                               | [7.34]                | [4.76]            | [3.24]  | [1.01]            |
| 8.87 4.68  | ************************************** | 1,445                 | 1,445             | 1,445   | 1,445             |
| 8.87   | - Just<br>- part<br>- Tus              | 0.01                  | 0.03              | 0.03    | 0.05              |
| 8.87   | F-Stat for Diff. in                    |                       |                   |         |                   |
|  | Asset-Specfic Coefs (P-value)          | .8<br>.00             | 87<br>(729)       | 4.00    | 68<br>306)        |

Figure 1 Shares of the French Asset Held by Cross-Border Investors

The figure shows cross-border holdings of the French asset from our numerical analysis. The Y-Axis shows the number of shares of the French asset (held by cross-border investors) divided by the number of shares of the French asset outstanding. The X-axis shows the degree of asset-specific information advantages. The four curves (labeled 0, 1, 2, 4) represent four levels of information advantage about the common components of payoffs. The top graph line (thin, purple, with "O" markings) represents the highest levels of asymmetry about the common components. Details of the numerical analysis are given in the text.

